30 settembre. 2025

PROBLEMA:

Come si definisce

a^x se $x \in \mathbb{R} \setminus \mathbb{Q}$

cioè,

se x è irrazionale ?

DEFINIZIONE DELLA FUNZIONE ESPONENZIALE IN $\mathbb{R} \setminus \mathbb{Q}$:

Esempio:

Come si definisce:

$$3^{\sqrt{2}} = ?$$

$$\left(\frac{1}{2}\right)^{\sqrt{6}} = ?$$

$$4^{\sqrt{2}} = ?$$

Discutiamo prima un caso particolare:

$$3^{\sqrt{2}} = ?$$

Idea:

Si approssima $\sqrt{2}$ con una successione crescente di numeri razionali.

$$\sqrt{2} = 1{,}41421356 23 \ldots$$

$$q_0 = 1$$

$$q_1 = 1{,}4$$

$$q_2 = 1{,}41$$

$$q_3 = 1{,}414$$

$$q_4 = 1{,}4142$$

$$q_5 = 1{,}41421$$

$$q_6 = 1{,}414213$$

$$\ldots\ldots$$

$$\Longrightarrow \quad q_n \xrightarrow[n]{} \sqrt{2}$$

Allora si ha che:

(I) $\Longrightarrow$ $3^{q_n} \leq 3^{q_{n+1}}$ poiché $q_n \leq q_{n+1}$

(II) $\Longrightarrow$ $3^{q_n} \leq 3^2 = 9$

poiché $q_n \leq 2 \ \forall n$

Dunque:

$(3^{q_n})_{n\in\mathbb{N}}$ è una successione crescente, superiormente limitata

DEF.: Si definisce $3^{\sqrt{2}}$ il limite della successione $(3^{q_n})_n$:

$$3^{\sqrt{2}} := \lim_{n\to+\infty} 3^{q_n}$$

(Si prova che il suddetto limite non dipende dalla successione $(q_n)_n$ scelta)

Caso generale: $r \in \mathbb{R} \setminus \mathbb{Q}$

$(q_n)_{n \in \mathbb{N}} \subseteq \mathbb{Q}$: $q_n \nearrow$

$q_n \longrightarrow r$

$(a > 0)$

Def.: $$a^r := \lim_{n \to +\infty} a^{q_n}$$

(Nota: si dimostra che il suddetto limite è indipendente dalla successione $(q_n)_n$ scelta per "approssimare" r)

Notazione:

$$\exp_a(x) := a^x \qquad 0 < a \neq 1$$

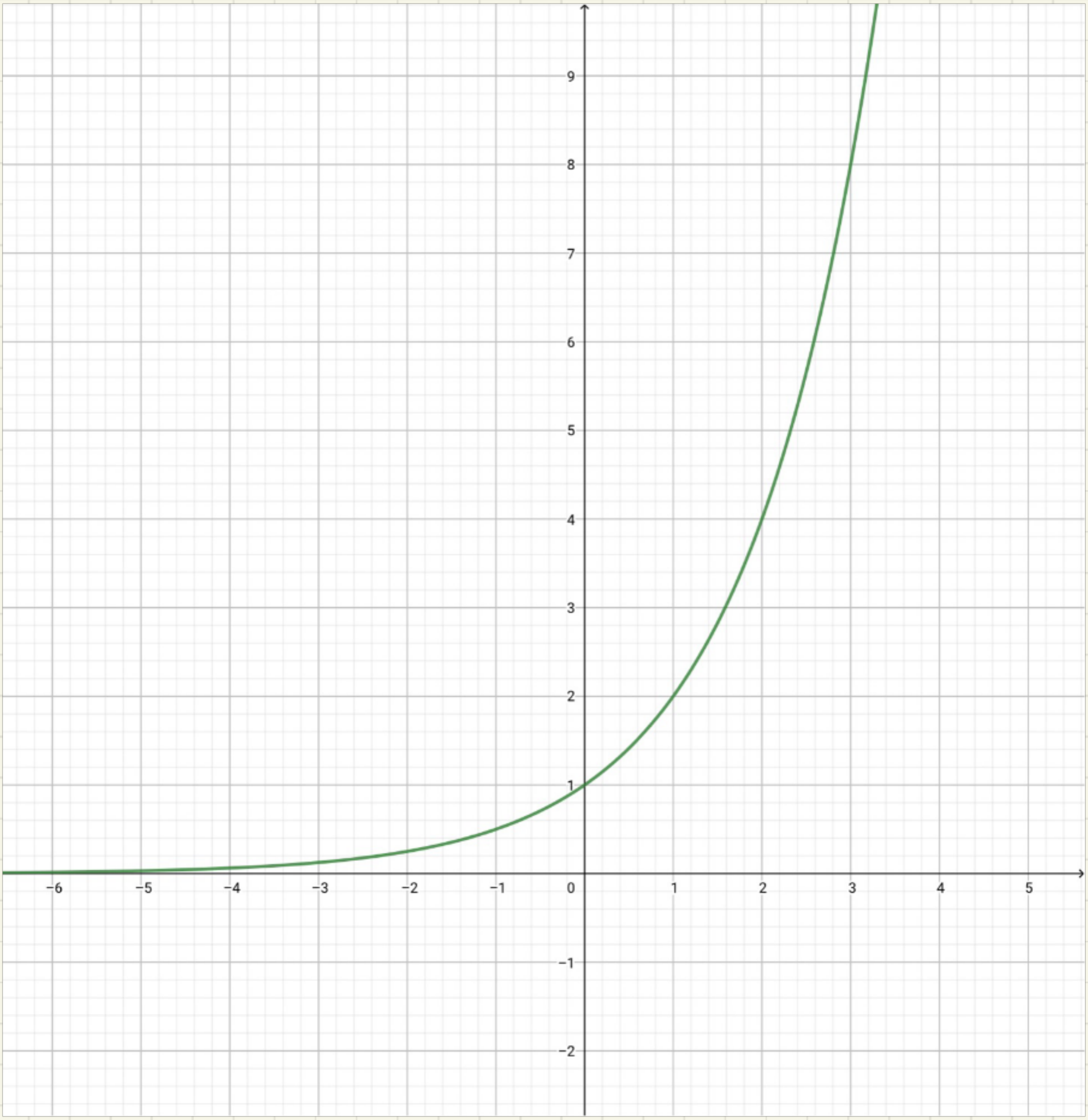
Se si sceglie e come base della funzione esponenziale: il numero e di Euler/Neper

e^x

$$\exp(x) := e^x$$

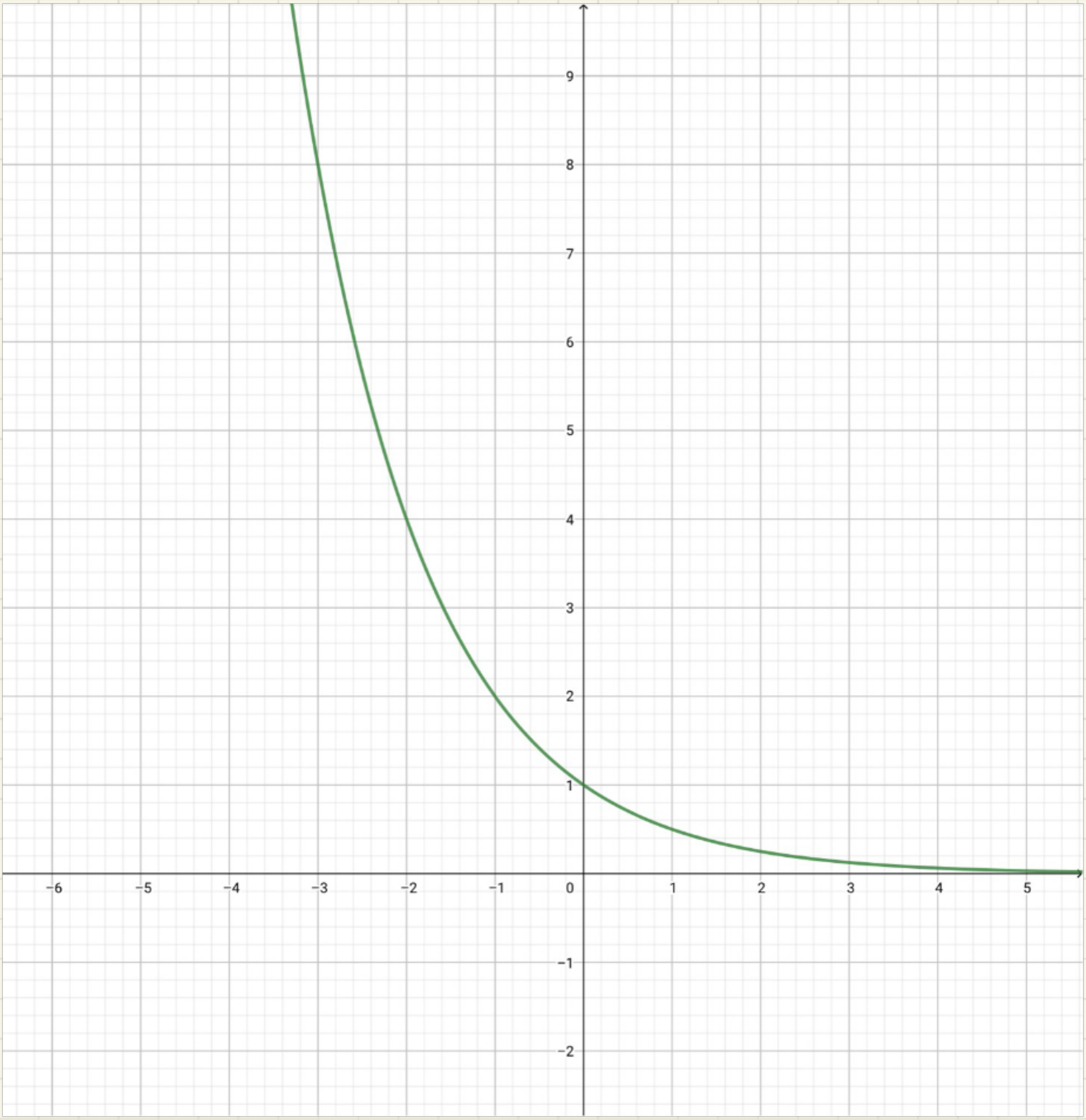
esponenziale a base naturale

GRAFICO DI a^x $(a > 1)$:



a^x è crescente (strettam.)
(caso $a = e$)

GRAFICO DI a^x $(0 < a < 1)$



a^x è decrescente (strett.)

FUNZIONE LOGARITMICA:

TEOR.:

$a \in \mathbb{R} \ : \quad 0 < a \ , \quad a \neq 1$

$\forall y \in \mathbb{R} : \ y > 0 \quad \exists! \, x \in \mathbb{R} :$

$$a^x = y$$

$\log_a y := x \longrightarrow a^{\log_a y} = y$

↑

logaritmo di y in base a

$\log_a : \mathbb{R}^*_+ \longrightarrow \mathbb{R}$

è una funzione di dominio

$$\mathbb{R}^*_+ = \{ r \in \mathbb{R} \mid r > 0 \}$$

Esempi :

$\log_2 16 = 4 \qquad (2^4 = 16)$

$\log_2 \frac{1}{32} = -5 \qquad \left(2^{-5} = \frac{1}{32}\right)$

$\log_{10} 0,001 = -3 \qquad \dots$

$\log_a 1 = 0 \qquad (a^0 = 1)$

$\log_{\frac{1}{3}} 81 = -4$

$\cancel{\log_a 0}$

$$\exp_a : \mathbb{R} \longrightarrow \mathbb{R}^*_+$$

$$\log_a : \mathbb{R}^*_+ \longrightarrow \mathbb{R}$$

Dal teorema precedente:

$$a^{\log_a y} = y \qquad \forall y \in \mathbb{R}^*_+$$

$$\log_a (a^x) = x \qquad \forall x \in \mathbb{R}$$

Dunque $\log_a$ è la funzione inversa di $\exp_a$

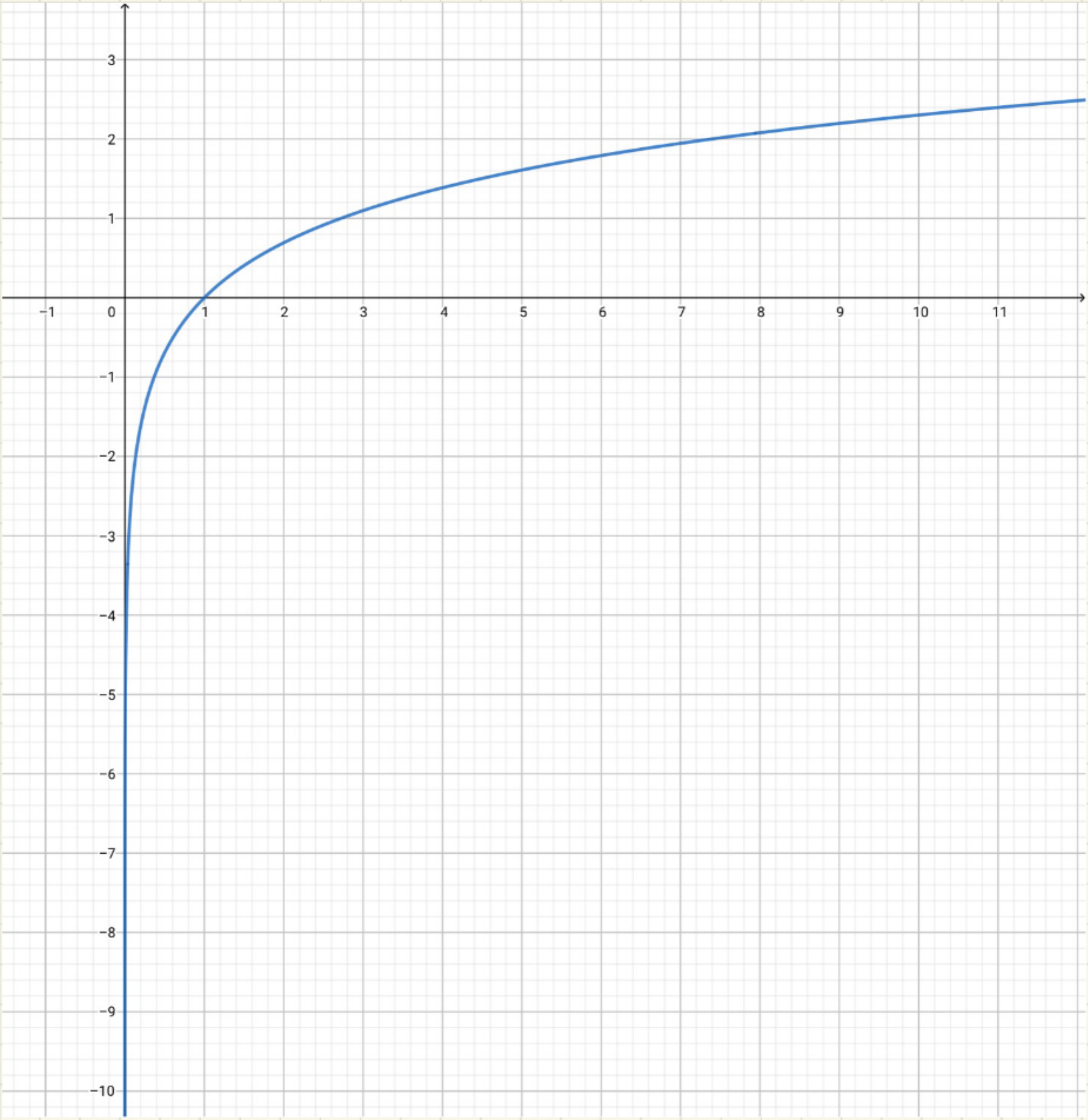
$$\mathbb{R}_+^* \xrightarrow{\log} \mathbb{R} \xrightarrow{\exp} \mathbb{R}_+^*$$

