Homework 1

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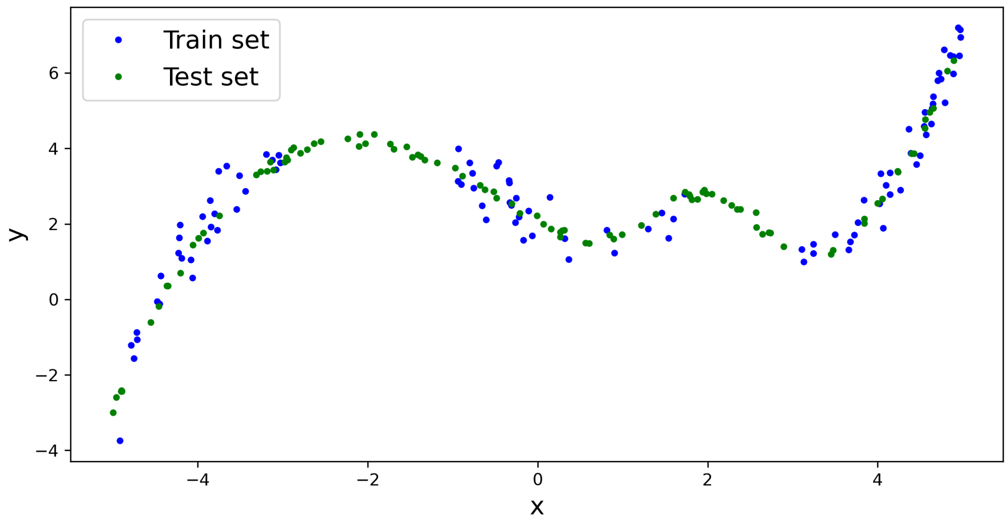
Regressions

Problem 1

The goal of this project is to develop a dense neural network able to approximate a scalar function:

$$f:\mathbb{R}\rightarrow\mathbb{R},\quad x\rightarrow f(x);\quad \text{network}(x)=g(x)\simeq f(x).$$

The train dataset is 100 points and the test dataset is 100 points. In both of them a noise is added on the y axis in the following way $y=f(x)+noise$. In the figure below they are both visualized in a two dimensional plane.



Methods 1

To approximate the target function $f(x)$ we use a dense neural network (DNN) with three hidden layers. The structure of the DNN is quite simple since the train dataset is small and the problem is not too complex.

Layer	Dim	Act fun
Input	1	sigmoid
Hidden 1	N_{H_1}	sigmoid
Hidden 2	N_{H_2}	sigmoid
Hidden 3	N_{H_3}	sigmoid
Output	1	

The loss is a Mean Square Error loss (MSE):

$$loss = \sum_i (y_i - g(x_i))^2$$

where $g(x)$ is the approximate function. We use a L2 regularization to avoid overfitting. Since the training set is small we use a K-fold method (with $k = 3$) to choose the best model. We use $pytorch$ to implement the neural network and $optuna$ to do the hyperparameter tuning. With $optuna$ we perform a random search with 25 trials. Batch size is 20 and the number of epochs is 1000. The hyperparameters are the parameters of the DNN, the learning rate, the optimization algorithm and the regularization.

