

# Applied Data Analysis for Public Policy Studies

## Differences-in-Differences

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# Recap from last week

- Applied inference tools to regression analysis
- *Standard error* of regression coefficients
- *Statistical significance* of regression coefficients



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- Applied inference tools to regression analysis
- *Standard error* of regression coefficients
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## Today: *Differences-in-differences*

- Exploits changes in policy over time that don't affect everyone
- Need to find (or construct) appropriate control group(s)
- *Key assumption:* parallel trends
- *Empirical application:* impact of *minimum wage* on *employment*



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  - *propensity-score matching*,
  - *differences-in-differences (DiD)*, and
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- In this lecture, we will cover a popular and rigorous program evaluation method: **differences-in-differences**.
- Next week we will look at **regression discontinuity designs**.



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## DiD Requirements:

- 2 time periods: before and after treatment.
- 2 groups:
  - *control group*: never receives treatment,
  - *treatment group*: initially untreated and then fully treated.
- Under certain assumptions, control group can be used as the counterfactual for treatment group



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- Seminal 1994 **paper** by prominent labor economists David Card and Alan Krueger entitled "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania"
- Estimates the effect of an increase in the minimum wage on the employment rate in the fast-food industry. Why this industry?



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Pennsylvania and New Jersey are **very similar**: similar institutions, similar habits, similar consumers, similar incomes, similar weather, etc.



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Let's take a closer at their data

```
library(devtools)
  # install package that contains the cleaned data
remotes::install_github("b-rodrigues/difffindiff")
  # load package
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ck1994 <- njmin
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```
library(tidyverse)
ck1994 %>%
  select(sheet,chain,state,observation,empft,emppt) %
  head(n=4)

## # A tibble: 4 x 6
##   sheet chain state      observation empft emppt
##   <chr> <chr> <chr>      <chr>     <dbl>  <dbl>
## 1 46    bk    Pennsylvania February  1992    30    15
## 2 49    kfc   Pennsylvania February  1992    6.5   6.5
## 3 506   kfc   Pennsylvania February  1992    3     7
## 4 56    wendys Pennsylvania February  1992    20    20
```



# Task 1 (10 minutes)

1. Take a look at the dataset and list the variables. Check the variable definitions with `? njmin`.
2. Tabulate the number of stores by `state` and by survey wave (`observation`). Does it match what's in *Table 1* of the paper?
3. Create a full-time equivalent (FTE) employees variable called `empfte` equal to `empft + 0.5*emppt + nmgrs`. `empft` and `emppt` correspond respectively to the number of full-time and part-time employees. `nmgrs` corresponds to the number of managers. This is how Card and Krueger compute their full-time equivalent (FTE) employment variable (p.775 of the paper).
4. Compute the average number of FTE employment, average percentage of FT employees (out of the number of FTE employees), and average starting wage (`wage_st`) by state and by survey wave. Compare your results with *Table 2* of the paper.
5. How different are New Jersey and Pennsylvania's fast-food restaurants before the minimum wage increase?



# Card and Krueger DiD: Tabular Results

Average Employment Per Store Before and After the Rise in NJ Minimum Wage

Variables	Pennsylvania	New Jersey
FTE employment before	23.33	20.44
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Change in mean FTE employment	-2.17	0.59



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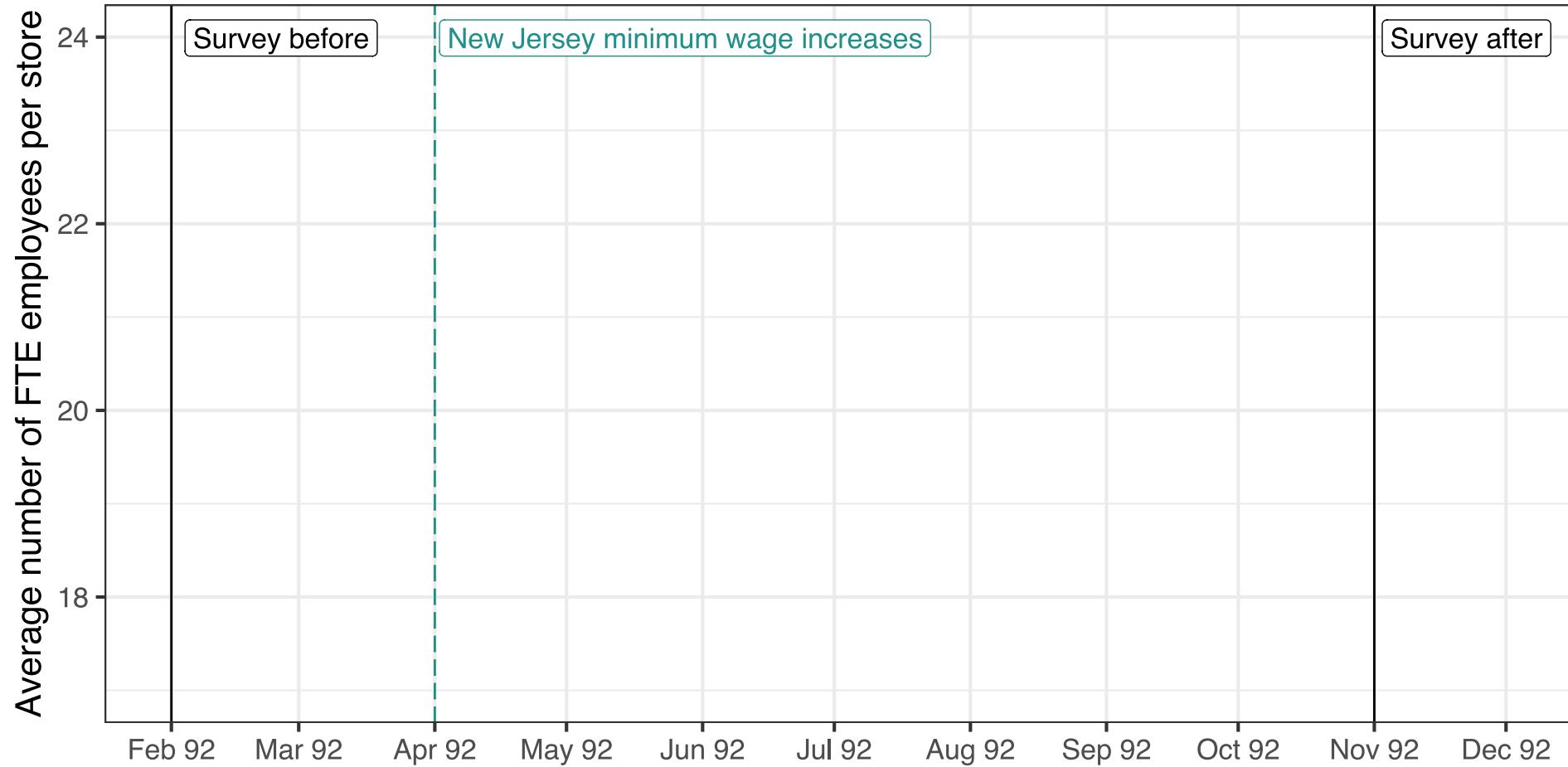
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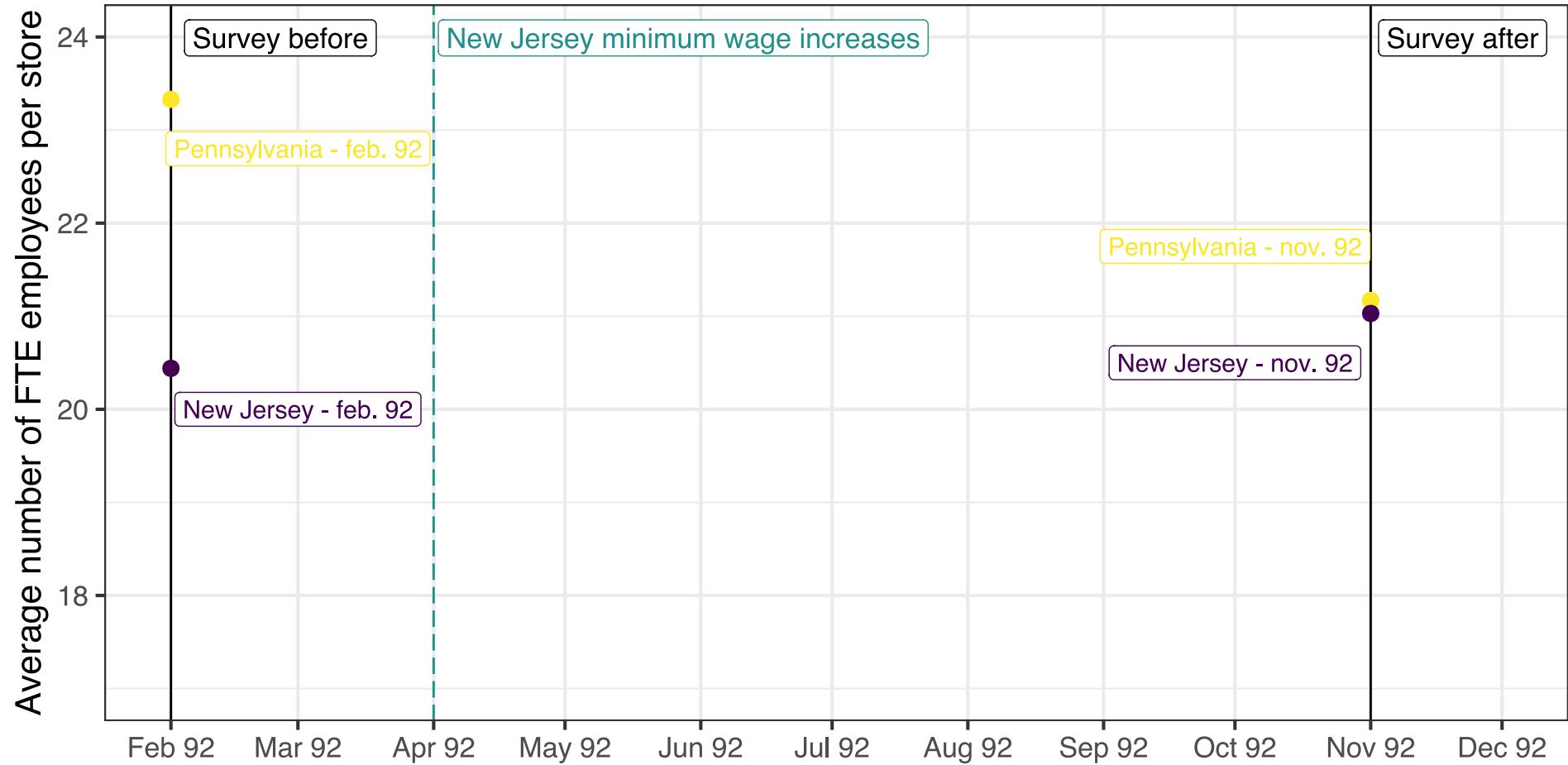
Let's look at these results graphically.



