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CPSC322
Assignment 4
# of late days used : 1

1.
a.

A	B	C	p
T	F	T	0.4

We can fix this row by making the probability equal to 0. In a joint distribution table, the sum of all probabilities is equal to 1. Without the above row, the table equals to 1.

b.

P(A)	Val
T	0.5
F	0.5

P(B)	Val
T	0.5
F	0.5

Are A and B independent?

Conditional Independence à A and B are independent given C
P(A|B,C) = P(A|C)
P(A=T | B=T, C=T) = P(A=T|C=T)

P(A=T|C=T)

A	B	C	p	p_e
T	T	T	0.1	0.33
T	F	T	0	0
F	T	T	0.1	0.33

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F	F	T	0.1	0.33

P(A=T|C=T)	**P(A=T|...)**	**P(A=F|...)**
C=T	= 0.33	=0.67
C=F	X	x

P(A|C) = 0.33

P(A=T|B=T, C=T)

A	**B**	**C**	**p**	**p_e**
T	T	T	0.1	0.5
F	T	T	0.1	0.5

P(A=T|B=T, C=T) = 0.5

P(A|C) =/= P(A=T|B=T, C=T)
Therefore, A and B are not independent given C

c.

Yes, both are equivalent due to Product rule and Bayes rule. We can show the two are equivalent with product rule; chain rule is derived by successive application of product rule.

P(A) * P(B|A) * P(C|A,B) = P(C) * P(B|C) * P(A|B,C)

$P(A) * \frac{P(B,A)}{P(A)} * \frac{P(A,B,C)}{P(A,B)} = P(C) * \frac{P(B,C)}{P(C)} * \frac{P(A,B,C)}{P(B,C)}$

P(A,B,C) = P(A,B,C)

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2.

a.

Law of Total Probability

P(A) = P(A | Bx)*P(Bx) + P(A|By)*P(By)

Bayes Rule

P(A|B) = $\frac{P(B|A)P(A)}{P(B)}$

P(Failure=T) = 0.0001 ⇒ P(Failure=F) = 0.9999

P(Report=T|Failure=T) = 0.95 ⇒ P(Report=F|Failure=T) = 0.05

P(Report=F|Failure=F) = 0.95 ⇒ P(Report=T|Failure=F) = 0.05

A → Report = T

Bx → Failure = T

By → Failure = F

P(Report = T)

=P(Report = T | Failure = T)*P(Failure = T) + P(Report = T | Failure = F)*P(Failure = F)

=0.95 * 0.0001 + 0.05 * 0.9999

=0.000095 + 0.049995

=0.05009

P(Failure = T | Report = T) $= \frac{P(Report = T | Failure = T) * P(Failure = T)}{P(Report = T)}$

$= \frac{0.95 * 0.0001}{0.05009}$

= 0.0019

b.

Law of Total Probability

P(A) = P(A | Bx)*P(Bx) + P(A|By)*P(By)

A → Report = T

Bx → Failure = T

By → Failure = F

P(Report = T)

=P(Report = T | Failure = T)*P(Failure = T) + P(Report = T | Failure = F)*P(Failure = F)

=0.95 * 0.01 + 0.05 * 0.99

=0.0095 + 0.0495

=0.059

P(Failure = T | Report = T) $= \frac{P(Report = T | Failure = T) * P(Failure = T)}{P(Report = T)}$

$= \frac{0.95 * 0.01}{0.059}$

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= 0.161

c.

Given the logged report of a problem, it is good news that the issue is so rare because exploding cars are bad and it would be costly to the manufacturer to recall all affected vehicles.

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3.

Possible probabilities
$P(L_1)$
$P(N)$
$P(L_2)$
$P(C_1 \mid L_1, N)$
$P(C_2 \mid N, L_2)$

BNet Cost

$P(L_1) + P(N) + P(L_2) + P(C_1 \mid L_1, N) + P(C_2 \mid N, L_2)$

= 1 + 50 + 1 + 5*51^2 + 5*51^2
= 26062

JPD Cost

2^2 * 51^3
= 530604

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4.

1.

Compute: P(S | Q=F)

Nodes to Prune

1. Z → Unobserved Leaf
2. Y → Unobserved Leaf
3. M → Unobserved Leaf
4. W → Conditionally Independent
5. V→ Unobserved Leaf after pruning Z

$f_1(O)$

O	P(O)
T	0.7
F	0.3

$f_2(NO)$

O	P(N = T| O)	P(N = F| O)
T	0.7	0.3
F	0.3	0.7

$f_3(RO)$

O	P(R = T| O)	P(R = F| O)
T	0.6	0.4
F	0.5	0.5

$f_4(QNR)$

N	R	P(Q = T| N,R)	P(Q = F| N,R)
T	T	0.1	0.9
T	F	0.2	0.8
F	T	0.3	0.7
F	F	0.4	0.6

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$f_5(UR)$

R	P(U = T| R)	P(U = F| R)
T	0.3	0.7
F	0.9	0.1

$f_6(SQU)$

Q	U	P(S = T| Q,U)	P(S = F| Q,U)
T	T	0.3	0.7
T	F	0.4	0.6
F	T	0.4	0.6
F	F	0.7	0.3

2.