Results 1

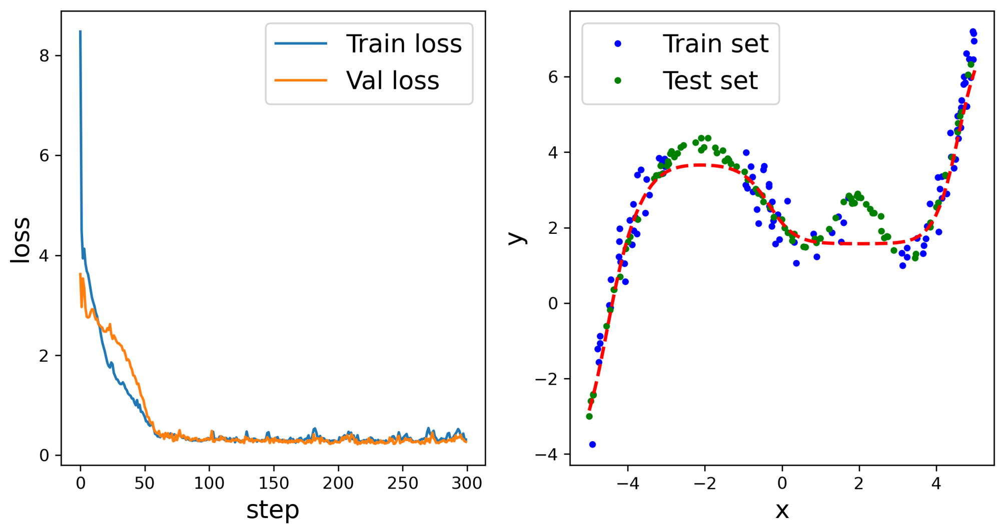
After the 25 trials the best model is :

Hyperparameter	Range	Best value
Hidden 1	10 - 50	45
Hidden 2	10 - 50	21
Hidden 3	3 - 30	23
learning rate	$10^{-3} - 10^{-1}$	0.016
L2 regularization	$10^{-4} - 10^{-1}$	0.0003
Optimizer	Adam, SGD, RMS	Adam

The test loss is:

$$testloss \simeq 0.27$$

Below the learning profile (left) and the best model approximation (right)



We visualize the weight distribution of each layer and the activation profile of each layer given different inputs. All this information can be found in the appendix.

Comments and possible improvements 1

As we can see in the figure above the predicted function it is not perfect and fails to approximate the test set in particular around -2 and 2. This problem can be partially solved by increasing the number of nodes of each layer and increasing the number of trials of the hyperparameter tuning.

Classification

Problem 2

The goal of this project is to classify images of clothes. For this task we use a DNN and a CNN.

The dataset we use is the fashionMNIST that is a collection of 28 x 28 images of clothes divided in 10 classes:

The train dataset has 60000 images and the test dataset 10000 images.

Methods 2

To classify the images we use two different networks, the first one is a DNN and the second is a CNN. The structure of the DNN is the same as the regression except for the output dimension that is 10. The structure of the CNN is the following:

Layer	Dim	Info	Act fun
Input	1 x 28 x 28		ReLU
Conv 1	N_{C_1}	kernel = 4, padding = 1	ReLU
Conv 2	N_{C_2}	kernel = 5, padding = 1	ReLU
Hidden 1	N_{H_1}		Sigmoid
Hidden 2	N_{H_2}		Sigmoid
Output	10		

The loss is the Cross Entropy :

$$loss = -\sum_{i=1}^{10} y_i \log p_i$$

where y_i is 1 if the prediction is correct otherwise is 0; p_i is the predicted probability.

We use a L2 regularization to avoid overfitting. Even in this case we use a K-fold method (with $k = 3$) to choose the best model. We use $pytorch$ to implement the neural network and $optuna$ to do the hyperparameter tuning. With $optuna$ we perform a random search with 10 trials. Batch size is 200 and the number of epochs is 20. The hyperparameters are the parameters of the CNN, the learning rate, the optimization algorithm and the regularization.

Results 2

After the 10 trials the best DNN model is :

Hyperparameter	Range	Best value
Hidden 1	10 - 50	49
Hidden 2	10 - 50	26
Hidden 3	3 - 30	24
learning rate	$10^{-3} - 10^{-1}$	0.002
L2 regularization	$10^{-4} - 10^{-1}$	0.0001
Optimizer	Adam, SGD, RMS	Adam

The test loss is:

$$testloss \simeq 0.35$$

Accuracy:

$$\frac{\text{right pred}}{\text{test set}} = 87$$

After the 10 trials the best CNN model is :

