Corso di Laurea in Informatica

FISICA GENERALE
Lezione 1 - Vettori

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Spazio e Tempo

- In fisica classica indichiamo con il termine evento, un fenomeno fisico (es. l'accensione di un led, un sasso che viene lanciato) che accade in uno dato punto dello spazio ad un dato istante di tempo.

Spazio e Tempo Newtoniani

- Prima di Newton lo spazio aveva un significato "relativo" alle cose: Io sono vicino alla fontana, Il libro è sul tavolo. Anche il tempo aveva un significato relativo alla successione di qualche accadimento (es. giorno, notte, giorno, notte).

- Newton introduce uno spazio "assoluto" che esiste indipendentemente dalle cose e un tempo assoluto che esiste indipendentemente dagli accadimenti.

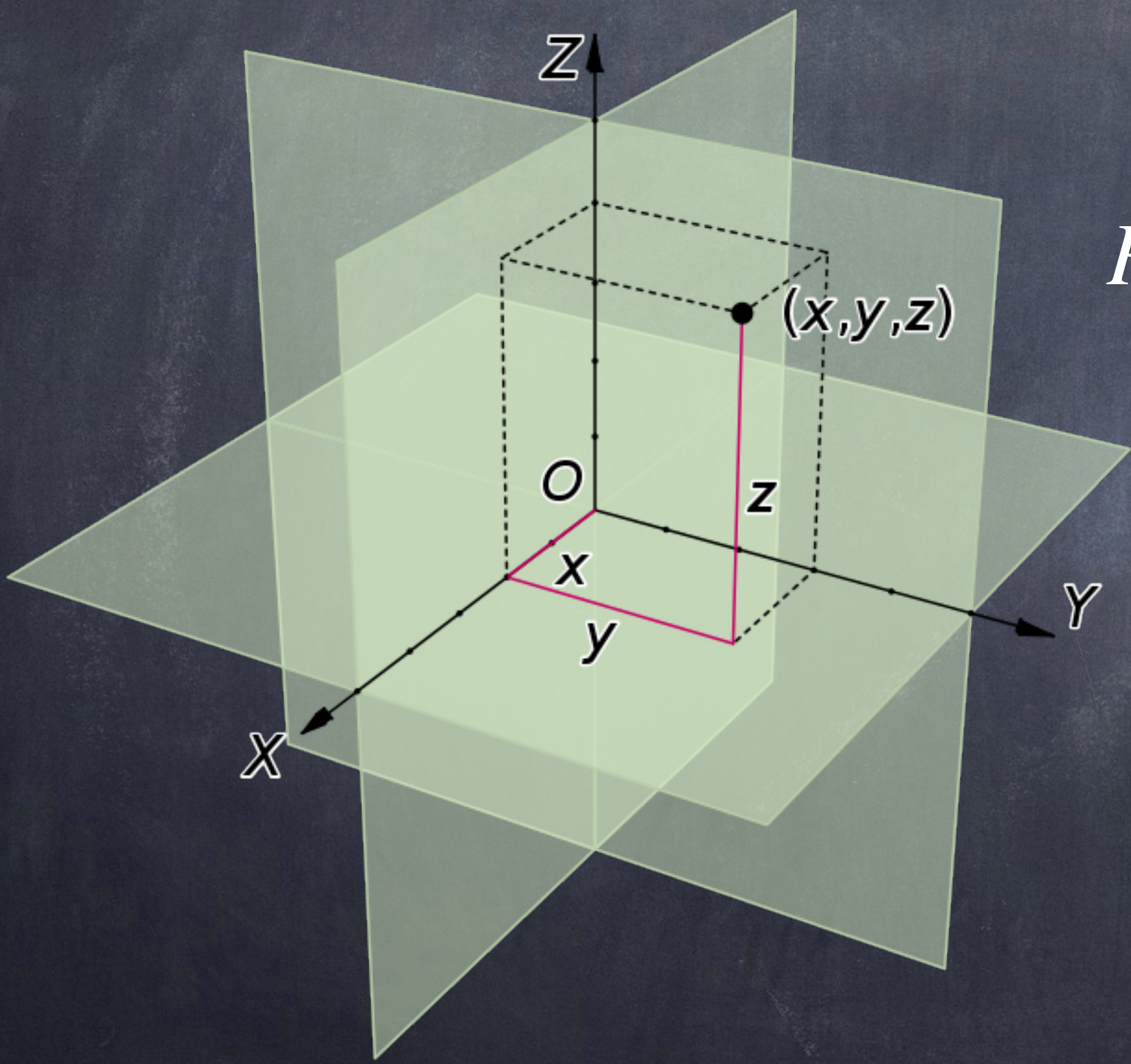
- Spazio e Tempo Newtoniani sono entità intrinseche indipendenti da tutto il resto.

Spazio e Tempo Newtoniani

- Newton ipotizzo che questo spazio "assoluto" avesse la struttura di uno spazio euclideo tridimensionale su cui era possibile definire un sistema di coordinate cartesiane: (x, y, z)

- Il tempo "assoluto" di Newton ha invece la struttura della retta reale parametrizzata da una variabile: t

Sistema di Coordinate

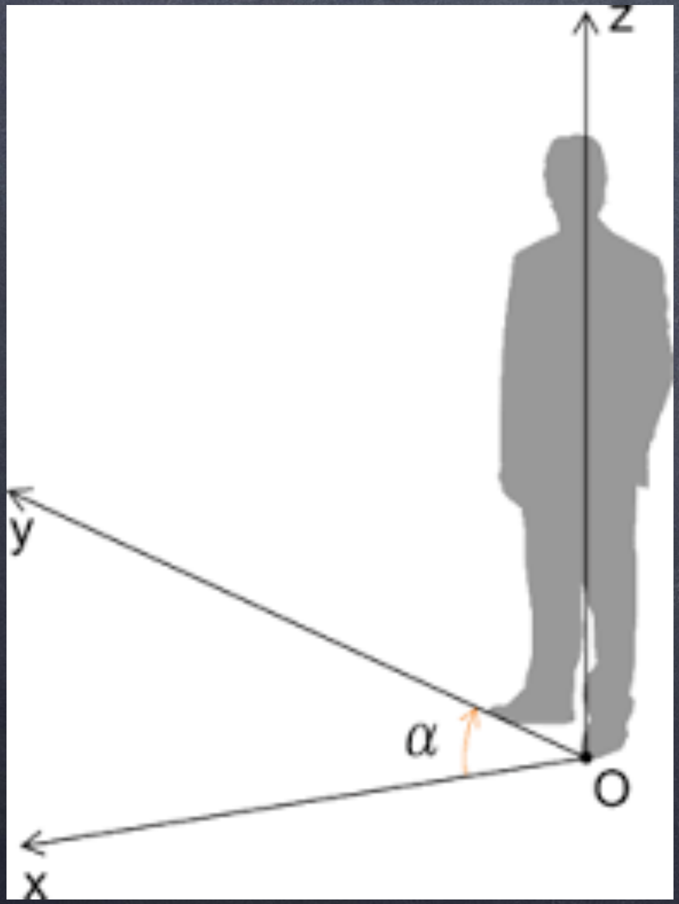


$$Punto\ dello\ Spazio --> P(x, y, z)$$

$$Evento --> E(x, y, z, t)$$

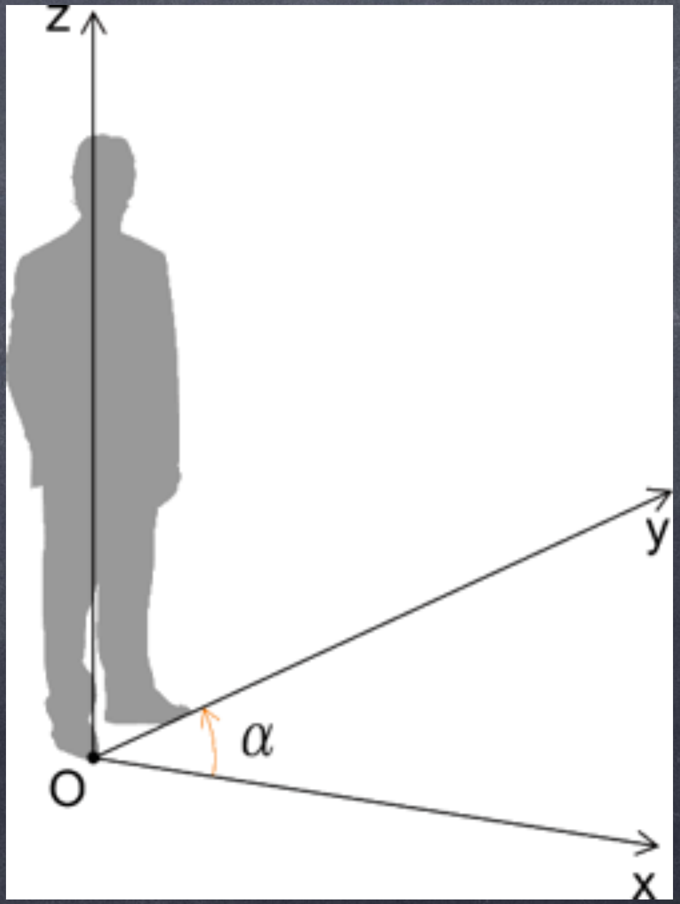
Sistema Destrorso o Sinistrorso

Sistema Destrorso



Senso Orario

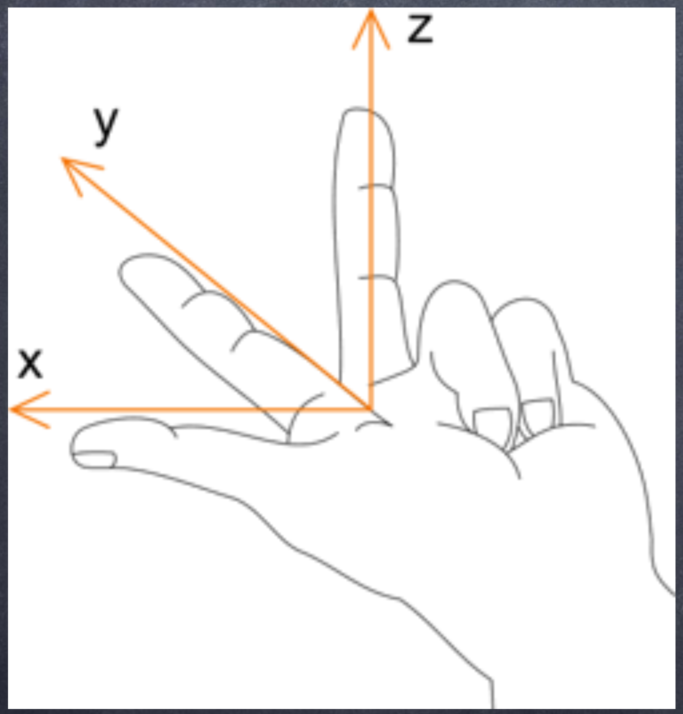
Sistema Sinistrorso



Senso Antiorario

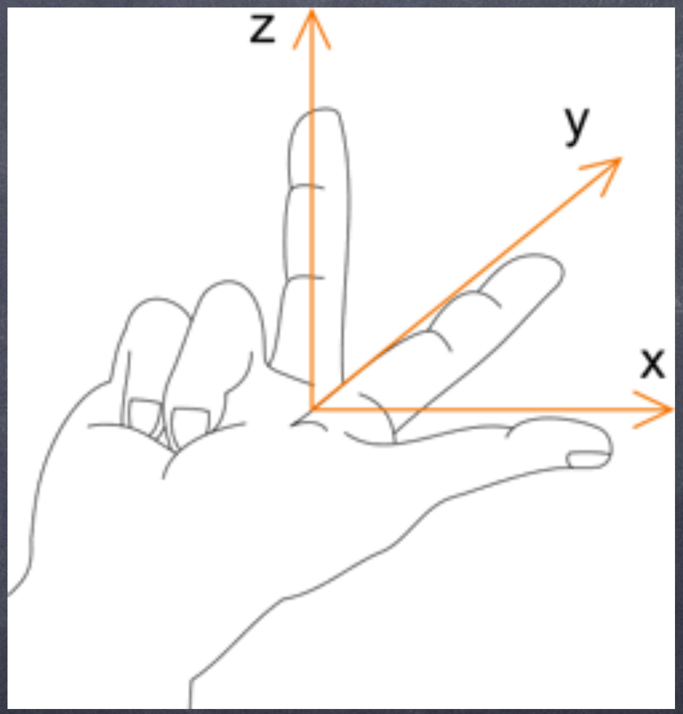
Sistema Destrorso o Sinistrorso

Sistema Destrorso



Mano Sinistra

Sistema Sinistrorso



Mano Destra

Grandezze Scalari e Vettoriali

- Scalare

Grandezze Scalari e Vettoriali

- Scalare
- Vettore

Definizione Vettore

- Segment Orientato $v = OP$

Definizione Vettore

- Segment Orientato $v = OP$
  - Direzione OP

Definizione Vettore

- Segment Orientato $v = OP$
    - Direzione OP
    - Verso da O a P

Definizione Vettore

- Segment Orientato $v = OP$
  - Direzione OP
  - Verso da O a P
  - Modulo v
    - Lunghezza, Norma

Algebra dei Vettori: Vettore Nullo

- Vettore Nullo $\mathbf{0}$
  - Modulo Nullo
  - Direzione e Verso Indeterminati

Algebra dei Vettori: Vettore Unitario

- Vettore Unitario o Versore
  - Modulo Unitario

Algebra dei Vettori: Vettore Unitario

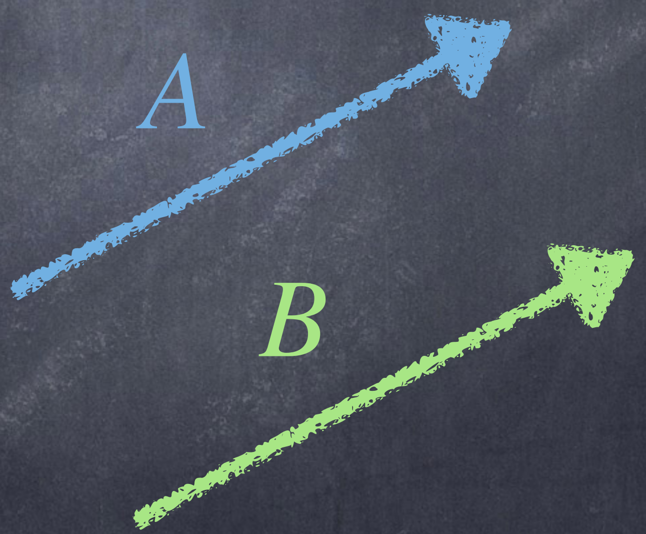
- Vettore Unitario o Versore
  - Modulo Unitario

- Versore $\hat{a}$ con Direzione e Verso di A

$$\hat{a} = \frac{A}{|A|}$$

Algebra dei Vettori: Vettori Equivalenti

- Vettori Equivalenti:
  - Stessa Direzione
  - Stesso Verso
  - Stesso Modulo



Algebra dei Vettori: Moltiplicazione per Scalare

- Vettore A, scalare β
- $B = \beta A \quad (|B| = \beta\, |A|)$

Algebra dei Vettori: Moltiplicazione per Scalare

- Vettore A, scalare β
- $B = \beta\, A \quad (|B| = \beta\, |A|)$
  - $\beta > 0$

Algebra dei Vettori: Moltiplicazione per Scalare

- Vettore A, scalare β
- $B = \beta\, A \quad (|B| = \beta\, |A|)$
  - $\beta > 0$
  - $\beta < 0$

Algebra dei Vettori: Moltiplicazione per Scalare

- Vettore A, scalare β

- $B = \beta\, A \quad (|B| = \beta\, |A|)$
  - $\beta > 0$

  - $\beta < 0$

  - $\beta = 0$

$B = \beta A = 0$

Algebra dei Vettori: Moltiplicazione per Scalare

- Proprieta'

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- Proprieta'
  - Commutativa: $\beta A = A\beta$

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- Proprieta'
  - Commutativa: $\beta A = A\beta$
  - Associativa: $\alpha(\beta A) = (\alpha\beta)A$

Algebra dei Vettori: Moltiplicazione per Scalare

- Proprieta'
  - Commutativa: $\beta A = A\beta$
  - Associativa: $\alpha(\beta A) = (\alpha\beta)A$
  - Distributiva: $(\alpha + \beta)A = \alpha A + \beta A$

$$\beta(A + B) = \beta A + \beta B$$

Algebra dei Vettori: Moltiplicazione per Scalare

- Proprieta'
  - Commutativa: $\beta A = A\beta$
  - Associativa: $\alpha(\beta A) = (\alpha\beta)A$
  - Distributiva: $(\alpha + \beta)A = \alpha A + \beta A$

    $\beta(A + B) = \beta A + \beta B$
- Vettore Opposto: $B = -A = \beta A(\beta = -1)$

Algebra dei Vettori: Somma di due Vettori

- Regola Parallelogramma

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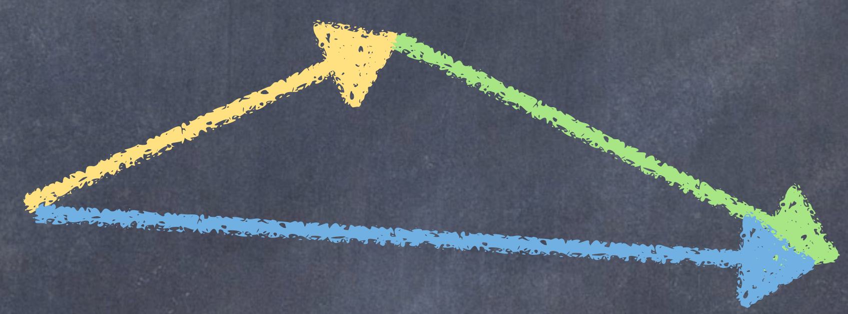
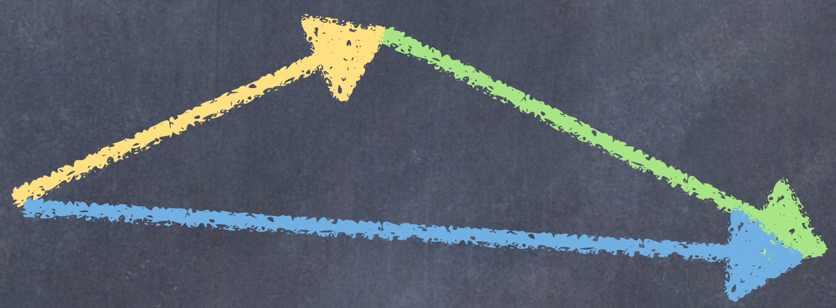
- Regola Parallelogramma