P(S | Q=F) =

$f_1(O)$

$f_2(NO)$

$f_3(RO)$

$f_4(QNR)$

$f_5(UR)$

$f_6(SQU)$

Before Observing Q

$P(S, Q=F) \quad = \sum_U f_6(SQU) \ \sum_R f_5(UR) \ \sum_O f_1(O)\, f_3(RO) \sum_N f_2(NO)\, f_4(QNR)$

After Observing Q

$P(S, Q=F) \quad = \sum_U f_8(SU) \ \sum_R f_5(UR) \ \sum_O f_1(O)\, f_3(RO) \sum_N f_2(NO)\, f_7(NR)$

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3.

$f_1(O)$	$f_4 \rightarrow f_7(NR)$
$f_2(NO)$	$f_6 \rightarrow f_8(SU)$
$f_3(RO)$	$f_2, f_7 \rightarrow f_9(OR)$
$f_4(QNR)$	$f_1, f_3, f_9 \rightarrow f_{10}(R)$
$f_5(UR)$	$f_5, f_{10} \rightarrow f_{11}(U)$
$f_6(SQU)$	$f_8, f_{11} \rightarrow f_{12}(S)$

$f_1(O)$

O	Value
T	.7
F	.3

$f_2(NO)$

N	O	Value
T	T	0.7
F	T	0.3
T	F	0.3
F	F	0.7

$f_3(RO)$

R	O	Value
T	T	0.6
F	T	0.4
T	F	0.5
F	F	0.5

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$f_4(QNR)$

Q	N	R	Value
T	T	T	0.1
F	T	T	0.9
T	T	F	0.2
F	T	F	0.8
T	F	T	0.3
F	F	T	0.7
T	F	F	0.4
F	F	F	0.6

$f_5(RO)$

U	R	Value
T	T	0.3
F	T	0.7
T	F	0.9
F	F	0.1

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$f_6(SQU)$ when Q=f

S	**Q**	**U**	**Value**
T	T	T	0.3
F	T	T	0.7
T	T	F	0.4
F	T	F	**0.6**
T	F	T	0.4
F	F	T	0.6
T	F	F	0.7
F	F	F	0.3

$f_7(NR)$ where Q=f

N	**R**	**Value**
T	T	0.9
F	T	0.7
T	F	0.8
F	F	0.6

$f_8(SU)$

S	**U**	**Value**
T	T	0.4
F	T	0.6
T	F	0.7
F	F	0.3

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$f_9(OR)$

O	R	Value
T	T	(0.7*0.9) + (0.3*0.7) = 0.84
F	T	(0.3*0.9) + (0.7*0.7) = 0.76
T	F	(0.7*0.8) + (0.3*0.6) = 0.74
F	F	(0.3*0.8) + (0.6*0.7) = 0.66

$f_{10}(R)$

R	Value
T	(0.7*0.6*0.84) + (0.3*0.5*0.76) =0.4668
F	(0.7*0.4*0.74) + (0.3*0.5*0.66) = 0.3062

$f_{11}(U)$

U	Value
T	(0.4668*0.3) + (0.3062*0.9) = 0.41562
F	(0.4668*0.7) + (0.3062*0.1) = 0.35738

$f_{12}(S)$

S	Value
T	(0.4*0.41562) + (0.7*0.35738) = 0.416414
F	(0.6*0.41562) + (0.3*0.35738) = 0.356586

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Normalize $f_{12}(S)$

$f_{12}(S)$

S	**Value**
T	0.416414/0.773 = 0.539
F	0.356586 / 0.773 = 0.461

$$
\begin{aligned}
P(S, Q=F) &= \sum_U f_8(SU) \sum_R f_5(UR) \sum_O f_1(O) f_3(RO) \sum_N f_2(NO) f_7(NR) \\
&= \sum_U f_8(SU) \sum_R f_5(UR) \sum_O f_1(O) f_3(RO) f_9(OR) \\
&= \sum_U f_8(SU) \sum_R f_5(UR) f_{10}(R) \\
&= \sum_U f_8(SU) f_{11}(U) \\
&= f_{12}(S)
\end{aligned}
$$