Hyperparameter	Range	Best value
N_{C_1}	4 - 10	4

Hyperparameter	Range	Best value
N_{C_1}	10 - 15	12
Hidden 1	20 - 50	38
Hidden 2	20 - 50	28
learning rate	$10^{-3} - 10^{-1}$	0.008
L2 regularization	$10^{-4} - 10^{-1}$	0.0001
Optimizer	Adam, SGD, RMS	Adam

The test loss is:

$$testloss \simeq 0.30$$

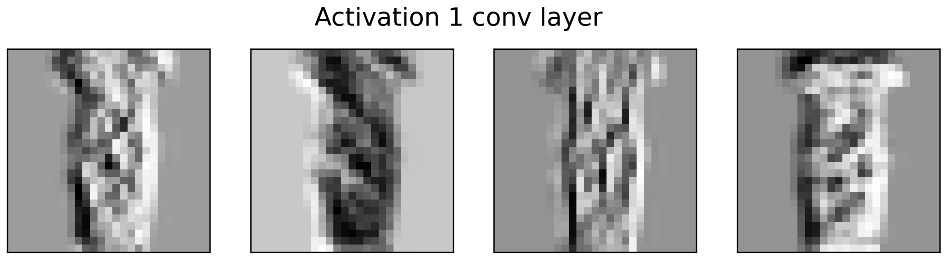
Accuracy:

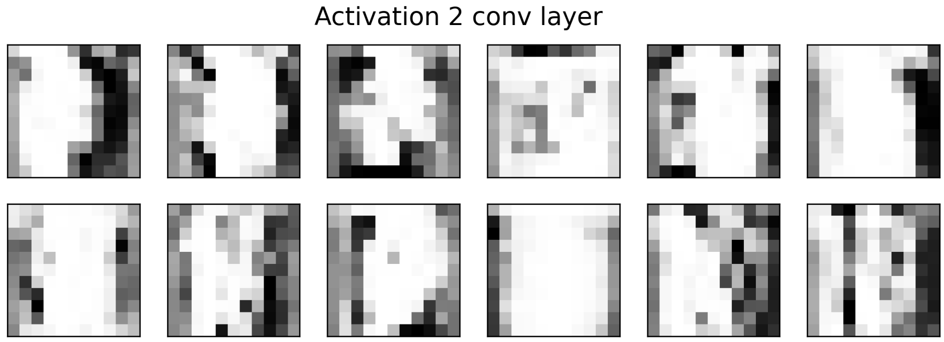
$$\frac{\text{right pred}}{\text{test set}} = 89$$

Confusion matrix:

For the CNN we do further analysis: we visualize the weight distribution of each dense layer (appendix), the receptive fields of the convolutional layers (appendix) and the activation profiles of the convolutional layers.

Activation profile:





If we look at the activation profiles we can see that the first convolutional layer captures the overall features of the image; in fact we can still distinguish the shapes of the original image. Whereas the second layer captures the more abstract features of the image and it is difficult to interpret.

Comments and possible improvements 2

With both the CNN and DNN we are able to predict the right class $\simeq 87$ of the times. This dataset is quite simple and so both models work very good. If we work with larger images with colors (3 channels) it is not possible to use a DNN and achieve good performances.