Algebra dei Vettori: Somma di due Vettori

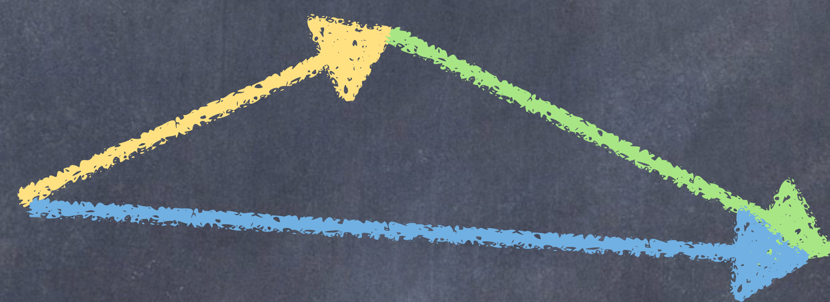
- Regola Parallelogramma

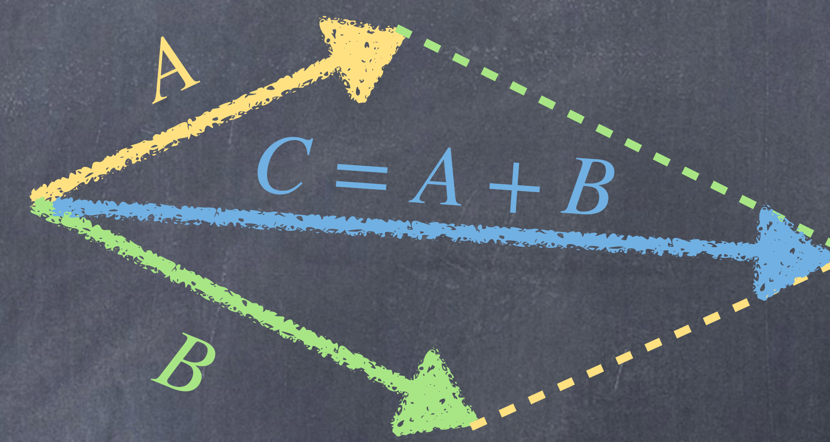


- Proprieta'

Algebra dei Vettori: Somma di due Vettori

- Regola Parallelogramma

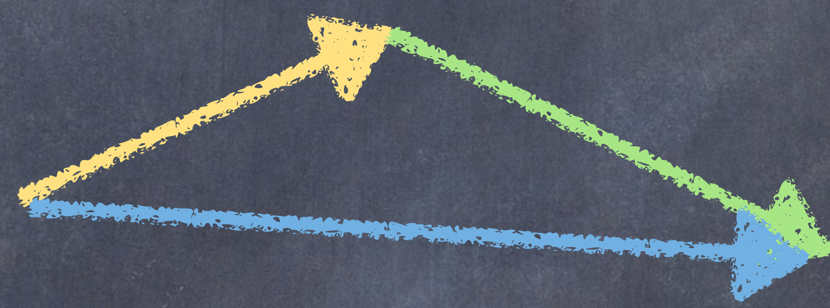


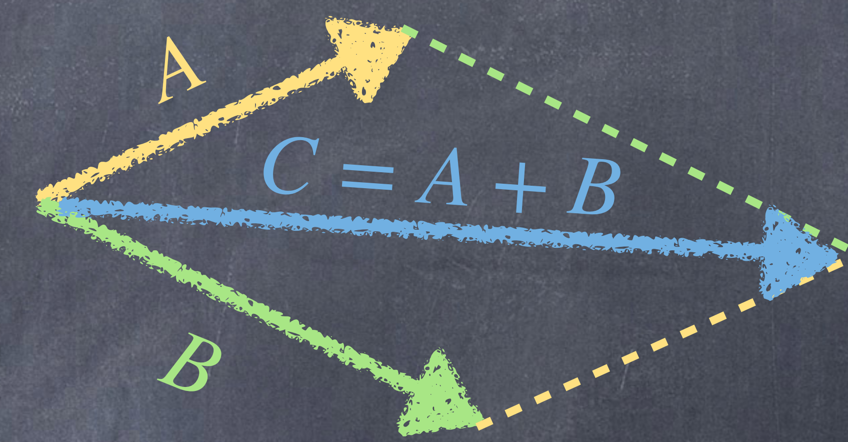


- Proprieta'
  - Commutativa $A + B = B + A$

Algebra dei Vettori: Somma di due Vettori

- Regola Parallelogramma

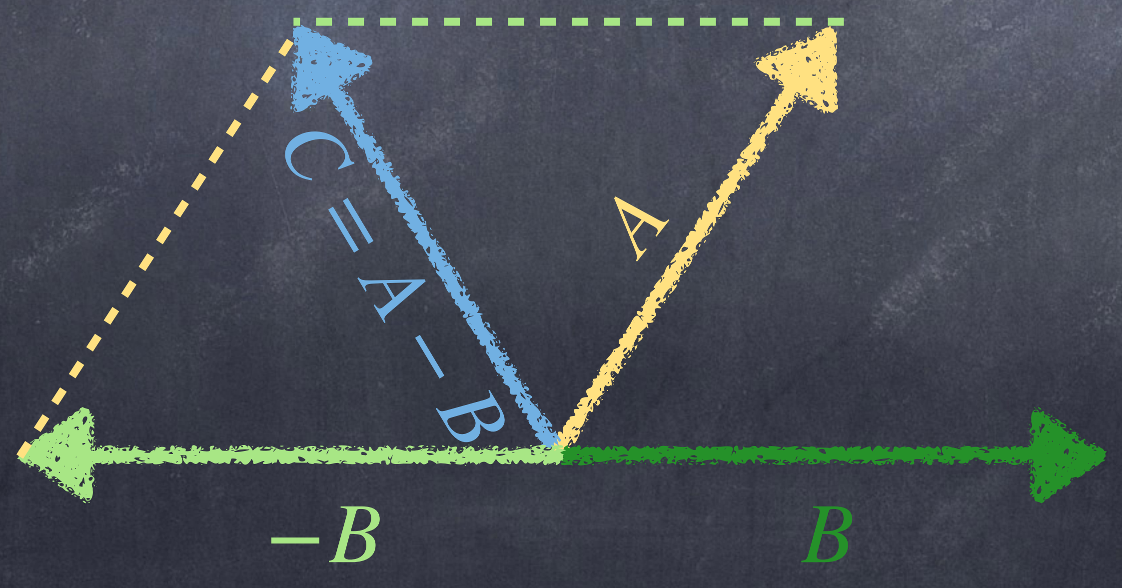




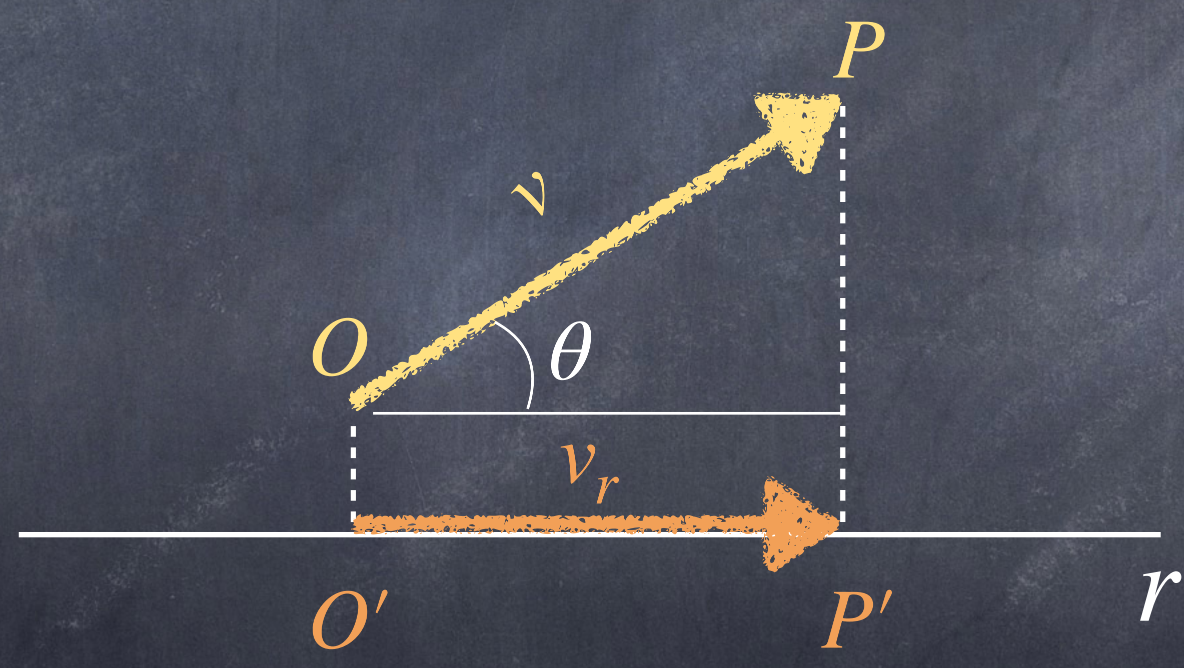
- Proprieta'
  - Commutativa $A + B = B + A$
  - Associativa $A + (B + C) = (A + B) + C$

Algebra dei Vettori: Differenza di due Vettori

- Differenza $A - B = A + (-B)$

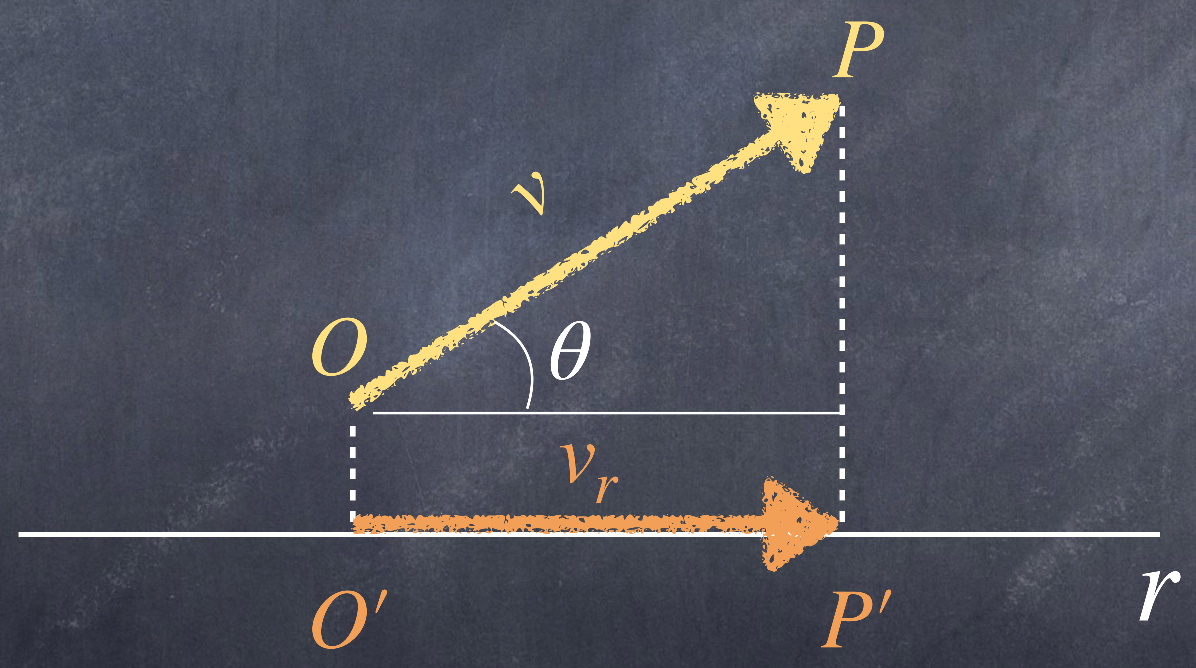


Componenti di un Vettore



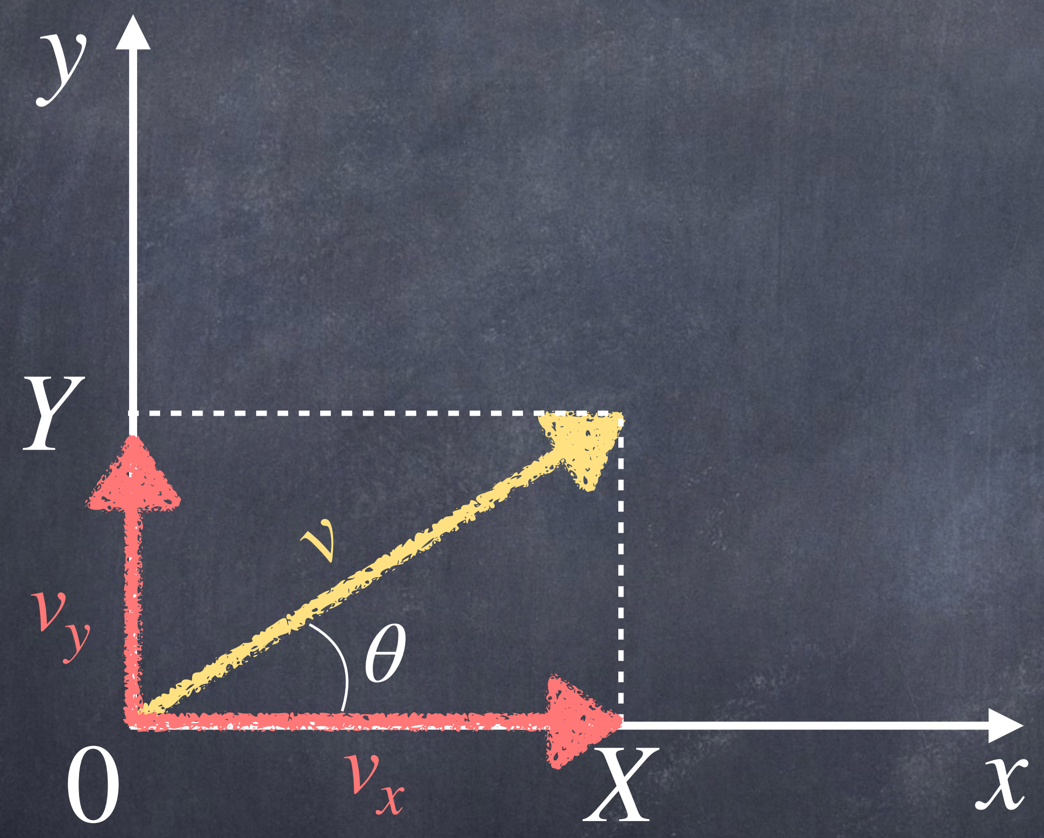
Componenti di un Vettore

Dato un vettore v individuato dal segmento orientato OP e una retta r, la proiezione ortogonale di OP su r è il segmento orientato O'P' (vettore v_r) che prende il nome di Componente del vettore v sulla retta r.



