

# Computational Quantum Physics

## Week 5

### Due on Week 6

#### Exercise 1: **Eigenproblem**

Consider a random Hermitian matrix  $A$  of size  $N$ .

- (a) Diagonalize  $A$  and store the  $N$  eigenvalues  $\lambda_i$  in crescent order.
- (b) Compute the normalized spacings between eigenvalues

$s_i = \Delta\lambda_i / \bar{\Delta\lambda}$  where

$$\Delta\lambda_i = \lambda_{i+1} - \lambda_i,$$

and  $\bar{\Delta\lambda}$  is the average  $\Delta\lambda_i$ .

- (c) Optional: Compute the average spacing  $\bar{\Delta\lambda}$  locally, i.e., over a different number of levels around  $\lambda_i$  (i.e.  $N/100, N/50, N/10 \dots N$ ) and compare the results of next exercise for the different choices.

#### Exercise 2: **Random Matrix Theory**

Study  $P(s)$ , the distribution of the  $s_i$  defined in the previous exercise, accumulating values of  $s_i$  from different random matrices of size at least  $N = 1000$ .

- (a) Compute  $P(s)$  for a random HERMITIAN matrix.
- (b) Compute  $P(s)$  for a DIAGONAL matrix with random real entries.
- (c) Fit the corresponding distributions with the function:

$$P(s) = as^\alpha \exp(-bs^\beta)$$

and report  $\alpha, \beta, a, b$ .

- (d) Optional: Compute and report the average  $\langle r \rangle$  of the following quantity

$$r_i = \frac{\min(\Delta\lambda_i, \Delta\lambda_{i+1})}{\max(\Delta\lambda_i, \Delta\lambda_{i+1})}$$

for the cases considered above. Compare the average  $\langle r \rangle$  that you obtain in the different cases.

*Hint:* if necessary neglect the first matrix eigenvalue.