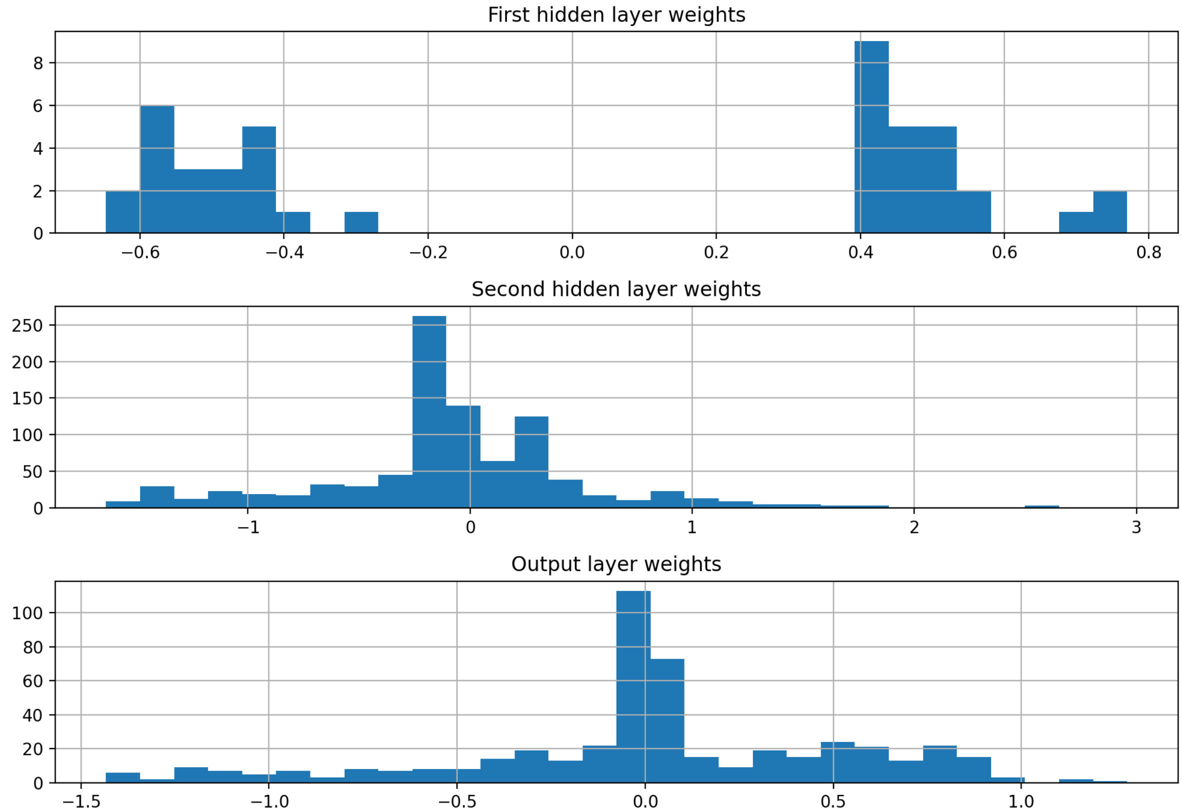
If we look at the confusion matrix (CNN) it is evident that the three problematic classes are: T-shirt, Pullover and Shirt. In particular the last one is often confused with one of the other two which make sense since they are hardly distinguishable.

Even in this case performances can be improved by adding more convolutional layers and/or performing more trials during hyperparameters tuning. In this homework I was limited by the computational power of my laptop.

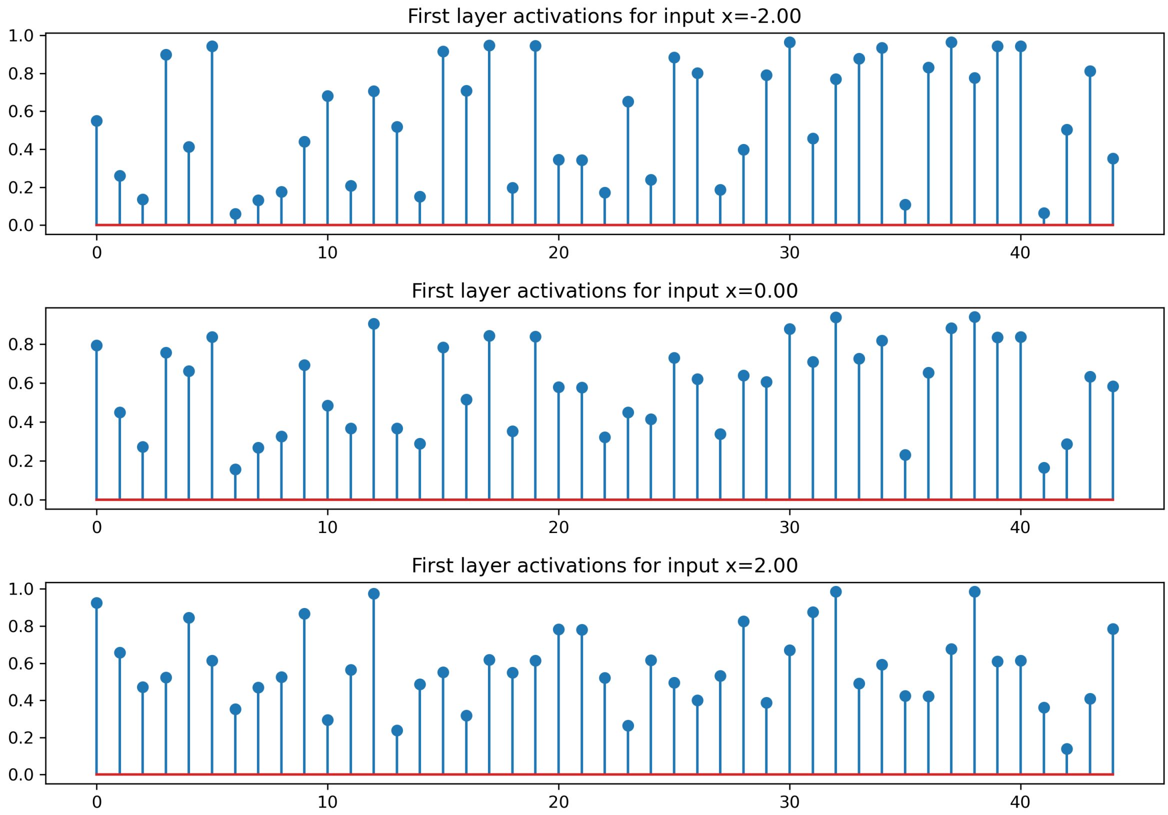
Appendix

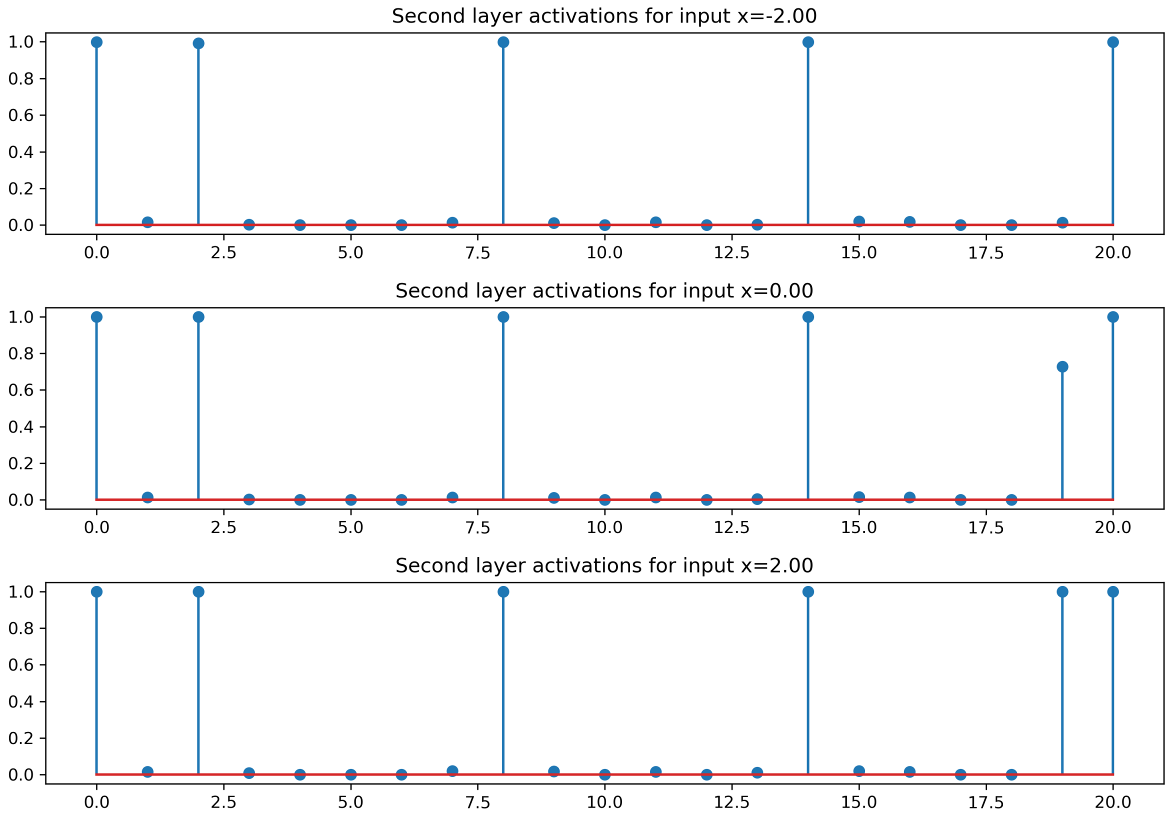
Regression

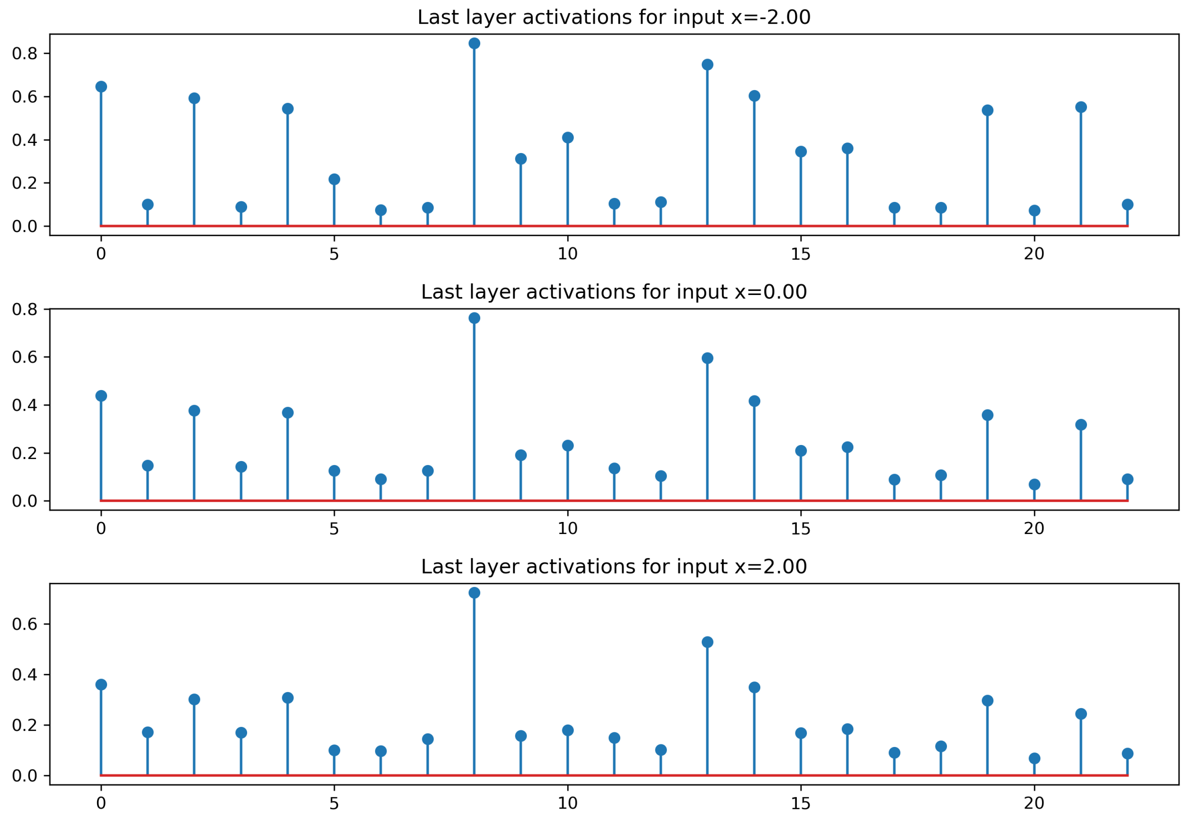
Distribution of weights:



Activation profiles:

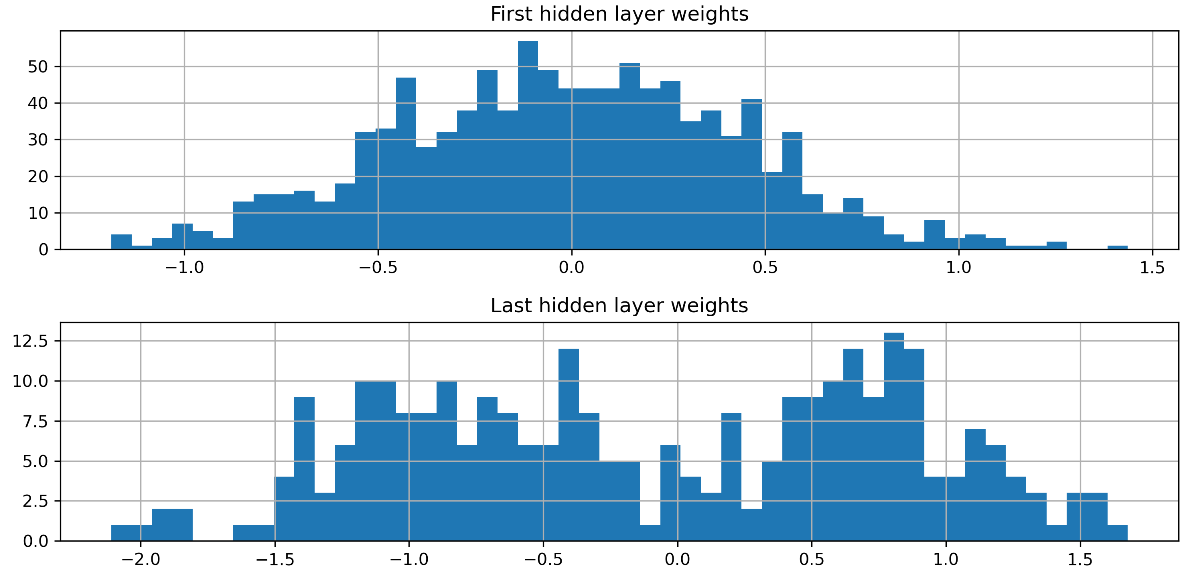






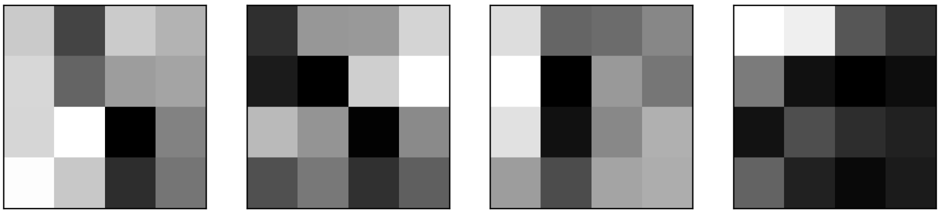
Classification (CNN)

Distribution of weights:



Receptive fields:

Receptive field 1 conv layer



Receptive fields 2 conv layer