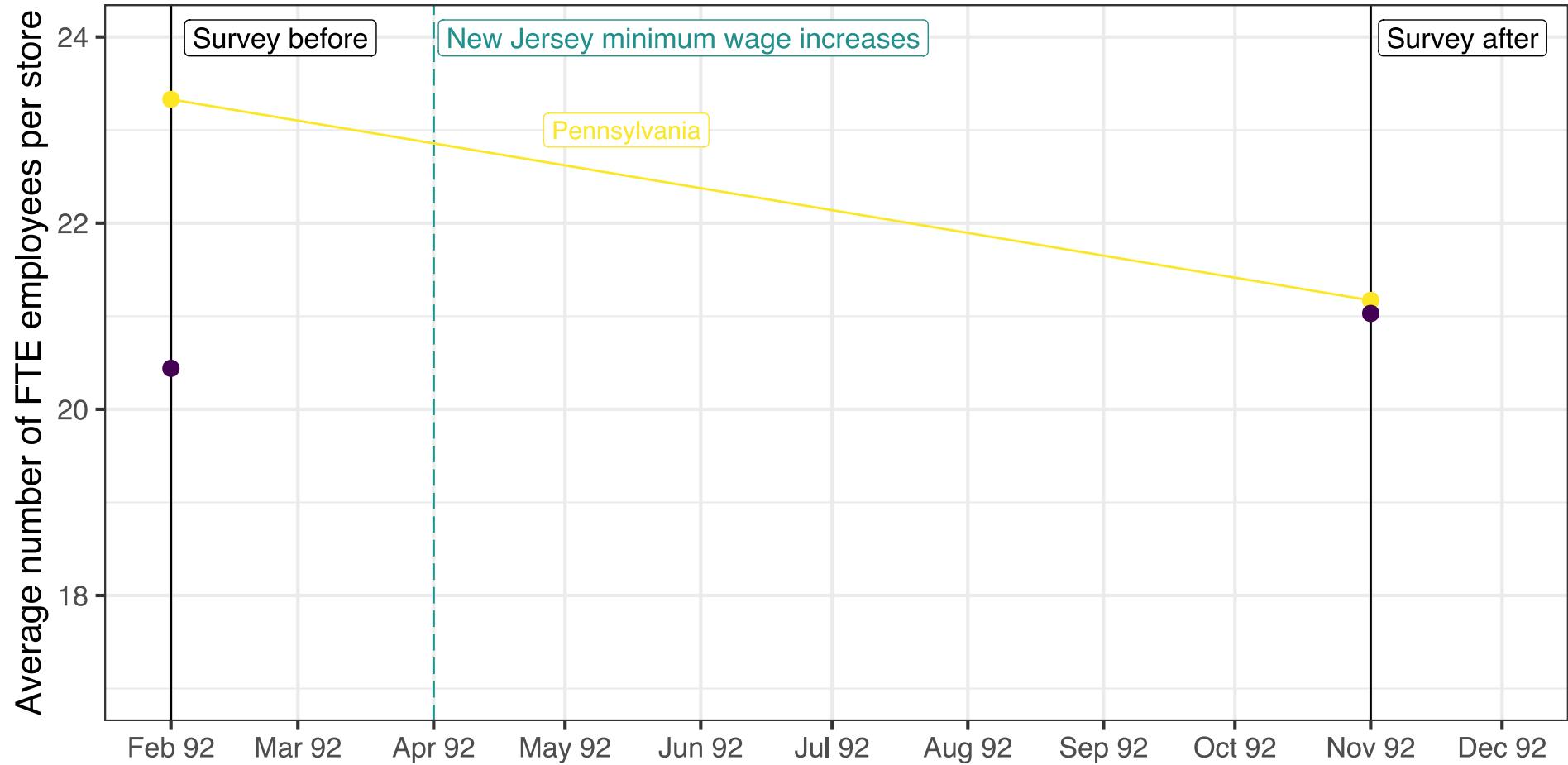
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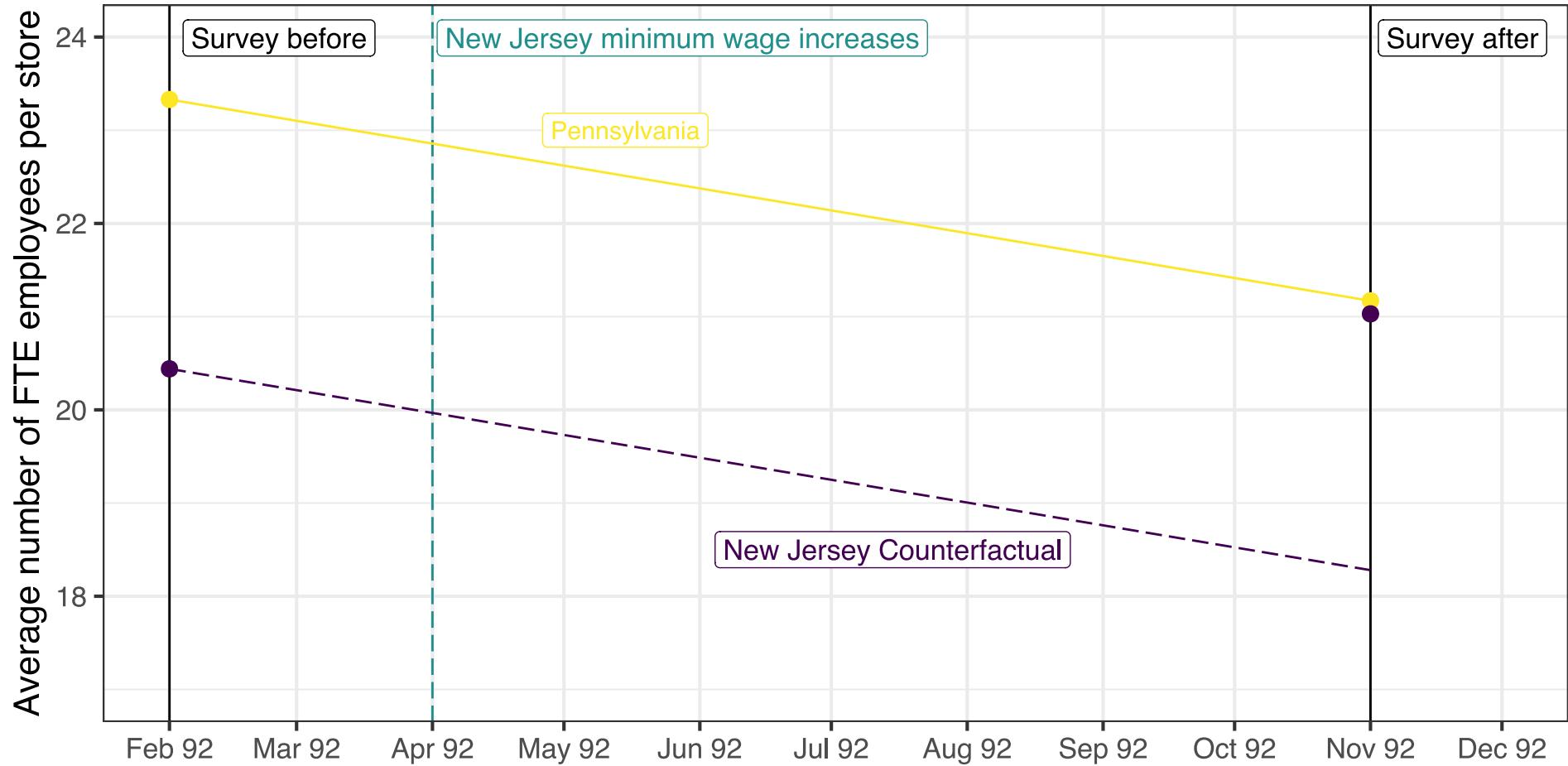
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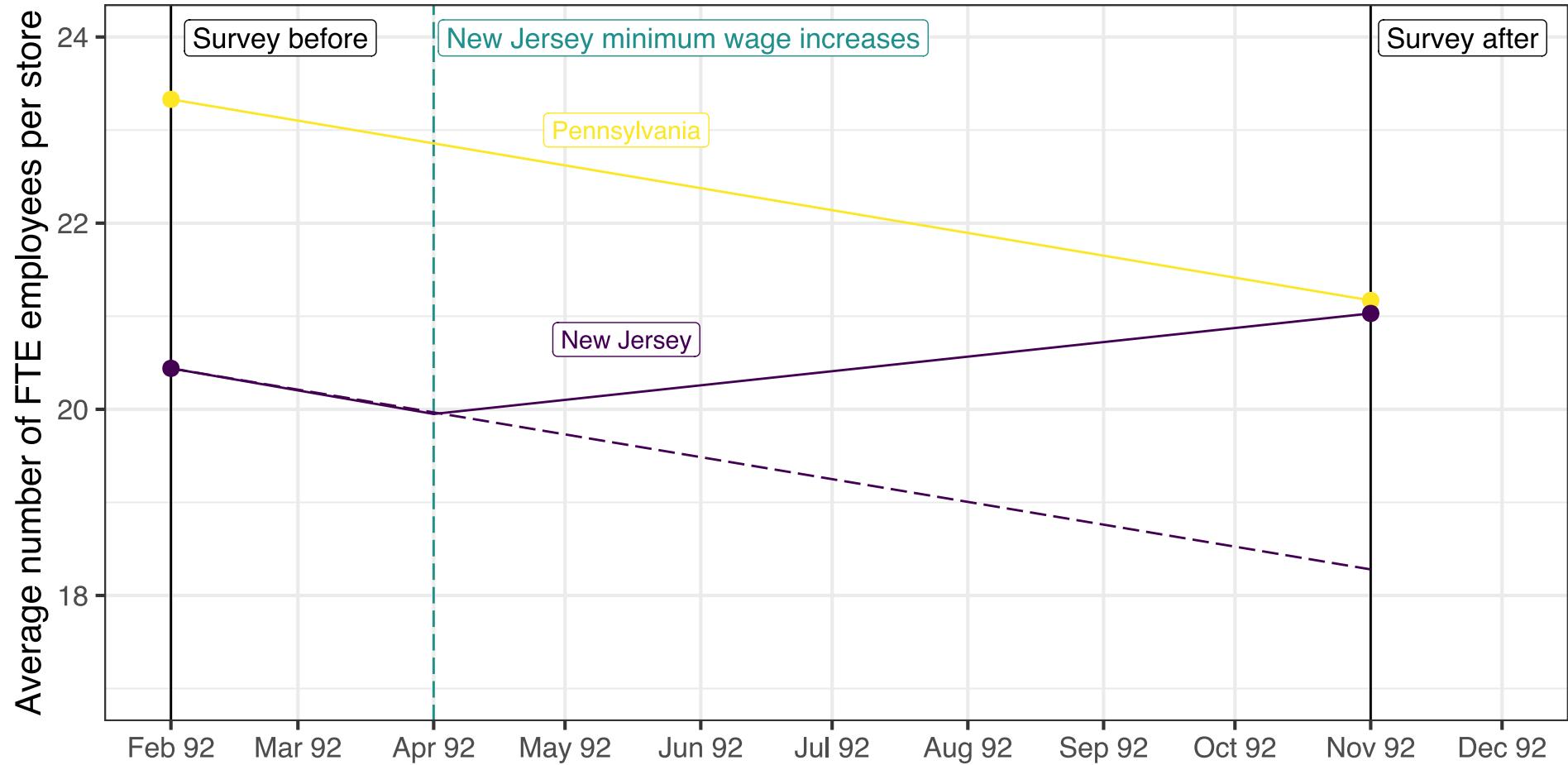
