Projekt

Analiza regulatorów strojonych metodą automatycznego strojenia przekaźnikowego dla zestawu problemów 1.

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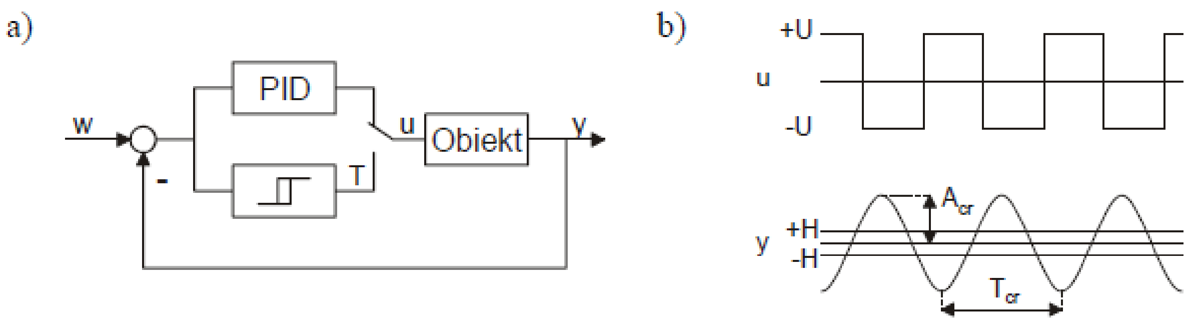
1. Wprowadzenie teoretyczne

Automatyczne strojenie (self-tuning) jest zazwyczaj przeprowadzane metodami odpowiedzi skokowej lub cyklu przekaźnikowego. Pierwszą z tych metod stosuje się wtedy, gdy w procesie występuje powtarzalny stan ustalony. Samostrojenie przekaźnikowe zaproponowane przez Astroma i Hagglunda jest metodą automatyzującą znany eksperyment Zieglera-Nicholsa, który pozwala określić nastawy regulatora PID.

Metoda Astroma-Hagglunda nie daje bezpośredniej informacji o tym, jak będzie wyglądać odpowiedź pętli, oprócz tego, że gwarantuje jej stabilność.

Klasyczny eksperyment Zieglera-Nicholsa nie jest wygodny ponieważ wzmocnienie musi być ustawiane ręcznie. Astroma i Hagglund zaproponowali automatyzację tego eksperymentu poprzez sterowanie przekaźnikowe w układzie z rys. 1a. Po przełączeniu na tryb T (tune) obiekt jest sterowany przez przekaźnik z histerezą.

Amplituda zmian wynosi - U, histereza - H (rys. 1b). W układzie powstają drgania ustalone, z których należy odczytać amplitudę A_{cr} oraz okres T_{cr} a następnie korzystając z funkcji opisującej przekaźnik lub reguł Zieglera-Nicholsa obliczyć nastawy.



Rys. 1. Strojenie przekaźnikowe: (a) układ; (b) typowe przebiegi.

Regulator	k_p	T_i	T_d
P	$0{,}5\ k_{cr}$		
PI	$0{,}45\ k_{cr}$	$0{,}85\ T_{cr}$	
PID	$0{,}6\ k_{cr}$	$0{,}5\ T_{cr}$	$0{,}125\ T_{cr}$

Tab. 1. Reguły Zieglera-Nicholsa nastawiania regulatorów PID metodą cyklu granicznego.

Strojenie przekaźnikowe jest szeroko stosowane ze względu na prostotę realizacji. Przebiega ono w układzie zamkniętym eliminując zakłócenia typu dryfu. Wydobywa informację tylko o dynamice obiektu, więc wyniki tej metody są na ogół gorsze niż dla odpowiedzi skokowej.

Niech R i I określają wartości bezwzględne części rzeczywistej i części urojonej transmitancji obiektu dla częstotliwości $\omega_{cr} = \frac{2\pi}{T_{cr}}$, tzn:

$$G_0(j\omega_{cr}) = -R - jI \tag{1}$$

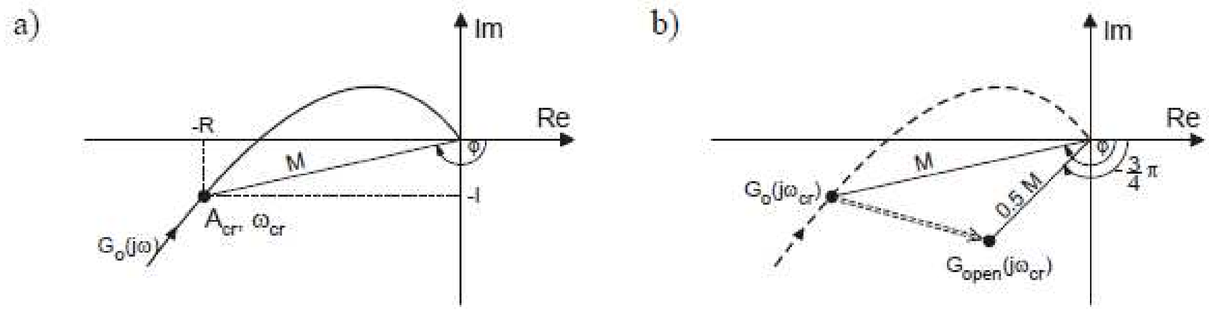
Korzystając z warunku Niquista określającego cykl graniczny przy sterowaniu przekaźnikowym:

$$R = \frac{\pi}{4U}\sqrt{A_{cr}^2 - H^2}, \quad I = \frac{\pi H}{4U} \tag{2}$$

otrzymuje się jeden punkt charakterystyki amplitudowo fazowej obiektu. Przyjmując że:

$$M = \sqrt{R^2 + I^2}, \quad \phi = \pi - arctg\left(\frac{I}{R}\right) \tag{3}$$

można ten punkt zaznaczyć na wykresach charakterystyki amplitudowo-fazowej.



Rys. 2. Charakterystyka amplitudowo-fazowa: (a) obiekt; (b) układ otwarty dla warunku Astroma-Hagglunda.

2. Projektowanie regulatora

Hagglund i Astrom w swojej pracy do wyznaczenia nastaw regulatora zaproponowali warunek:

$$G_0(j\omega_{cr})G_{PID}(j\omega_{cr}) = 0.5e^{-j\frac{3}{4}\pi} \tag{4}$$

Warunek ten stanowi kombinację dwukrotnego zapasu modułu i zapasu fazy równego 45°. Czułość tak dobranego regulatora będzie mniejsza niż w przypadku nastaw dobranych według reguł Zieglera-Nicholsa.

2.1 Regulator PI

Analiza warunku (4) pokazuje, że wiążą się z nim istotne ograniczenia. Dodatnie wzmocnienie k_p oraz czas całkowania T_i regulatora PI o transmitancji $k_p\left(1 + \frac{1}{T_i s}\right)$ otrzymuje się tylko dla amplitud A_{cr} spełniających nierówność:

$$A_{cr} < 1.2H \tag{5}$$

podchodząc do problemu praktycznie, nierówność ta jest bardzo trudna do spełnienia. Wymaga ona precyzyjnego doboru amplitudy sterowania U, czyli kilku prób dopasowywujących.

Wynikiem strojenia przekaźnikowego na podstawie warunku (4), nie może być regulator PI.

2.2 Regulator PID

Warunek (4) pozwala określić tylko dwa związki między trzema nastawami k_p, T_i, T_d regulatora PID. W celu jednoznacznego wyznaczenia nastaw regulatora, przyjęto, że $T_d = \frac{T_i}{4}$.

W wyniku podstawień oraz obliczeń uzyskujemy następujące wzory na nastawy regulatora PID.

$$T_i = \frac{2}{\omega_{cr}} tg\left(\frac{\phi}{2} - \frac{\pi}{8}\right), \quad k_p = \frac{2}{M}\frac{\omega_{cr}T_i}{(\omega_{cr}T_i)^2 + 4}, \quad T_d = \frac{T_i}{4} \tag{6}$$

3. Rozwiązanie

Celem projektu jest nastrojenie regulatorów dla następujących obiektów metodą automatycznego strojenie przekaźnikowego:

$$P_1(s) = \frac{e^{-s}}{1+sT} \tag{3.1}$$

gdzie:

$T = 0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 1, 1.2, 1.5, 2, 4, 6, 8, 10, 20, 50, 100, 200, 500, 1000$

$$P_2(s) = \frac{e^{-s}}{(1+sT)^2} \tag{3.2}$$

gdzie:

$T = 0.01, 0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 1, 1.2, 1.5, 2, 4, 6, 8, 10, 20, 50, 100, 200, 500$

$$P_3(s) = \frac{e^{-s}}{(1+s)(1+sT)^2} \tag{3.3}$$

gdzie:

$T = 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 2, 5, 10$

$$P_4(s) = \frac{1}{(1+sT)^n} \tag{3.4}$$

gdzie:

$n = 3, 4, 5, 6, 7, 8$

$$P_5(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)} \tag{3.5}$$

gdzie:

$\alpha = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$

$$P_6(s) = \frac{e^{-sL}}{s(1+sT)} \tag{3.6}$$

gdzie:

$T = 0.01, 0.02, 0.05, 0.1, 0.3, 0.5, 0.7, 1 \quad T + L = 1$

$$P_7(s) = \frac{Te^{-sL}}{(1+sT)(1+sT1)} \tag{3.7}$$

gdzie:

$T = 1, 2, 5, 10$

$L = 0.01, 0.02, 0.05, 0.1, 0.3, 0.5, 0.7, 1 \quad T1 + L = 1$

$$P_8(s) = \frac{1-\alpha s}{(1+s)^3} \tag{3.8}$$

gdzie:

$\alpha = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.1$

$$P_9(s) = \frac{1}{(1+s)\left((sT)^2 + 1.4sT + 1\right)} \tag{3.9}$$

gdzie:

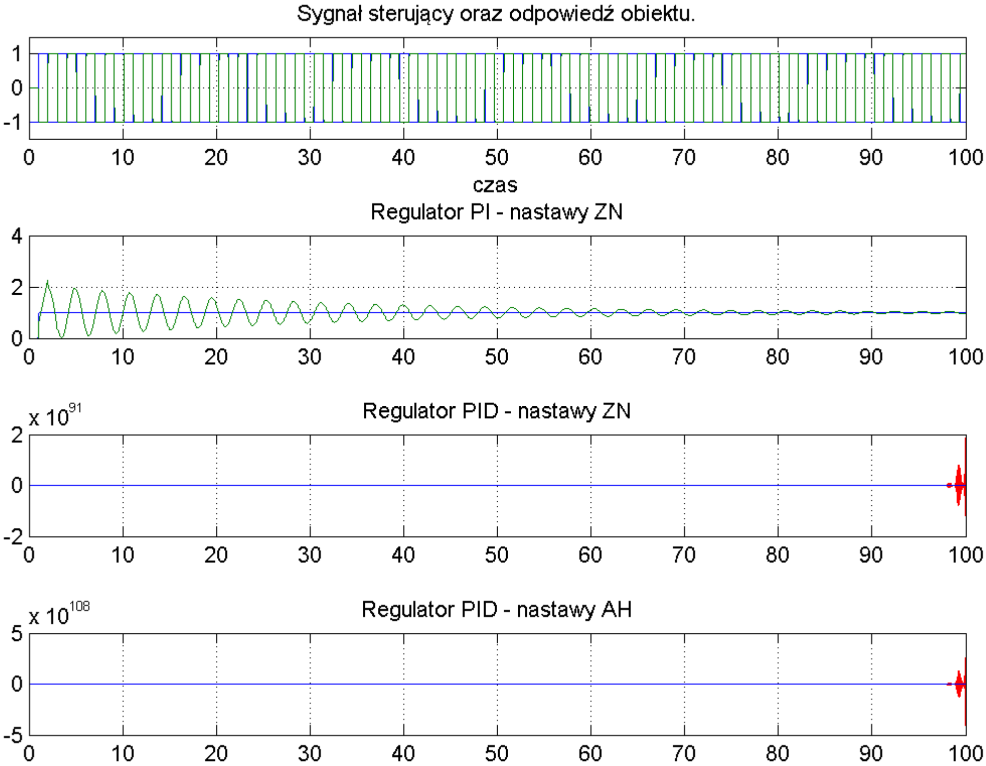
$T = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1$

W dalszej kolejności przystąpiono do utworzenia powyższych modeli, skryptu będącego implementacją metody automatycznego strojenia przekaźnikowego oraz generowania wykresów.

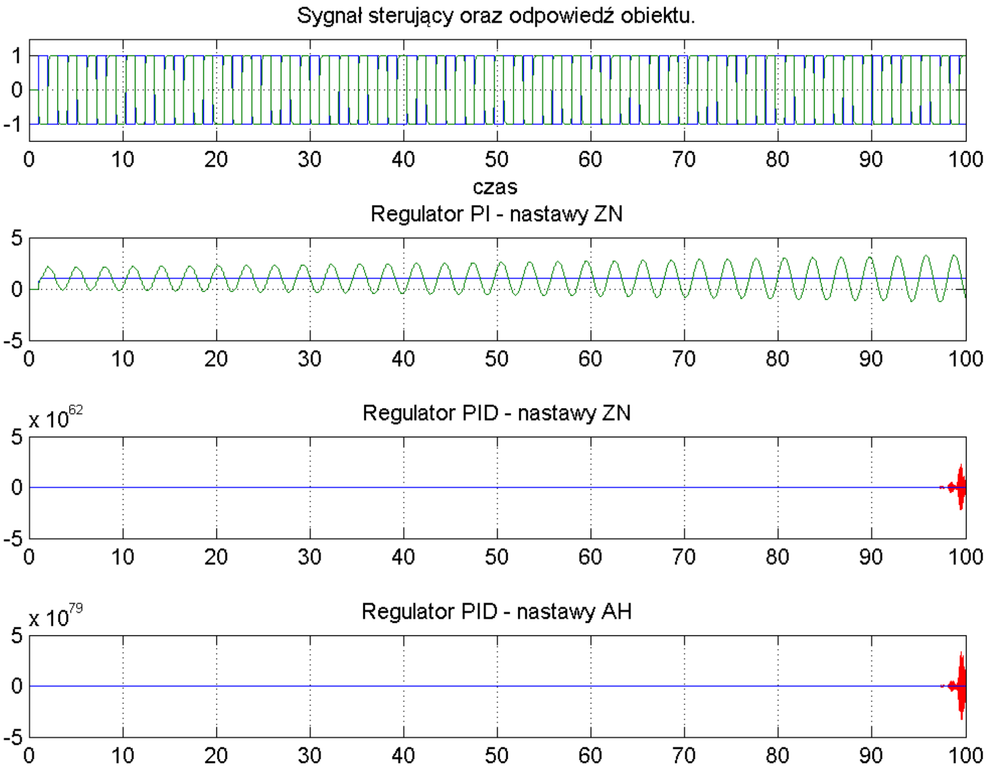
Na poniższych rysunkach przedstawiono reakcję obiektu z regulatorem PI oraz PID objętego ujemnym sprzężeniem zwrotnym. Na wejście podawano skok jednostkowy. Nastawy były dobierane według reguły Zieglera-Nicholsa (ZL) - Tab. 1, oraz według wzorów (6) zaproponowanych przez Hagglunda i Astroma (AH).

Wykresy odpowiadają wszystkim możliwym obiektom zawartym w zadanym zestawie problemów.

Obiekt 3.1.



Obiekt G1, T = 0.02



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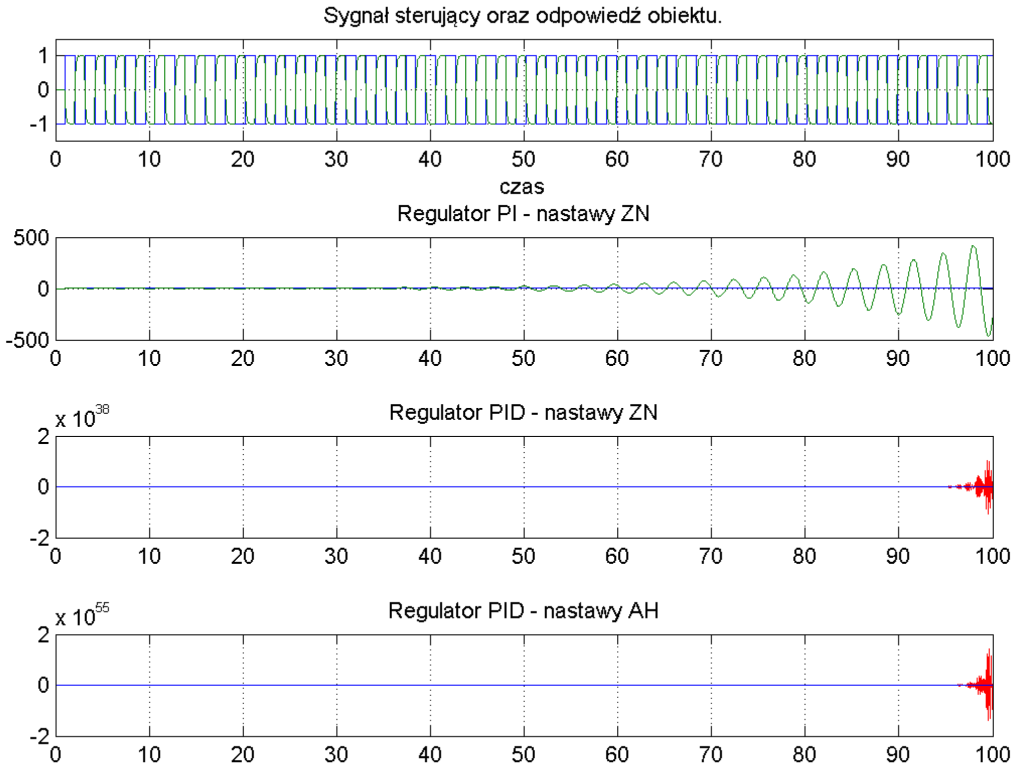
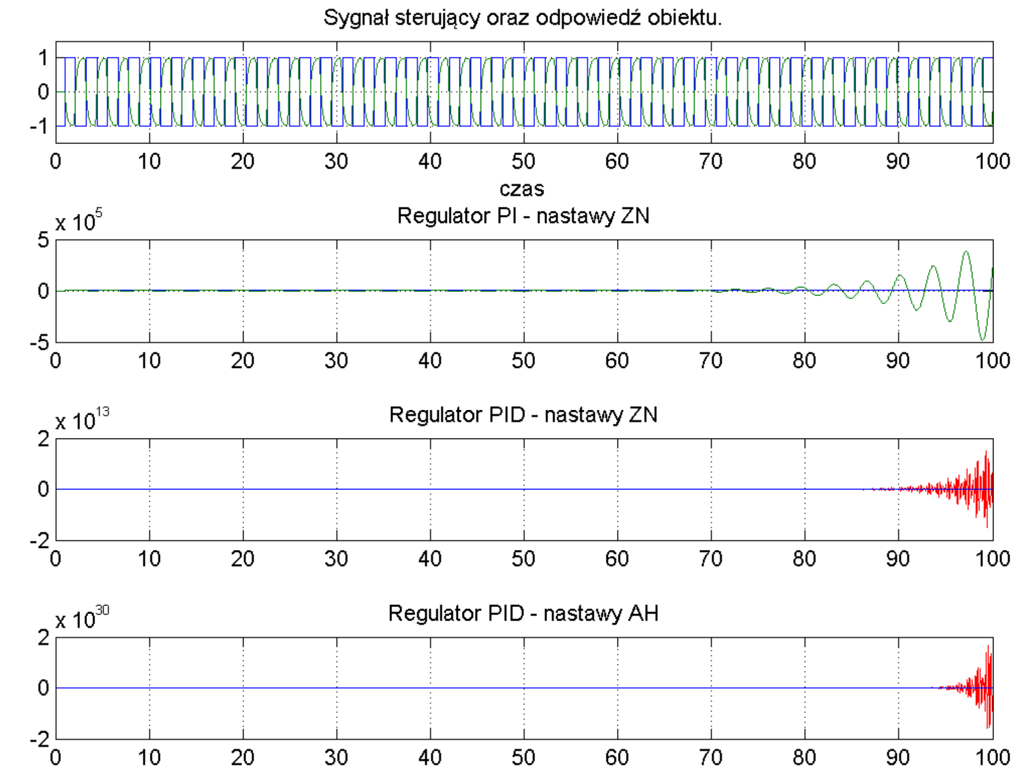
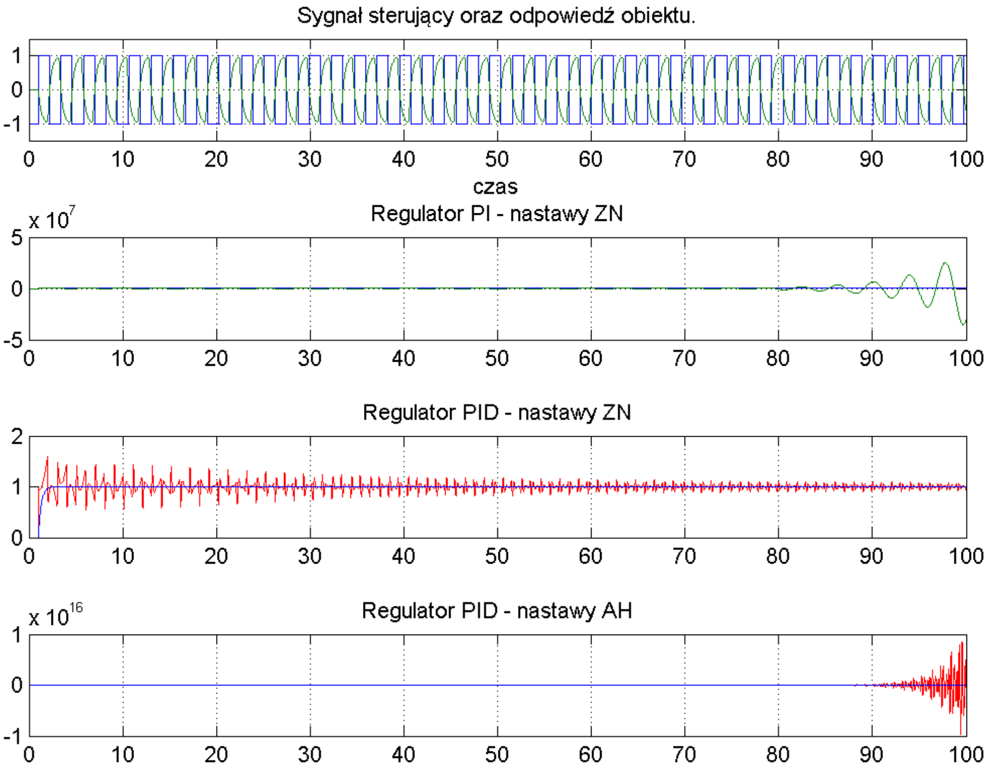


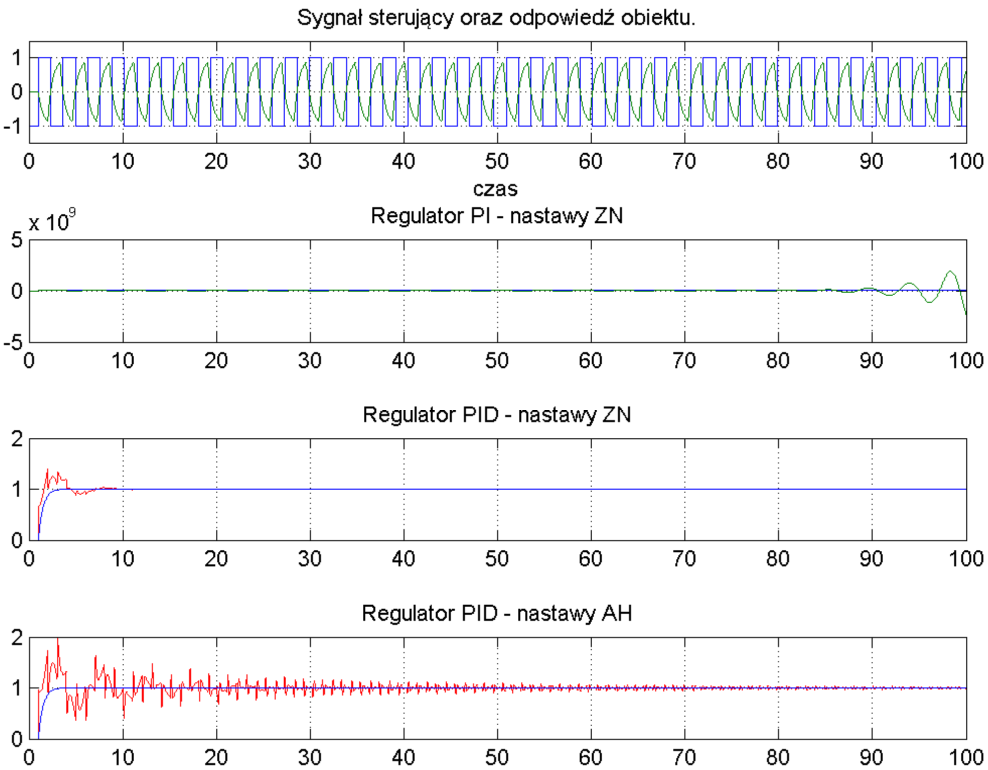
Obiekt G1, T = 0.1



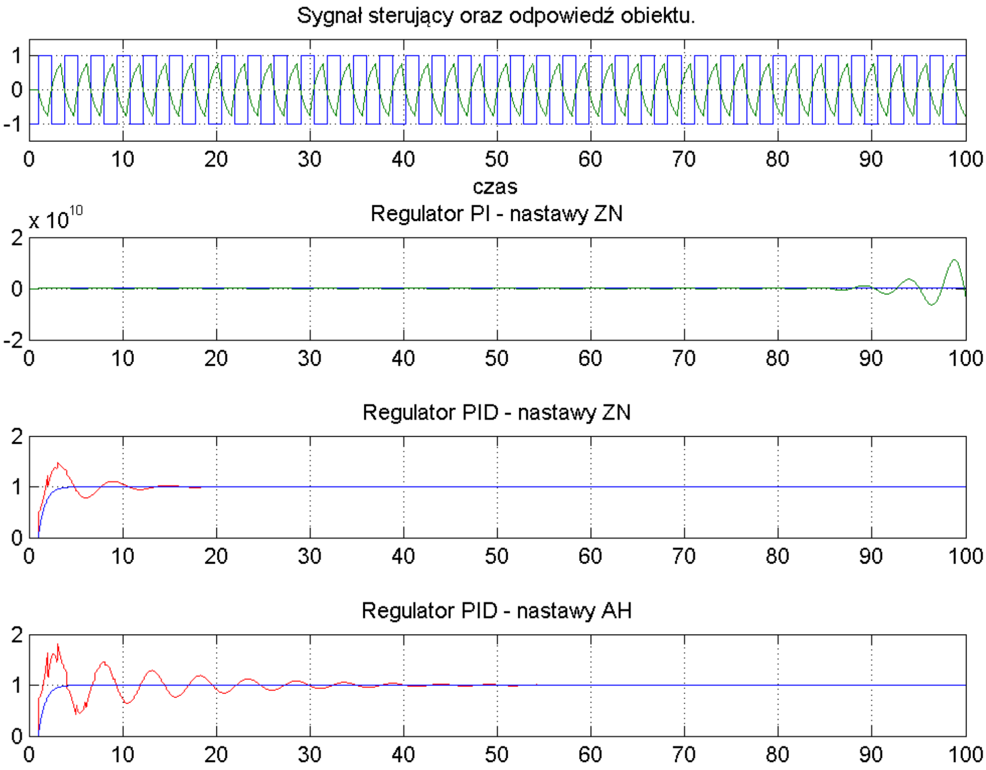
Obiekt G1, T = 0.2



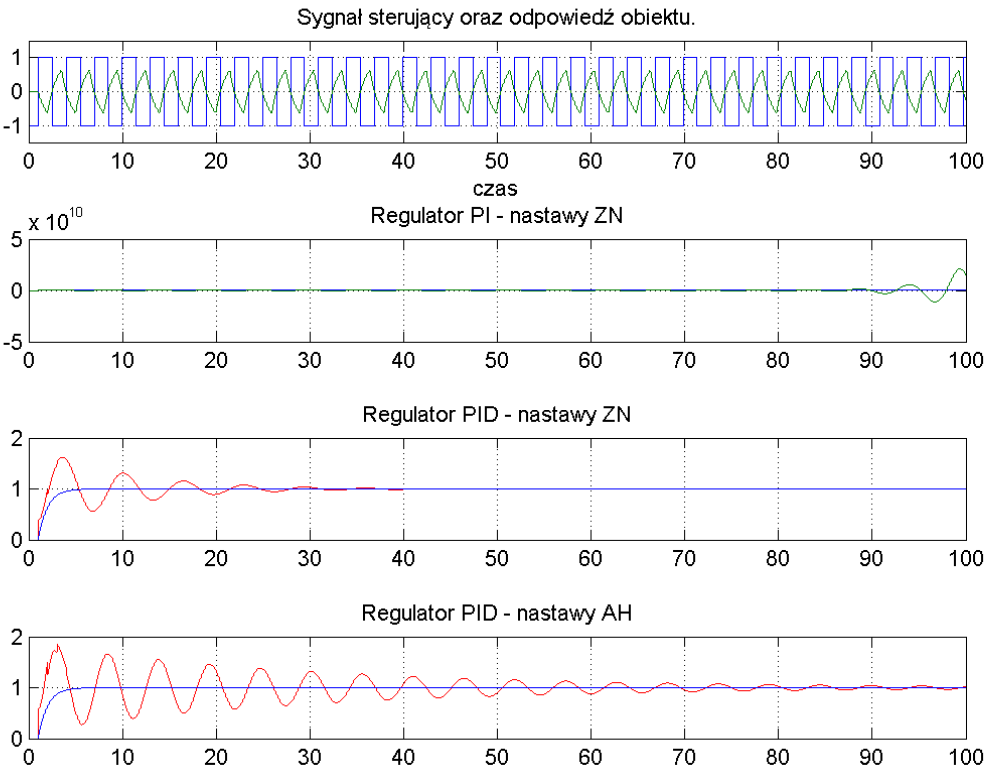
Obiekt G1, T = 0.3



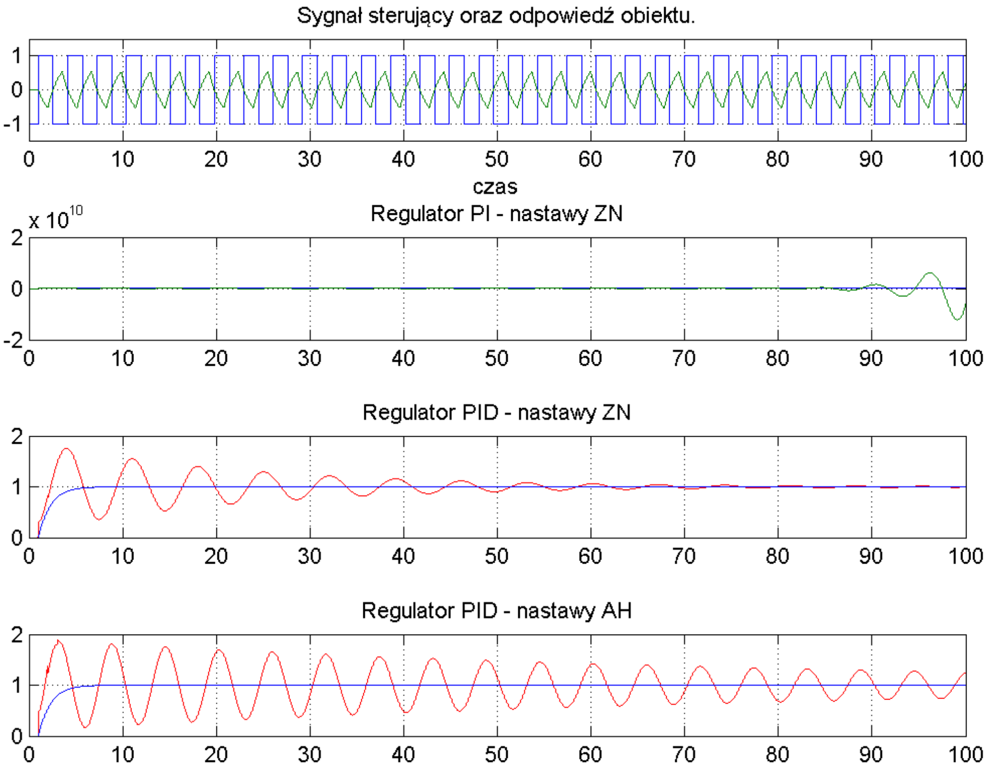
Obiekt G1, T = 0.5



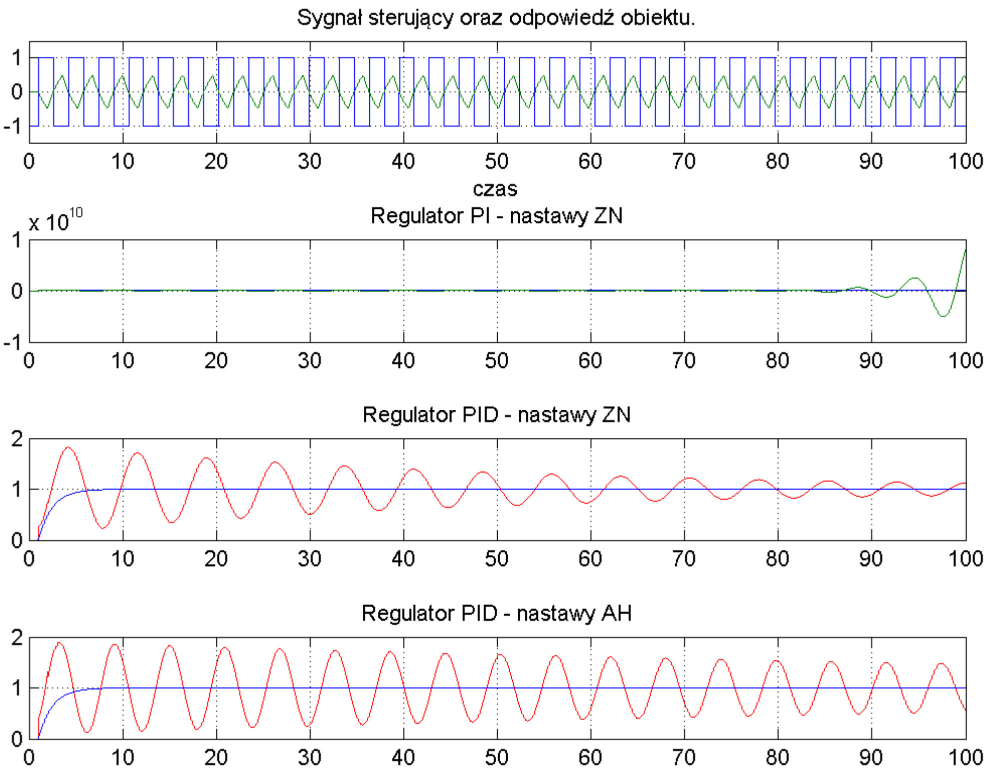
Obiekt G1, T = 0.7



Obiekt G1, T = 1



Obiekt G1, T = 1.2



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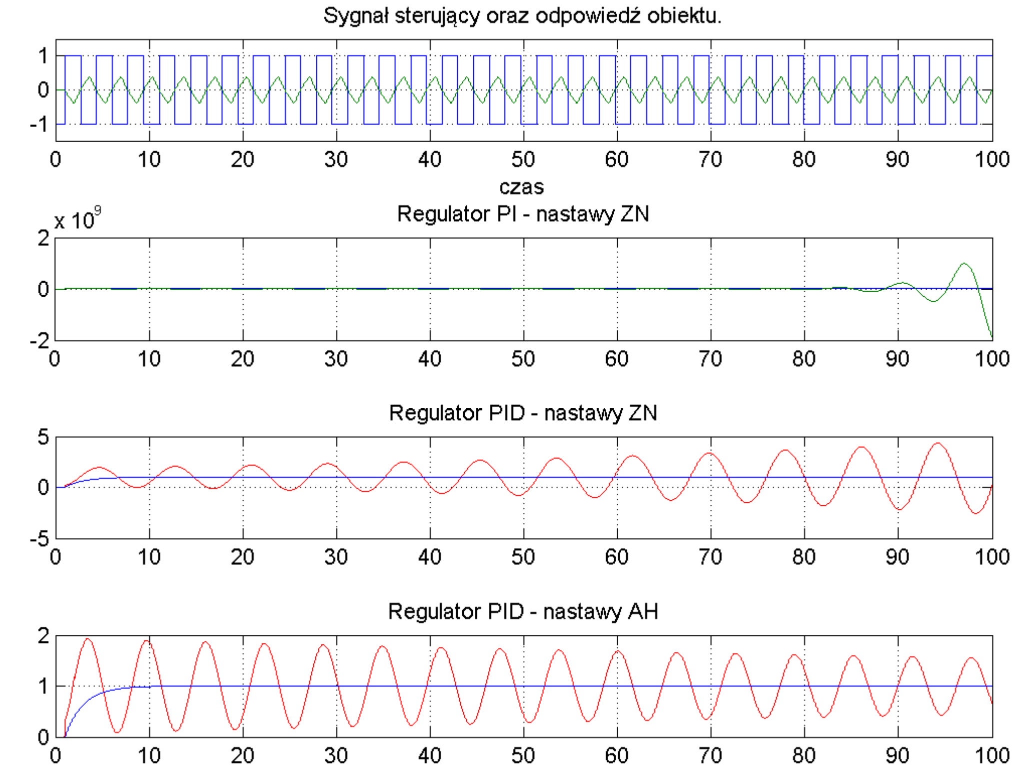
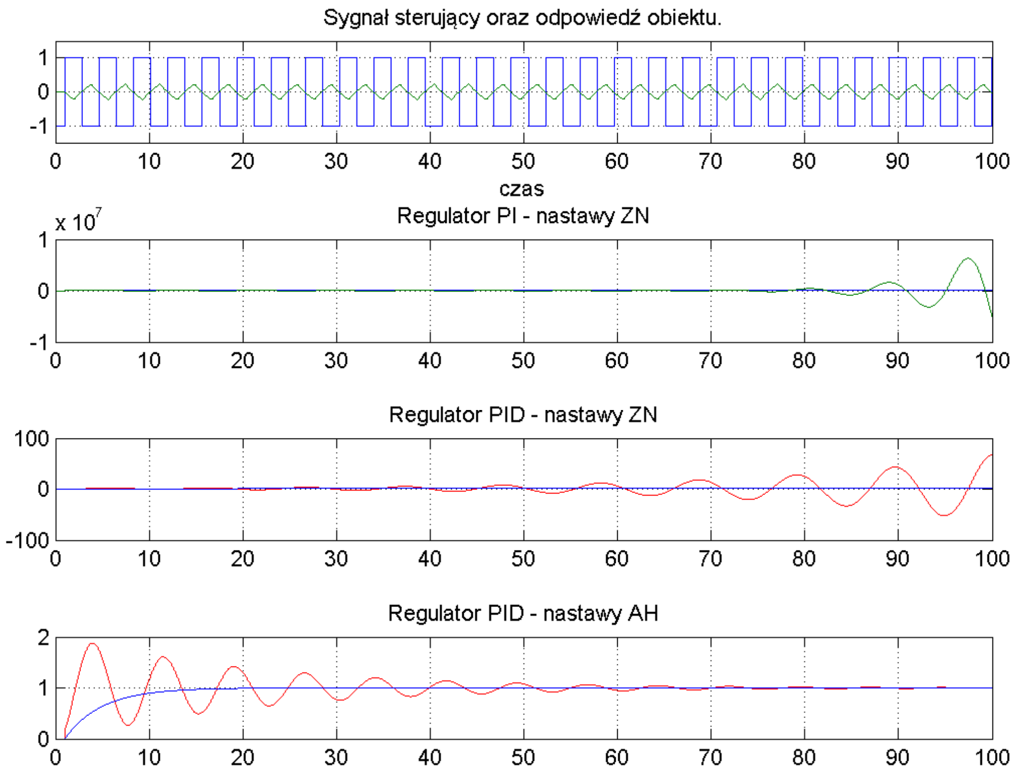
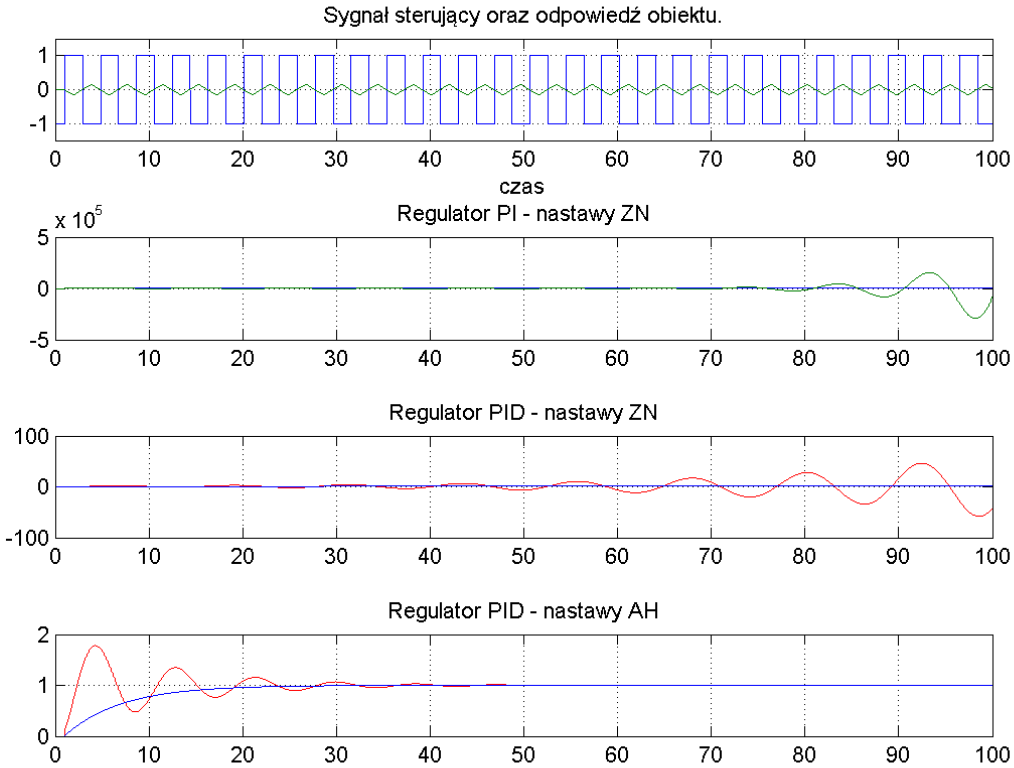


