degeneracyCount

December 10, 2025

[31]:
```
import numpy as np
import matplotlib.pyplot as plt
import tqdm
```

[32]:
```
import sys, pathlib
sys.path.insert(0, str(pathlib.Path.cwd().parent))
```

[33]:
```
from decoding.beliefPropagation import performBeliefPropagationFast
from decoding.OSD import performOSD
```

[34]:
```
codes = [
    "[[72, 12, 6]]",
    "[[90, 8, 10]]",
    "[[108, 8, 10]]",
    "[[144, 12, 12]]",
    "[[288, 12, 18]]",
]

trials = 1000
physicalErrorRates = np.logspace(-3.2, -1.3, 8)
results_BP = {}
```

[35]:
```
np.random.seed(0)
```

[36]:
```
for code in tqdm.tqdm(codes):
    oc = np.load(f'../codes/{code}.npz')
    name = code
    results_BP[name] = {}
    code = oc['Hx']
    Lx = oc['Lx']
    distance = oc['distance']
    n = len(code[0])
    logicalErrorRates = []
    BPs_fault_rates = []
    BPs_miscorrected_rates = []
    incorrectable_rates = []
    degeneracies = []
```

```
    for errorRate in physicalErrorRates:

        initialBeliefs = [np.log((1 - errorRate) / errorRate)] * n
        logical_error = 0
        BPs_fault = 0
        BPs_miscorrected = 0
        incorrectable = 0
        degenerateErrors = 0


        for _ in range(trials):

            #### CODE CAPACITY ERROR MODEL ####
            # non-trivial pythonic way to generate random bitstring with given␣
↪error rate
            error = (np.random.random(n) < errorRate).astype(int)

            syndrome = (error @ code.T) % 2

            #### SIMPLE PHENOMENOLOGICAL ERROR MODEL ####
            # measurementError = (np.random.random(len(syndrome)) < errorRate).
↪astype(int)
            # syndrome = (syndrome + measurementError) % 2

            detection, isSyndromeFound, llrs =␣
↪performBeliefPropagationFast(code, syndrome, initialBeliefs, verbose=False)

            # if not isSyndromeFound:
                # logical_error += 1
                # BPs_fault += 1

                # detection = performOSD(code, syndrome, llrs, detection)



            # This is the XOR, between the actual error and the detected error.␣
↪We are simulating the correction of the error
            residual = (detection + error) % 2

            syndromeLogic = (Lx @ residual) % 2

            if isSyndromeFound and not np.any(syndromeLogic) and (np.
↪array_equal(detection, error) == False):
                degenerateErrors += 1

            if np.any(syndromeLogic):
                logical_error += 1
```

```
                error_weight = np.sum(error)
                if error_weight < (distance // 2):
                    BPs_miscorrected += 1
                else:
                    incorrectable += 1


        ler = logical_error / trials
        logicalErrorRates.append(ler)
        BPs_fault_rates.append(BPs_fault)
        BPs_miscorrected_rates.append(BPs_miscorrected)
        incorrectable_rates.append(incorrectable)
        degeneracies.append(degenerateErrors)

    results_BP[name]['ler'] = logicalErrorRates
    results_BP[name]['BPs_fault'] = BPs_fault_rates
    results_BP[name]['BPs_miscorrected'] = BPs_miscorrected_rates
    results_BP[name]['incorrectable'] = incorrectable_rates
    results_BP[name]['degeneracies'] = degeneracies
```

```
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```
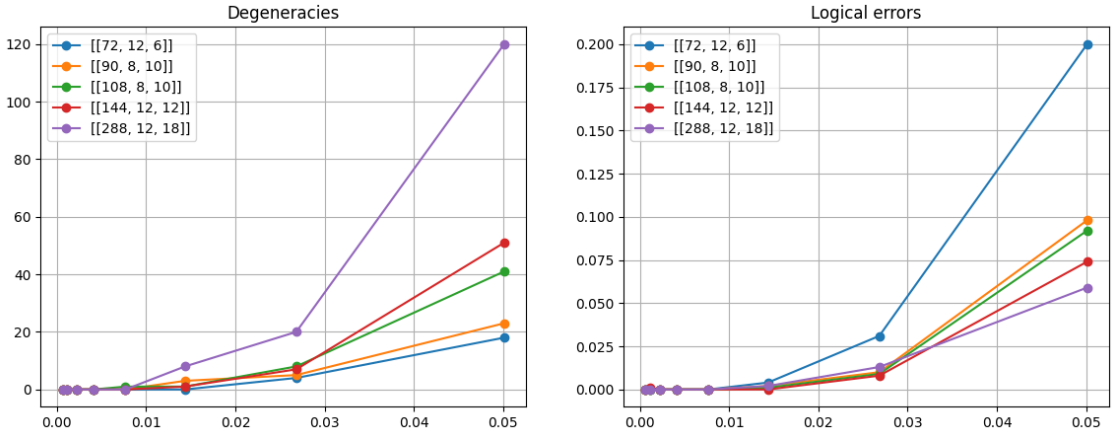

[37]:
```
fig, axes = plt.subplots(1, 2, figsize=(14, 5))