$$y \longmapsto \log_2 y \longmapsto e^{\log_2 y} = y$$

$$\mathbb{R} \xrightarrow{\exp} \mathbb{R}_+^* \xrightarrow{\log} \mathbb{R}$$

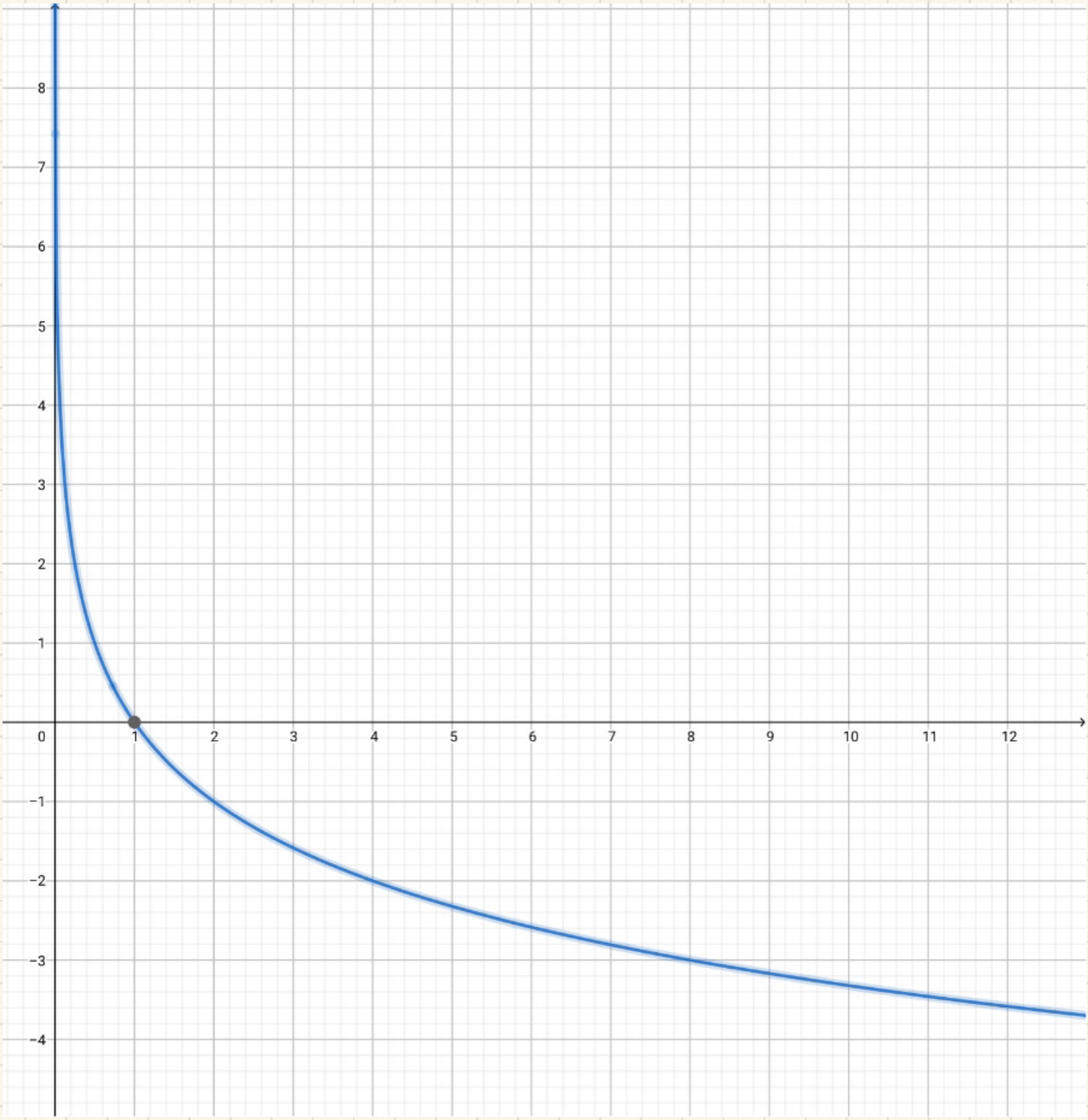
$$x \longrightarrow a^x \longmapsto \log_a(a^x) = x$$

GRAFICO DI $\log_a x$ $(a > 1)$



$\log_a x$ è crescente

GRAFICO DI $\log_a x$ $(0 < a < 1)$



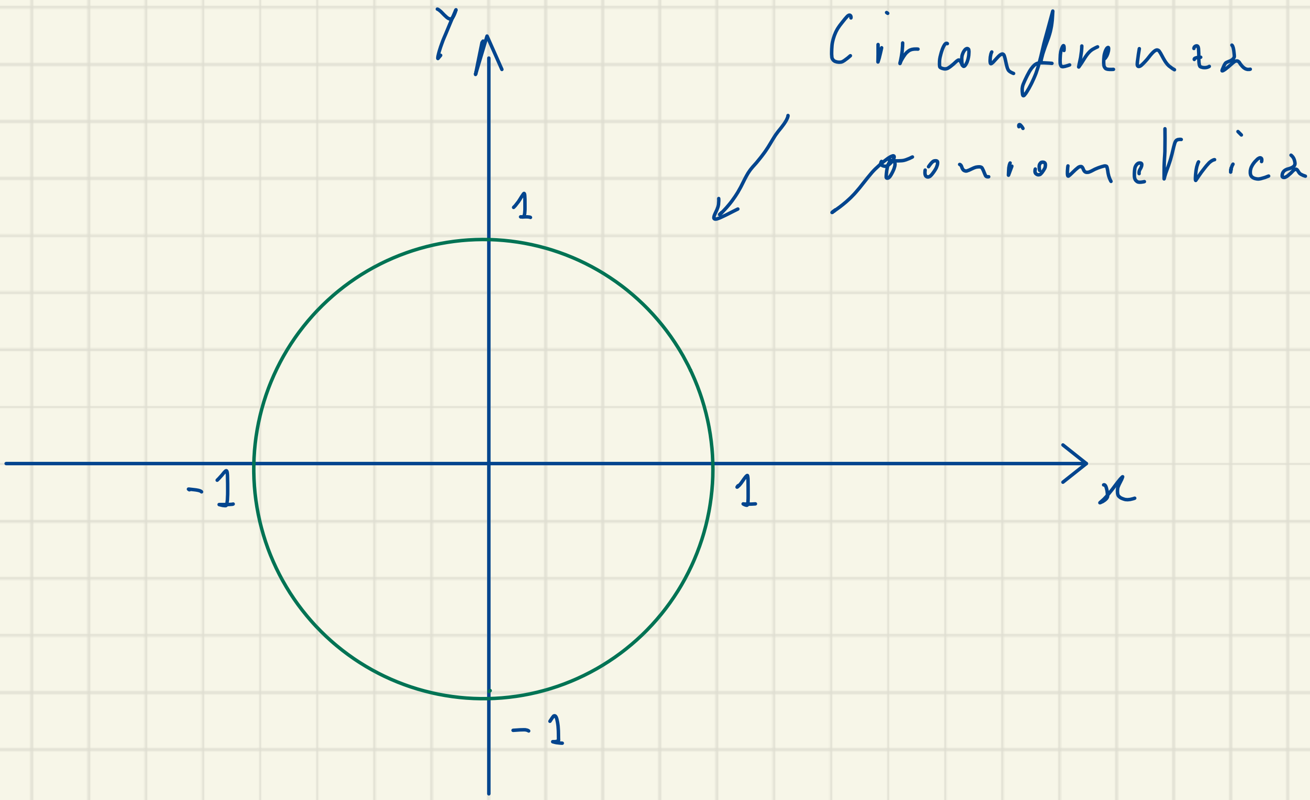
$\log_a x$ è decrescente

Breve ripasso

di trigonometria:

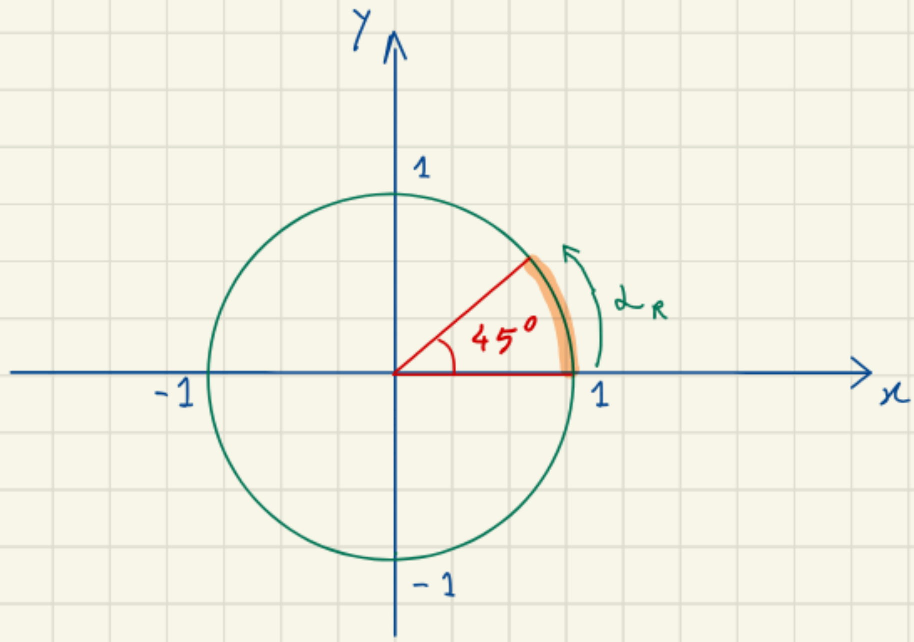
FUNZIONI GONIOMETRICHE:

Angoli in radianti:



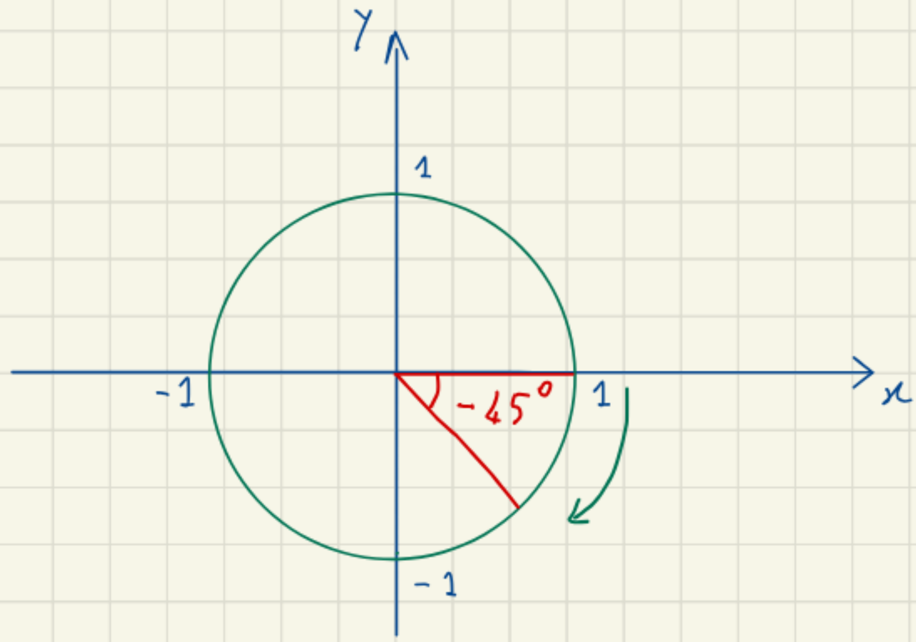
$x^2 + y^2 = 1$ Eq. della circonf. goniometrica