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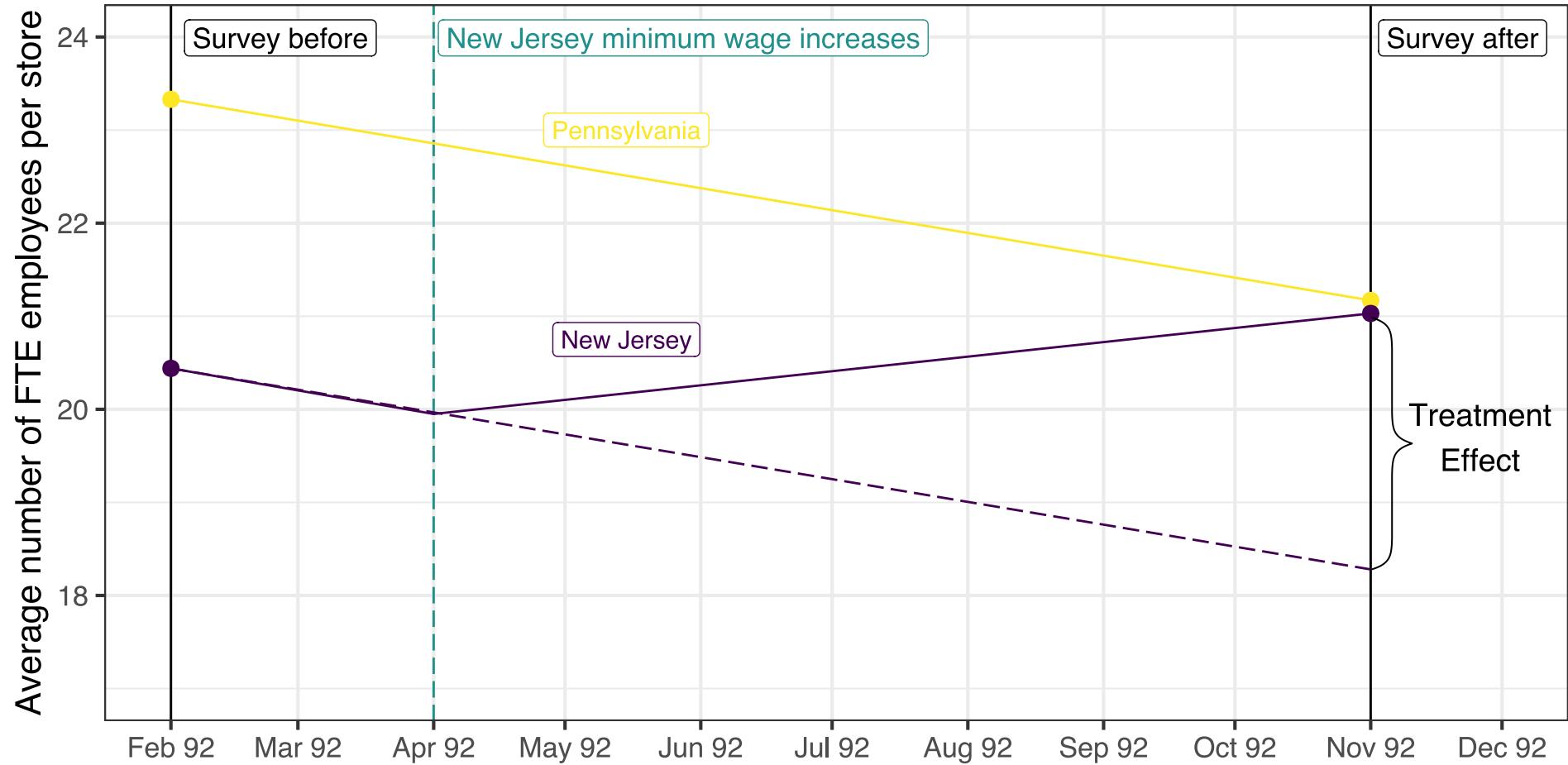
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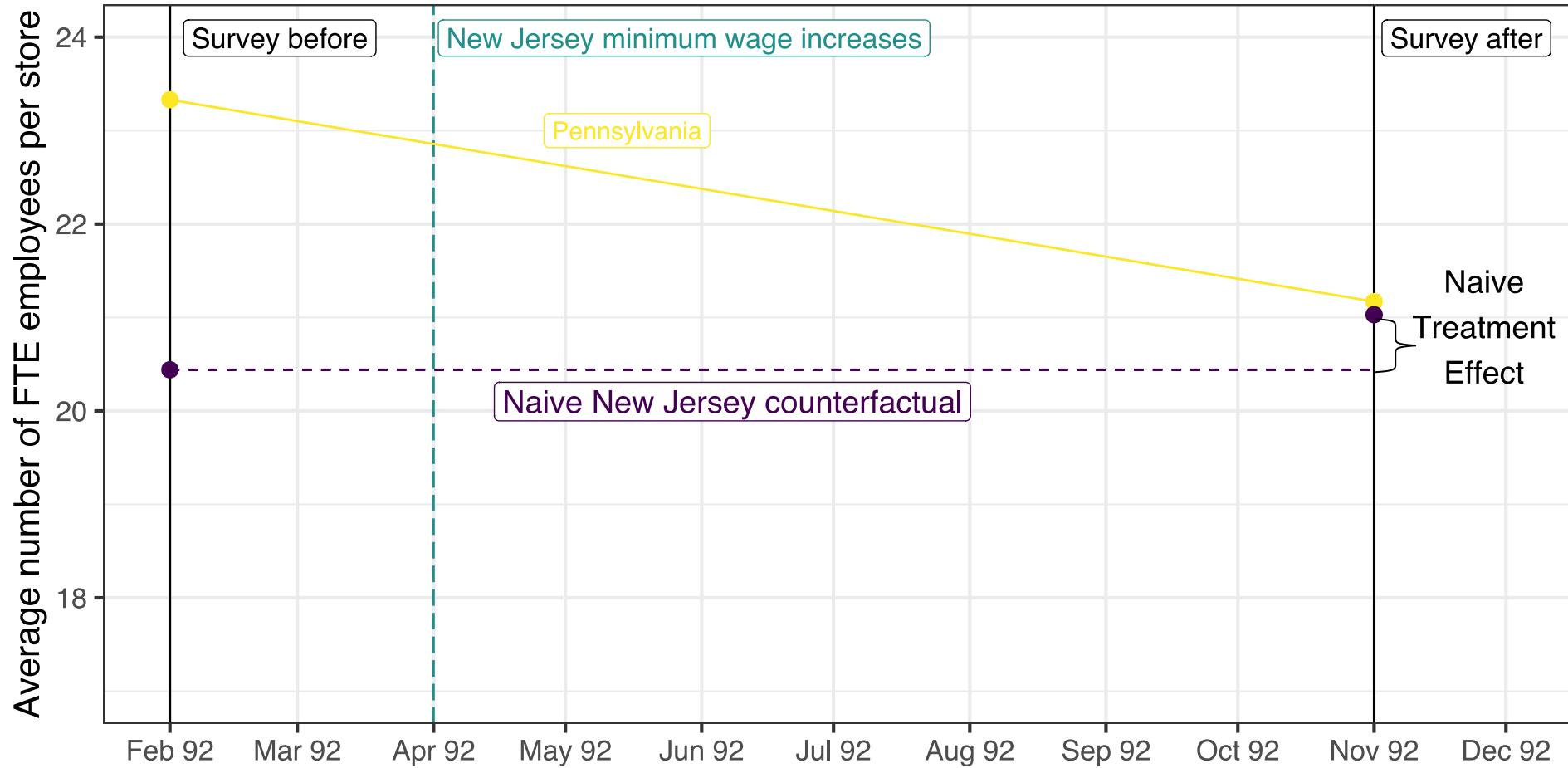
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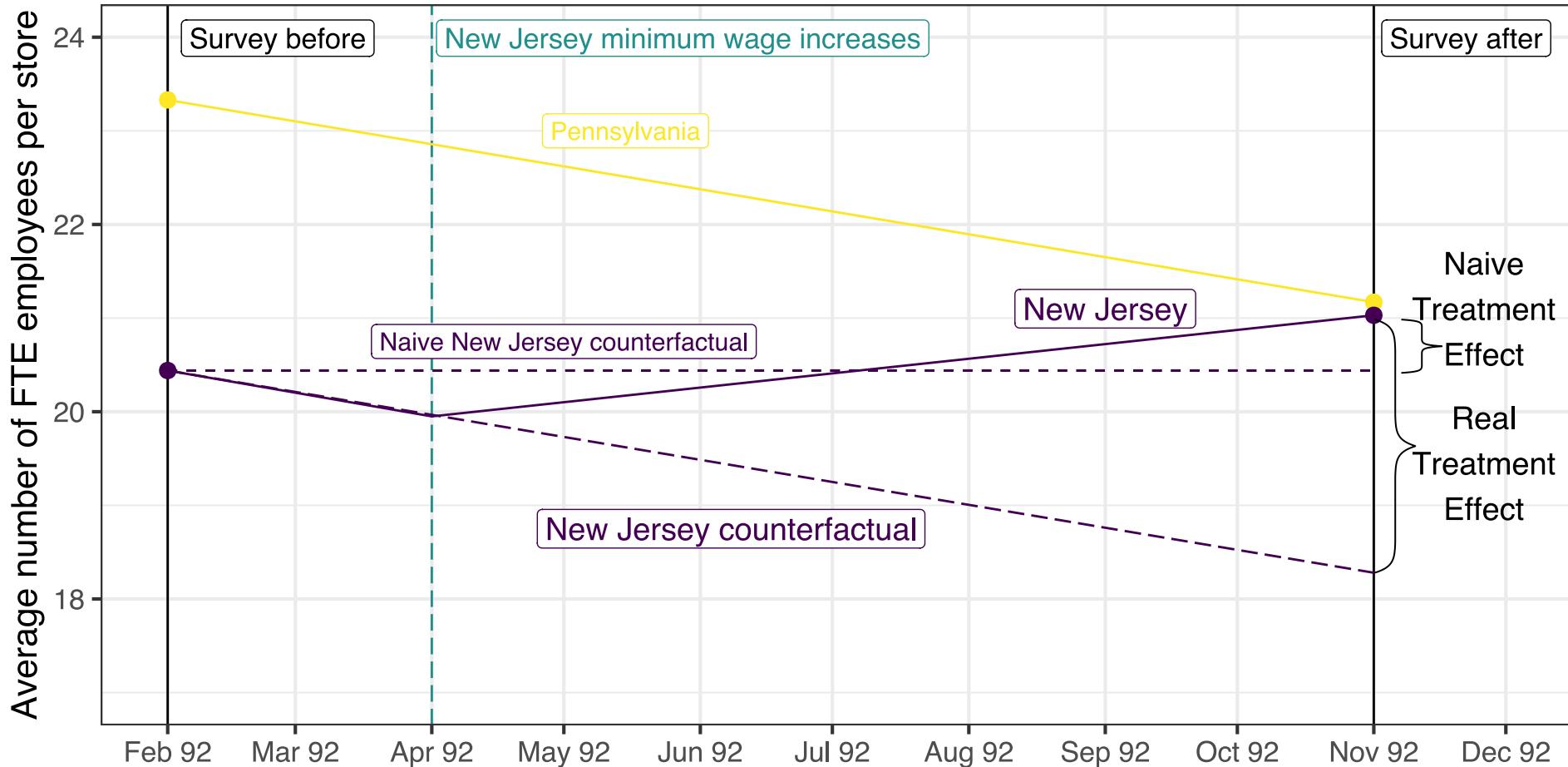
