

Laboratory Session 07 : May 18, 2020

Exercises due : June 3, 2020

Exercise 1

- a researcher has collected $n = 15$ observations that are supposed to come from a **Normal** distribution with known variance $\sigma^2 = 16$:

26.8	26.3	28.3	28.5	16.3
31.9	28.5	27.2	20.9	27.5
28.0	18.6	22.3	25.0	31.5

- assuming a normal prior for μ , $\text{Norm}(m = 20, s^2 = 25)$,
- (a) determine the posterior distribution $P(\mu \mid y_1 \dots y_{15})$ and find the posterior mean and standard deviation
 - (b) find the 95% credibility interval for μ
 - (c) plot the posterior distribution, indicating on the same plot: the mean value, the standard deviation, and the 95% credibility interval
 - (d) repeat the analysis using a different prior $\text{Norm}(m = 30, s^2 = 16)$ and plot, on the same graph the likelihood, the prior and the posterior.
 - (e) compare the credibility intervals obtained with the two priors

Exercise 2

- a researcher has collected $n = 16$ observations that are supposed to come from a **Normal** distribution with known variance $\sigma^2 = 4$:

4.09	4.68	1.87	2.62	5.58	8.68	4.07	4.78
4.79	4.49	5.85	5.09	2.40	6.27	6.30	4.47

- assuming the prior is a step function:

$$g(\mu) = \begin{cases} \mu & \text{for } 0 < \mu \leq 3, \\ 3 & \text{for } 3 < \mu \leq 5, \\ 8 - \mu & \text{for } 5 < \mu \leq 8, \\ 0 & \text{for } \mu > 8. \end{cases}$$

- (a) find the posterior distribution, the posterior mean and standard deviation
- (b) find the 95% credibility interval for μ
- (c) plot the posterior distribution, indicating on the same plot: the mean value, the standard deviation, and the 95% credibility interval
- (d) plot, on the same graph, the prior, the likelihood and the posterior distribution