Lunghezza della circonferenza gon.
$= 2\pi \cdot 1 = 2\pi$



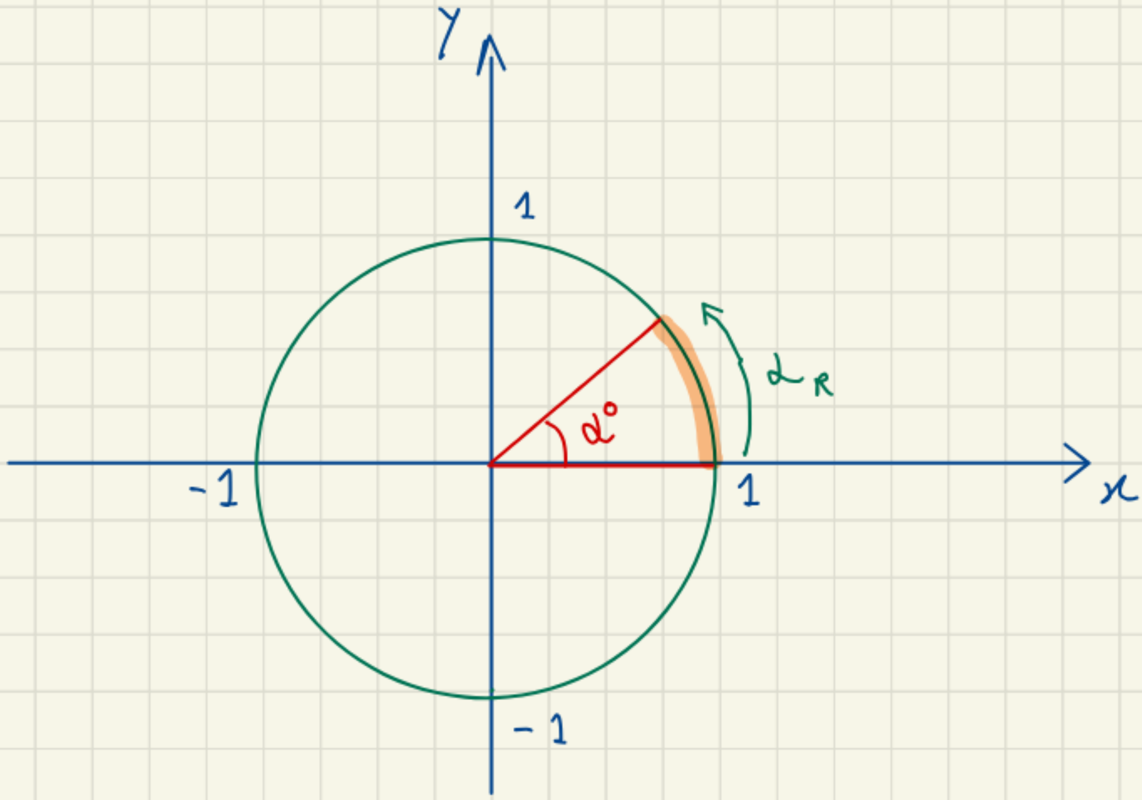
Orientamento positivo =

= senso antiorario



Orientamento negativo =

= senso negativo



$\alpha°$ = misura dell'angolo in gradi

α_R = misura dell'angolo in radianti

$$\alpha° : 360° = \alpha_R : 2\pi$$

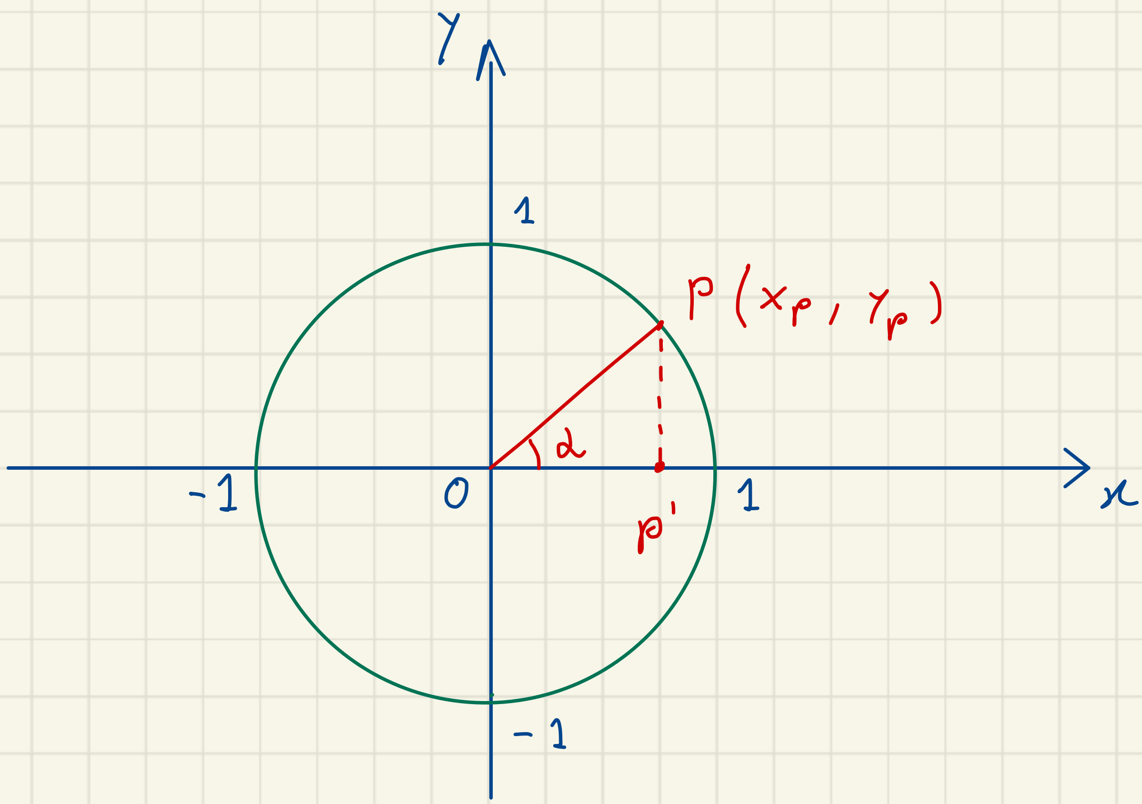
$$\alpha^\circ : 360 = \alpha_R : 2\pi$$

$$\Rightarrow \quad \alpha_R = \alpha^\circ \cdot \frac{2\pi}{360} = \alpha^\circ \cdot \frac{\pi}{180^\circ}$$

$$\boxed{\alpha_R = \alpha^\circ \cdot \frac{\pi}{180}}$$

α°	α_R
0	0
45°	$\frac{\pi}{4}$
90°	$\frac{\pi}{2}$
180°	π
270°	$\frac{3\pi}{2}$
360°	2π

Funzioni seno e coseno:



Sia P un punto sulla circonf. goniometrica: $P(x_P, y_P)$

Sia α l'angolo che il raggio OP forma con l'asse delle x.

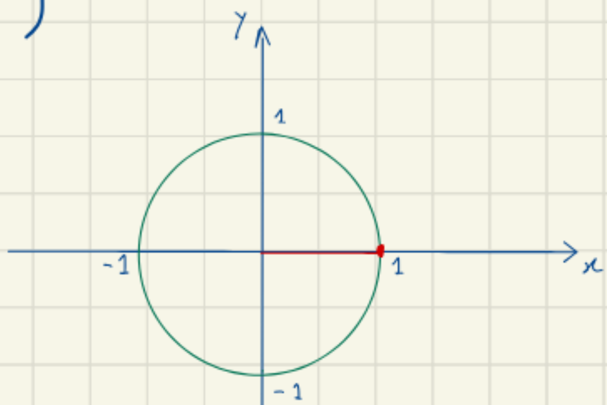
DEF.: Si definiscono:

$$\sin \alpha := y_P$$

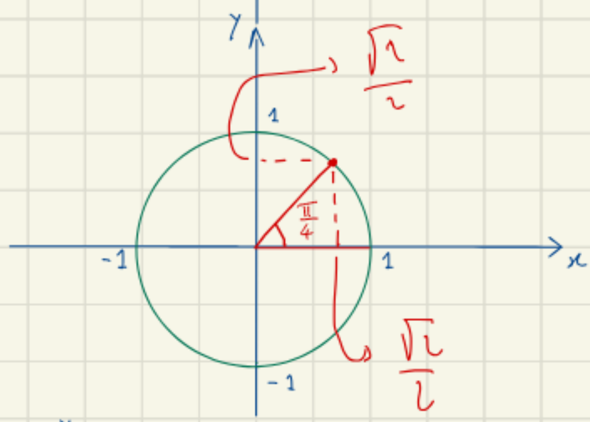
$$\cos \alpha := x_P$$

Esempi: (in radianti)

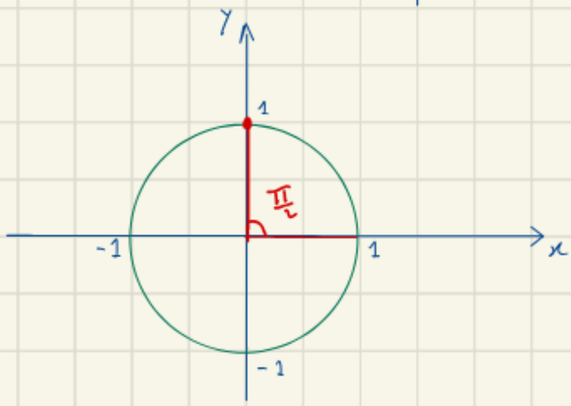
$$\cos 0 = 1$$
$$\sin 0 = 0$$



$$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$



$$\cos \frac{\pi}{2} = 0$$
$$\sin \frac{\pi}{2} = 1$$



Siccome (x_p, y_p) giace sulla circonferenza goniometrica:

$$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$$

Per costruzione, il seno e il coseno sono funzioni periodiche di periodo 2π:

$$\sin(\alpha \pm 2\pi) = \sin\alpha \quad , \quad \forall \alpha \in \mathbb{R}$$

$$\cos(\alpha \pm 2\pi) = \cos\alpha \quad , \quad \forall \alpha \in \mathbb{R}$$

Inoltre: $\forall x \in \mathbb{R}$

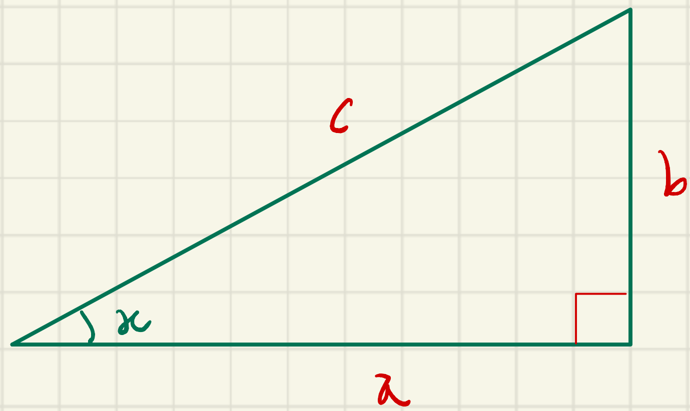
$$|\sin x| \leq 1 \quad , \quad |\cos x| \leq 1$$

Si è così provato che:

$$\boxed{\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}}$$

$$\mathbb{D}(\operatorname{tg}) = \{\alpha \in \mathbb{R} \mid \cos\alpha \neq 0\}$$

$$= \{\alpha \in \mathbb{R} \mid \alpha \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\}$$



$$a = c \cdot \cos x$$

$$b = c \cdot \sin x$$

$$\frac{b}{a} = \tan x \implies b = a \cdot \tan x$$

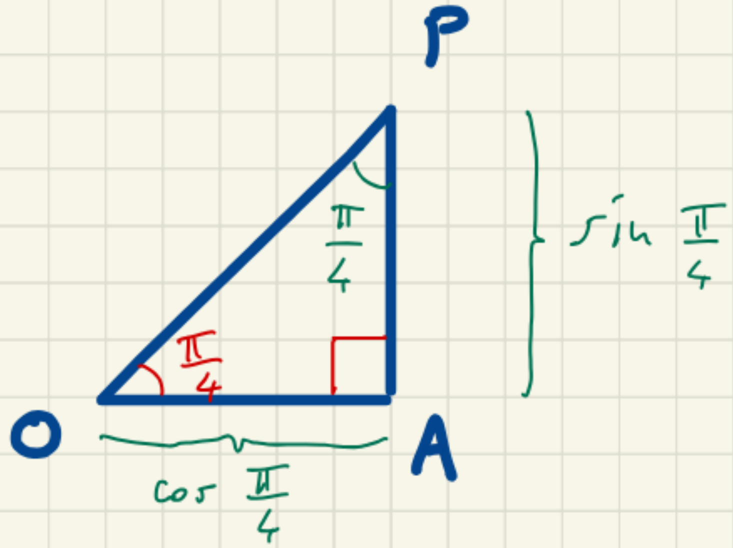
TABELLA RIASSUNTIVA: $\frac{\sin\alpha}{\cos\alpha} = \operatorname{tg}\alpha$

α	$\sin\alpha$	$\cos\alpha$	$\operatorname{tg}\alpha$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	//
π	0	-1	0
$\frac{3\pi}{2}$	-1	0	//

DIMOSTRAZIONE TAVOLA RIASSUNTIVA

$\alpha = \frac{\pi}{4}$

$\overline{OP} = 1$



$\hat{A} = \frac{\pi}{2}$ $\qquad \hat{O} = \frac{\pi}{4}$

$\Longrightarrow \quad \hat{P} = \pi - \hat{A} - \hat{O} = \pi - \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$O\hat{P}A$ triangolo rettangolo isoscele

$\overline{OA} = \overline{PA}$

Dal T. di Pitagora:

$$\underset{\underset{1}{\shortparallel}}{\overline{OP}^2} = \overline{OA}^2 + \overline{PA}^2 = 2\,\overline{OA}^2$$