$$|v_r| = |v| cos\theta$$

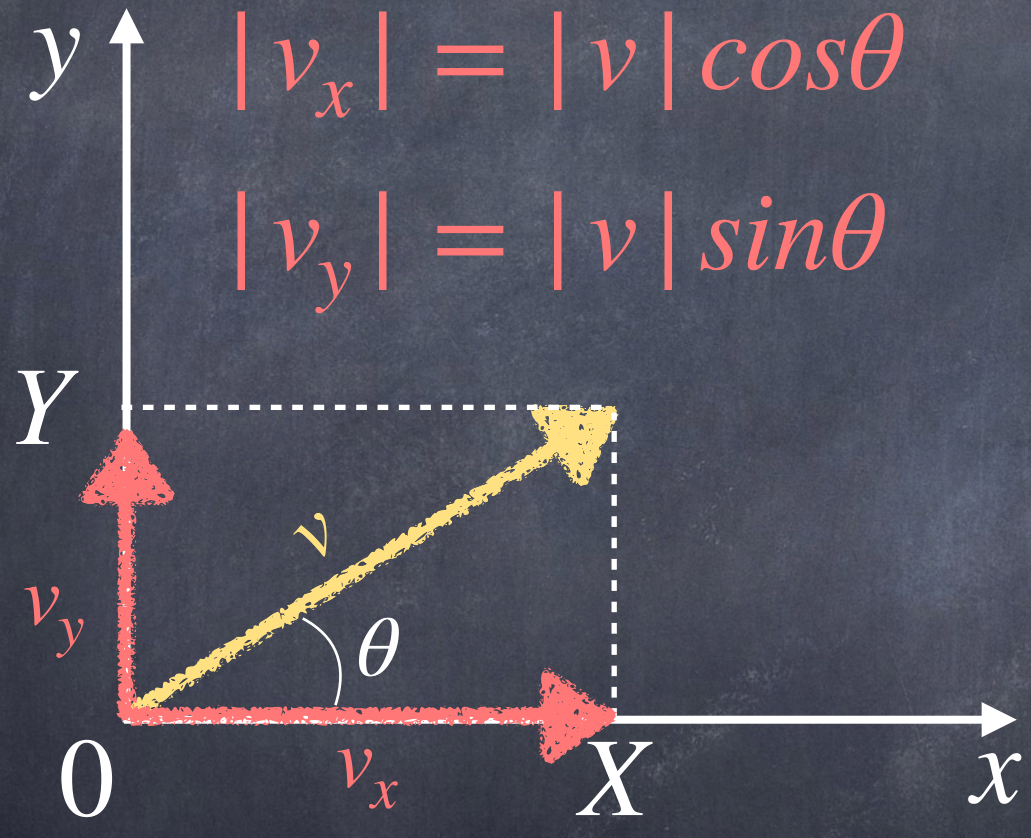
Componenti di un Vettore



Componenti di un Vettore

$$|v_x| = |v|cos\theta$$

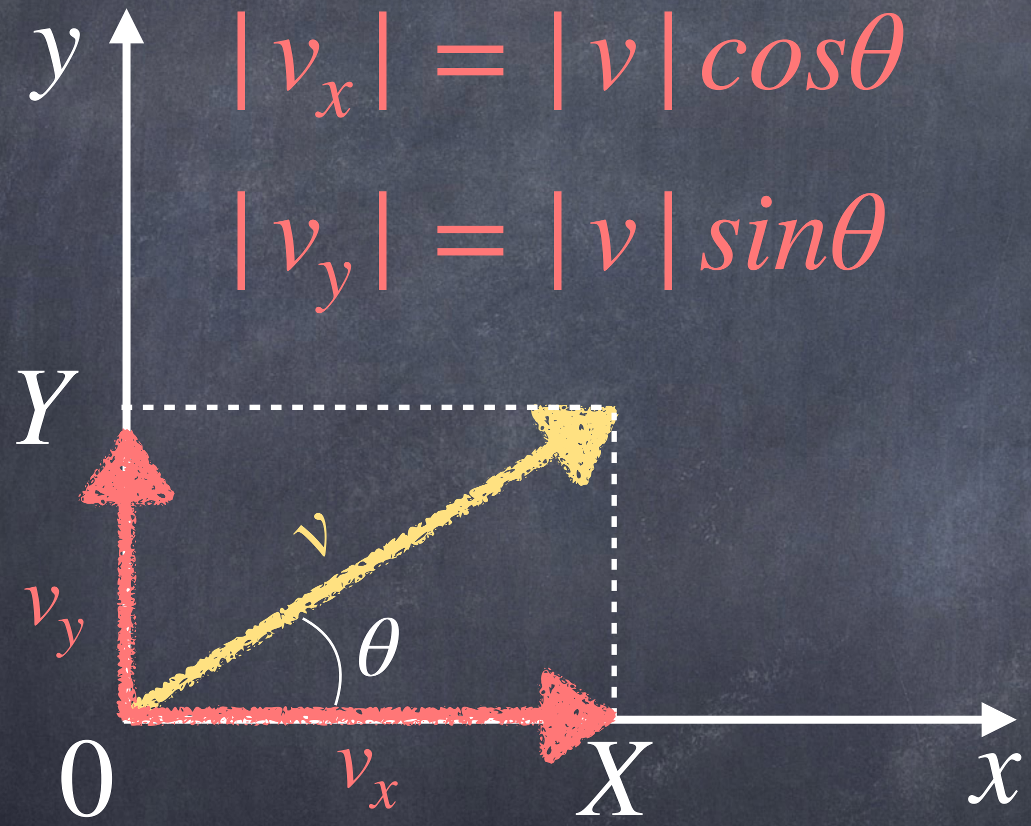
$$|v_y| = |v|sin\theta$$



Componenti di un Vettore

$$|v_x| = |v|cos\theta$$

$$|v_y| = |v|sin\theta$$

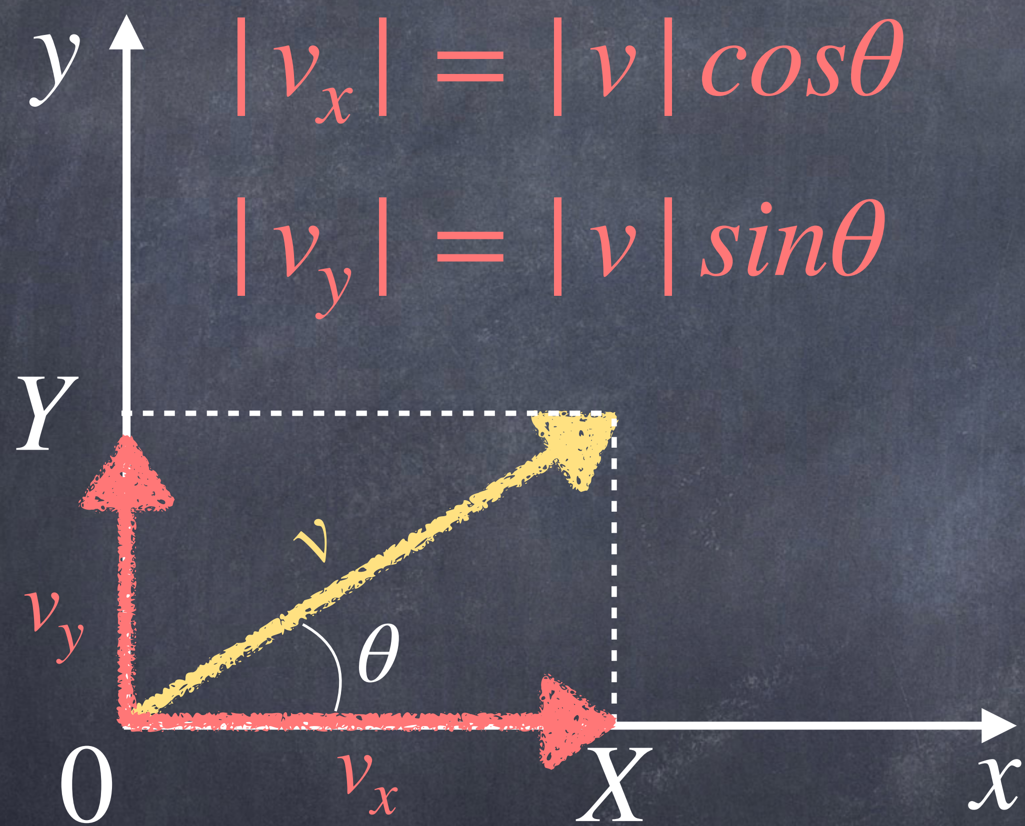


$$v = v_x + v_y$$

Componenti di un Vettore

$$|v_x| = |v|cos\theta$$

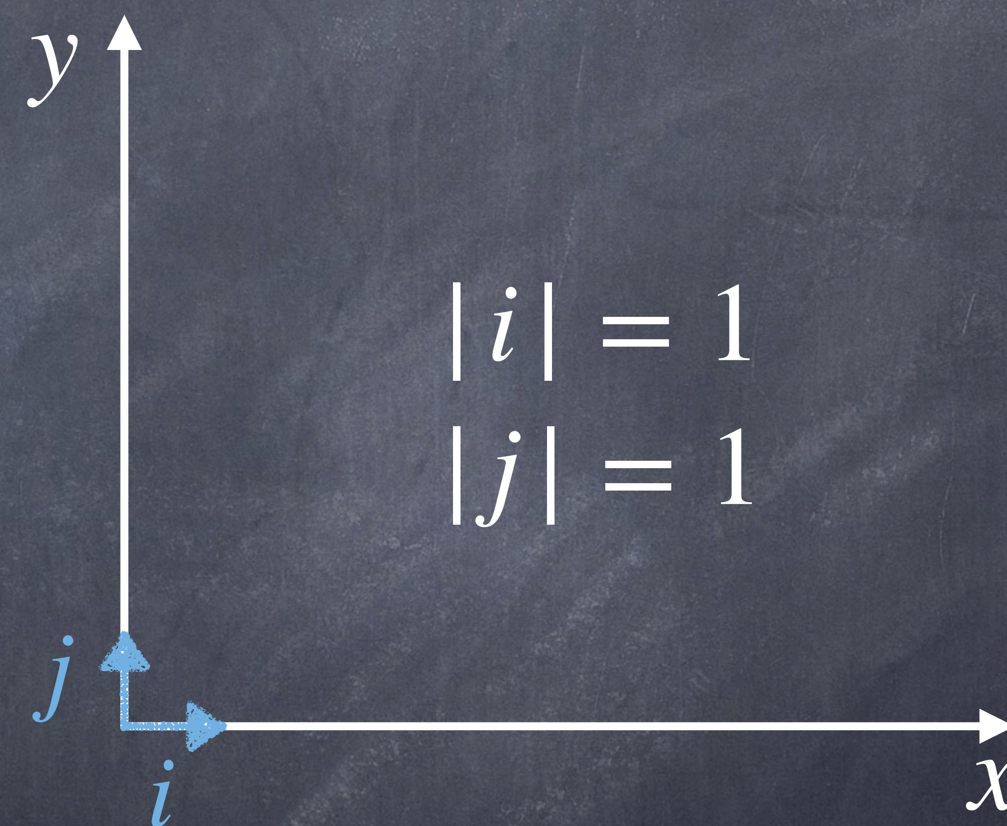
$$|v_y| = |v|sin\theta$$



$$v = v_x + v_y$$

$$|i| = 1$$

$$|j| = 1$$

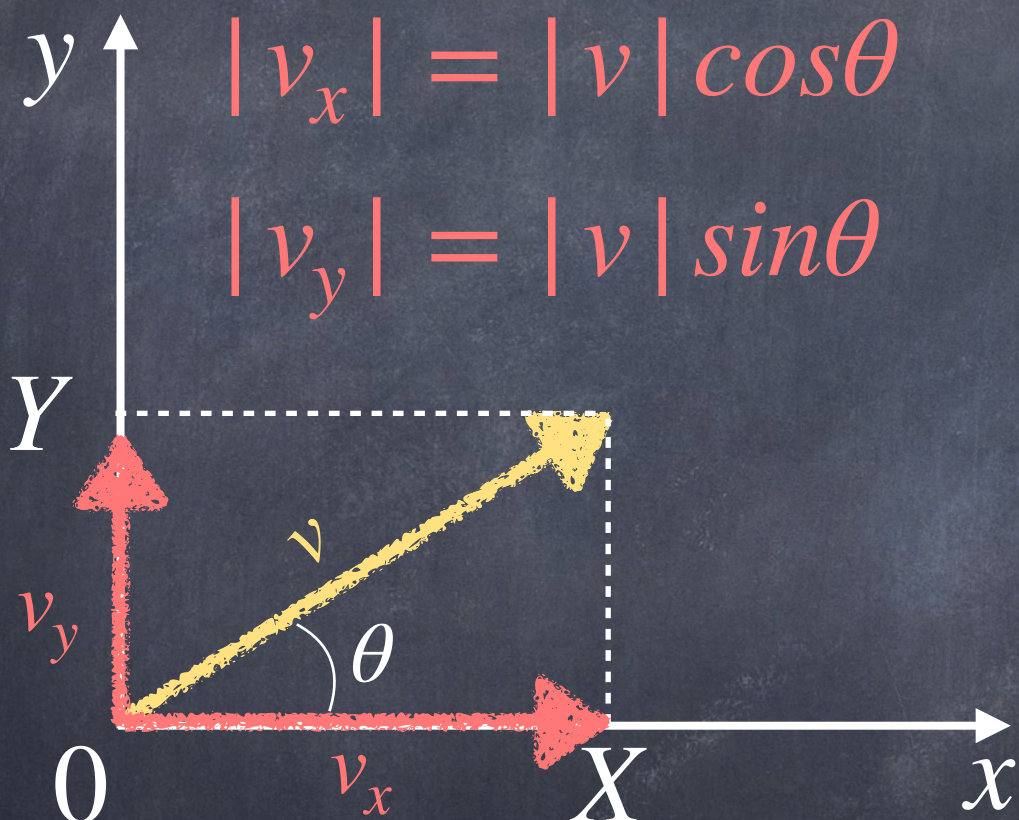


Versori degli assi x e y

Componenti di un Vettore

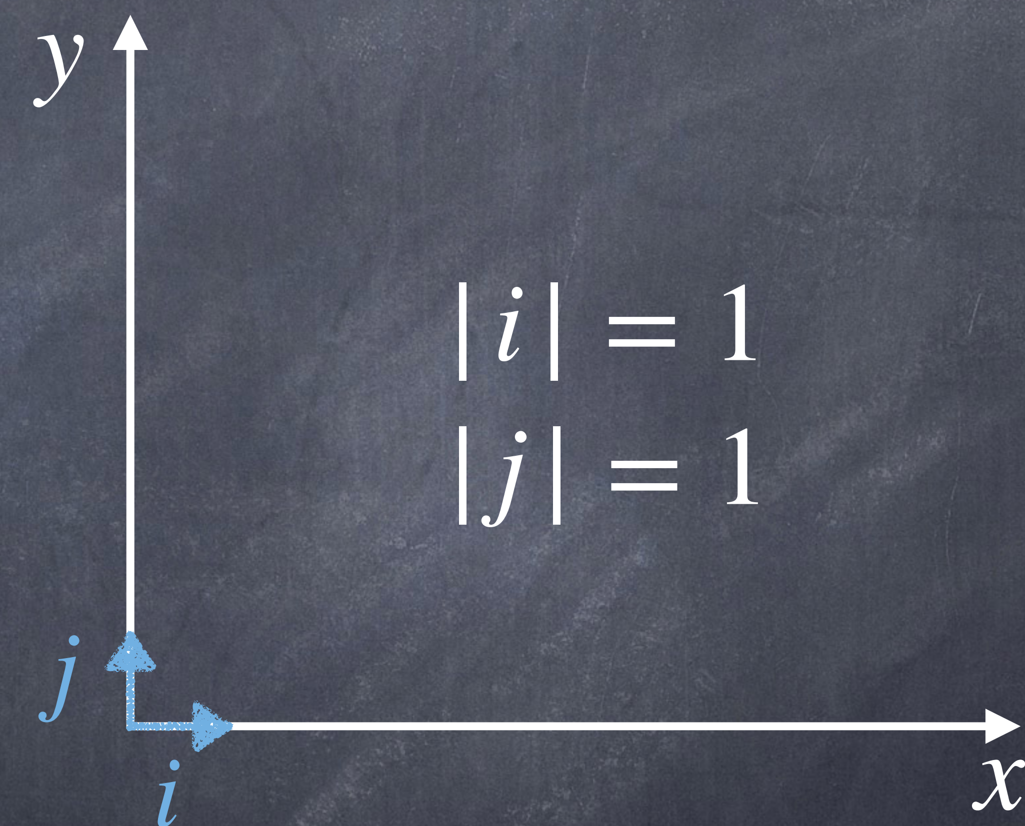
$$|v_x| = |v|\cos\theta$$

$$|v_y| = |v|\sin\theta$$



$$|i| = 1$$

$$|j| = 1$$



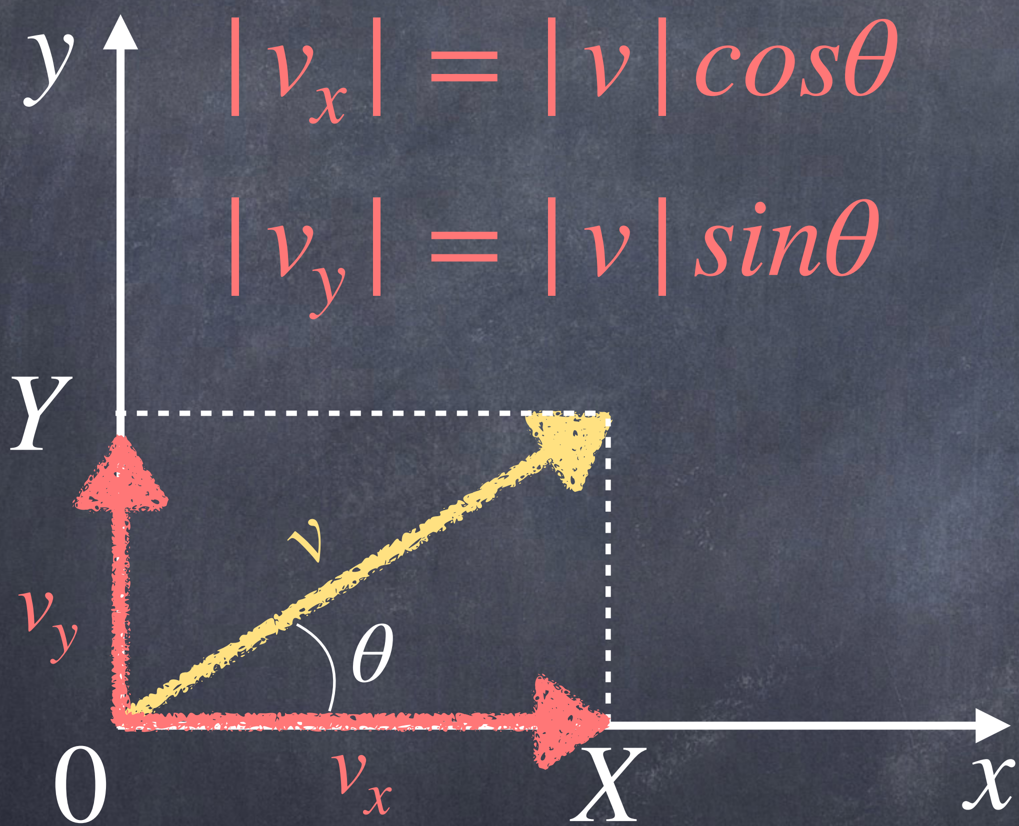
$$v_x = i\,|v_x| \qquad v_y = j\,|v_y|$$

$$v = v_x + v_y$$

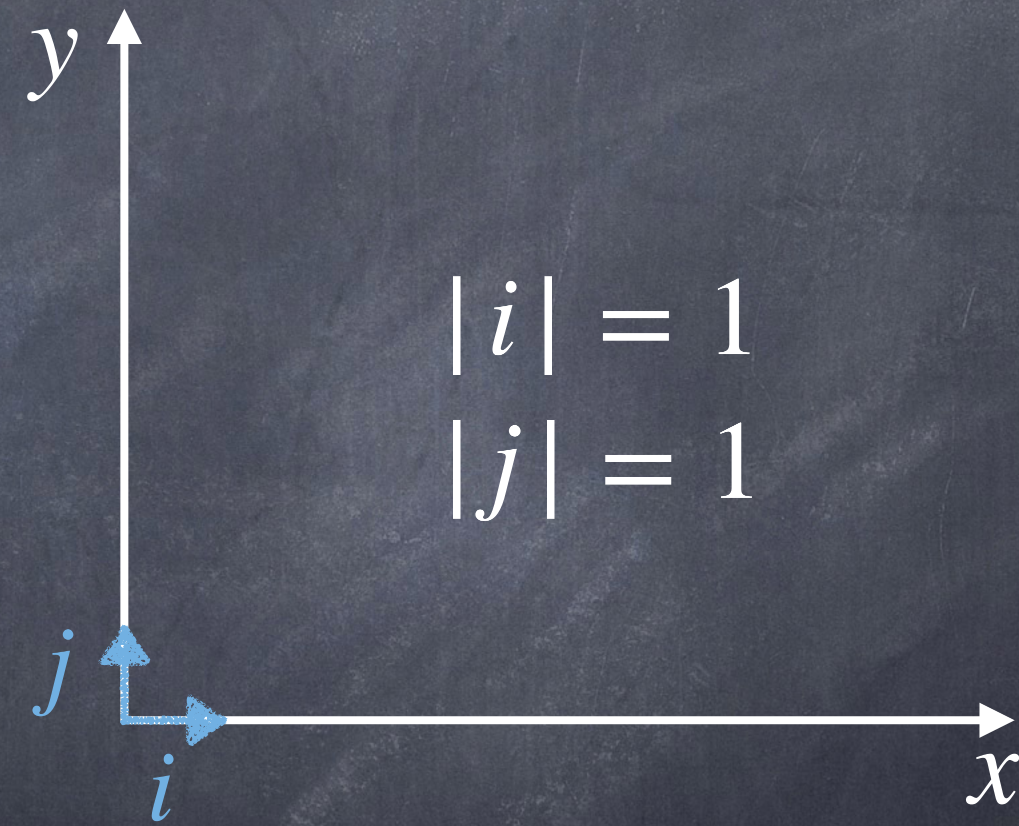
Componenti di un Vettore

$$|v_x| = |v|\cos\theta$$

$$|v_y| = |v|\sin\theta$$



$$v = v_x + v_y$$



$$|i| = 1$$

$$|j| = 1$$

$$v_x = i\,|v_x| \qquad v_y = j\,|v_y|$$

$$v = i\,|v_x| + j\,|v_y|$$

Componenti di un Vettore

$$A(A_x, A_y) = iA_x + jA_y$$

Componenti di un Vettore

$$A(A_x, A_y) = iA_x + jA_y$$

Somma

$$A(A_x, A_y) + B(B_x, B_y) = C(A_x + B_x, A_y + B_y)$$

Componenti di un Vettore

$$A(A_x, A_y) = iA_x + jA_y$$

Somma

$$A(A_x, A_y) + B(B_x, B_y) = C(A_x + B_x, A_y + B_y)$$

Differenza

$$A(A_x, A_y) - B(B_x, B_y) = C(A_x - B_x, A_y - B_y)$$

Componenti di un Vettore

$$A(A_x, A_y) = iA_x + jA_y$$

Somma

$$A(A_x, A_y) + B(B_x, B_y) = C(A_x + B_x, A_y + B_y)$$

Differenza

$$A(A_x, A_y) - B(B_x, B_y) = C(A_x - B_x, A_y - B_y)$$

Moltiplicazione per Scalare

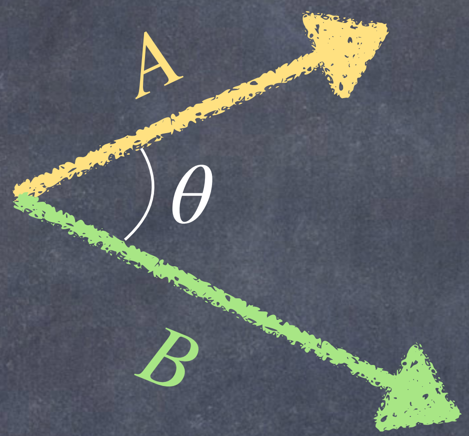
$$\beta A(A_x, A_y) = C(\beta A_x, \beta A_y)$$

