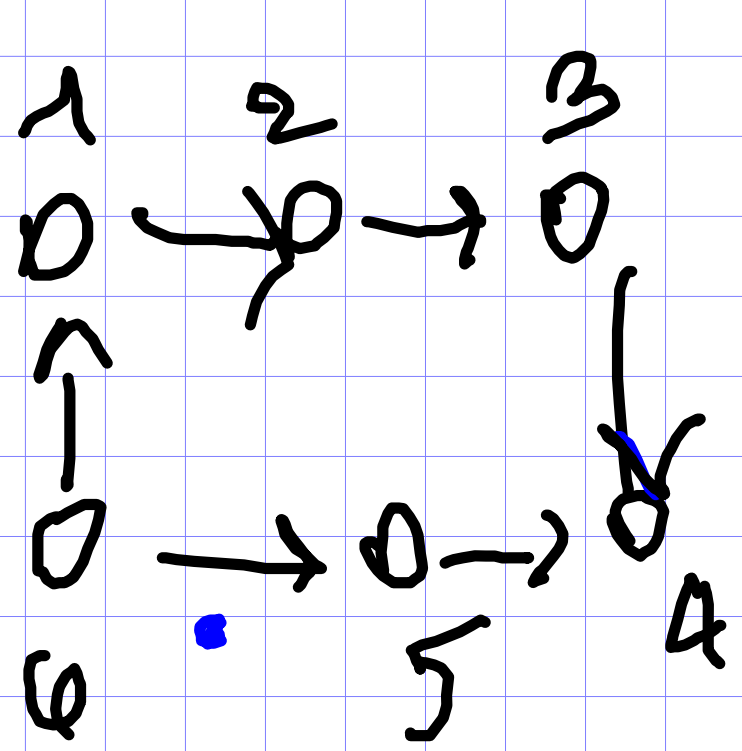
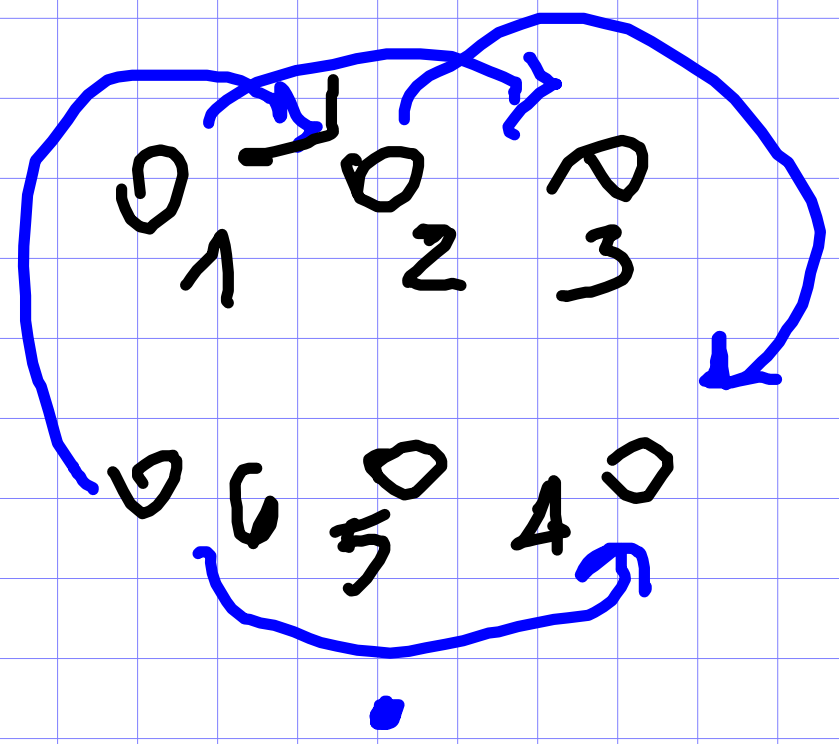
Calcolare il quadrato di $G=(V,E)$