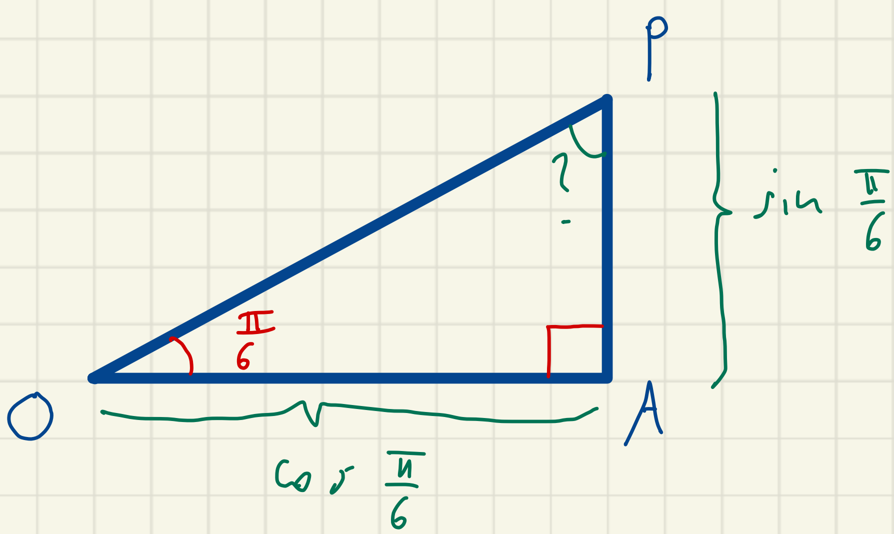
$$\overline{OA}^2 = \frac{1}{2} \Longrightarrow \overline{PA} = \overline{OA} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$$

$$\operatorname{tg}\frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = 1$$

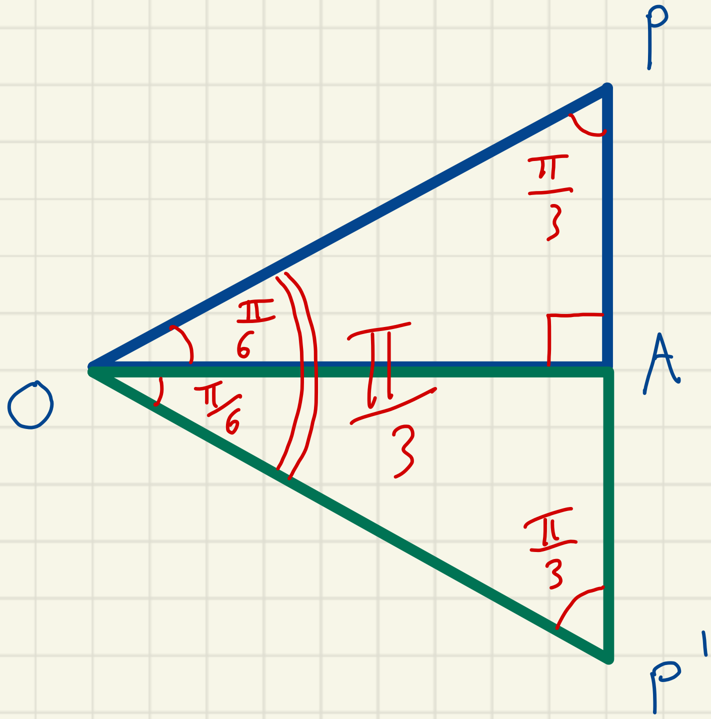
$$\overline{OP} = 1$$

$$\alpha = \frac{\pi}{6}$$



$$\hat{P} = \pi - \hat{O} - \hat{A} = \pi - \frac{\pi}{6} - \frac{\pi}{2} =$$

$$= \frac{\pi}{3}$$



$O\overset{\triangle}{P}P'$ è un triangolo equilatero

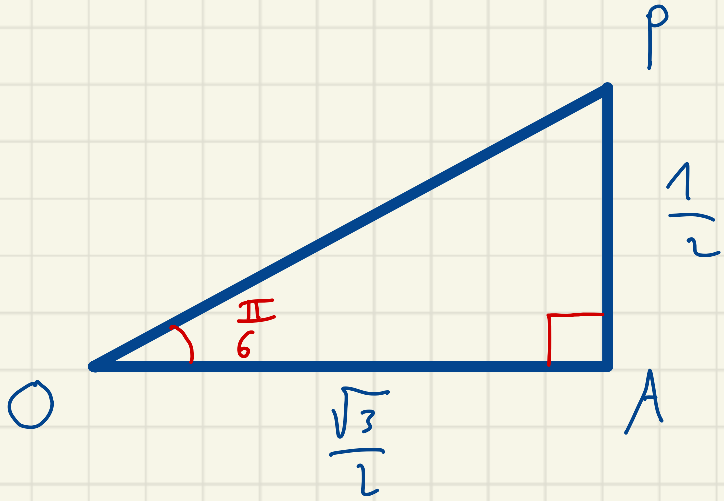
$$\Longrightarrow \quad \overline{PP'} = \overline{OP} = 1$$

$$\overline{PA} = \frac{1}{2}\,\overline{PP'} = \frac{1}{2}$$

Dal T. di Pitagora:

$$\overline{OA} = \sqrt{\overline{OP}^2 - \overline{AP}^2} = \sqrt{1 - \left(\frac{1}{2}\right)^2} =$$

$$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

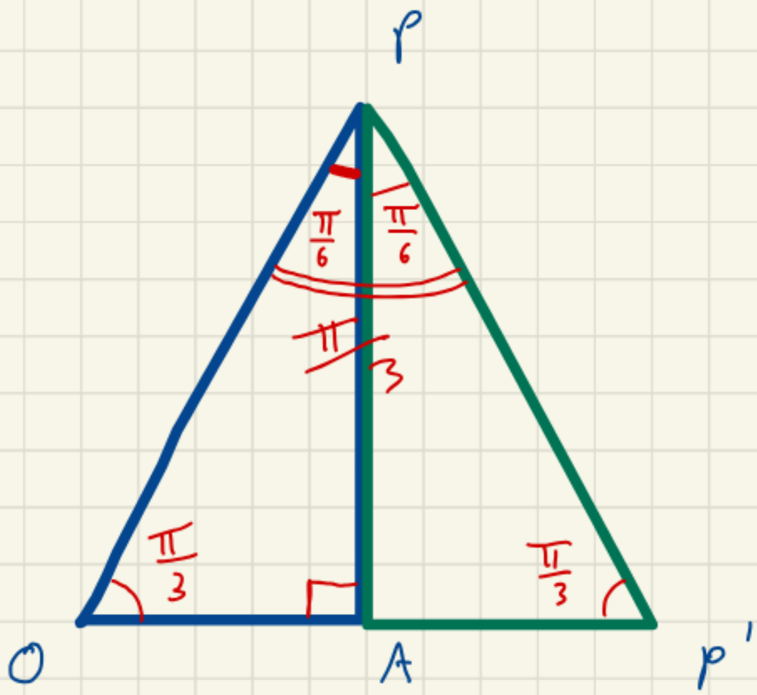


$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sin\frac{\pi}{6} = \frac{1}{2}$$

$$\tan\frac{\pi}{6} = \frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$\alpha = \frac{\pi}{3}$



$\overset{\Delta}{OPP'}$ è un triangolo equilatero

$\overline{OA} = \frac{1}{2} \overline{OP'} = \frac{1}{2}$

$\overline{AP} = \sqrt{\overline{OP}^2 - \overline{OA}^2} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\Longrightarrow \operatorname{tg} \frac{\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

$\cos \frac{\pi}{3} = \frac{1}{2}$

GRAFICO DEL SENO

$y = \sin x$

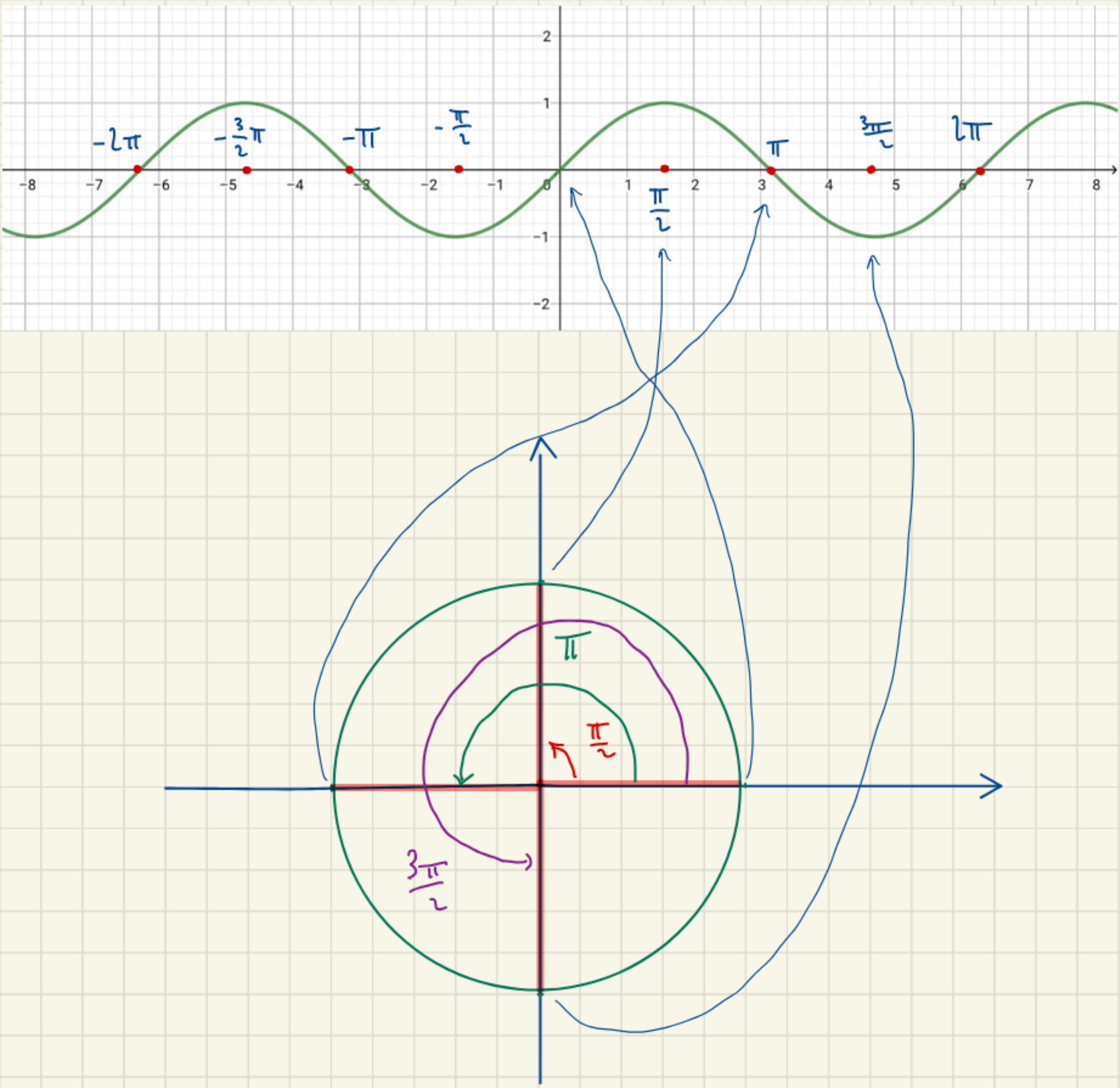


GRAFICO DEL COSENO:

$y = \cos x$

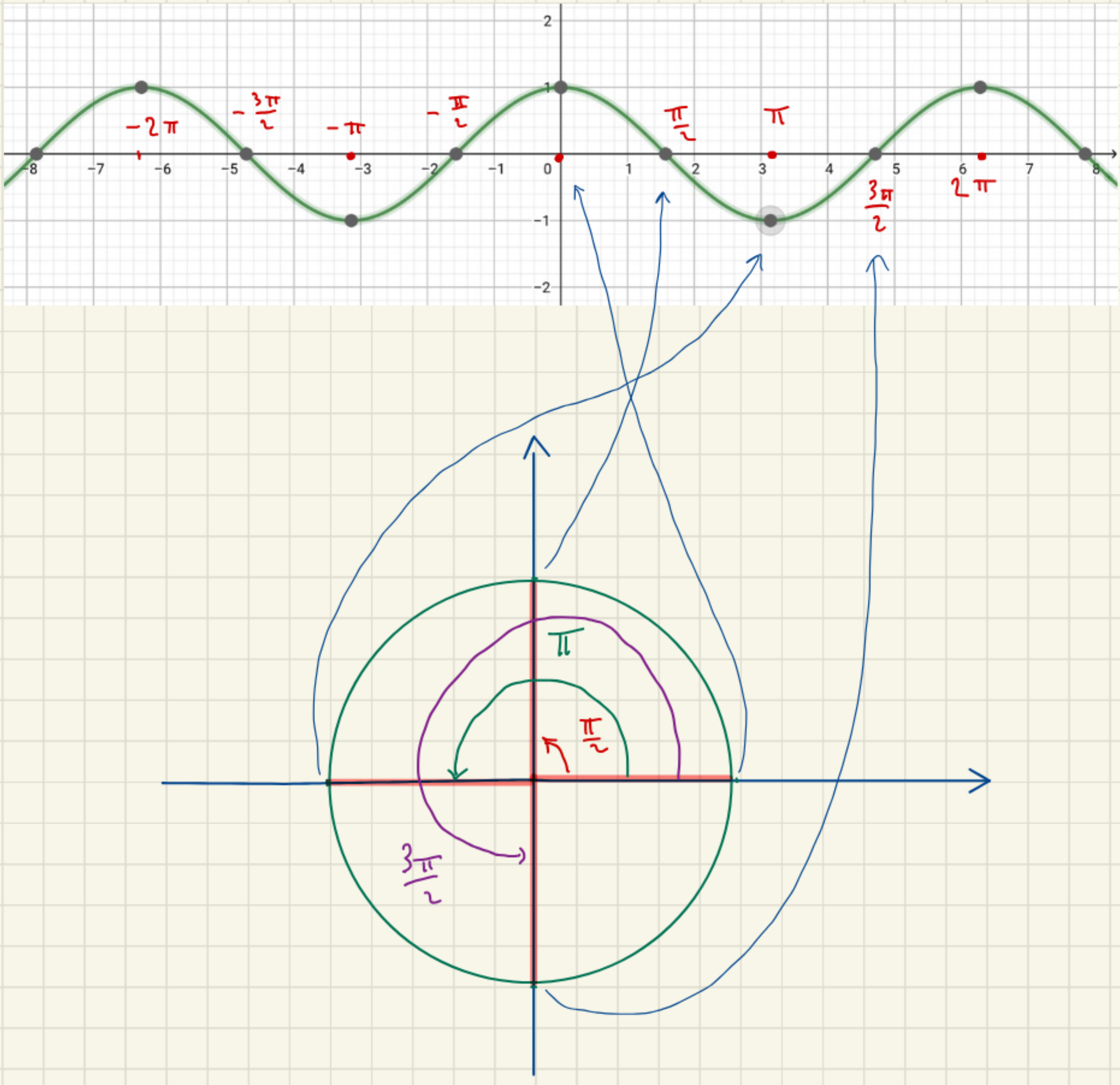
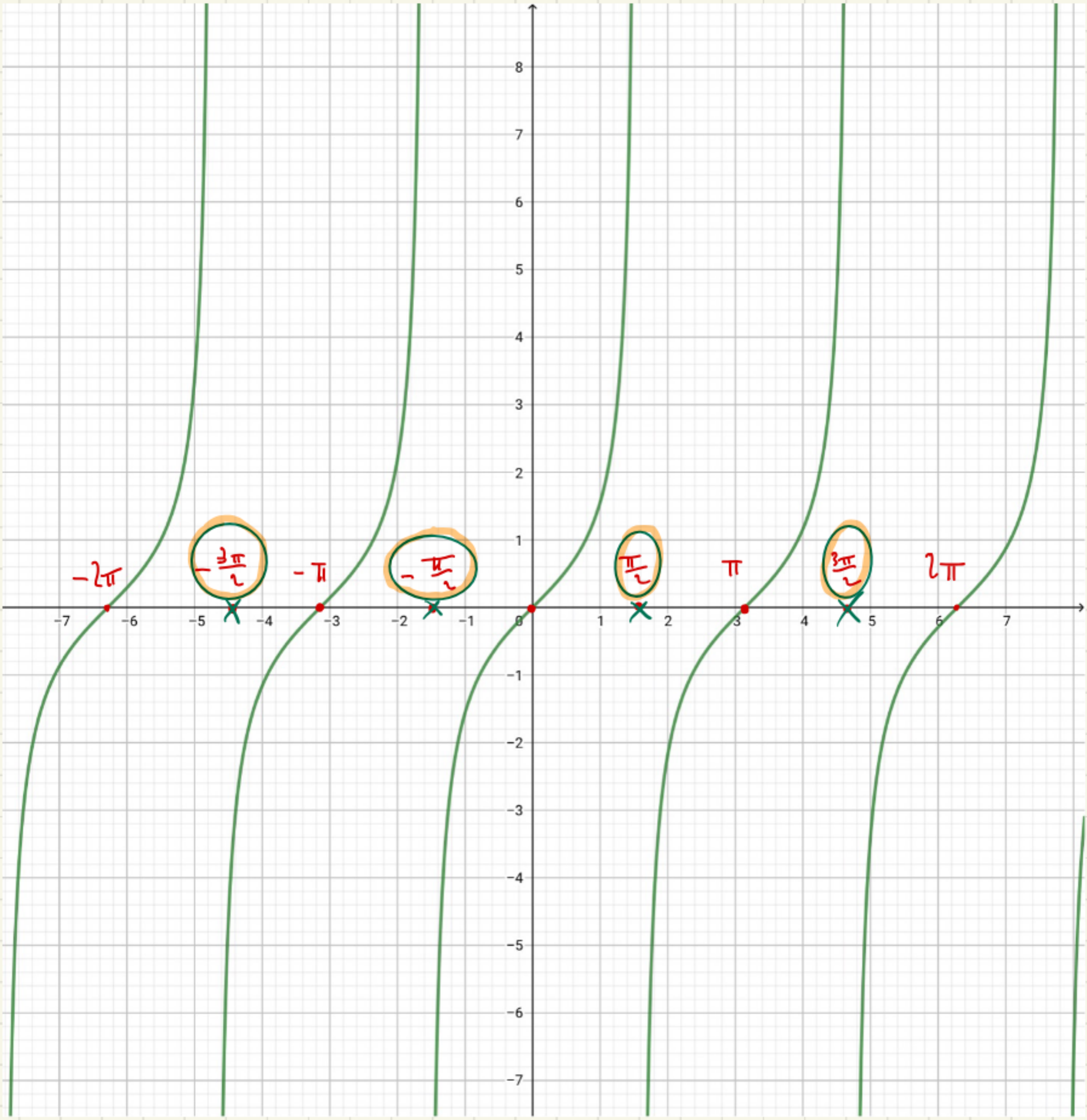


GRAFICO DELLA TANGENTE

$y = \tan x = \dfrac{\sin x}{\cos x}$ $\qquad x \neq \dfrac{\pi}{2} + k\pi$



FORMULE DI ADDIZIONE E SOTTRAZIONE:

$$\boxed{\cos(\alpha-\beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta}$$

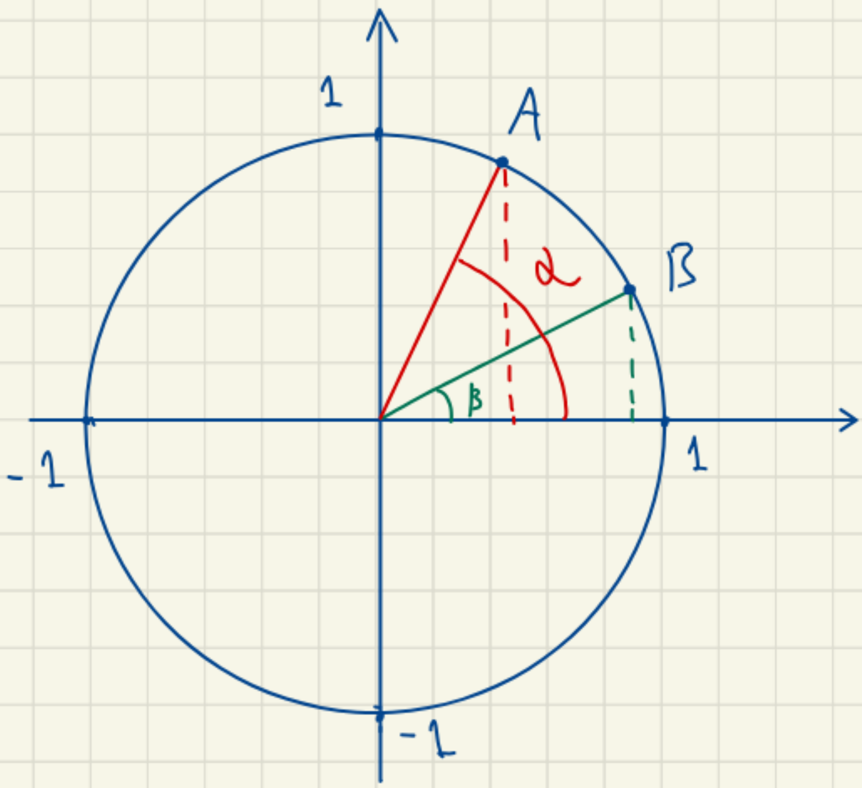
$$\boxed{\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta}$$

$$\boxed{\sin(\alpha-\beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta}$$

$$\boxed{\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta}$$

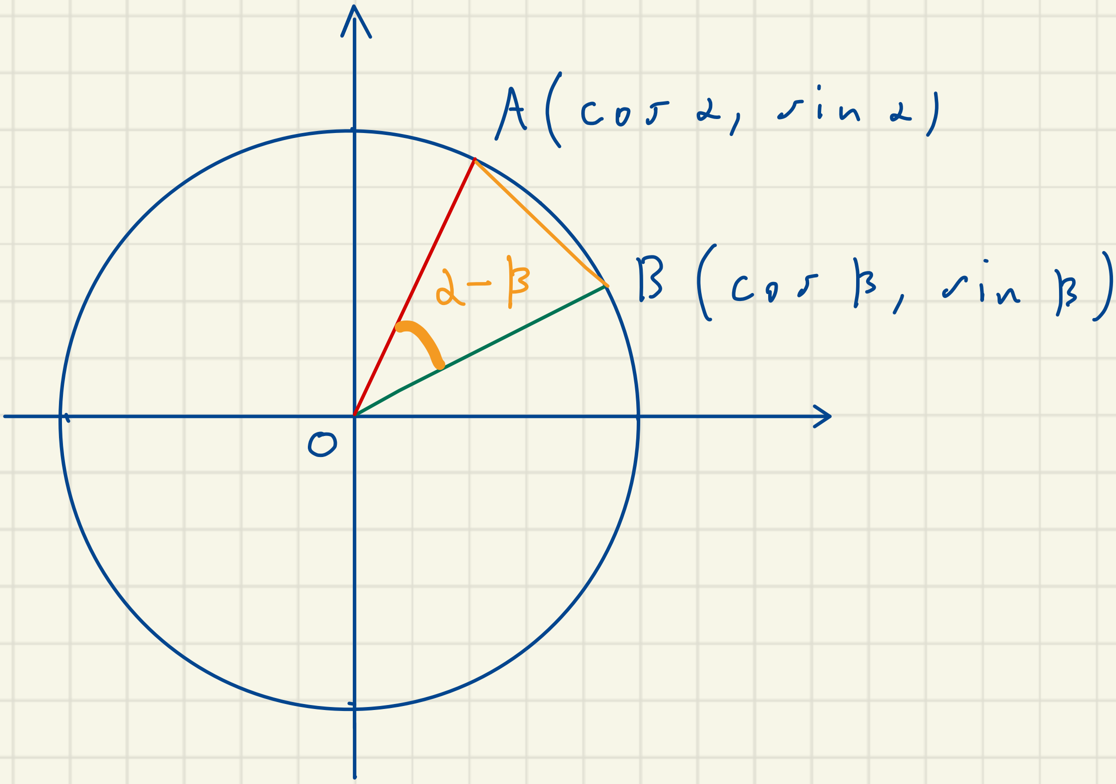
DIMOSTRAZIONE FORMULE DI ADDIZIONE E SOTTRAZIONE:

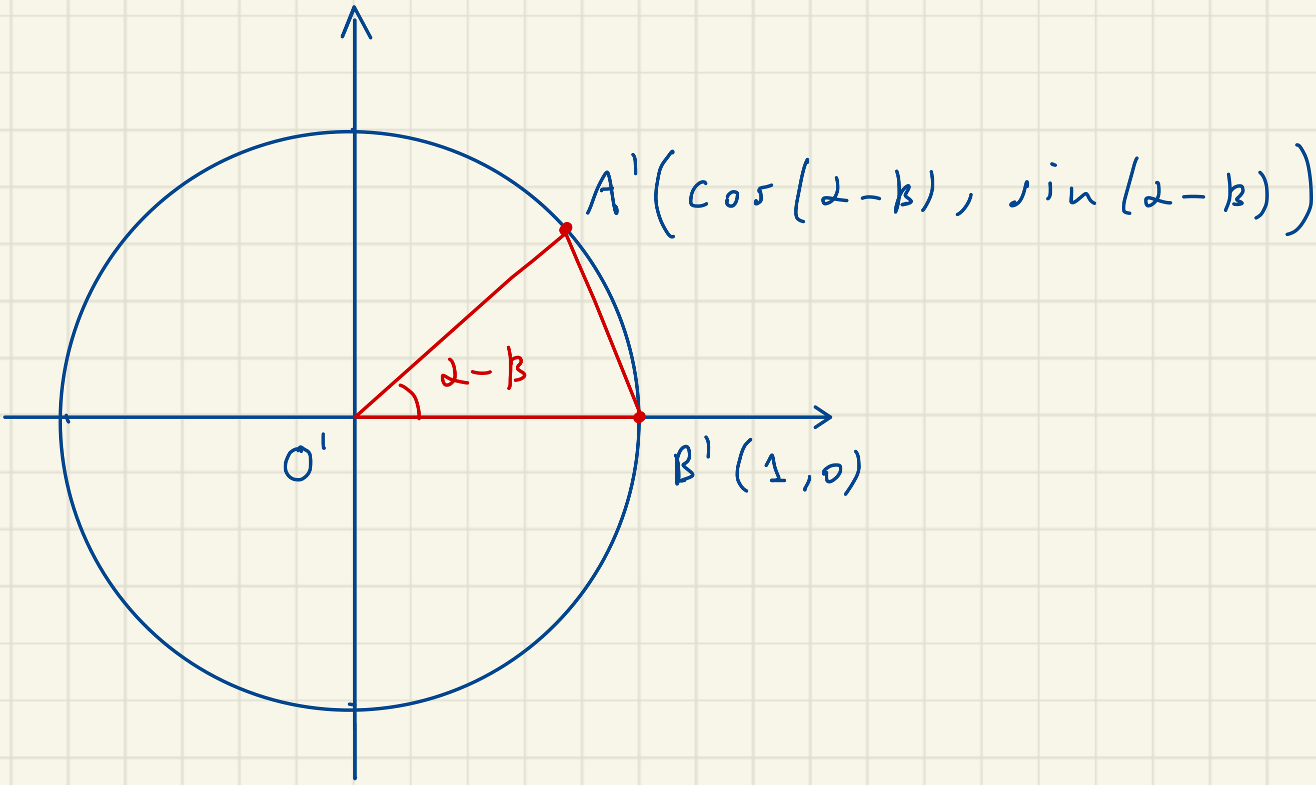
$\cos(\alpha - \beta) = \ ?$



$B(\cos\beta, \sin\beta)$

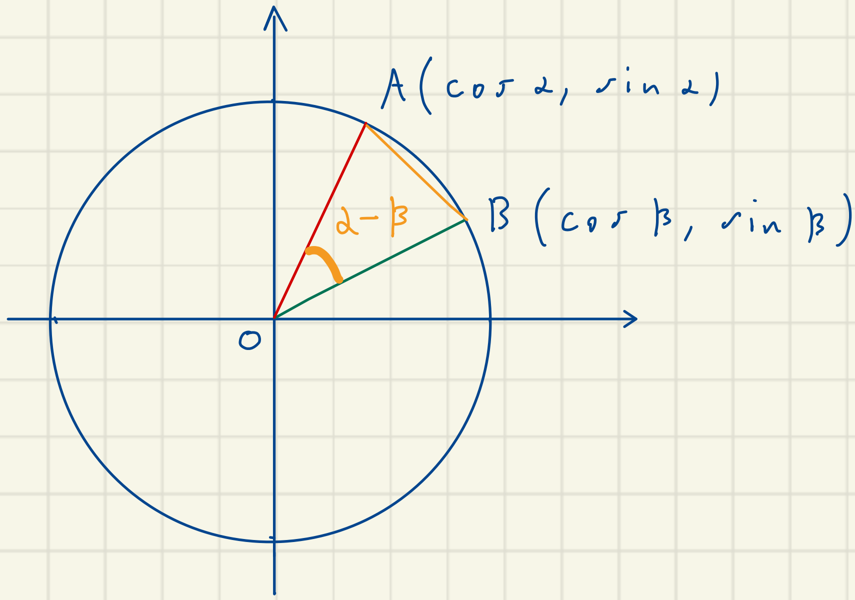
$A(\cos\alpha, \sin\alpha)$





$\overline{OA} = \overline{O'A'}$, $\overline{OB} = \overline{O'B'}$, $A\hat{O}B \simeq A'\hat{O'}B'$

$\Longrightarrow \quad \overset{\triangle}{AOB} \simeq \overset{\triangle}{A'O'B'} \quad \Longrightarrow \quad \boxed{\overline{AB} = \overline{A'B'}}$

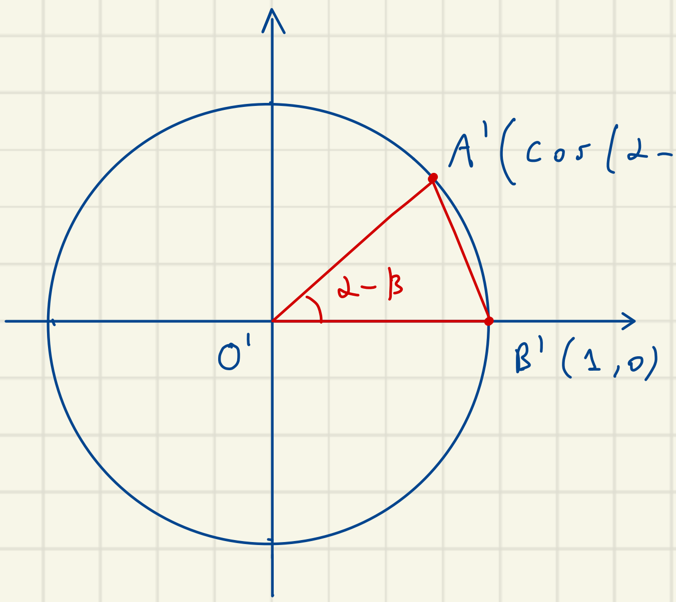


$$\overline{AB}^2 = (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$$

$$= \cos^2\alpha - 2\cos\alpha\cdot\cos\beta + \cos^2\beta + \sin^2\alpha$$

$$- 2\sin\alpha\cdot\sin\beta + \sin^2\beta =$$

$$= 2 - 2\cos\alpha\cdot\cos\beta - 2\sin\alpha\cdot\sin\beta$$



$$\overline{A'B'}^2 = (\cos(\alpha-\beta)-1)^2 + \sin^2(\alpha-\beta) =$$

$$= \cos^2(\alpha-\beta) + 1 - 2\cos(\alpha-\beta) + $$

$$+ \sin^2(\alpha-\beta)$$

$$= 2 - 2\cos(\alpha-\beta)$$

$$\overline{AB}^2 = \overline{A'B'}^2$$

$$\cancel{2} - 2\cos\alpha \cdot \cos\beta - 2\sin\alpha \cdot \sin\beta =$$

$$= \cancel{2} - 2\cos(\alpha - \beta)$$

$$\boxed{\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta}$$

Usando il fatto che coseno è una funzione pari:

$$\cos(\alpha+\beta) = \cos(\alpha - (-\beta)) =$$

$$= \cos\alpha \cdot \underbrace{\cos(-\beta)}_{= \cos\beta} + \sin\alpha \cdot \underbrace{\sin(-\beta)}_{= -\sin\beta}$$

$$= \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

Quindi:

$$\boxed{\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta}$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\alpha = \frac{\pi}{2}$$

$$\Downarrow$$

$$\cos\left(\frac{\pi}{2} - \beta\right) = \overbrace{\cos\frac{\pi}{2}}^{0} \cdot \cos\beta + \overbrace{\sin\frac{\pi}{2}}^{1} \cdot \sin\beta$$

$$\Longrightarrow \quad \boxed{\cos\left(\frac{\pi}{2} - \beta\right) = \sin\beta} \qquad (A)$$

Se poniamo: $\gamma := \frac{\pi}{2} - \beta \rightarrow \beta = \frac{\pi}{2} - \gamma$

$$\cos\gamma = \sin\left(\frac{\pi}{2} - \gamma\right)$$

$$\Longrightarrow \quad \boxed{\sin\left(\frac{\pi}{2} - \gamma\right) = \cos\gamma} \qquad (B)$$

Esercizio:

Provare in modo analogo che:

$$\boxed{\sin(\alpha-\beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta}$$

Suggerimento:

Usare la relazione (A) di prima:

$$\sin(\alpha-\beta) = \cos\left(\frac{\pi}{2} - (\alpha-\beta)\right) =$$

$$= \cos\left(\left(\frac{\pi}{2} - \alpha\right) + \beta\right)$$

e analogamente dimostrare che:

$$\boxed{\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \sin\beta}$$

$$\sin(\alpha-\beta) = ?$$

$$\sin(\alpha-\beta) = \cos\left(\frac{\pi}{2} - (\alpha-\beta)\right) =$$

$$= \cos\left(\left(\frac{\pi}{2} - \alpha\right) + \beta\right) =$$

$$= \cos\left(\frac{\pi}{2} - \alpha\right) \cdot \cos\beta - \sin\left(\frac{\pi}{2} - \alpha\right) \cdot \sin\beta$$

$$= \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

FORMULE DI DUPLICAZIONE:

Si provano usando le formule di addizione:

$$\cos(2\alpha) = \cos(\alpha+\alpha) =$$
$$= \cos^2\alpha - \sin^2\alpha$$

$$= (1-\sin^2\alpha) - \sin^2\alpha =$$
$$= 1 - 2\sin^2\alpha$$

$$= \cos^2\alpha - (1-\cos^2\alpha) =$$
$$= 2\cos^2\alpha - 1$$

$$\sin(2\alpha) = \sin(\alpha+\alpha) =$$
$$= \sin\alpha\cdot\cos\alpha + \cos\alpha\cdot\sin\alpha =$$
$$= 2\sin\alpha\cdot\cos\alpha$$

FUNZIONI GONIOMETRICHE "INVERSE":

La funzione seno:

$$\sin : \mathbb{R} \longrightarrow \mathbb{R}$$

NON è invertibile, poiché non è né SU né 1-1.
Tuttavia se restringiamo il suo codominio:

$$\sin : \mathbb{R} \longrightarrow [-1, 1]$$

è SURIETTIVA
(ma non INIETTIVA)

Se però sostituiamo alla funzione seno la sua restrizione all'interv. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$:

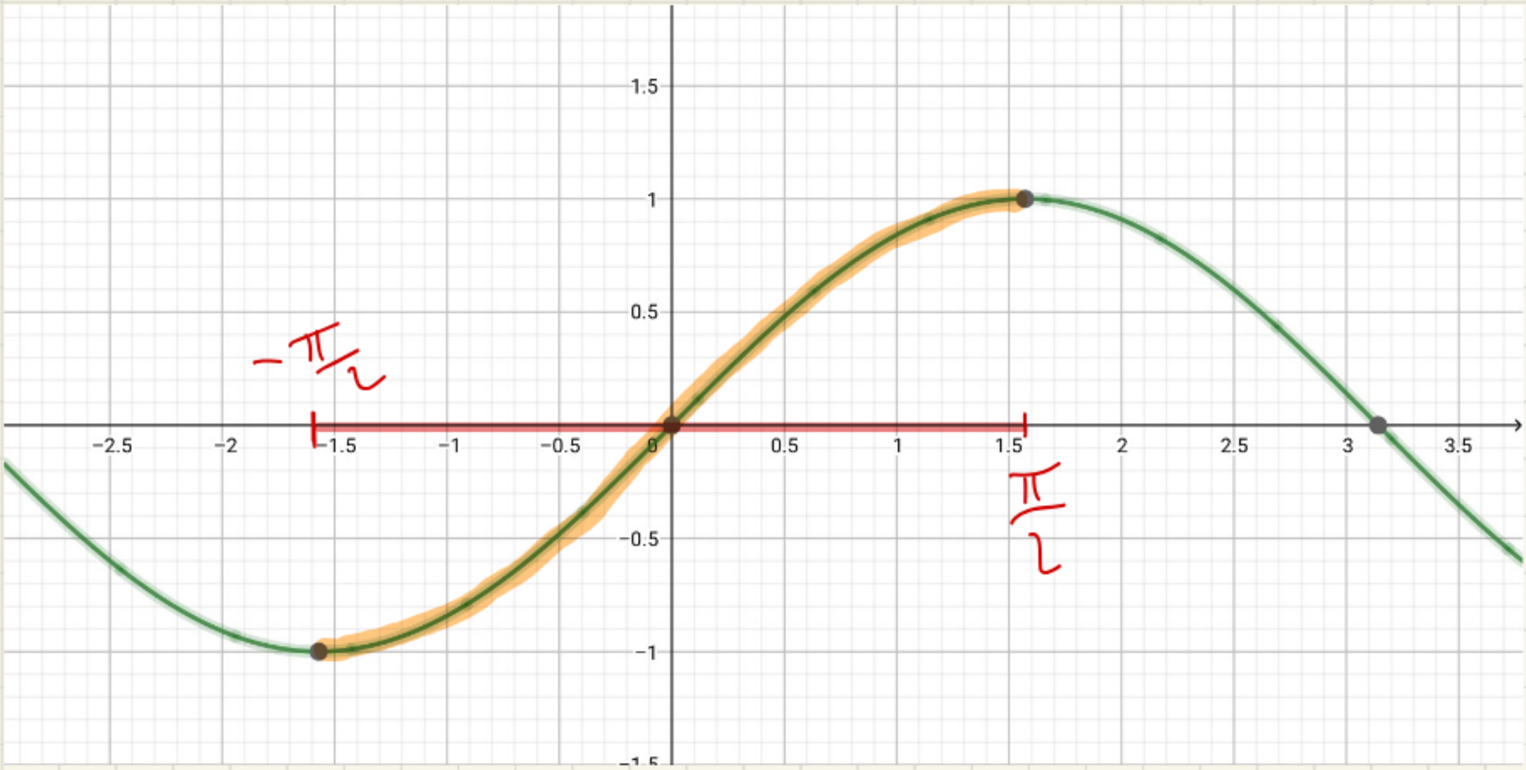
$$\sin\Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$$

$\Longrightarrow$ $\sin\Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$ è SU e 1-1

$\Longrightarrow$ è invertibile

$$\sin\Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$$

funzione inversa



1.5
1
0.5
−0.5
−1
−2.5
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
2.5
3
3.5
$-\pi/2$
$\pi/2$

$$\left(\sin\big|_{[-\frac{\pi}{2},\frac{\pi}{2}]}\right)^{-1}(y) =: \arcsin y$$

arcoseno di y

$$\arcsin : [-1,1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$y \longmapsto \arcsin y$$

ATTENZIONE:

Il fatto che arcsin sia l'inversa di una restrizione del seno ha delle conseguente:

$\forall y \in [-1, 1]:$

$$\sin(\arcsin y) = y$$

$\forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (e non in $\mathbb{R}$!)

$$\arcsin(\sin x) = x$$

(<u>Esempio</u>:

$x = \pi$ (nota che $\pi \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$)