Componenti di un Vettore

Tre Dimensioni

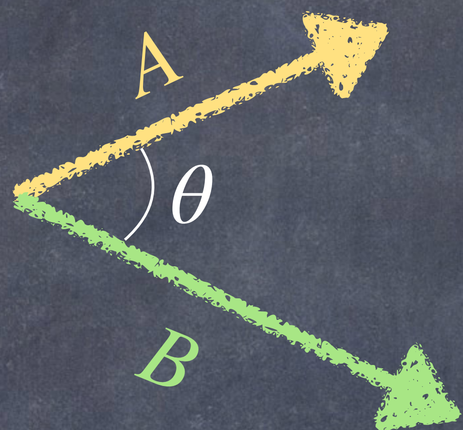
$$v = i|v_x| + j|v_y| + k|v_k|$$

Prodotto Scalare



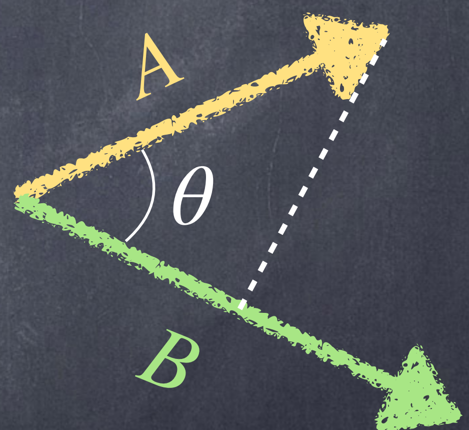
$$c = A \cdot B = |A||B|cos\theta$$

Prodotto Scalare

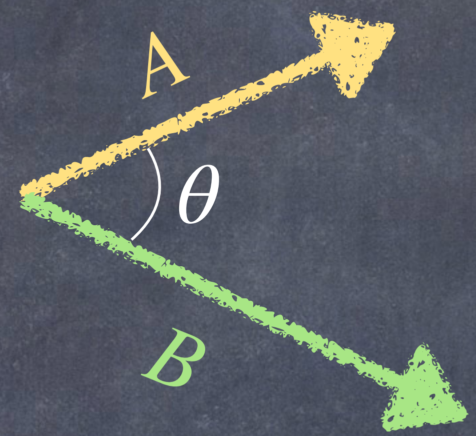


$$c = A \cdot B = |A||B| cos\theta$$

$$c = A \cdot B = (|A| cos\theta)|B|$$

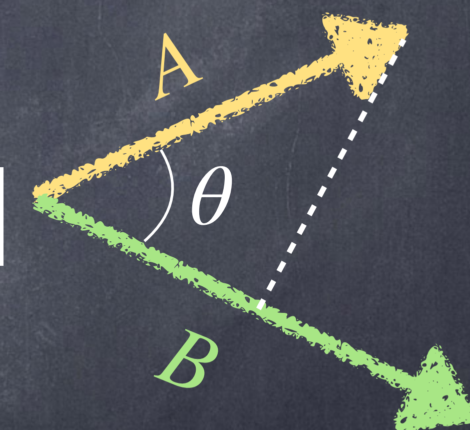


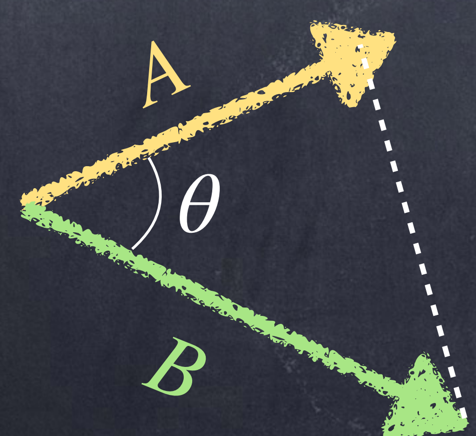
Prodotto Scalare



$$c = A \cdot B = |A||B|cos\theta$$

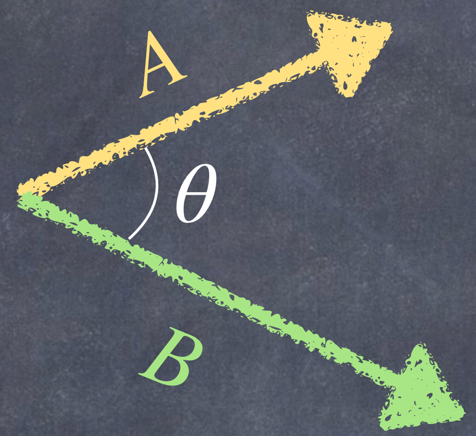
$$c = A \cdot B = (|A|cos\theta)|B|$$





$$c = A \cdot B = |A|(|B|cos\theta)$$

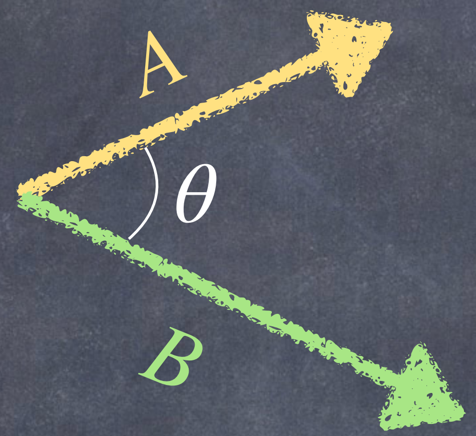
Prodotto Scalare



$$c = A \cdot B = |A||B|cos\theta$$

Proprieta' Commutativa $A \cdot B = B \cdot A$

Prodotto Scalare



$$c = A \cdot B = |A||B| cos\theta$$

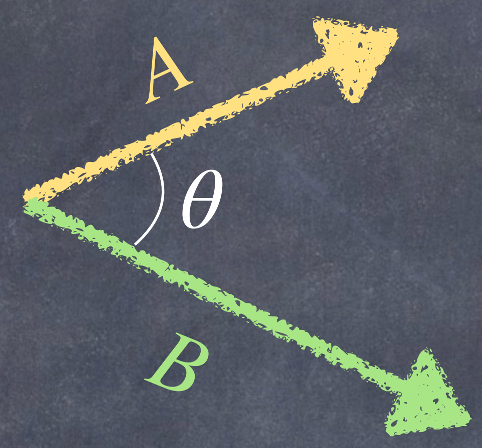
Proprieta' Commutativa $A \cdot B = B \cdot A$

Vettori Paralleli $(\theta = 0)$

$$A \cdot B = |A||B|$$

$$A \cdot A = |A|^2$$

Prodotto Scalare



$$c = A \cdot B = |A||B|cos\theta$$

Proprieta' Commutativa $A \cdot B = B \cdot A$

Vettori Paralleli $(\theta = 0)$

$$A \cdot B = |A||B|$$

$$A \cdot A = |A|^2$$

Vettori Ortogonali $(\theta = \frac{\pi}{2})$

$$A \cdot B = 0$$

Prodotto Scalare

Versori Terna Assi Ortogonale i, j, k

$$i \cdot i = 1 \qquad j \cdot j = 1 \qquad k \cdot k = 1$$

$$i \cdot j = 0 \qquad j \cdot k = 0 \qquad i \cdot k = 0$$

Prodotto Scalare

Versori Terna Assi Ortogonale i, j, k

$$i \cdot i = 1 \qquad j \cdot j = 1 \qquad k \cdot k = 1$$

$$i \cdot j = 0 \qquad j \cdot k = 0 \qquad i \cdot k = 0$$

$$A = iA_x + jA_y + kA_z \qquad B = iB_x + jB_y + kB_z$$

Prodotto Scalare

Versori Terna Assi Ortogonale i, j, k

$$i \cdot i = 1 \qquad j \cdot j = 1 \qquad k \cdot k = 1$$

$$i \cdot j = 0 \qquad j \cdot k = 0 \qquad i \cdot k = 0$$

$$A = iA_x + jA_y + kA_z \qquad B = iB_x + jB_y + kB_z$$

$$A \cdot B = (iA_x + jA_y + kA_z) \cdot (iB_x + jB_y + kB_z)$$

Prodotto Scalare

Versori Terna Assi Ortogonale i, j, k

$$i \cdot i = 1 \qquad j \cdot j = 1 \qquad k \cdot k = 1$$

$$i \cdot j = 0 \qquad j \cdot k = 0 \qquad i \cdot k = 0$$

$$A = iA_x + jA_y + kA_z \qquad B = iB_x + jB_y + kB_z$$

$$A \cdot B = (iA_x + jA_y + kA_z) \cdot (iB_x + jB_y + kB_z)$$
$$= (A_xB_x + A_yB_y + A_zB_z)$$

Prodotto Scalare

Versori Terna Assi Ortogonale i, j, k

$$i \cdot i = 1 \qquad j \cdot j = 1 \qquad k \cdot k = 1$$

$$i \cdot j = 0 \qquad j \cdot k = 0 \qquad i \cdot k = 0$$

$$A = iA_x + jA_y + kA_z \qquad B = iB_x + jB_y + kB_z$$

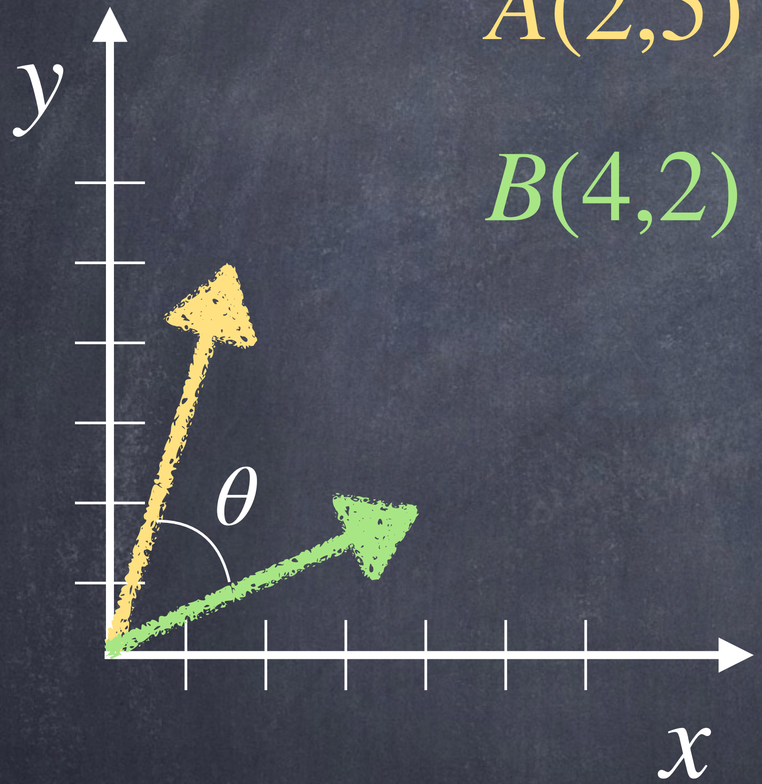
$$A \cdot B = (iA_x + jA_y + kA_z) \cdot (iB_x + jB_y + kB_z)$$
$$= (A_xB_x + A_yB_y + A_zB_z)$$