for name in results_BP:
    axes[0].plot(physicalErrorRates, results_BP[name]['degeneracies'],␣
 ↪label=name, marker='o')
    axes[0].grid(True)
    axes[0].legend()
    axes[0].set_title('Degeneracies')

for name in results_BP:
    axes[1].plot(physicalErrorRates, results_BP[name]['ler'], label=name,␣
 ↪marker='o')
    axes[1].grid(True)
    axes[1].legend()
    axes[1].set_title('Logical errors')
```



[38]:
```
results_OSD = {}

for code in tqdm.tqdm(codes):
    oc = np.load(f'../codes/{code}.npz')
    name = code
    results_OSD[name] = {}
    code = oc['Hx']
    Lx = oc['Lx']
    distance = oc['distance']
    n = len(code[0])
    logicalErrorRates = []
    BPs_fault_rates = []
    BPs_miscorrected_rates = []
    incorrectable_rates = []
    degeneracies = []

    for errorRate in physicalErrorRates:

        initialBeliefs = [np.log((1 - errorRate) / errorRate)] * n
        logical_error = 0
        BPs_fault = 0
        BPs_miscorrected = 0
        incorrectable = 0
        degenerateErrors = 0


        for _ in range(trials):

            #### CODE CAPACITY ERROR MODEL ####
            # non-trivial pythonic way to generate random bitstring with given␣
 ↪error rate
```

```
            error = (np.random.random(n) < errorRate).astype(int)

            syndrome = (error @ code.T) % 2

            #### SIMPLE PHENOMENOLOGICAL ERROR MODEL ####
            # measurementError = (np.random.random(len(syndrome)) < errorRate).
↪astype(int)
            # syndrome = (syndrome + measurementError) % 2

            detection, isSyndromeFound, llrs =␣
↪performBeliefPropagationFast(code, syndrome, initialBeliefs, verbose=False)

            if not isSyndromeFound:
                # logical_error += 1
                # BPs_fault += 1

                detection = performOSD(code, syndrome, llrs, detection)



            # This is the XOR, between the actual error and the detected error.␣
↪We are simulating the correction of the error
            residual = (detection + error) % 2

            syndromeLogic = (Lx @ residual) % 2

            osd_syndrome_check = (detection @ code.T) % 2
            is_valid_osd = np.array_equal(osd_syndrome_check, syndrome)

            if is_valid_osd and not np.any(syndromeLogic) and (np.
↪array_equal(detection, error) == False):
                degenerateErrors += 1

            if np.any(syndromeLogic):
                logical_error += 1

                error_weight = np.sum(error)
                if error_weight < (distance // 2):
                    BPs_miscorrected += 1
                else:
                    incorrectable += 1


        ler = logical_error / trials
        logicalErrorRates.append(ler)
        BPs_fault_rates.append(BPs_fault)
        BPs_miscorrected_rates.append(BPs_miscorrected)
```

```
        incorrectable_rates.append(incorrectable)
        degeneracies.append(degenerateErrors)

    results_OSD[name]['ler'] = logicalErrorRates
    results_OSD[name]['BPs_fault'] = BPs_fault_rates
    results_OSD[name]['BPs_miscorrected'] = BPs_miscorrected_rates
    results_OSD[name]['incorrectable'] = incorrectable_rates
    results_OSD[name]['degeneracies'] = degeneracies
```