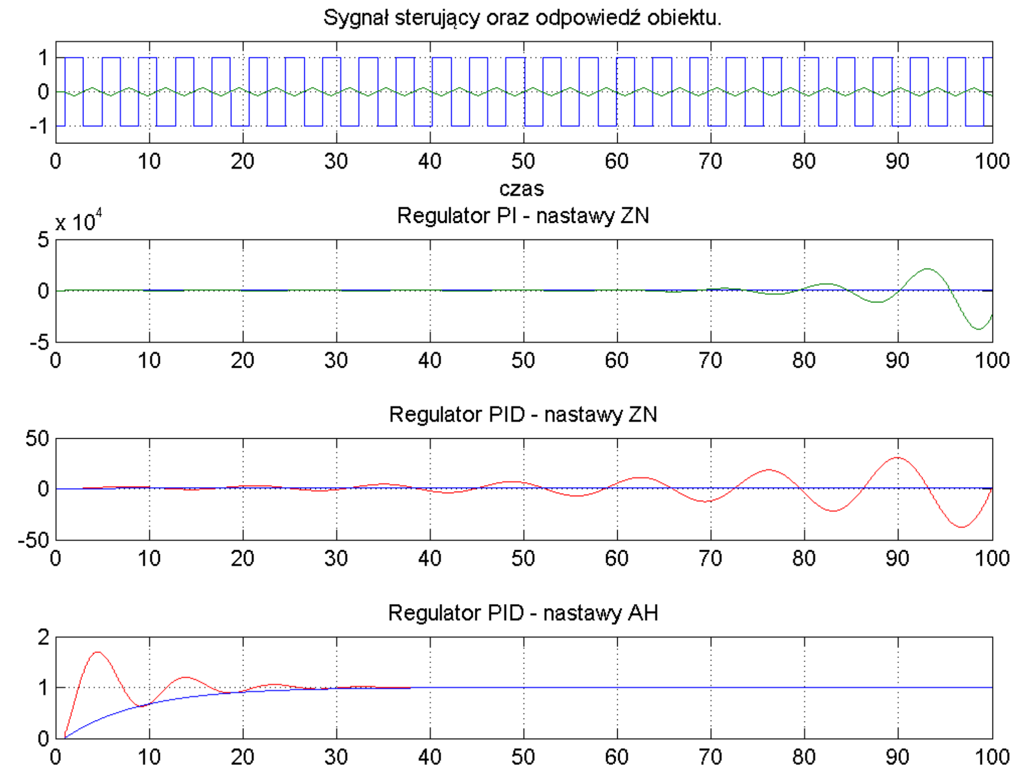
Obiekt G1, T = 2



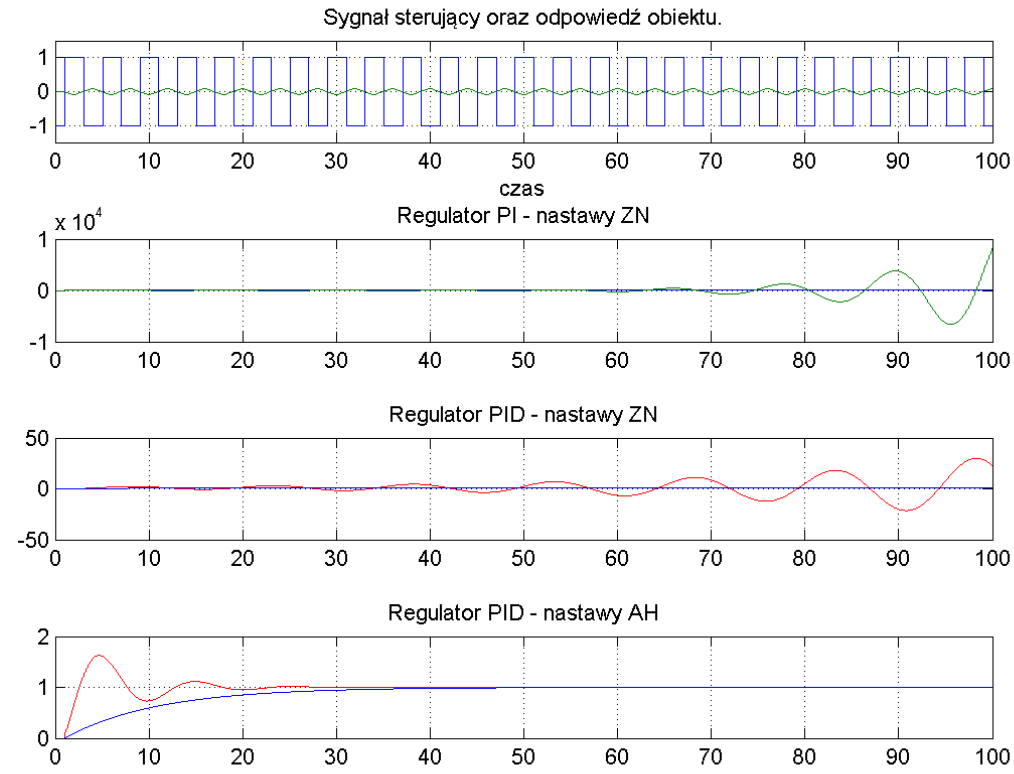
Obiekt G1, T = 4



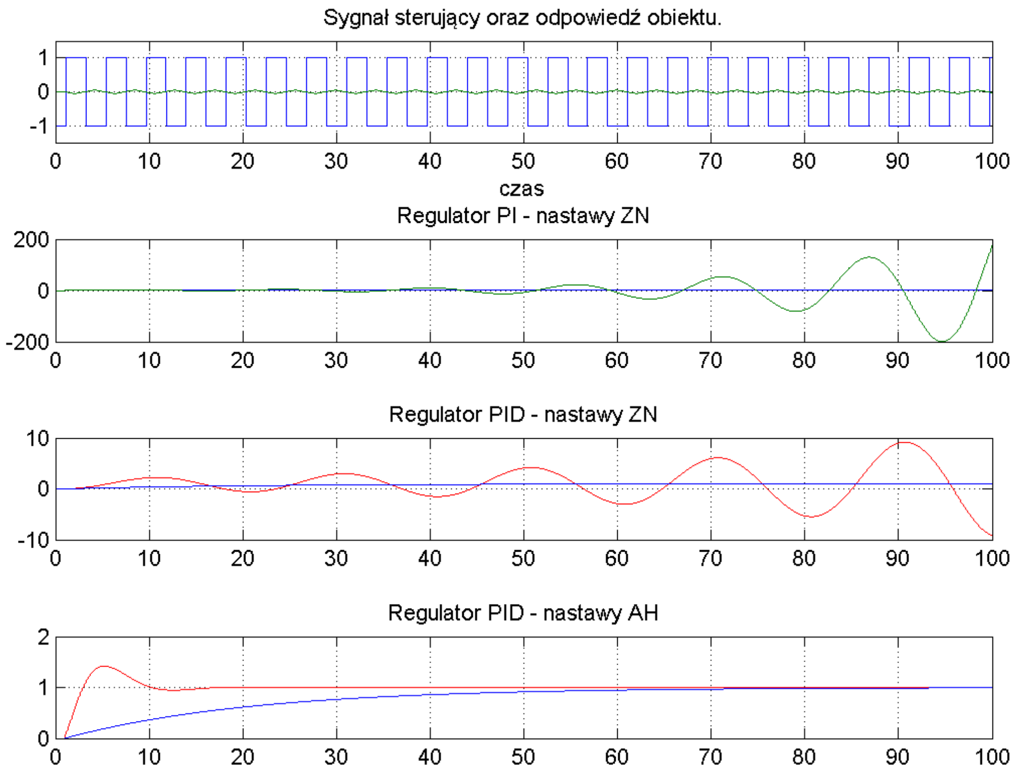
Obiekt G1, T = 6



Obiekt G1, T = 8



Obiekt G1, T = 10



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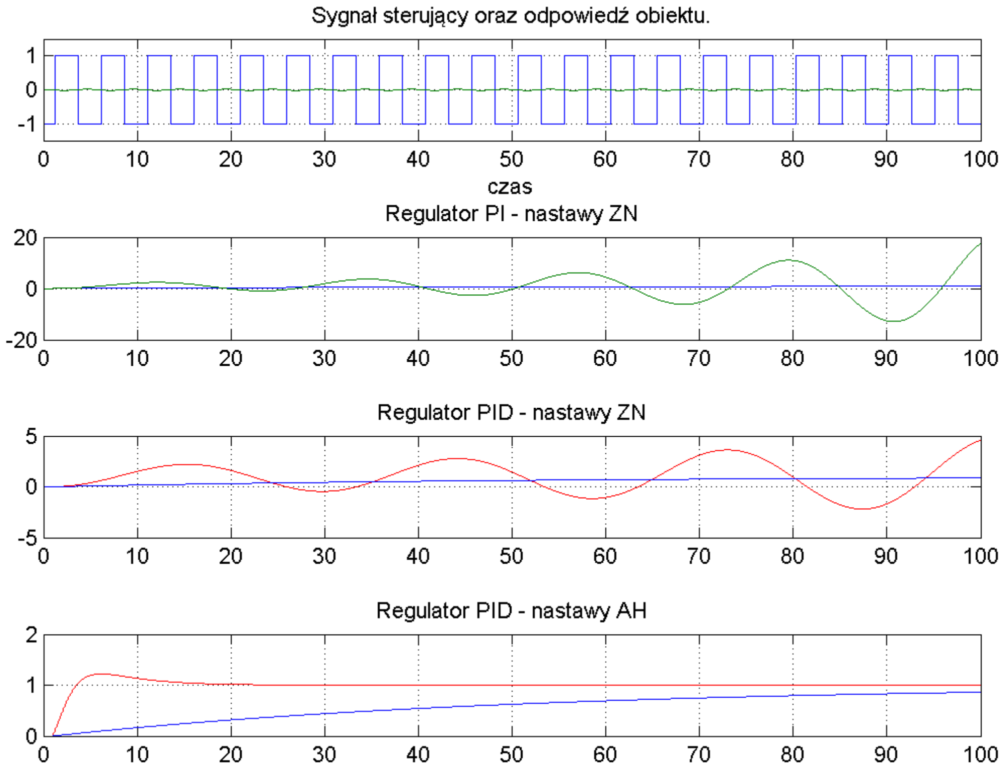

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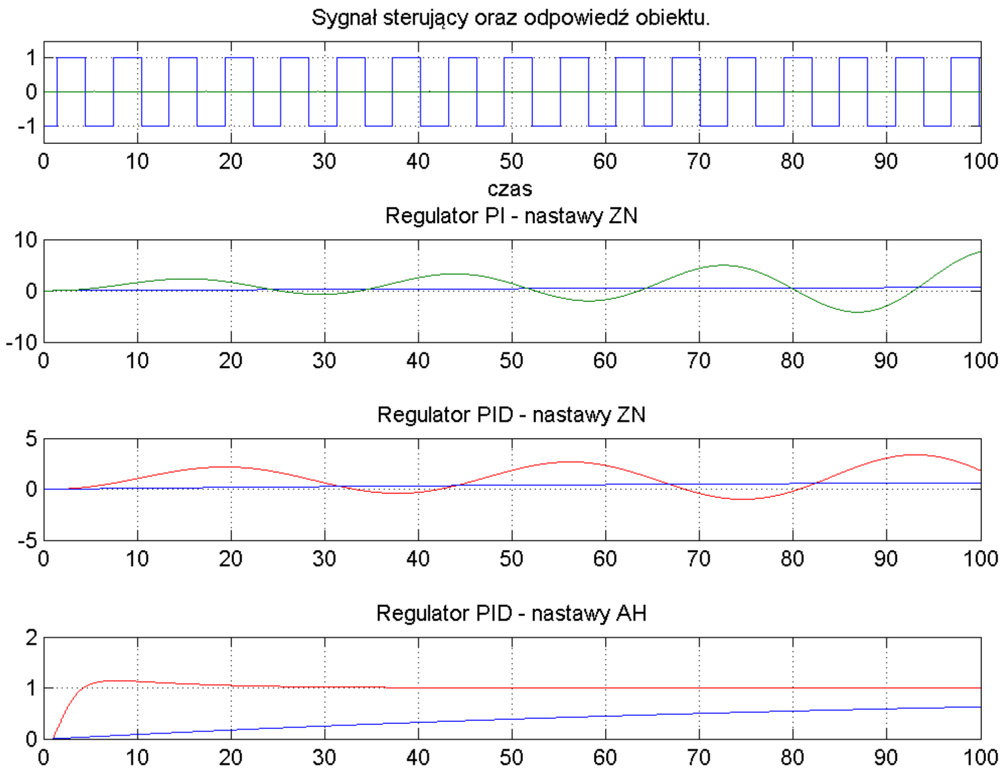
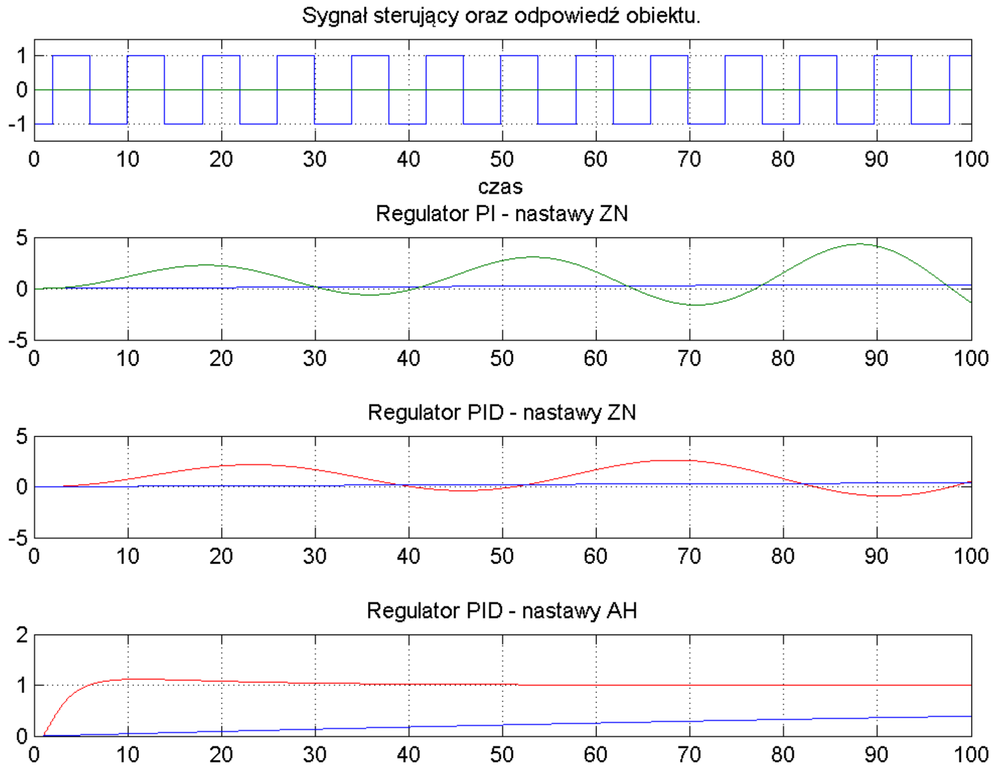


Obiekt G1, T = 100



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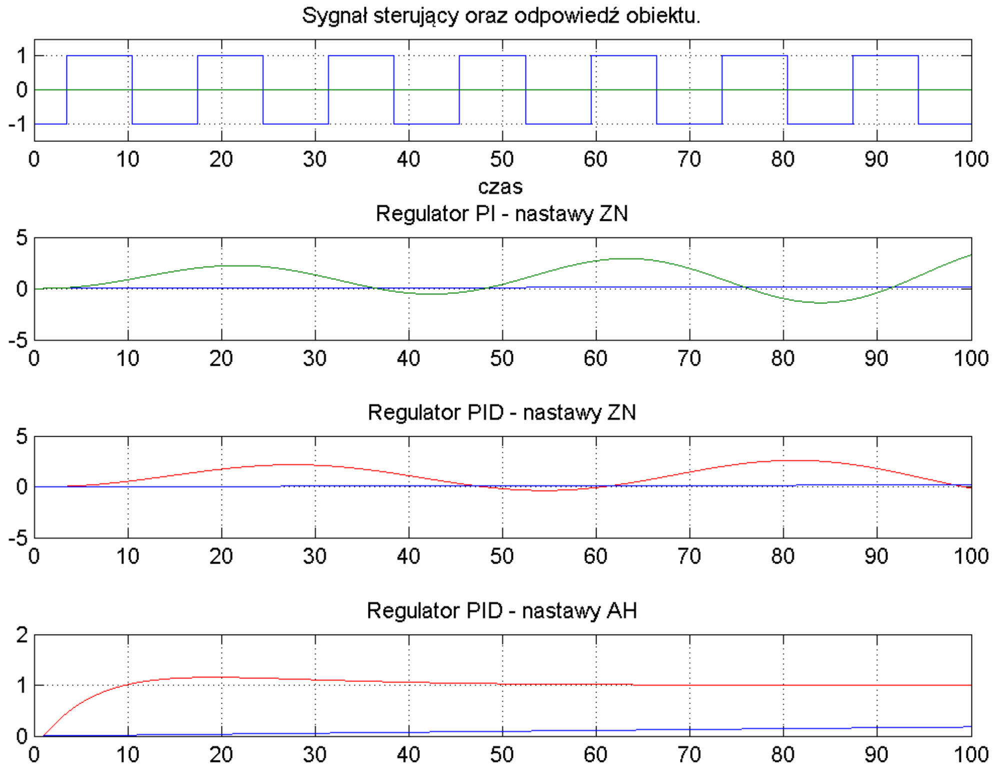

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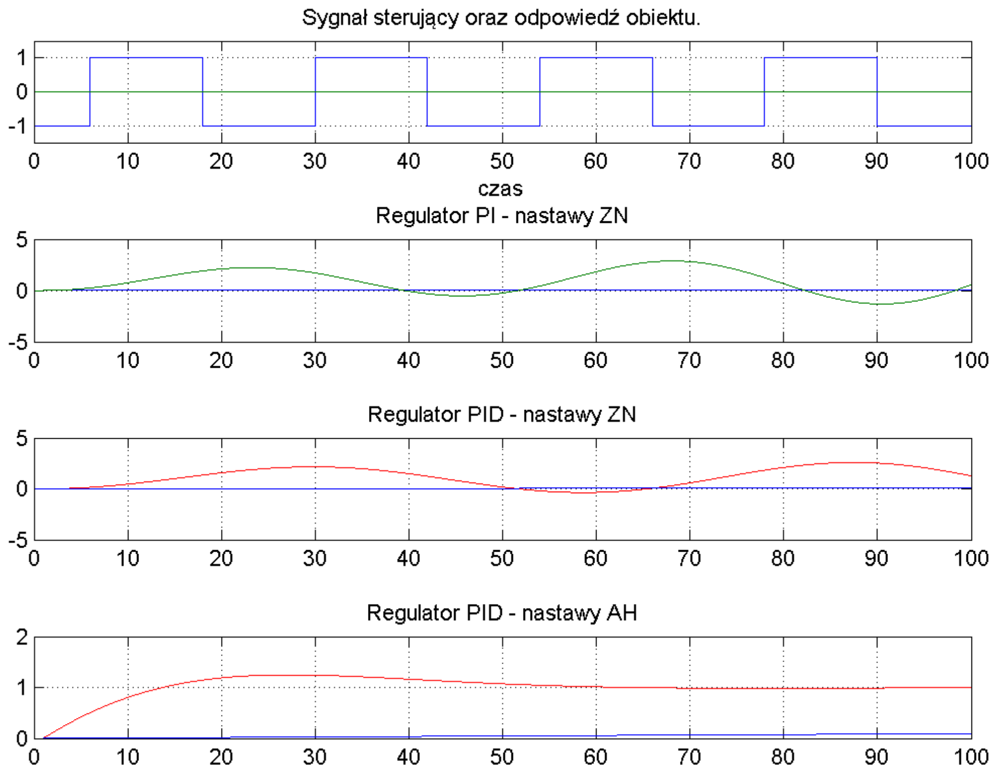

Obiekt G1, T = 1000

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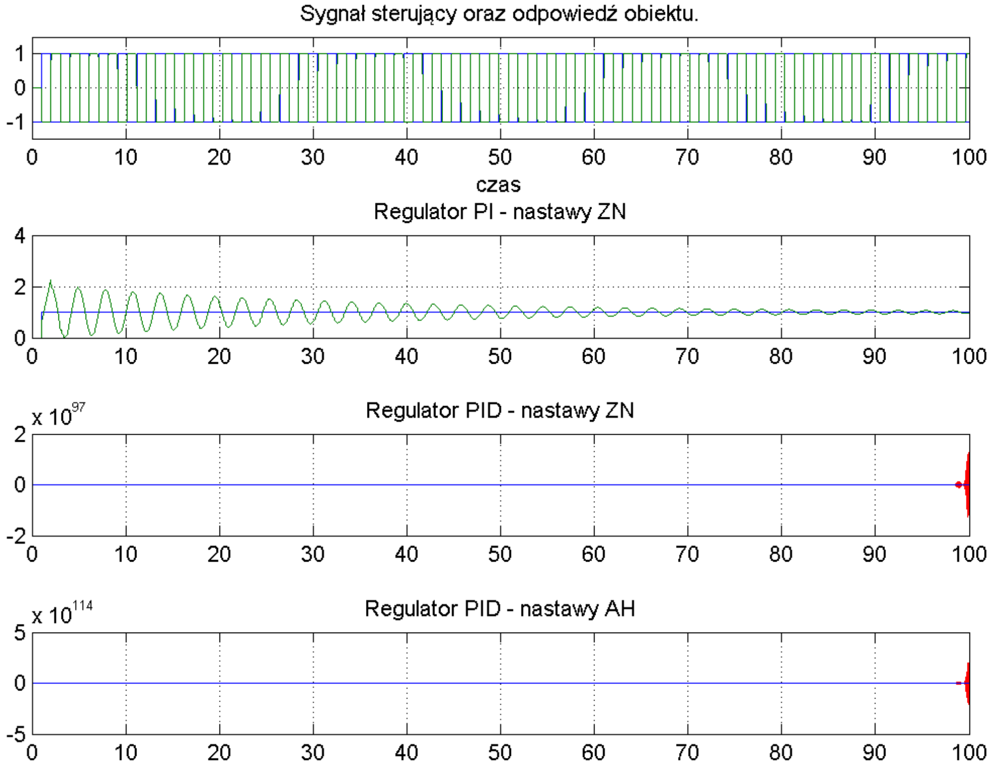
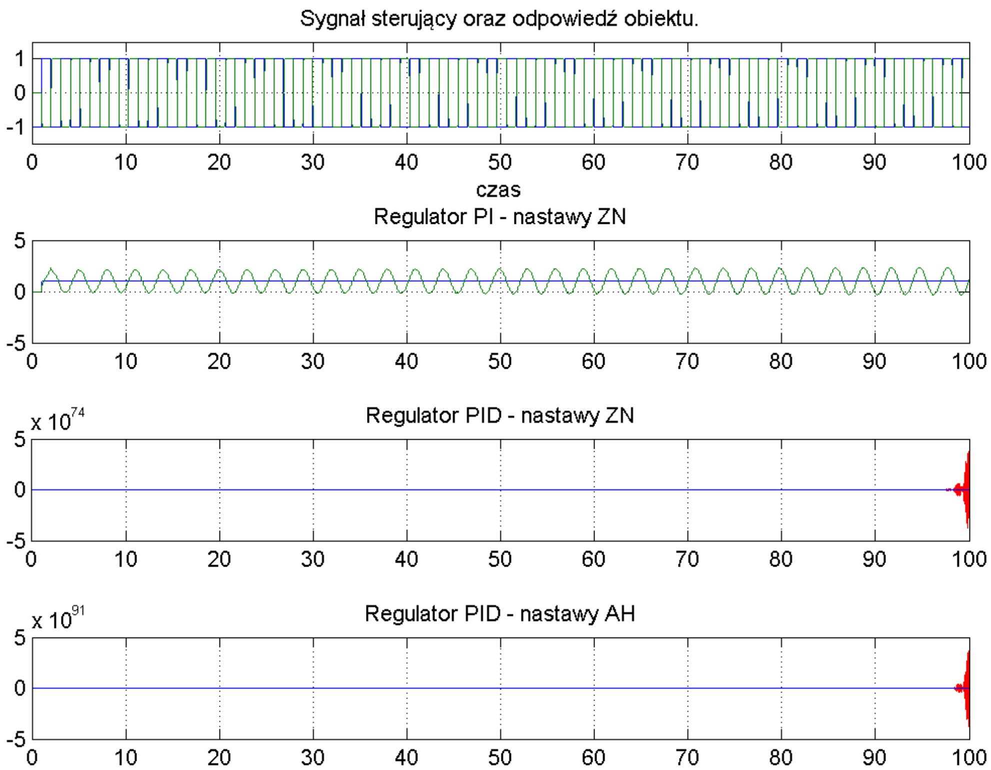


Obiekt G2, T = 0.01



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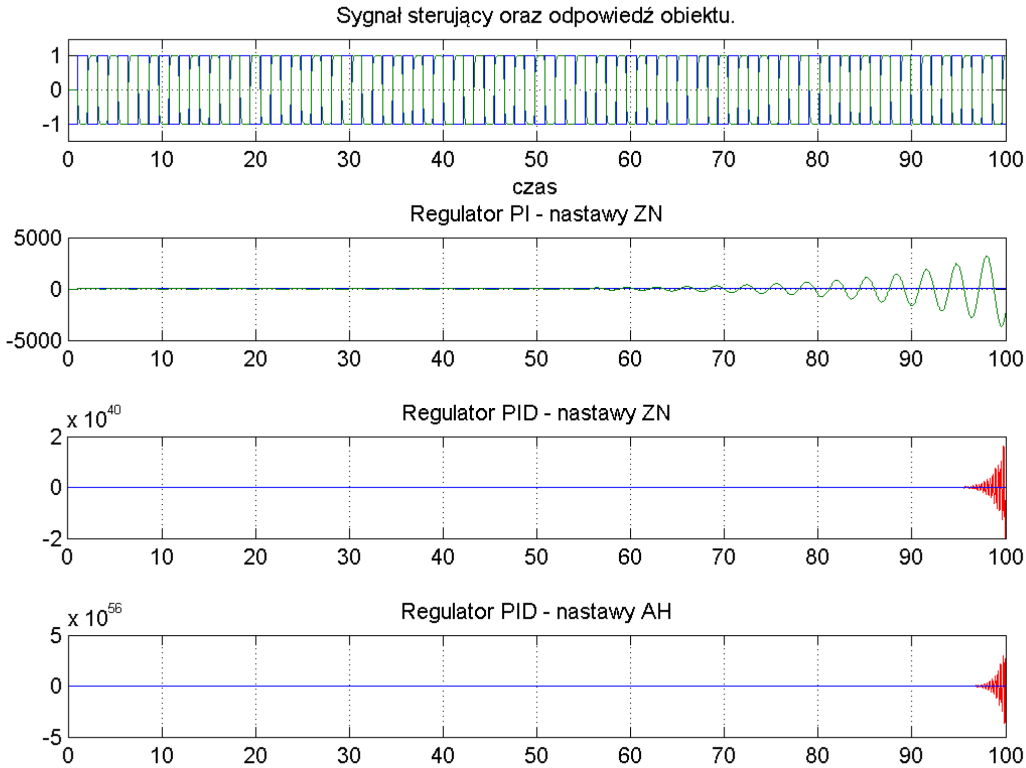

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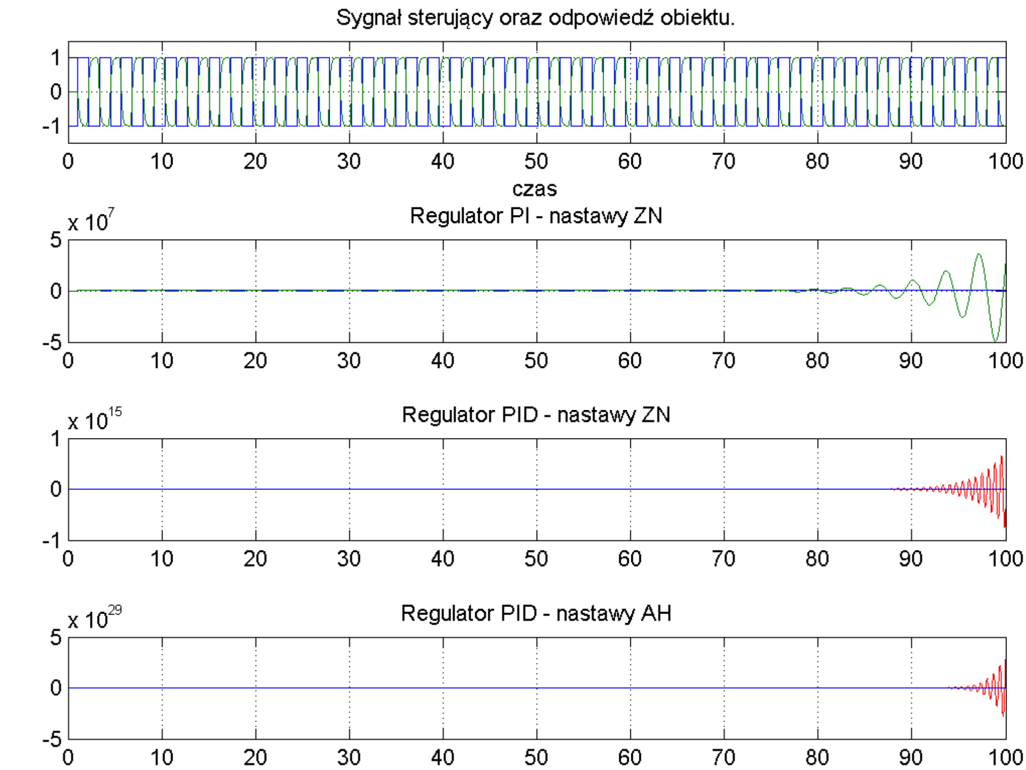

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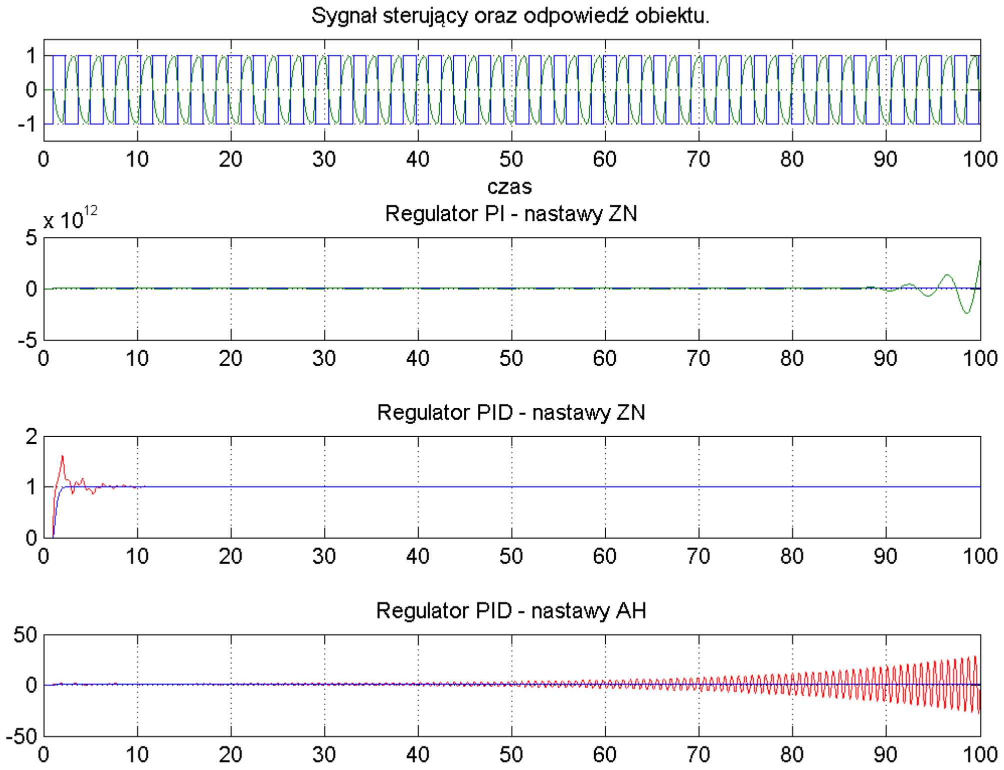
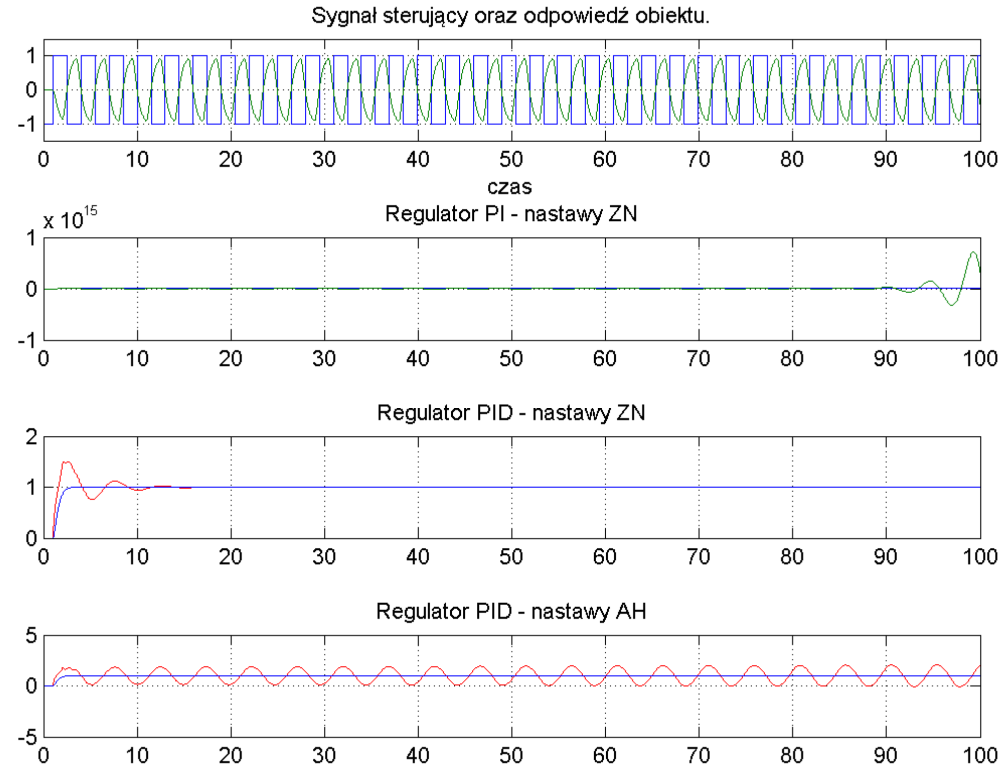


Obiekt G2, T = 0.2



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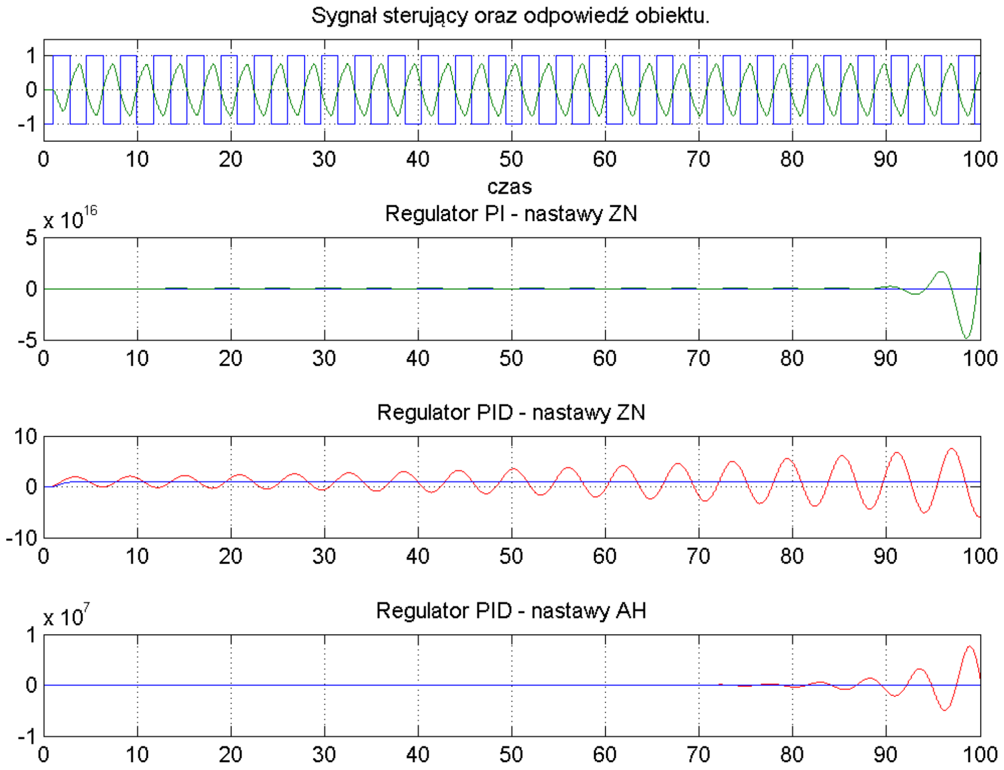

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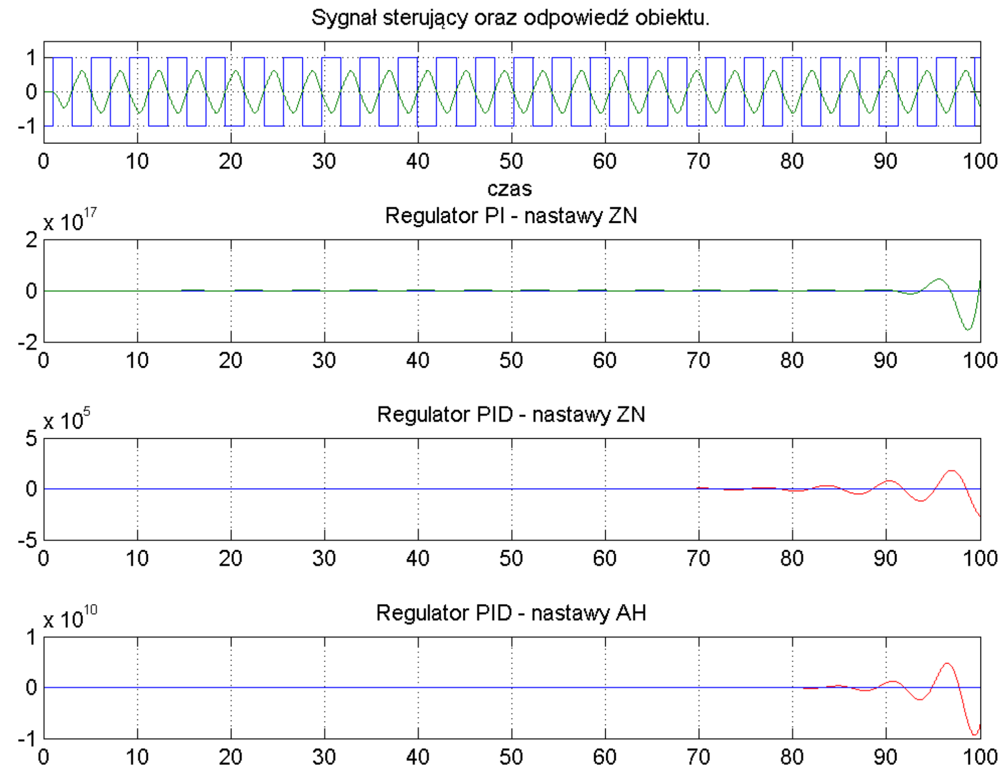
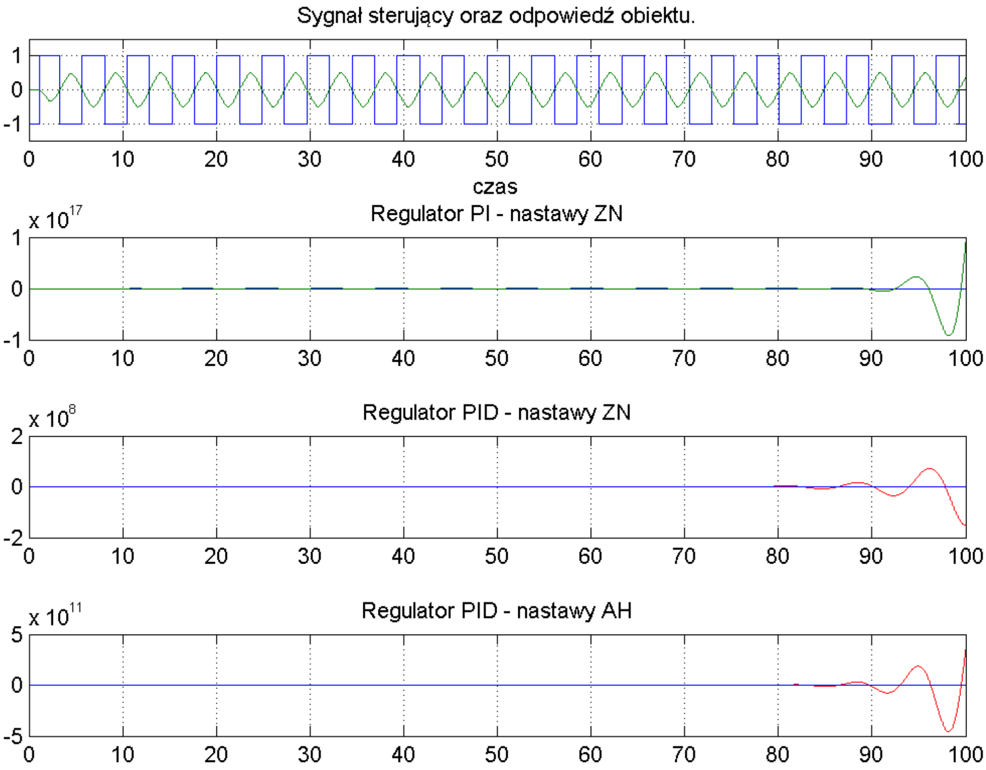
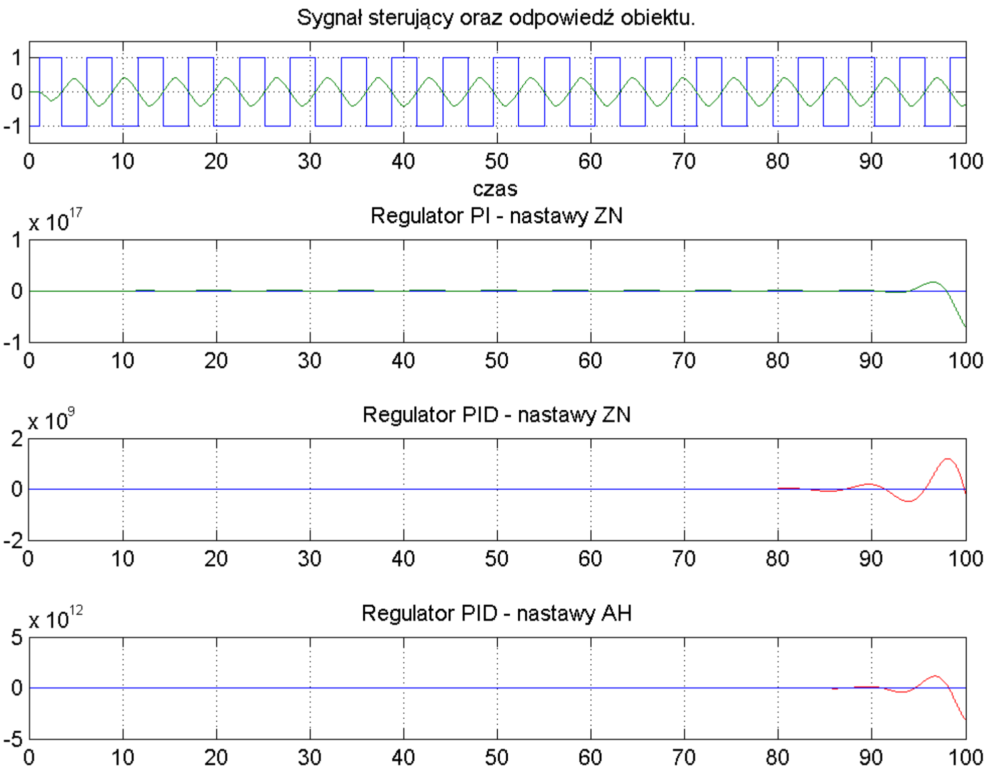


Obiekt G2, T = 0.7



Obiekt G2, T = 1



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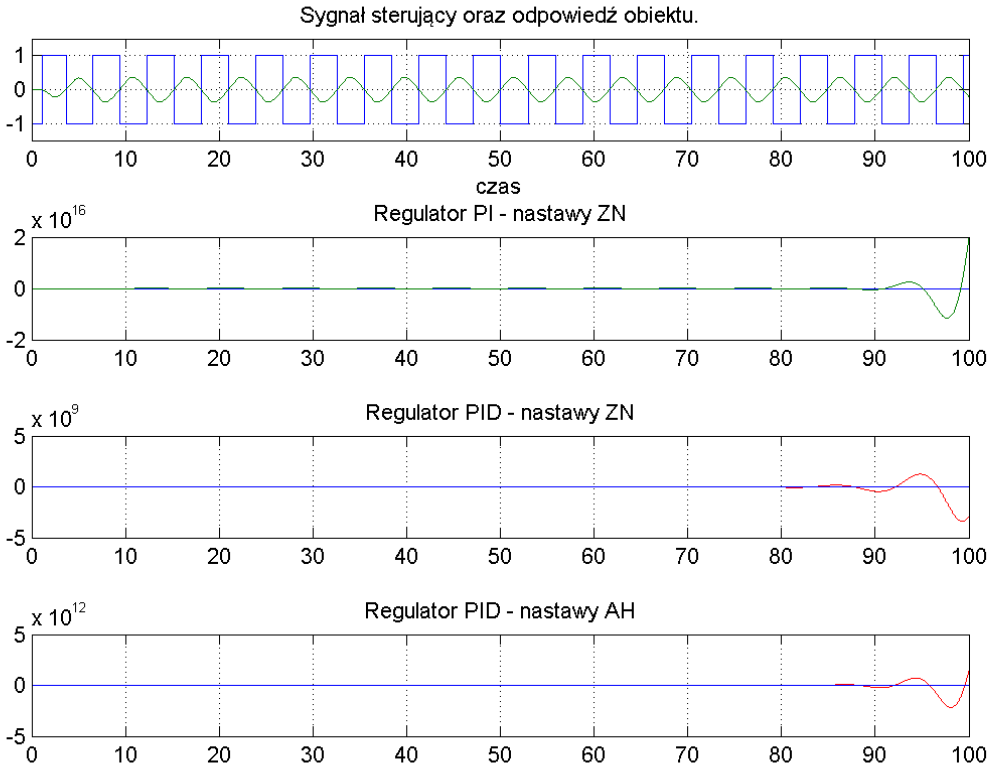

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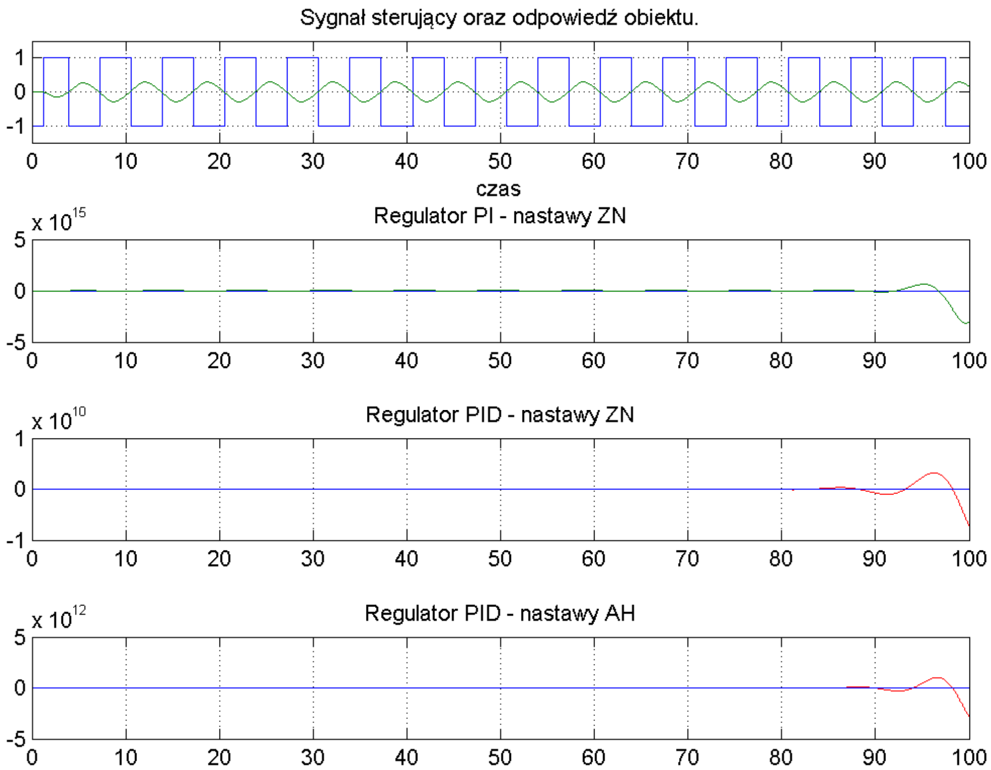

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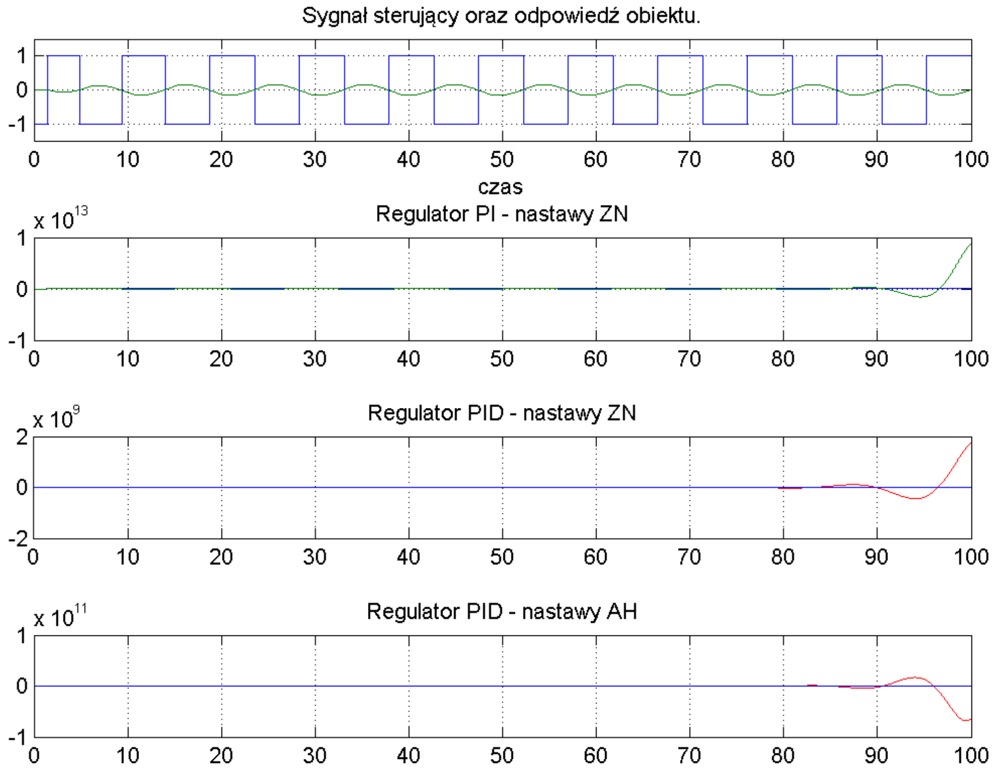

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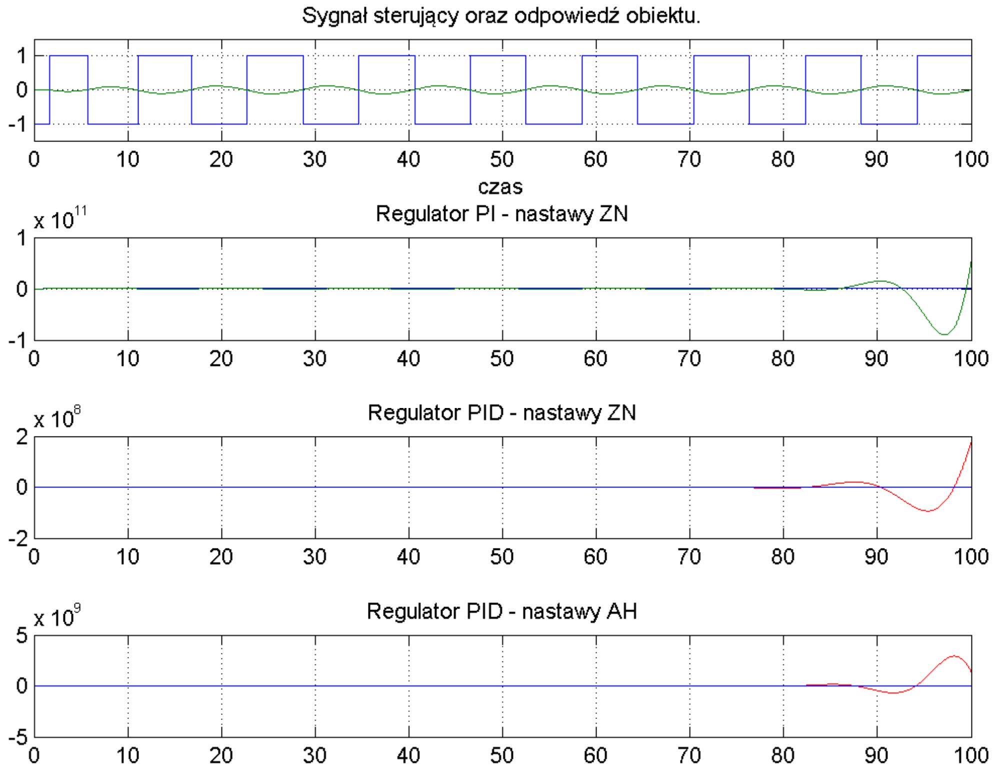
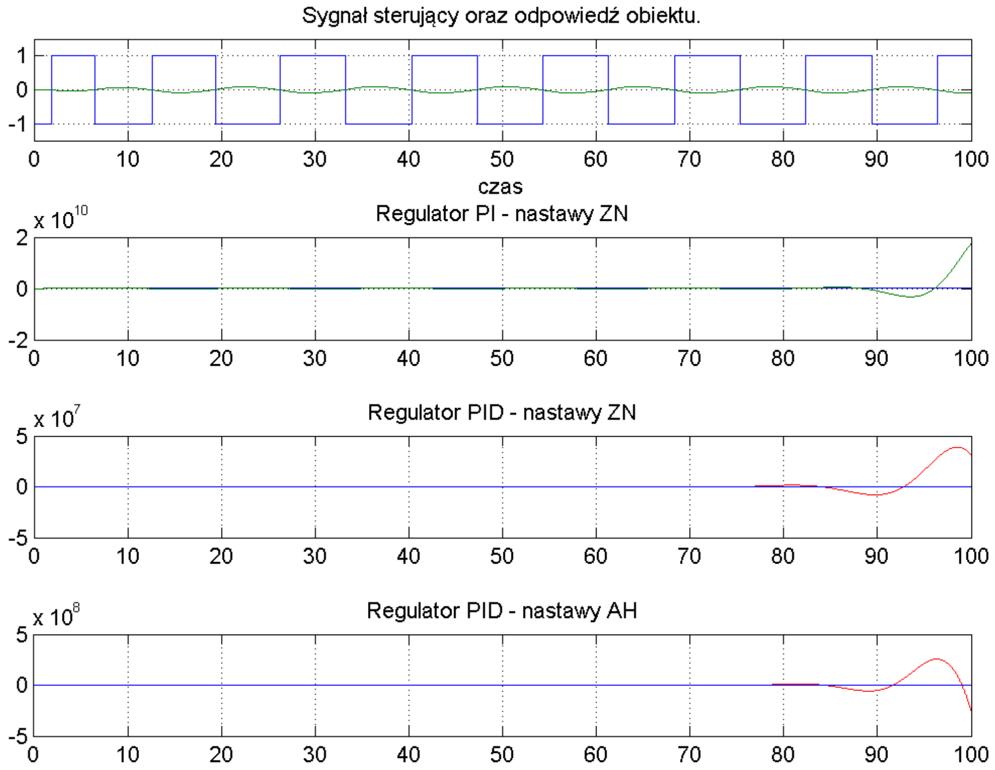
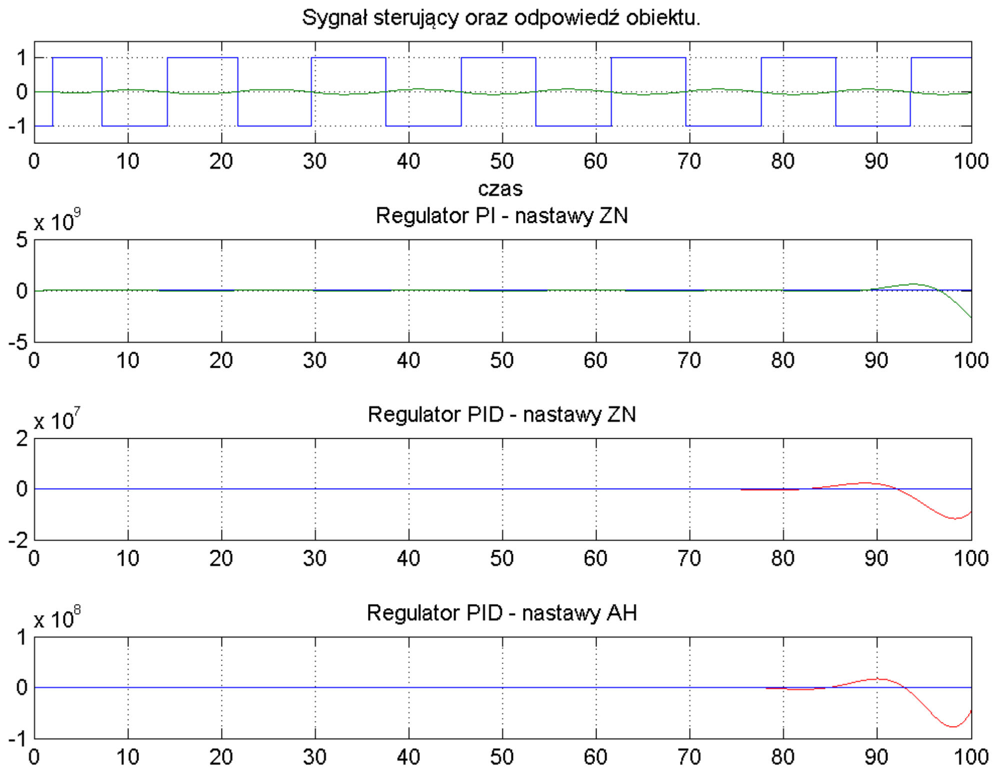


Obiekt G2, T = 6



Obiekt G2, T = 8



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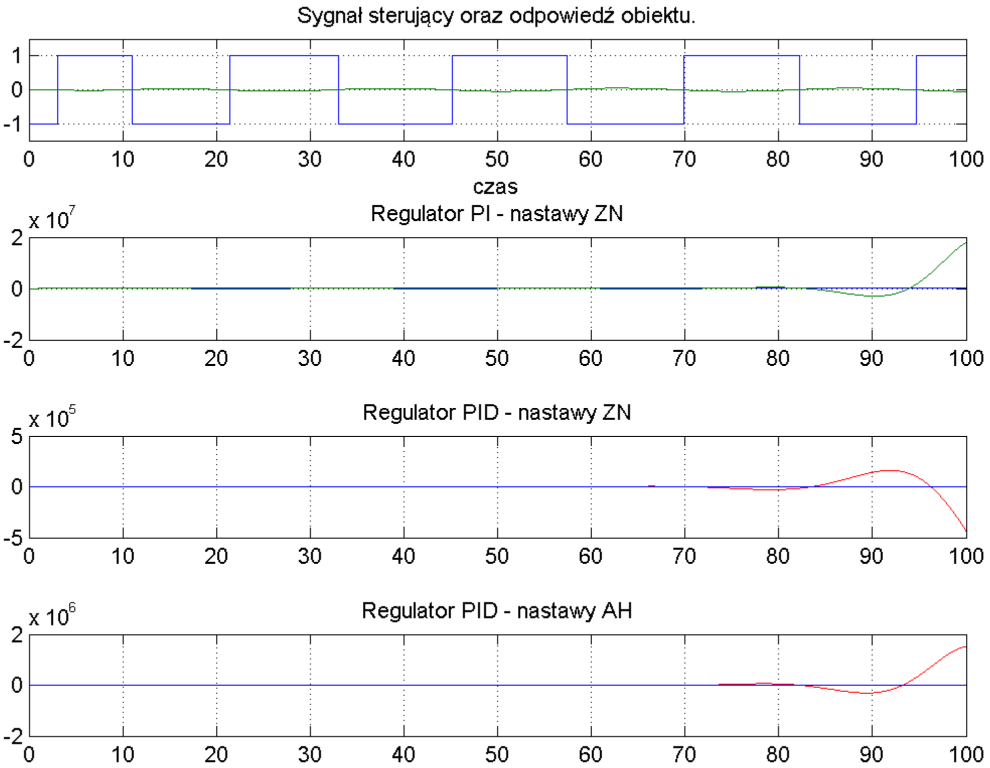

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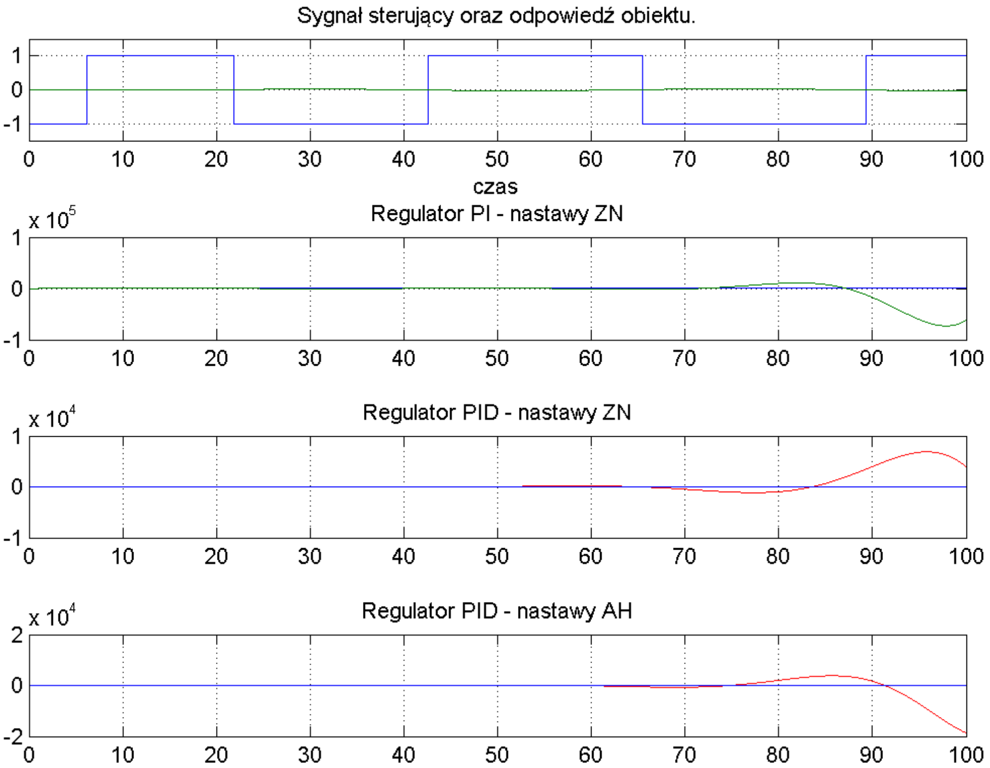

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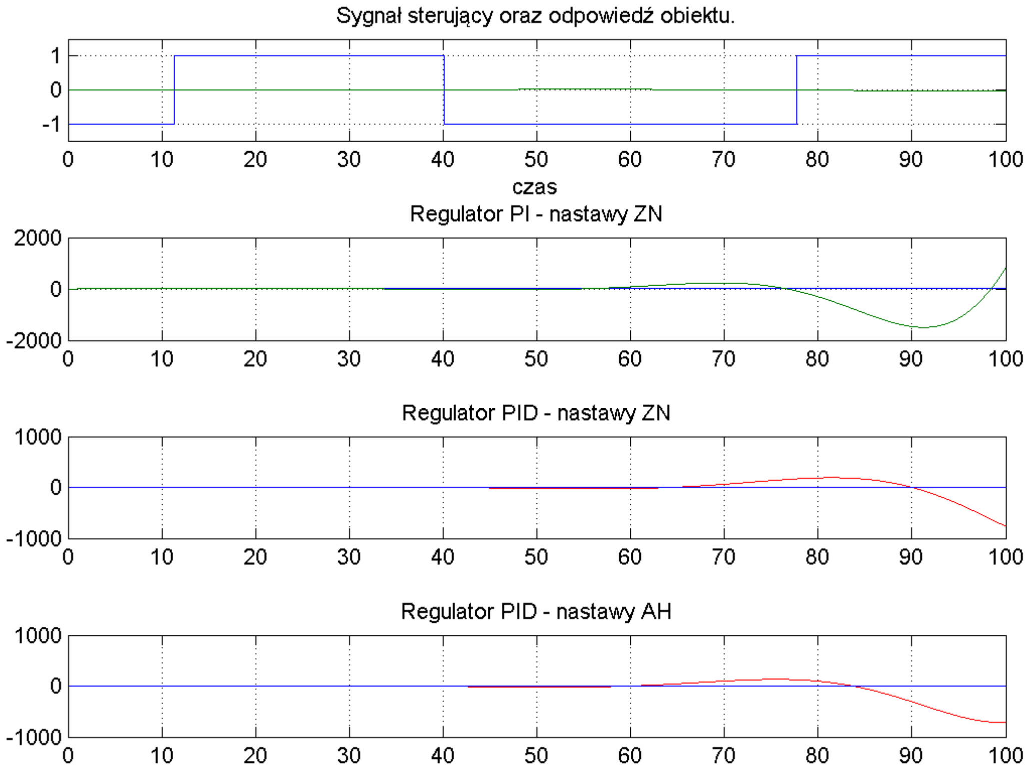

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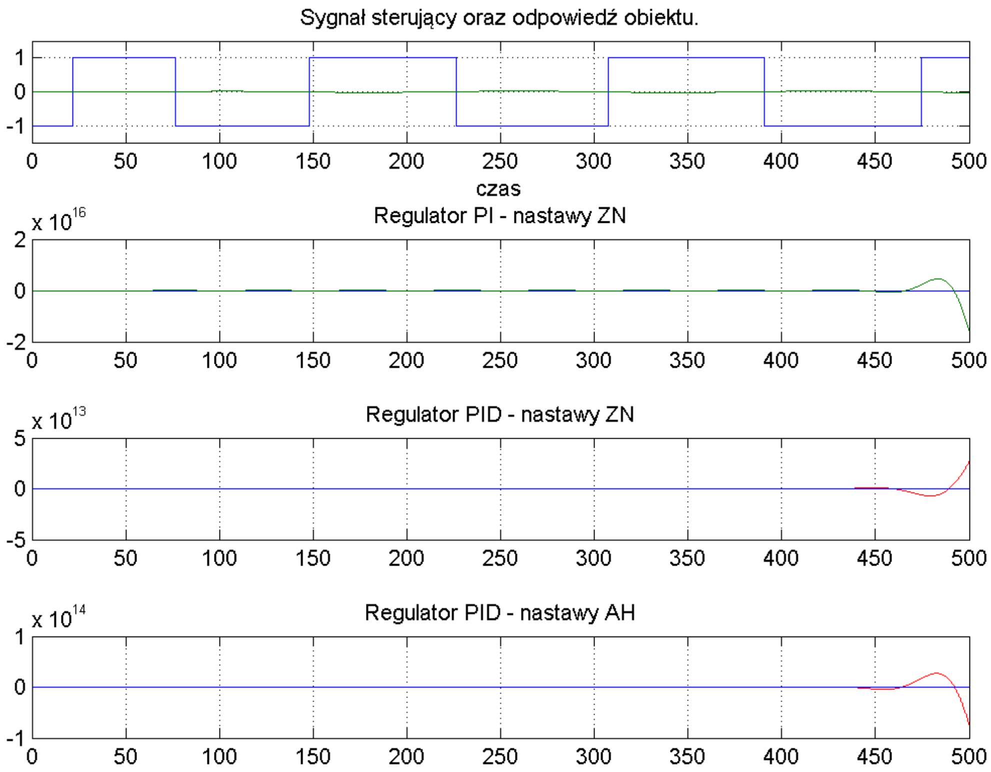

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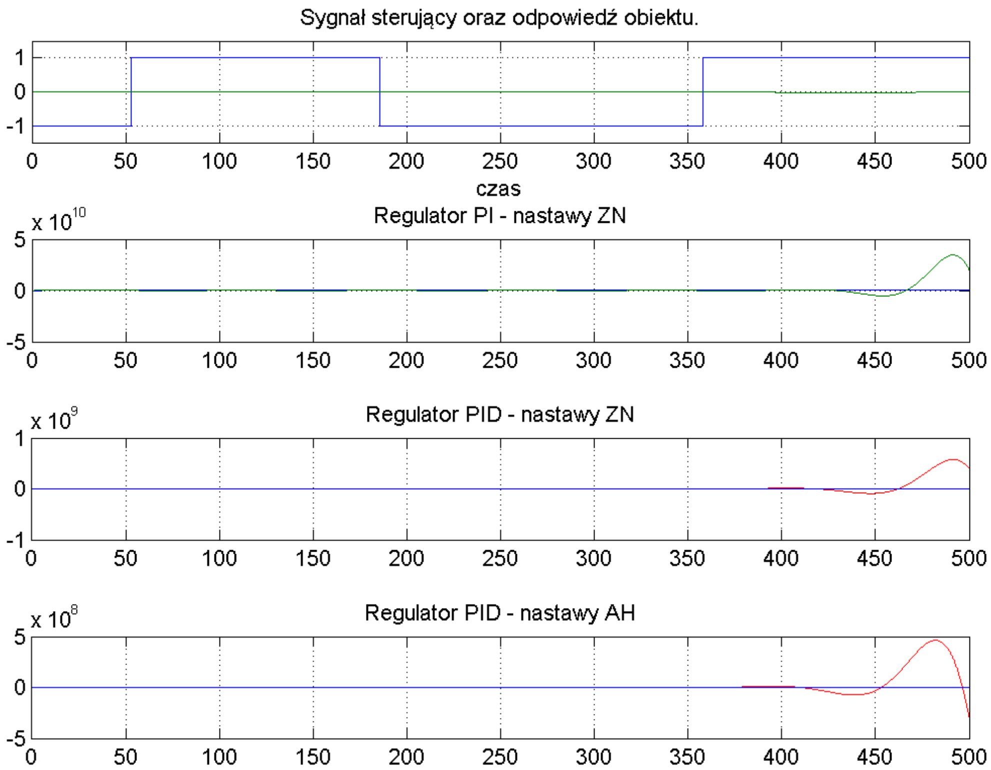
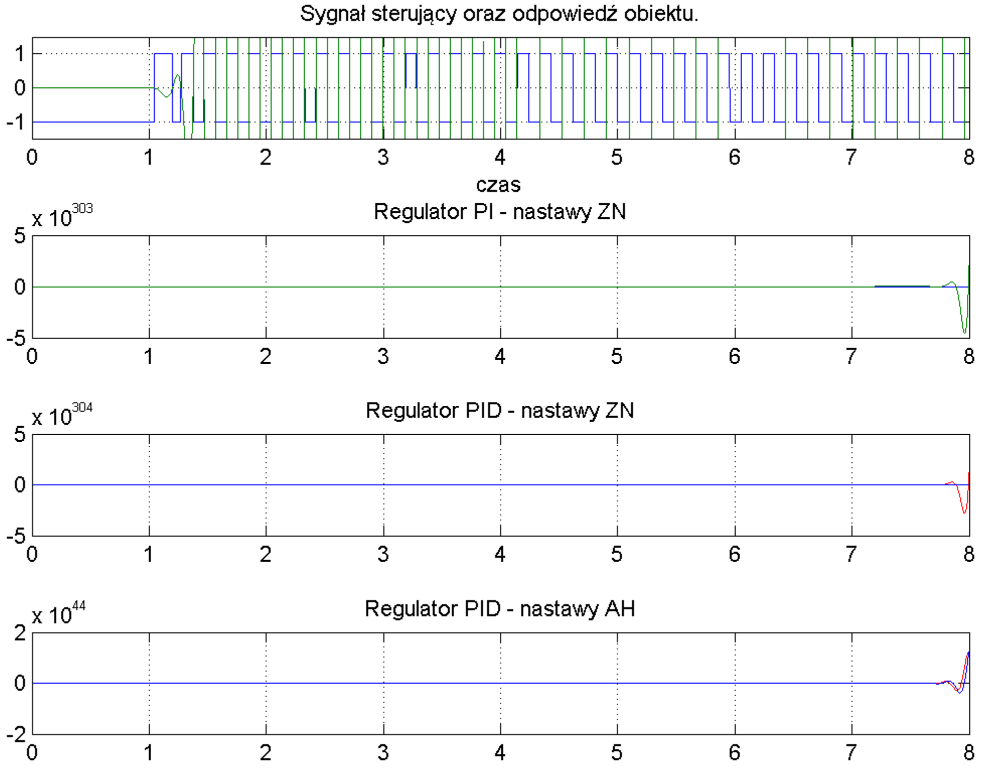
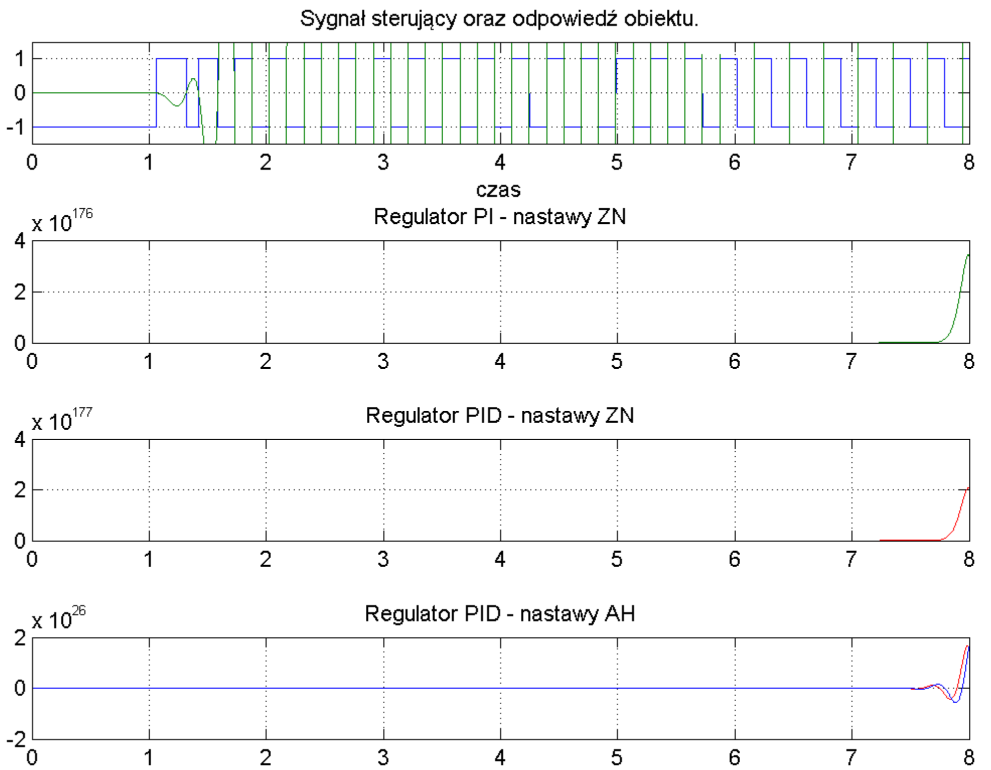


Obiekt G2, T = 500

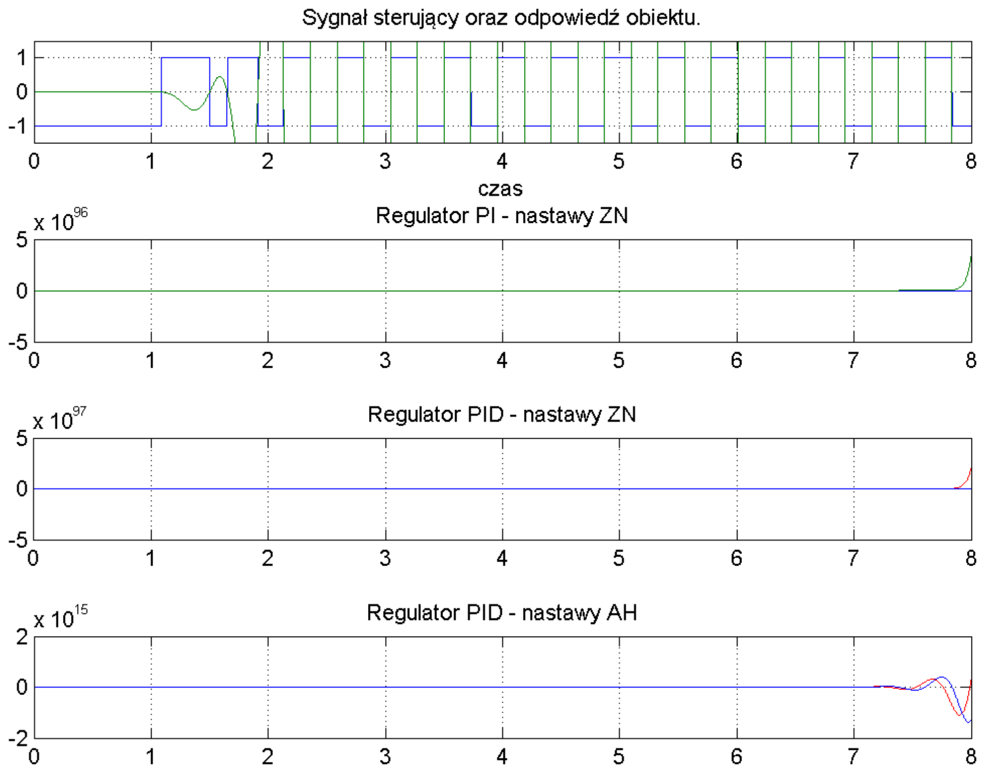
Obiekt 3.3.