$$A \cdot A = A_x^2 + A_y^2 + A_z^2 = |A|^2$$

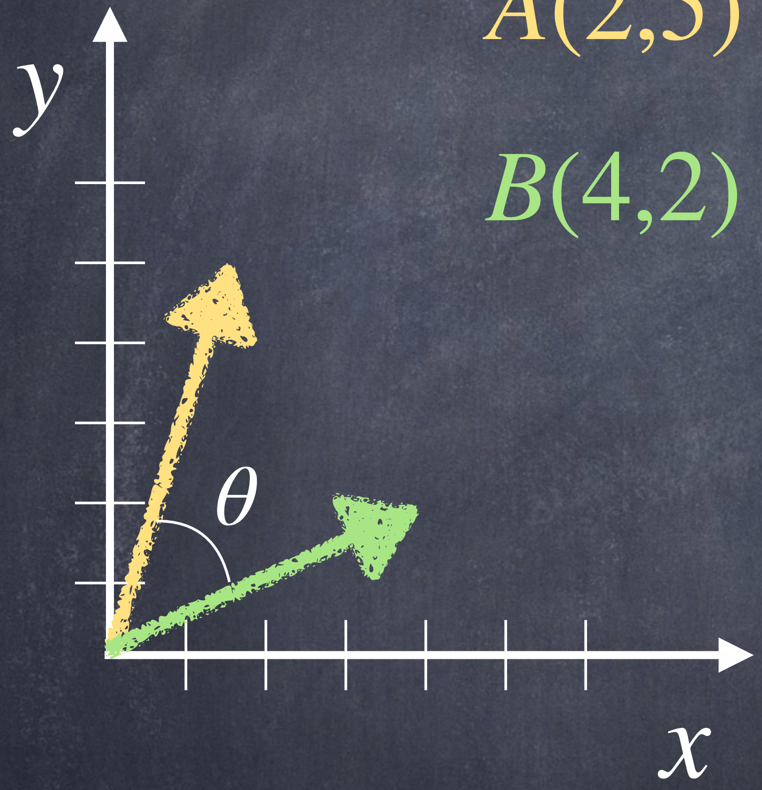
Esercizi Prodotto Scalare

$A(2,5)$

$B(4,2)$



Esercizi Prodotto Scalare

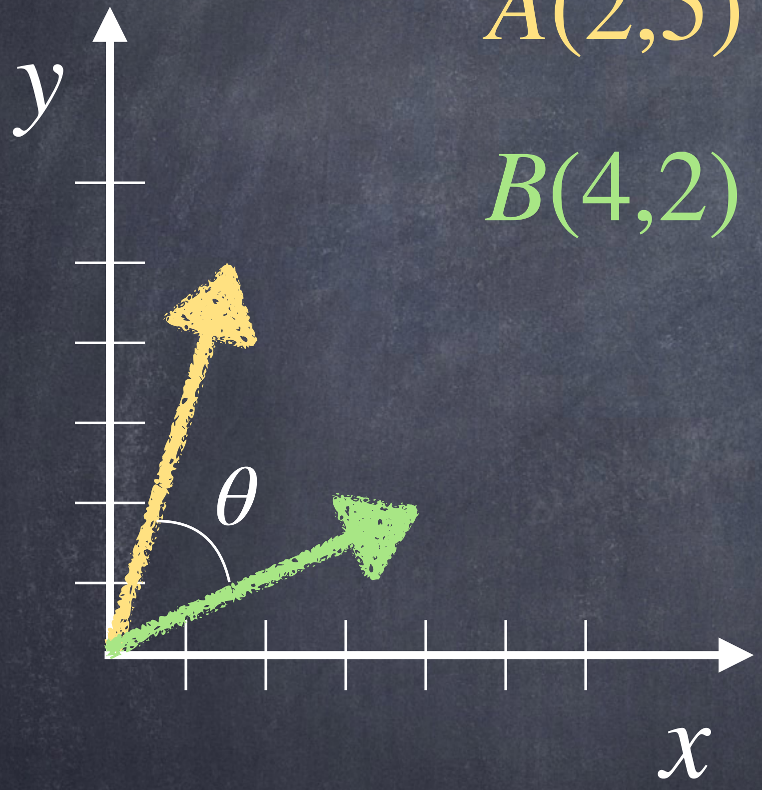


$A(2,5)$

$B(4,2)$

$$A \cdot B = 2 \times 4 + 5 \times 2 = 18$$

Esercizi Prodotto Scalare



$A(2,5)$

$B(4,2)$

$$A \cdot B = 2 \times 4 + 5 \times 2 = 18$$

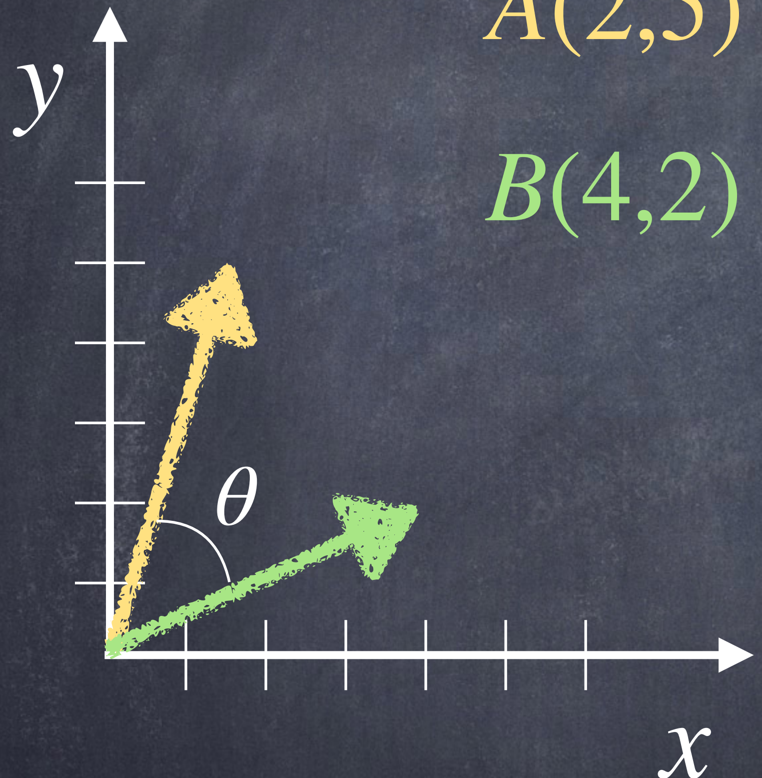
$$A \cdot B = |A||B| cos\theta$$

$$cos\theta = \frac{A \cdot B}{|A||B|}$$

Esercizi Prodotto Scalare

$A(2,5)$

$B(4,2)$



$$A \cdot B = 2 \times 4 + 5 \times 2 = 18$$

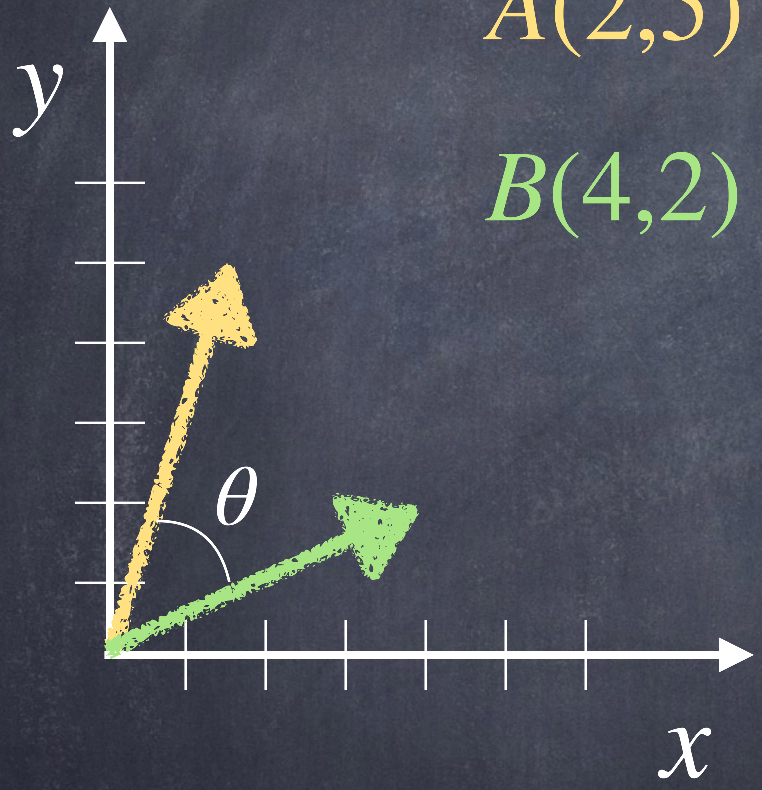
$$A \cdot B = |A||B| cos\theta$$

$$cos\theta = \frac{A \cdot B}{|A||B|}$$

$$|A| = \sqrt{A_x^2 + A_y^2} = \sqrt{2^2 + 5^2} = 5.38$$

$$|B| = \sqrt{B_x^2 + B_y^2} = \sqrt{4^2 + 2^2} = 4.47$$

Esercizi Prodotto Scalare



$A(2,5)$

$B(4,2)$

$$A \cdot B = 2 \times 4 + 5 \times 2 = 18$$

$$A \cdot B = |A||B| cos\theta$$

$$cos\theta = \frac{A \cdot B}{|A||B|}$$

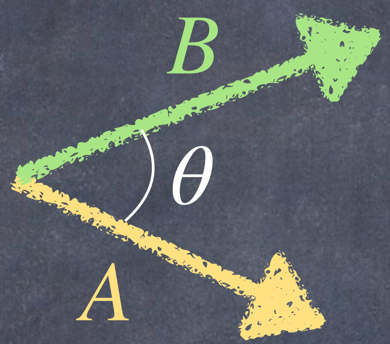
$$|A| = \sqrt{A_x^2 + A_y^2} = \sqrt{2^2 + 5^2} = 5.38$$

$$|B| = \sqrt{B_x^2 + B_y^2} = \sqrt{4^2 + 2^2} = 4.47$$

$$cos\theta = 0.75$$

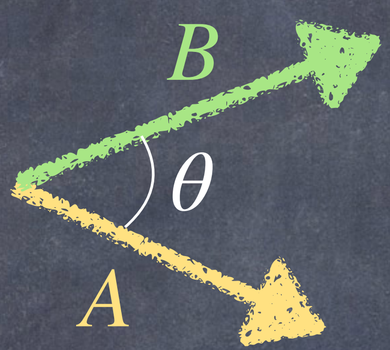
$$\theta = 42.8^\circ$$

Prodotto Vettoriale



$$C = A \times B$$

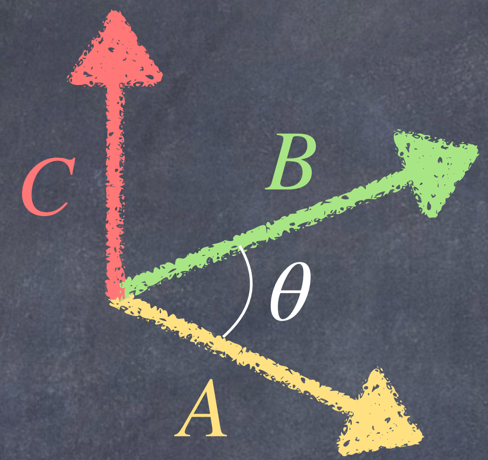
Prodotto Vettoriale



$$C = A \times B$$

$$|C| = |A||B|\, sin\theta$$

Prodotto Vettoriale

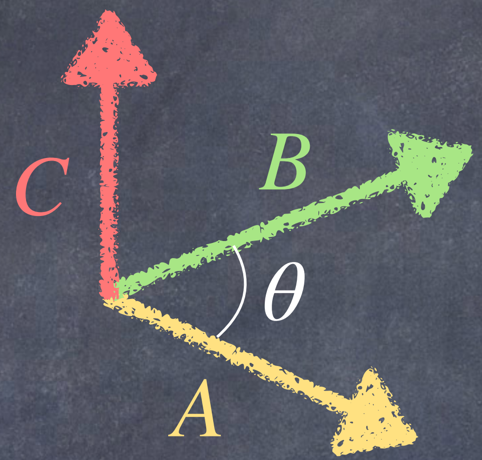


$$C = A \times B$$

$$|C| = |A||B|\, sin\theta$$

- Direzione: perpendicolare ad A e B

Prodotto Vettoriale

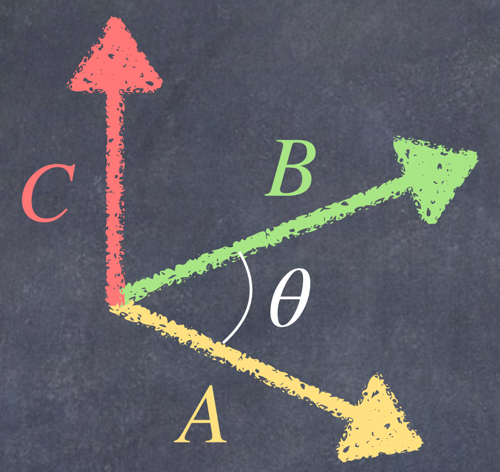


$$C = A \times B$$

$$|C| = |A||B| \sin\theta$$

- Direzione: perpendicolare ad A e B
- Se A e B sono paralleli $|C| = 0$

Prodotto Vettoriale

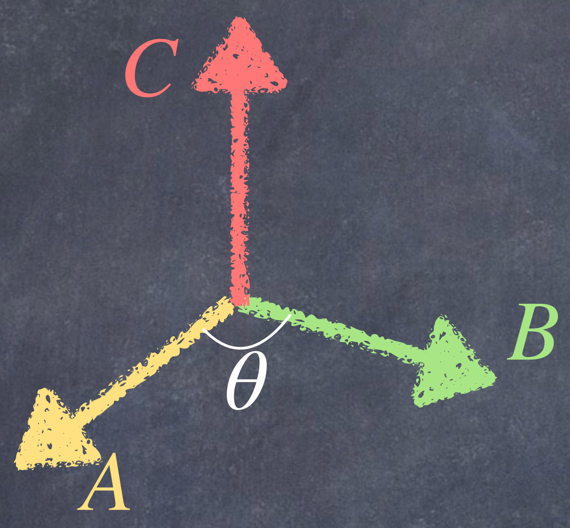


$$C = A \times B$$

$$|C| = |A||B| \; sin\theta$$

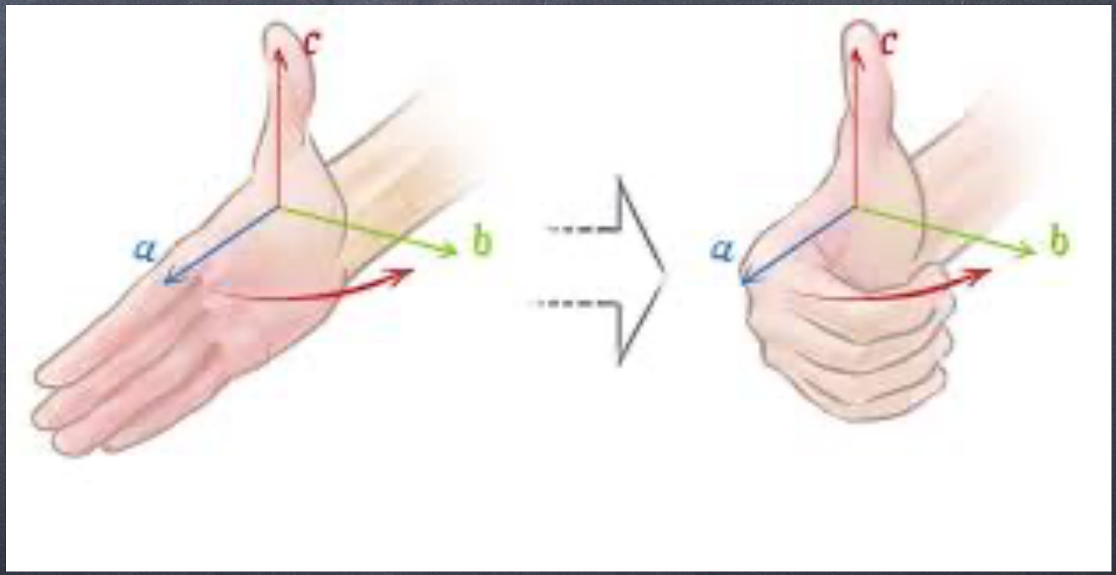
- Direzione: perpendicolare ad A e B
- Se A e B sono paralleli $|C| = 0$
- Verso: regola della mano destra

Prodotto Vettoriale

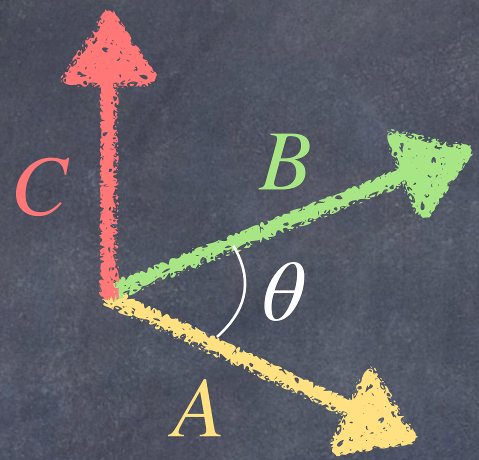


$$C = A \times B$$

Regola della
Mano Destra

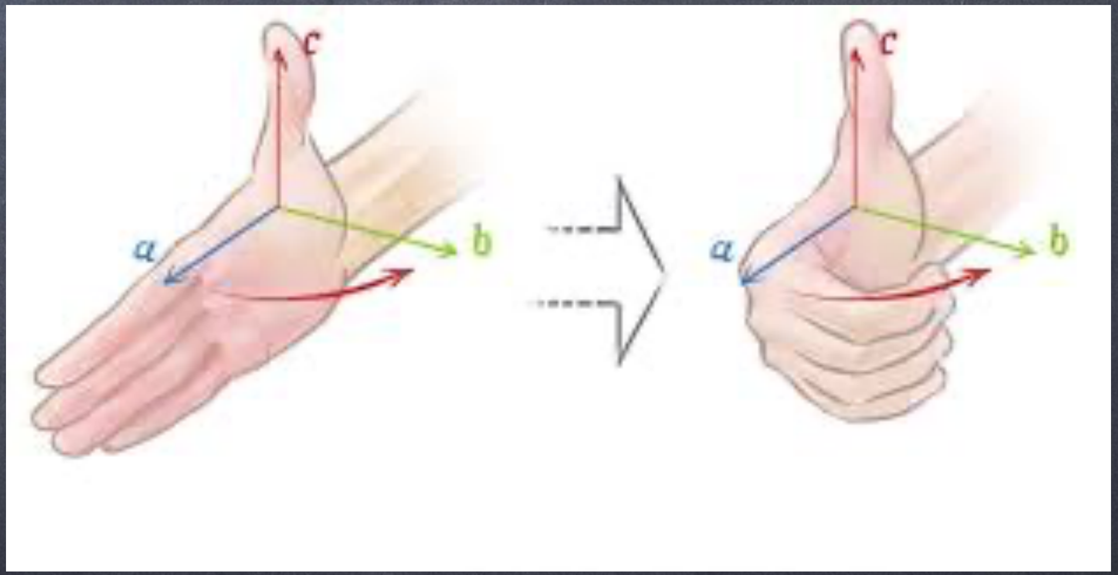


Prodotto Vettoriale



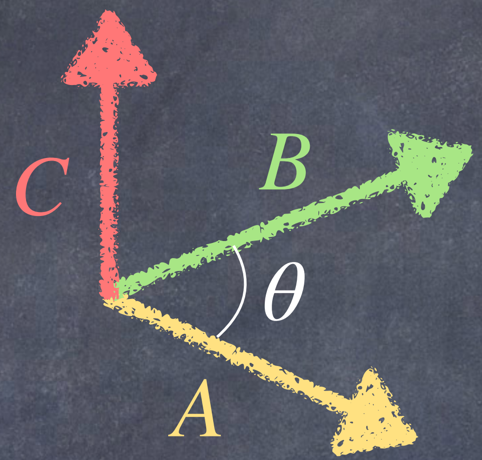
$$C = A \times B$$

Regola della
Mano Destra



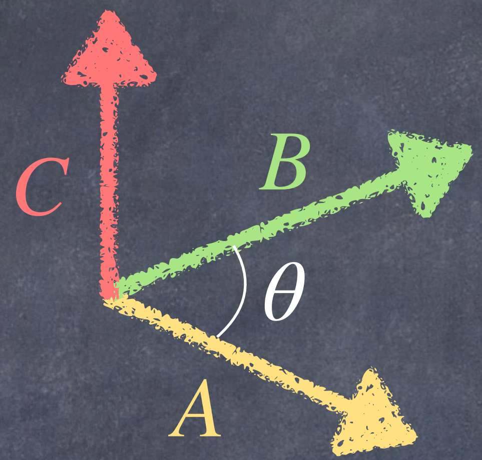
Visto da C la rotazione di A su B
e' in senso Antiorario

Prodotto Vettoriale



$$C = A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

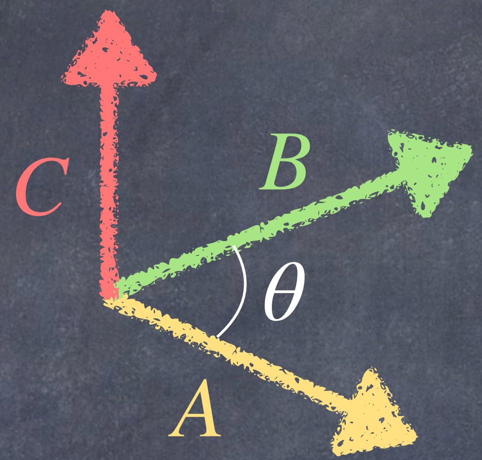
Prodotto Vettoriale



$$C = A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

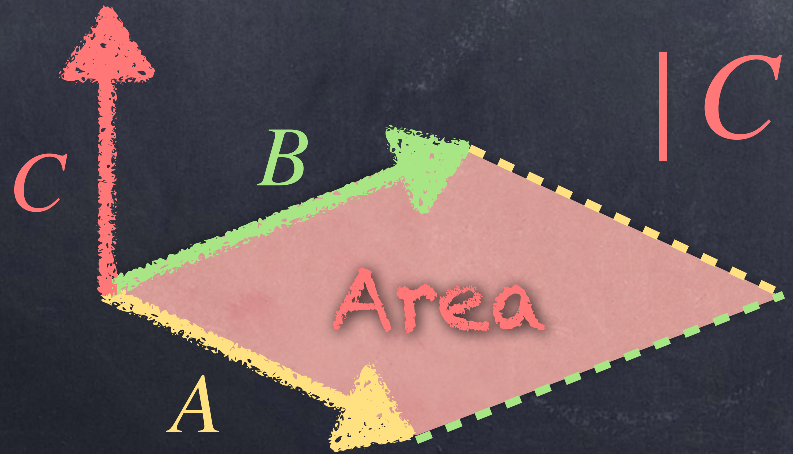
$$= (A_yB_z - B_yA_z)\ \hat{i} + (A_zB_x - B_zA_x)\ \hat{j} + (A_xB_y - B_xA_y)\ \hat{k}$$

