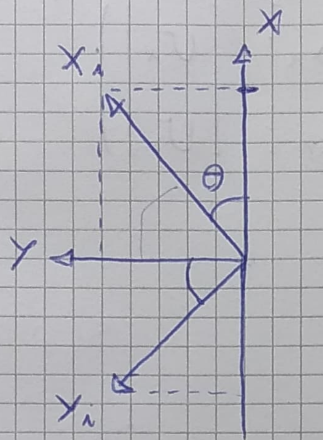
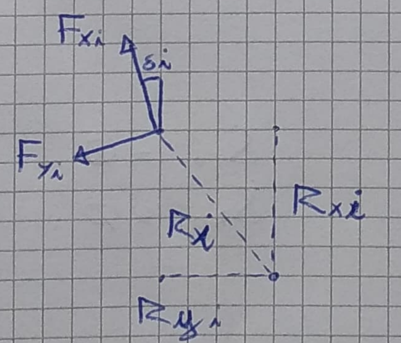
$$\begin{bmatrix} m\dot{v}_x \\ m\dot{v}_y \\ J\dot{\omega} \end{bmatrix} = \begin{bmatrix} F_x + m\omega v_y - \frac{1}{2}\rho S v^2 C_D \\ F_y - m\omega v_x \\ \tau \end{bmatrix}$$

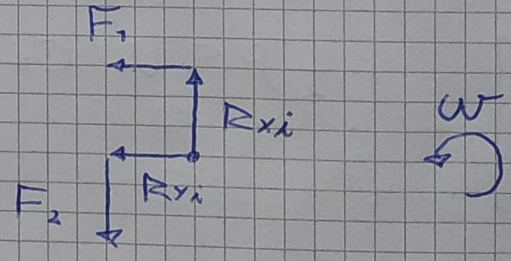


$$\begin{cases} X \rightarrow +X_i \cos\theta \\ Y \rightarrow +X_i \sin\theta \\ X \rightarrow -Y_i \sin\theta \\ Y \rightarrow Y_i \cos\theta \end{cases}$$

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} c\theta_{wi} & -s\theta_{wi} \\ s\theta_{wi} & c\theta_{wi} \end{bmatrix} \begin{bmatrix} F_{xi} \\ F_{yi} \end{bmatrix}$$

from WHEEL to BODY



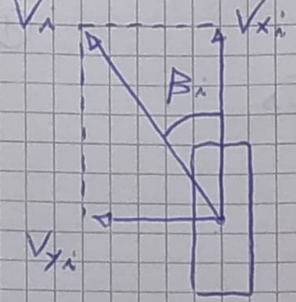


$$\begin{cases} F_1 = F_{yi}\cos\delta + F_{xi}\sin\delta \\ F_2 = F_{yi}\sin\delta - F_{xi}\cos\delta \end{cases}$$

$$T = F_1 R_{xi} + F_2 R_{yi}$$

$$\begin{cases} F_{xi} = N_i\,\mu(\lambda_i, \theta_i) \\ F_{yi} = -N_i\,\mu(\beta_i, \theta_{\beta i}) \end{cases}$$

$$\mu(s, \theta_s) = \theta_{1s}\left(1 - e^{-s\theta_{2s}}\right) - s\,\theta_{3s}$$

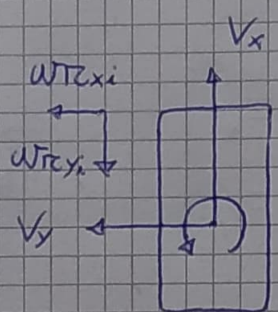


$$\beta_i = TG^{-1}\left(\frac{V_{yi}}{V_{xi}}\right)$$

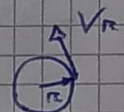
$$\begin{bmatrix} V_{xi} \\ V_{yi} \end{bmatrix} = \begin{bmatrix} C\,\delta_{wi} & S\,\delta_{wi} \\ -S\,\delta_{wi} & C\,\delta_{wi} \end{bmatrix} \left(\omega \begin{bmatrix} -R_{yi} \\ R_{xi} \end{bmatrix} + \begin{bmatrix} V_x \\ V_y \end{bmatrix} \right)$$

rotation from BODY to WHEEL

$$\begin{cases} V_x - \omega R_{yi} \\ V_y + \omega R_{xi} \end{cases}$$



because $V_R = R\omega$



EQUILIBRIUM trajectory

$$\begin{bmatrix} F_{x0} \\ F_{y0} \\ \tau_0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\rho S V_0^2 C_D \\ 0 \\ 0 \end{bmatrix} \qquad \begin{matrix} \delta_{F0} = 0 & \lambda_3 = 0 \\ \delta_{R0} = 0 & \lambda_4 = 0 \end{matrix}$$

$$F_{x0} = \sum_{1}^{2} {}_i \, N_i \, \mu(\lambda_i, \theta_{\lambda i}) \qquad \text{FWD}$$

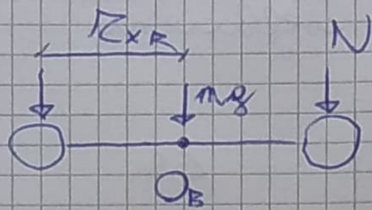
$$\tau_0 = \sum_{1}^{2} {}_i \, -N_i \, \mu(\lambda_i, \theta_{\lambda i}) R_{yi}$$

$$\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} = \begin{bmatrix} R_{xR} \\ R_{xR} \\ R_{xF} \\ R_{xF} \end{bmatrix} \frac{m g}{2(R_{xF} + R_{xR})} + \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \frac{C_D R_z - C_m (R_{xF} + R_{xR})}{2(R_{xF} + R_{xR})} \frac{1}{2}\rho S V_0^2$$

$R_{x1} = R_{x2} = R_{xF}$ $\qquad$ $R_{y1} = R_{y4} = R_y$

$R_{x3} = R_{x4} = -R_{xR}$ $\qquad$ $R_{y2} = R_{y3} = -R_y$

• Vehicle MASS :

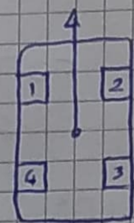


$$N_1 = \frac{1}{2} \frac{R_{xR}}{R_{xF} + R_{xR}} \cdot mg$$

R_{xF} , R_{xR} , $R_y > \emptyset$

IMPORTANT !

$$\begin{cases} R_{x1}, R_{x2} > \emptyset \\ R_{x3}, R_{x4} < \emptyset \\ R_{y1}, R_{y4} > \emptyset \\ R_{y2}, R_{y3} < \emptyset \end{cases}$$



. Pitch MOMENTUM :

$$N_x = \boxed{*} \cdot \frac{1}{2} \cdot \frac{1}{2} \rho S v_0^2$$

$*$ → FRONT $= - \dfrac{C_D R_z - C_M (R_{xF} + R_{xR})}{(R_{xF} + R_{xR})}$

→ REAR $= \dfrac{C_D R_z - C_M (R_{xF} + R_{xR})}{(R_{xF} + R_{xR})}$

C_m = aerodynamic PITCH MOMENTUM coefficient

MATLAB CODE

ESPNCSIM_V1

$$\underline{\underline{F_{AB}}} = \frac{1}{2} \rho S v^2 \begin{bmatrix} C_{x0} & \cos(\beta) \\ C_{y\beta} & \sin(\beta) \end{bmatrix} \qquad 22$$

$$V_{wi} = \begin{bmatrix} C(\delta_{wi}) & S(\delta_{wi}) \\ -S(\delta_{wi}) & C(\delta_{wi}) \end{bmatrix} \left(\omega \begin{bmatrix} -R_{yi} \\ R_{xi} \end{bmatrix} + v \begin{bmatrix} \cos(\beta) \\ \sin(\beta) \end{bmatrix} \right) \qquad 46$$

$$\beta_i = \mathrm{ATAN} \left(\frac{V_{xi}}{V_{xi}} \right) \qquad 50$$

$$\mu_L = \mu_{LONG}\,(KIND,\ LL(i))$$

$$\mu_S = \mu_{LONG}\,(KIND,\ \beta_i / (\pi/2))$$

WHEEL FRICTION coefficient

$$\mu_B = \begin{bmatrix} C\,\delta_{wi} & -S\,\delta_{wi} \\ S\,\delta_{wi} & C\,\delta_{wi} \end{bmatrix} \begin{bmatrix} \mu_L + CR \\ \mu_S \end{bmatrix} \qquad 61$$

$$\underline{\underline{\tau_{Bi}}} = \left[(S\,\delta_{wi} R_{xi} - C\,\delta_{wi} R_{yi}) \quad (C\,\delta_{wi} R_{xi} + S\,\delta_{wi} R_{yi}) \right] \mu_B$$

Reference the μ COMPONENTS in the BODY reference frame 66

SLOPE

$$\underline{\underline{F_z W}} = H^{+} \cdot \left(\frac{R_z}{2} \rho S U^2 \begin{bmatrix} 0 \\ C_{y\beta} \sin\beta \\ -C_{x0} \cos\beta \end{bmatrix} + \begin{bmatrix} m g \cos\theta \\ -R_z\, m U W \cos\beta \\ \circledast \end{bmatrix} \right)$$

$\circledast$ $- m g \sin\theta - R_z\, m U W \sin\beta$

$$\begin{cases} F_B = \sum_{1}^{4}{}_i\ \mu_{B(i)} F_z W(i) + 100 \cdot \begin{bmatrix} V_0 - U \\ 0 \end{bmatrix} + F_{eB} \\ \tau_B = \sum_{1}^{4} \tau_{Bi}(i) F_z w(i) + \tau_{EQ} \end{cases} \qquad 82$$

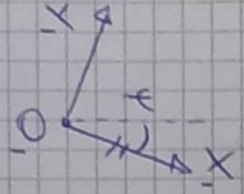
(!) CONTROLS

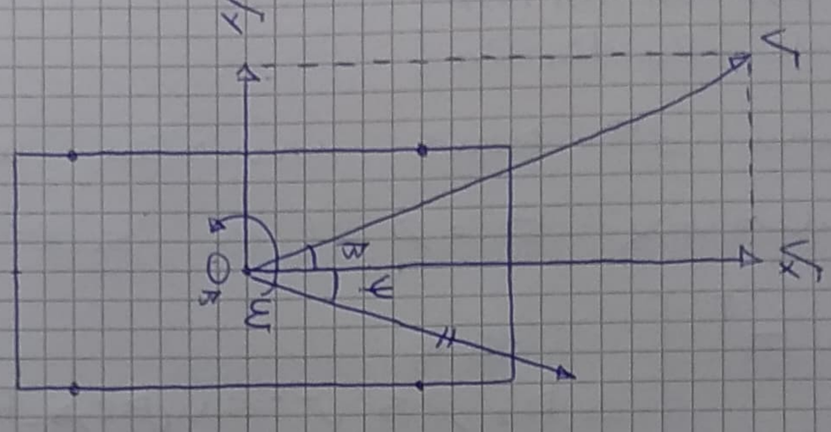
PLANT

$$dV = \begin{bmatrix} m^{-1} & 0 & 0 \\ 0 & (mU)^{-1} & 0 \\ 0 & 0 & J^{-1} \end{bmatrix} \left(\begin{bmatrix} C\beta & S\beta & 0 \\ -S\beta & C\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} F_B \\ \tau_B \end{bmatrix} - \begin{bmatrix} 0 \\ mU \\ 0 \end{bmatrix} W \right) \qquad 90$$

$$dP = \begin{bmatrix} U \cos(\psi + \beta) \\ U \sin(\psi + \beta) \end{bmatrix} \qquad d\psi = W$$

$$dx = \begin{bmatrix} dP \\ dV .* [0;1;1] \\ d\psi \\ \text{TRACK\_ERROR} \\ \text{DOT\_HAT\_X} \end{bmatrix}$$



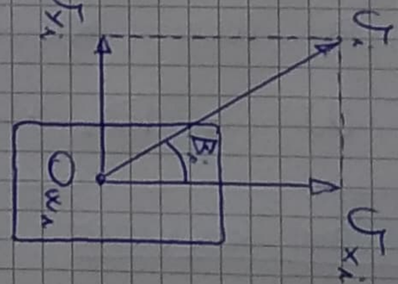


F_{aB} is ALONG the U DIRECTION

C_{xo} = DRAG coefficient along X

C_{yB} = DRAG coefficient along Y

$\left.\begin{matrix} V_{wi} \\ \beta_i \end{matrix}\right\}$ is CONSISTENT between NOTES and MATLAB ☺



$$\tau_{Bi} = \begin{bmatrix} R_{xi} & R_{yi} \end{bmatrix} \begin{bmatrix} S\,\delta_{wi} & C\,\delta_{wi} \\ -C\,\delta_{wi} & S\,\delta_{wi} \end{bmatrix} \begin{bmatrix} C\,\delta_{wi} & -S\,\delta_{wi} \\ S\,\delta_{wi} & C\,\delta_{wi} \end{bmatrix} \begin{bmatrix} u_L + C_R \\ u_S \end{bmatrix}$$

$$\begin{bmatrix} 2SC & -S^2+C^2 \\ -C^2+S^2 & 2SC \end{bmatrix}$$

not coherent with the NOTES (2 ROTATIONS instead of 1)

⇒ suppose NOTES CORRECT

In MATLAB

$R_{xF} > 0 \qquad R_{xR} < 0 \qquad R_{yF} > 0$

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ R_{yF} & -R_{yF} & -R_{yF} & R_{yF} \\ -R_{xF} & -R_{xF} & -R_{xR} & -R_{xR} \end{bmatrix}$$

2^{ND} and 3^{RD} rows inverted wrt NOTES

Considering $\theta = 0$ (no SLOPE)

$$F_z W = H^{\dagger} \begin{bmatrix} m g \\ \frac{1}{2} R_z \rho S v^2 \sin(\beta) C_{y\beta} - R_z m v \omega \cos\beta \\ -\frac{1}{2} R_z \rho S v^2 \cos(\beta) C_{x0} - R_z m v \omega \sin\beta \end{bmatrix}$$

What changes :

- ON NOTES , the AIR RESISTANCE is only along x (no y component $\rightarrow \beta = 0$)
- ON NOTES is also present the C_{r} component
- ON MATLAB we have the action of :

$R_z m v \omega = R_z \cdot$ ANGULAR MOMENTUM WHY ?

EQUILIBRIUM SOLVER

LAMBDA0S = FSOLVE (@LONG_EQ , 0) ;

LAMBDA0 = [LAMBDA0S ; LAMBDA0S ; 0 ; 0]

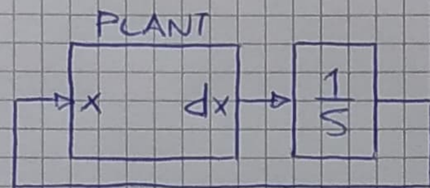
$$\text{LONG\_EQ}(x) = \frac{1}{2} \rho S V_0^2 C_{x0} + F_z W \;.\!* \begin{bmatrix} \mu_L(\text{KIND}, x) + CR \\ \mu_L(\text{KIND}, x) + CR \\ CR \\ CR \end{bmatrix}$$

Back to the PLANT:

$$dV = \left[F_{Bx} \cos\beta + F_{By} \sin\beta \right] \frac{1}{m}$$

$$\underline{\underline{d\beta}} = \left[-F_{Bx} \sin\beta + F_{By} \cos\beta \right] \frac{1}{mV} - \omega \qquad *$$

Instead of having V_x and V_y, here are V and β



$$X = \left[P_x \quad P_y \quad V \quad \beta \quad \omega \quad \psi \quad r \quad \hat{x} \right]$$

$$dx = \left[dP_x \quad dP_y \quad \boxed{dV \quad d\beta \quad d\omega} \quad \omega \quad \text{TRACK ERR} \quad \dot{\hat{x}} \right]$$

multiplied by ∅ WHY?

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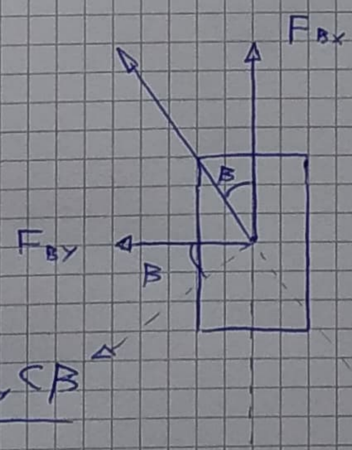
*



$$\underbrace{\frac{-F_{Bx} S\beta + F_{By} C\beta}{m}}_{dV_{TANGENTIAL}}$$

$$\Rightarrow d\beta = \frac{dV_{TAN}}{V} - \omega$$

Strange, the measurement unit is not consistent