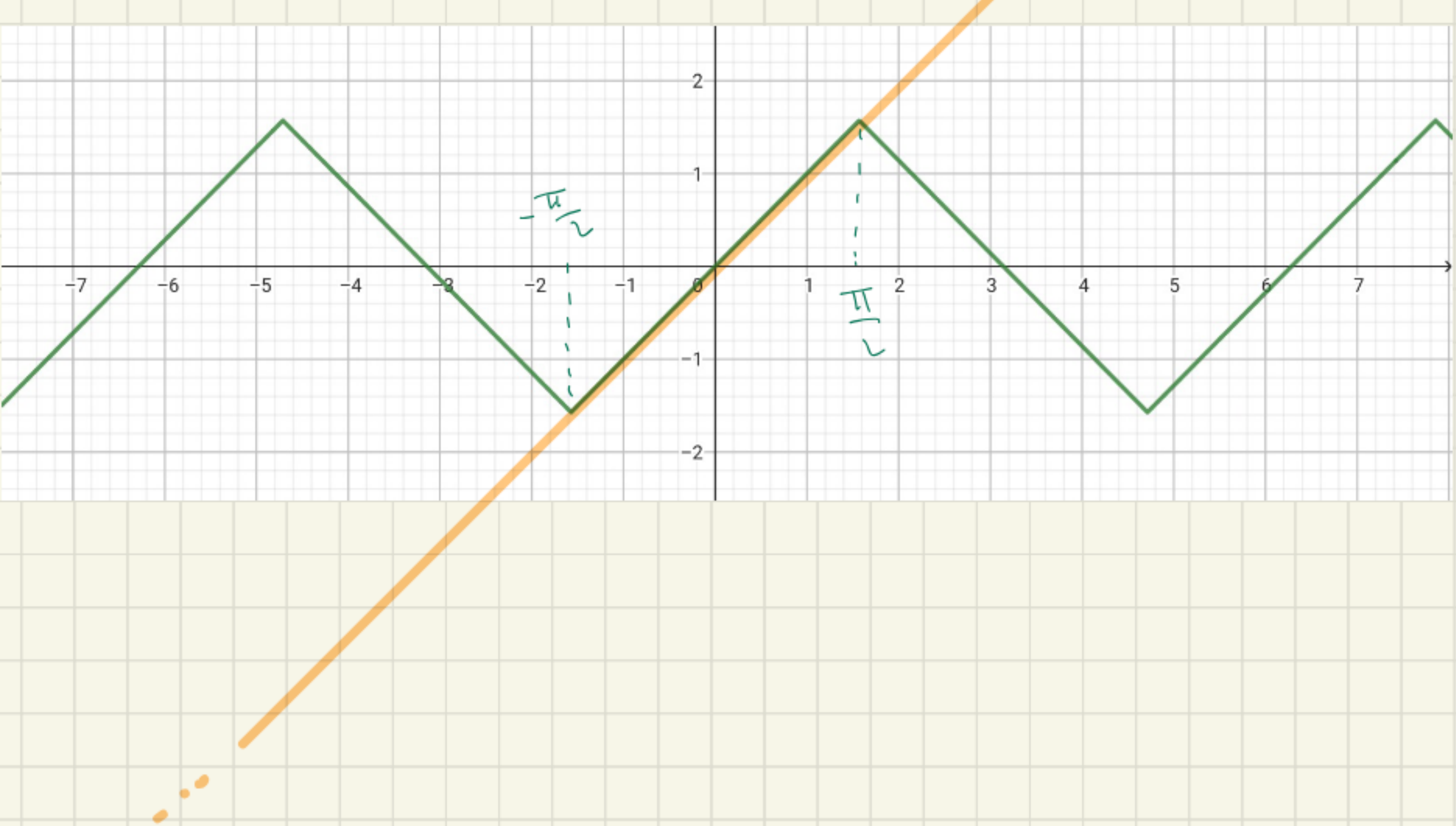
$$\arcsin(\sin \pi) =$$

$$= \arcsin(0) = 0 \neq \pi$$

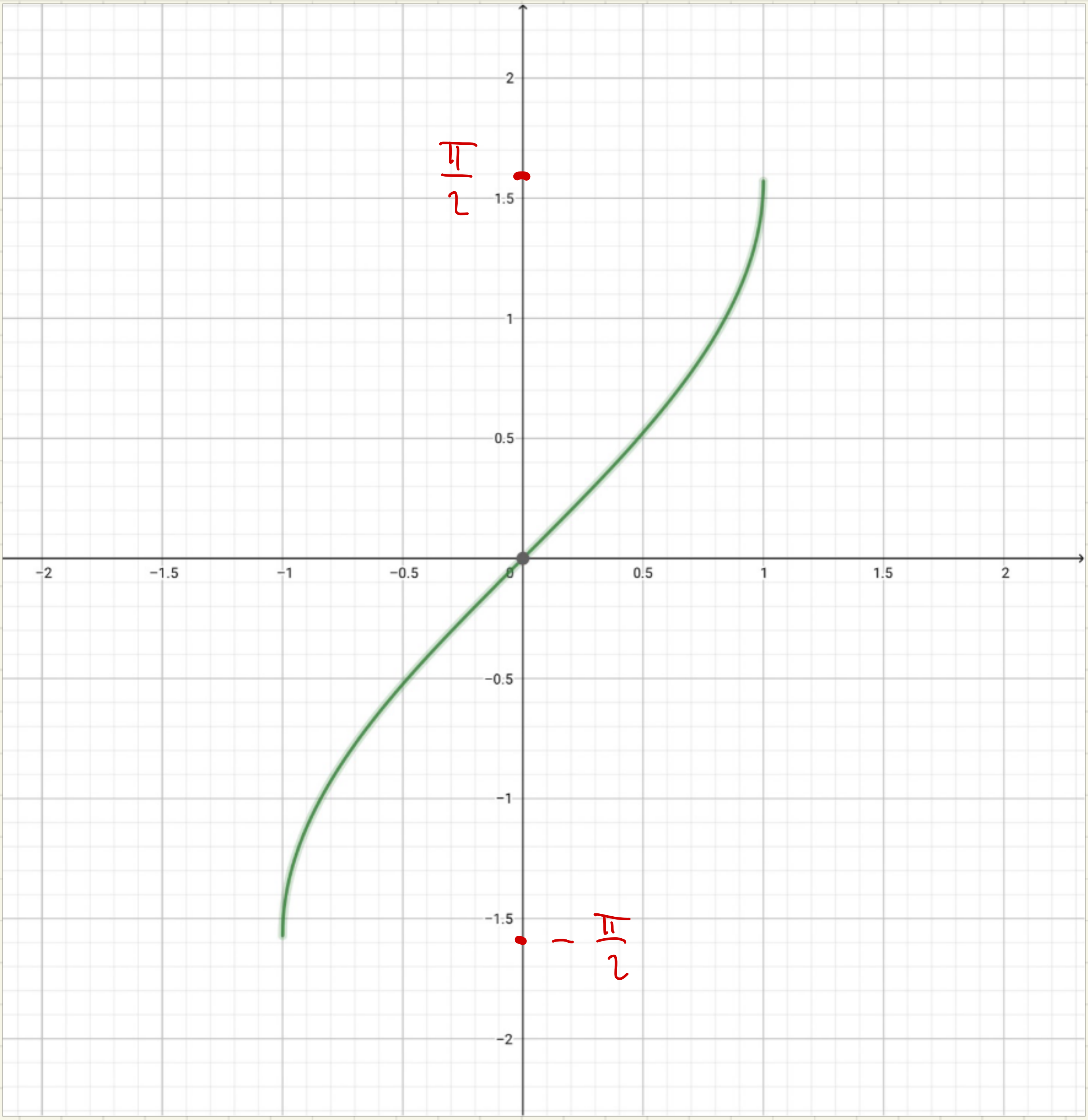
)

$y = \arcsin(\sin x)$

$y = x$



$y = \arcsin x$



Anche la funzione coseno non è invertibile; consideriamo la sua restrizione a $[0, \pi]$:

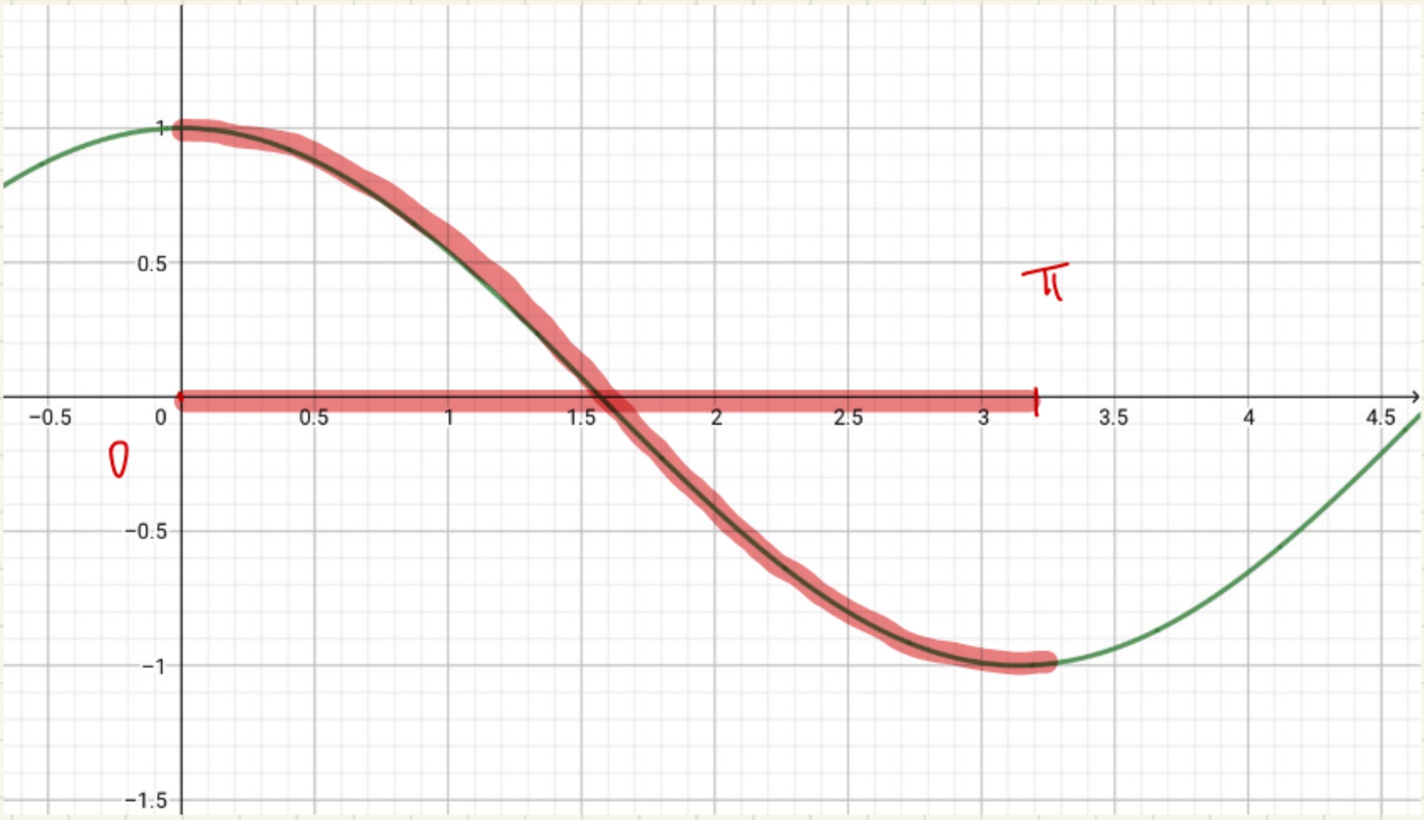
$$\cos\big|_{[0,\pi]} : [0, \pi] \longrightarrow [-1, 1]$$

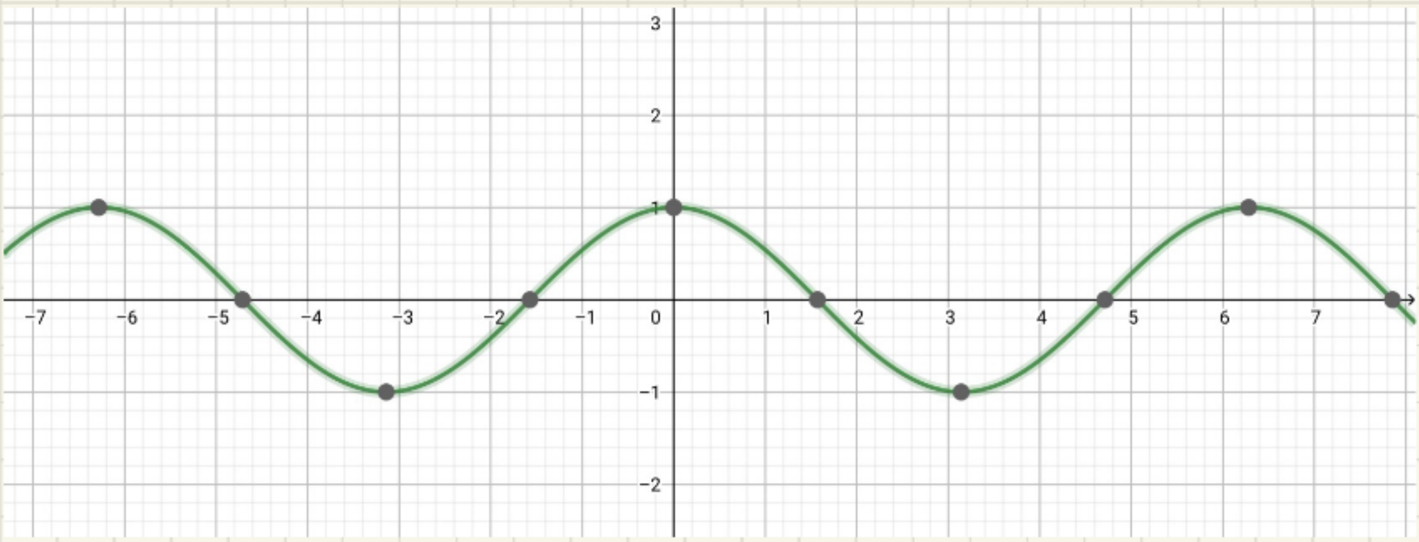
$\Longrightarrow$ $\cos\big|_{[0,\pi]}$ è su e 1-1

$\Longrightarrow$ è invertibile

$$\cos\big|_{[0,\pi]} : [0, \pi] \longrightarrow [-1, 1]$$

funzione inversa


π
0
−0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
1
0.5
−0.5
−1
−1.5


−7
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
7
3
2
1
−1
−2

$$\left(\cos|_{[0,\pi]}\right)^{-1}(y) =: \arccos y$$

arcocoseno di y

$$\arccos : [-1,1] \longrightarrow [0,\pi]$$

$$y \longmapsto \arccos y$$

<u>ATTENZIONE</u>:

Il fatto che arccos sia l'inversa di una restrizione del coseno ha delle conseguenze come prima.

