Grammar

Base Type

$\tau ::= \sigma \mid r$

$\sigma ::= \mathit{float} \mid \sigma \times \sigma \mid \eta \cdot \sigma$

(Meta) Natural Number

$\eta ::= 0 \mid 1 \mid ...$

Range

$r ::= \eta..\eta \mid r \cdot r$

Term

$t ::= l \mid p \mid x \mid \text{for } i : l \text{ in } t \mid t.\text{fst} \mid t.\text{snd} \mid \text{let } x = t \text{ in } t \mid (t, t)$

Literal

$l ::= \text{nat} \mid \text{float}$

$\text{nat} = 0 \mid 1 \mid ...$

$\text{float} = 0.0 \mid -4.21 \mid 523.215 \mid ...$

Range Literal

$\text{rl} ::= \text{range}(\text{nat}, \text{nat}) \mid \text{range}(\text{nat}, \text{nat}) \cdot \text{rl}$

Place Expression

$p ::= x \mid p[t] \mid p\langle t \rangle \mid p.\text{fst} \mid p.\text{snd}$

Environment

Type Environment

$\Gamma ::= \bullet \mid \Gamma, (x : \tau)$

Kind Environment

$\Delta ::= \bullet$

- there are no contents to be used.

Typing Rules

$$\frac{\Delta; \Gamma \vdash t : \sigma \quad \neg\exists\tau.\Gamma(x) = \tau \quad \Delta; \Gamma, (x : \sigma) \vdash t_{\text{body}} : \sigma_{\text{body}}}{\Delta; \Gamma \vdash \text{let } x = t \text{ in } t_{\text{body}} : \sigma_{\text{body}}} \text{ T-LET}$$

$$\frac{\Gamma \vdash [\![\text{rl}]\!] = r \quad \Delta; \Gamma, (i : r) \vdash t_{\text{body}} : \sigma}{\Delta; \Gamma \vdash \text{for } i : \text{rl} \text{ in } t_{\text{body}} : r \cdot \sigma} \text{ T-FOR}$$

$$\frac{\Delta; \Gamma \vdash t : \overline{\eta_i \cdot \sigma} \quad \Delta; \Gamma \vdash [\![\text{rl}]\!] = \overline{\eta_i'..\eta_i''} \quad \overline{\eta_i''} \leq \overline{\eta_i}}{\Delta; \Gamma \vdash t\langle \text{rl} \rangle : \overline{(\eta_i'' - \eta_i')} \cdot \sigma} \text{ T-SLICE}$$

$$\frac{\Delta; \Gamma \vdash t : \eta_1 \cdot \sigma \quad \Delta \vdash [\![nat]\!] = \eta_2 \quad \eta_1 > \eta_2}{\Delta; \Gamma \vdash t[nat] : \sigma} \text{ T-INDEX-NAT}$$

$$\frac{\Delta; \Gamma \vdash t : \overline{\eta_i} \cdot \sigma \qquad \Delta \vdash t_{\text{index}} : \overline{\eta_i' .. \eta_i''} \qquad \overline{\eta_i''} \leq \overline{\eta_i}}{\Delta; \Gamma \vdash t[t_{\text{index}}] : \sigma} \text{ T-INDEX-RANGE}$$

$$\frac{\Delta; \Gamma \vdash t : \sigma_1 \times \sigma_2}{\Delta; \Gamma \vdash t.\text{fst} : \sigma_1} \text{ T-FST}$$

$$\frac{\Delta; \Gamma \vdash t : \sigma_1 \times \sigma_2}{\Delta; \Gamma \vdash t.\text{snd} : \sigma_2} \text{ T-SND}$$

$$\frac{}{\Delta; \Gamma \vdash \text{float} : \mathit{float}} \text{ T-FLOAT-LIT}$$

$$\frac{\Delta; \Gamma \vdash t_1 : \sigma_1 \qquad \Delta; \Gamma \vdash t_2 : \sigma_2}{\Delta; \Gamma \vdash (t_1, t_2) : \sigma_1 \times \sigma_2} \text{ T-TUPLE-LIT}$$

Kinding rules

$$\frac{}{\Delta \vdash [\![\text{nat}]\!] = \eta} \text{ K-NAT-LIT}$$

$$\frac{\Delta \vdash [\![\text{nat}_1]\!] = \eta_1 \qquad \Delta \vdash [\![\text{nat}_2]\!] = \eta_2 \qquad \eta_1 \leq \eta_2}{\Delta \vdash [\![\text{range}(\text{nat}_1, \text{nat}_2)]\!] = \eta_1 .. \eta_2} \text{ K-RANGE-ONE}$$

$$\frac{\Delta \vdash [\![\text{rl}_1]\!] = r_1 \qquad \Delta \vdash [\![\text{rl}_2]\!] = r_2}{\Delta \vdash [\![\text{rl}_1 \cdot \text{rl}_2]\!] = r_1 \cdot r_2} \text{ K-RANGE-MUL}$$

Examples

For expression

```
for i: range(0,5) . range(0,6) . range(0.7) in 4.2
```

This results in a value of type $(0..5) \cdot (0..6) \cdot (0..7) \cdot float$

```
for i : range(0,5) in for j: range(0,10) in 1.2
```

This results in a value of type $(0..5) \cdot (0..10) \cdot float$

Indexing by a value of type range

```
for i: range(0,5) in a[0][i]
```

This is equivalent to: `a[0][0:5]`

Slicing

```
a[range(0,10) . range(0,5)]
```

This is of type $(0..10) \cdot (0..5) \cdot \sigma$ where σ is the type of $a[0][0]$

let in

```
let arr =
  for i: range(0,5) in
    for j : range(0,5) in
      3.14159
  in arr[range(0,2).range(0,1)]
```

This is of type $(0..2) \cdot (0..1) \cdot float$

let in, for, and tuple

tuple

```
let arr_1 =
  for i: range(0,5) in
    for j: range(0,5) in
      3.14159 in
let arr_2 =
  for i: range(2,4) in
    for j: range(1,3) in
      arr_1[i][j] in
(arr_1, arr_2)
```

This is of type $((0..5) \cdot (0..5) \cdot float) \times ((0..2) \cdot (0..2) \cdot float)$

nested tuple/array

```
let tup = (3.14159, for i : range(0,5) in 6.25) in
  for i : range(0,10) in
    tup
```

This is of type $(0..10) \cdot (float \times ((0..5) \cdot float))$