

NTE Queueing Networks

Michele Luca Puzzo 1783133

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1 Exercise 3

Jackson networks

Exercise 3

Consider an ISP network with J links connecting routers.

- The offered traffic matrix and the routing in the network are such that it is possible to maintain link utilization equal to a constant value $\rho_0 = 0.7$ for all links.
- The volume of the overall input traffic is increased by 10% (i.e., the mean rate of each flow of packets offered to the network is increased by 10%).

Calculate the percentage increase of the mean delay of a packet through the network.

In the steady state in an Open Jackson Network I have: $\lambda = \gamma[\mathbf{I} - \mathbf{R}]^{-1}$ and the mean time of a customer through the network is:

$$E[D] = \frac{1}{\gamma} \sum_{j=1}^J \left(\frac{\rho_0}{1-\rho_0} + \lambda_j \sum_{h=1}^J r_{jh} \tau_{jh} \right)$$

At a certain point the volume of the overall input traffic is increased by 10%:

$$\gamma_{new} = \gamma + 0.1\gamma = 1.1\gamma$$

Consequently:

$$\lambda_{new} = 1.1\gamma[\mathbf{I} - \mathbf{R}]^{-1} = 1.1\lambda$$

$$\rho_{new} = \frac{\lambda_{new}}{\mu} = 1.1 \cdot \rho_0 = 1.1 \cdot 0.7 = 0.77$$

Without considering the propagation delay through the network I have:

$$E[D_{new}] = \frac{1}{1.1\gamma} \sum_{j=1}^J \left(\frac{1.1\rho_0}{1-1.1\rho_0} \right)$$

$$\frac{E[D_{new}]}{E[D]} = \frac{\frac{1}{1.1\gamma}}{\frac{1}{\gamma}} \cdot \frac{\sum_{j=1}^J \left(\frac{1.1\rho_0}{1-1.1\rho_0} \right)}{\sum_{j=1}^J \left(\frac{\rho_0}{1-\rho_0} \right)} = \frac{1}{1.1} \cdot \frac{\frac{1.1 \cdot \rho_0}{1-1.1 \cdot \rho_0} \cdot J}{\frac{\rho_0}{1-\rho_0} \cdot J} = \frac{(1-\rho_0)}{1-1.1 \cdot \rho_0} \approx \mathbf{1.304}$$

So the percentage increase of the mean delay of a packet through the network is **30.4%**

Considering the propagation delay through the network:

$$\begin{aligned} E[D_{new}] &= \frac{1}{1.1\gamma} \sum_{j=1}^J \left(\frac{1.1\rho_0}{1-1.1\rho_0} + 1.1\lambda_j \sum_{h=1}^J r_{jh}\tau_{jh} \right) \\ \frac{E[D_{new}]}{E[D]} &= \frac{\frac{1}{1.1\gamma}}{\frac{1}{\gamma}} \cdot \frac{\sum_{j=1}^J \left(\frac{1.1\rho_0}{1-1.1\rho_0} \right)}{\sum_{j=1}^J \left(\frac{\rho_0}{1-\rho_0} \right)} \cdot \frac{\sum_{j=1}^J \left(1.1\lambda_j \sum_{h=1}^J r_{jh}\tau_{jh} \right)}{\sum_{j=1}^J \left(\lambda_j \sum_{h=1}^J r_{jh}\tau_{jh} \right)} = \frac{1}{1.1} \cdot \frac{\frac{1.1 \cdot \rho_0}{1-1.1 \cdot \rho_0} \cdot J}{\frac{\rho_0}{1-\rho_0} \cdot J} \cdot 1.1 \frac{E[T]}{E[T]} \\ \frac{E[D_{new}]}{E[D]} &= \frac{1.1 \cdot (1-\rho_0)}{1-1.1 \cdot \rho_0} \approx \mathbf{1.435} \end{aligned}$$

So the percentage increase of the mean delay of a packet through the network is **43.5%**

I can conclude that the increasing the overall input traffic by 10% involves an higher increasing in percentage of the mean delay through the network, even higher if I consider also the propagation delay.