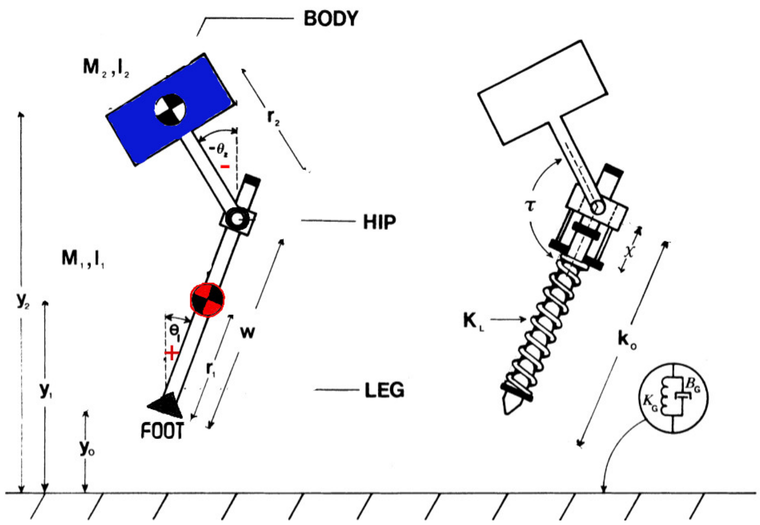
1 The model : spring/mass + gravity/mass oscillator

The model consists of a body, an actuated hinge-type hip, and a leg. The leg is massful and springy, and its length can be controlled by a position actuator. We separate control of the model into a vertical hopping part and a horizontal balance part. Vertical control takes advantage of the springy leg to achieve resonant hopping motion. The control system measures vertical energy to control the thrust delivered on each hop to reach the desired height.



1.1 Parameters

Standard units are meters [**m**], kilograms [**Kg**] and seconds [**s**]

Simulation Parameters

$M_1 = 1$ kg $\qquad M_2 = 10$ kg

$I_1 = 1$ kg-m^2 $\qquad I_2 = 10$ kg-m^2

$r_1 = 0.5$ m $\qquad r_2 = 0.4$ m

$k_0 = 1$ m

$K_L = 10^3$ Nt/m

$K_{L2} = 10^5$ Nt/m $\qquad B_{L2} = 125$ Nt-s/m

$K_G = 10^4$ Nt/m $\qquad B_G = 75$ Nt-s/m

$H = 0.4$ m

$$K_P = \begin{cases} 1800 \text{ Nt-m/rad} & \text{for } y_0 \leqslant 0 \\ 1200 \text{ Nt-m/rad} & \text{otherwise} \end{cases}$$

$$K_V = \begin{cases} 200 \text{ Nt-m-s/rad} & \text{for } y_0 \leqslant 0 \\ 60 \text{ Nt-m-s/rad} & \text{otherwise.} \end{cases}$$

$$W = w - r_1$$

$$F_K = \begin{cases} K_L(k_0 - w + \chi) & \text{for } (k_0 - w + \chi) > 0 \\ K_{L2}(k_0 - w + \chi) - B_{L2}\dot{w} & \text{otherwise} \end{cases} \tag{45}$$

$$F_x = \begin{cases} -K_G(x_0 - x_{TD}) - B_G\dot{x}_0 & \text{for } y_0 < 0 \\ 0 & \text{otherwise} \end{cases} \tag{46}$$

$$F_y = \begin{cases} -K_G y_0 - B_G y_0 & \text{for } y_0 < 0 \\ 0 & \text{otherwise.} \end{cases} \tag{47}$$

- **H** = hopping height measured at the foot $\rightarrow$ y0. Put H = $\frac{\pi^2 M_2 g}{8 K_L}$ = **0.12** for duration of support = duration of flight (6)
- ω_n = natural frequency when leg provide support = $\sqrt{KL/M2} = 31.62$. (3)
- $\boldsymbol{T_{stance}}$ = stance interval during repetitive hopping = $\frac{\pi}{\omega_n}$ = 0.0993 s. (4)
- $0 < \chi_{min} < \boldsymbol{\chi} < \chi_{max} \rightarrow$ values not specified.
- !! $\boldsymbol{\Delta w = 0.1078\ m}$ for H = 0.4 m. Max compression of leg spring during stance $\sim M_2, g, K_L, M_1$ **(16)**.

1.2 Hopping stages

4 stages: **STANCE** = when leg provides support at stages 1,3,4

1. **Lift-Off** : foot leaves the ground
2. **Top** : peak altitude, vertical motion changes downward
3. **Touchdown**: contact with the ground
4. **Bottom** : minimum altitude, vertical motion changes to upward

2 Vertical controller : generate resonant oscillations and control height

It's suggested to use a finite state sequencer machine.

2.0.1 How to control height of each hop?

Each time BOTTOM occurs, indicated by $\dot{y}_0$ changing sign from negative to positive, you predict your current and desired energy, then you get how much to push or pull with $\Delta\chi$.
To hop to height H total vertical desired energy must be:

$$(14)\ \boldsymbol{E_H = M_1 g(H + r1) + M_2 g(H + k_0 + r2)}$$

The total energy during flight is the result of the energy at stance before lift off - dissipation when you push the actuator at lift-off :

$$\boldsymbol{E_{Flight} = (E_{LO-} - E_{LO-Loss})} =$$

$$E_{\text{FLIGHT}} = \frac{M_2}{M_1 + M_2}\Big[M_1 g y_1 + M_2 g y_2 + \frac{1}{2}\left(M_1 \dot{y}_1^2\right) + \frac{1}{2}\left(M_2 \dot{y}_2^2\right) + \frac{1}{2}\left(K_L (k_0 - w + \chi)^2\right) + \frac{1}{2}\left(K_G y_0^2\right)\Big] \qquad (13)$$

You compute $\boldsymbol{\Delta E_H = E_H - E_{Flight}}$ that's the energy you miss, then your input will be

$$\Delta\chi = -(\chi - w + k_0) + \sqrt{(\chi - w + k_0)^2 + \frac{2\Delta E_H}{K_L}}\,. \qquad (15)$$

ΔE_H can be negative of positive:

- if $\Delta E_H > 0$ you need to **inject energy** (do positive work) $\rightarrow$ push ($\chi > 0$) during support AND pull ($\chi < 0$) during flight
- if $\Delta E_H < 0$ you need to **remove energy** (do negative work) $\rightarrow$ pull ($\chi < 0$) during support AND push ($\chi > 0$) during flight

The magnitude of energy you remove depends on w and $\Delta\chi$ see equation (11).

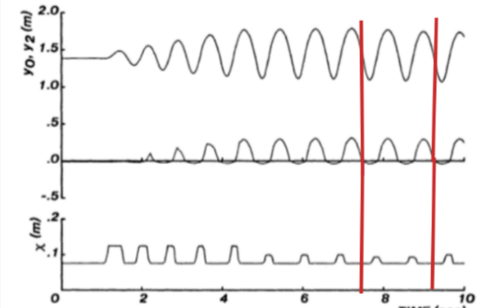
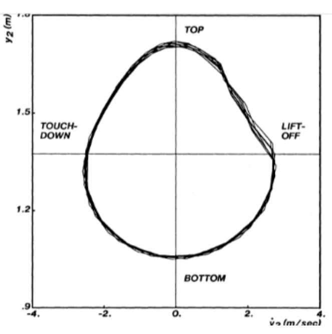
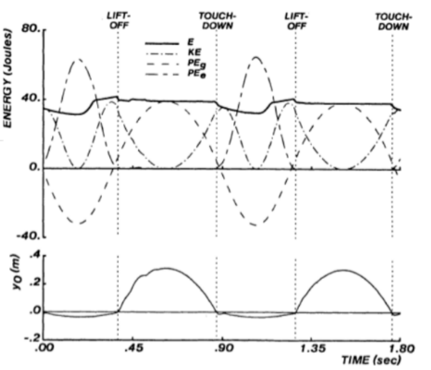


Fig. 2. Vertical hopping. Starting from rest, vertical energy was increased until desired hopping height was attained. Hopping height was regulated through the position actuator that acts in series with the leg spring. Note different vertical scales. Top curve—elevation of hip. Middle curve—elevation of foot. Bottom curve—length of leg actuator.





ig. 4. Vertical energy for two hopping cycles at constant hopping height. Total energy, kinetic energy, gravitational potential, and elastic potential are shown. Primary losses of energy occur at TOUCHDOWN and LIFT-OFF, indicated by the vertical dotted lines. Data are from Fig. 2, t = 7.5 to 9.3 s.

- χ : we can consider it an ideal actuator with no constraints, further $\chi(t) = \chi_0 + kt^2$ (a finite response actuator).