Comparison of the effects of the numerical scheme on the simulation of a flow around a square cylinder

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1 Introduction

In this report we will consider some results from the scientific literature concerning the flow around a square cylinder and we will investigate the effects of the variation of the numerical convection scheme on the aerodynamic coefficients and the main quantities of the flow. A RANS approach was followed to simulate the air flow around the cylinder by using a RNG k-ϵ model. Time advancement is made by a second-order implicit Euler scheme, the pressure-velocity coupling is solved by a PISO algorithm, while two case studies defined by the use of QUICK and first order upwind (UDS) schemes were considered for the convective term. An analysis of the flow was conducted until it is fully established. We verified that its stationarity by evaluating the difference in the values of two consecutive maxima of the lift coefficient with a transient overshoot threshold $\alpha = 3\text{e} - 3$. Comparison of the two case studies shows that for UDS the end of the transient is reached at time $T_0 = 181.20$, which corresponds to 1.2 times the duration of the transient for QUICK. By assuming the perfect periodicity of flow, we derived the period of vortex detachment $T \approx 7.52$, coincident with the period of lift oscillation and we computed the associated Strouhal number, that in this adimensional model is given by $St = T^{-1}$.

2 Theoretical introduction: truncation error

The differences between UDS and QUICK schemes must be sought in their truncation error. Let us take a generic mono-dimensional problem:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} - \nu\frac{\partial^2 u}{\partial x^2} = g, \quad x \in \mathbb{R}\,, t \geq 0 \tag{1}$$

where a is the flow velocity and ν the kinematic viscosity. The truncation errors can be written as

$$\begin{aligned} \tau_j^c &= -\frac{a}{24}\frac{\partial^3 u}{\partial x^3}(x_j)\Delta x^2 + o(\Delta x^2) \qquad && \text{QUICK} \\ \tau_j^c &= \frac{a}{2}\frac{\partial^2 u}{\partial x^2}(x_j)\Delta x + o(\Delta x) && \text{UDS} \end{aligned} \tag{2}$$

QUICK gives a second order approximation of the convective flux, with a third order numerical dispersive term. Instead, UDS gives a first order approximation of the convective flux, with a second order numerical diffusion term. Adding the latter to the physical diffusion term, the effect is an

Case	St	$\overline{C}_D$	$\widetilde{C}_D$	$\widetilde{C}_L$
Lyn	0.135	2.05÷2.23	n.a.	n.a.
Sakamoto	0.134	2.22	0.132	1.45
Durao	0.139	n.a.	n.a.	n.a.
Murakami	0.132	2.09	0.13	1.60
QUICK	0.138	1.90	0.056	1.39
UDS	0.133	1.57	0.0082	0.364

Table 1: Aerodynamic coefficients obtained by experiments and simulations for a flow around a square cylinder.

over-prediction of viscosity, hence of dissipation: $\nu_{tot} = \nu_{num} + \nu_{air} = \nu_{num} + \mathrm{Re}^{-1}$. In addition, because of the increased diffusivity, the gradients of the velocity field are flattened.

3 Aerodynamic coefficients

By observing Table 1, it is evident that the results obtained with the QUICK convection scheme exhibit a strong correlation with the available data. Conversely, when switching to the UDS convection scheme, the aerodynamic coefficients are consistently underestimated, particularly the fluctuating ones. This discrepancy becomes evident when examining Figure 1, which clearly illustrates the difference in coefficient amplitudes. Although the Strouhal number is slightly underestimated, it remains comparable to the results of other experiments [1, 2, 7, 9].
The reason behind these underestimation lies in the characteristics of the UDS scheme. As previously mentioned, this scheme encounters difficulties in accurately predicting fluxes with steep gradients, resulting in their flattening. Since the velocity gradient is closely linked to the forces generated around the body, the aerodynamic coefficients are consequently underestimated. This behaviour was expected, as a similar underestimation of coefficients was observed in a previous analogous study [3], upon switching from the QUICK to the UDS convective scheme.