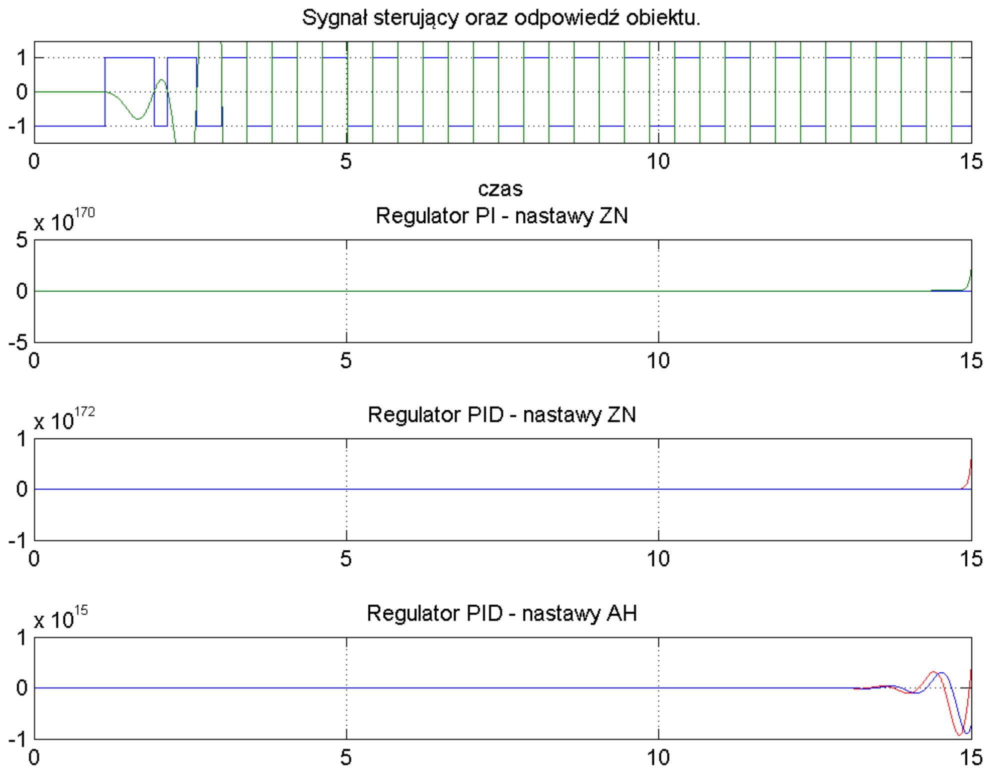
Obiekt G3, T = 0.005



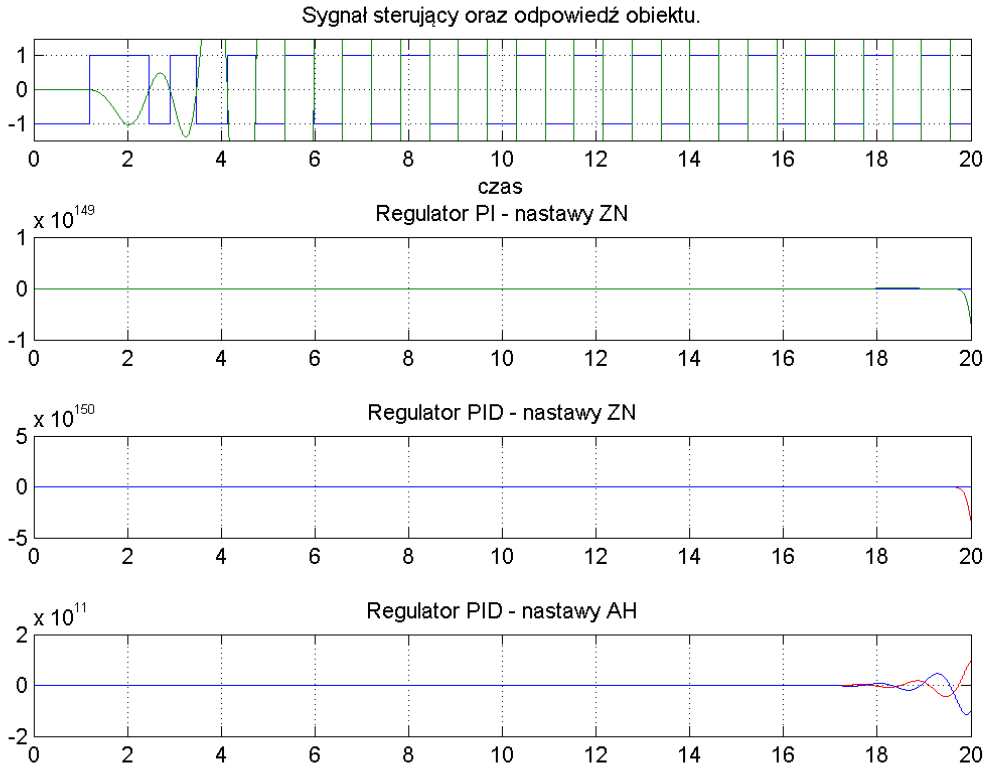
Obiekt G3, T = 0.01



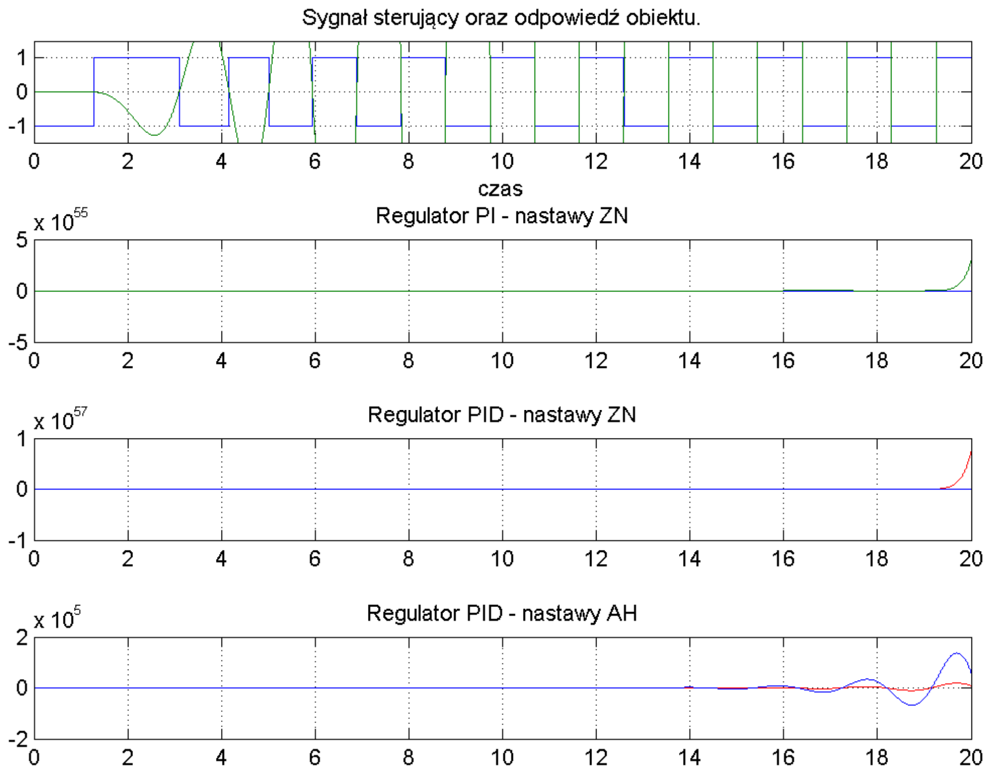
Obiekt G3, T = 0.02



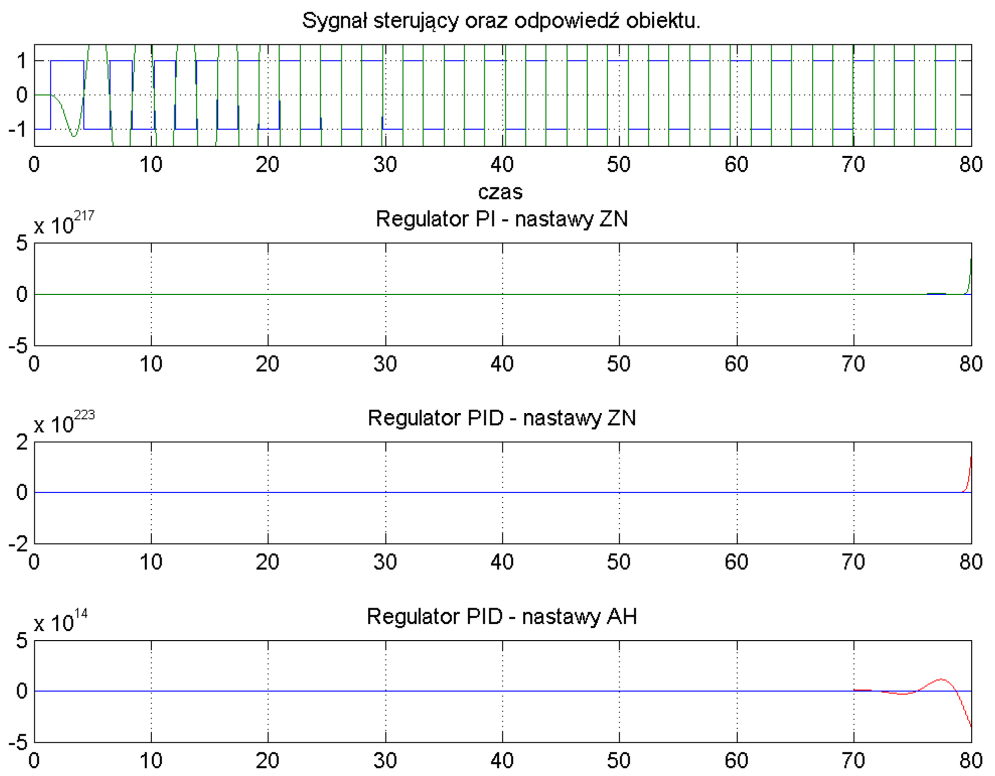
Obiekt G3, T = 0.05



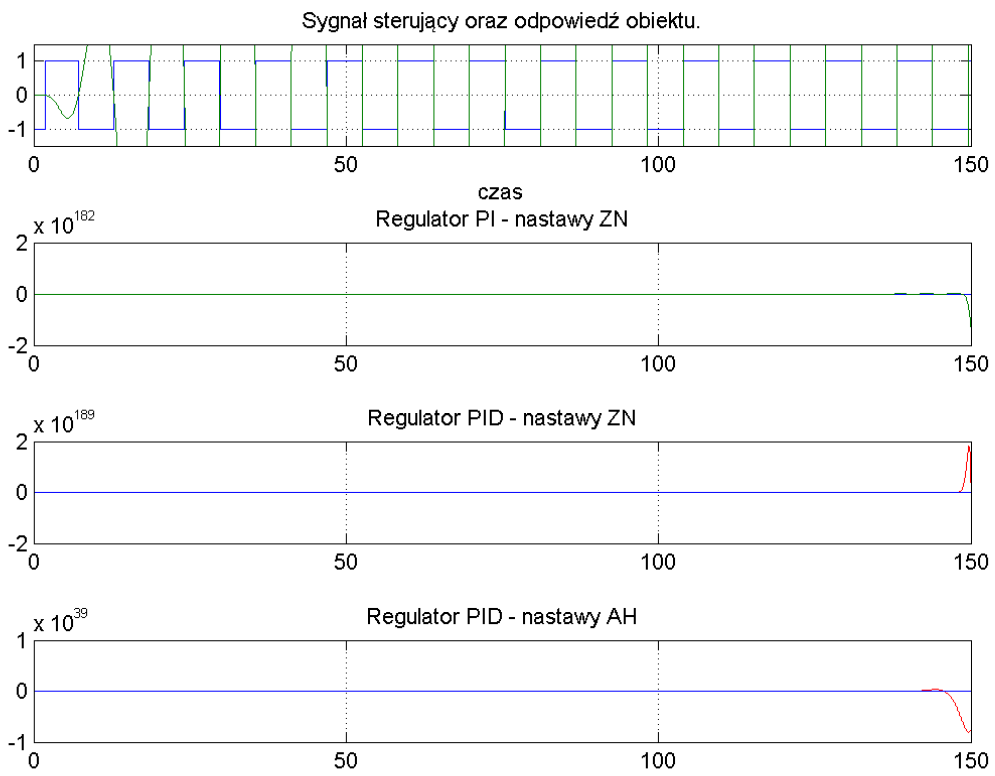
Obiekt G3, T = 0.1



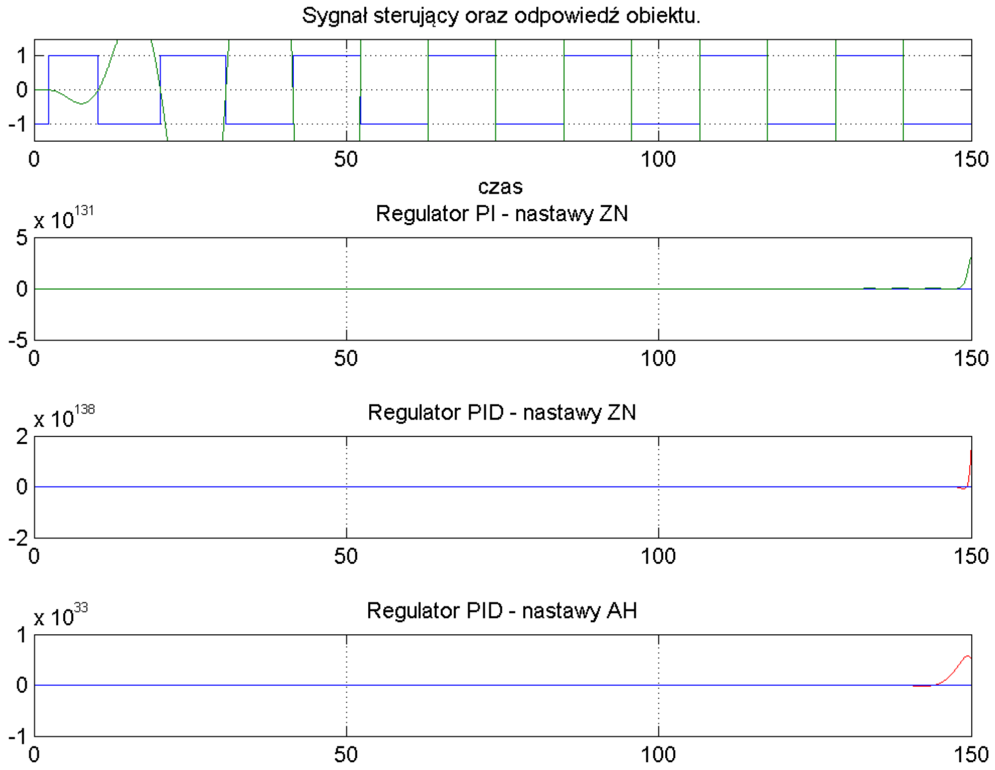
Obiekt G3, T = 0.2



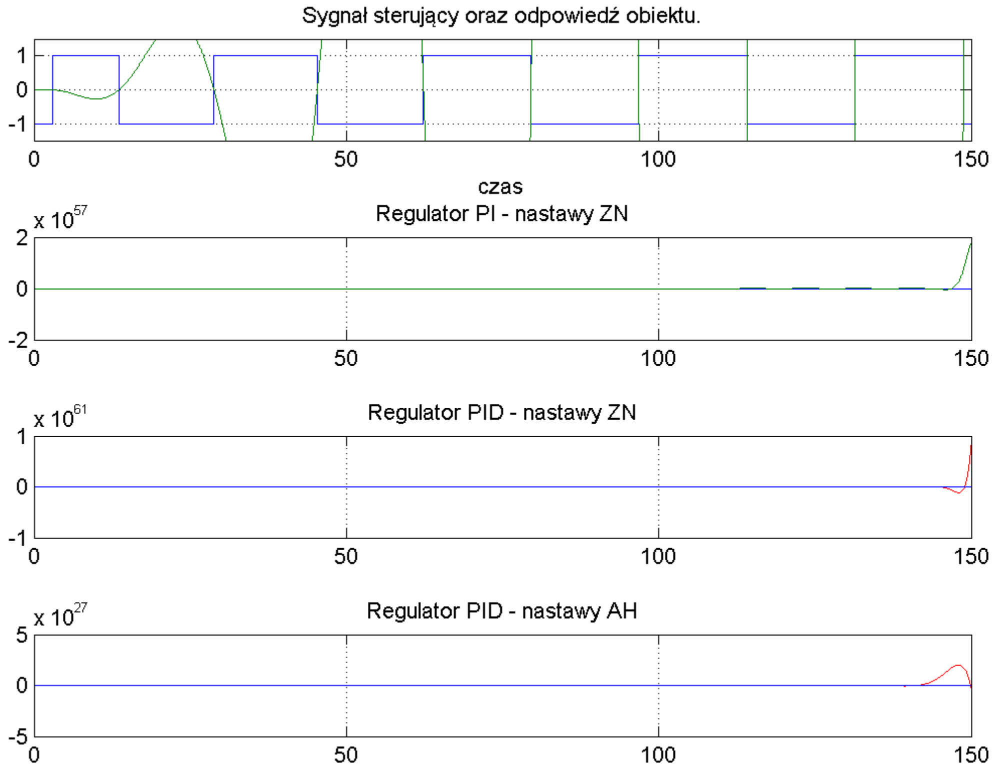
Obiekt G3, T = 0.5



Obiekt G3, T = 2

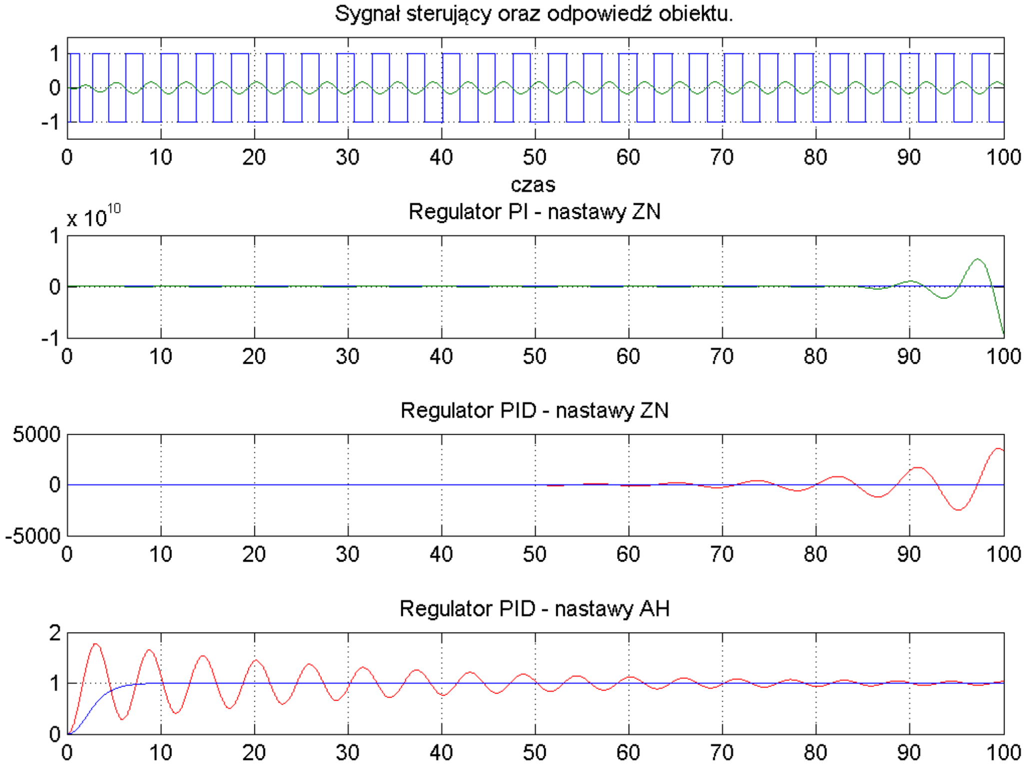


Obiekt G3, T = 5

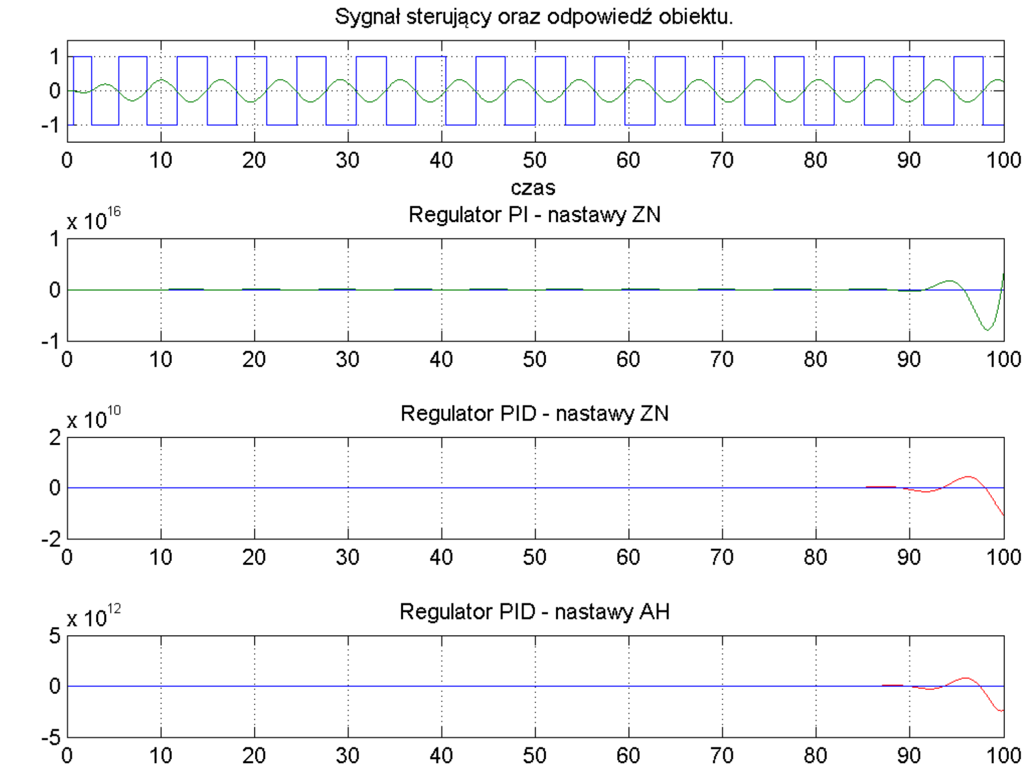


Obiekt G3, T = 10

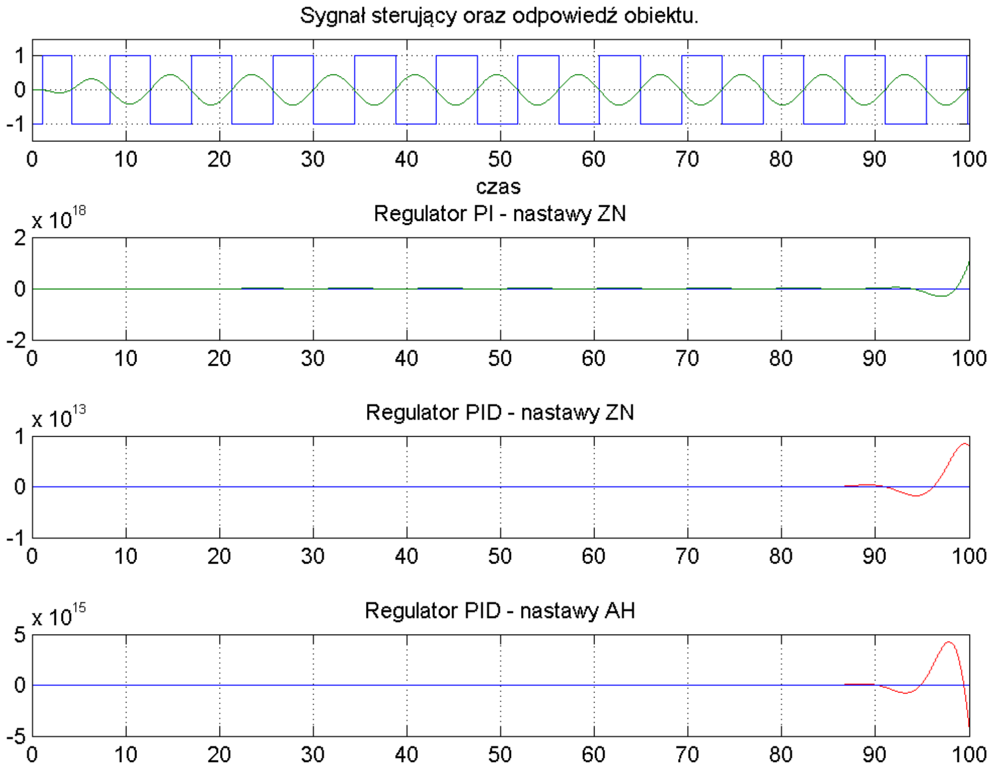
Obiekt 3.4.



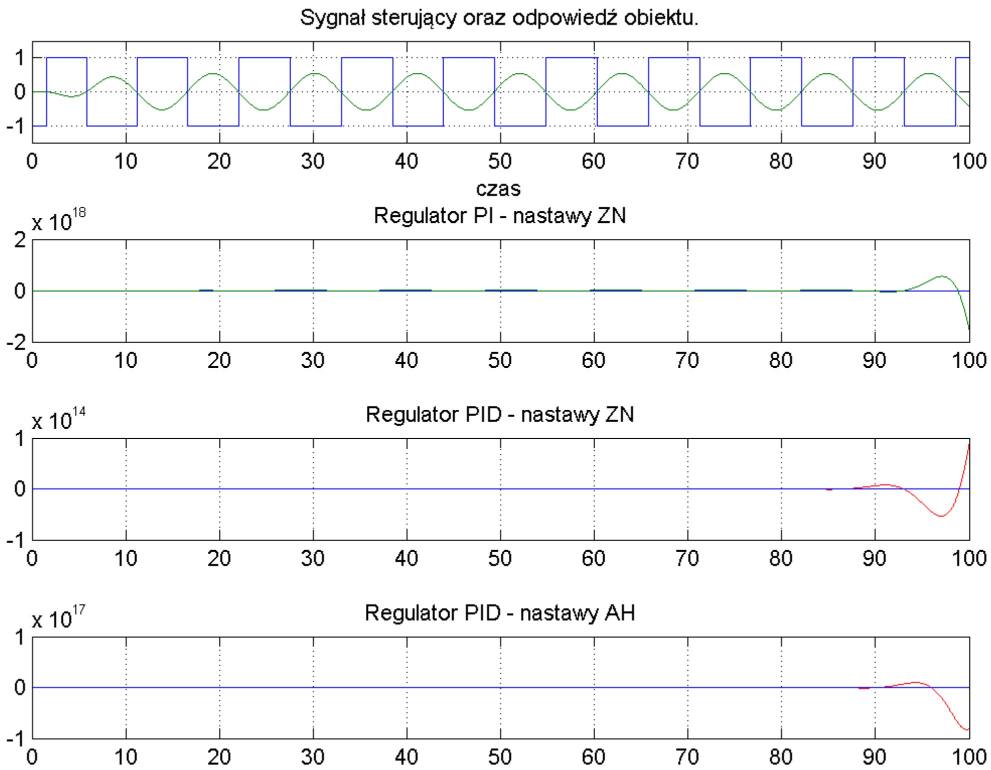
Obiekt G4, n = 3



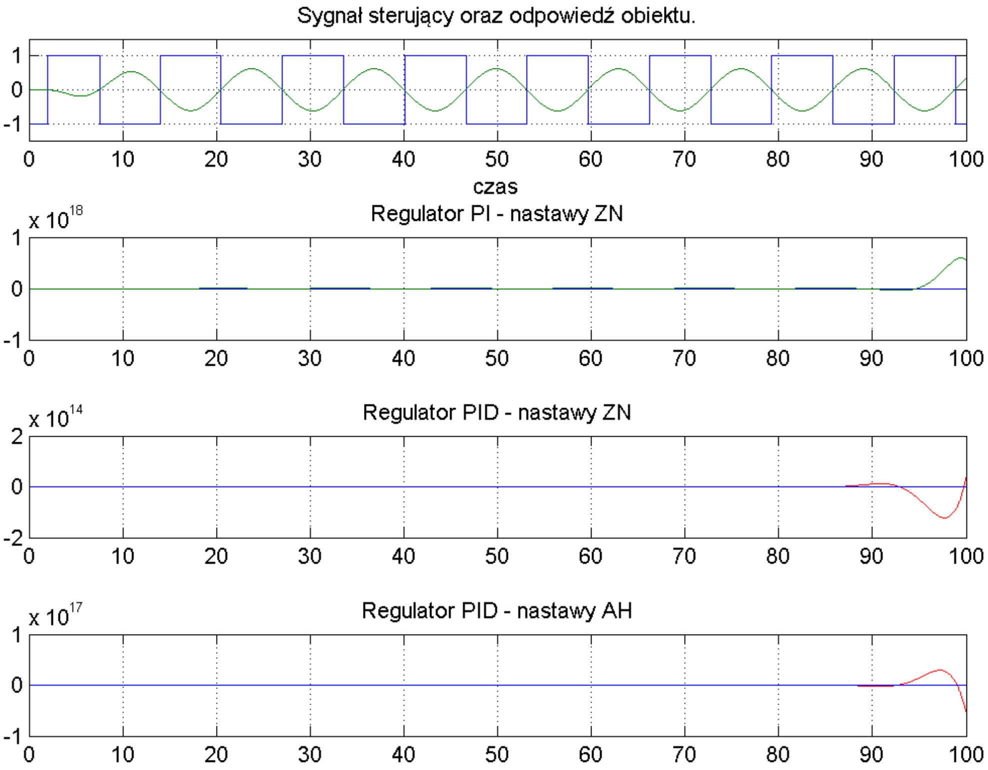
Obiekt G4, n = 4



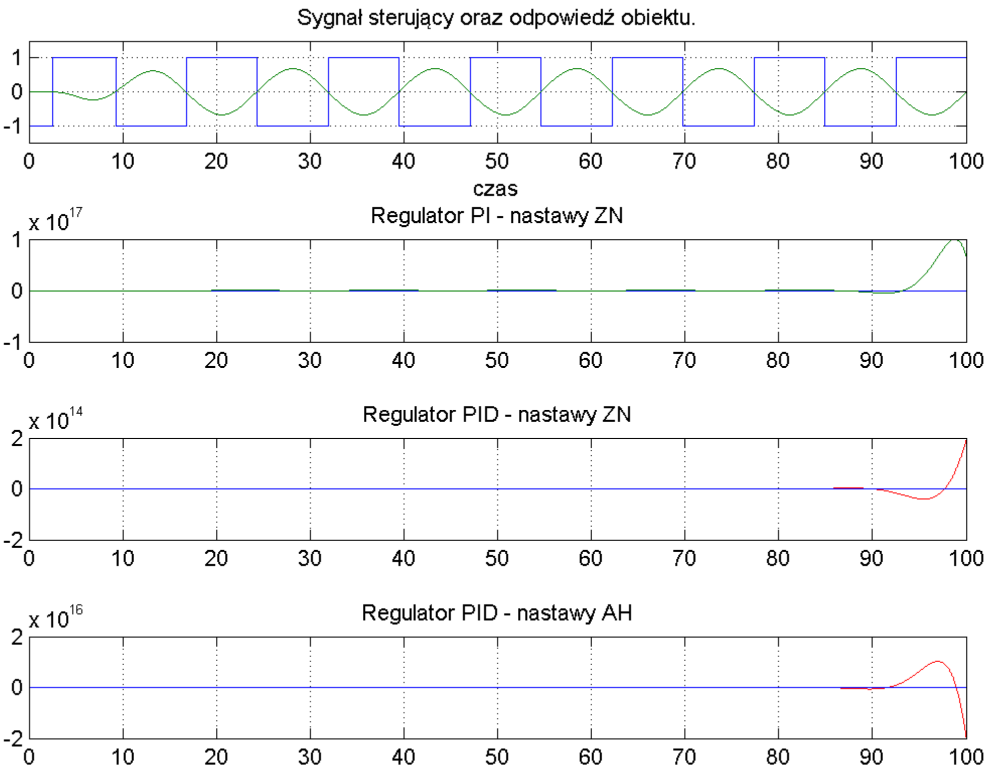
Obiekt G4, n = 5



Obiekt G4, n = 6

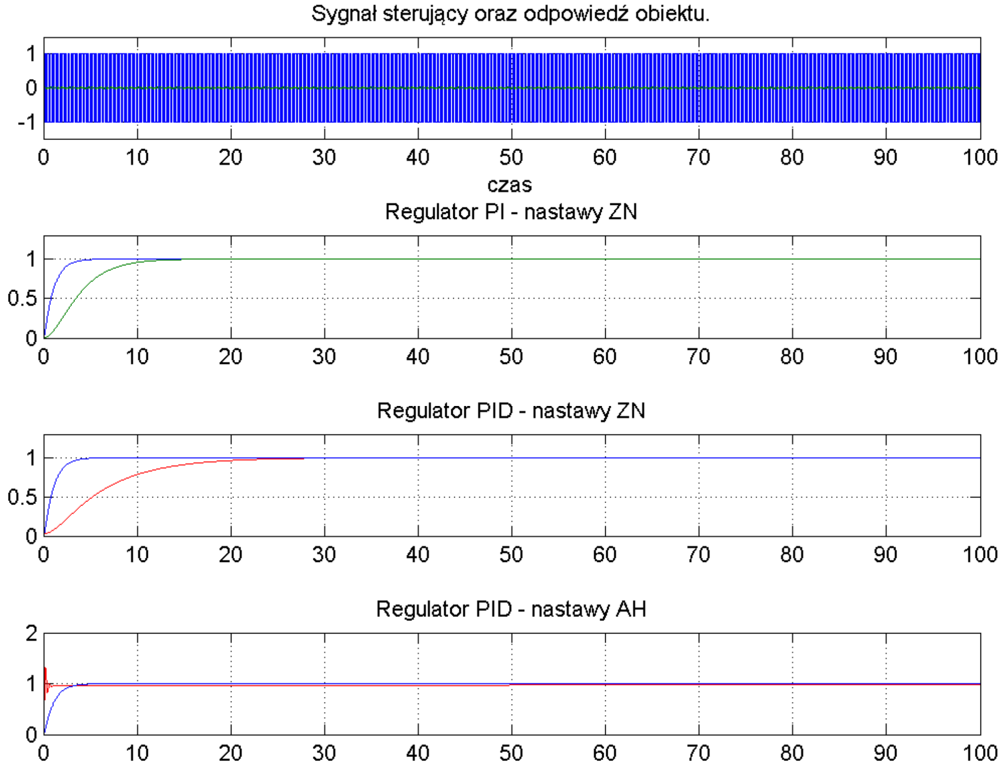


Obiekt G4, n = 7

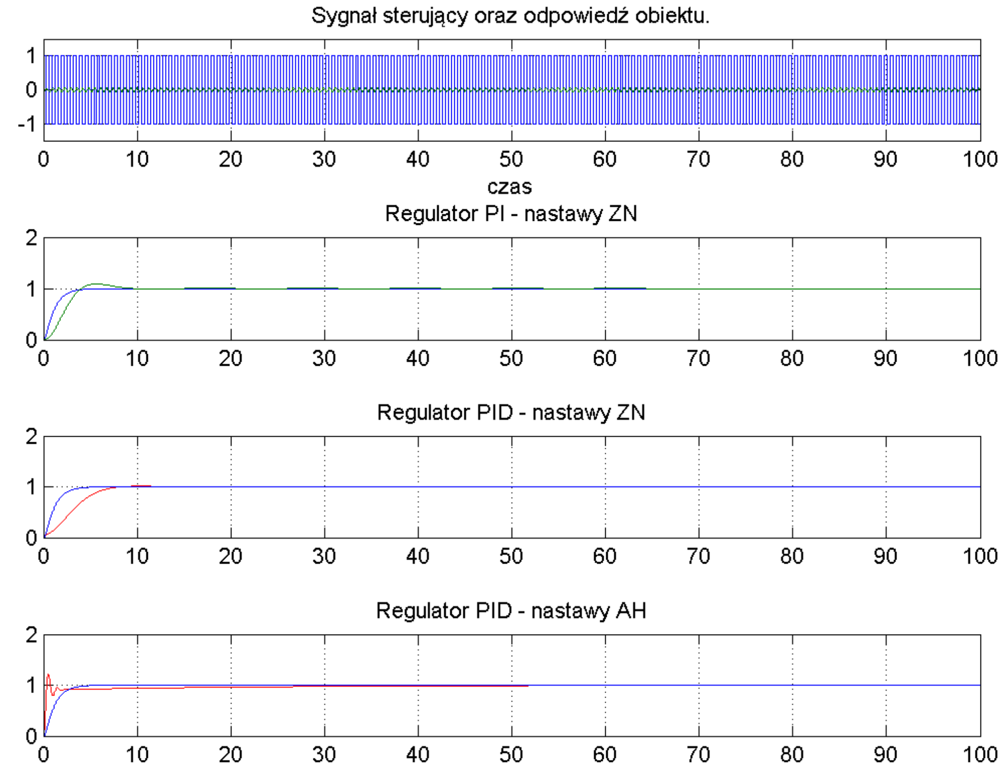


Obiekt G4, n = 8

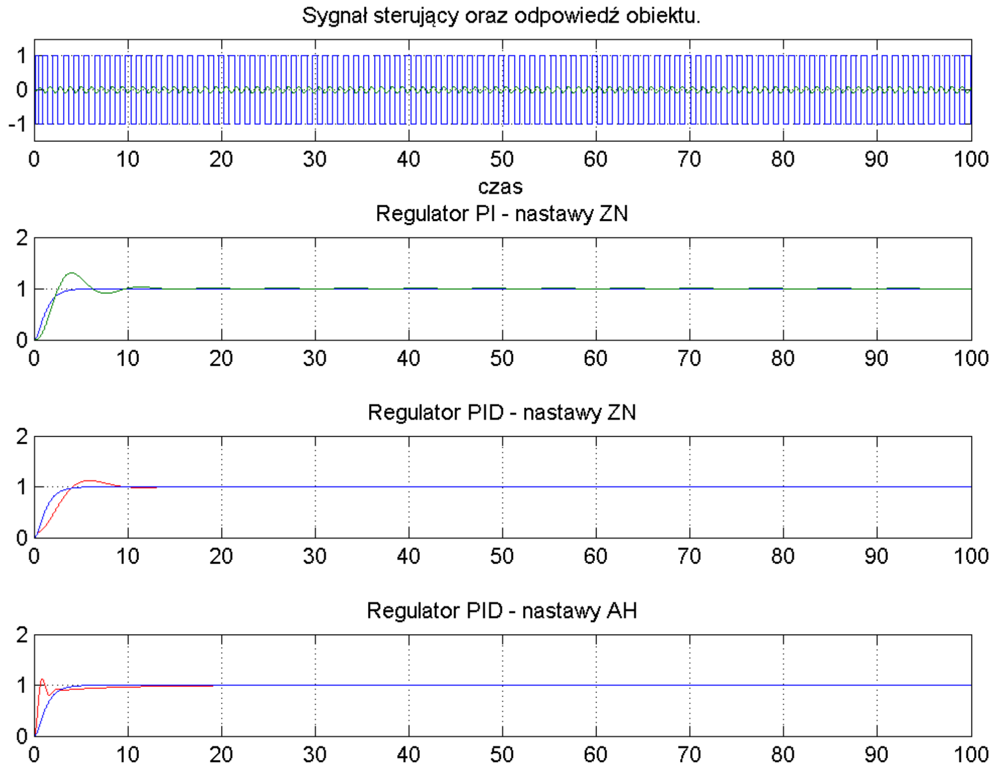
Obiekt 3.5.