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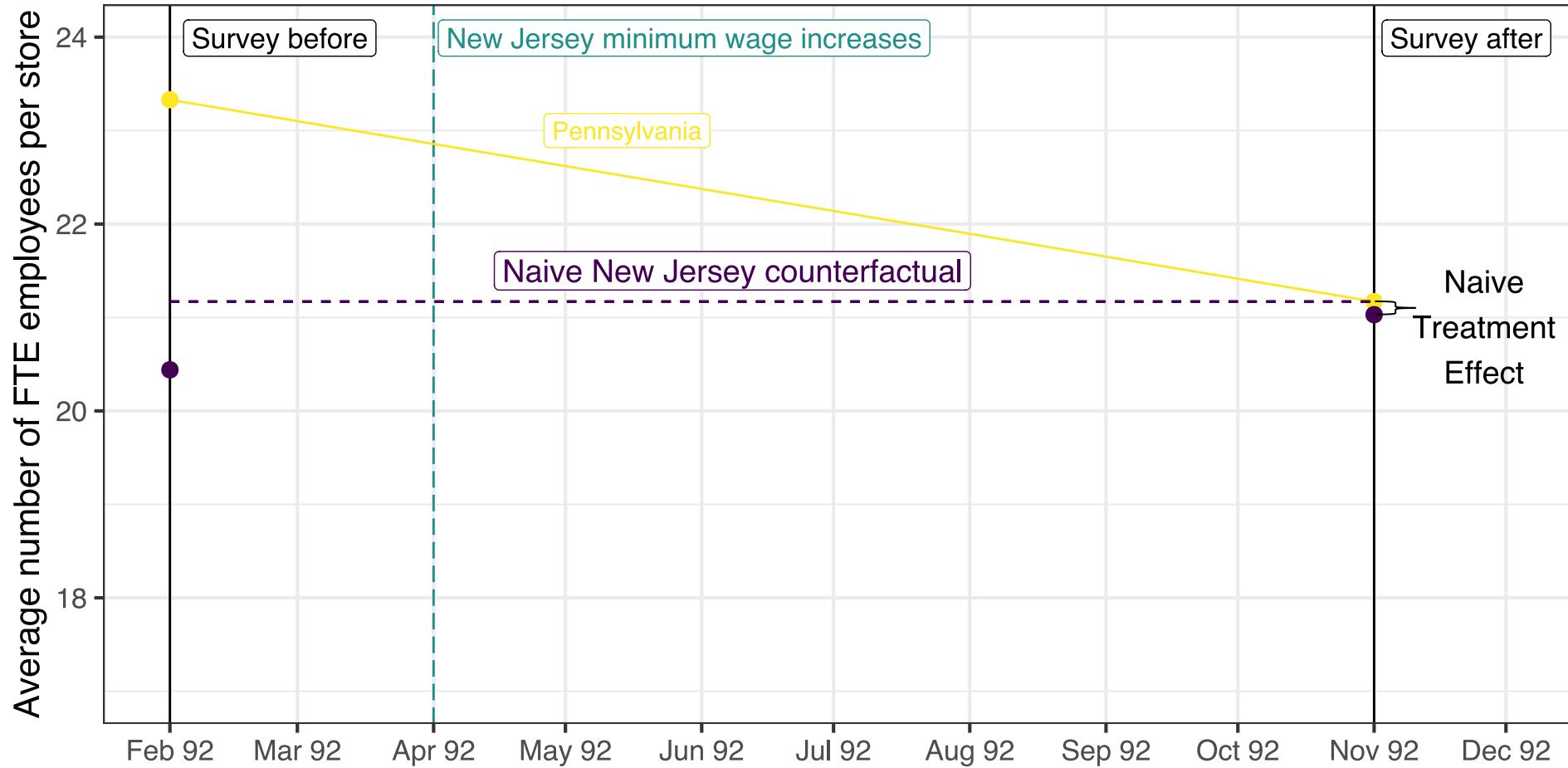
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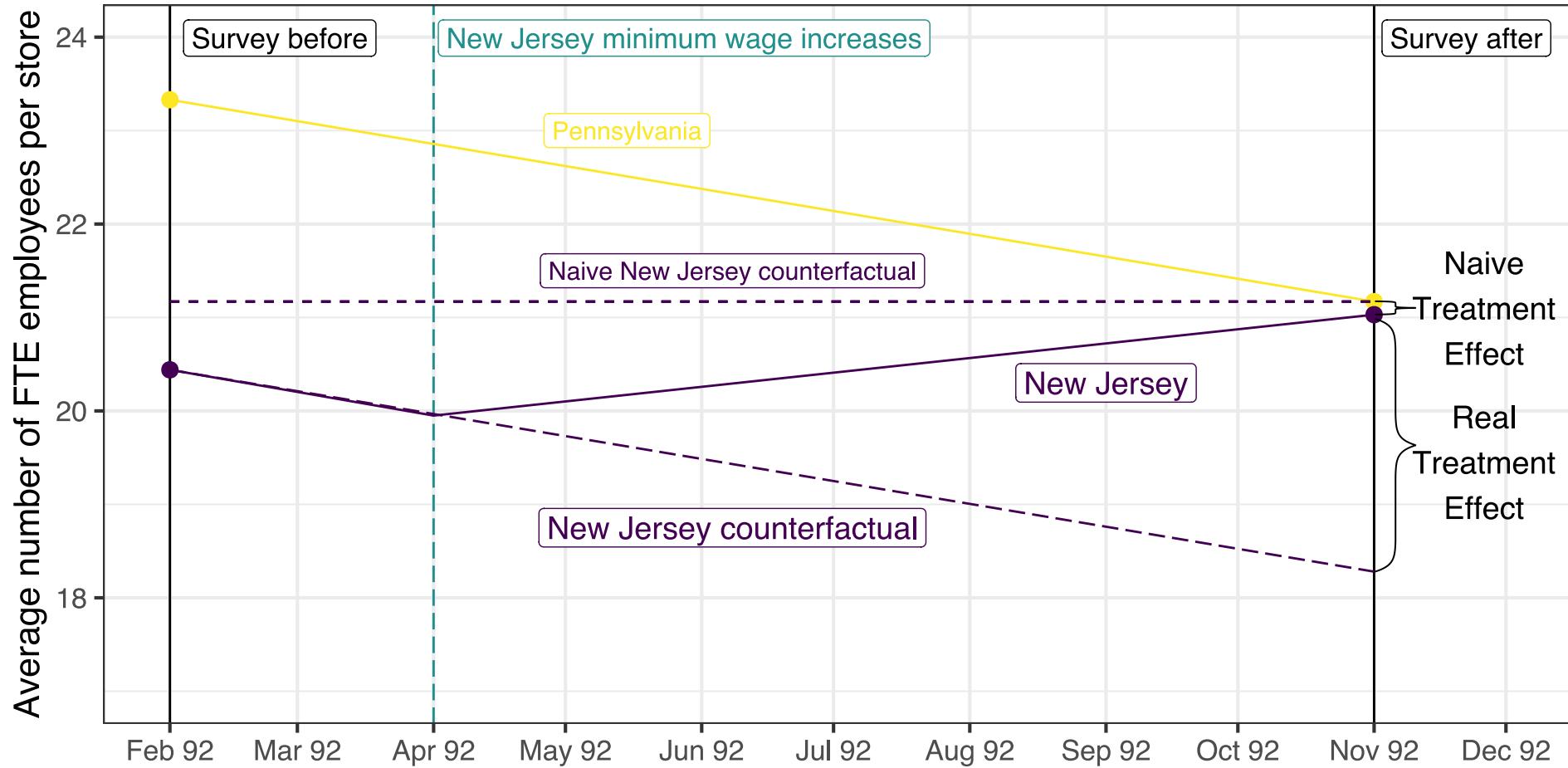
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# Estimation

