

## Grammar

### Base Type

$\tau ::= \sigma \mid r$

$\sigma ::= \text{float} \mid \sigma \times \sigma \mid \eta \cdot \sigma$

### Natural Numbers

$\eta ::= 0 \mid 1 \mid \dots$

### Range

$r ::= \eta..\eta \mid r \cdot r$

### Term

$t ::= l \mid p \mid \text{for } i : r \text{ in } t \mid t.\text{fst} \mid t.\text{snd} \mid \text{let } x = t \text{ in } t \mid (t, t)$

•  $i$  and  $x$  are identifiers.

### Literal

$\text{fl} = 0.0 \mid -4.21 \mid 523.215 \mid \dots$

### Place Expression

$p ::= x \mid p[t] \mid p\langle t \rangle \mid p.\text{fst} \mid p.\text{snd}$

## Environment

### Type Environment

$\Gamma ::= \bullet \mid \Gamma, (x : \tau)$

## Typing Rules

$$\frac{\Gamma \vdash t : \sigma \quad \Gamma, (x : \sigma) \vdash t_{\text{body}} : \sigma_{\text{body}}}{\Gamma \vdash \text{let } x = t \text{ in } t_{\text{body}} : \sigma_{\text{body}}} \text{ T-LET}$$

$$\frac{r : \text{ok} \quad \Gamma, (i : r) \vdash t_{\text{body}} : \sigma}{\Gamma \vdash \text{for } i : r \text{ in } t_{\text{body}} : r \cdot \sigma} \text{ T-FOR}$$

$$\frac{\begin{array}{l} \Gamma \vdash t : \eta_1 \cdot \eta_2 \dots \cdot \eta_n \cdot \sigma \\ r = (\eta'_1..\eta''_1) \cdot (\eta'_2..\eta''_2) \dots \cdot (\eta'_n..\eta''_n) \\ r : \text{ok} \quad \forall i \in \{1, 2, \dots, n\}. \eta''_i \leq \eta_i \end{array}}{\Gamma \vdash t\langle r \rangle : (\eta''_1 - \eta'_1) \cdot (\eta''_2 - \eta'_2) \cdot \dots \cdot (\eta''_n - \eta'_n) \cdot \sigma} \text{ T-SLICE}$$

$$\frac{\Gamma \vdash t[\eta..(\eta + 1)]}{\Gamma \vdash t[\eta] : \sigma} \text{ T-INDEX-NAT}$$

$$\frac{\begin{array}{l} \Gamma \vdash t : \overline{\eta_i} \cdot \sigma \\ \Gamma \vdash t_{\text{index}} : (\eta'_1..\eta''_1) \cdot (\eta'_2..\eta''_2) \cdot \dots \cdot (\eta'_n..\eta''_n) \\ \forall i \in \{1, 2, \dots, n\}. \eta''_i \leq \eta_i \end{array}}{\Gamma \vdash t[t_{\text{index}}] : \sigma} \text{ T-INDEX-RANGE}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\text{fst} : \sigma_1} \text{ T-FST}$$

$$\begin{array}{c}
\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\text{snd} : \sigma_2} \text{T-SND} \\
\\
\frac{}{\Gamma \vdash \text{fl} : \textit{float}} \text{T-FLOAT-LIT} \\
\\
\frac{\Gamma \vdash t_1 : \sigma_1 \quad \Gamma \vdash t_2 : \sigma_2}{\Gamma \vdash (t_1, t_2) : \sigma_1 \times \sigma_2} \text{T-TUPLE-LIT}
\end{array}$$

## Well-formedness rules

$$\begin{array}{c}
\frac{\eta_1 \leq \eta_2}{\eta_1.. \eta_2 : \text{ok}} \text{W-RANGE-ONE} \\
\\
\frac{r_1 : \text{ok} \quad r_2 : \text{ok}}{r_1 \cdot r_2 : \text{ok}} \text{W-RANGE-MUL}
\end{array}$$

## Examples

### For expression

```
for i: range(0,5) . range(0,6) . range(0,7) in 4.2
```

This results in a value of type  $5 \cdot 6 \cdot 7 \cdot \text{float}$

```
for i : range(0,5) in for j: range(0,10) in 1.2
```

This results in a value of type  $5 \cdot 10 \cdot \text{float}$

### Indexing by a value of type range

```
for i: range(0,5) in a[0][i]
```

This is equivalent to: `a[0][0:5]`

### Slicing

```
a[range(0,10) . range(0,5)]
```

This is of type  $10 \cdot 5 \cdot \sigma$  where  $\sigma$  is the type of `a[0][0]`

### let in

```
let arr =  
  for i: range(0,5) in  
    for j : range(0,5) in  
      3.14159  
  in arr[range(0,2).range(0,1)]
```

This is of type  $2 \cdot 1 \cdot \text{float}$

### let in, for, and tuple

#### tuple

```
let arr_1 =  
  for i: range(0,5) in  
    for j: range(0,5) in  
      3.14159 in  
let arr_2 =  
  for i: range(2,4) in  
    for j: range(1,3) in  
      arr_1[i][j] in  
(arr_1, arr_2)
```

This is of type  $(5 \cdot 5 \cdot \text{float}) \times (2 \cdot 2 \cdot \text{float})$

#### nested tuple/array

```
let tup = (3.14159, for i : range(0,5) in 6.25) in  
  for i : range(0,10) in  
    tup
```

This is of type  $10 \cdot (\text{float} \times (5 \cdot \text{float}))$