

Laboratory Session 05 : April 28, 2021

Exercises due : May 16, 2021

## Exercise 1

- A publishing company has recently launched a new journal. In order to determine how effective it is in reaching its possible audience, a market survey company selects a random sample of people from a possible target audience and interviews them. Out of 150 interviewed people, 29 have read the last issue of the journal.
- a) What kind of distribution would you assume for  $y$ , the number of people that have seen the last issue of the journal ?
- b) Assuming a uniform prior, what is the posterior distribution for  $y$  ?
- c) Plot both posterior and likelihood distributions functions

## Exercise 2

- Three students want to construct their prior probability about the proportion of residents that support the building of a new concert hall in their small town.
- Anna thinks that her prior is a beta distribution with mean 0.2 and a standard deviation of 0.08.
- Benny moved only recently to this new town and therefore he does not have the slightest idea about it. Therefore he decides to use a uniform prior.
- Chris believes that his prior should have a trapezoidal shape

$$f(X) = \begin{cases} 20x & 0 \leq x < 0.1 \\ 2 & 0.1 \leq x < 0.3 \\ 5 - 10x & 0.3 \leq x < 0.5 \\ 0 & x \geq 0.5 \end{cases}$$

- a) Draw and compare the three prior distributions.
- The next day the three students decide to interview a sample of 100 citizens of the small town, asking for their opinion. Out of the interviewed sample, 26 support the building of the new concert hall.
- b) Evaluate and draw the three posterior distributions.
- c) Give an estimate of the most probable value and the 95% credibility interval.

## Exercise 3

- A coin is flipped  $n = 30$  times with the following outcomes:

T, T, T, T, T, H, T, T, H, H, T, T, H, H, H, T, H, T, H, T, H, H, T, H, T, H, T, H, H, H

- a) Assuming a flat prior, and a beta prior, plot the likelihood, prior and posterior distributions for the data set.
- b) Evaluate the most probable value for the coin probability  $p$  and, integrating the posterior probability distribution, give an estimate for a 95% credibility interval.
- c) Repeat the same analysis assuming a sequential analysis of the data. Show how the most probable value and the credibility interval change as a function of the number of coin tosses (i.e. from 1 to 30).
- d) Do you get a different result, by analyzing the data sequentially with respect to a one-step analysis (i.e. considering all the data as a whole) ?