Prodotto Vettoriale



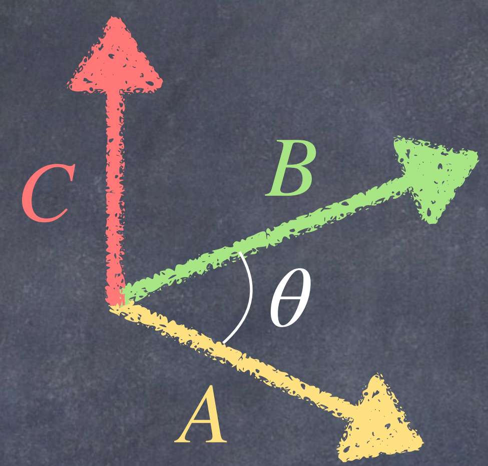
$$C = A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - B_y A_z)\,\hat{i} + (A_z B_x - B_z A_x)\,\hat{j} + (A_x B_y - B_x A_y)\,\hat{k}$$



$|C| =$ Area

Prodotto Vettoriale



$$C = A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - B_y A_z)\ \hat{i} + (A_z B_x - B_z A_x)\ \hat{j} + (A_x B_y - B_x A_y)\ \hat{k}$$

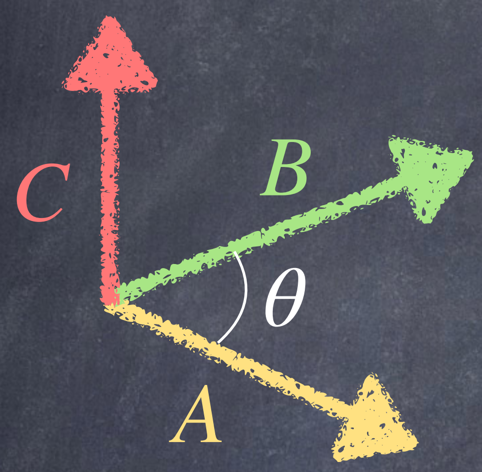
$|C| = \text{Area}$

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

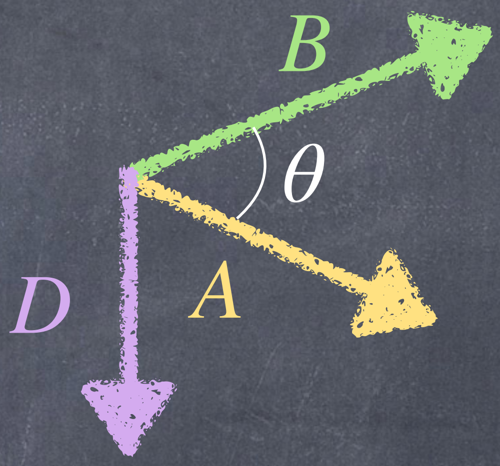
$$\hat{k} \times \hat{i} = \hat{j}$$

Prodotto Vettoriale

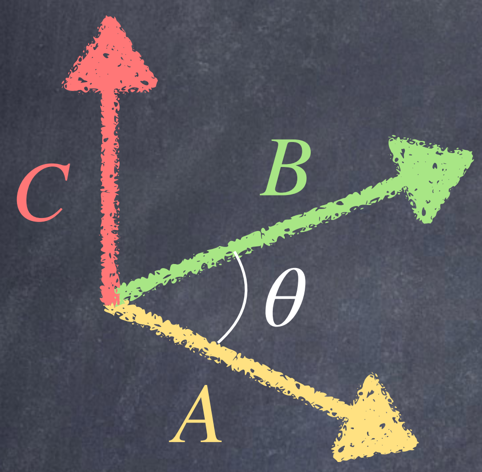


$$C = A \times B = -B \times A$$

$$D = B \times A = -C$$

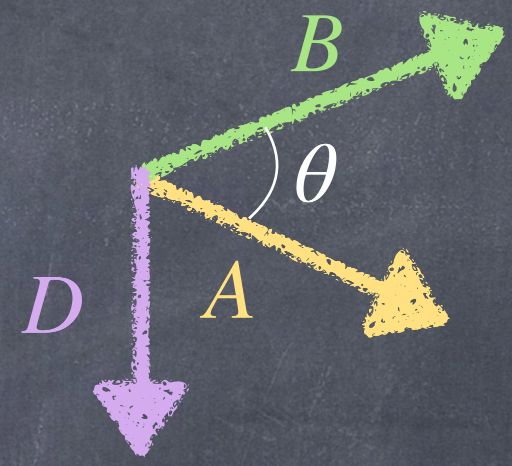


Prodotto Vettoriale



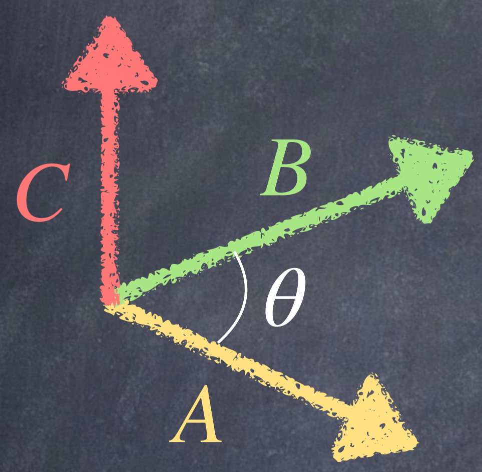
$$C = A \times B = -B \times A$$

$$D = B \times A = -C$$



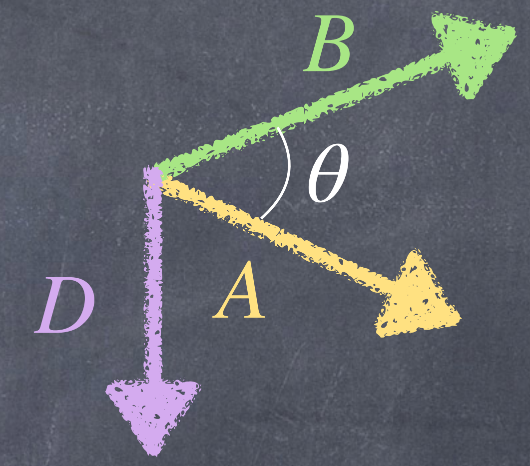
$$(\lambda A) \times B = \lambda(A \times B) = A \times (\lambda B)$$

Prodotto Vettoriale



$$C = A \times B = -B \times A$$

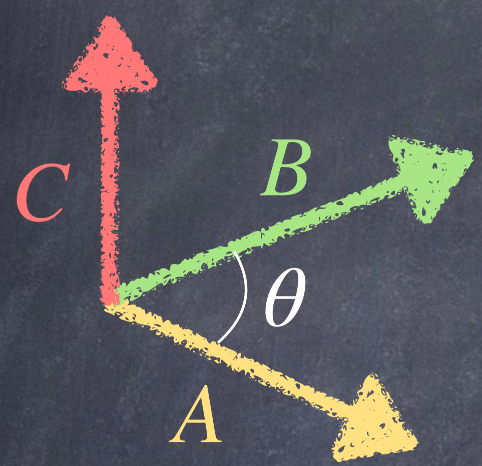
$$D = B \times A = -C$$



$$(\lambda A) \times B = \lambda (A \times B) = A \times (\lambda B)$$

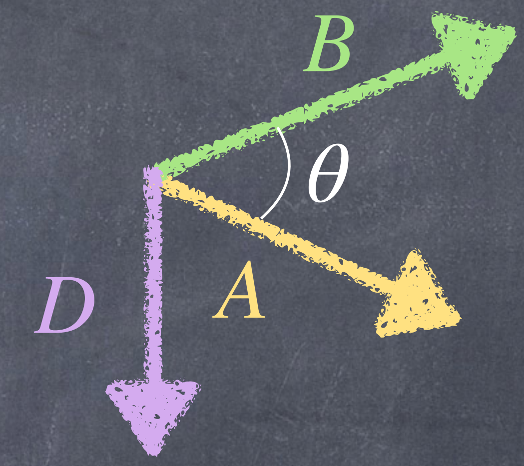
$$A \times (B + C) = A \times B \ + \ A \times C \qquad (A + B) \times C = A \times C \ + \ B \times C$$

Prodotto Vettoriale



$$C = A \times B = -B \times A$$

$$D = B \times A = -C$$

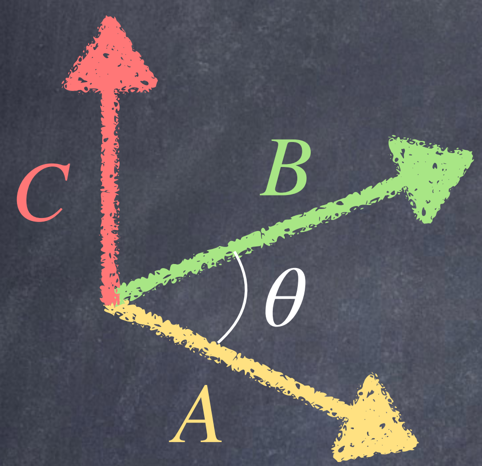


$$(\lambda A) \times B = \lambda (A \times B) = A \times (\lambda B)$$

$$A \times (B + C) = A \times B + A \times C \qquad (A + B) \times C = A \times C + B \times C$$

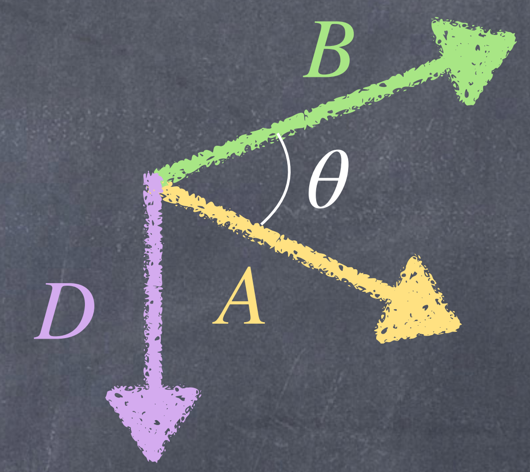
$$A \cdot (B \times C) = B \cdot (C \times A) = C \cdot (A \times B)$$

Prodotto Vettoriale



$$C = A \times B = -B \times A$$

$$D = B \times A = -C$$



$$(\lambda A) \times B = \lambda (A \times B) = A \times (\lambda B)$$

$$A \times (B + C) = A \times B + A \times C \qquad (A + B) \times C = A \times C + B \times C$$

$$A \cdot (B \times C) = B \cdot (C \times A) = C \cdot (A \times B)$$

$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$$

Prodotto Vettoriale

$$A \times B = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

Prodotto Vettoriale

$$A \times B = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\hat{i} \times \hat{j} = \hat{k}$$
$$\hat{j} \times \hat{k} = \hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j}$$

Prodotto Vettoriale

$$\hat{i} \times \hat{j} = \hat{k}$$
$$\hat{j} \times \hat{k} = \hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j}$$

$$A \times B = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$A_x\hat{i} \times B_x\hat{i} = 0$$

$$A_x\hat{i} \times B_y\hat{j} = A_xB_y\hat{k}$$

$$A_x\hat{i} \times B_z\hat{k} = -A_xB_z\hat{j}$$

Prodotto Vettoriale

$$\hat{i} \times \hat{j} = \hat{k}$$
$$\hat{j} \times \hat{k} = \hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j}$$

$$A \times B = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$A_x\hat{i} \times B_x\hat{i} = 0$$

$$A_x\hat{i} \times B_y\hat{j} = A_xB_y\hat{k}$$

$$A_x\hat{i} \times B_z\hat{k} = -A_xB_z\hat{j}$$

$$A_y\hat{j} \times B_x\hat{i} = -A_yB_x\hat{k}$$

$$A_y\hat{j} \times B_y\hat{j} = 0$$

$$A_y\hat{j} \times B_z\hat{k} = A_yB_z\hat{i}$$

Prodotto Vettoriale

$$A \times B = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\hat{i} \times \hat{j} = \hat{k}$$
$$\hat{j} \times \hat{k} = \hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j}$$

$$A_x\hat{i} \times B_x\hat{i} = 0$$
$$A_x\hat{i} \times B_y\hat{j} = A_xB_y\hat{k}$$
$$A_x\hat{i} \times B_z\hat{k} = -A_xB_z\hat{j}$$

$$A_y\hat{j} \times B_x\hat{i} = -A_yB_x\hat{k}$$
$$A_y\hat{j} \times B_y\hat{j} = 0$$
$$A_y\hat{j} \times B_z\hat{k} = A_yB_z\hat{i}$$

$$A_z\hat{k} \times B_x\hat{i} = A_zB_x\hat{j}$$
$$A_z\hat{k} \times B_y\hat{j} = -A_zB_y\hat{i}$$
$$A_z\hat{k} \times B_z\hat{k} = 0$$

Prodotto Vettoriale

$$A \times B = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\hat{i} \times \hat{j} = \hat{k}$$
$$\hat{j} \times \hat{k} = \hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j}$$

$$A_x\hat{i} \times B_x\hat{i} = 0 \qquad A_y\hat{j} \times B_x\hat{i} = -A_yB_x\hat{k} \qquad A_z\hat{k} \times B_x\hat{i} = A_zB_x\hat{j}$$

$$A_x\hat{i} \times B_y\hat{j} = A_xB_y\hat{k} \qquad A_y\hat{j} \times B_y\hat{j} = 0 \qquad A_z\hat{k} \times B_y\hat{j} = -A_zB_y\hat{i}$$

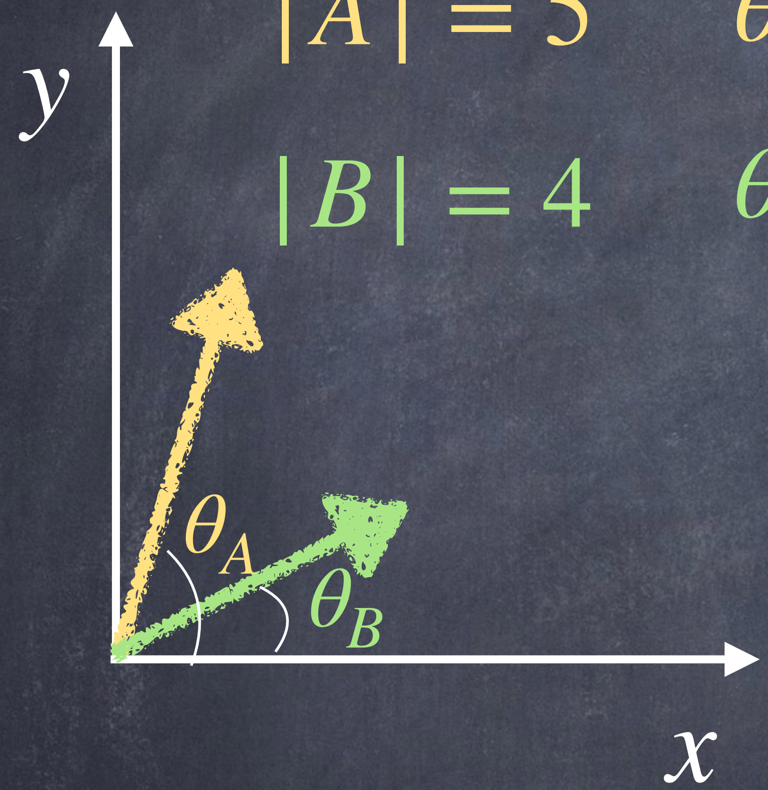
$$A_x\hat{i} \times B_z\hat{k} = -A_xB_z\hat{j} \qquad A_y\hat{j} \times B_z\hat{k} = A_yB_z\hat{i} \qquad A_z\hat{k} \times B_z\hat{k} = 0$$

$$A \times B = (A_yB_z - B_yA_z)\ \hat{i} + (A_zB_x - B_zA_x)\ \hat{j} + (A_xB_y - B_xA_y)\ \hat{k}$$

Esercizi

$|A| = 5 \quad \theta_A = 60^\circ$

$|B| = 4 \quad \theta_B = 20^\circ$



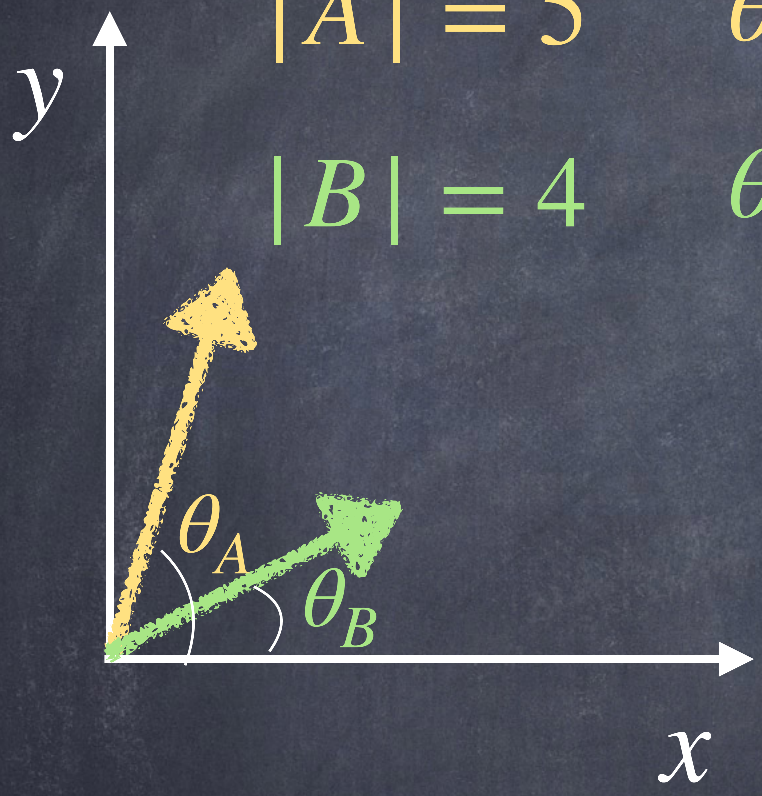
Calcolare: $A \cdot B$

$A \times B$

Esercizi

$|A| = 5 \quad \theta_A = 60^\circ$

$|B| = 4 \quad \theta_B = 20^\circ$



Calcolare: $A \cdot B$

$A \times B$

$$\theta_{AB} = \theta_A - \theta_B = 40^\circ$$

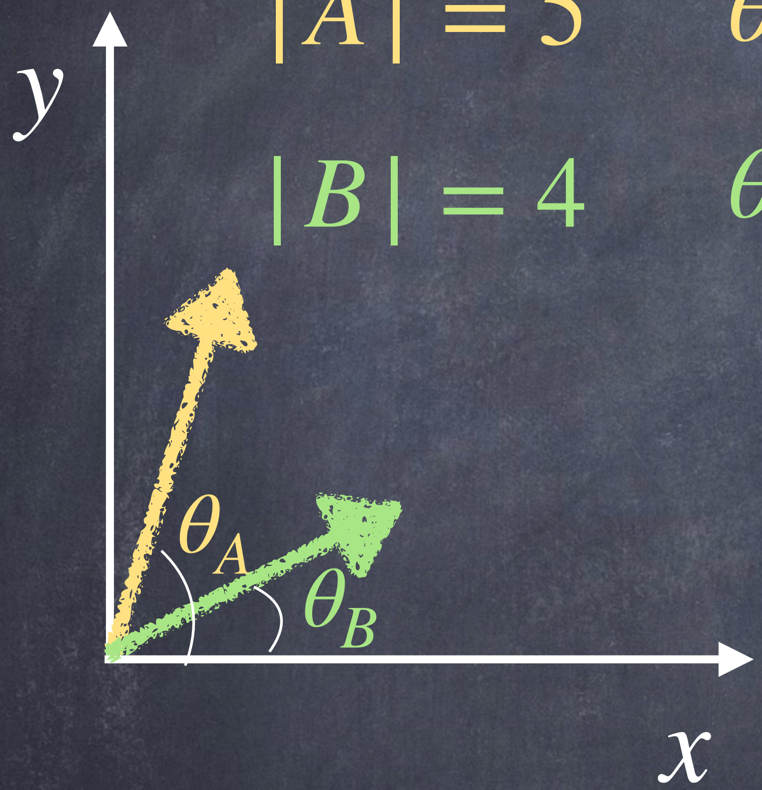
Esercizi

$|A| = 5 \quad \theta_A = 60°$

$|B| = 4 \quad \theta_B = 20°$

Calcolare: $A \cdot B$

$A \times B$



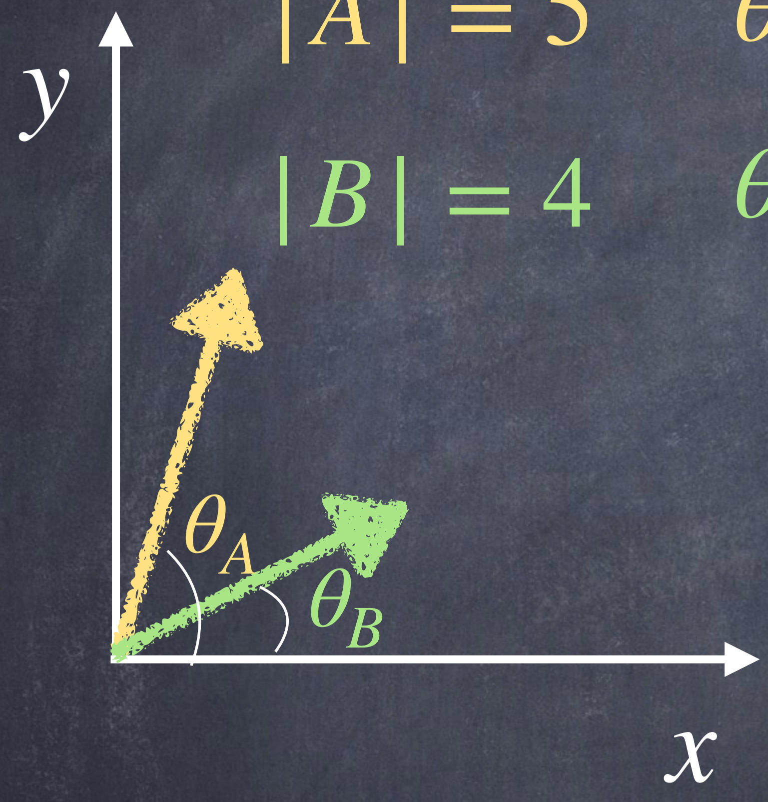
$$\theta_{AB} = \theta_A - \theta_B = 40°$$

$$A \cdot B = |A||B| cos\theta_{AB}$$

Esercizi

$|A| = 5 \quad \theta_A = 60°$

$|B| = 4 \quad \theta_B = 20°$



Calcolare: $A \cdot B$

$A \times B$

$$\theta_{AB} = \theta_A - \theta_B = 40°$$

$$A \cdot B = |A||B| cos\theta_{AB} \quad = 5 \cdot 4cos\left(40\frac{\pi}{180}\right)$$

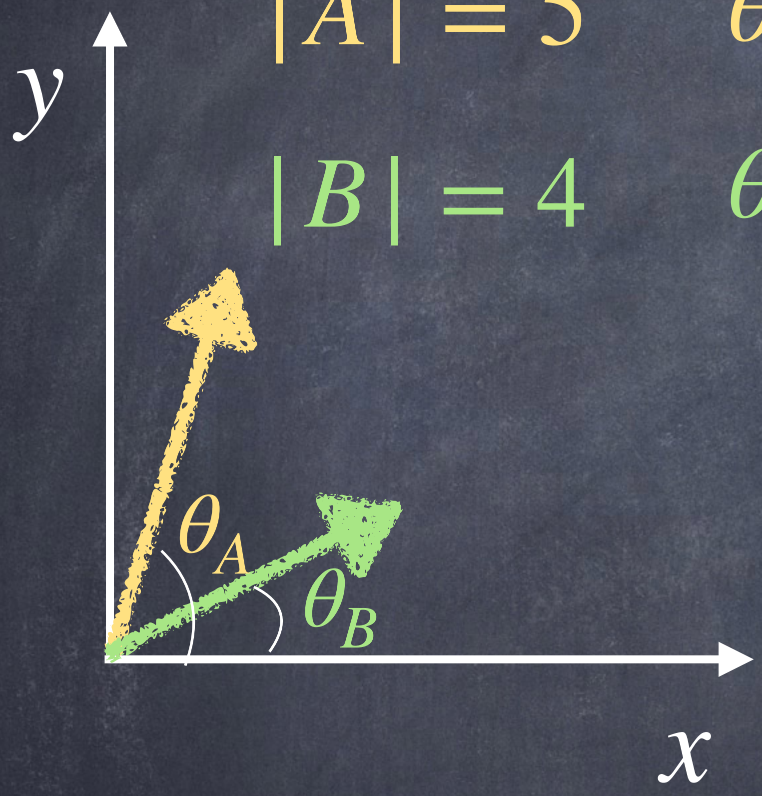
Esercizi

Calcolare: $A \cdot B$

$A \times B$

$|A| = 5 \quad \theta_A = 60^\circ$

$|B| = 4 \quad \theta_B = 20^\circ$



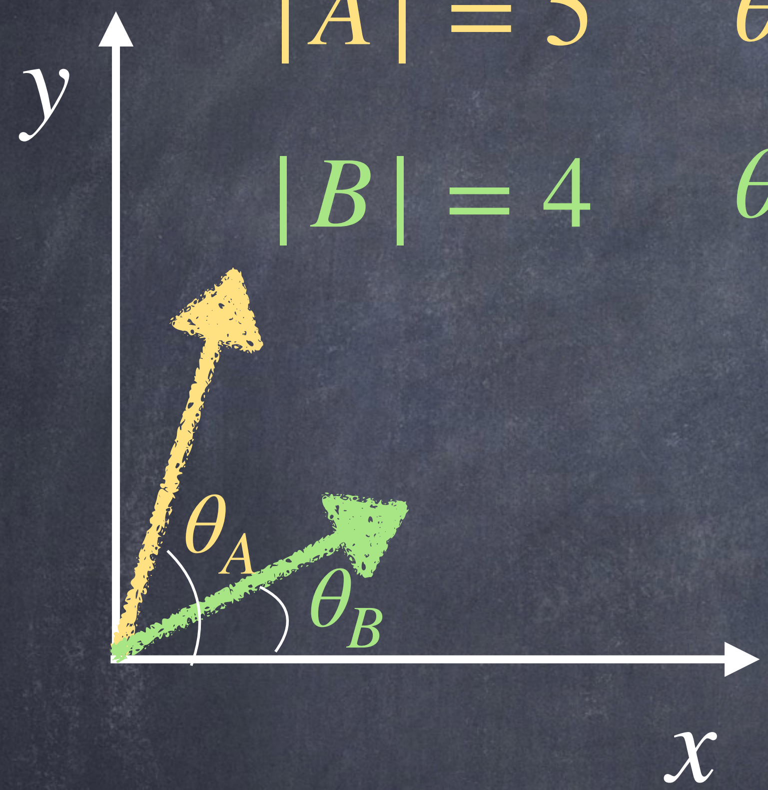
$$\theta_{AB} = \theta_A - \theta_B = 40^\circ$$

$$A \cdot B = |A||B| cos\theta_{AB} \quad = 5 \cdot 4cos\left(40\frac{\pi}{180}\right) \quad = 15.32$$

Esercizi

$|A| = 5 \quad \theta_A = 60^\circ$

$|B| = 4 \quad \theta_B = 20^\circ$



Calcolare: $A \cdot B$

$A \times B$

$$\theta_{AB} = \theta_A - \theta_B = 40^\circ$$

$$A \cdot B = |A||B| cos\theta_{AB} \quad = 5 \cdot 4cos\left(40\frac{\pi}{180}\right) \quad = 15.32$$

$$|A \times B| = |A||B| sin\theta_{AB}$$

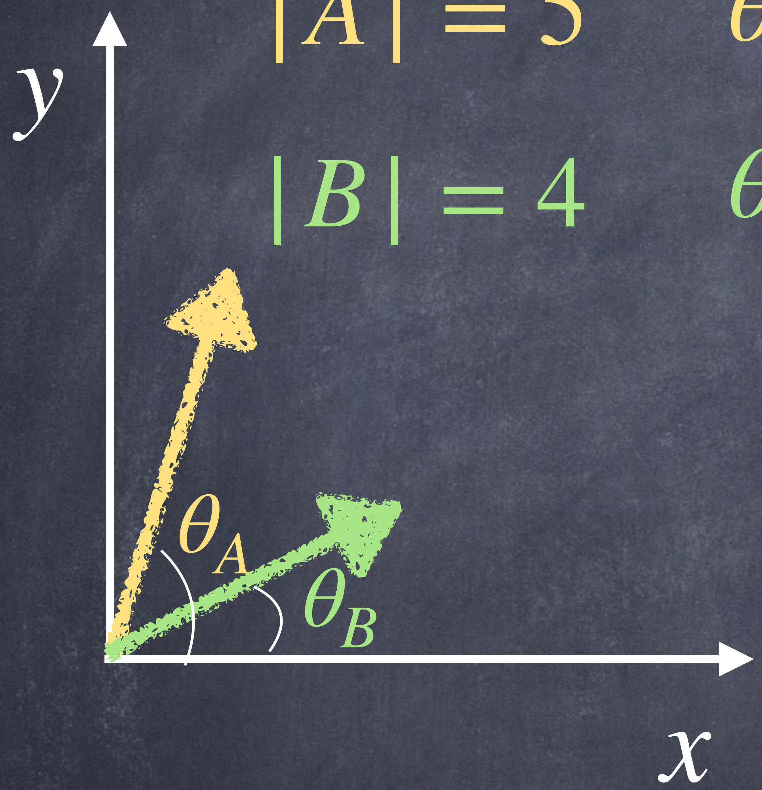
Esercizi

Calcolare: $A \cdot B$

$A \times B$

$|A| = 5 \quad \theta_A = 60^\circ$

$|B| = 4 \quad \theta_B = 20^\circ$



$$\theta_{AB} = \theta_A - \theta_B = 40^\circ$$

$$A \cdot B = |A||B| cos\theta_{AB} \quad = 5 \cdot 4cos\left(40\frac{\pi}{180}\right) \quad = 15.32$$

$$|A \times B| = |A||B| sin\theta_{AB} \quad = 5 \cdot 4sin\left(40\frac{\pi}{180}\right)$$

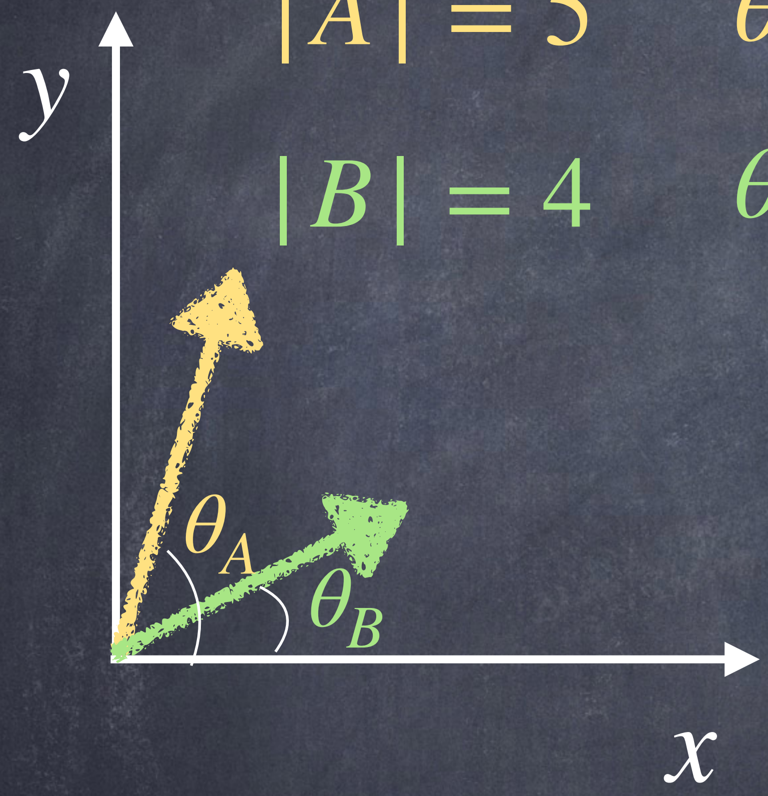
Esercizi

$|A| = 5 \quad \theta_A = 60^\circ$

$|B| = 4 \quad \theta_B = 20^\circ$

Calcolare: $A \cdot B$

$A \times B$



$$\theta_{AB} = \theta_A - \theta_B = 40^\circ$$

$$A \cdot B = |A||B| cos\theta_{AB} \quad = 5 \cdot 4cos\left(40\frac{\pi}{180}\right) \quad = 15.32$$

$$|A \times B| = |A||B| sin\theta_{AB} \quad = 5 \cdot 4sin\left(40\frac{\pi}{180}\right) \quad = 12.85$$

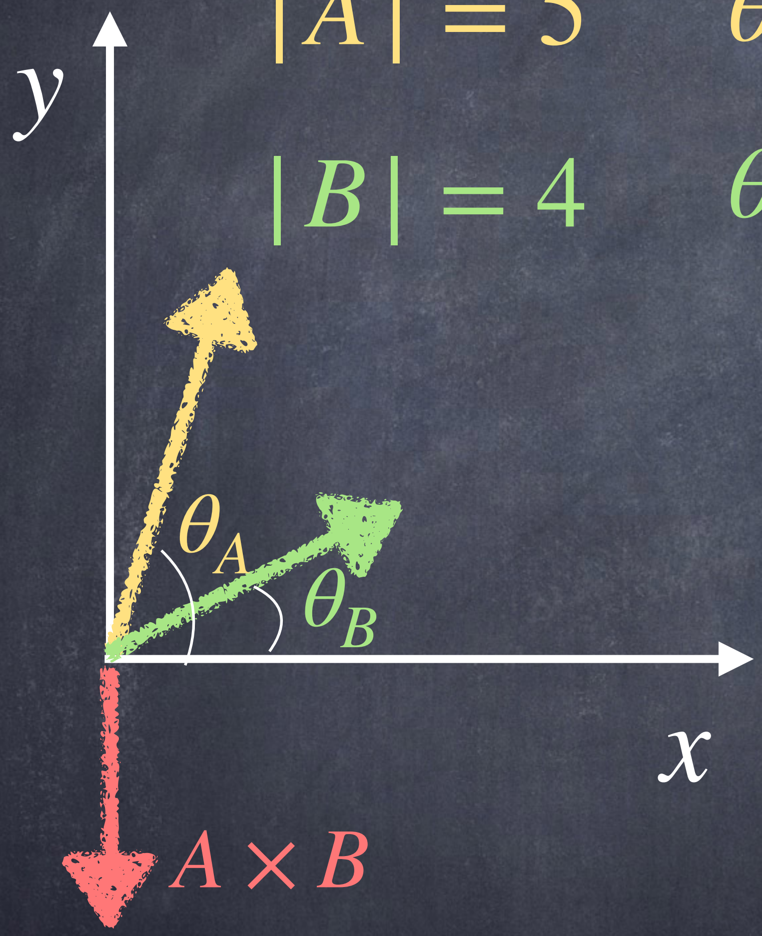
Esercizi

Calcolare: $A \cdot B$

$A \times B$

$|A| = 5 \quad \theta_A = 60^\circ$

$|B| = 4 \quad \theta_B = 20^\circ$

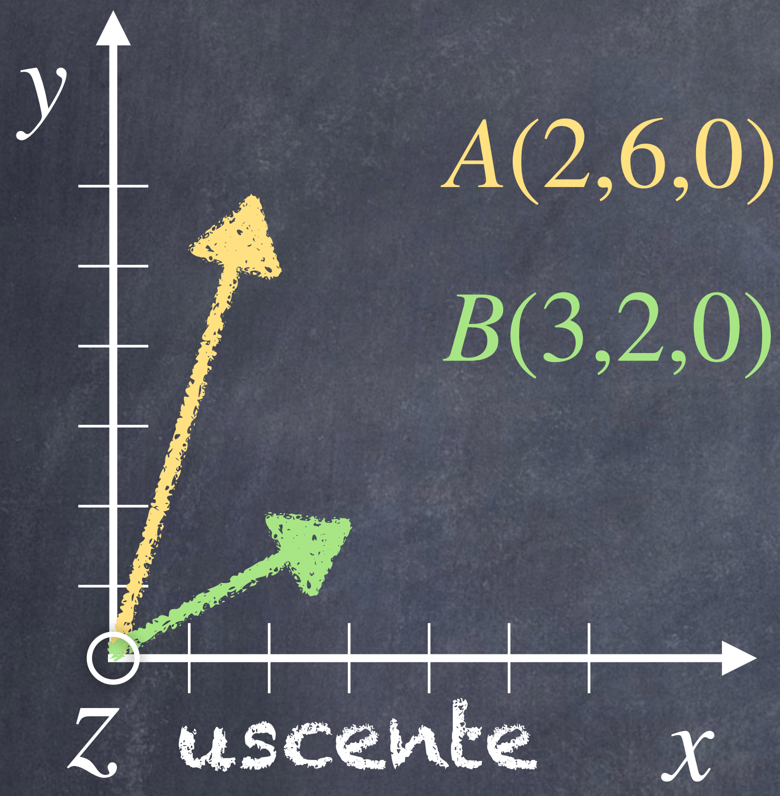


$$\theta_{AB} = \theta_A - \theta_B = 40^\circ$$

$$A \cdot B = |A||B| cos\theta_{AB} \quad = 5 \cdot 4cos\left(40\frac{\pi}{180}\right) \quad = 15.32$$

$$|A \times B| = |A||B| sin\theta_{AB} \quad = 5 \cdot 4sin\left(40\frac{\pi}{180}\right) \quad = 12.85$$