```
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```
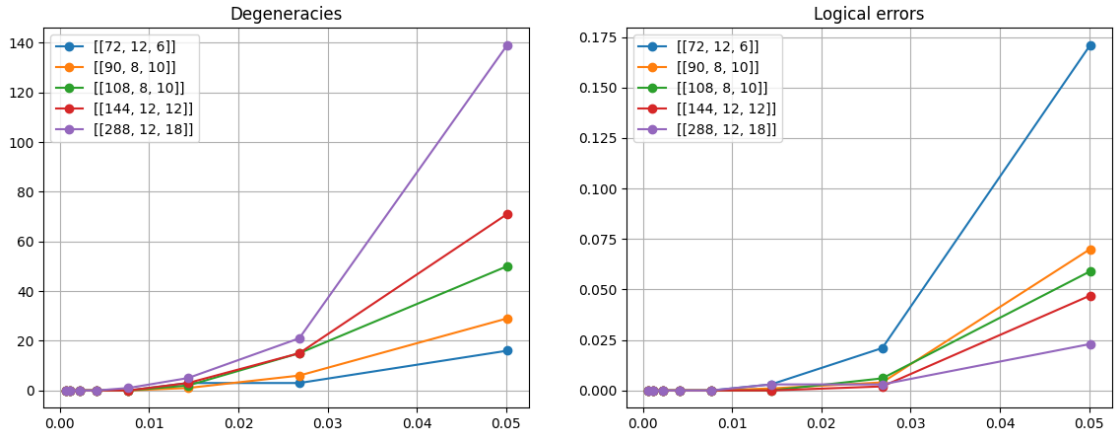
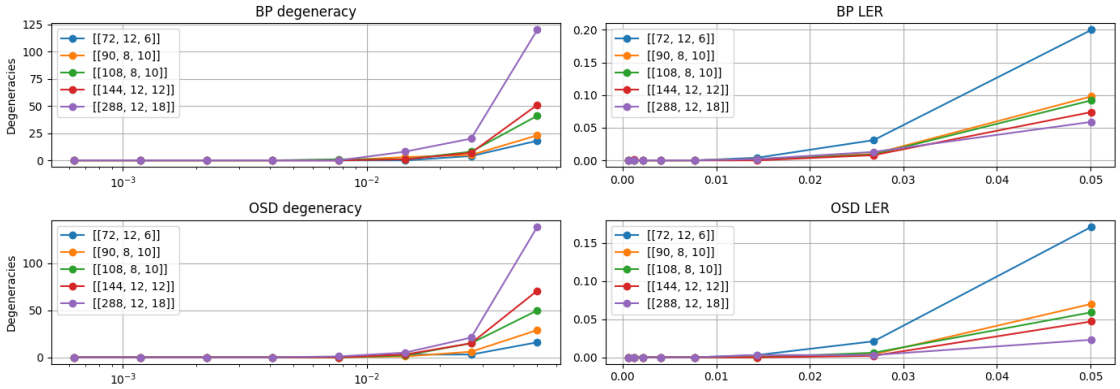

[39]:
```
fig, axes = plt.subplots(1, 2, figsize=(14, 5))

for name in results_OSD:
    axes[0].plot(physicalErrorRates, results_OSD[name]['degeneracies'],␣
 ↪label=name, marker='o')
    axes[0].grid(True)
    axes[0].legend()
    axes[0].set_title('Degeneracies')

for name in results_OSD:
    axes[1].plot(physicalErrorRates, results_OSD[name]['ler'], label=name,␣
 ↪marker='o')
    axes[1].grid(True)
    axes[1].legend()
    axes[1].set_title('Logical errors')
```