cioè

$$G^2=(V,E^2)$$ dove

$$e \in E^2 \iff e = e_1 \cdot e_2 \text{ con } e_1 \in E,\ e_2 \in E$$



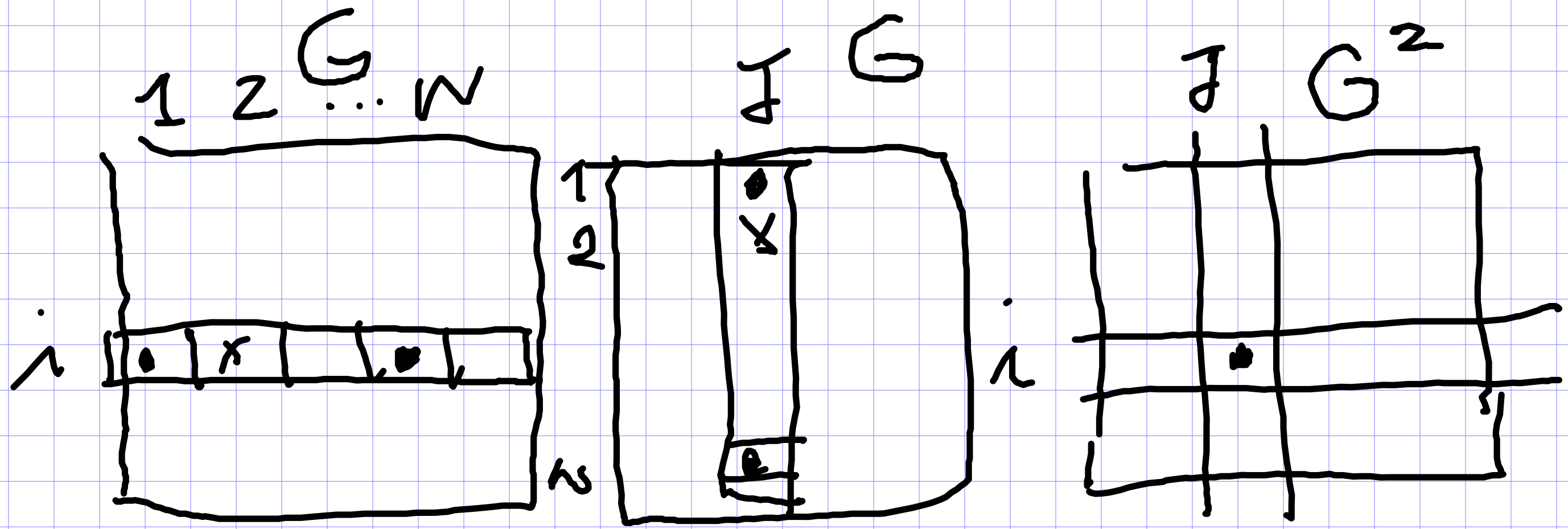


$(i,j) \in E^2$

$\Updownarrow$

$(i,w) \in E$

$(w,j) \in E$



$$(i,N) \in E \iff (i,J) \in G^2$$

$$(N,J) \in E$$

$$G^2 D(i,J) = \bigvee_{w=1}^{N} (i,w) \text{ AND } (w,J)$$

$$\exists w \in V :$$

Il calcolo di G^2 richiama il calcolo del prodotto di due matrici

$m_1 \times m_2$ $P = A \cdot B$

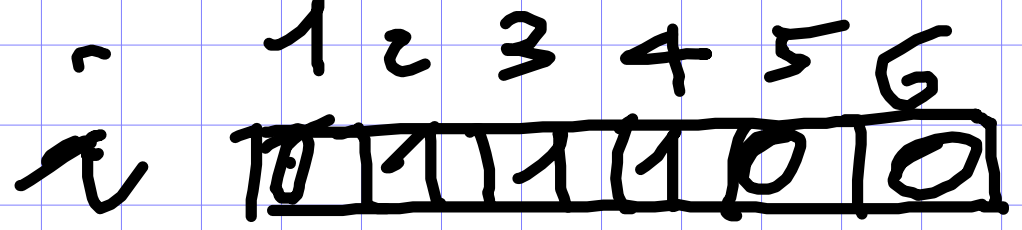
$A = m_1 \times \underline{n}$

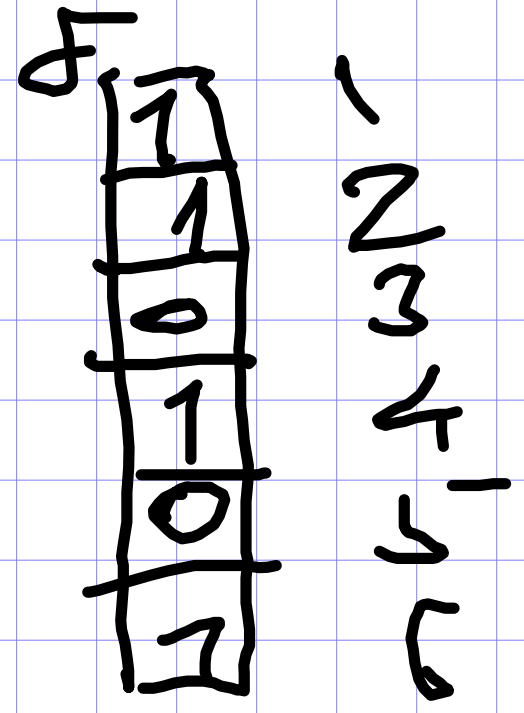
$B = \underline{n} \times m_2$

$$p_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$$

$\wedge$ AND

$\vee$ OR