4 Pressure statistics

Pressure statistics around the cylinder are important in terms of structural safety and as a measure of radiated acoustic noise. Looking at Figure 2 we note that mean and rms pressure coefficients, obtained from the QUICK simulation, are in reasonable agreement with the available data. More specifically, one can observe that the mean pressure at the back of the cylinder is in good agreement with the LES 3D results, but both are slightly overestimated compared with the experimental data. On the other hand, the UDS simulation predicts the highest side and back pressure with respect to experiments and simulations, while underestimating the fluctuating component. The overestimation of the back pressure was observed in [3] for all k-ϵ models, but the increase, obtained while maintaining the same turbulence model and varying numerical scheme, suggests another cause related to the overdiffusion of the UDS scheme. One can note that the results obtained are similar

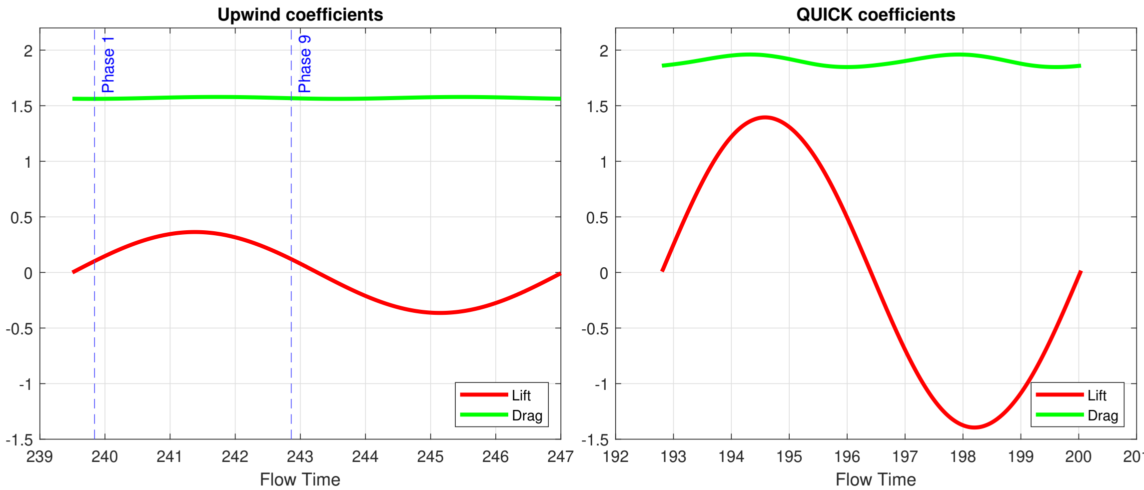

Figure 1: Graphs of the aerodynamic coefficients for both schemes.

to the trend of the pressure coefficient of the standard k-ϵ model in [3]. In fact we recall that the latter model induces an overestimation of the turbulent diffusion on the mean flow, which as also mentioned in [3] generates a similar effect to the one caused by the variation in the convection scheme from QUICK to UDS, described in Section 2.

5 Velocity in the wake region along the centerline

The Figure 3 shows that QUICK results match very well with the LES 3D results under all aspects, but also with the experimental data of Durao [2] and Lyn [1] for the length of the detaching vortex. On the other hand UDS presents 2 problems. Firstly, it predicts the detaching vortex too large by a factor of around 1.5 with respect to QUICK and Lyn [1]: this is motivated by the over-predicted viscosity, which is known to cause a diffusion of vortexes. Then it shows a slower recovery of the undisturbed flow velocity in the wake region that can be attributed to the excessive dissipation of kinetic energy in such region.

In Figure 4 we can clearly see the over-diffusive effect of UDS from the flattening of velocity gradients, but this time we can also note the dispersive effect of QUICK by looking at the oscillations around the regions with steep variations of the vertical velocity.