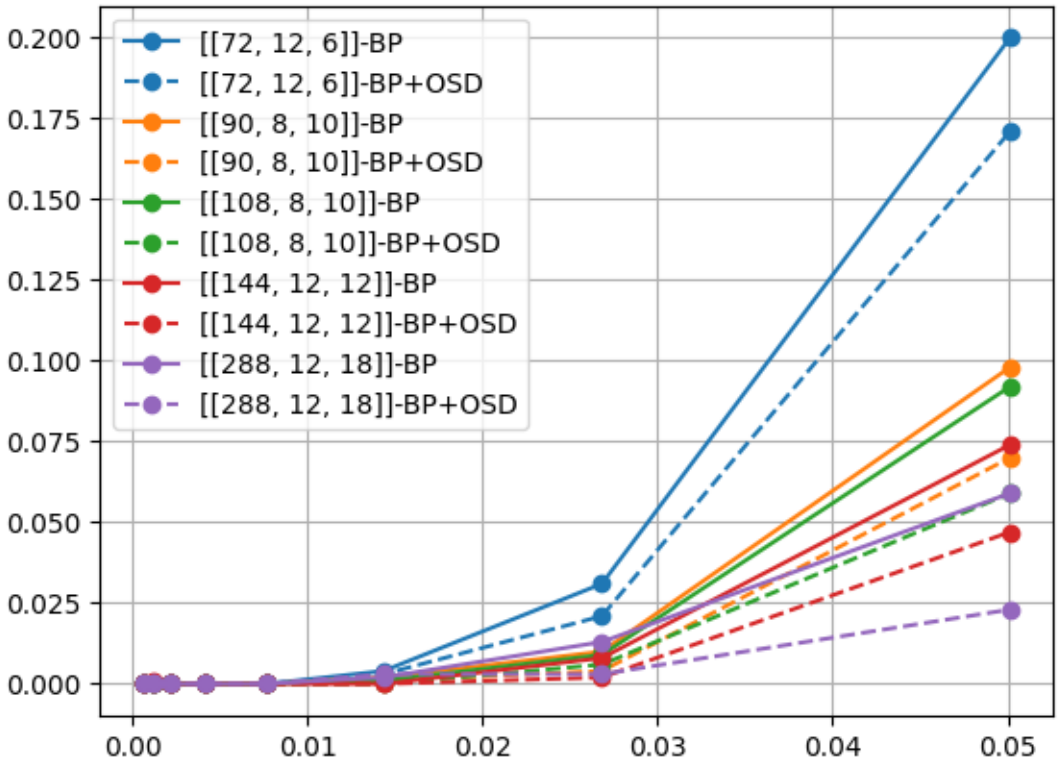


[40]:
```
fig, axes = plt.subplots(2, 2, figsize=(14, 5))

# Left plot: BP results
for name in results_BP:
```

