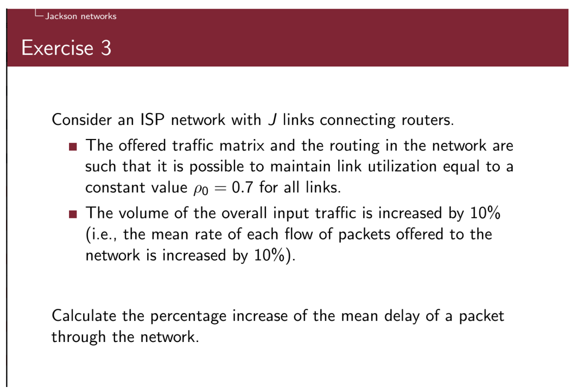
NTE Queueing Networks

Michele Luca Puzzo 1783133

April 2022

1 Exercise 3



In the steady state in an Open Jackson Network I have: $\lambda = \gamma[\mathbf{I} - \mathbf{R}]^{-1}$ and the mean time of a customer through the network is:
$E[D] = \frac{1}{\gamma} \sum_{j=1}^{J} \left(\frac{\rho_0}{1-\rho_0} + \lambda_j \sum_{h=1}^{J} r_{jh}\tau_{jh} \right)$

At a certain point the volume of the overall input traffic is increased by 10%:

$\gamma_{new} = \gamma + 0.1\gamma = 1.1\gamma$

Consequently:

$\lambda_{new} = 1.1\gamma[\mathbf{I} - \mathbf{R}]^{-1} = 1.1\lambda$

$\rho_{new} = \frac{\lambda_{new}}{\mu} = 1.1 \cdot \rho_0 = 1.1 \cdot 0.7 = 0.77$

Without considering the propagation delay through the network I have:

$E[D_{new}] = \frac{1}{1.1\gamma} \sum_{j=1}^{J} \left(\frac{1.1\rho_0}{1-1.1\rho_0} \right)$

$$\frac{E[D_{new}]}{E[D]} = \frac{\frac{1}{1.1\gamma}}{\frac{1}{\gamma}} \cdot \frac{\sum_{j=1}^{J}\left(\frac{1.1\rho_0}{1-1.1\rho_0}\right)}{\sum_{j=1}^{J}\left(\frac{\rho_0}{1-\rho_0}\right)} = \frac{1}{1.1} \cdot \frac{\frac{1.1\cdot\rho_0}{1-1.1\cdot\rho_0}\cdot J}{\frac{\rho_0}{1-\rho_0}\cdot J} = \frac{(1-\rho_0)}{1-1.1\cdot\rho_0} \approx \mathbf{1.304}$$

So the percentage increase of the mean delay of a packet through the network is $\mathbf{30.4}\%$

Considering the propagation delay through the network:

$$E[D_{new}] = \frac{1}{1.1\gamma}\sum_{j=1}^{J}\left(\frac{1.1\rho_0}{1-1.1\rho_0} + 1.1\lambda_j \sum_{h=1}^{J} r_{jh}\tau_{jh}\right)$$

$$\frac{E[D_{new}]}{E[D]} = \frac{\frac{1}{1.1\gamma}}{\frac{1}{\gamma}} \cdot \frac{\sum_{j=1}^{J}\left(\frac{1.1\rho_0}{1-1.1\rho_0}\right)}{\sum_{j=1}^{J}\left(\frac{\rho_0}{1-\rho_0}\right)} \cdot \frac{\sum_{j=1}^{J}\left(1.1\lambda_j \sum_{h=1}^{J} r_{jh}\tau_{jh}\right)}{\sum_{j=1}^{J}\left(\lambda_j \sum_{h=1}^{J} r_{jh}\tau_{jh}\right)} = \frac{1}{1.1} \cdot \frac{\frac{1.1\cdot\rho_0}{1-1.1\cdot\rho_0}\cdot J}{\frac{\rho_0}{1-\rho_0}\cdot J} \cdot 1.1\frac{E[T]}{E[T]}$$

$$\frac{E[D_{new}]}{E[D]} = \frac{1.1\cdot(1-\rho_0)}{1-1.1\cdot\rho_0} \approx \mathbf{1.435}$$

So the percentage increase of the mean delay of a packet through the network is $\mathbf{43.5}\%$

I can conclude that the increasing the overall input traffic by 10% involves an higher increasing in percentage of the mean delay through the network, even higher if I consider also the propagation delay.