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3. **Interaction term between the two:**  $TREAT_s \times POST_t$  ↗ the *coefficient on this term is the DiD causal effect!*



# DiD in Regression Form

Treatment dummy variable

$$TREAT_s = \begin{cases} 0 & \text{if } s = \text{Pennsylvania} \\ 1 & \text{if } s = \text{New Jersey} \end{cases}$$



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**Which observations correspond to  $TREAT_s \times POST_t = 1$ ?**

- Let's put all these ingredients together:

$$EMP_{st} = \alpha + \beta TREAT_s + \gamma POST_t + \delta(TREAT_s \times POST_t) + \varepsilon_{st}$$

- $\delta$ : causal effect of the minimum wage increase on employment



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$$[\mathbb{E}(EMP_{st} \mid TREAT_s = 1, POST_t = 1) - \mathbb{E}(EMP_{st} \mid TREAT_s = 1, POST_t = 0)] - \\ [\mathbb{E}(EMP_{st} \mid TREAT_s = 0, POST_t = 1) - \mathbb{E}(EMP_{st} \mid TREAT_s = 0, POST_t = 0)] = \delta$$



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In table form:

	Pre mean	Post mean	$\Delta(\text{post} - \text{pre})$
Pennsylvania (PA)	$\alpha$	$\alpha + \gamma$	$\gamma$
New Jersey (NJ)	$\alpha + \beta$	$\alpha + \beta + \gamma + \delta$	$\gamma + \delta$
$\Delta(\text{NJ} - \text{PA})$	$\beta$	$\beta + \delta$	$\delta$



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$\Delta(\text{NJ} - \text{PA})$	$\beta$	$\beta + \delta$	$\delta$

This table generalizes to other settings by substituting *Pennsylvania* with *Control* and *New Jersey* with *Treatment*



# Task 2 (10 minutes)

1. Create a dummy variable, `treat`, equal to `FALSE` if `state` is Pennsylvania and `TRUE` if New Jersey.
2. Create a dummy variable, `post`, equal to `FALSE` if `observation` is February 1992 and `TRUE` otherwise.
3. Estimate the following regression model. Do you obtain the same results as in slide 9?

$$empfte_{st} = \alpha + \beta treat_s + \gamma post_t + \delta(treat_s \times post_t) + \varepsilon_{st}$$



# Identifying Assumptions

# DiD Crucial Assumption: Parallel Trends

**Common or parallel trends assumption:** absent any minimum wage increase, Pennsylvania's fast-food employment trend would have been what we should have expected to see in New Jersey.



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- This assumption states that Pennsylvania's fast-food employment trend between February and November 1992 provides a reliable counterfactual employment trend New Jersey's fast-food industry *would have experienced* had New Jersey not increased its minimum wage.



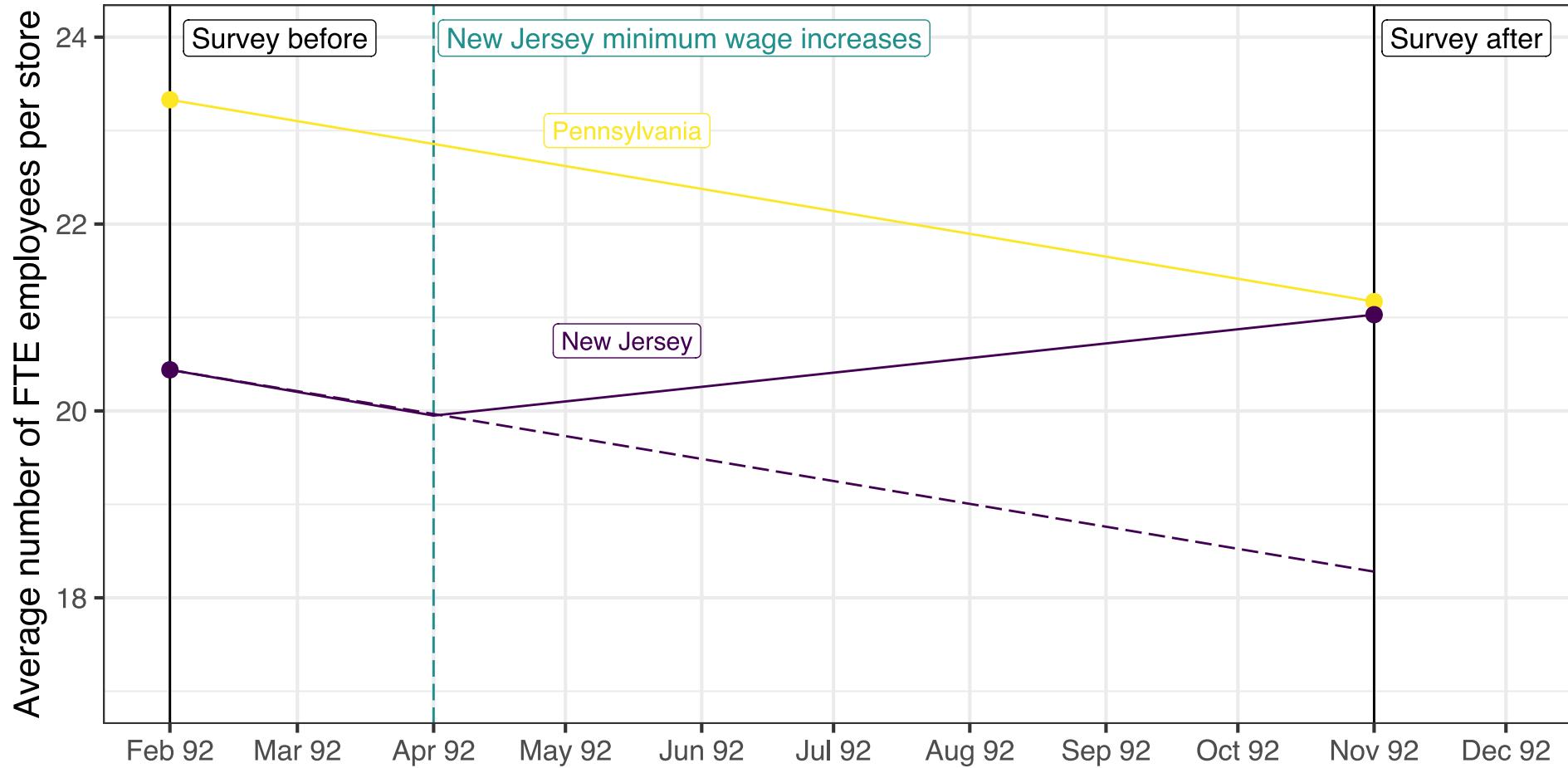
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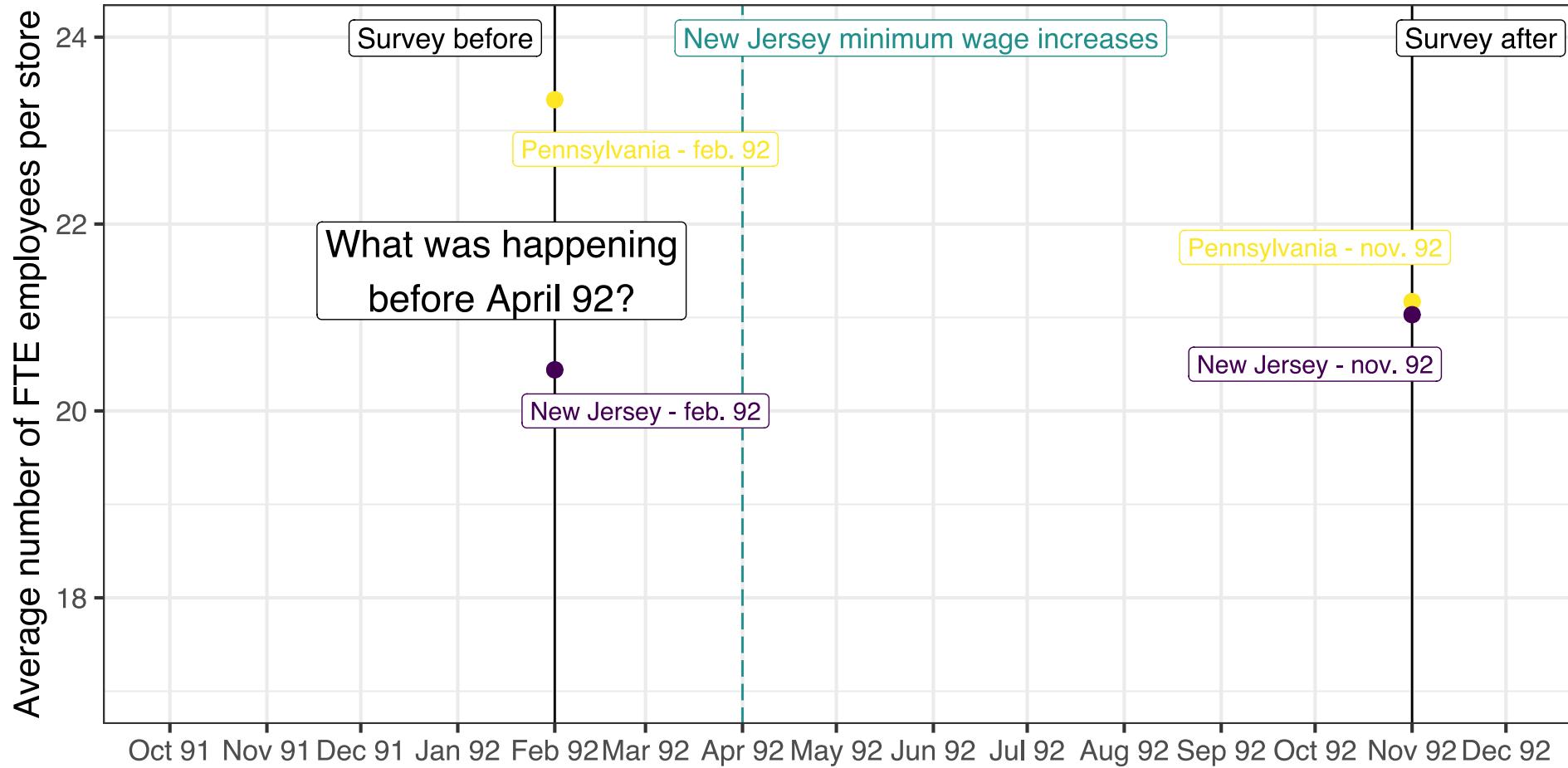
- This assumption states that Pennsylvania's fast-food employment trend between February and November 1992 provides a reliable counterfactual employment trend New Jersey's fast-food industry *would have experienced* had New Jersey not increased its minimum wage.
- Impossible to completely validate or invalidate this assumption.
- *Intuitive check:* compare trends before policy change (and after policy change if no expected medium-term effects)



