Grammar

Base Type

$\tau ::= \sigma \mid r$

$\sigma ::= \mathit{float} \mid \sigma \times \sigma \mid \eta \cdot \sigma$

Natural Numbers

$\eta ::= 0 \mid 1 \mid ...$

Range

$r ::= \eta..\eta \mid r \cdot r$

Term

$t ::= \text{fl} \mid p \mid \text{for } i : r \text{ in } t \mid t.\text{fst} \mid t.\text{snd} \mid \text{let } x = t \text{ in } t \mid (t, t)$

- i and x are identifiers.

Literal

$\text{fl} = 0.0 \mid -4.21 \mid 523.215 \mid ...$

Place Expression

$p ::= x \mid p[t] \mid p\langle t \rangle \mid p.\text{fst} \mid p.\text{snd}$

Environment

Type Environment

$\Gamma ::= \bullet \mid \Gamma, (x : \tau)$

Typing Rules

$$\frac{\Gamma \vdash t : \sigma \quad \Gamma, (x : \sigma) \vdash t_{\text{body}} : \sigma_{\text{body}}}{\Gamma \vdash \text{let } x = t \text{ in } t_{\text{body}} : \sigma_{\text{body}}} \text{ T-LET}$$

$$\frac{r : \text{ok} \quad \Gamma, (i : r) \vdash t_{\text{body}} : \sigma}{\Gamma \vdash \text{for } i : r \text{ in } t_{\text{body}} : r \cdot \sigma} \text{ T-FOR}$$

$$\frac{\begin{array}{c}\Gamma \vdash t : \eta_1 \cdot \eta_2 ... \cdot \eta_n \cdot \sigma \\ r = (\eta_1'..\eta_1'') \cdot (\eta_2'..\eta_2'') ... \cdot (\eta_n'..\eta_n'') \\ r : \text{ok} \quad \forall i \in \{1, 2, ..., n\}.\eta_i'' \leq \eta_i\end{array}}{\Gamma \vdash t\langle r \rangle : (\eta_1'' - \eta_1') \cdot (\eta_2'' - \eta_2') \cdot ... \cdot (\eta_n'' - \eta_n') \cdot \sigma} \text{ T-SLICE}$$

$$\frac{\Gamma \vdash t[\eta..(\eta + 1)]}{\Gamma \vdash t[\eta] : \sigma} \text{ T-INDEX-NAT}$$

$$\frac{\begin{array}{c}\Gamma \vdash t : \overline{\eta_i} \cdot \sigma \\ \Gamma \vdash t_{\text{index}} : (\eta_1'..\eta_1'') \cdot (\eta_2'..\eta_2'') \cdot ... \cdot (\eta_n'..\eta_n'') \\ \forall i \in \{1, 2, ..., n\}.\eta_i'' \leq \eta_i\end{array}}{\Gamma \vdash t[t_{\text{index}}] : \sigma} \text{ T-INDEX-RANGE}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\text{fst} : \sigma_1} \text{ T-FST}$$

$$\frac{\Gamma \vdash t : \sigma_1 \times \sigma_2}{\Gamma \vdash t.\mathrm{snd} : \sigma_2}\ \text{T-SND}$$

$$\frac{}{\Gamma \vdash \mathrm{fl} : \mathit{float}}\ \text{T-FLOAT-LIT}$$

$$\frac{\Gamma \vdash t_1 : \sigma_1 \quad \Gamma \vdash t_2 : \sigma_2}{\Gamma \vdash (t_1, t_2) : \sigma_1 \times \sigma_2}\ \text{T-TUPLE-LIT}$$

Well-formedness rules

$$\frac{\eta_1 \leq \eta_2}{\eta_1..\eta_2 : \mathrm{ok}}\ \text{W-RANGE-ONE}$$

$$\frac{r_1 : \mathrm{ok} \quad r_2 : \mathrm{ok}}{r_1 \cdot r_2 : \mathrm{ok}}\ \text{W-RANGE-MUL}$$

Examples

For expression

```
for i: range(0,5) . range(0,6) . range(0.7) in 4.2
```

This results in a value of type $5 \cdot 6 \cdot 7 \cdot float$

```
for i : range(0,5) in for j: range(0,10) in 1.2
```

This results in a value of type $5 \cdot 10 \cdot float$

Indexing by a value of type range

```
for i: range(0,5) in a[0][i]
```

This is equivalent to: `a[0][0:5]`

Slicing

```
a[range(0,10) . range(0,5)]
```

This is of type $10 \cdot 5 \cdot \sigma$ where σ is the type of $a[0][0]$

let in

```
let arr =
  for i: range(0,5) in
    for j : range(0,5) in
      3.14159
  in arr[range(0,2).range(0,1)]
```

This is of type $2 \cdot 1 \cdot float$

let in, for, and tuple

tuple

```
let arr_1 =
  for i: range(0,5) in
    for j: range(0,5) in
      3.14159 in
let arr_2 =
  for i: range(2,4) in
    for j: range(1,3) in
      arr_1[i][j] in
(arr_1, arr_2)
```

This is of type $(5 \cdot 5 \cdot float) \times (2 \cdot 2 \cdot float)$

nested tuple/array

```
let tup = (3.14159, for i : range(0,5) in 6.25) in
  for i : range(0,10) in
    tup
```

This is of type $10 \cdot (float \times (5 \cdot float))$