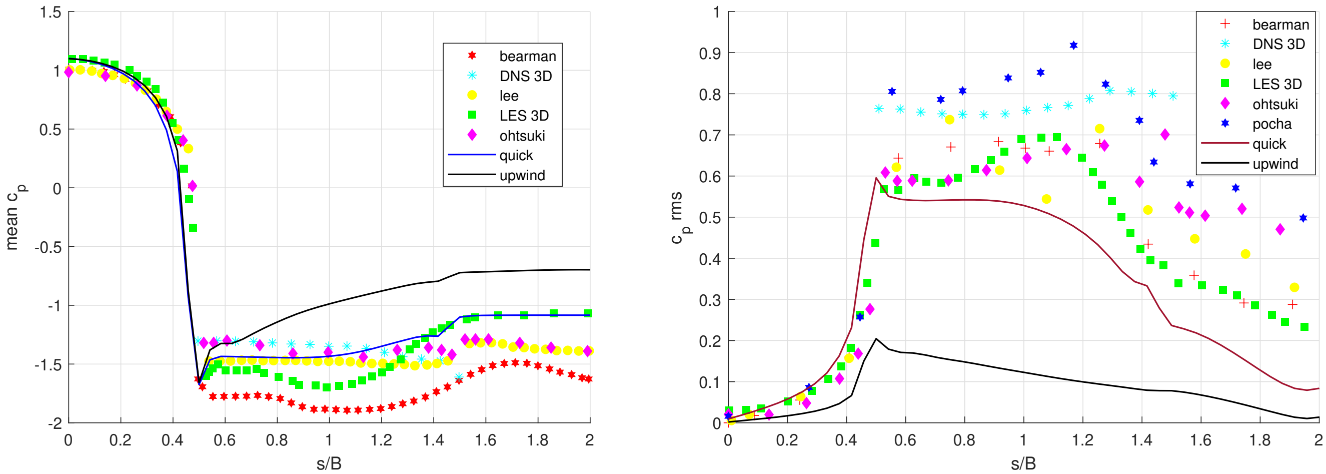

Figure 2: Mean and rms pressure coefficient distribution around the cylinder. The horizontal axis is the curvilinear abscissa along the cylinder surface, where $s/B = 0$ corresponds to the stagnation point, while $s/B = 0.5$ and 1.5 corresponds to the windward and leeward corner of the cylinder, respectively.

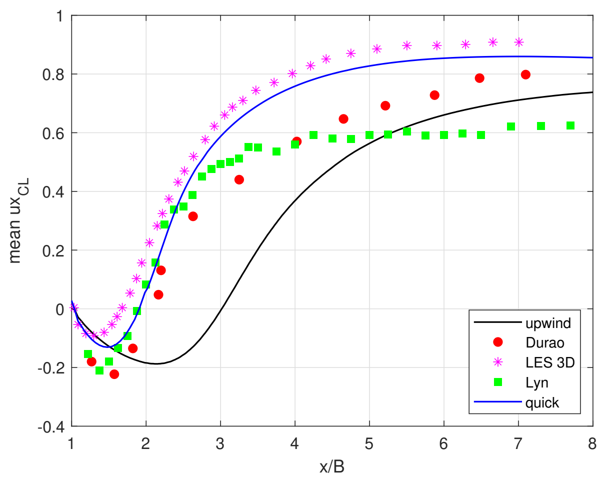

Figure 3: Mean horizontal velocity along the centerline ($y/B = 0$). The longitudinal axis is the position in the space, where $x/B = 0$ and 1 represents the upstream and the downstream side face of the cylinder, respectively.

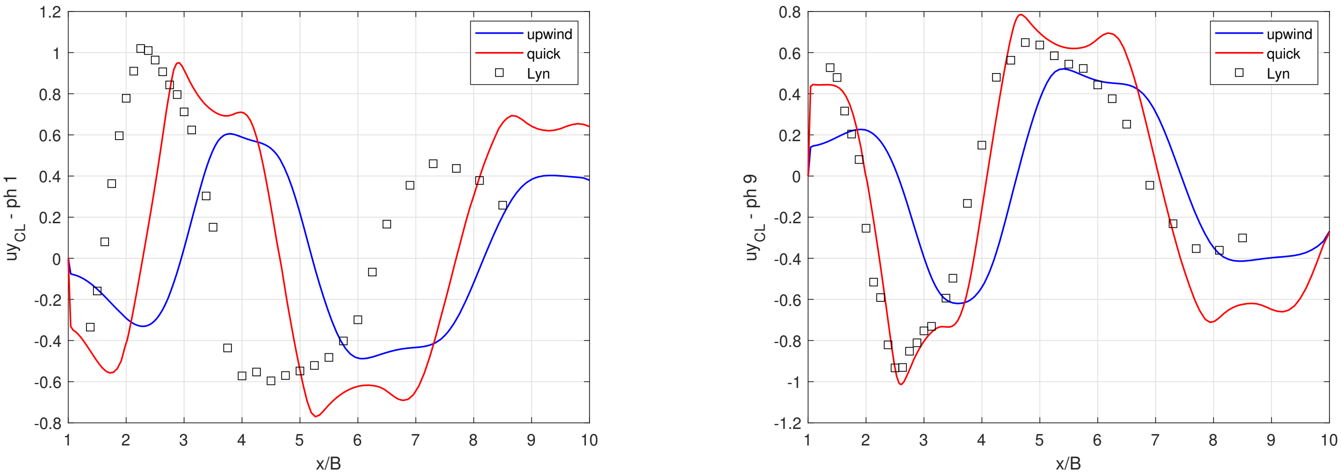

Figure 4: Instantaneous vertical velocity along the centerline, for phases 1 and 9.

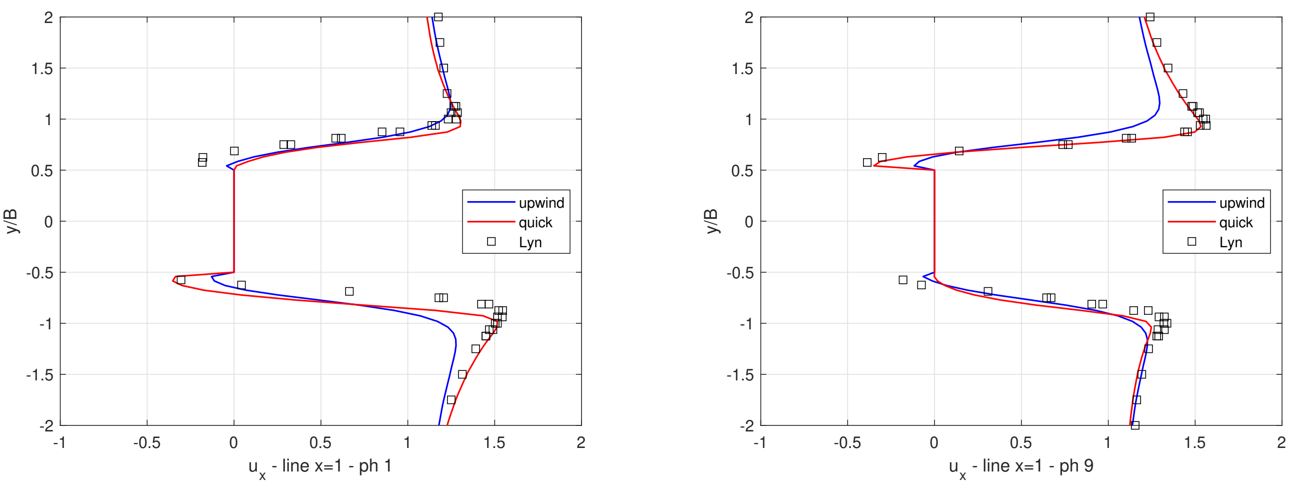

Figure 5: Distribution of the longitudinal component of the instantaneous velocity, along the vertical axis passing through the back face of the cylinder ($x/B = 1$), for phases 1 and 9.

6 Velocity distribution on the line $x/B = 1$

Predictions obtained by QUICK convection scheme fit well with Lyn's experiments, while again we observe a decreased slope of the velocity simulated by UDS. But what it is more interesting to observe is that UDS, according with Lyn's experiment, predicts qualitatively a small reversed flow also on the side in which the vortex is not detaching, whereas QUICK fixes the velocity to zero in the same point. This holds for both phase 1 and phase 9. Thus, even though QUICK remains the most appropriate scheme on average, UDS catches one more characteristic of the phenomenology of the flow.

7 Summary and conclusion

The QUICK convective scheme is found to adequately reproduce the aerodynamic coefficients and all the other main quantities of the flow, both mean in time and instantaneous, with respect to the available data. Instead, UDS introduces numerical diffusion and tends to underestimate aerodynamic coefficients and the fluctuating part of the pressure coefficient, while overestimating the side and back pressure and the size of the detaching vortex. Despite this behaviour, UDS could be helpful when using an approach affected by numerical instability, but as a downside it gives lower accuracy of the first order. Consequently UDS could be fine in predicting stationary fluxes with low gradients, but for the examined case of study, which is turbulent and time-dependent, it is not efficient, since we would need a higher spacial resolution to make an accurate prediction, which could be computationally too expensive.

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