NTE Scheduling and LB

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1 Exercise 2

Exercise 2

- A server receives service demands from two flows
  - Flow 1 is made of small jobs, arriving with mean rate $\lambda_1 = 0.25$ *job/s* and deterministic service time of duration $1/\mu_1 = 1$ *s*; the second flow has mean rate $\lambda_2 = 1$ *job/s*, with mean service time $1/\mu_2 = 0.7$ *s* and $COV_2 = 10$.
  - Arrivals follow Poisson processes and service time for each flow are i.i.d. r.v.s.
- What is the worst case delay of class 2 for any non-preemptive service policy?
- Evaluate $\mathrm{E}[W_1]$ and $\mathrm{E}[W_2]$ in case the static priority HOL policy is used, assigning high priority level to flow 1.

1.1 Question 1

Since jobs belonging to first has deterministic service time then:

$\sigma^2_{X_1} = 0 \Rightarrow E[X_1^2] = (E[X_1])^2 = 1$

From the definition of the COV:it is possibile to obtain the second moment of the service time of jobs belonging to second flow:

$COV = \frac{\sigma_{X_2}}{E[X_2]} = \frac{\sqrt{E[X_2^2]-(E[X_2])^2}}{E[X_2]}$

$\Rightarrow \mathrm{E}[\mathrm{X}_2^2] = (COV \cdot E[X_2])^2 + (E[X_2])^2 = (10 \cdot 0.7)^2 + 0.7^2 = 49.49s^2$

The utilization factor for the flow 1 is:

$\rho_1 = \lambda_1 \cdot E[X_1] = 0.25 \cdot 1 = 0.25$

The utilization factor for the flow 2 is:

$\rho_2 = \lambda_2 \cdot E[X_2] = 1 \cdot 0.7 = 0.7$

The overall utilization factor is:

$\rho = \lambda_2 \cdot E[X_2] + \lambda_1 \cdot E[X_1] = 0.7 + 0.25 = 0.95$

The residual service time found by the arriving customer is:

$W_0 = \frac{1}{2}\sum_{i=1}^{2} \lambda_i \cdot E[X_i^2] = \frac{1}{2}(\lambda_1 \cdot E[X_1^2] + \lambda_2 \cdot E[X_2^2])$

$\mathrm{W}_0 = 0.5 \cdot (0.25 \cdot 1 + 1 \cdot 49.5) = 24.875s$

Applying the Conservation law for HOL, non-anticipative systems:

$\sum_{i=1}^{2} \frac{\rho_i}{\rho} E[W_i] = E[W_{FCFS}] = \frac{W_0}{1-\rho} \rightarrow (\frac{\rho_1}{\rho} E[W_1] + \frac{\rho_2}{\rho} E[W_2]) = \frac{W_0}{1-\rho}$

Since I am looking for the worst case delay of class 2 then I am going to consider $E[W_1] = 0$ indeed "The conservation law says that improving one class comes to the detriment of the performance experienced by other classes" and $E[W_1] = 0$ is the best possible improving for class 1. Then:

$E[W_2] = \frac{W_0}{1-\rho} \cdot \frac{\rho}{\rho 2} = \frac{24.875}{1-0.95} \cdot \frac{0.95}{0.7} = \mathbf{675.18s}$

With Conservation law I am taking into account FCFS and Static Priority policies because they are HOL,non-anticipative and non-preemptive ones. I have also consider SJF because it is a non-preemptive policy but I know that it is anticipative so it has an additional piece of information respect to the other two policies indeed the overall satisfaction of customers is higher (not all tasks, the longer ones, will be happier), the overall average on the entire population of the waiting line is less than FCFS: $\overline{W}_{FCFS} \geq \overline{W}_{SJF}$ and so also the worst case will not be worse of FCFS.

1.2 Question 2

$E[W_1] = \frac{W_0}{1-\rho_1} = \frac{24.875}{1-0.25} \approx \mathbf{33.2s}$

$E[W_2] = \frac{W_0}{(1-\rho_1)\cdot(1-\rho)} = \frac{24.875}{(1-0.25)\cdot(1-0.95)} \approx \mathbf{663.3s}$

We have obtained a value for $E[W_2]$ that is indeed less than the worst case delay. The considerable value for $E[W_1]$ is due to "high variance service times of low priority customers affect also high priority customers".