LE-Degeneracy

December 4, 2025

[27]:
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
```

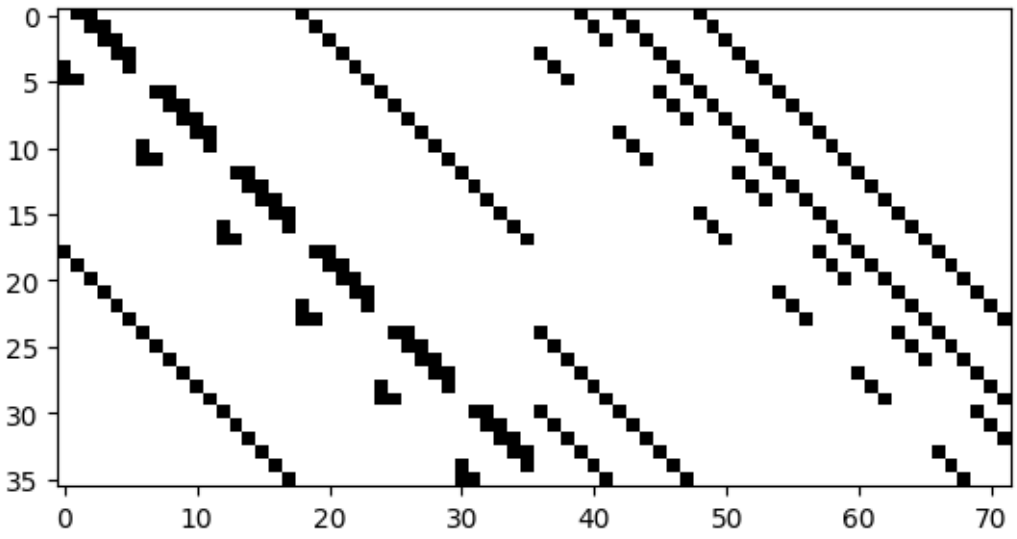
[28]:
```
from beliefPropagation import performBeliefPropagation
```

[29]:
```
code = "[[72, 12, 6]]"

matrix = np.load(f"codes/{code}.npz")
n = 72
k = 12
```

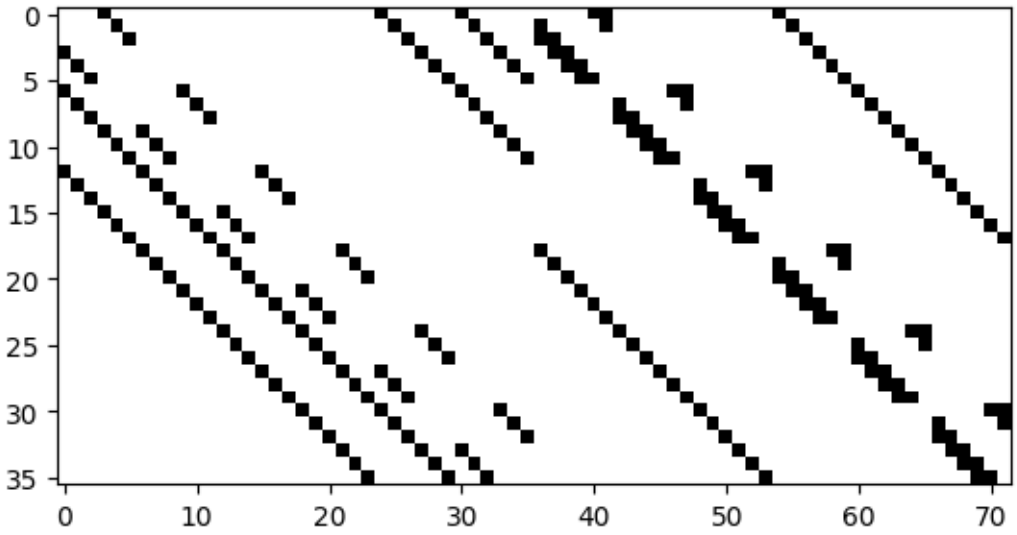
[30]:
```
Hx = matrix["Hx"]
plt.imshow(Hx, cmap='Greys', interpolation='nearest')
```

[30]:
```
<matplotlib.image.AxesImage at 0x115157650>
```



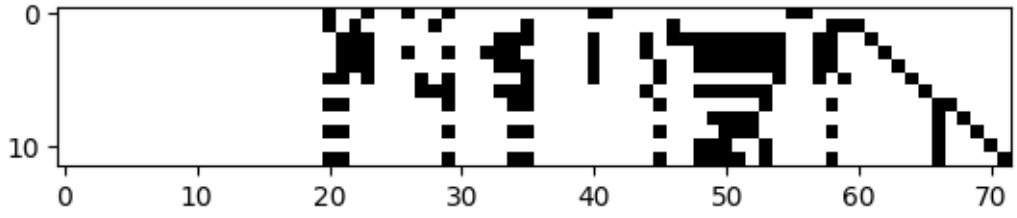
[31]:
```
Hz = matrix['Hz']
plt.imshow(Hz, cmap="Greys", interpolation="nearest")
```

[31]: <matplotlib.image.AxesImage at 0x1151d1eb0>



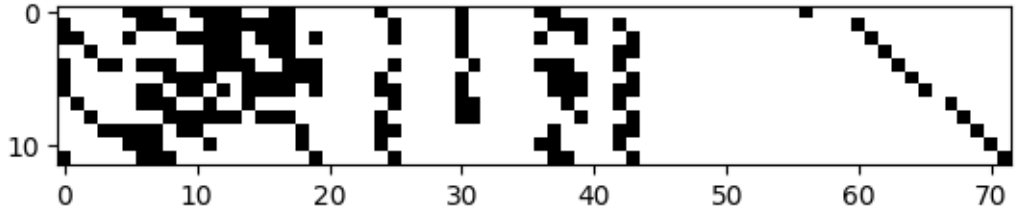
[32]:
```
Lx = matrix['Lx']
plt.imshow(Lx, cmap="Greys", interpolation="nearest")
```

[32]: <matplotlib.image.AxesImage at 0x1218c6090>



[33]:
```
Lz = matrix['Lz']
plt.imshow(Lz, cmap="Greys", interpolation="nearest")
```

[33]: <matplotlib.image.AxesImage at 0x12198b9b0>



[34]:
```
print(Hx.shape)
print(Lx.shape)
```

```
(36, 72)
(12, 72)
```

[35]:
```
# logical operator anticommute
# np.sum((Hz @ Lx.T) % 2 )
```

[36]:
```
# np.sum((Hx @ Lz.T) % 2)
```

[37]:
```
errorRate = 0.05
```

[38]:
```
# set numpy random seed
np.random.seed(0)
```

[39]:
```
error = (np.random.rand(n) < errorRate).astype(int)
print(error)

# trivial error
# error = Hz[0].copy()
```

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

[40]:
```
# e_actual != e_decoded
# but
# e_actual + e_decoded \in S

e_simple = np.zeros(n, dtype=int)
e_simple[0] = 1

stabilizer = Hz[0].copy()
e_actual = (e_simple + stabilizer) % 2
error = e_actual.copy()
```

[41]:
```
# non-trivial error

# e_simple = np.zeros(n, dtype=int)
# e_simple[0] = 1

# logical_op = Lz[0].copy()
# e_actual = (e_simple + logical_op) % 2
# error = e_actual.copy()
```

[42]:
```
print(error.reshape(1, -1)) # this is just for plt, since it wants a matrix
```

```
[[1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0
  0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]]
```

[43]:
```python
cmap = ListedColormap(['white', 'red'])
plt.imshow(error.reshape(1, -1), cmap=cmap, interpolation="nearest")
plt.yticks([])
```

[43]: ([], [])

[44]:
```python
initialBeliefs = [np.log((1 - errorRate) / errorRate)] * n
print(initialBeliefs)
```

```
[np.float64(2.9444389791664403), np.float64(2.9444389791664403),
np.float64(2.9444389791664403), np.float64(2.9444389791664403),
np.float64(2.9444389791664403), np.float64(2.9444389791664403),
np.float64(2.9444389791664403), np.float64(2.9444389791664403),
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np.float64(2.9444389791664403), np.float64(2.9444389791664403),
```

```
np.float64(2.9444389791664403), np.float64(2.9444389791664403),
np.float64(2.9444389791664403), np.float64(2.9444389791664403),
np.float64(2.9444389791664403), np.float64(2.9444389791664403),
np.float64(2.9444389791664403), np.float64(2.9444389791664403),
np.float64(2.9444389791664403), np.float64(2.9444389791664403),
np.float64(2.9444389791664403), np.float64(2.9444389791664403)]
```

[45]:
```
plt.imshow(np.asarray(initialBeliefs).reshape(1, -1), interpolation='nearest')
plt.yticks([])
```

[45]: ([], [])

[46]:
```
detection, isSyndromeFound, lastBeliefs = performBeliefPropagation(Hx, error,␣
↪initialBeliefs)
```

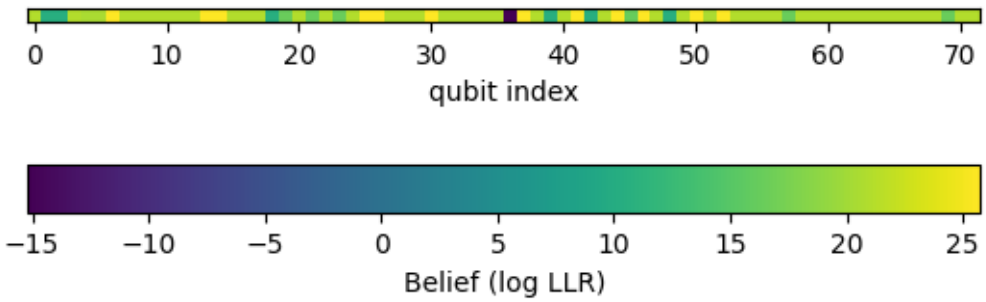
```
Initial syndrome: [1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1
0 0 0 0 0 0
 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

