Quick Permutation Test: feature filtering of n-gram data

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Introduction

N-grams (k-tuples) are vectors of n characters derived from input sequence(s). They may form continuous sub-sequences or be discontinuous. Another important n-gram parameter is its position. Instead of just counting n-grams, one may want to count how many n-grams occur at a given position in multiple (e.g. related) sequences.

	P1	P2	P3	P4	P5	P6
S1	3	1	2	1	2	2
S2	1	2	3	3	4	1
S3	4	1	3	1	1	1

Sample sequences.

1	2	3	4
2	3	1	0
2	1	2	1
4	0	1	1

Unigram counts.

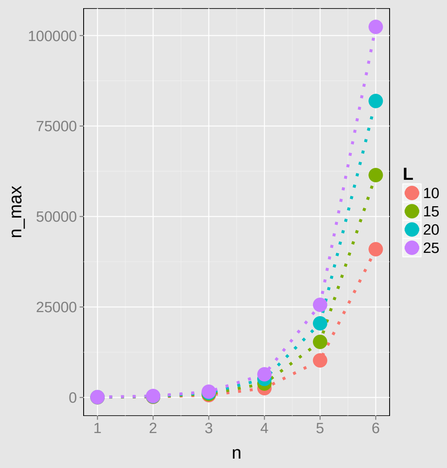
P1_1	P2_1	P3_1	P4_1	P5_1	P6_1	P1_2	P2_2	P3_2	P4_2	P5_2
0	1	0	1	0	0	0	0	1	0	1
1	0	0	0	0	1	0	1	0	0	0
0	1	0	1	1	1	0	0	0	0	0

A fraction of possible unigrams with position information.

Curse of dimensionality

Number of possible positioned n-grams (not taking into account distances between elemnts of n-gram):

$$\boldsymbol{n_{max} = L \times m^n}$$



Permutation test

During permutation tests class labels are randomly exchanged during computation of significance statistic. p-values are defined as:

$$\text{p-value} = \frac{\boldsymbol{N_{T_P > T_R}}}{\boldsymbol{N}}$$

where $\boldsymbol{N_{T_P > T_R}}$ is number of times when $\boldsymbol{T_P}$ (permuted test statistic) was more extreme than $\boldsymbol{T_R}$ (test statistic for non-permuted data).
Permutation tests are model and statistic independent, but computationally expensive (especially precise estimation of low p-values, because the number of permutations is inversely proportional to the interval between p-values).

QuiPT algorithm

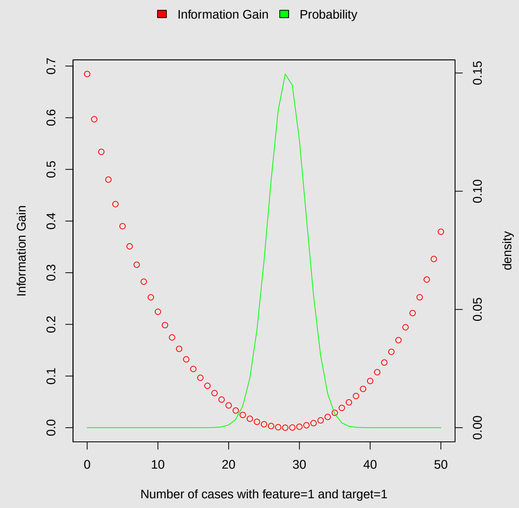
If probability that target equals 1 is $\boldsymbol{p}$ and probability that feature equals 1 is $\boldsymbol{q}$ and feature and target are independent then each of them has the following probabilities

$$\boldsymbol{P(Target, Feature) = (1,1)) = p \cdot q}$$
$$\boldsymbol{P(Target, Feature) = (1,0)) = p \cdot (1-q)}$$
$$\boldsymbol{P(Target, Feature) = (0,1)) = (1-p) \cdot q}$$
$$\boldsymbol{P(Target, Feature) = (0,0)) = (1-p) \cdot (1-q)}$$

$$\boldsymbol{F(n_{1,1}, n_{1,0}, n_{0,1}, n_{0,0}) = \binom{n}{n_{1,1}} (p \cdot q)^{n_{1,1}} n - n_{1,1} \binom{}{n_{1,0}} (p \cdot (1-q))^{n_{1,0}} n - n_{1,1} - n_{1,0} \binom{}{n_{0,1}} ((1-p) \cdot q)^{n_{0,1}} \binom{n - n_{1,1} - n_{1,0} - n_{0,1}}{n_{0,0}} ((1-p) \cdot (1-q))^{n_{0,0}}}$$

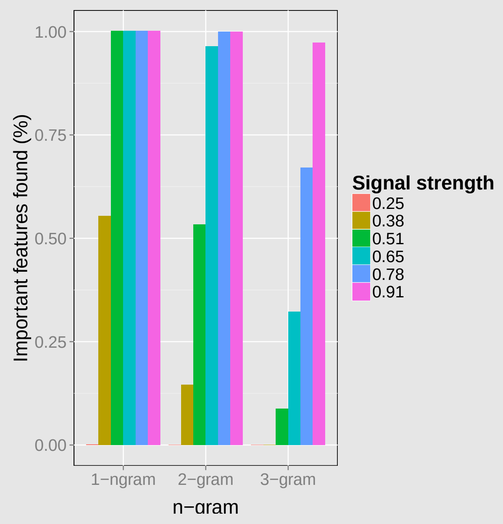
In addition to this: $\boldsymbol{n_{1,.} = n_{1,1} + n_{1,0}}$ and $\boldsymbol{n_{.,1} = n_{1,1} + n_{0,1}}$ are known and fixed.

Validation procedure

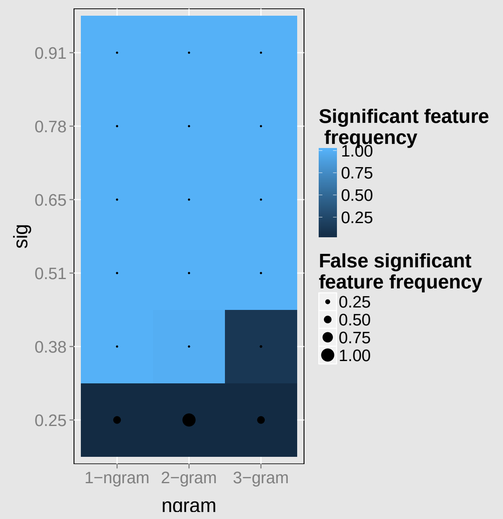


	Target	Feature	Freq
1	0	0	40
2	1	0	10
3	0	1	25
4	1	1	40

Test power



False discoveries



Summary

Quick permutation test is a powerful and quick equivalent of permutation test in binary feature-binary target testing scenario.

Avaibility

biogram R package:
`http://cran.r-project.org/web/packages/biogram/`

Bibliography