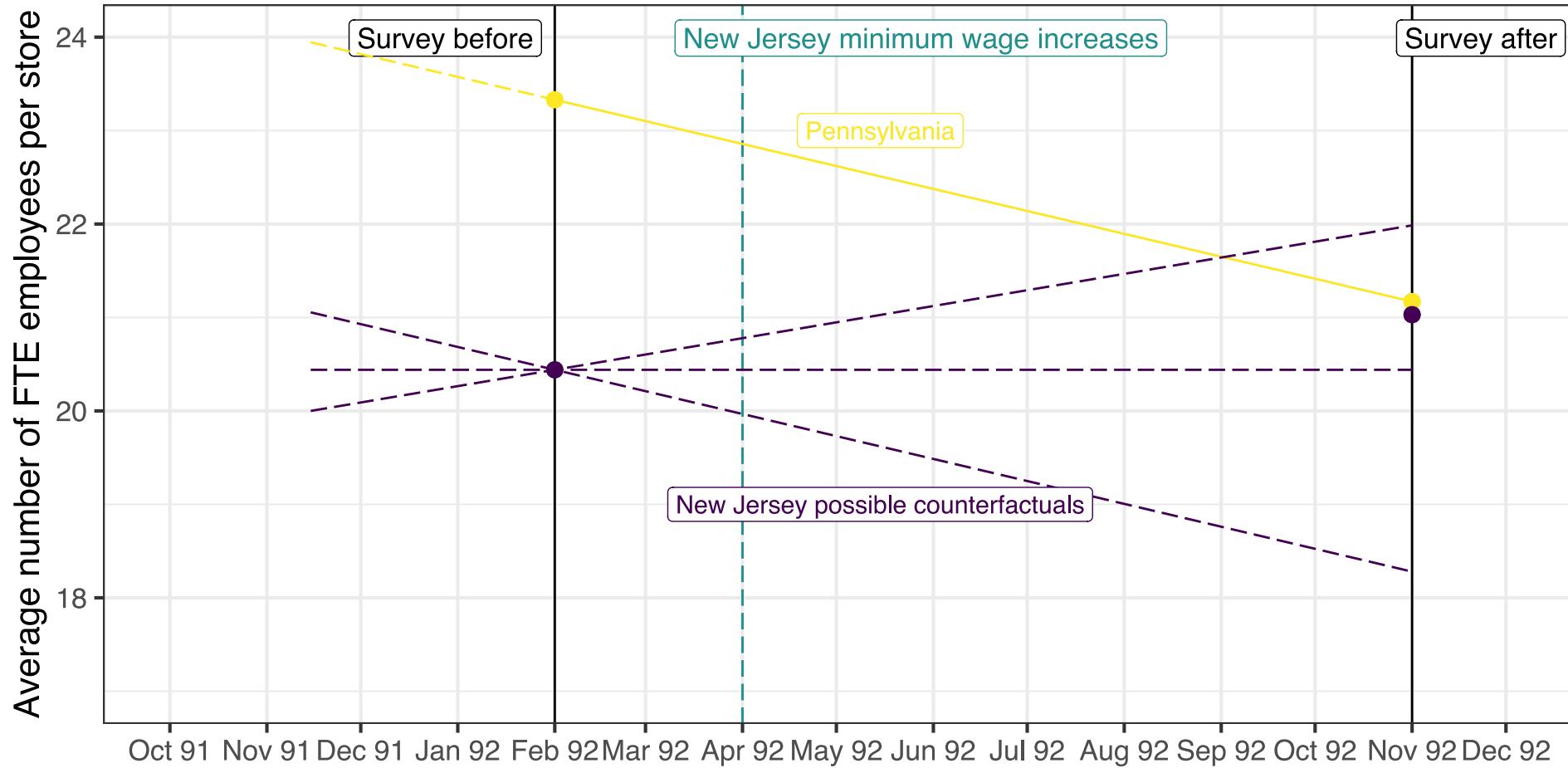
# Parallel Trends: Graphically



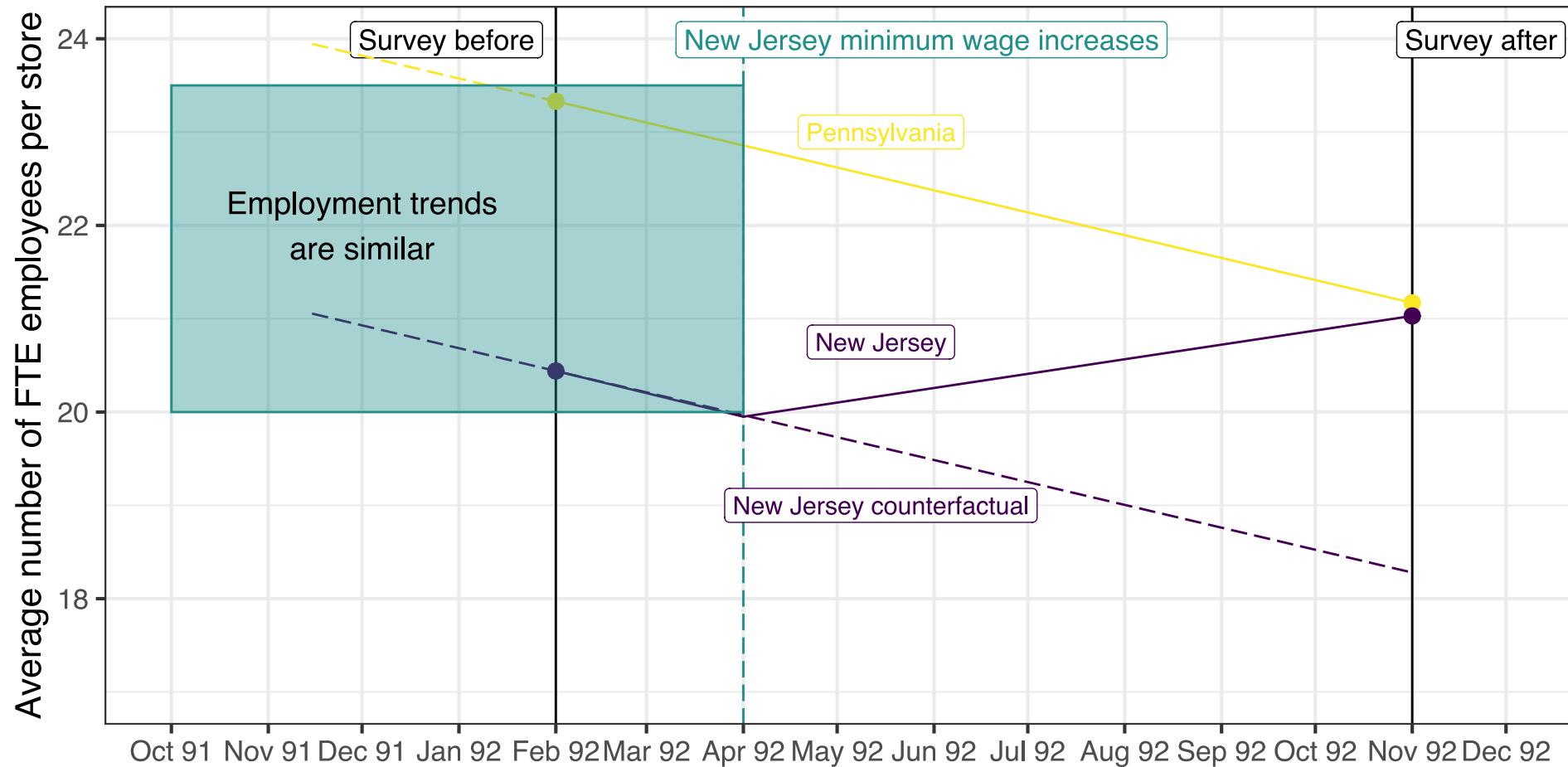
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