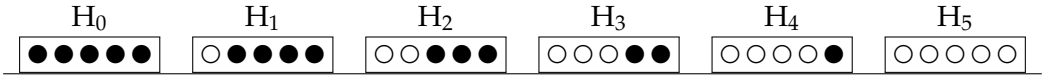
Laboratory Session 04 : April 21, 2021

Exercises due on : May 9, 2021

Exercise 1 - Six Boxes Toy Model : inference

- The six boxes toy model is described in reference [1].
- Labeling the boxes as follows:



- write a program in R that:

1) allows the user to insert the color of a randomly extracted box and

3) prints on the standard output the probability of selecting each box

4) plots the probability for each box as a function of the extraction step

Exercise 2 - Six Boxes Toy Model : simulation

- consider again the six boxes toy model of the previous exercise and write a simulation program that:

1) selects a random box

2) makes random sampling from the box

3) prints on the standard output the probability of selecting each box

4) plots the probability for each box as a function of the number of trial

Exercise 3

- An important property of the gamma distribution is the so-called *reproductive property*
- given a sequence of independent random variable $X_j \sim \text{Gamma}(\alpha_j, \beta)$, it follows that

$$Y = \sum_{j=1}^{n} X_j \rightarrow Y \sim \text{Gamma}(\alpha, \beta) \quad \text{where} \quad \alpha = \sum_{j=1}^{n} \alpha_j$$

- if $\alpha = m$ is an integer, a random variable from gamma distribution $\text{Gamma}(m, \beta)$ (also known as Erlang distribution) can be obtained by summing m independent exponential random variables $X_j \sim \text{Exp}(\beta)$:

$$Y = \frac{1}{\beta} \sum_{j=1}^{n} (-\ln U_j) = -\frac{1}{\beta} \ln \prod_{j=1}^{n} U_j$$

a) write an algorithm to sample variables from an Erlang distribution $\text{Gamma}(m, \beta)$

Exercise 4

- one of the first random number generator was proposed by von Neumann, the so-called *middle square* algorithm
- write R code to implement this type of generator and, given a fixed digit number input, square it an remove the leading and trailing digits, in order to return a number with the same number of digits as the original number
- *Suggestion :* after having squared the number, convert it to a list of characters
  (`number <- unlist(strsplit(as.character(x.squared),""))`)
  and, after having removed the head and tail of the list, convert it back to a number
  (`as.numeric(paste(number.after.trimming, collapse=""))`)

Bibliography

[1] G. D'Agostini, *Probability, propensity and probabilities of propensities (and of probabilities)*, https://arxiv.org/pdf/1612.05292.pdf

G. D'Agostini, *More lessons form the six box toy experiment*, https://arxiv.org/pdf/1701.01143.pdf