Self-Assessment American Options

1 Exercise 1: Lattice Methods

a) Compute and visualize the $Cox - Ross - Rubinstein$ binomial tree (without dividends) of the underlying, the European call and put and the American call and put.

b) Compute and visualize the *Early Exercise Boundary* for an American put option. Assuming a sufficiently small time-step, use Monte Carlo simulation to compute the Risk-Neutral probability that an American put option is early exercised and plot the *Optimal Stopping Time* distribution for an ITM put option.

c) Compute and visualize the Cox-Ross-Rubinstein *Trinomial Tree* (without dividends) of the underlying, and the American Put Option. Compare the pricing engine of the American Option using binomial tree with the one using trinomial tree as you increase the number of steps. For which pricing engine is the convergence faster? Find an explanation of the result.

d) Price an American Lookback put with payoff: $max_{\{0<s<t\}}S(s) - S(t)$ using the *Forward Shooting Grid* algorithm built over the trinomial tree. Show how the *Richardson Extrapolation* can be used to achieve higher computational efficiency.

2 Exercise 2: Finite-Difference method

a) Through a change of variables of your choice, transform the $Black - Scholes$ PDE into the $1D$ *Heat equation*. Using the *theta method* with asymptotic Dirichlet Boundary conditions for a European call or put, study the stability of *discretization scheme*, focusing on $\theta = 1$ (Implicit Euler), $\theta = 1/2$ (Crank-Nicolson), $\theta = 0$ (Explicit Euler).

b) Price an American put with the theta method using $Brennan - Schwarz$ projection. Compute and visualize Δ and Γ of an American put in the continuation region and compare it with the greeks of a European put.

3 Exercise 3: LSMC

a) Take *Bates* Stochastic-Volatility-Jump diffusion model under the $\mathbb{Q} - measure$:

$$\frac{dS_t}{S_{t^-}} = \big(r - q - \lambda\kappa_J\big)\,dt + \sqrt{v_t}\,dW_{1,t} + (J-1)\,dN_t$$

$$dv_t = \gamma\big(\theta - v_t\big)\,dt + \sigma_v\sqrt{v_t}\,dW_{2,t}$$

$$corr(dW_{1,t}, dW_{2,t}) = \rho$$

$$\kappa_J = \mathbb{E}[J-1] = e^{\mu_J + \frac{1}{2}\sigma_J^2} - 1$$

$$J_t \sim Lognorm(\mu_J, \sigma_J), \quad N_t \sim Poisson\ Process\ with\ intensity\ \lambda > 0$$

Price an American call Option using *Longstaff and Schwartz Least Squares Monte Carlo* method where the Continuation Value is estimated via polynomial regressions.

b) Set $q = 0$ and introduce in *Bates* dynamics discrete dividends in the form of deterministic dividend payout ratio. Use $LSMC$ to obtain the optimal stopping distribution for an ATM call and comment it.

c) Assume multidimensional *Bates* SVJ dynamics, injecting correlation in price shocks and in the Compound Poisson Processes and price an American worst-of down-and-out option using **Neural** LSMC.