$\forall y \in [-1, 1]$:

$$\cos(\arccos y) = y$$

$\forall x \in [0, \pi]$ (e non in $\mathbb{R}$!)

$$\arccos(\cos x) = x$$

(<u>Esempio</u>:

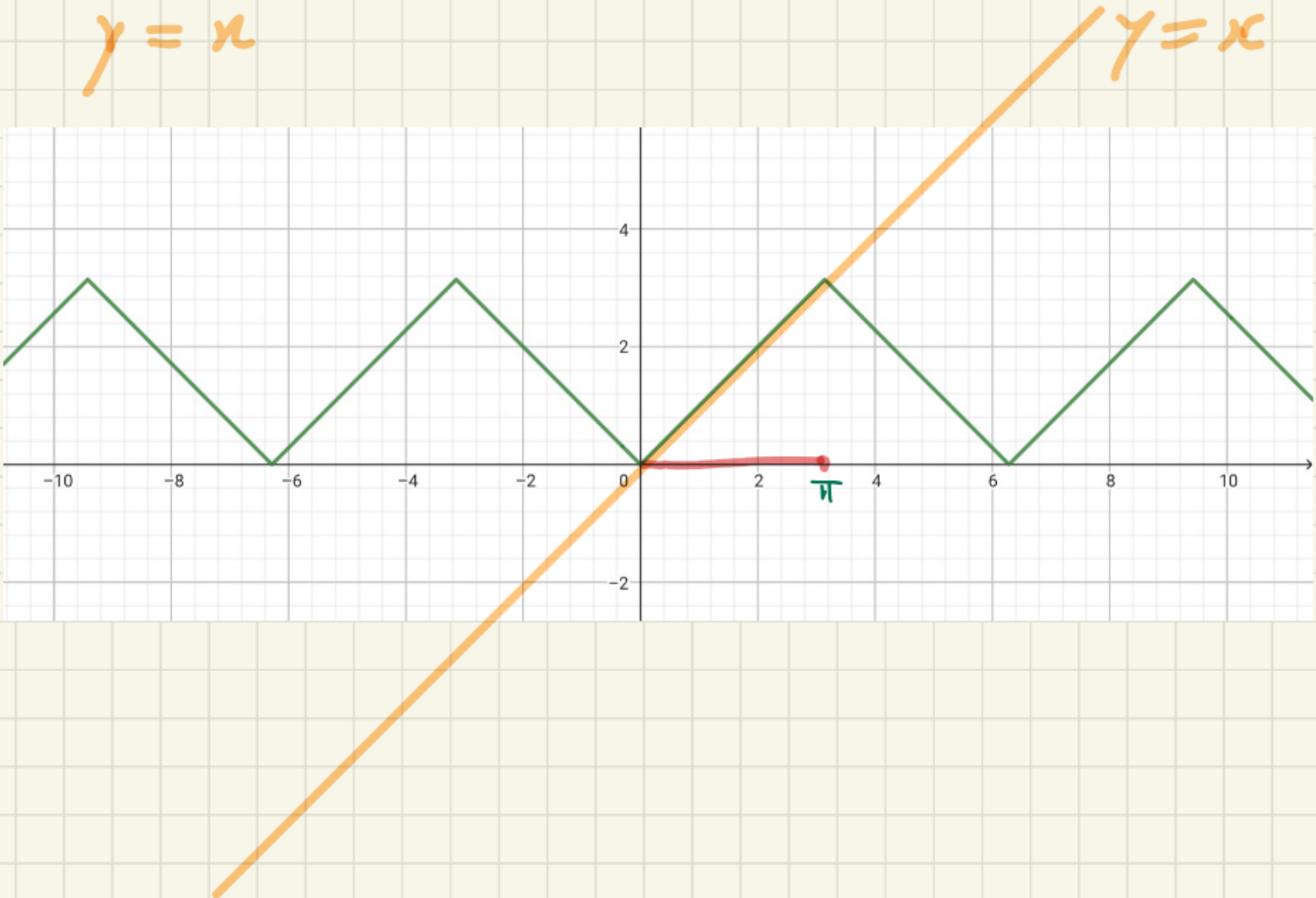
$x = \frac{3\pi}{2}$ (nota che $\frac{3\pi}{2} \notin [0, \pi]$)

$$\arccos\left(\cos \frac{3\pi}{2}\right) =$$

$$= \arccos(0) = \frac{\pi}{2} \neq \frac{3\pi}{2}$$)

$y = \arccos(\cos x)$

$y = x$



$y = \arccos x$

3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
−1.6
−1.4
−1.2
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
π
π/2

Anche la funzione tangente <u>non</u> è invertibile (non è biunivoca) ma lo è la sua restrizione:

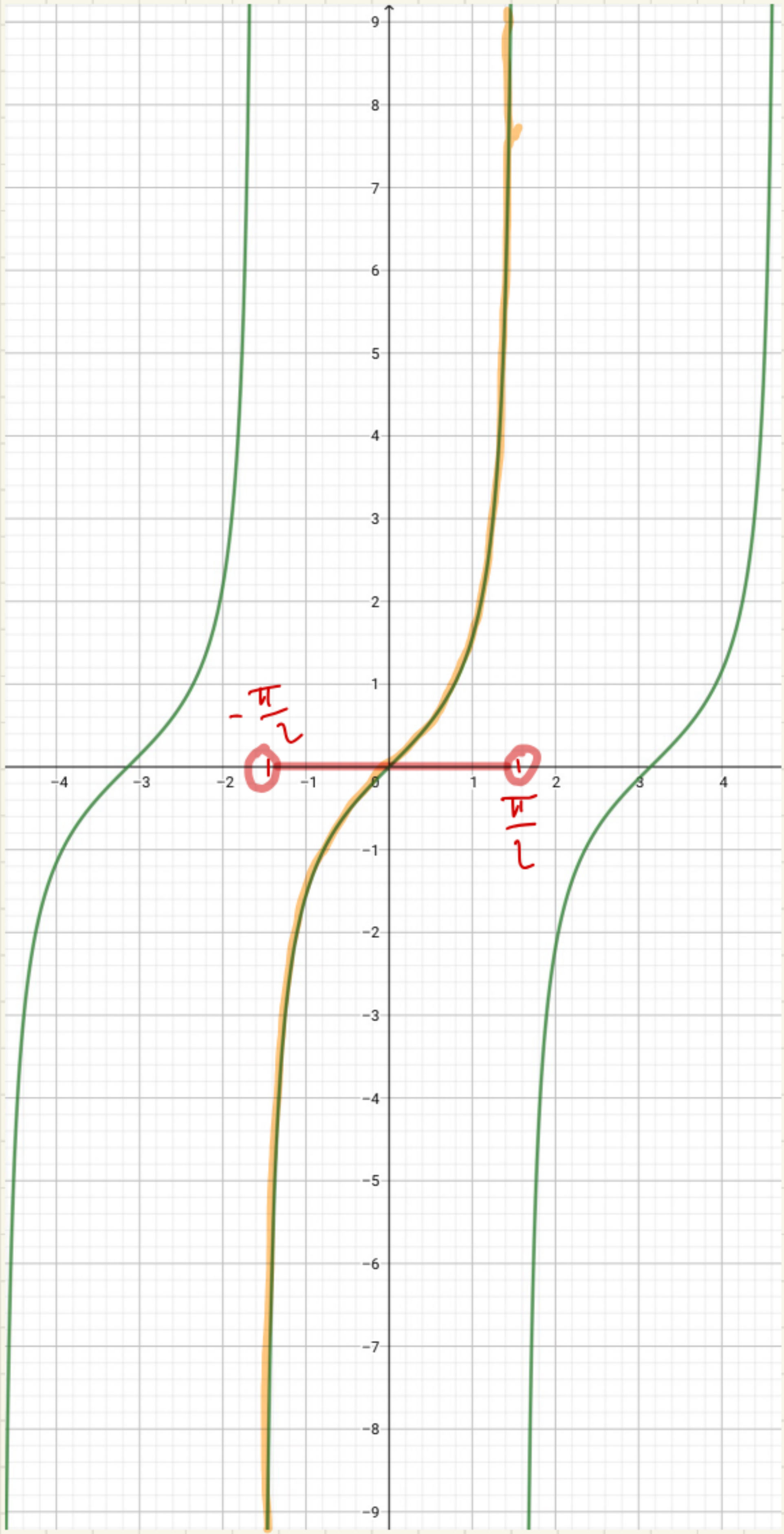
$$\tan\Big|_{]-\frac{\pi}{2},\frac{\pi}{2}[} : \left]-\frac{\pi}{2},\frac{\pi}{2}\right[\longrightarrow \mathbb{R}$$

$\Longrightarrow$ $\tan\Big|_{]-\frac{\pi}{2},\frac{\pi}{2}[}$ è su e 1-1

$\Longrightarrow$ è invertibile

$$\tan\Big|_{]-\frac{\pi}{2},\frac{\pi}{2}[} : \left]-\frac{\pi}{2},\frac{\pi}{2}\right[\longrightarrow \mathbb{R}$$

funzione inversa



$-\frac{\pi}{2}$
$\frac{\pi}{2}$

(arctg y)

$$\left(\tan\big|_{]-\frac{\pi}{2},\frac{\pi}{2}[}\right)^{-1}(y) =: \arctan y$$

arcotangente di y

$$\arctan : \mathbb{R} \longrightarrow \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$$

$$y \longmapsto \arctan y$$

ATTENZIONE:

Il fatto che arctg sia l'inversa di una restrizione della tangente ha delle conseguenze!

$\forall y \in \mathbb{R}$:

$$\tan(\arctan y) = y$$

$\forall x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ (e non in $\mathbb{R}$!)

$$\arctan(\tan x) = x$$

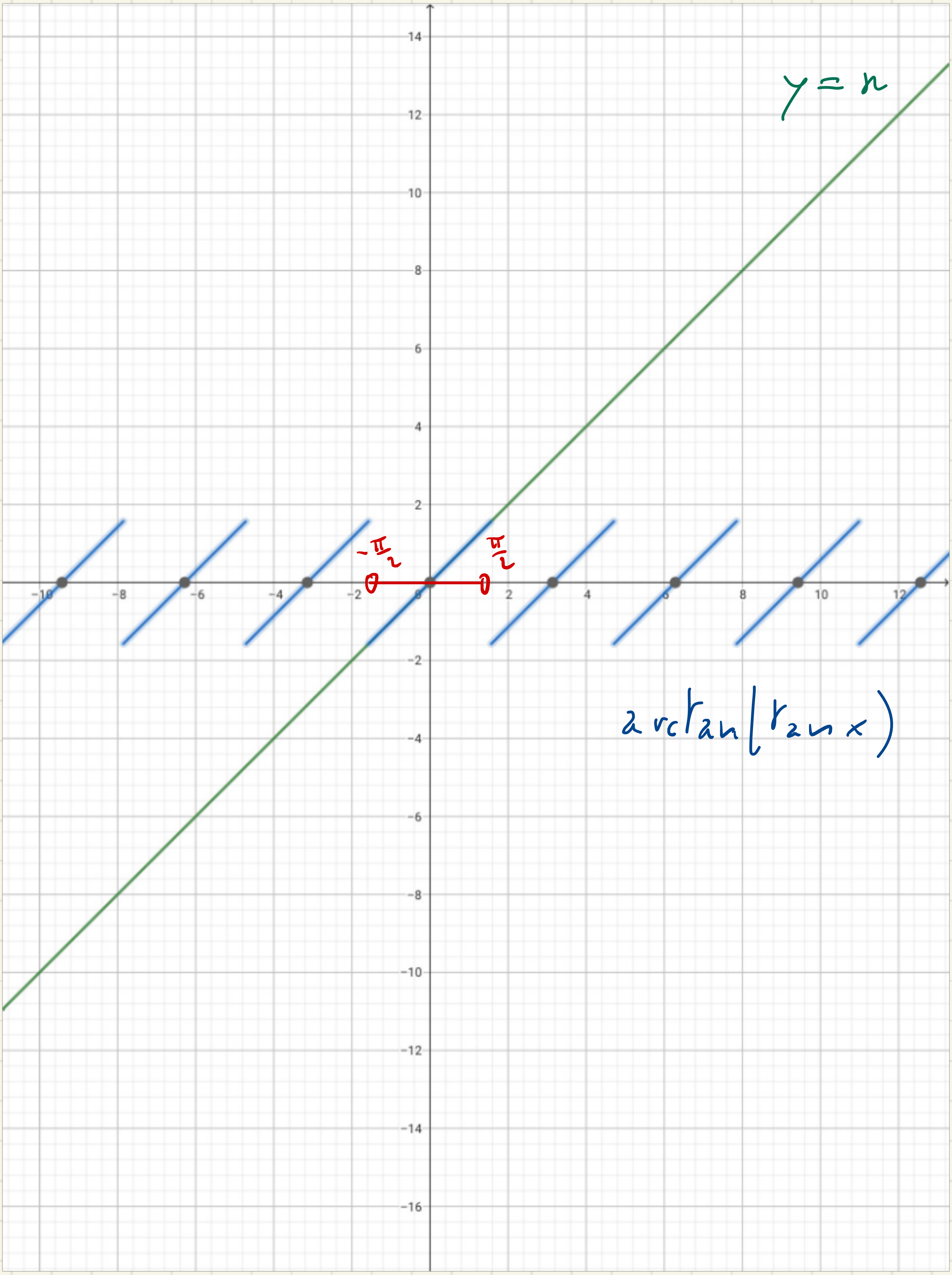

y = x
arctan(tan x)
-π/2
π/2
14
12
10
8
6
4
2
-2
-4
-6
-8
-10
-12
-14
-16
-10
-8
-6
-4
-2
0
2
4
6
8
10
12

Grafico di $y = \operatorname{arctg} x$

$(= \arctan x)$