[47]:
```
print(detection, isSyndromeFound)
```

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0] False
```

[48]:
```
im = plt.imshow(np.asarray(lastBeliefs).reshape(1, -1), interpolation='nearest')
cbar = plt.colorbar(im, orientation='horizontal', pad=0.2)
cbar.set_label('Belief (log LLR)')
plt.xlabel('qubit index')
plt.yticks([])
```

[48]: ([], [])



[49]:
```
residual = (detection + error) % 2
print(residual)
```

```
[1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1
 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

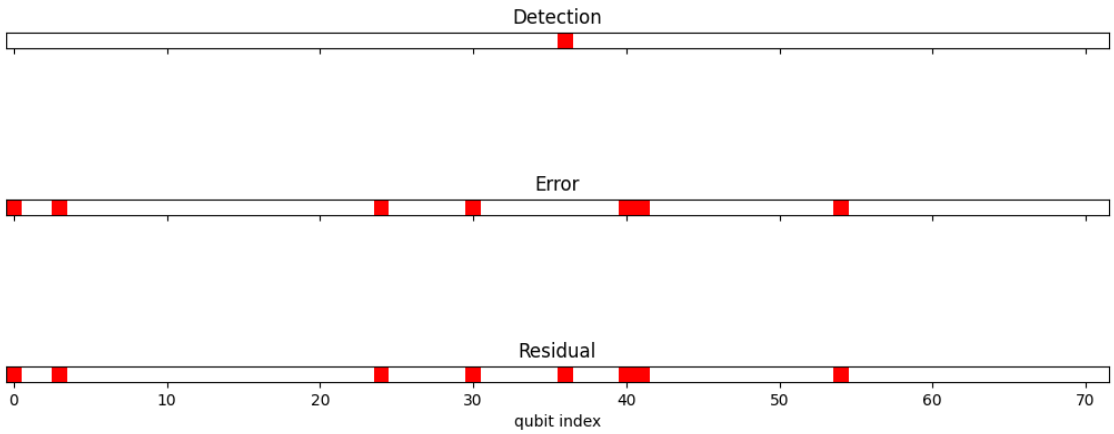
[50]:
```
fig, axs = plt.subplots(3, 1, figsize=(10, 4.5), sharex=True,␣
 ↪constrained_layout=True)

axs[0].imshow(detection.reshape(1, -1), cmap=cmap, interpolation="nearest")
axs[0].set_yticks([])
axs[0].set_title("Detection")

axs[1].imshow(error.reshape(1, -1), cmap=cmap, interpolation="nearest")
axs[1].set_yticks([])
axs[1].set_title("Error")

axs[2].imshow(residual.reshape(1, -1), cmap=cmap, interpolation="nearest")
axs[2].set_yticks([])
axs[2].set_title("Residual")
axs[2].set_xlabel("qubit index")
```

[50]:
```
Text(0.5, 0, 'qubit index')
```

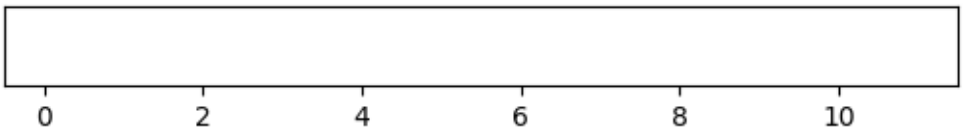


[51]:
```
# Lx is a 12 x 72 matrix, and residual is a 1 x 72 vector
print(Lx.shape, residual.shape)
syndromeLogicError = (Lx @ residual) % 2 # we do not need the transpose since @␣
 ↪does the broadcasting. I think is considered as a trivial case
print(syndromeLogicError)
```

```
(12, 72) (72,)
[0 0 0 0 0 0 0 0 0 0 0 0]
```

[52]:
```
plt.imshow(syndromeLogicError.reshape(1, -1), cmap=cmap,␣
  ↪interpolation="nearest")
plt.yticks([])
```

[52]: ([], [])



[ ]: