

Grammar

Base Type

$\tau ::= \sigma \mid r$

$\sigma ::= \text{float} \mid \sigma \times \sigma \mid \eta \cdot \sigma$

(Meta) Natural Number

$\eta ::= 0 \mid 1 \mid \dots$

Range

$r ::= \eta.. \eta \mid r \cdot r$

Term

$t ::= l \mid p \mid x \mid \text{for } i : l \text{ in } t \mid t.\text{fst} \mid t.\text{snd} \mid \text{let } x = t \text{ in } t \mid (t, t)$

Literal

$l ::= \text{nat} \mid \text{float}$

$\text{nat} = 0 \mid 1 \mid \dots$

$\text{float} = 0.0 \mid -4.21 \mid 523.215 \mid \dots$

Range Literal

$\text{rl} ::= \text{range}(\text{nat}, \text{nat}) \mid \text{range}(\text{nat}, \text{nat}) \cdot \text{rl}$

Place Expression

$p ::= x \mid p[t] \mid p\langle t \rangle \mid p.\text{fst} \mid p.\text{snd}$

Environment

Type Environment

$\Gamma ::= \bullet \mid \Gamma, (x : \tau)$

Kind Environment

$\Delta ::= \bullet$

• there are no contents to be used.

Typing Rules

$$\frac{\Delta; \Gamma \vdash t : \sigma \quad \neg \exists \tau. \Gamma(x) = \tau}{\Delta; \Gamma, (x : \sigma) \vdash t_{\text{body}} : \sigma_{\text{body}}} \text{T-LET}$$
$$\frac{\Gamma \vdash \llbracket \text{rl} \rrbracket = r \quad \Delta; \Gamma, (i : r) \vdash t_{\text{body}} : \sigma}{\Delta; \Gamma \vdash \text{for } i : \text{rl} \text{ in } t_{\text{body}} : r \cdot \sigma} \text{T-FOR}$$
$$\frac{\Delta; \Gamma \vdash t : \overline{\eta_i} \cdot \overline{\sigma} \quad \Delta; \Gamma \vdash t_{\text{range}} : \overline{\eta'_i} \cdot \overline{\eta''_i} \quad \overline{\eta''_i} \leq \overline{\eta_i}}{\Delta; \Gamma \vdash t\langle t_{\text{range}} \rangle : (\overline{\eta'_i} - \overline{\eta''_i} + 1) \cdot \sigma} \text{T-SLICE}$$
$$\frac{\Delta; \Gamma \vdash t : \eta_1 \cdot \sigma \quad \Delta \vdash \llbracket \text{nat} \rrbracket = \eta_2 \quad \eta_1 > \eta_2}{\Delta; \Gamma \vdash t[\text{nat}] : \sigma} \text{T-INDEX-NAT}$$

$$\begin{array}{c}
\frac{\Delta; \Gamma \vdash t : \overline{\eta_i} \cdot \sigma \quad \Delta \vdash t_{\text{index}} : \overline{\eta'_i \cdot \eta''_i} \quad \overline{\eta''_i} \leq \overline{\eta_i}}{\Delta; \Gamma \vdash t[t_{\text{index}}] : \sigma} \text{T-INDEX-RANGE} \\
\\
\frac{\Delta; \Gamma \vdash t : \sigma_1 \times \sigma_2}{\Delta; \Gamma \vdash t.\text{fst} : \sigma_1} \text{T-FST} \\
\\
\frac{\Delta; \Gamma \vdash t : \sigma_1 \times \sigma_2}{\Delta; \Gamma \vdash t.\text{snd} : \sigma_2} \text{T-SND} \\
\\
\frac{}{\Delta; \Gamma \vdash \text{float} : \text{float}} \text{T-FLOAT-LIT} \\
\\
\frac{\Delta; \Gamma \vdash t_1 : \sigma_1 \quad \Delta; \Gamma \vdash t_2 : \sigma_2}{\Delta; \Gamma \vdash (t_1, t_2) : \sigma_1 \times \sigma_2} \text{T-TUPLE-LIT}
\end{array}$$

Kinding rules

$$\begin{array}{c}
\frac{}{\Delta \vdash \llbracket \text{nat} \rrbracket = \eta} \text{K-NAT-LIT} \\
\\
\frac{\Delta \vdash \llbracket \text{nat}_1 \rrbracket = \eta_1 \quad \Delta \vdash \llbracket \text{nat}_2 \rrbracket = \eta_2 \quad \eta_1 \leq \eta_2}{\Delta \vdash \llbracket \text{range}(\text{nat}_1, \text{nat}_2) \rrbracket = \eta_1 .. \eta_2} \text{K-RANGE-ONE} \\
\\
\frac{\Delta \vdash \llbracket \text{rl}_1 \rrbracket = r_1 \quad \Delta \vdash \llbracket \text{rl}_2 \rrbracket = r_2}{\Delta \vdash \llbracket \text{rl}_1 \cdot \text{rl}_2 \rrbracket = r_1 \cdot r_2} \text{K-RANGE-MUL}
\end{array}$$