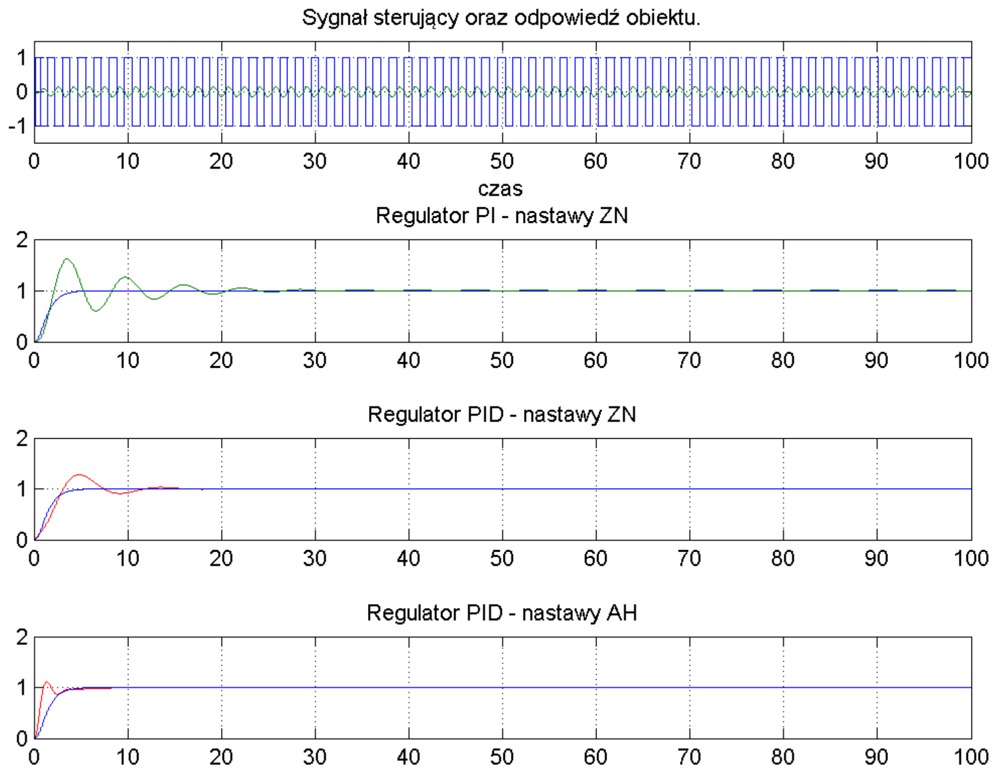
Obiekt G5, alfa = 0.1



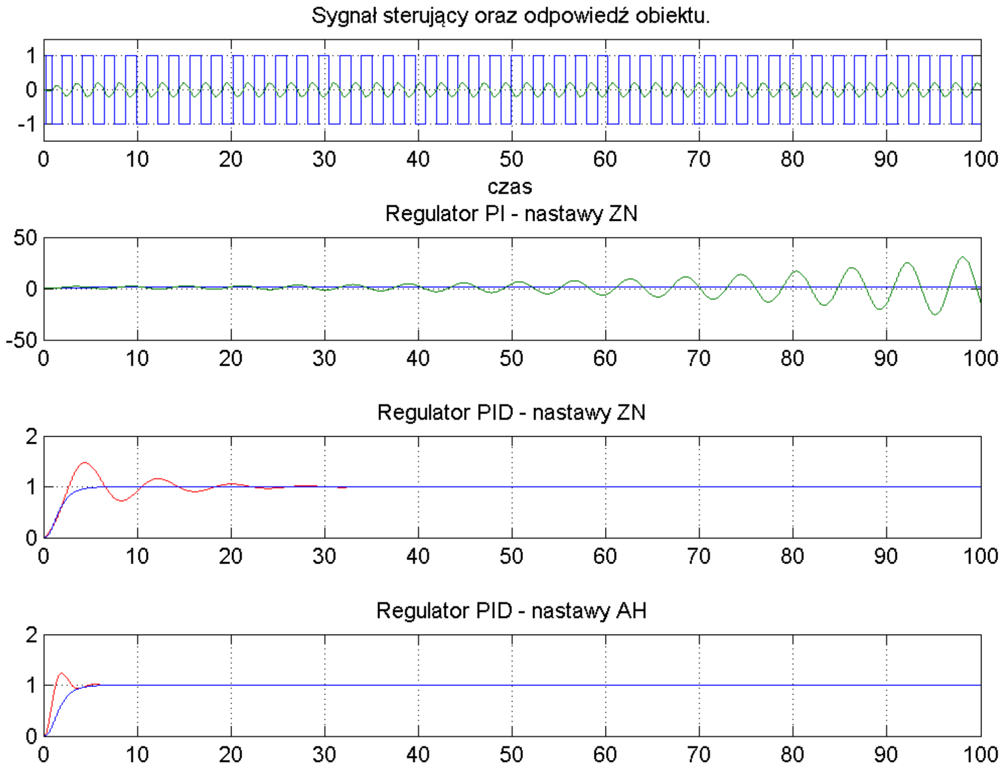
Obiekt G5, alfa = 0.2



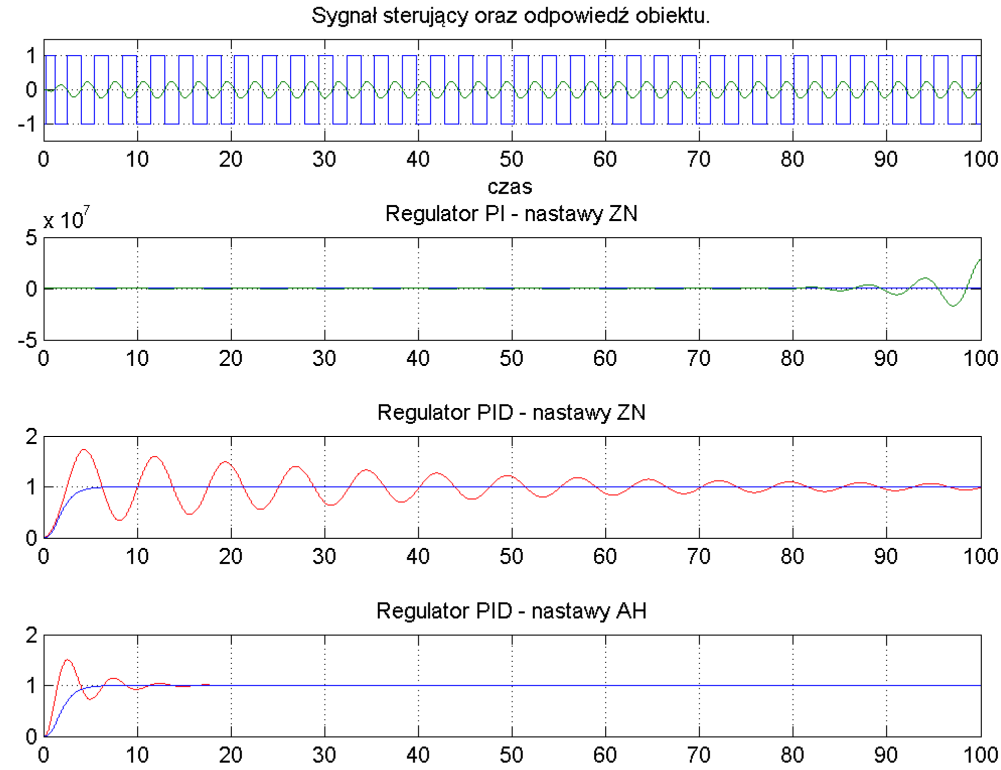
Obiekt G5, alfa = 0.3



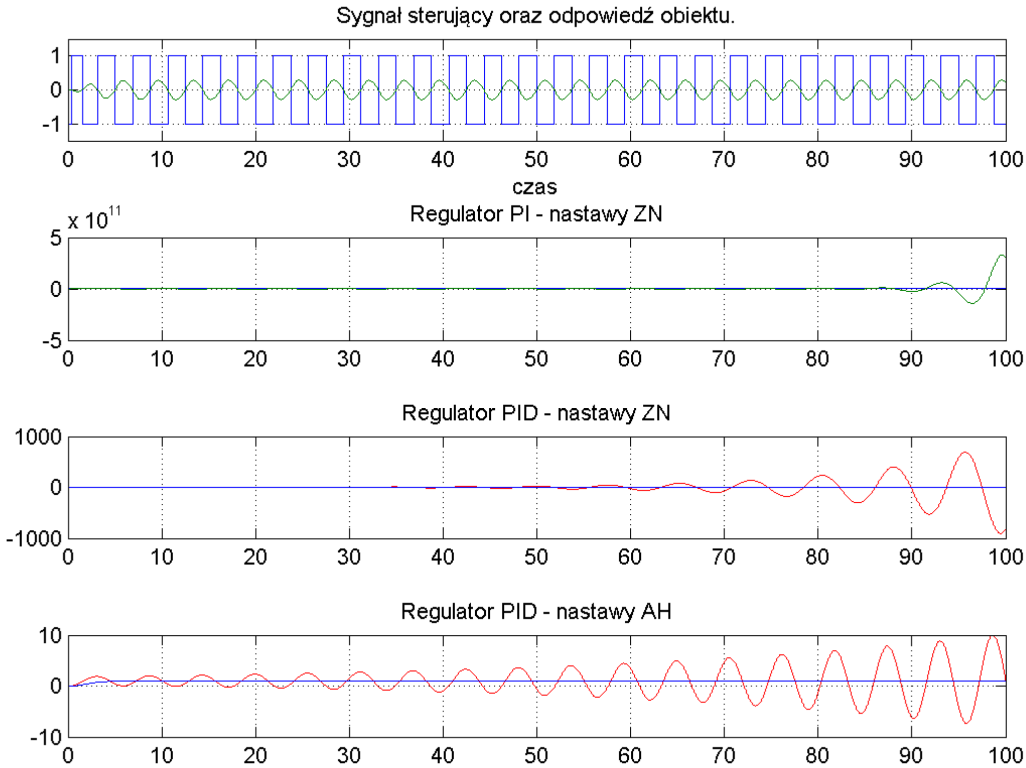
Obiekt G5, alfa = 0.4



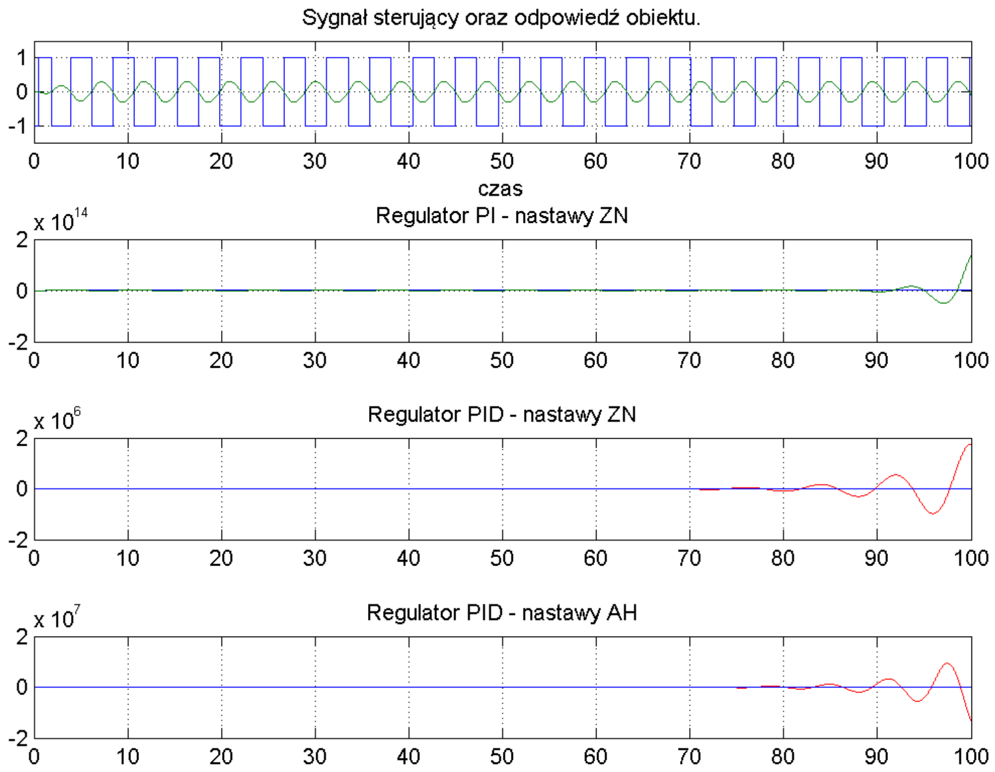
Obiekt G5, alfa = 0.5



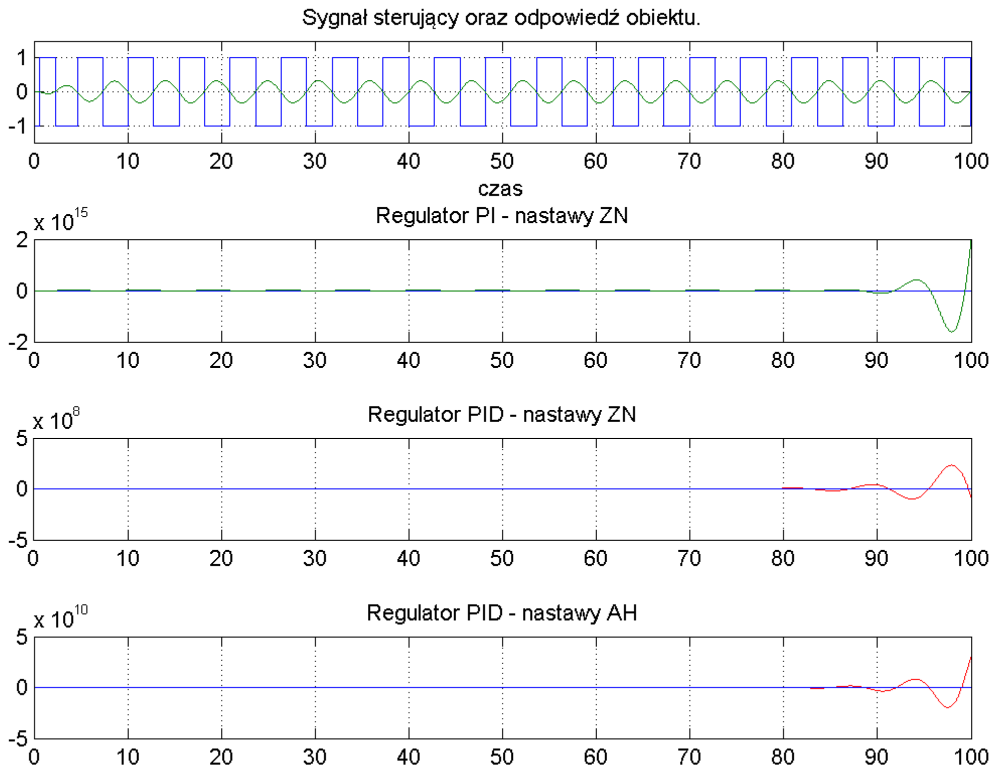
Obiekt G5, alfa = 0.6



Obiekt G5, alfa = 0.7

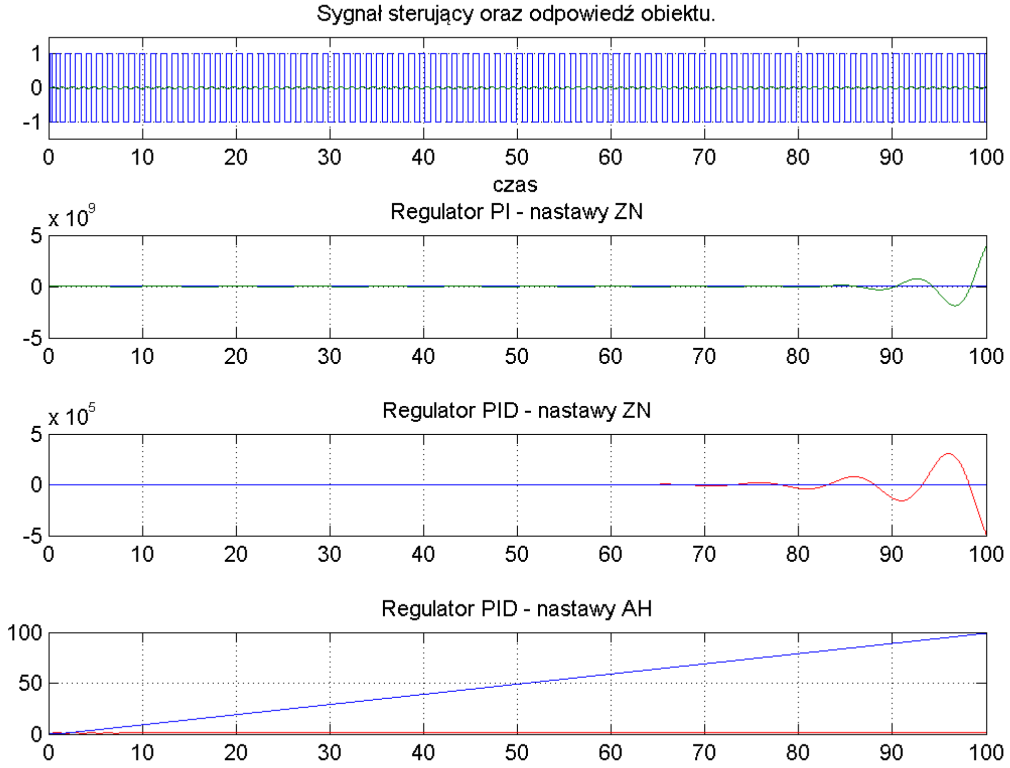


Obiekt G5, alfa = 0.8

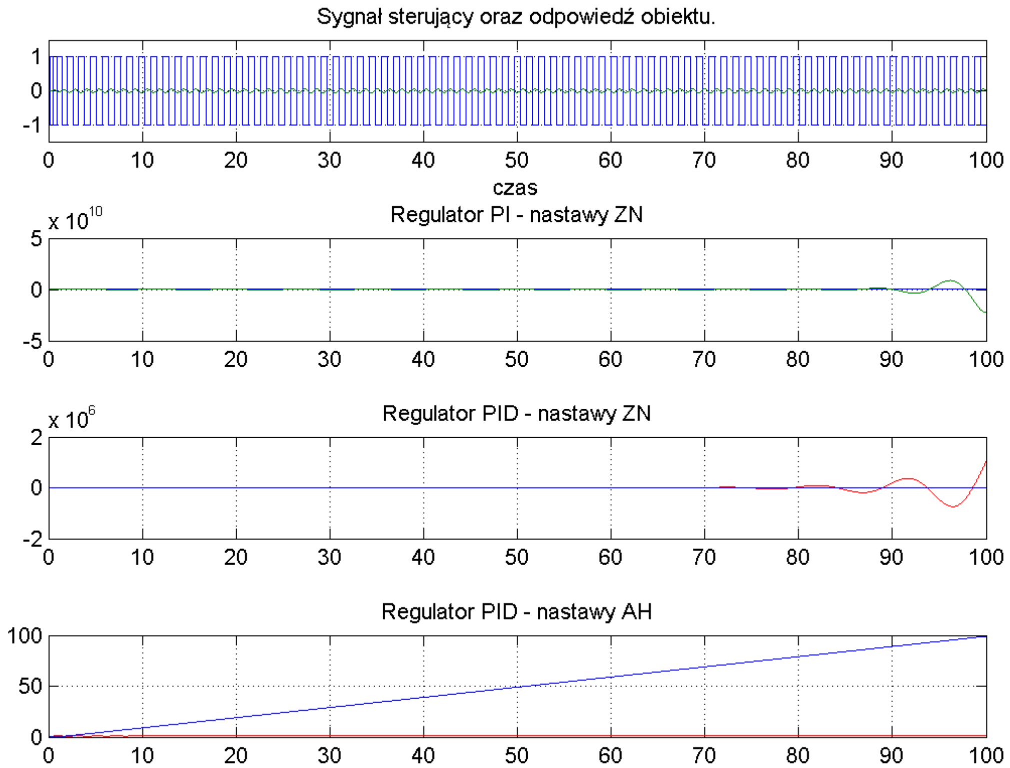


Obiekt G5, alfa = 0.9

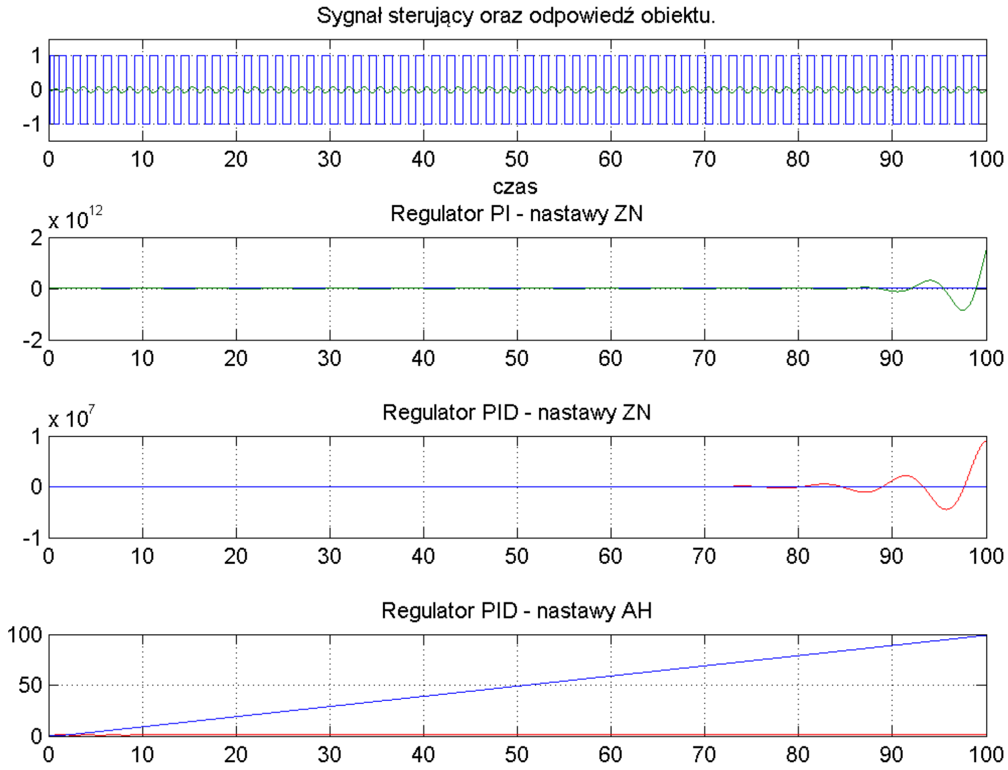
Obiekt 3.6.



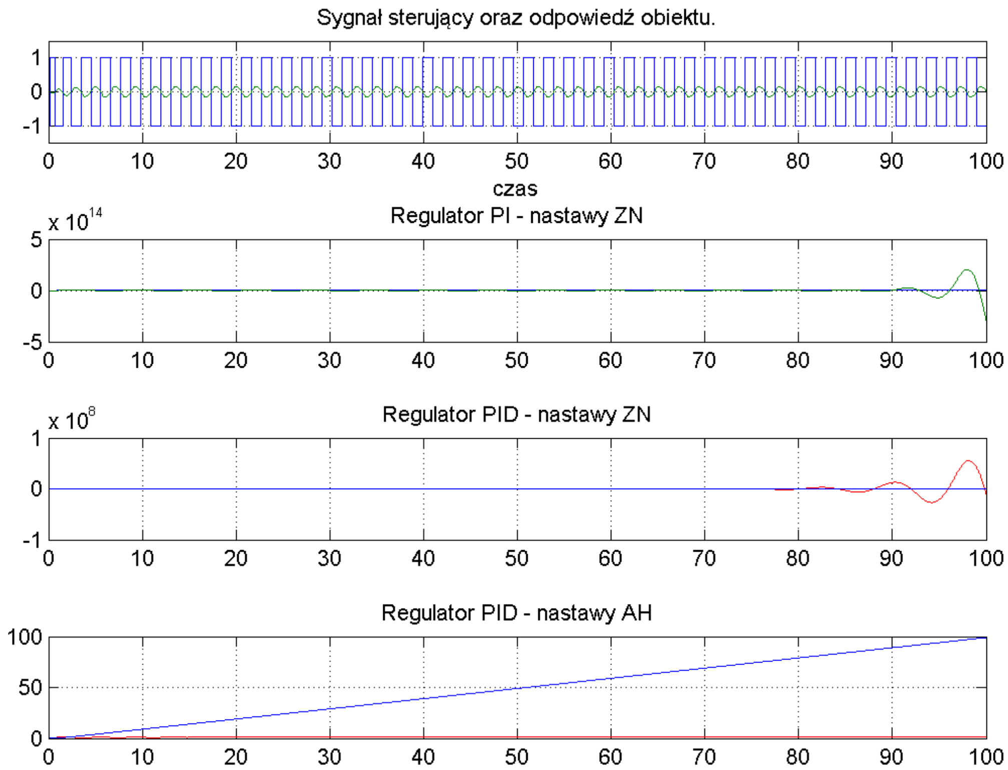
Obiekt G6, L = 0.01, T = 0.99



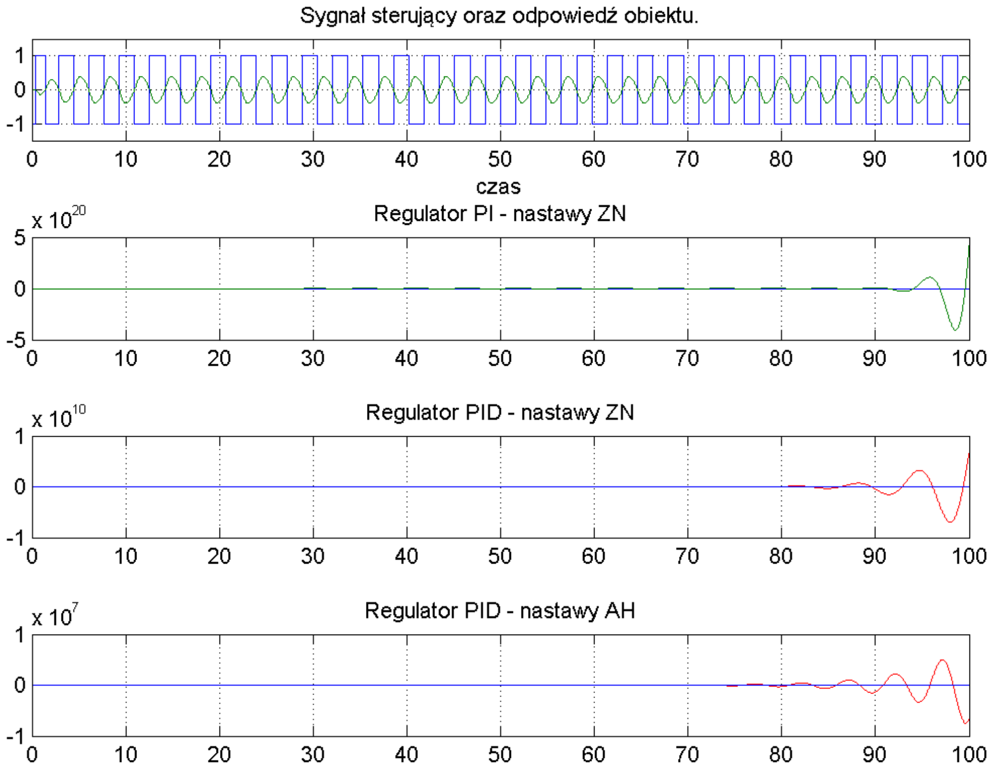
Obiekt G6, L = 0.02, T = 0.98



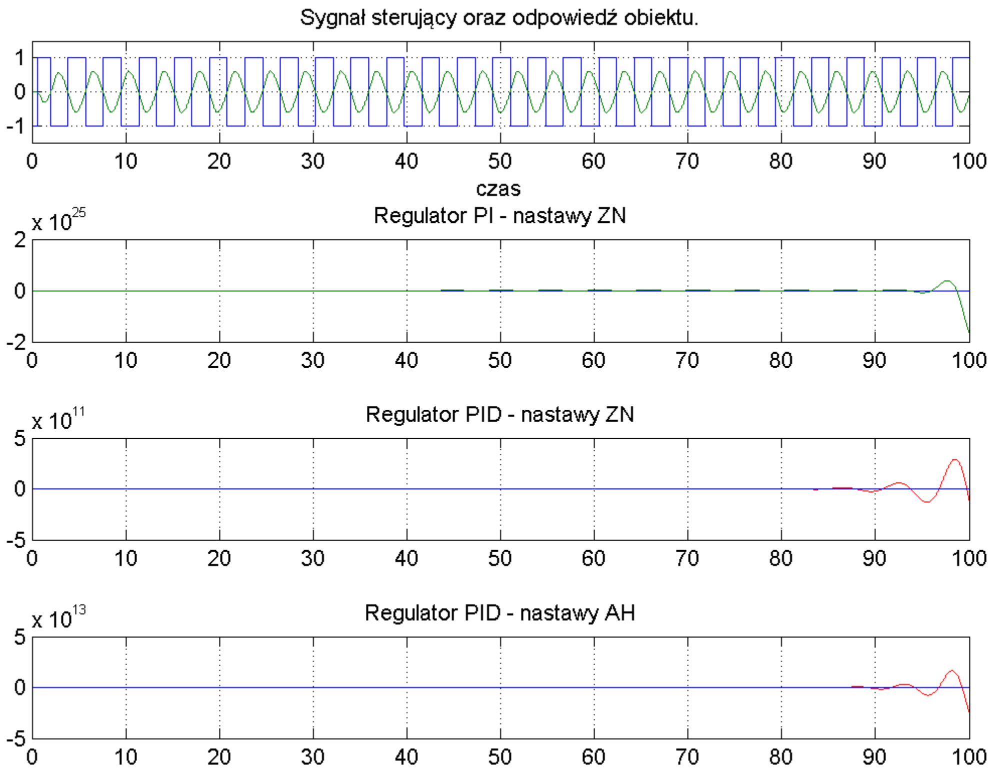
Obiekt G6, L = 0.05, T = 0.95



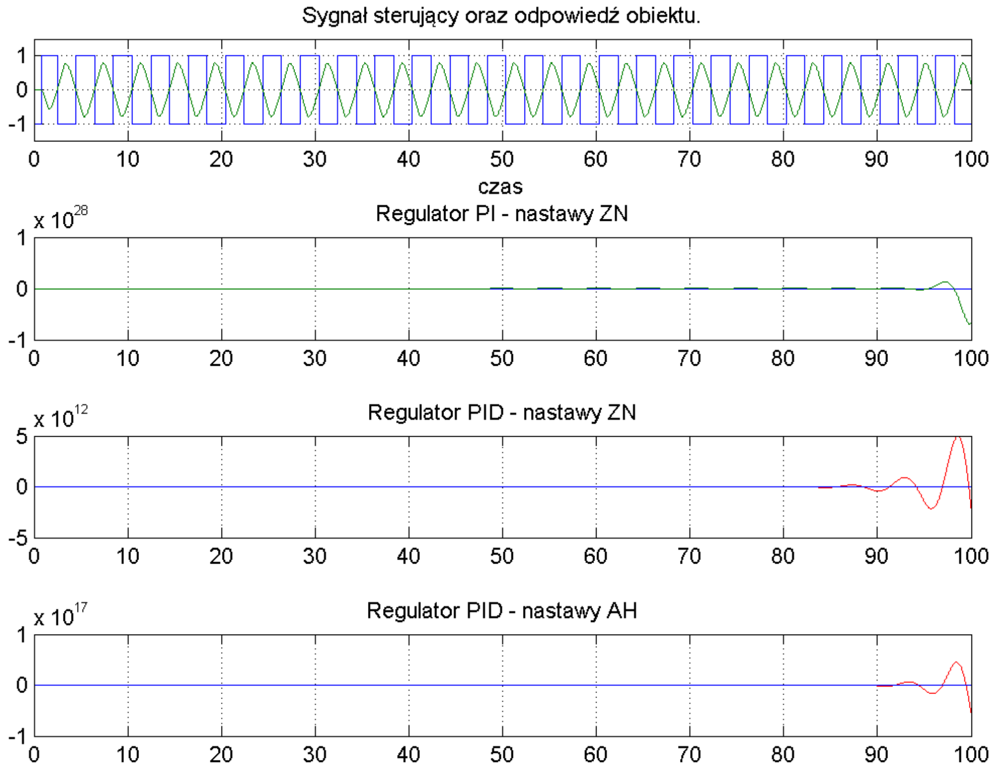
Obiekt G6, L = 0.1, T = 0.9



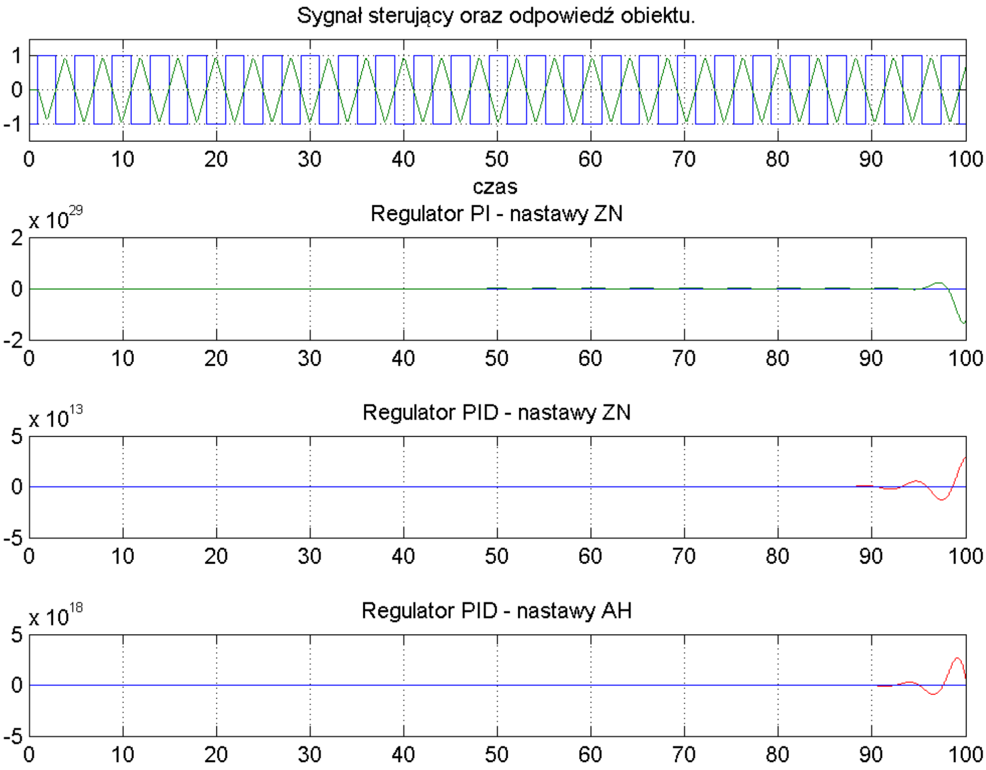
Obiekt G6, L = 0.2, T = 0.8



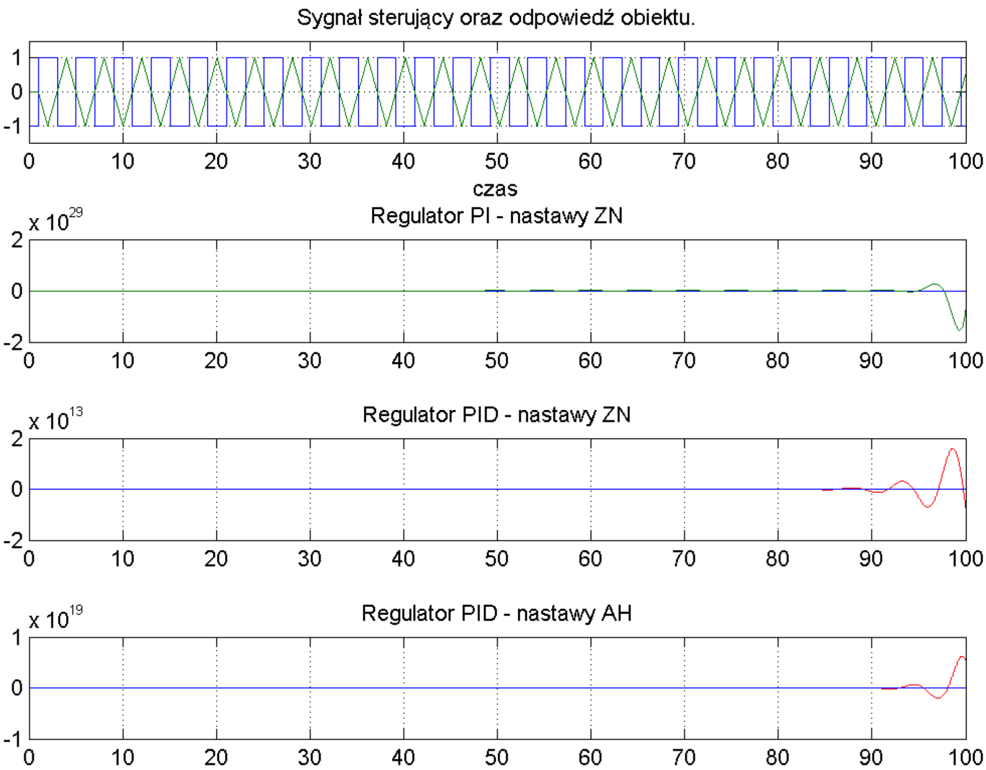
Obiekt G6, L = 0.3, T = 0.7



Obiekt G6, L = 0.5, T = 0.5

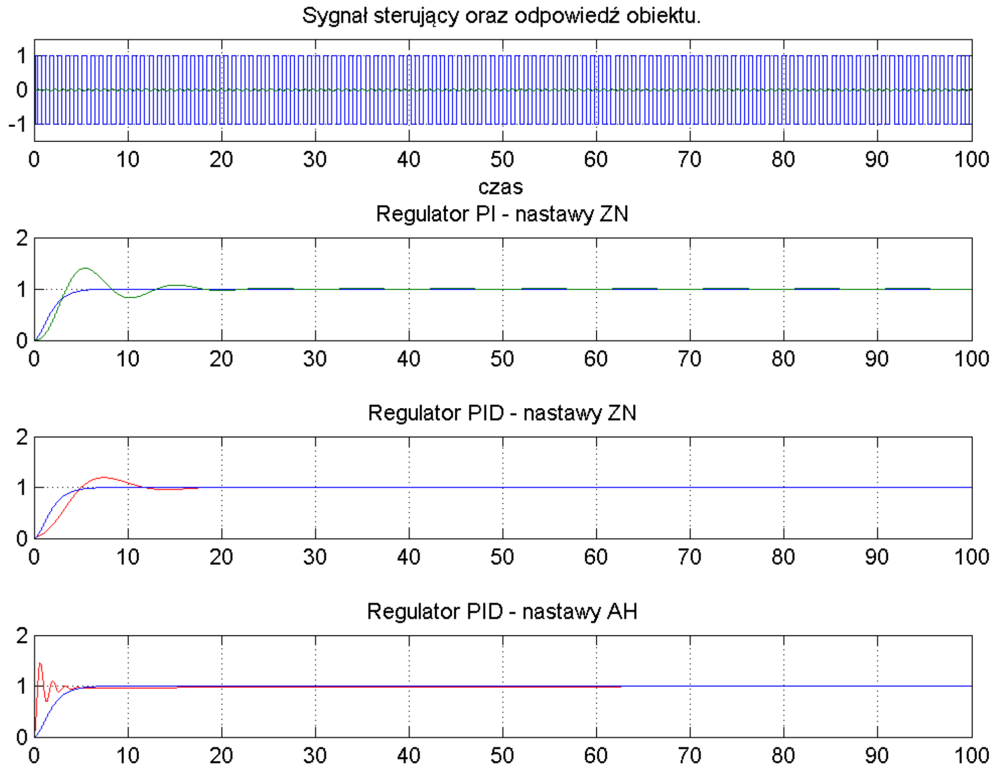


Obiekt G6, L = 0.7, T = 0.3

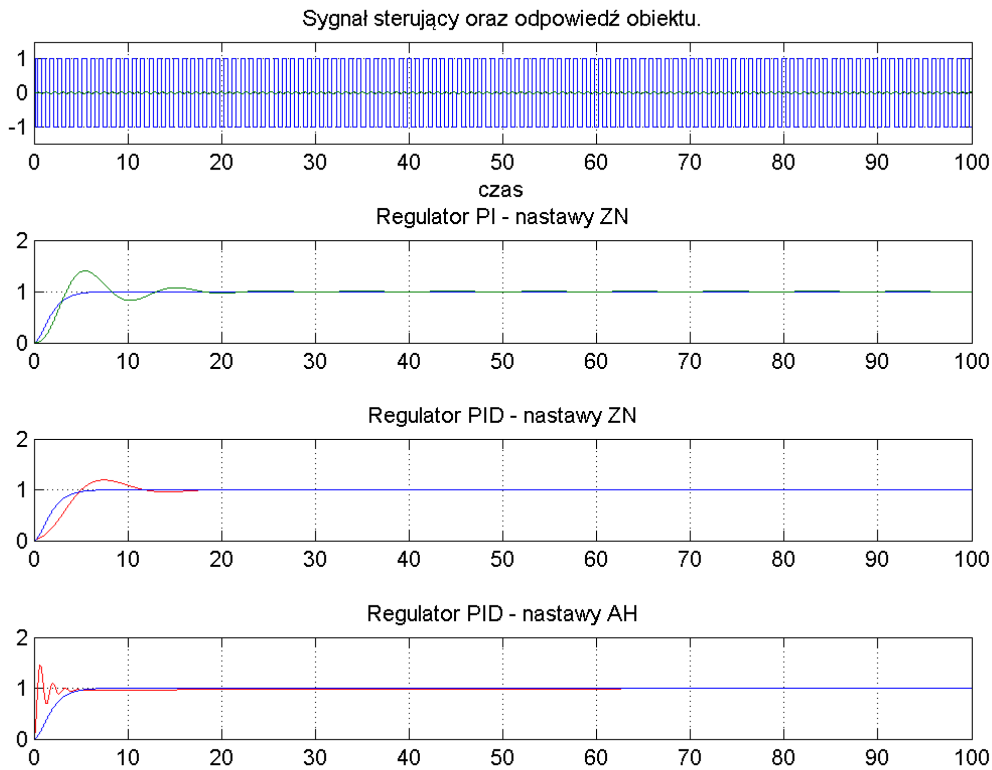


Obiekt G6, L = 1, T = 0

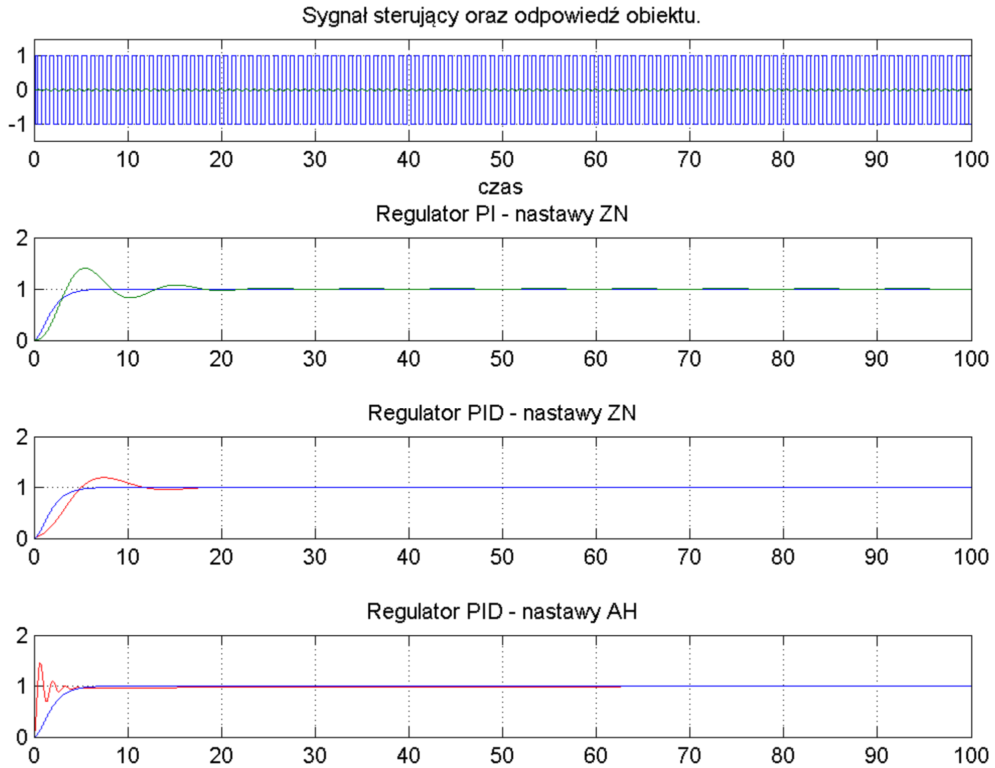
Obiekt 3.7.