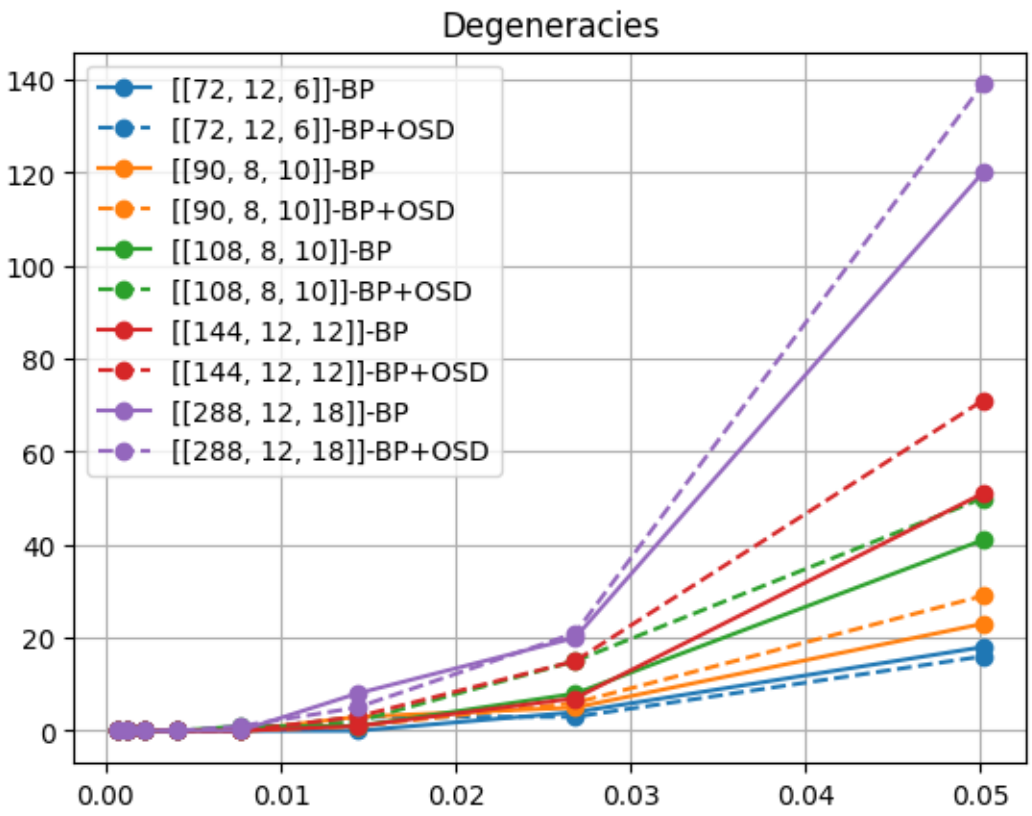
```
    axes[0][0].plot(physicalErrorRates, results_BP[name]['degeneracies'],␣
 ↪label=name, marker='o')
axes[0][0].grid(True)
axes[0][0].legend()
# axes[0][0].set_xlabel('Physical error rate')
axes[0][0].set_xscale('log')
axes[0][0].set_ylabel('Degeneracies')
axes[0][0].set_title('BP degeneracy')

# Right plot: OSD results
for name in results_OSD:
    axes[1][0].plot(physicalErrorRates, results_OSD[name]['degeneracies'],␣
 ↪label=name, marker='o')
axes[1][0].grid(True)
axes[1][0].legend()
# axes[1][0].set_xlabel('Physical error rate')
axes[1][0].set_xscale('log')
axes[1][0].set_ylabel('Degeneracies')
axes[1][0].set_title('OSD degeneracy')

for name in results_BP:
    axes[0][1].plot(physicalErrorRates, results_BP[name]["ler"], label=name,␣
 ↪marker="o")
axes[0][1].grid(True)
axes[0][1].set_title("BP LER")
axes[0][1].legend()

for name in results_OSD:
    axes[1][1].plot(physicalErrorRates, results_OSD[name]['ler'], label=name,␣
 ↪marker="o")
axes[1][1].grid(True)
axes[1][1].set_title("OSD LER")
axes[1][1].legend()

plt.tight_layout()
plt.show()
```



[ ]:
```
colors = plt.rcParams['axes.prop_cycle'].by_key()['color']
for i, name in enumerate(results_BP):
    c = colors[i % len(colors)]
    plt.plot(physicalErrorRates, results_BP[name]['ler'], label=f"{name}-BP",␣
 ↪marker='o', color=c)
    plt.plot(physicalErrorRates, results_OSD[name]['ler'],␣
 ↪label=f"{name}-BP+OSD", marker='o', linestyle='dashed', color=c)
plt.legend()
plt.grid(True)
plt.title("Logical Error Rate")
plt.show()
```



[43]:
```
colors = plt.rcParams['axes.prop_cycle'].by_key()['color']
for i, name in enumerate(results_BP):
    c = colors[i % len(colors)]
    plt.plot(physicalErrorRates, results_BP[name]['degeneracies'],␣
 ↪label=f"{name}-BP", marker='o', color=c)
    plt.plot(physicalErrorRates, results_OSD[name]['degeneracies'],␣
 ↪label=f"{name}-BP+OSD", marker='o', linestyle='dashed', color=c)
plt.legend()
plt.grid(True)
plt.title("Degeneracies")
plt.show()
```


Degeneracies
[[72, 12, 6]]-BP
[[72, 12, 6]]-BP+OSD
[[90, 8, 10]]-BP
[[90, 8, 10]]-BP+OSD
[[108, 8, 10]]-BP
[[108, 8, 10]]-BP+OSD
[[144, 12, 12]]-BP
[[144, 12, 12]]-BP+OSD
[[288, 12, 18]]-BP
[[288, 12, 18]]-BP+OSD
140
120
100
80
60
40
20
0
0.00
0.01
0.02
0.03
0.04
0.05

[ ]: