Quick Permutation Test (QuiPT)

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Outline

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n-grams (k-tuples) are sets of n characters derived from the input sequence(s). They may form continuous sub-sequences or be discontinuous.

	X1	X2	X3	X4	X5	X6
1	1	2	1	4	2	1
2	3	1	4	3	3	2
3	4	1	3	3	4	2

Sample sequences.

1	2	3	4
3	2	0	1
1	1	3	1
1	1	2	2

Unigrams.

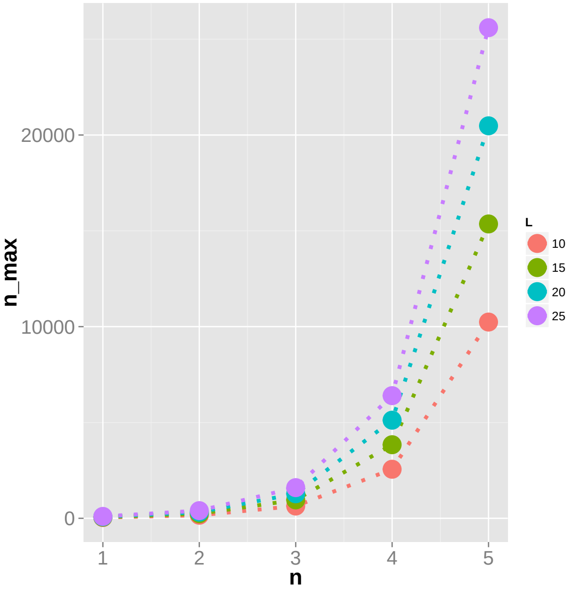
X1_1_0	X2_1_0	X3_1_0	X4_1_0	X5_1_0	X6_1_0	X1_2_0
1	0	1	0	0	1	0
0	1	0	0	0	0	0
0	1	0	0	0	0	0

A fraction of possible unigrams with position information.

Positioned n-gram data is binary.

Number of possible positioned n-grams:

$$n_{max} = L \times m^n$$


20000
10000
0
n_max
1
2
3
4
5
n
L
10
15
20
25

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4. Calculate p-value using:

$$\text{p-value} = \frac{N_{T_P > T_R}}{N}$$

$N_{T_P > T_R}$ is number of times when T_P was bigger than T_R

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- Statistic independent.

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- Unfeasible precise estimation of low p-values.

The number of permutations is inversely proportional to the interval between p-values.

Example: with 10×10^6 permutation the smallest possible p-values are: 0, 1×10^{-6}, 2×10^{-6} and so on.

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The binary positioned n-gram data tabulated by binary label can be easily described in 2d contingency table.

sequence ID	feature	target
1	1	0
2	1	0
3	0	0
4	1	1
5	0	1
...	...	...

Positioned n-grams with a label.

	target	feature
0	$n_{1,1}$	$n_{1,0}$
1	$n_{0,1}$	$n_{0,0}$

Contingency table.

Test statistics used by QuiPT (information gain, Kullback-Leibler divergence) measure inbalance of contingency tables.

If probability that target equals 1 is p and probability that feature equals 1 is q and feature and target are independent then each of them has the following probabilities

$$P(Target, Feature) = (1, 1)) = p \cdot q$$

$$P(Target, Feature) = (1, 0)) = p \cdot (1 - q)$$
$$P(Target, Feature) = (0, 1)) = (1 - p) \cdot q$$
$$P(Target, Feature) = (0, 0)) = (1 - p) \cdot (1 - q)$$

$$
\begin{aligned}
F(n_{1,1}, n_{1,0}, n_{0,1}, n_{0,0}) = \binom{n}{n_{1,1}} (p \cdot q)^{n_{1,1}} \binom{n - n_{1,1}}{n_{1,0}} (p \cdot (1-q))^{n_{1,0}} \\
\binom{n - n_{1,1} - n_{1,0}}{n_{0,1}} ((1-p) \cdot q)^{n_{0,1}} \\
\binom{n - n_{1,1} - n_{1,0} - n_{0,1}}{n_{0,0}} ((1-p) \cdot (1-q))^{n_{0,0}}
\end{aligned}
$$

In addition to this: $n_{1,\cdot} = n_{1,1} + n_{1,0}$ and $n_{\cdot,1} = n_{1,1} + n_{0,1}$ are known and fixed.

- $n_{1,1}$ is from range $[0, min(n_{\cdot,1}, n_{1,\cdot})]$.

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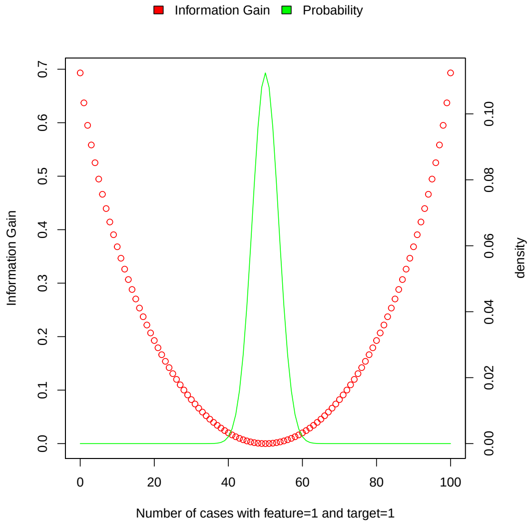
- The test statistic is computed for each possible value of $n_{1,1}$.
- The distribution of test statistics under hypothesis that target and feature are independant is computed using values from 3.



	Target	Feature	Freq
1	0	0	50
2	1	0	50
3	0	1	50
4	1	1	50

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- Using the exact distribution of possible values of the criterion QuiPT yields precise small p-values without increasing the computation time.

Outline

1. n-grams
   - n-gram definition
   - Positioned n-grams
2. Permutation test
   - Testing framework
   - Advantages
   - Drawbacks
   - p-value resolution
3. QuiPT
   - Contingency tables
   - Multinomial distribution of target-feature relationship
   - Advantages over permutation test
4. Simulation scheme
   - Power of the test
   - False significant features
5. Conclusion

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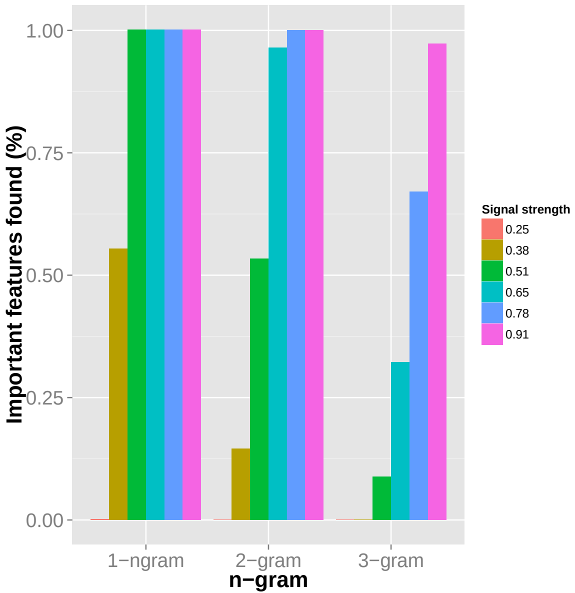
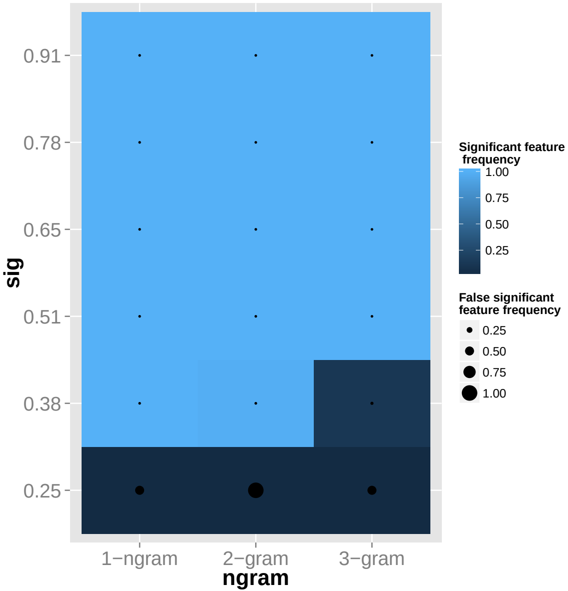
5. Iterate steps 1-4 over other values of p_d - 0.38, 0.51, 0.65, 0.78, 0.91.
6. Repeat steps 1-5 200 times.


1.00
0.75
0.50
0.25
0.00
Important features found (%)
1−ngram
2−gram
3−gram
n−gram
Signal strength
0.25
0.38
0.51
0.65
0.78
0.91


0.91
0.78
0.65
0.51
0.38
0.25
sig
1−ngram
2−gram
3−gram
ngram
Significant feature frequency
1.00
0.75
0.50
0.25
False significant feature frequency
0.25
0.50
0.75
1.00

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Quick permutation test is a powerful and quick equivalent of permutation test in binary feature-binary target testing scenario.