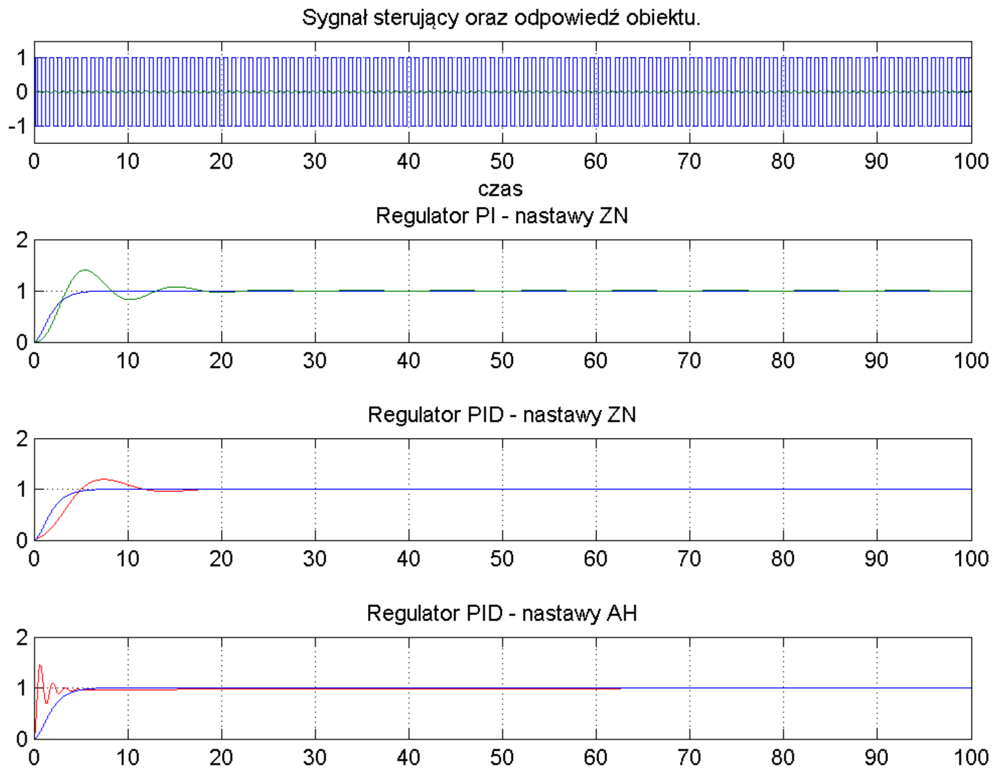
Obiekt G7, T = 1, L = 0.01, T1 = 0.99



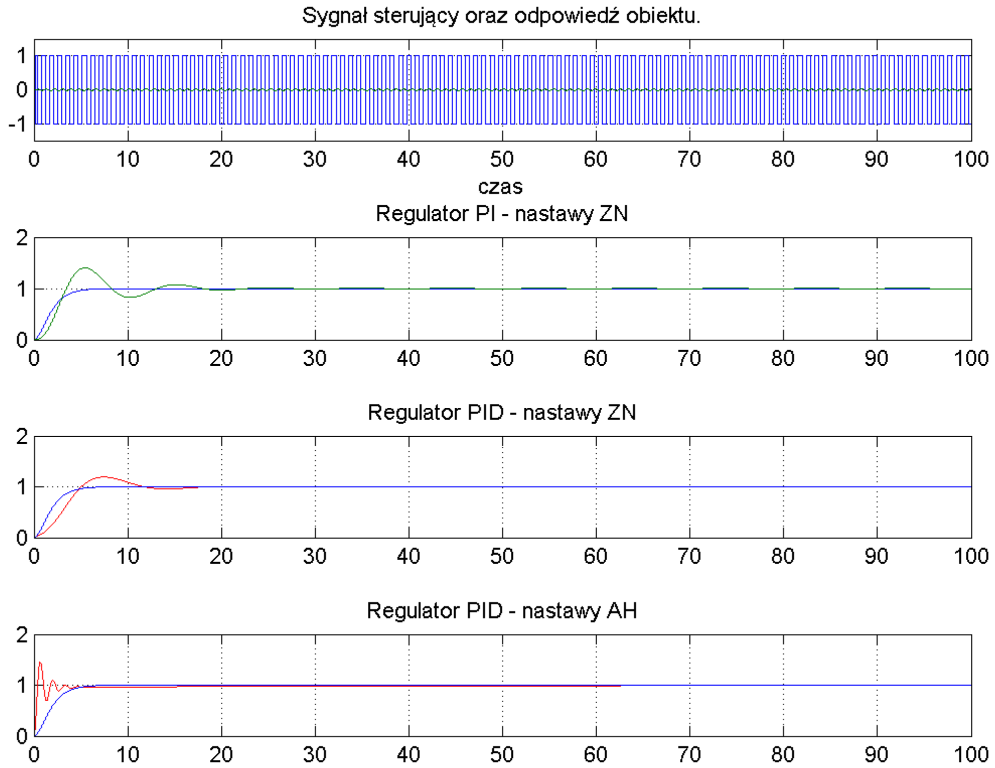
Obiekt G7, T = 1, L = 0.02, T1 = 0.98



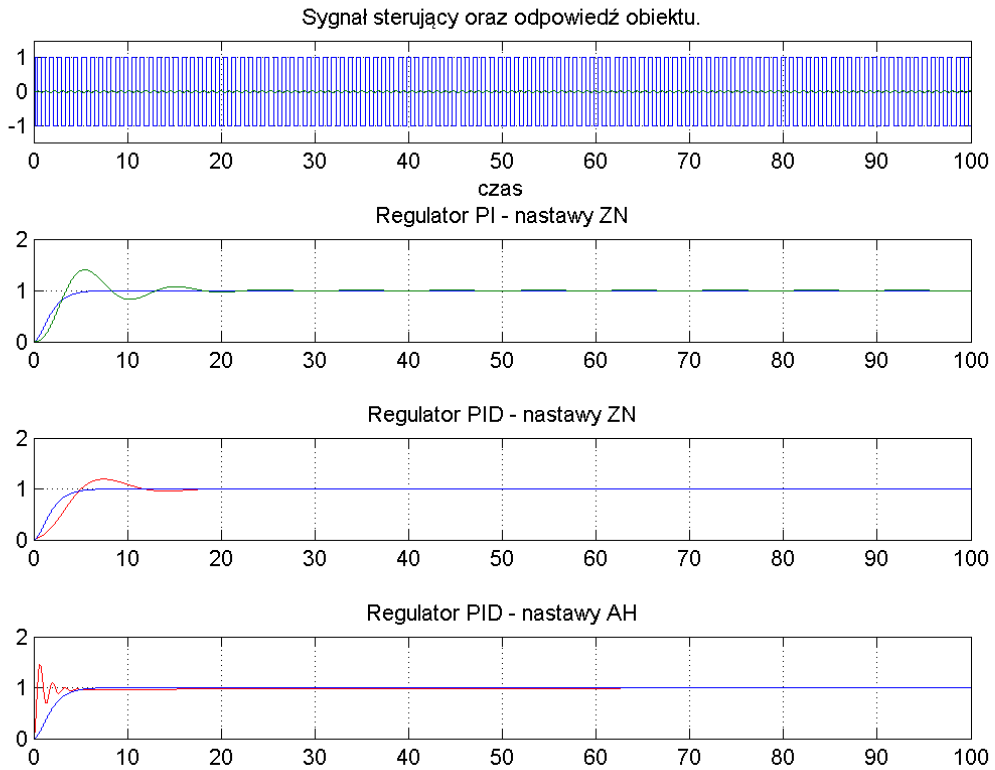
Obiekt G7, T = 1, L = 0.05, T1 = 0.95



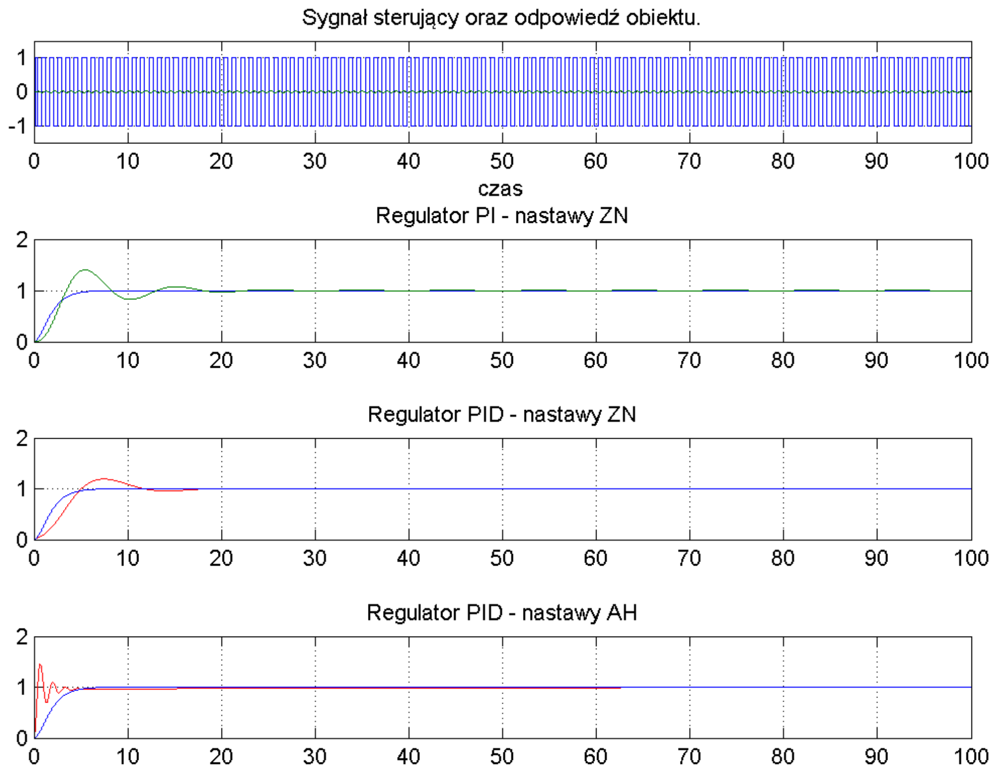
Obiekt G7, T = 1, L = 0.1, T1 = 0.9



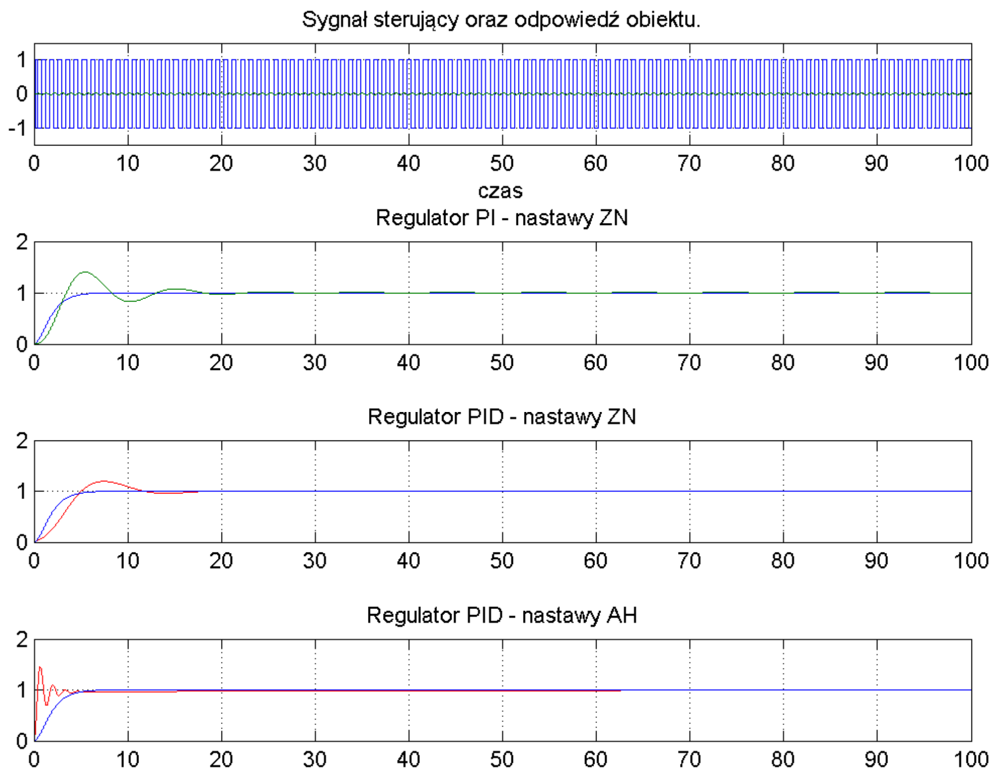
Obiekt G7, T = 1, L = 0.2, T1 = 0.8



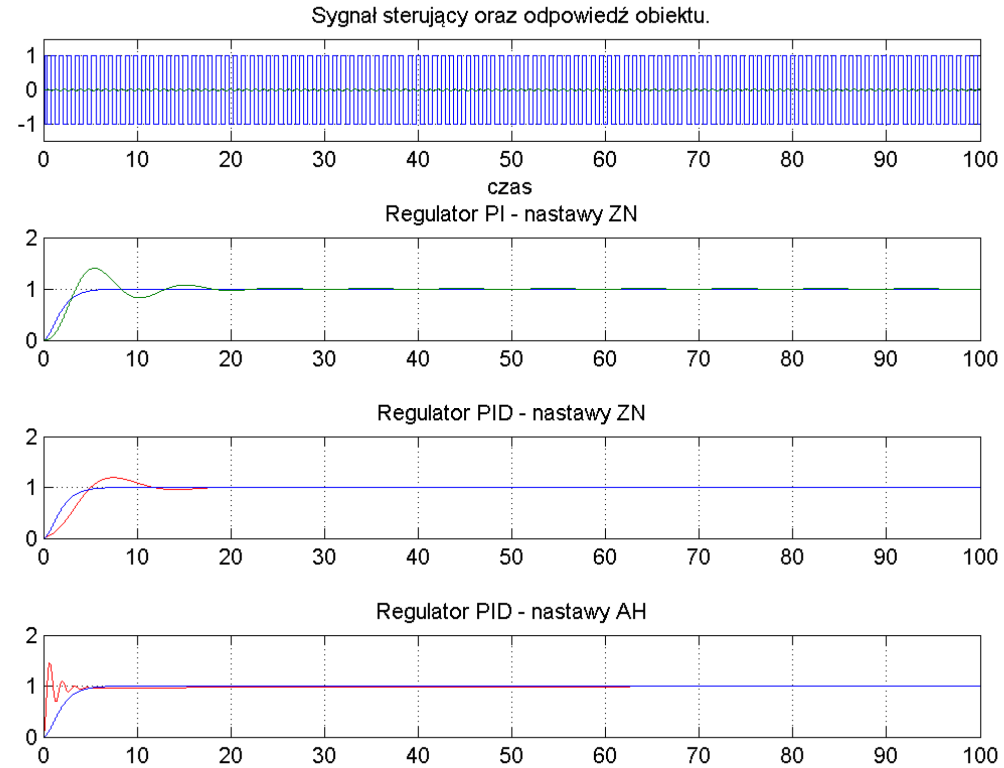
Obiekt G7, T = 1, L = 0.3, T1 = 0.7



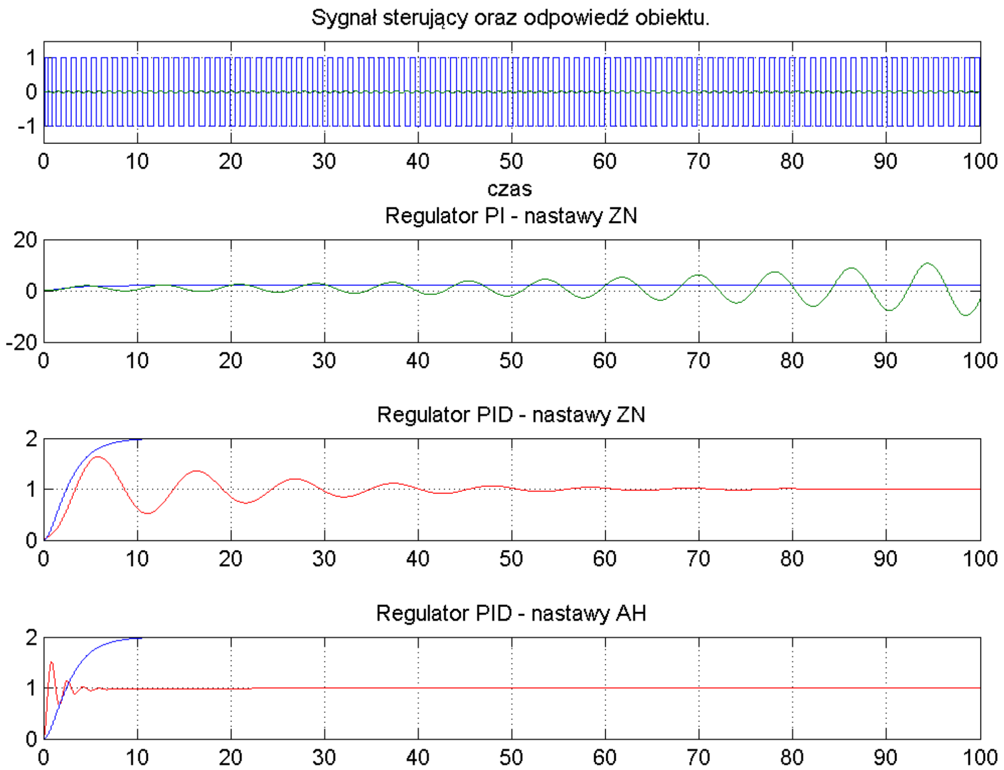
Obiekt G7, T = 1, L = 0.5, T1 = 0.5



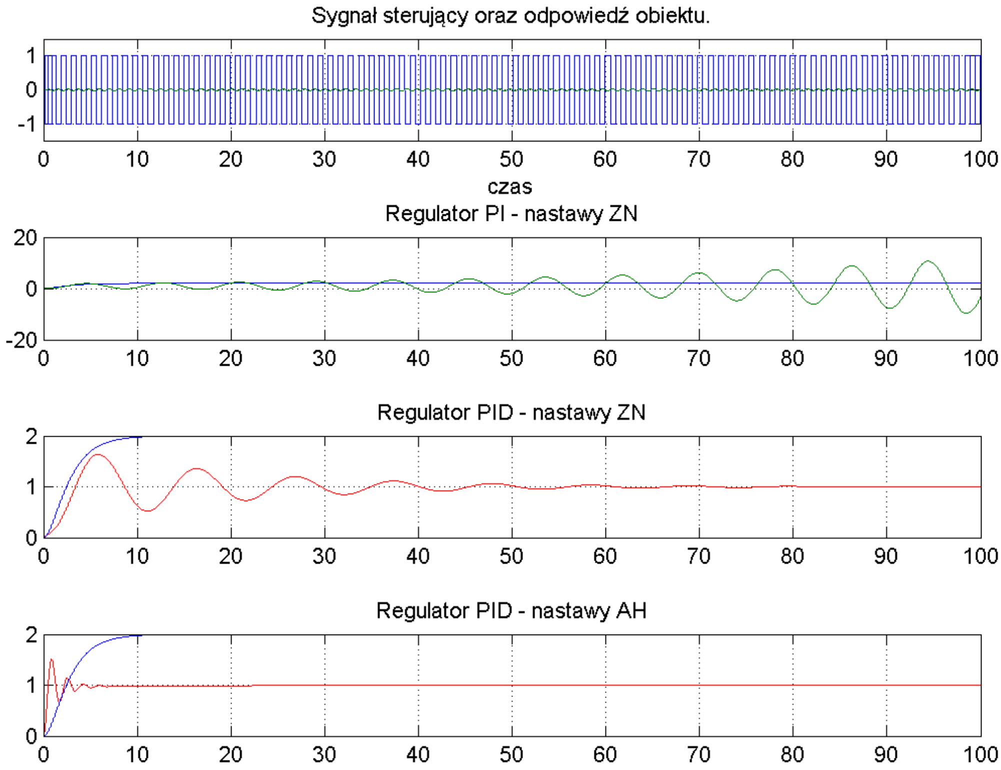
Obiekt G7, T = 1, L = 0.7, T1 = 0.3



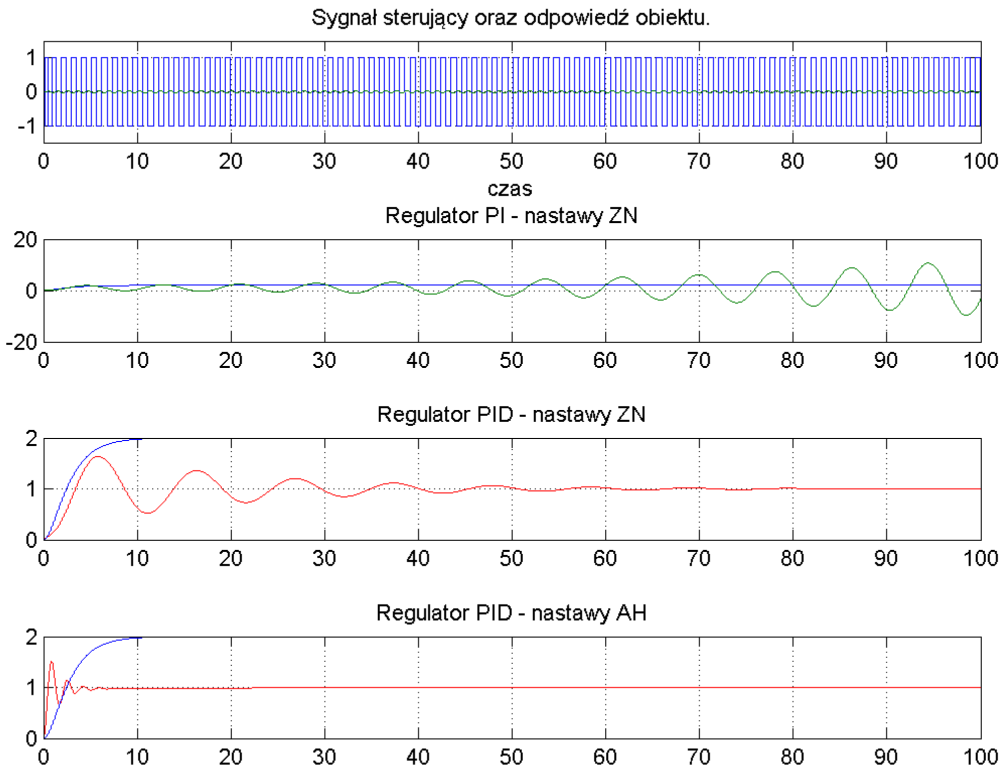
Obiekt G7, T = 1, L = 1, T1 = 0



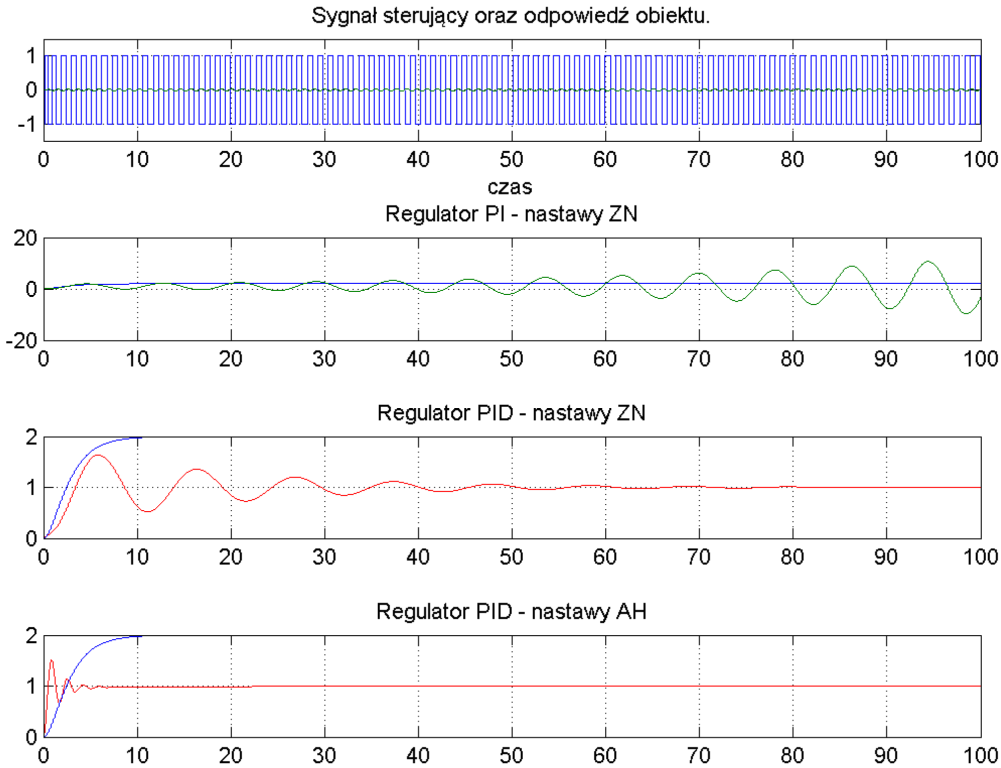
Obiekt G7, T = 2, L = 0.01, T1 = 0.99



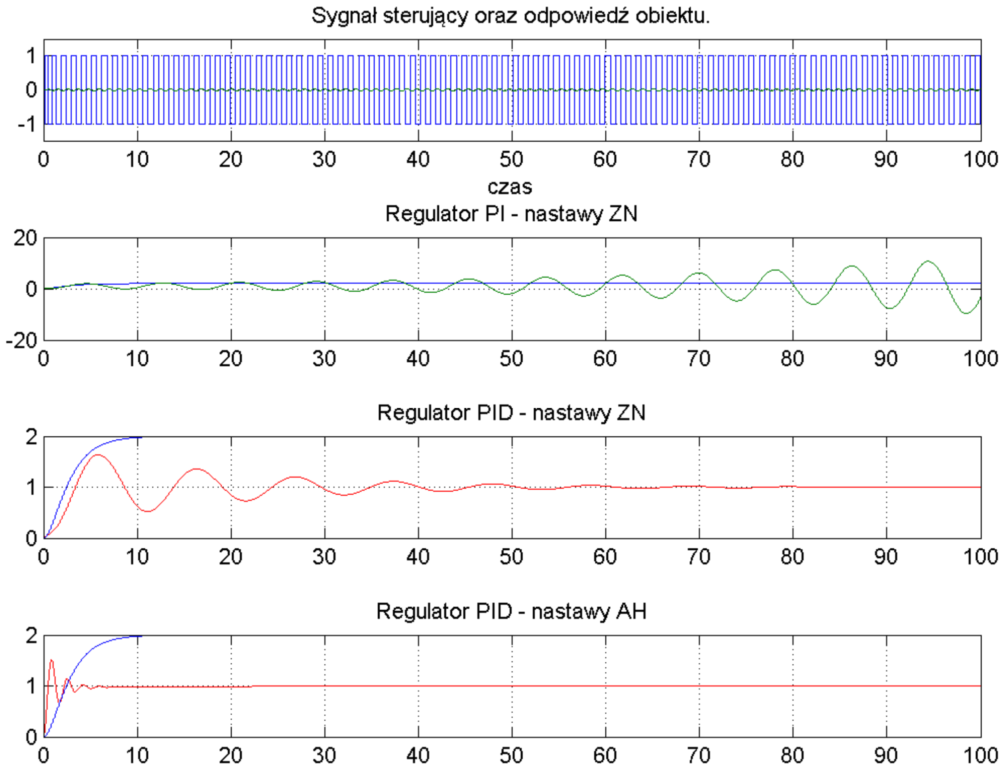
Obiekt G7, T = 2, L = 0.02, T1 = 0.98



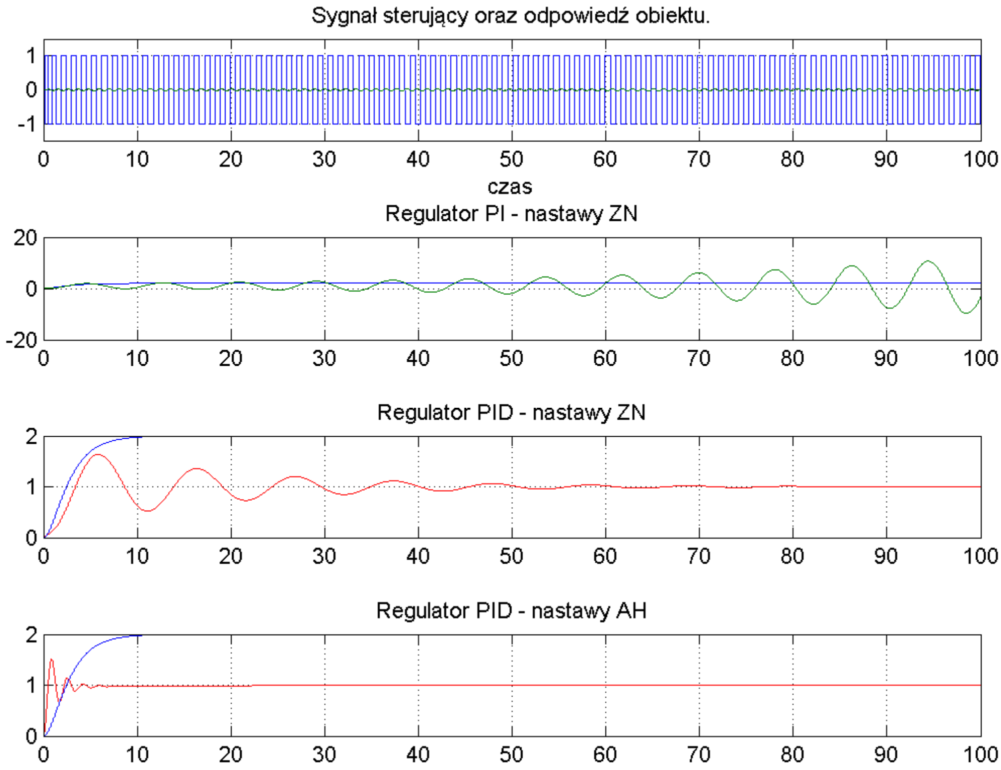
Obiekt G7, T = 2, L = 0.05, T1 = 0.95



Obiekt G7, T = 2, L = 0.1, T1 = 0.9



Obiekt G7, T = 2, L = 0.2, T1 = 0.8



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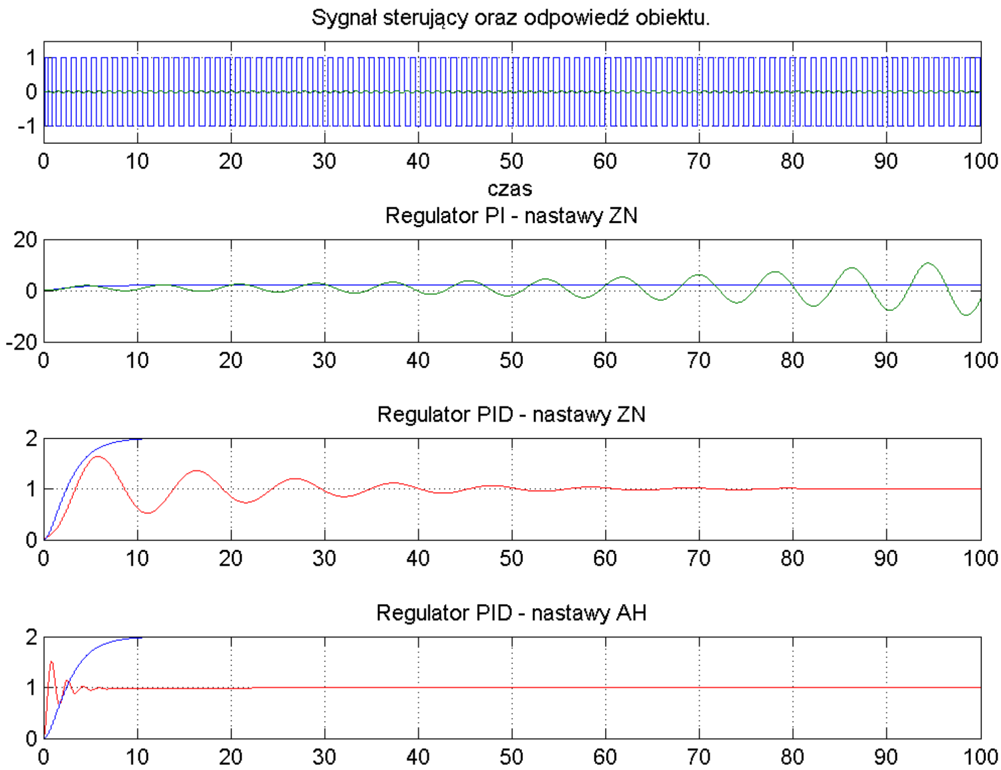
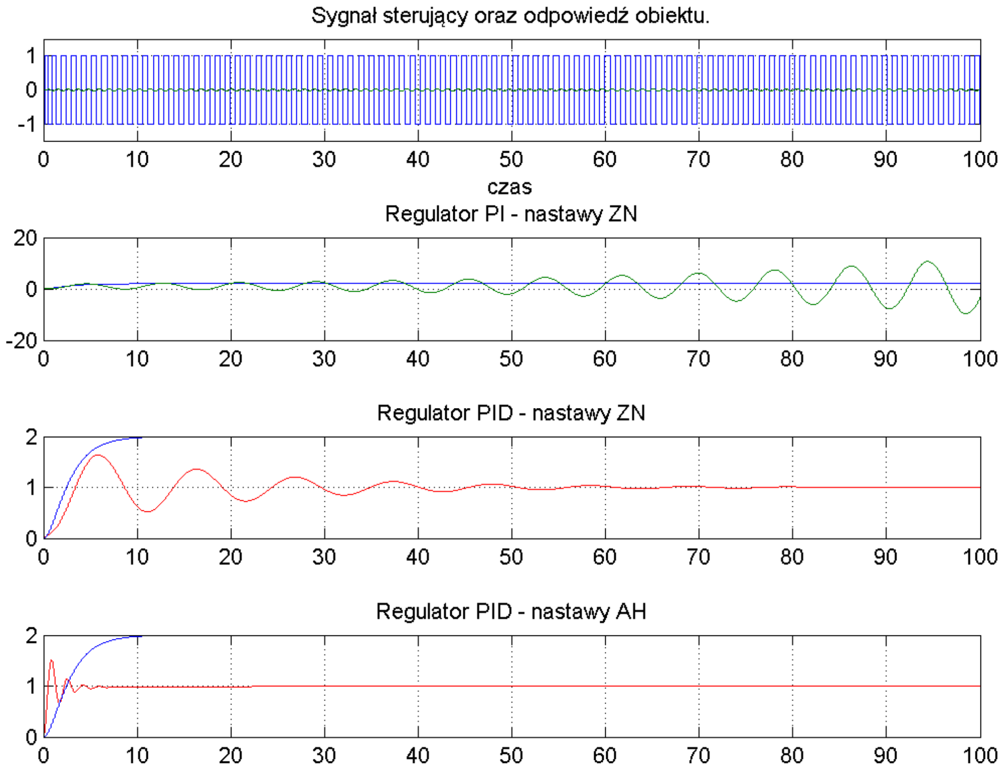
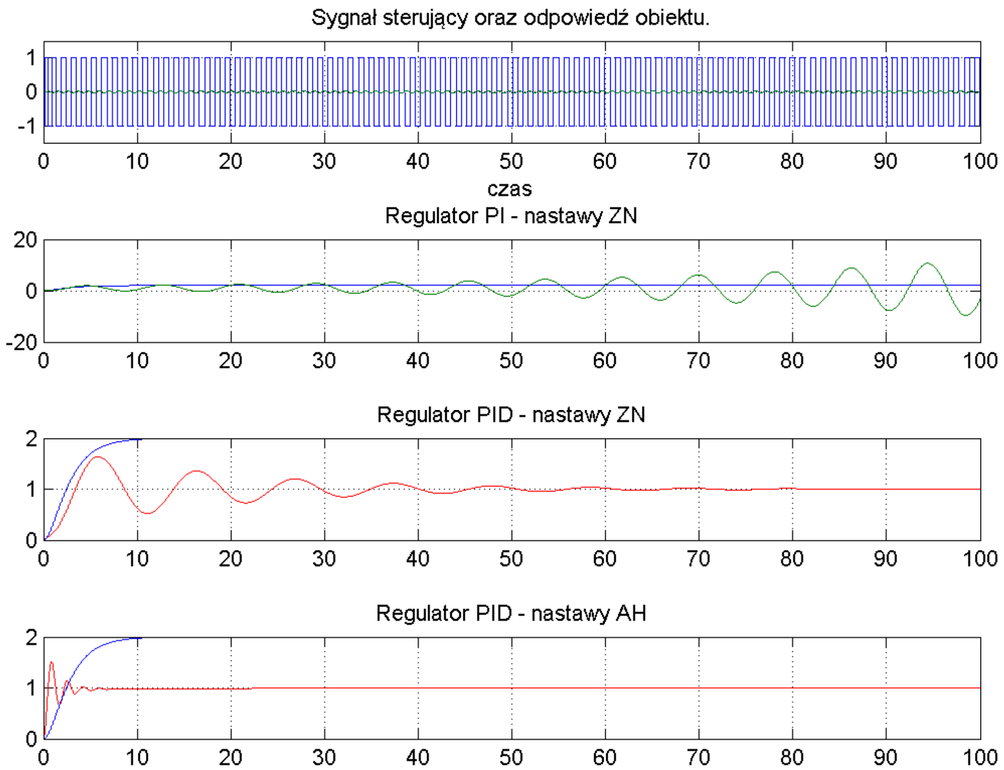


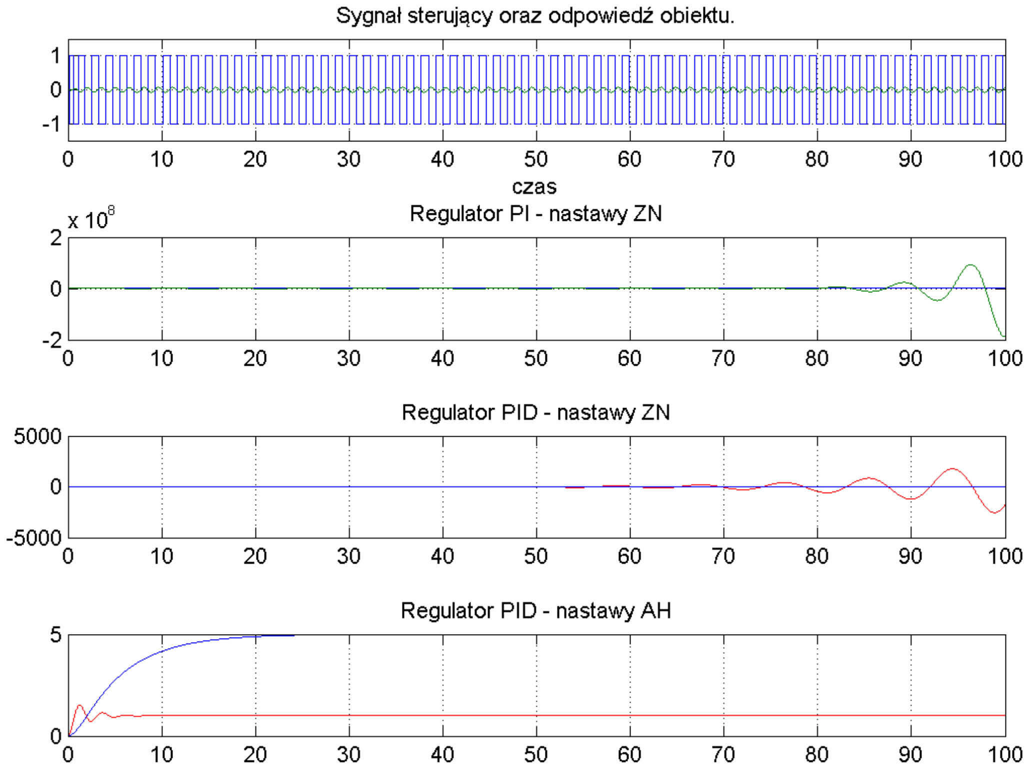
Obiekt G7, T = 2, L = 0.5, T1 = 0.5



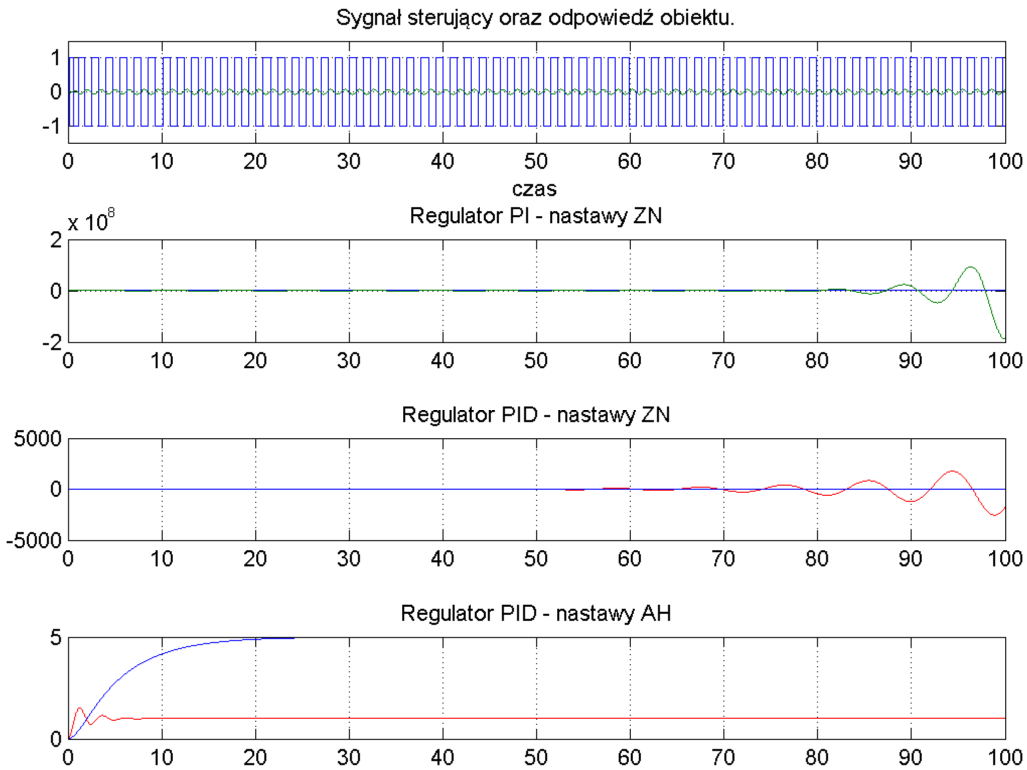
Obiekt G7, T = 2, L = 0.7, T1 = 0.3



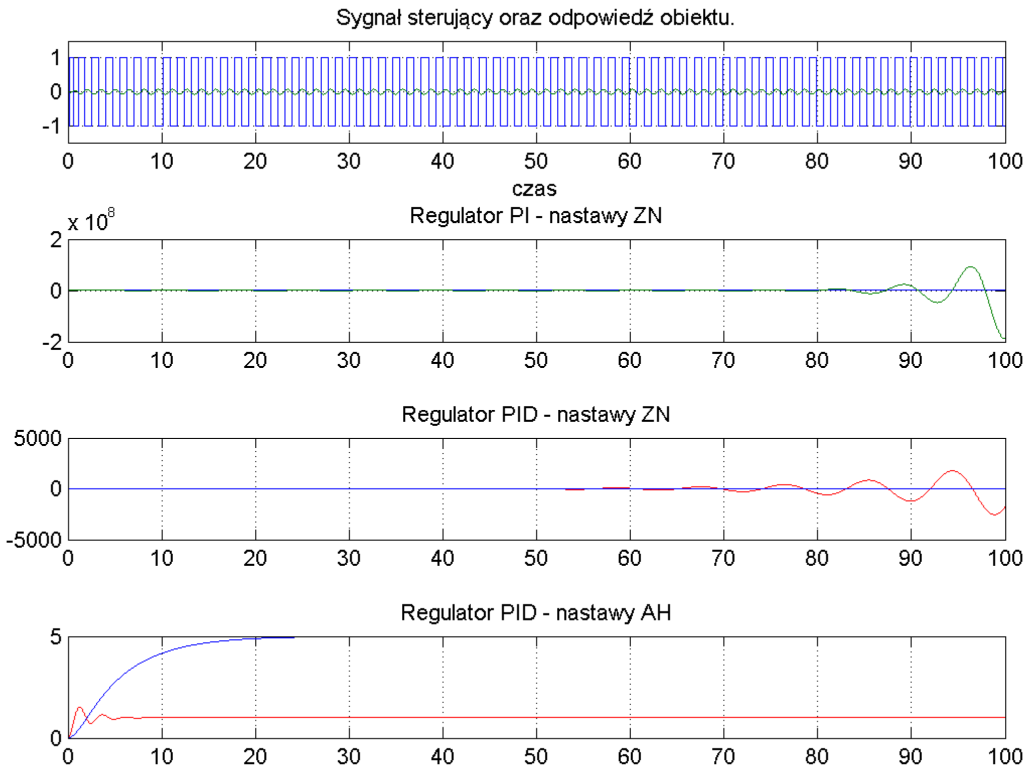
Obiekt G7, T = 2, L = 1, T1 = 0



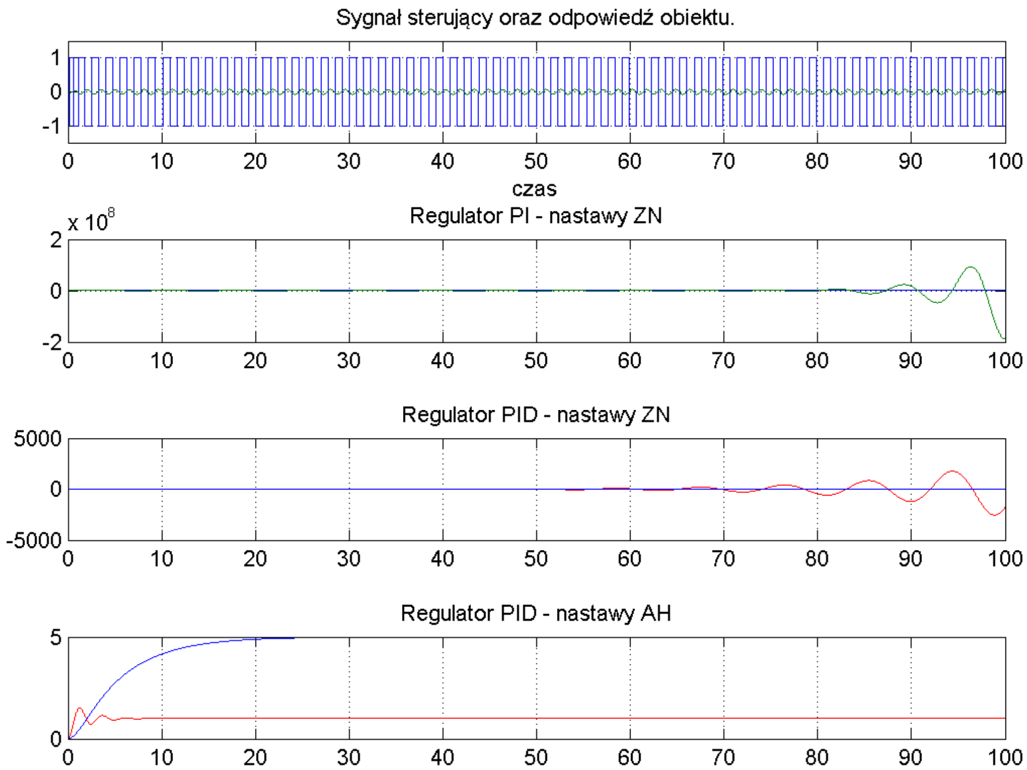
Obiekt G7, T = 5, L = 0.01, T1 = 0.99



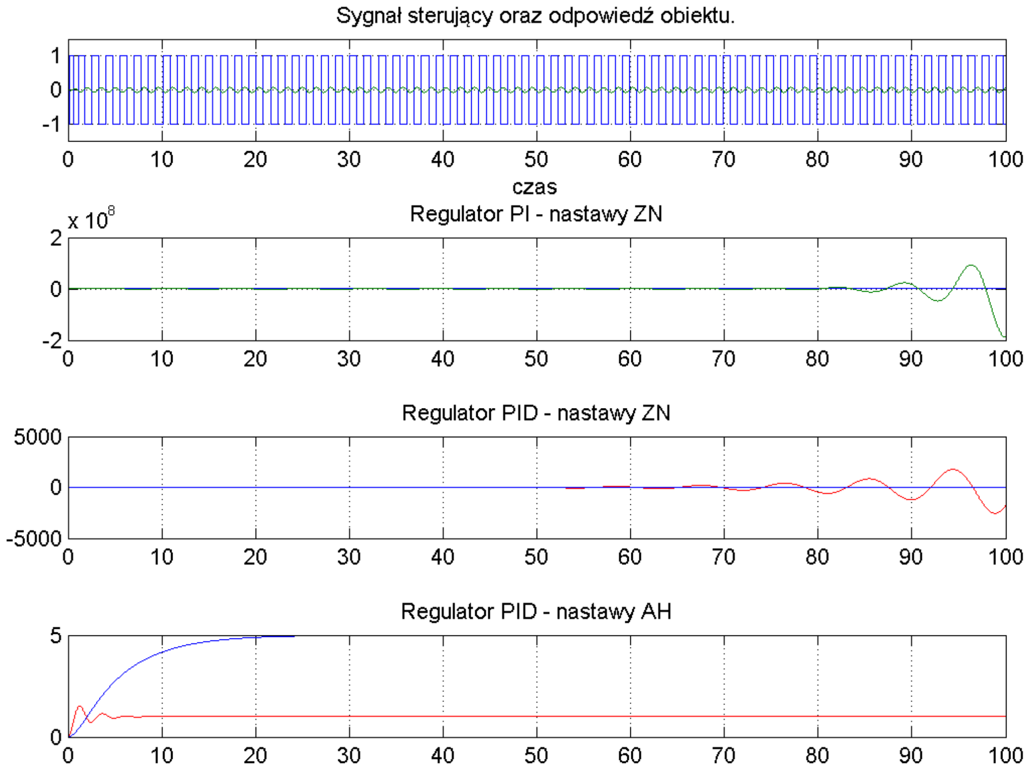
Obiekt G7, T = 5, L = 0.02, T1 = 0.98



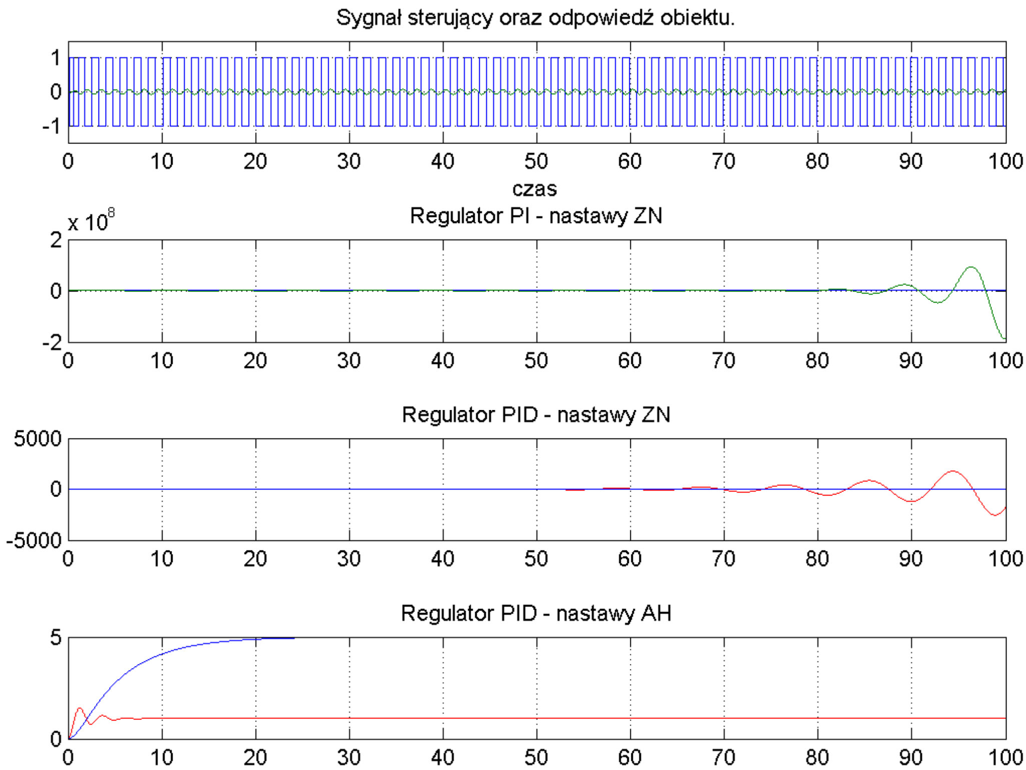
Obiekt G7, T = 5, L = 0.05, T1 = 0.95



Obiekt G7, T = 5, L = 0.1, T1 = 0.9



Obiekt G7, T = 5, L = 0.2, T1 = 0.8



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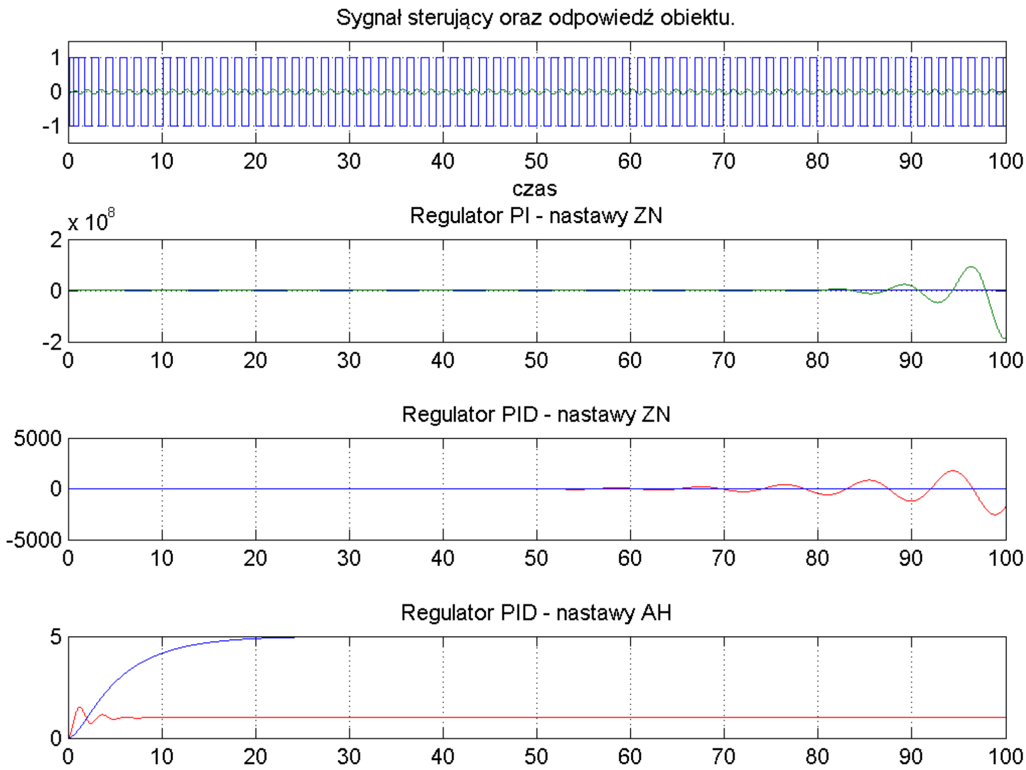

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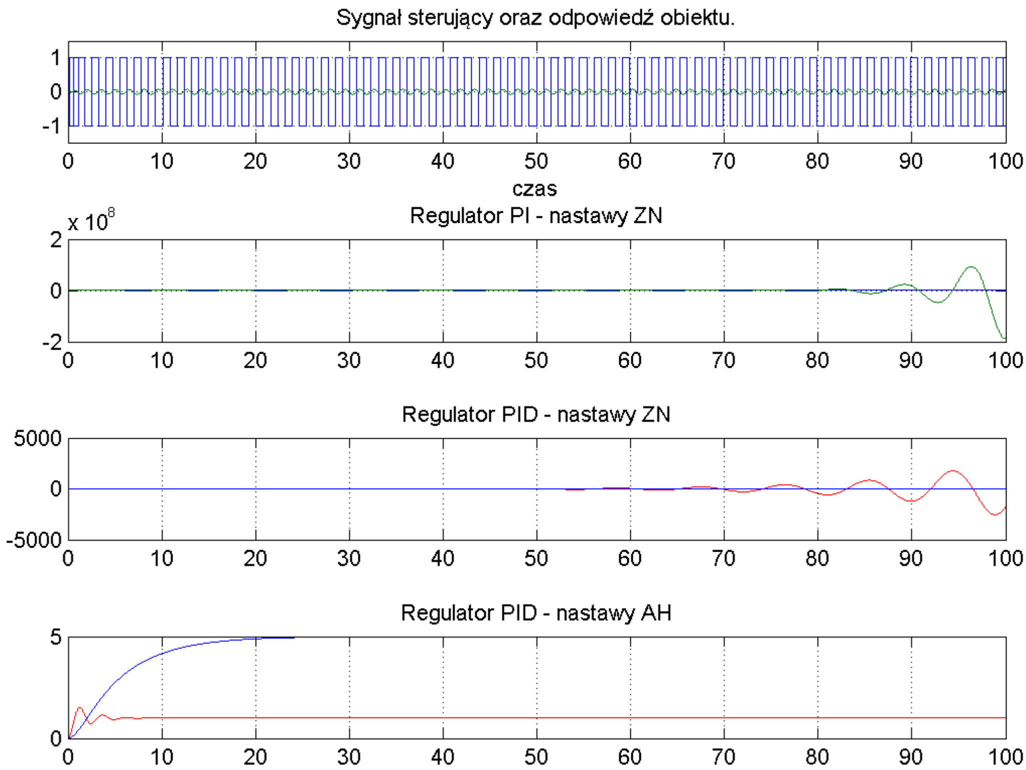
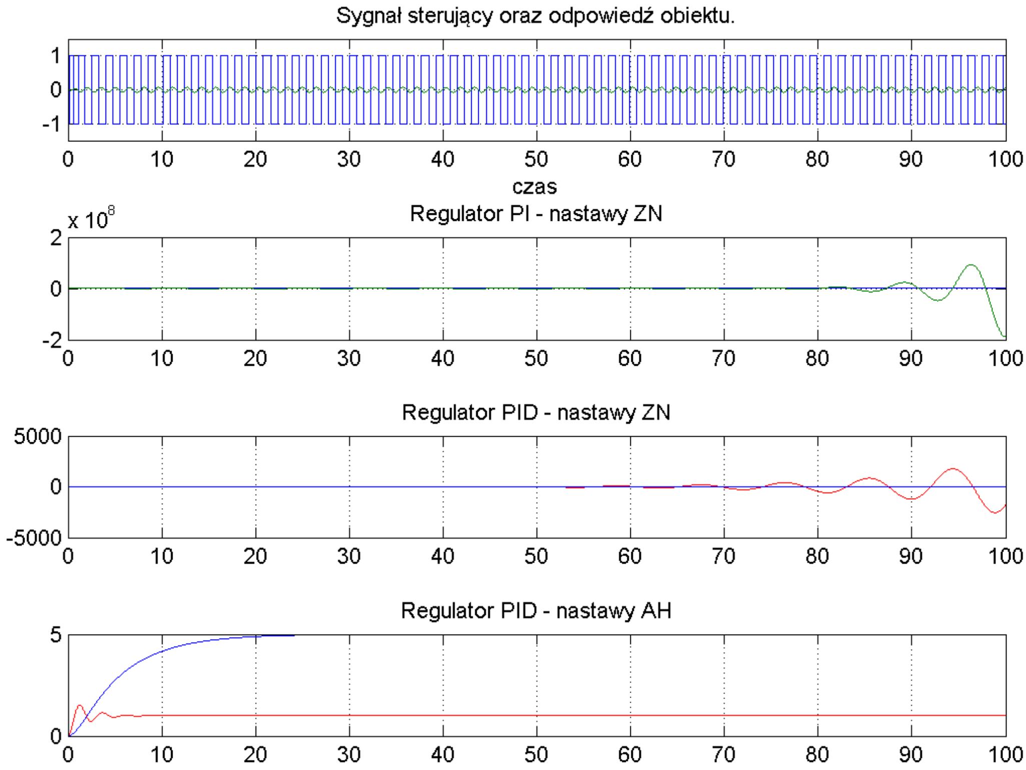
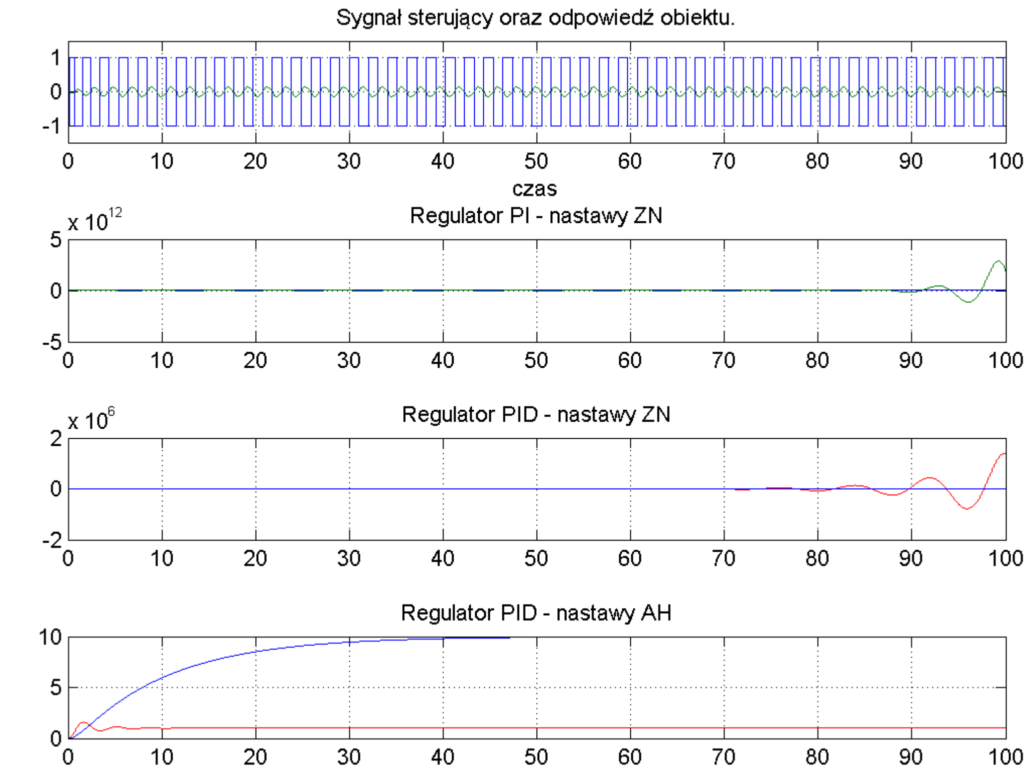


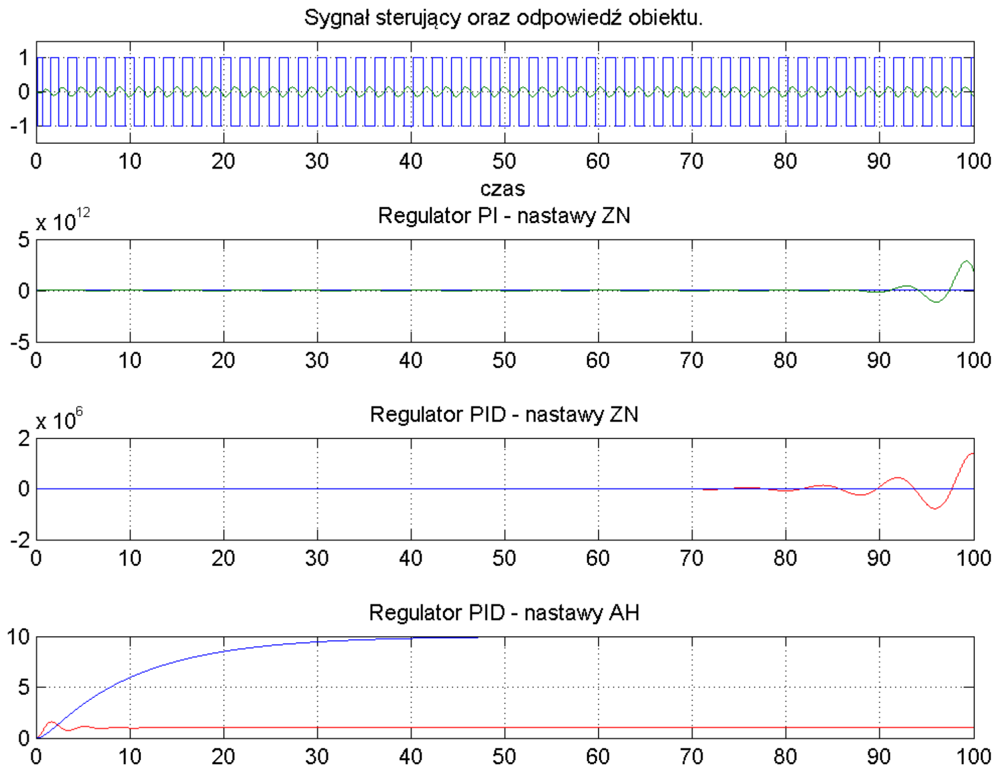
Obiekt G7, T = 5, L = 0.7, T1 = 0.3



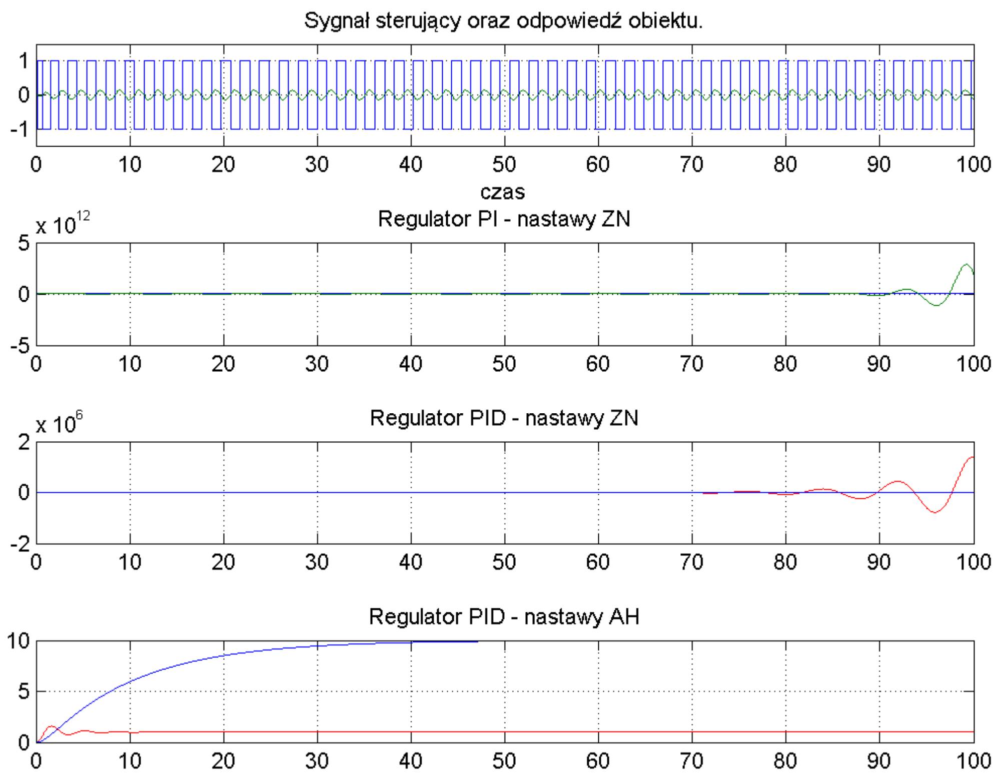
Obiekt G7, T = 5, L = 1, T1 = 0



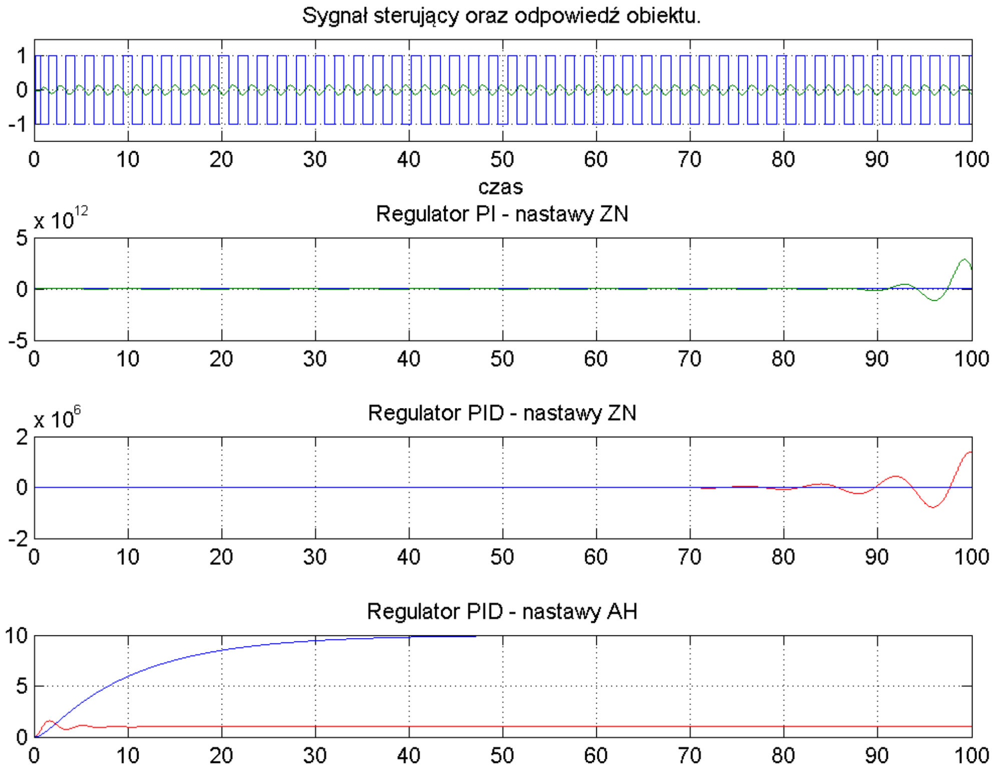
Obiekt G7, T = 10, L = 0.01, T1 = 0.99



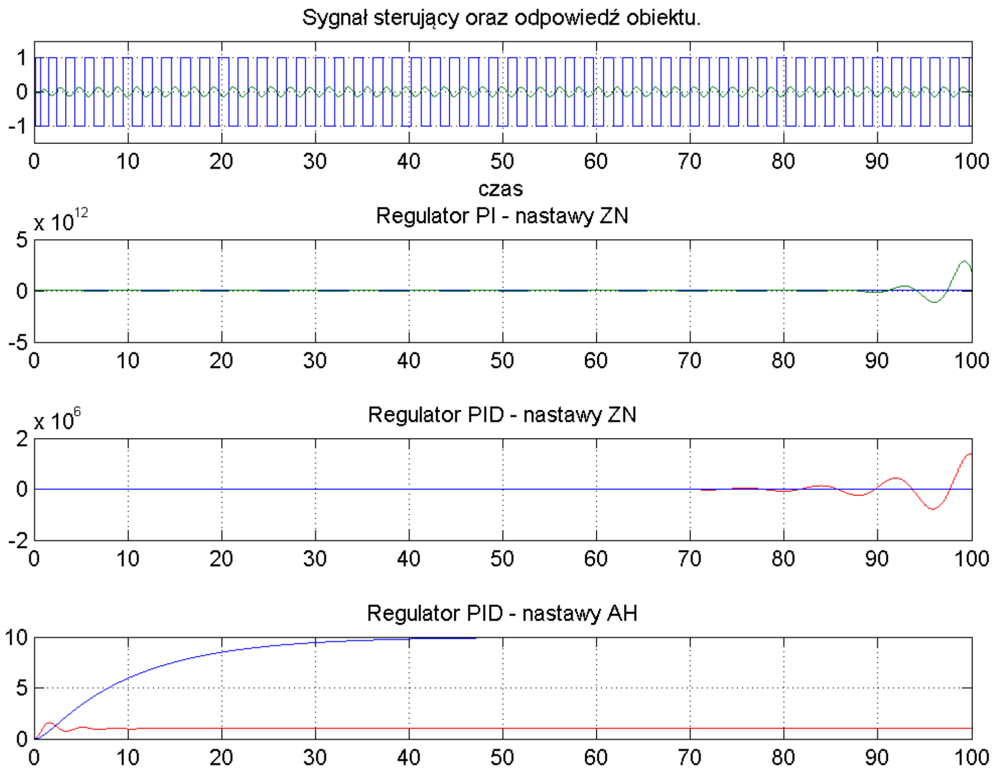
Obiekt G7, T = 10, L = 0.02, T1 = 0.98



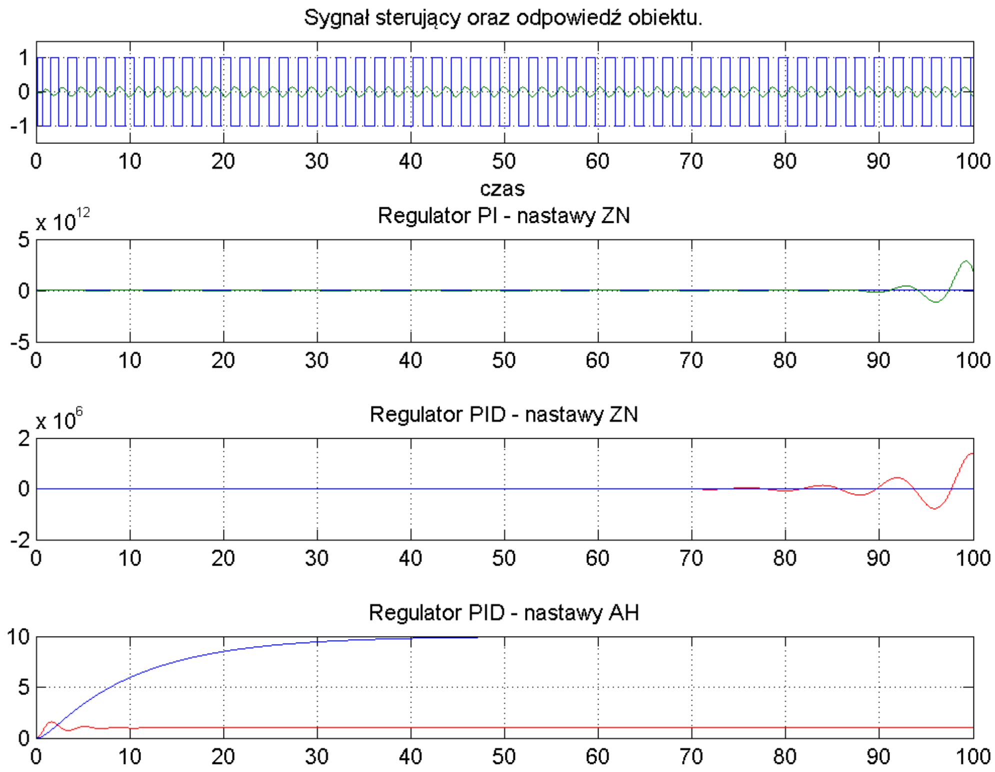
Obiekt G7, T = 10, L = 0.05, T1 = 0.95



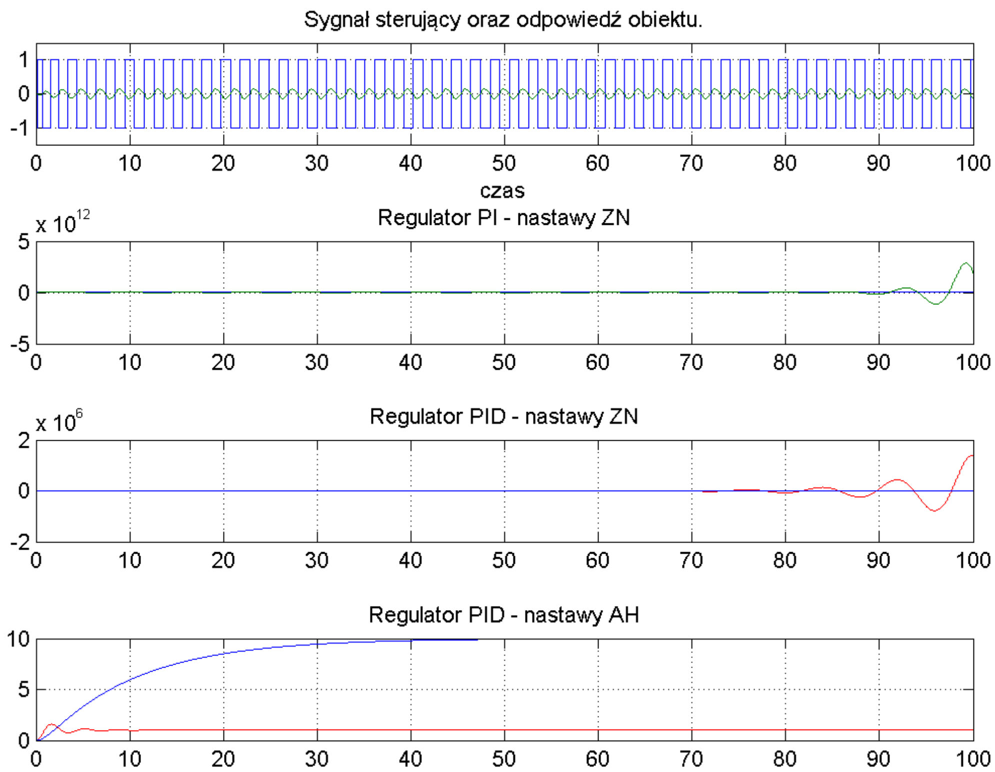
Obiekt G7, T = 10, L = 0.1, T1 = 0.9



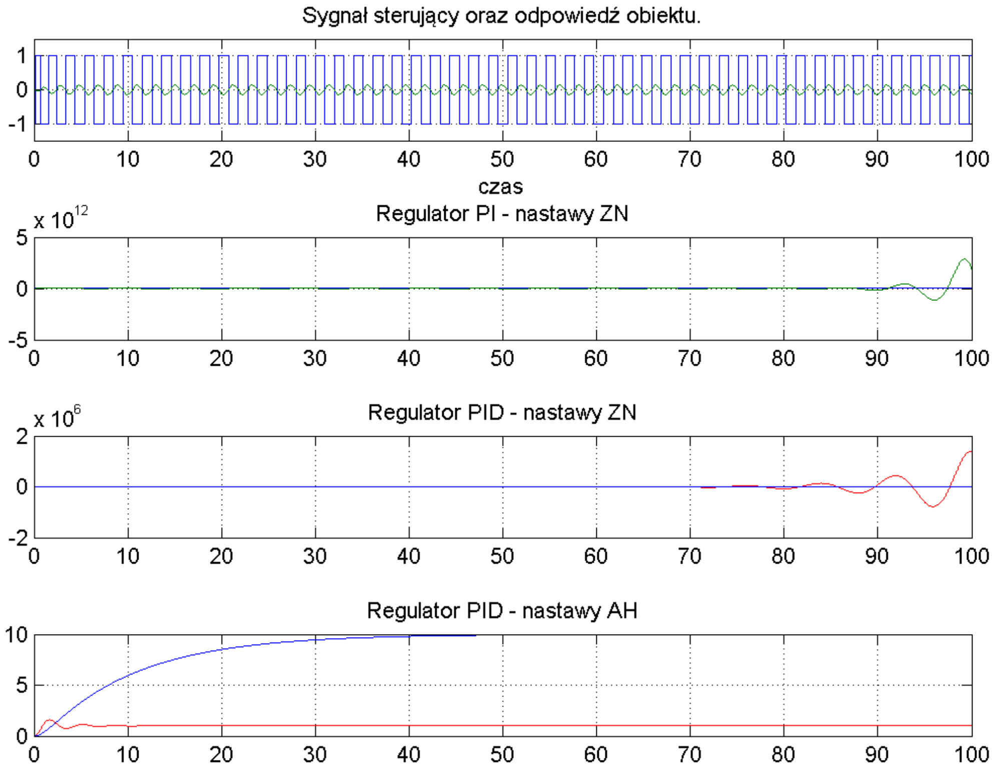
Obiekt G7, T = 10, L = 0.2, T1 = 0.8



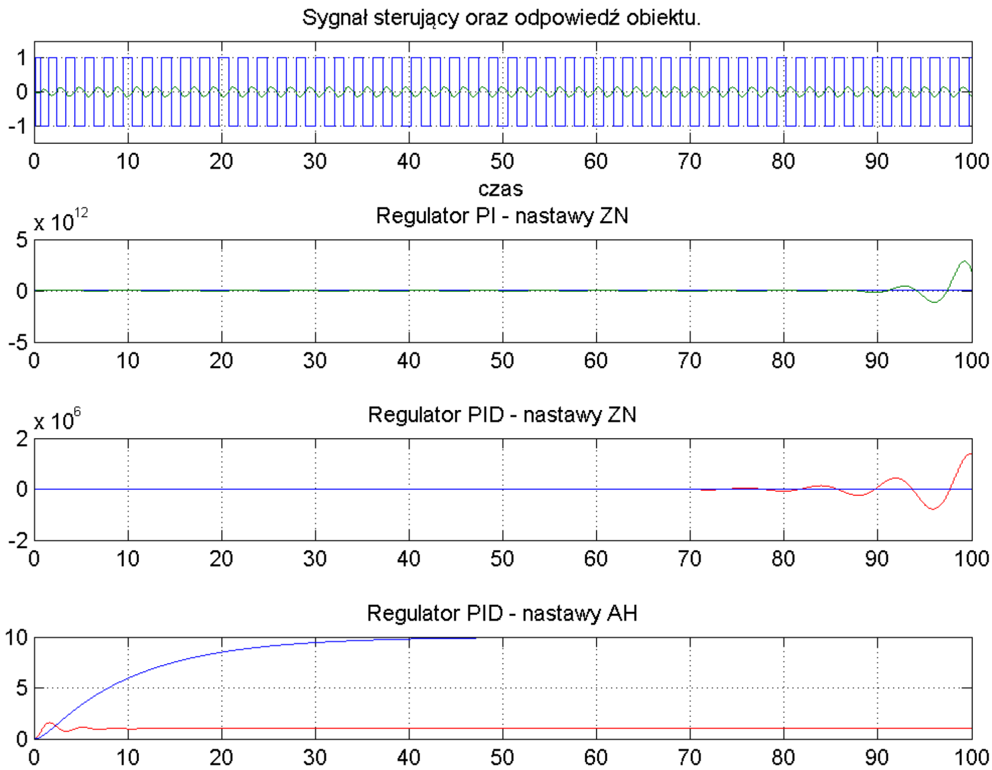
Obiekt G7, T = 10, L = 0.3, T1 = 0.7



Obiekt G7, T = 10, L = 0.5, T1 = 0.5

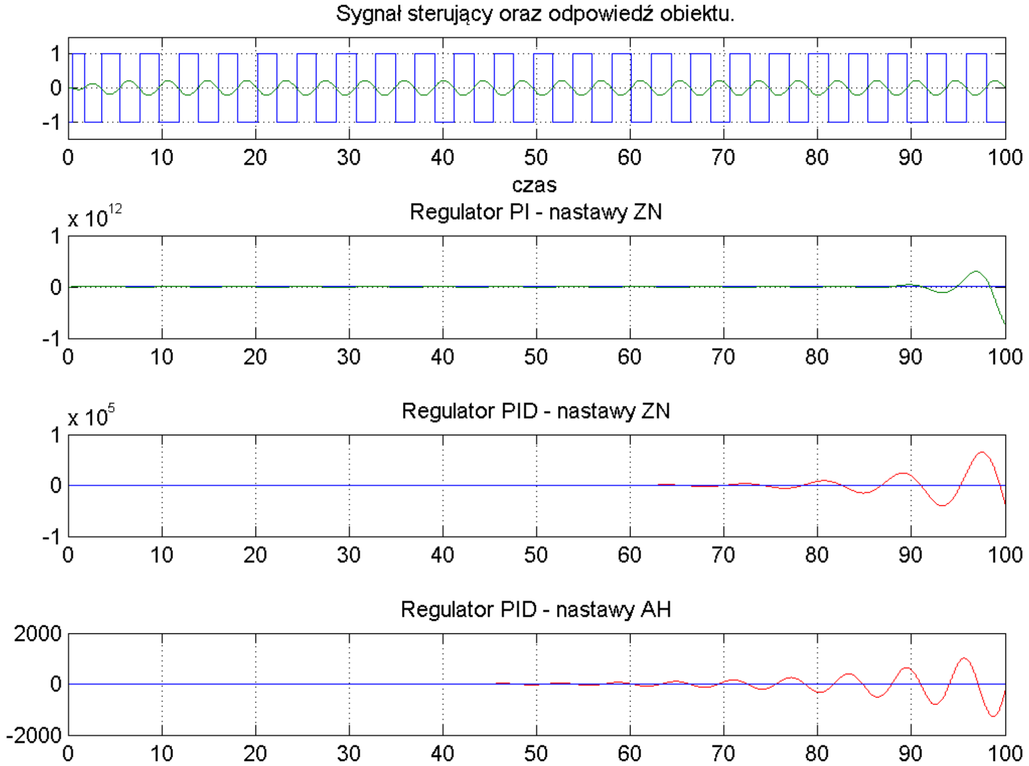


Obiekt G7, T = 10, L = 0.7, T1 = 0.3

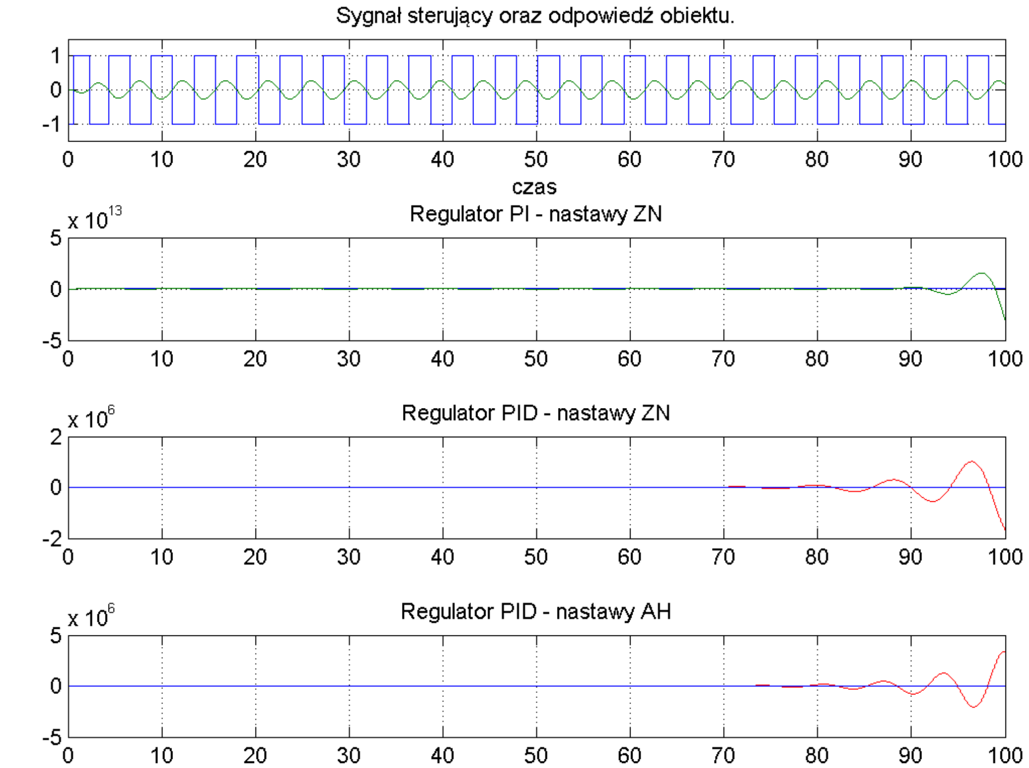


Obiekt G7, T = 10, L = 1, T1 = 0

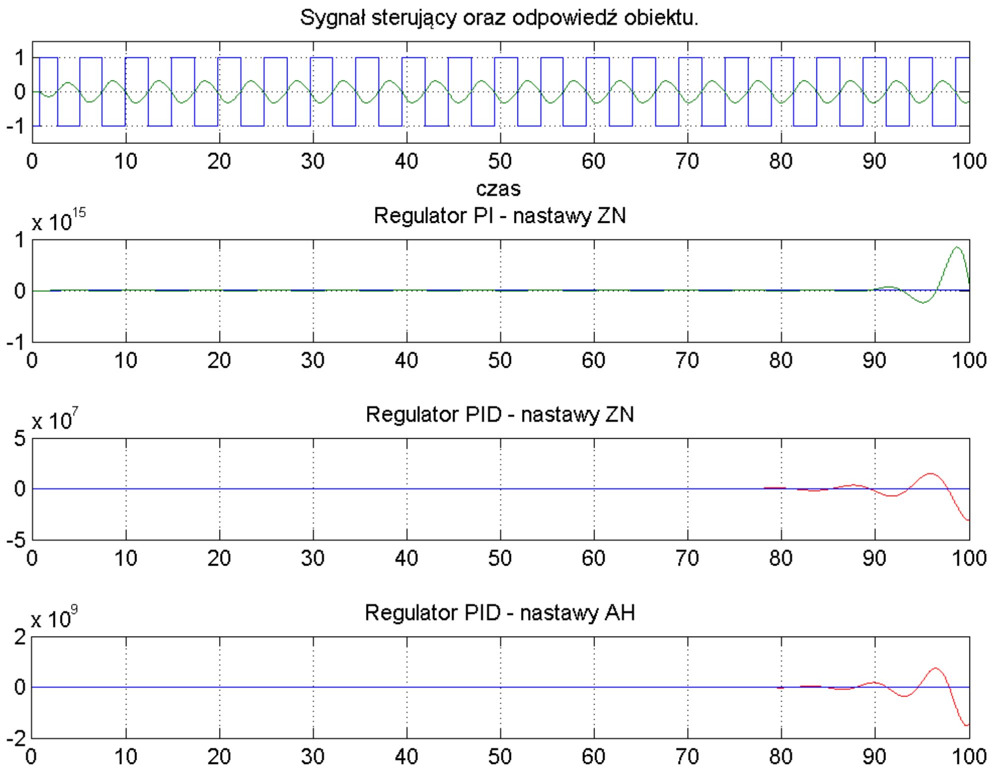
Obiekt 3.8.