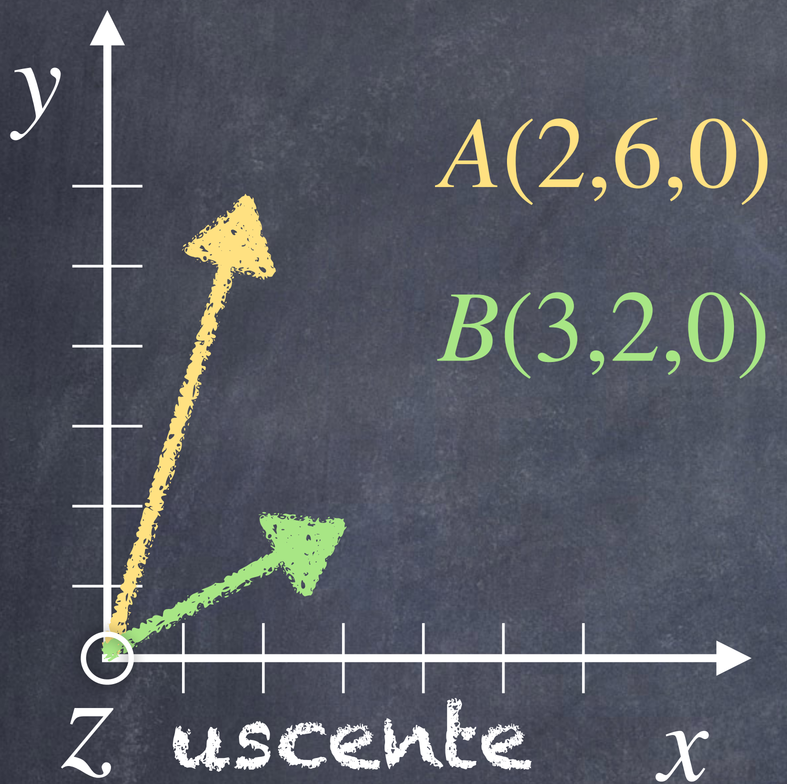
Esercizi



Calcolare: $A \cdot B$

$A \times B$

Esercizi

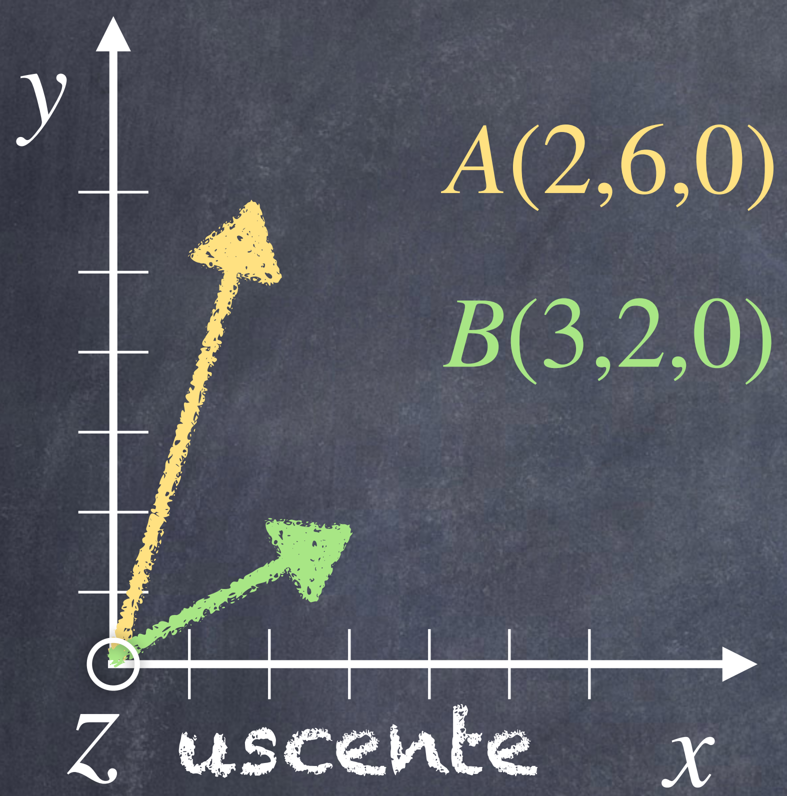


Calcolare: $A \cdot B$

$A \times B$

$$A \cdot B = (A_x B_x + A_y B_y + A_z B_z)$$

Esercizi

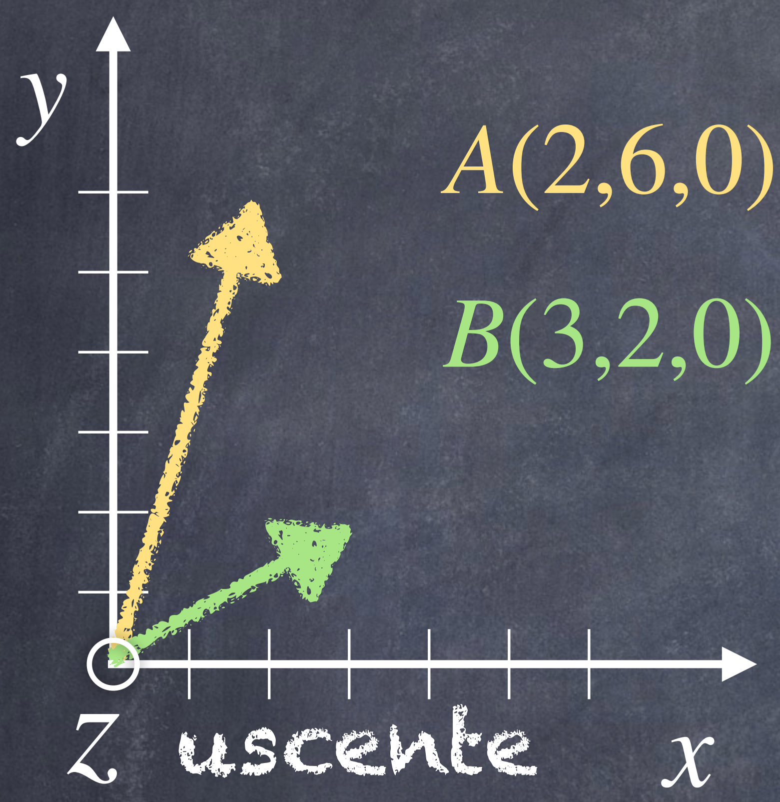


Calcolare: $A \cdot B$

$A \times B$

$$A \cdot B = (A_x B_x + A_y B_y + A_z B_z)$$
$$= 2 \cdot 3 + 6 \cdot 2 + 0 \cdot 0$$

Esercizi

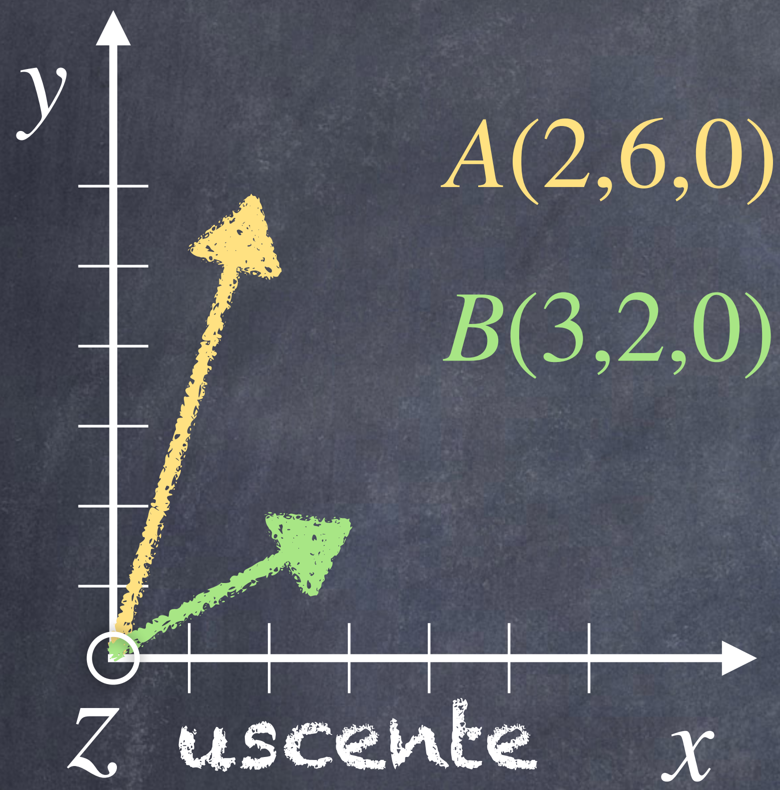


Calcolare: $A \cdot B$

$A \times B$

$$A \cdot B = (A_x B_x + A_y B_y + A_z B_z)$$
$$= 2 \cdot 3 + 6 \cdot 2 + 0 \cdot 0 \qquad = 18$$

Esercizi



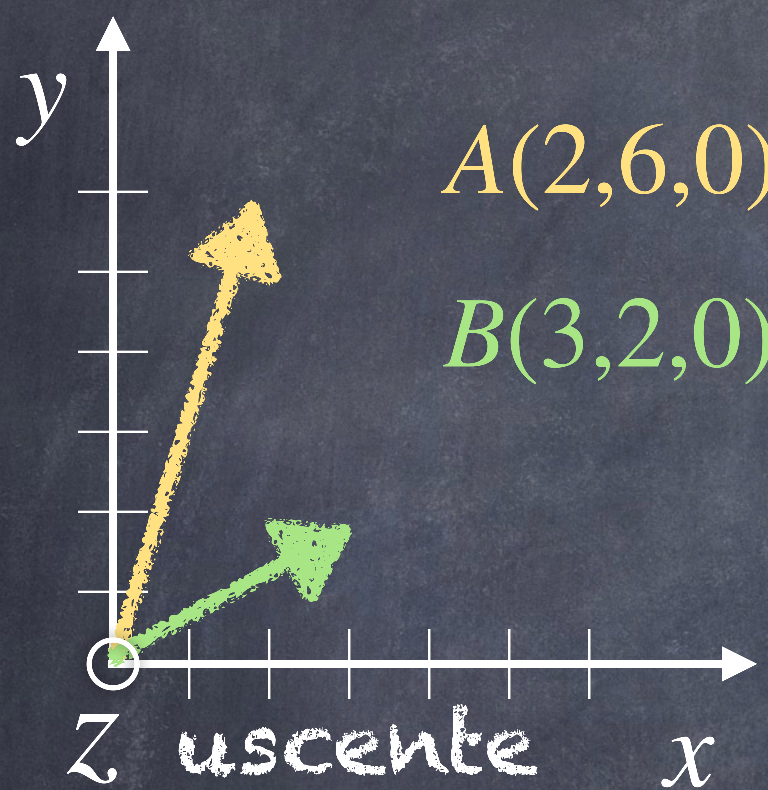
Calcolare: $A \cdot B$

$A \times B$

$$A \cdot B = (A_x B_x + A_y B_y + A_z B_z)$$
$$= 2 \cdot 3 + 6 \cdot 2 + 0 \cdot 0 \qquad = 18$$

$$A \times B = (A_y B_z - B_y A_z)\ \hat{i} + (A_z B_x - B_z A_x)\ \hat{j} + (A_x B_y - B_x A_y)\ \hat{k}$$

Esercizi



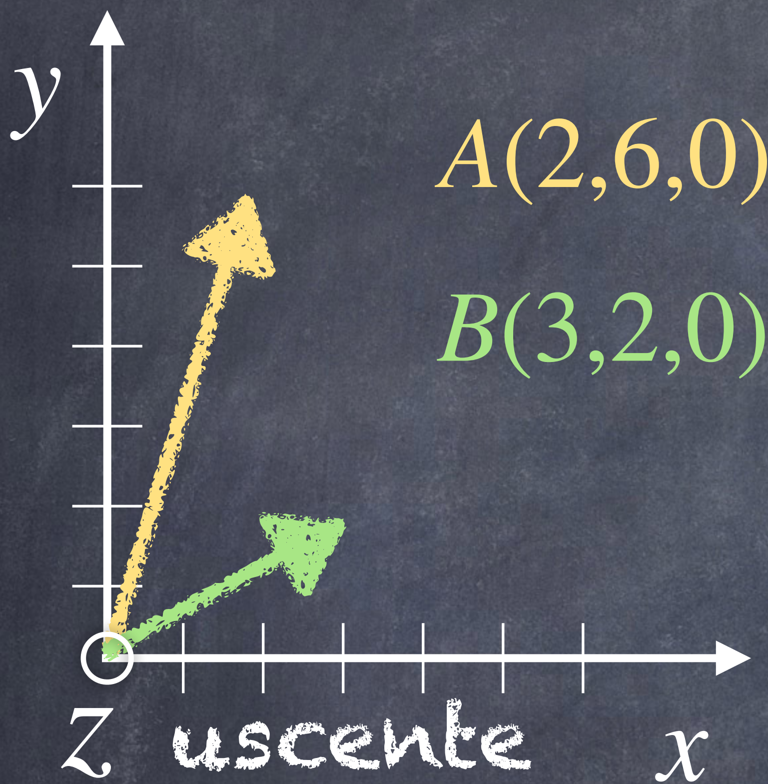
Calcolare: $A \cdot B$

$A \times B$

$$A \cdot B = (A_x B_x + A_y B_y + A_z B_z)$$
$$= 2 \cdot 3 + 6 \cdot 2 + 0 \cdot 0 \qquad = 18$$

$$A \times B = (A_y B_z - B_y A_z)\ \hat{i} + (A_z B_x - B_z A_x)\ \hat{j} + (A_x B_y - B_x A_y)\ \hat{k}$$
$$= (6 \cdot 0 - 2 \cdot 0)\ \hat{i} + (0 \cdot 3 - 0 \cdot 2)\ \hat{j} + (2 \cdot 2 - 3 \cdot 6)\ \hat{k}$$

Esercizi



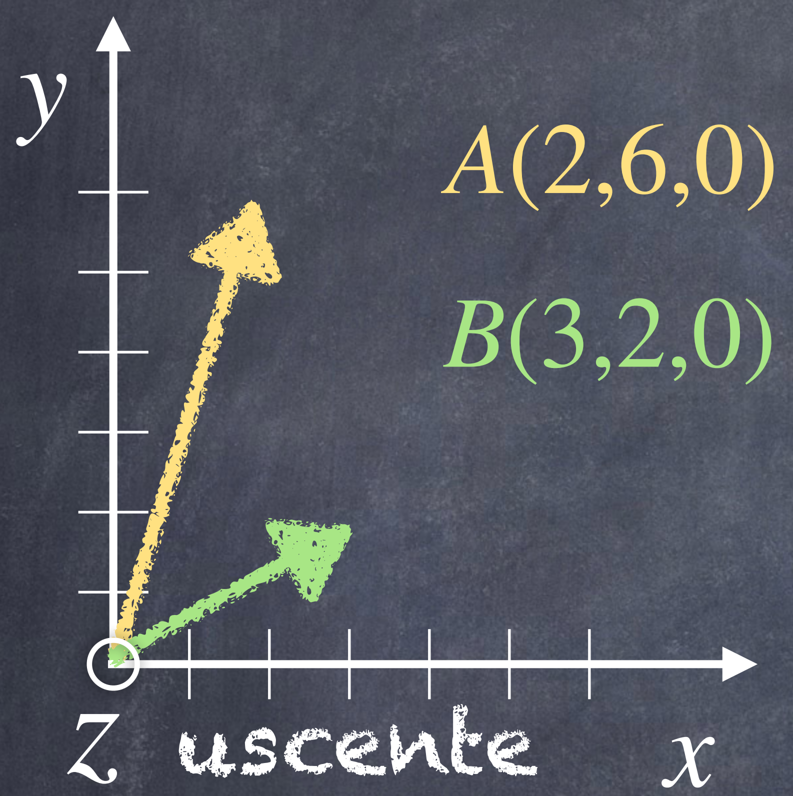
Calcolare: $A \cdot B$

$A \times B$

$$A \cdot B = (A_x B_x + A_y B_y + A_z B_z)$$
$$= 2 \cdot 3 + 6 \cdot 2 + 0 \cdot 0 \qquad = 18$$

$$A \times B = (A_y B_z - B_y A_z)\ \hat{i} + (A_z B_x - B_z A_x)\ \hat{j} + (A_x B_y - B_x A_y)\ \hat{k}$$
$$= (6 \cdot 0 - 2 \cdot 0)\ \hat{i} + (0 \cdot 3 - 0 \cdot 2)\ \hat{j} + (2 \cdot 2 - 3 \cdot 6)\ \hat{k}$$
$$= 0\ \hat{i} + 0\ \hat{j} - 14\ \hat{k}$$

Esercizi



Calcolare: $A \cdot B$

$A \times B$

$$A \cdot B = (A_x B_x + A_y B_y + A_z B_z)$$
$$= 2 \cdot 3 + 6 \cdot 2 + 0 \cdot 0 \qquad = 18$$

$$A \times B = (A_y B_z - B_y A_z)\,\hat{i} + (A_z B_x - B_z A_x)\,\hat{j} + (A_x B_y - B_x A_y)\,\hat{k}$$
$$= (6 \cdot 0 - 2 \cdot 0)\,\hat{i} + (0 \cdot 3 - 0 \cdot 2)\,\hat{j} + (2 \cdot 2 - 3 \cdot 6)\,\hat{k}$$
$$= 0\,\hat{i} + 0\,\hat{j} - 14\,\hat{k} \qquad = -\,14\,\hat{k}$$