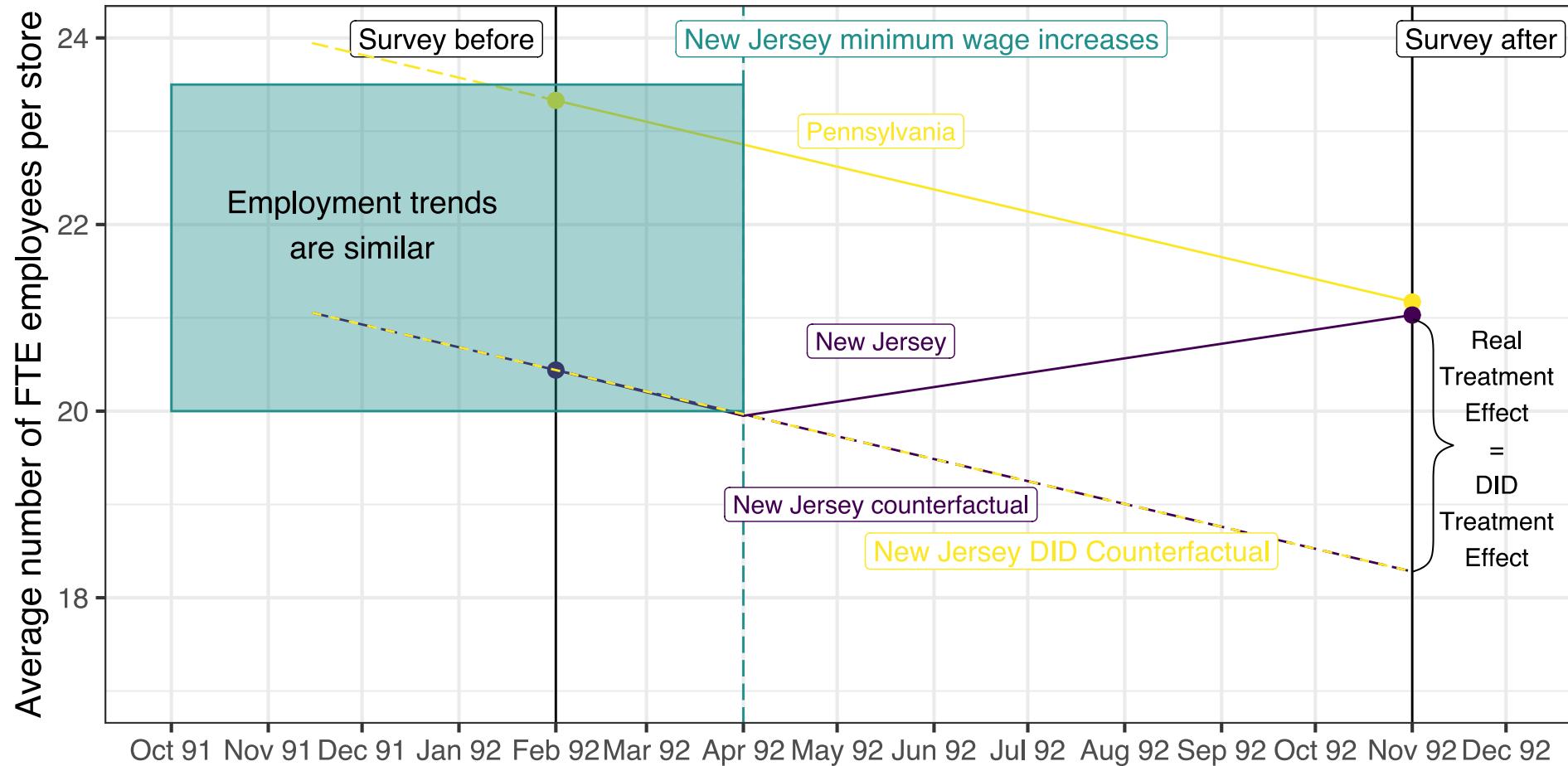
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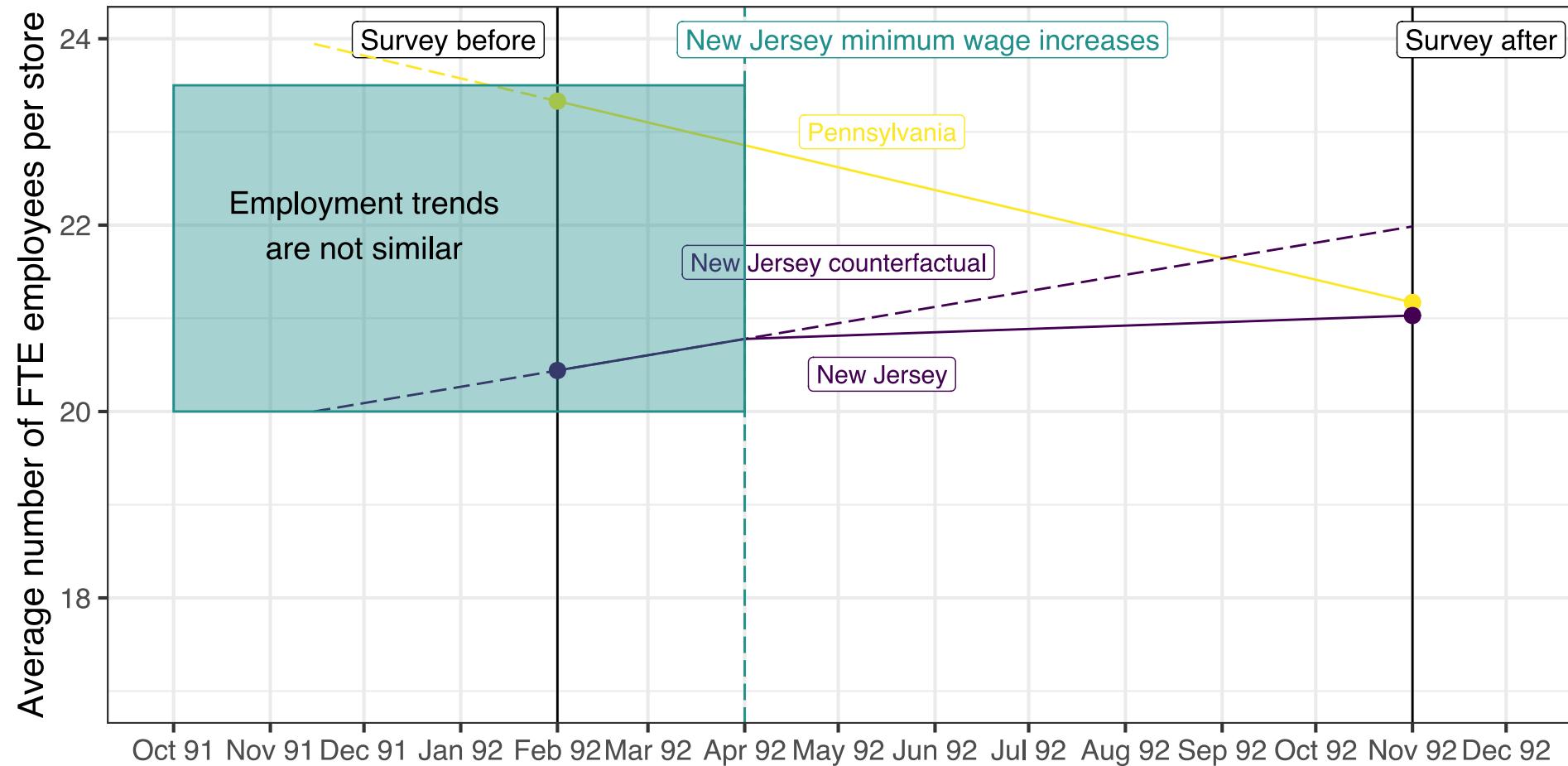
# Parallel trends assumption → Verified ✓