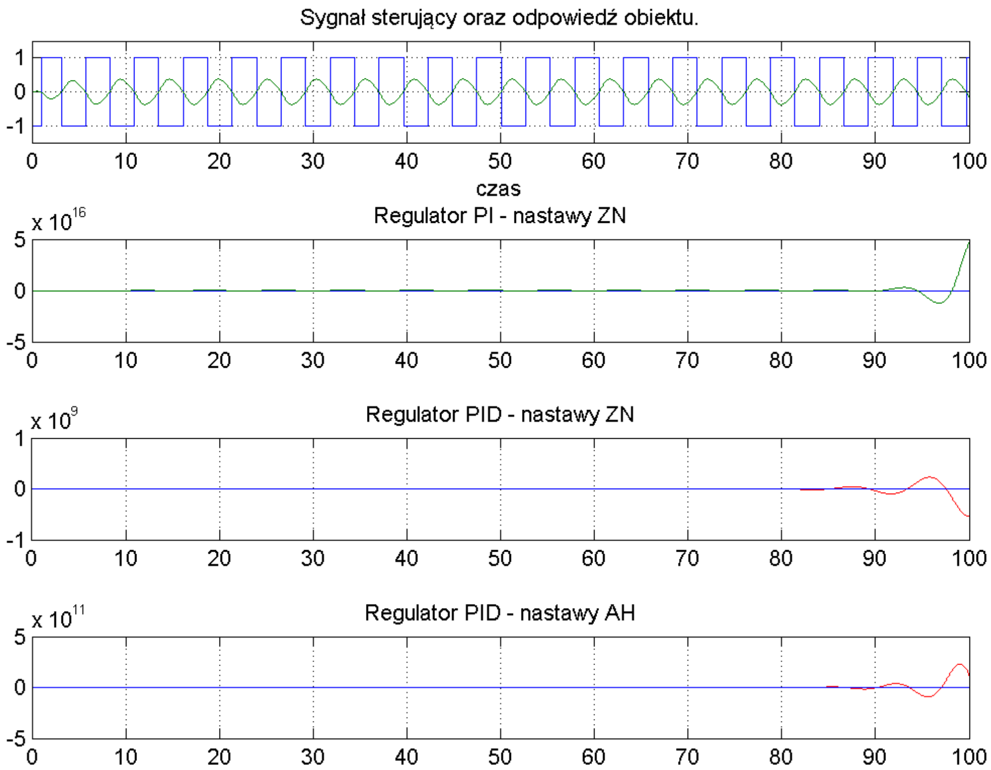
Obiekt G8, alfa = 0.1



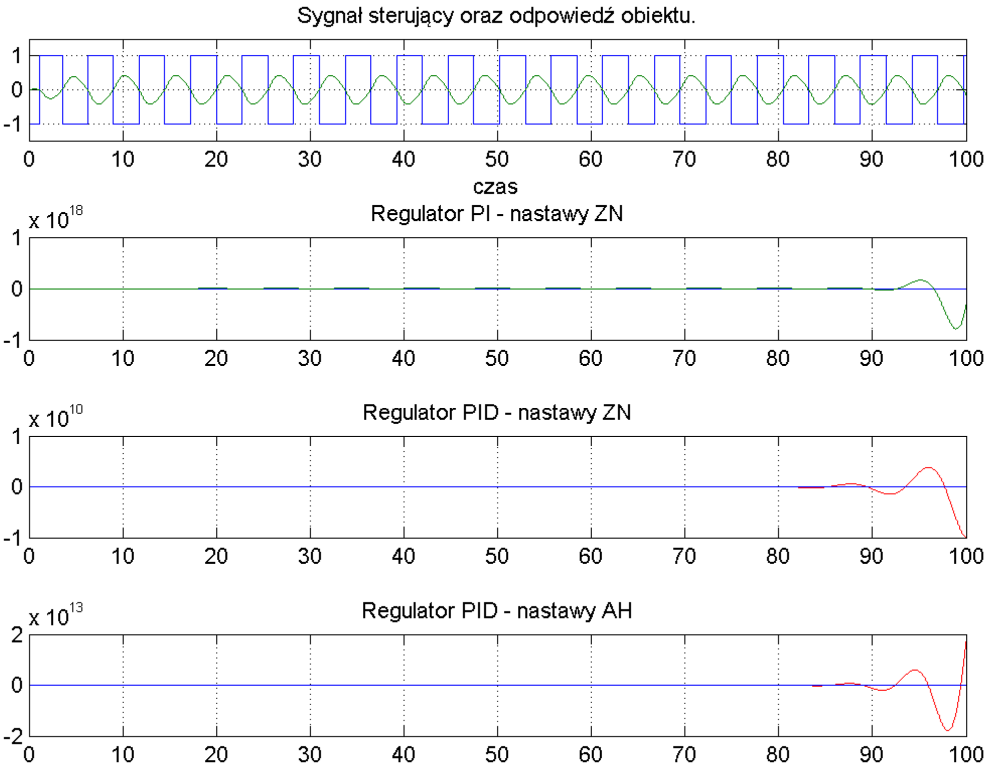
Obiekt G8, alfa = 0.2



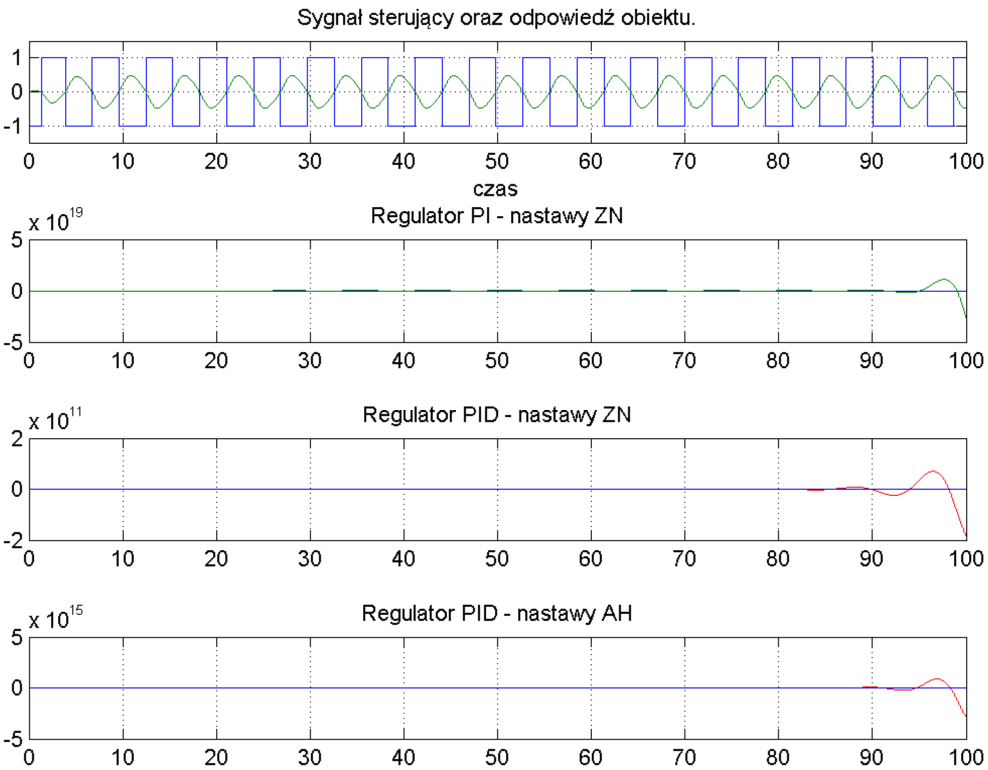
Obiekt G8, alfa = 0.3



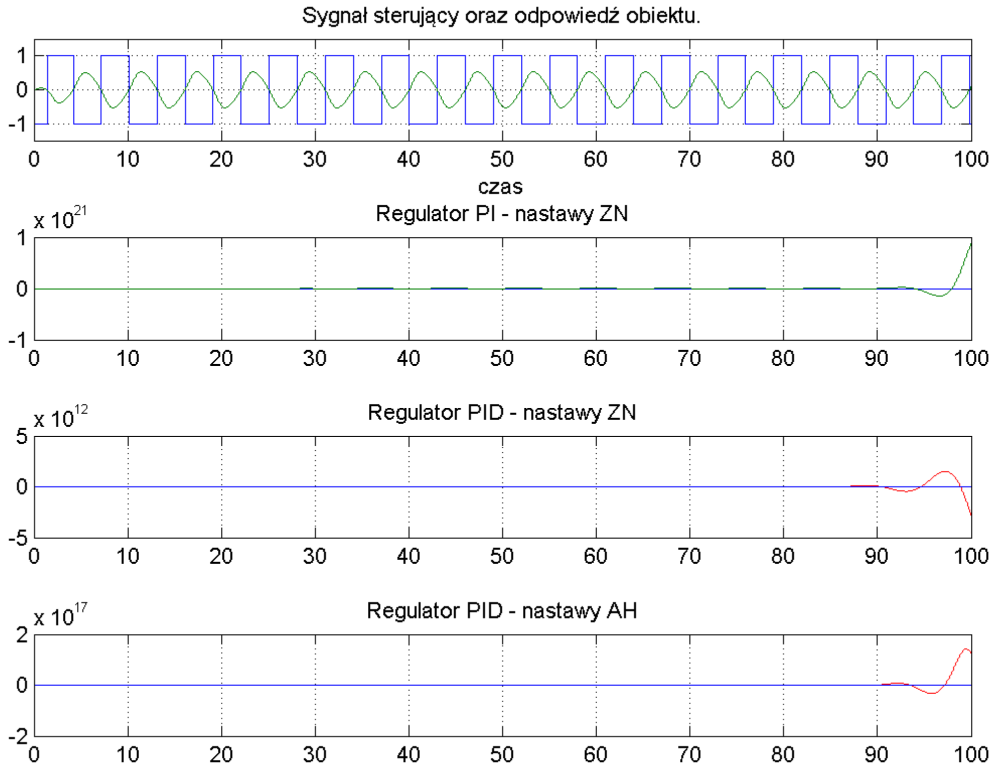
Obiekt G8, alfa = 0.4



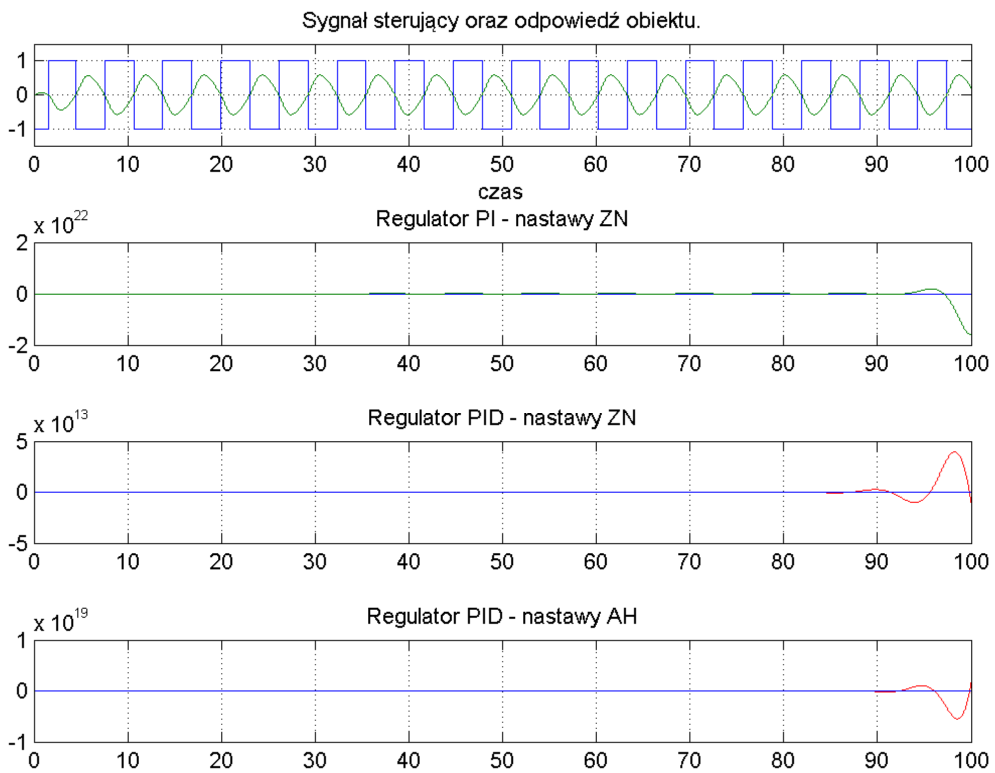
Obiekt G8, alfa = 0.5



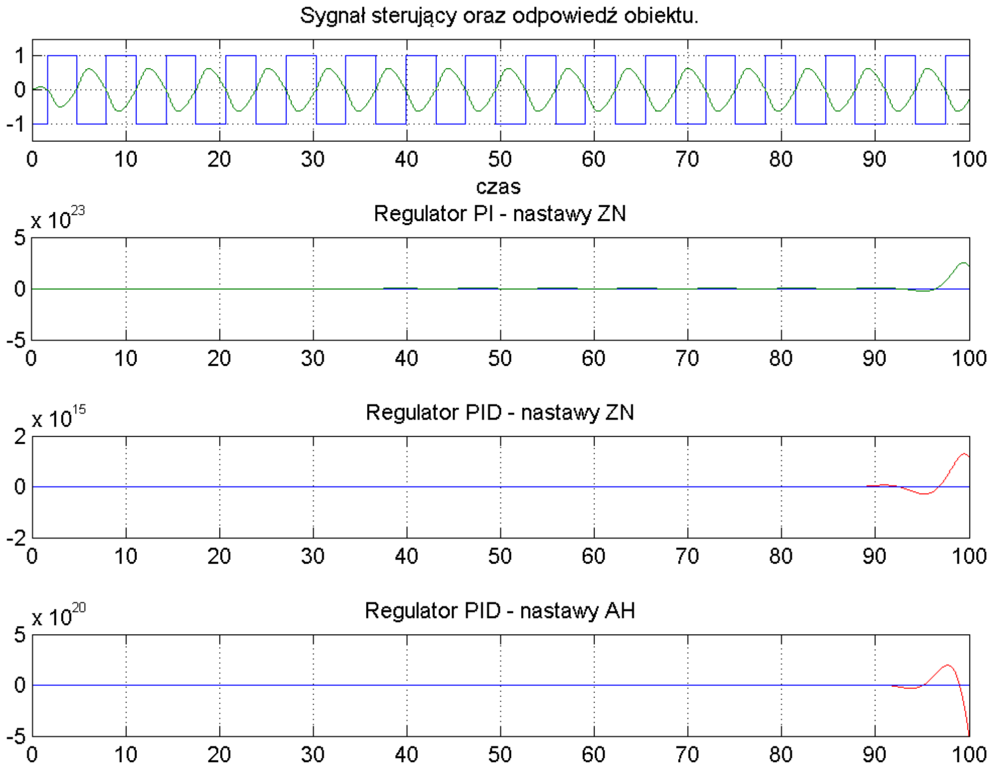
Obiekt G8, alfa = 0.6



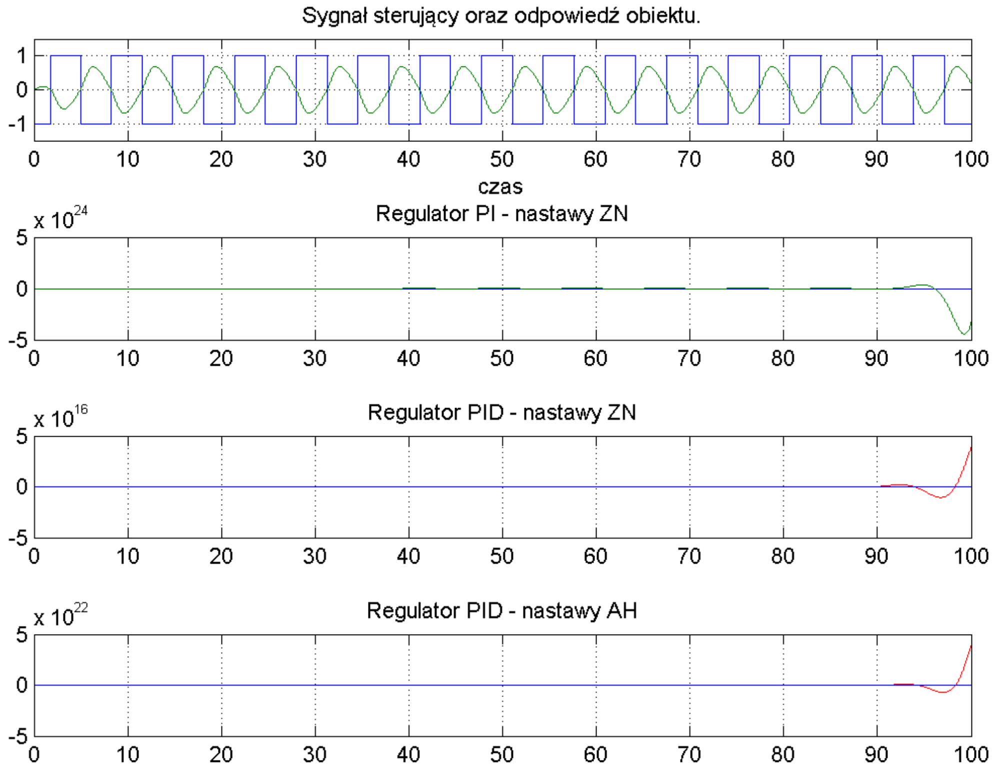
Obiekt G8, alfa = 0.7



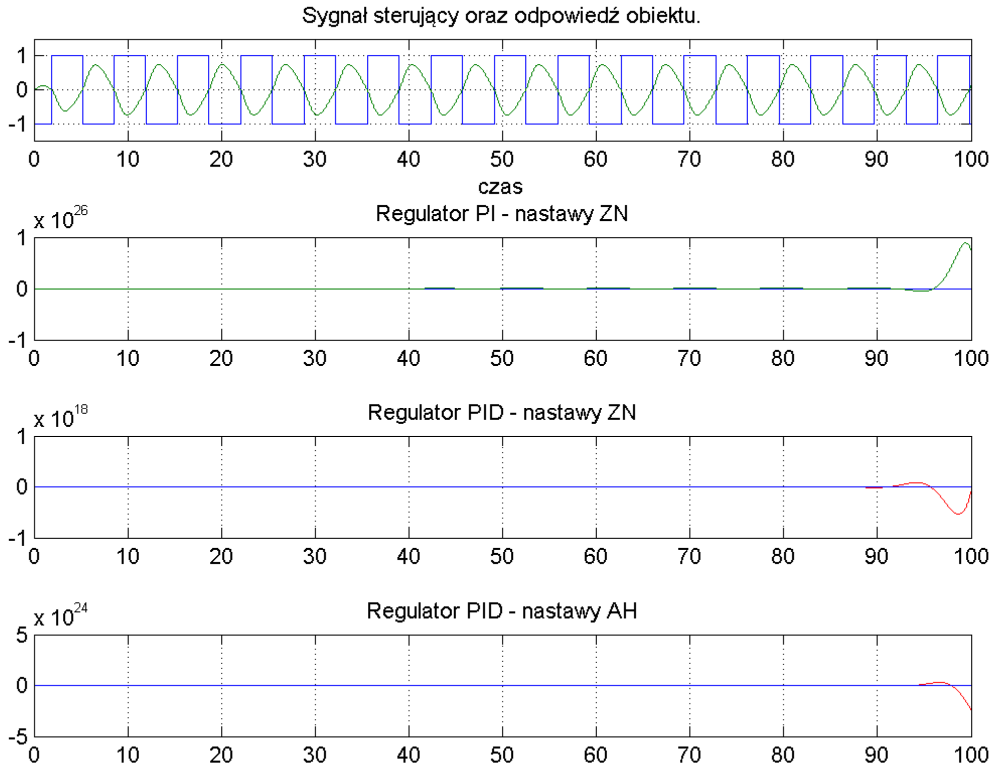
Obiekt G8, alfa = 0.8



Obiekt G8, alfa = 0.9

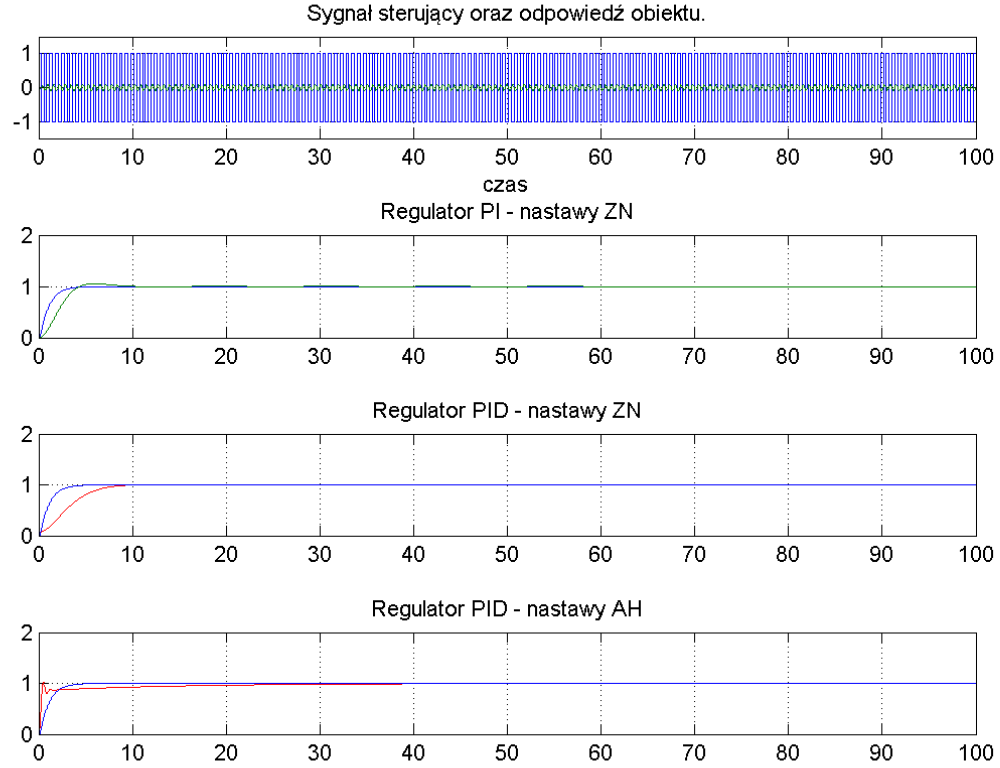


Obiekt G8, alfa = 1

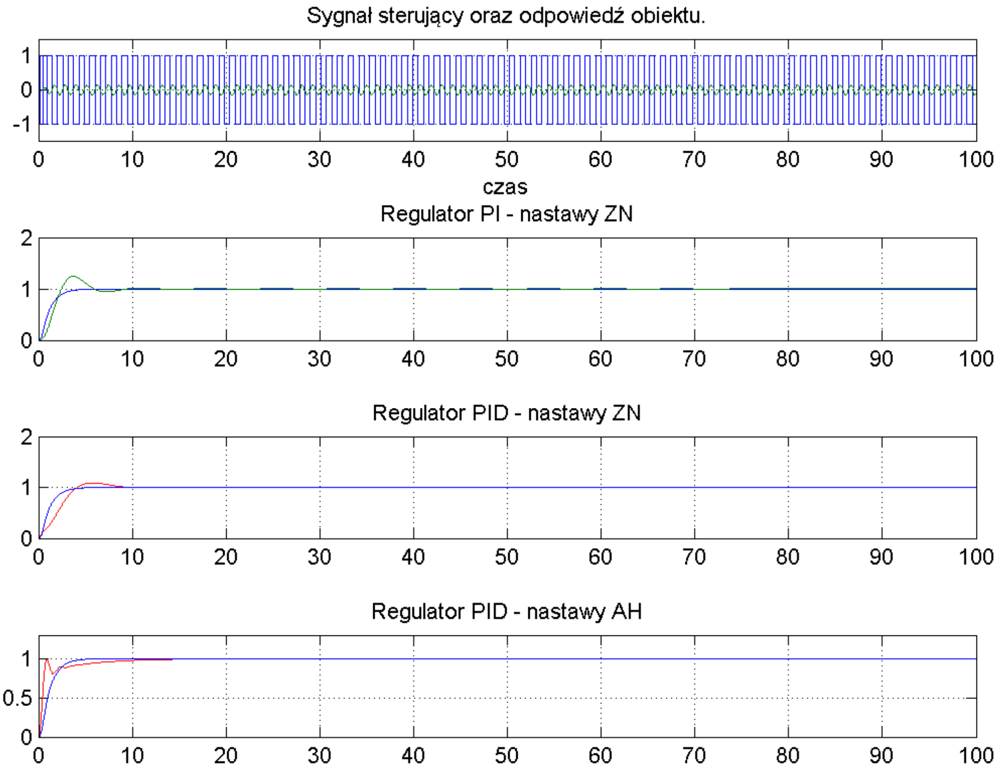


Obiekt G8, alfa = 1.1

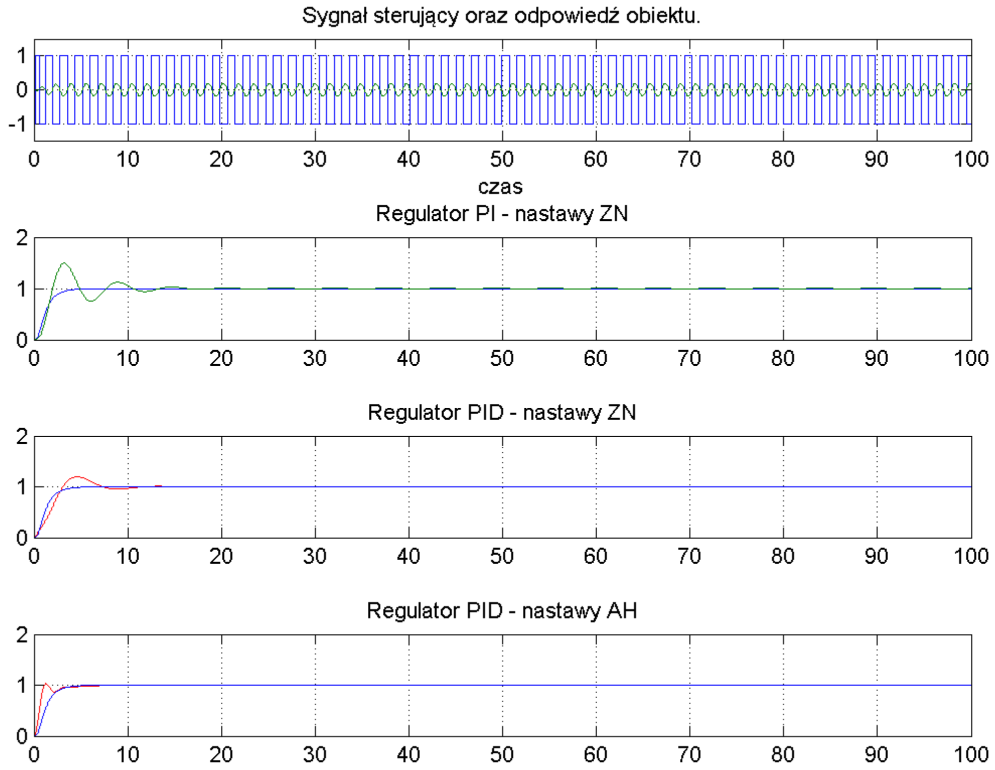
Obiekt 3.9.



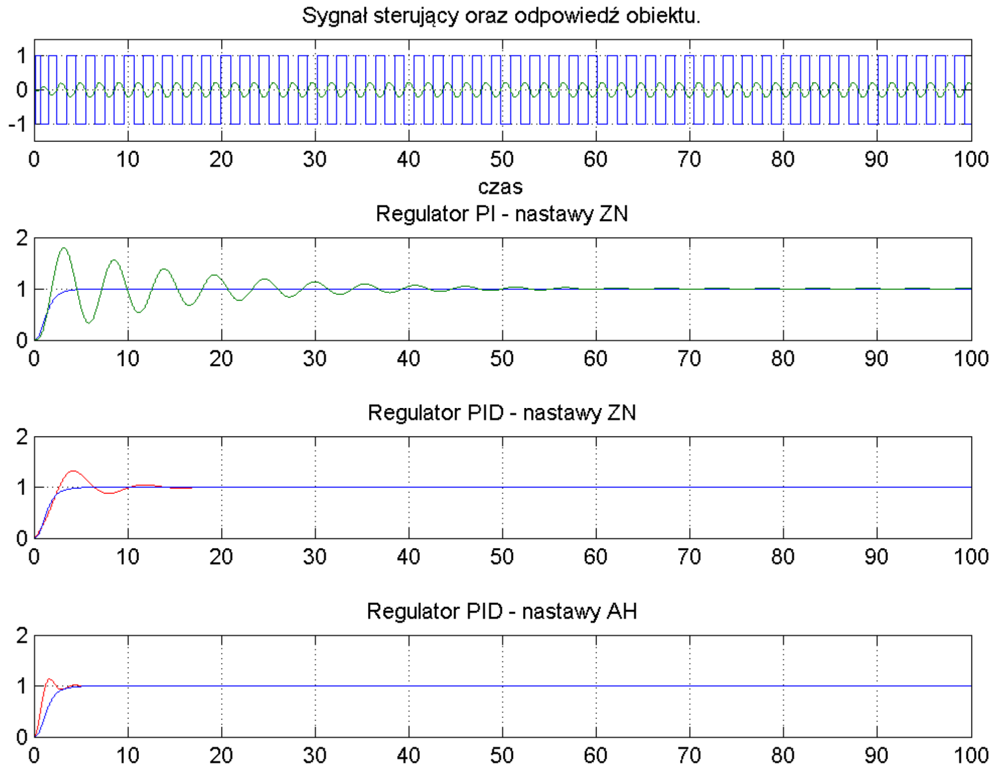
Obiekt G9, T = 0.1



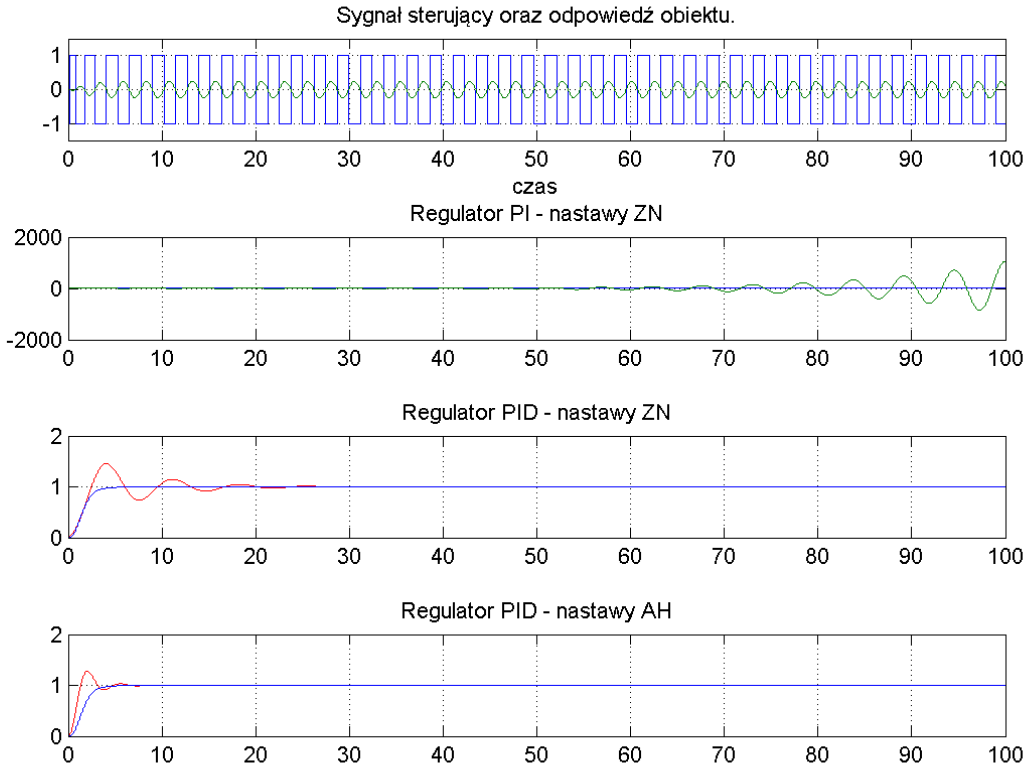
Obiekt G9, T = 0.2



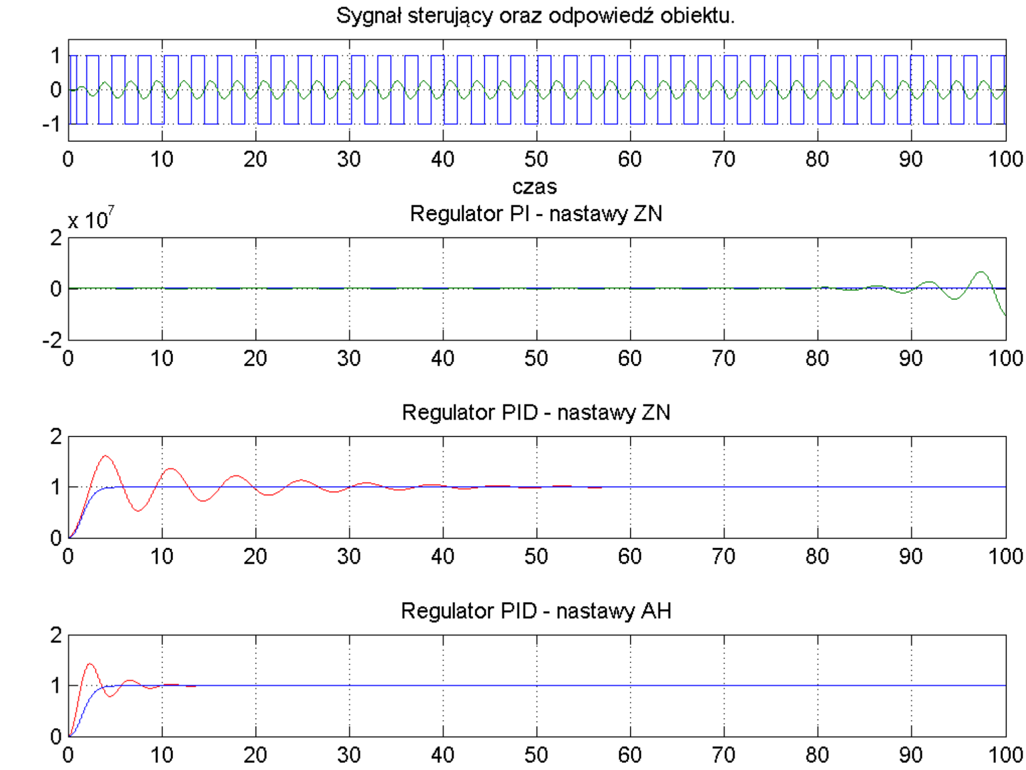
Obiekt G9, T = 0.3



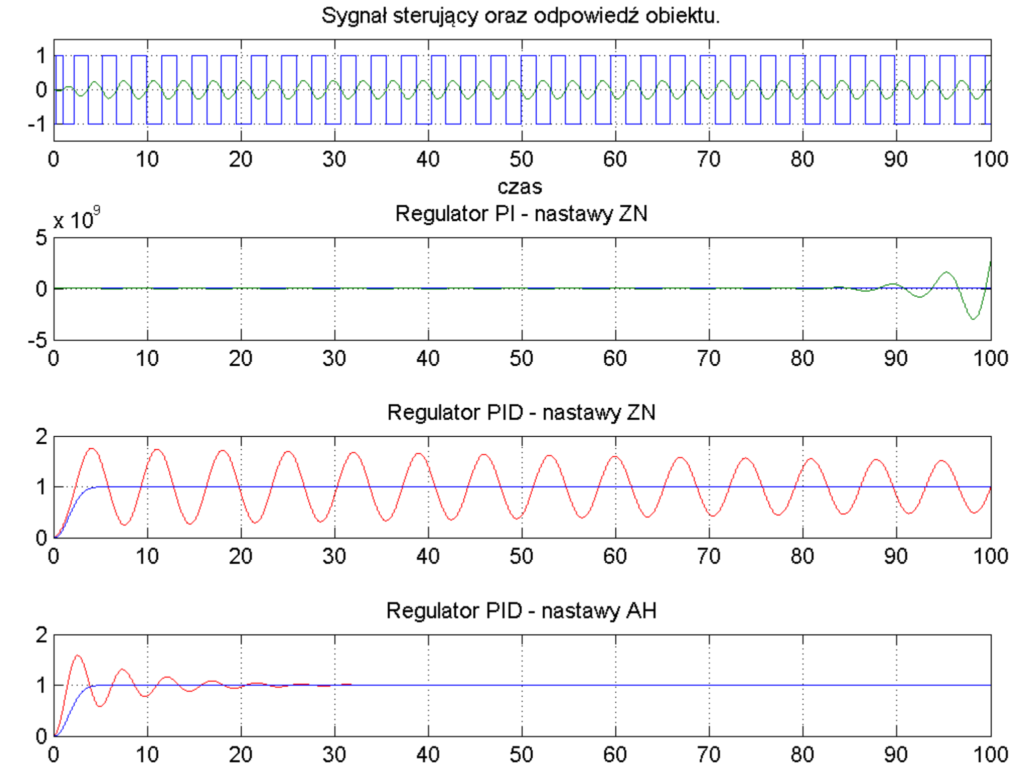
Obiekt G9, T = 0.4



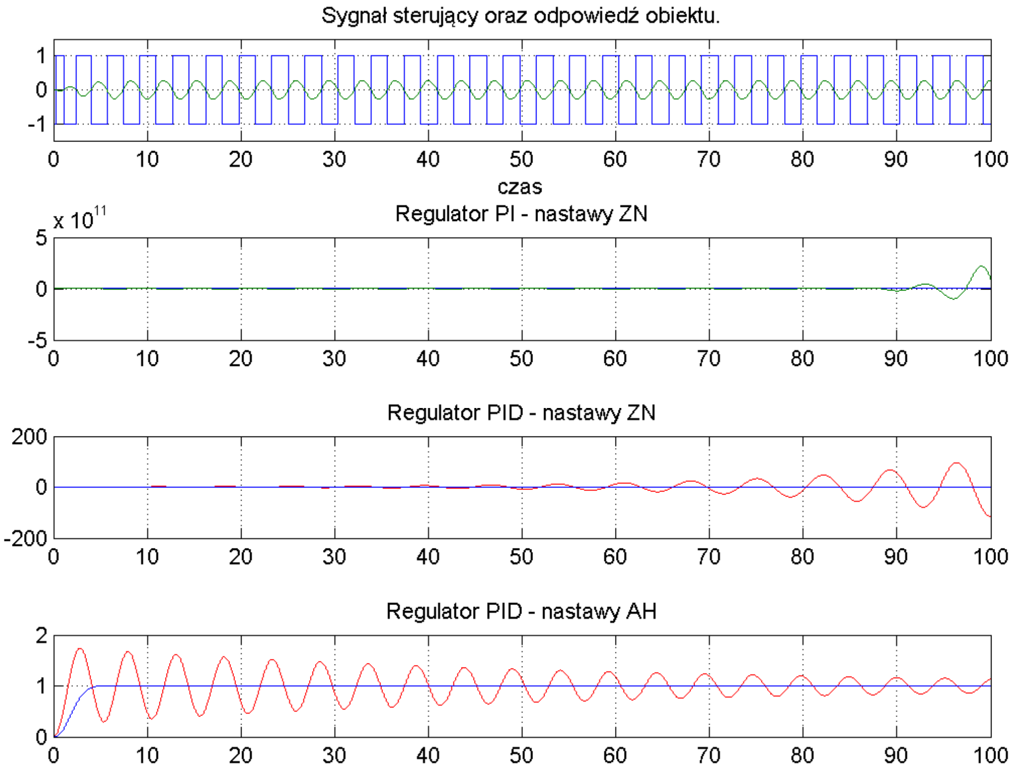
Obiekt G9, T = 0.5



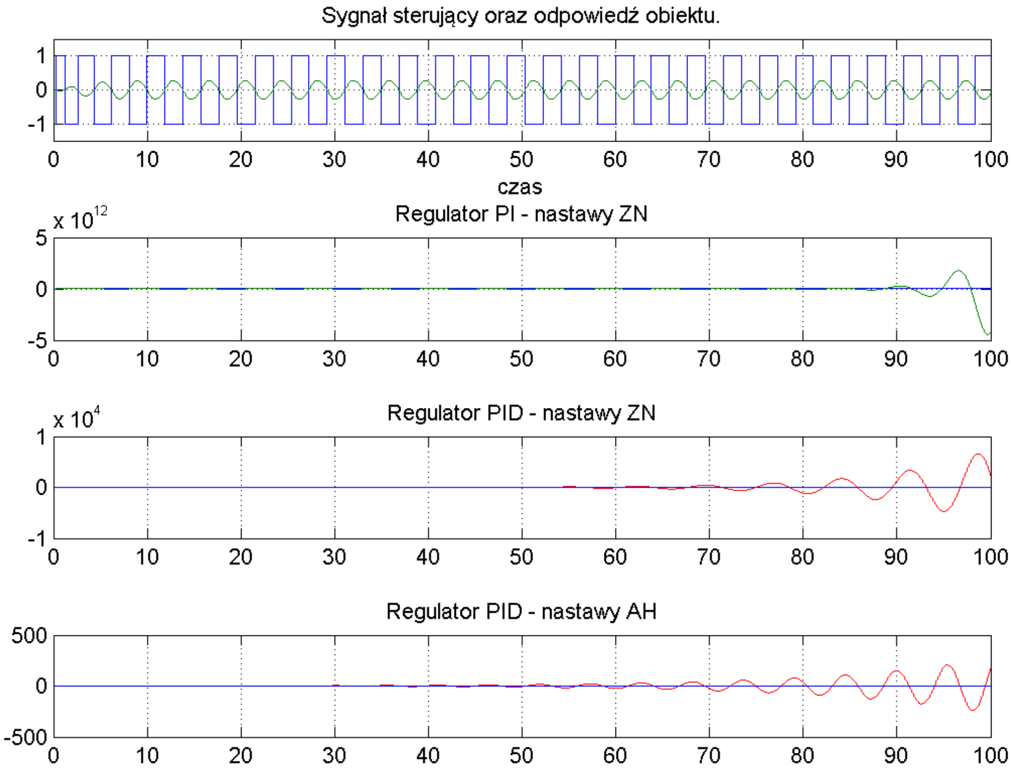
Obiekt G9, T = 0.6



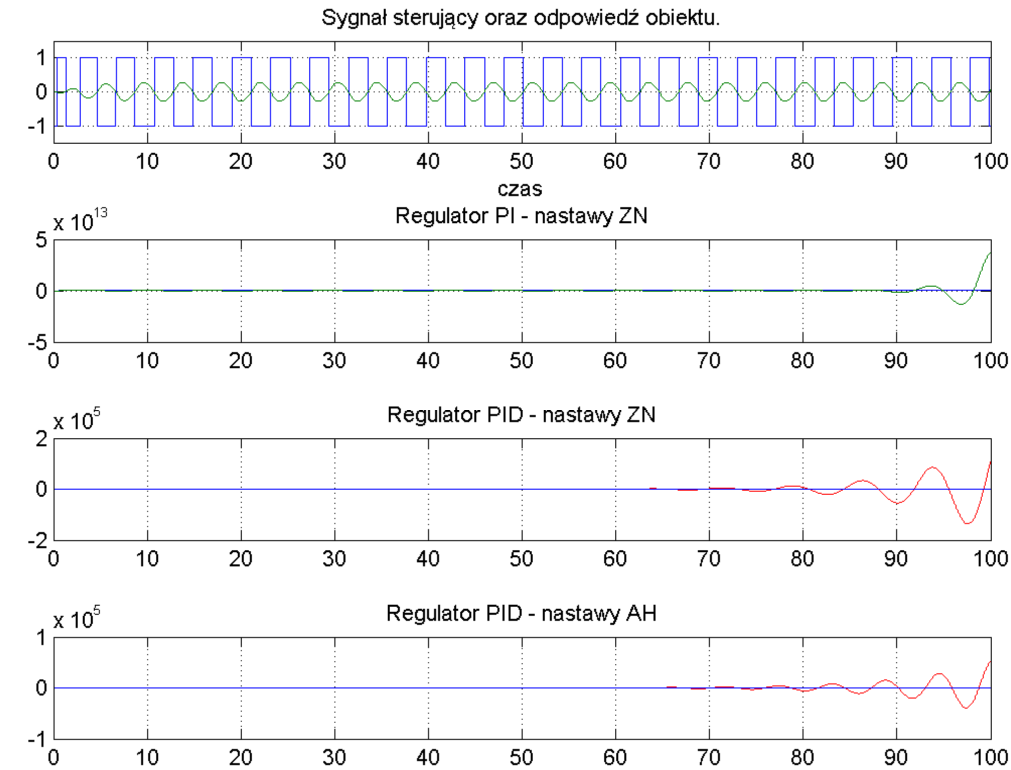
Obiekt G9, T = 0.7



Obiekt G9, T = 0.8



Obiekt G9, T = 0.9



Obiekt G9, T = 1

4. Wnioski

Automatyczne strojenie (self-tuning) jest zazwyczaj przeprowadzane metodami odpowiedzi skokowej lub cyklu przekaźnikowego.