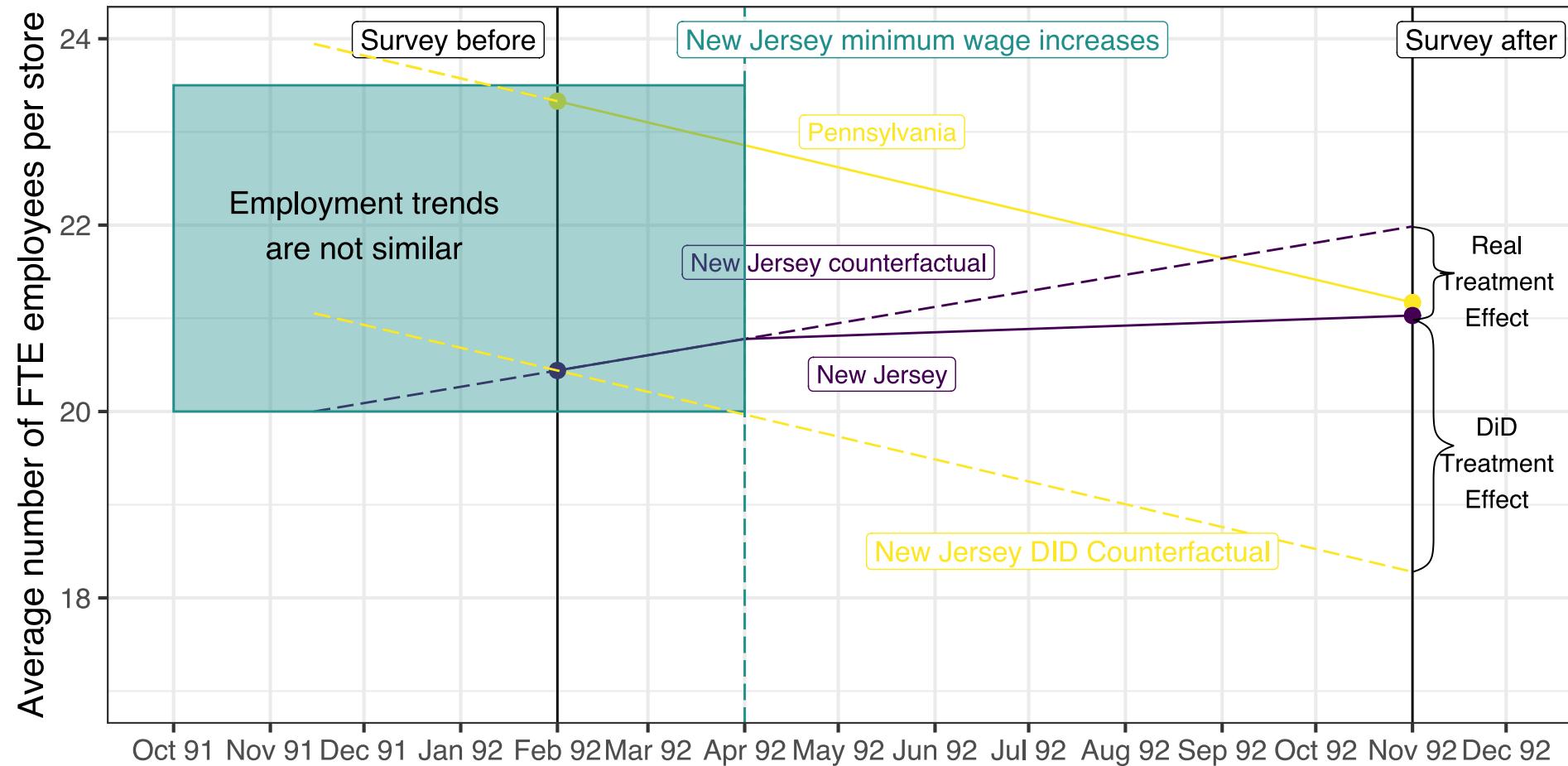
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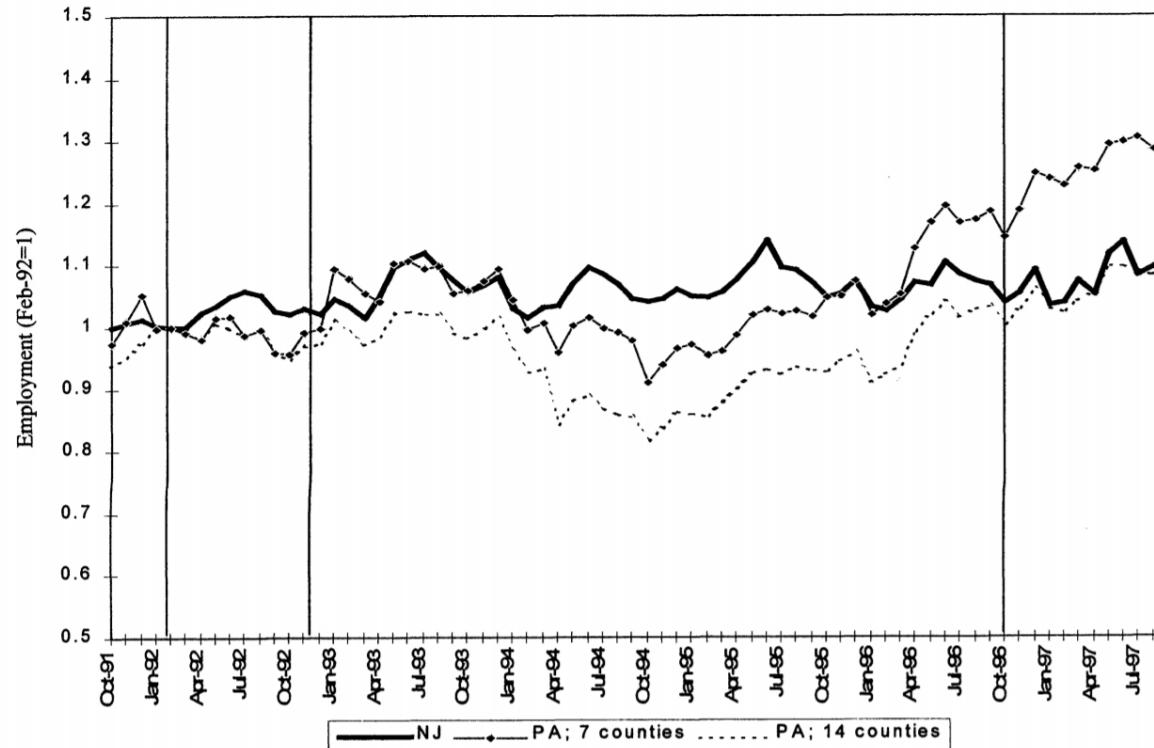


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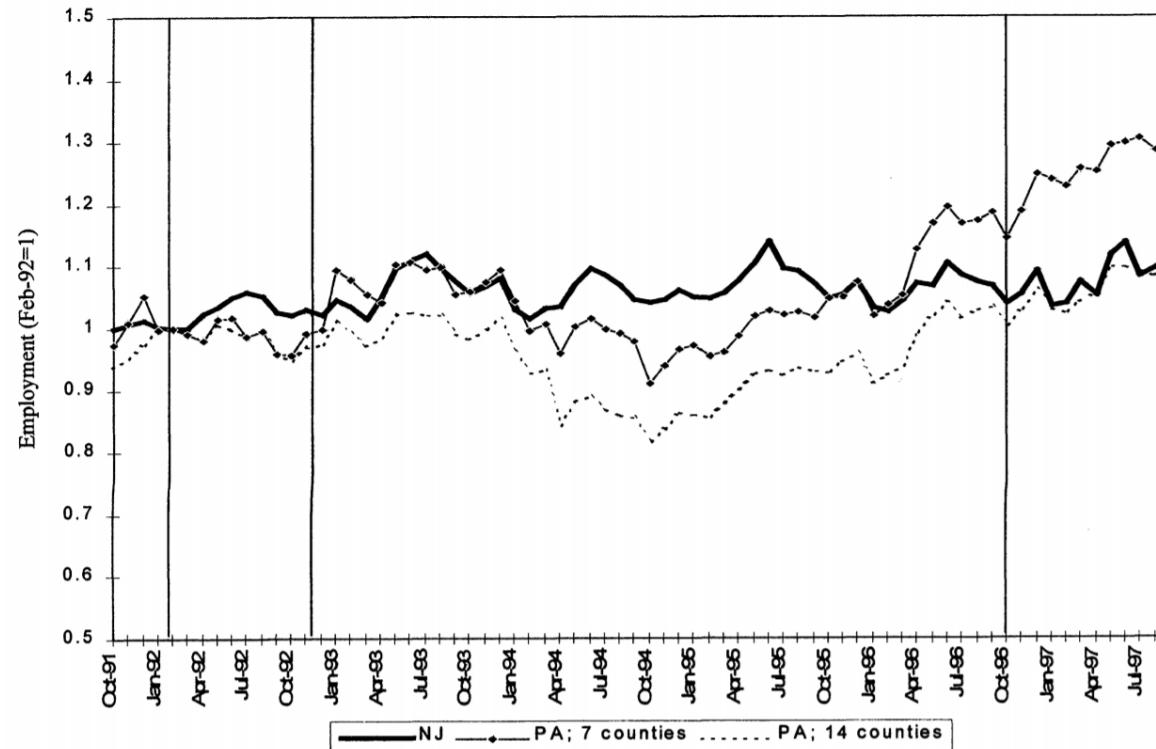
# Parallel Trends Assumption: Card and Krueger (2000)

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- Is the common trend assumption likely to be verified?



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The key assumption underlying DiD estimation is that, in the no-treatment state, restaurant  $i$ 's outcome in state  $s$  at time  $t$  is given by:

$$\mathbb{E}[Y_{ist}^0 | s, t] = \gamma_s + \lambda_t$$

2 implicit assumptions:

1. **Selection bias**: relates to fixed state characteristics ( $\gamma$ )
2. **Time trend**: same time trend for treatment and control group ( $\lambda$ )



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→ the comparison group allows to estimate the **time trend**.



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Therefore we have:

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$$\begin{aligned} DD &= \mathbb{E}[Y_{ist}|s = \text{NJ}, t = \text{Nov}] - \mathbb{E}[Y_{ist}|s = \text{NJ}, t = \text{Feb}] \\ &\quad - \left( \mathbb{E}[Y_{ist}|s = \text{PA}, t = \text{Nov}] - \mathbb{E}[Y_{ist}|s = \text{PA}, t = \text{Feb}] \right) \\ &= \delta + \lambda_{Nov} - \lambda_{Feb} - (\lambda_{Nov} - \lambda_{Feb}) \\ &= \delta \end{aligned}$$



END

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