$(i1) \cdot (1j) = \phi \cdot (1) = \phi$ V

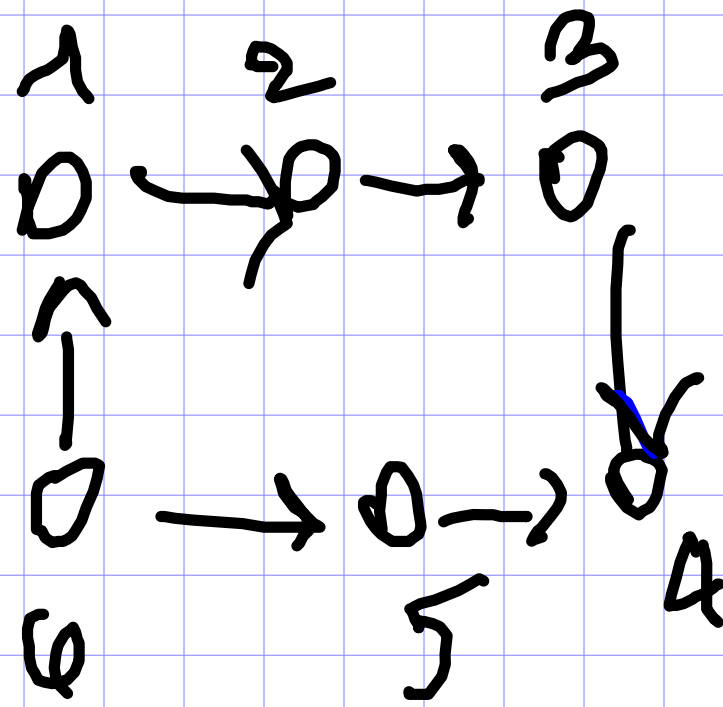
$(i2) \cdot (2j) = 1 \cdot 1 = 1$ V

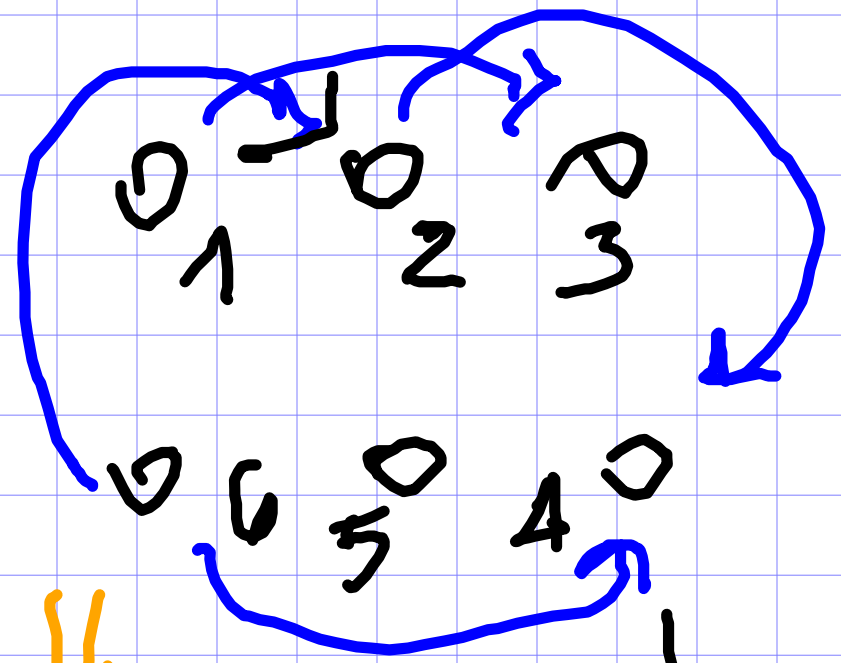
$(i3) \cdot (30) = 1 \cdot 0 = \phi$ V

$(i4) \cdot (4j) = 1 \cdot 1 = 1$

$(i5) \cdot (5j) = 0 \cdot 0 = 0$

$(i6) \cdot (6j) = 0 \cdot 1 = 0$



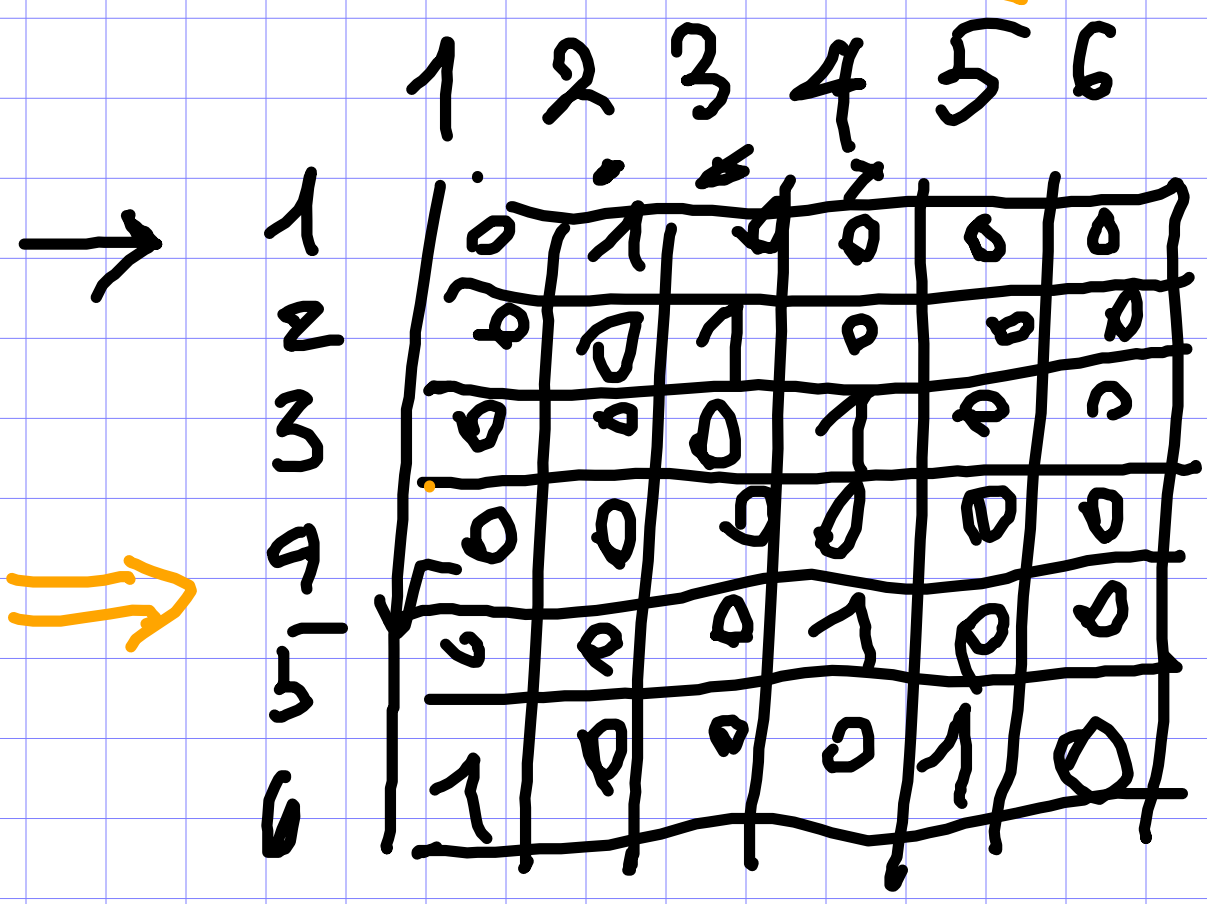


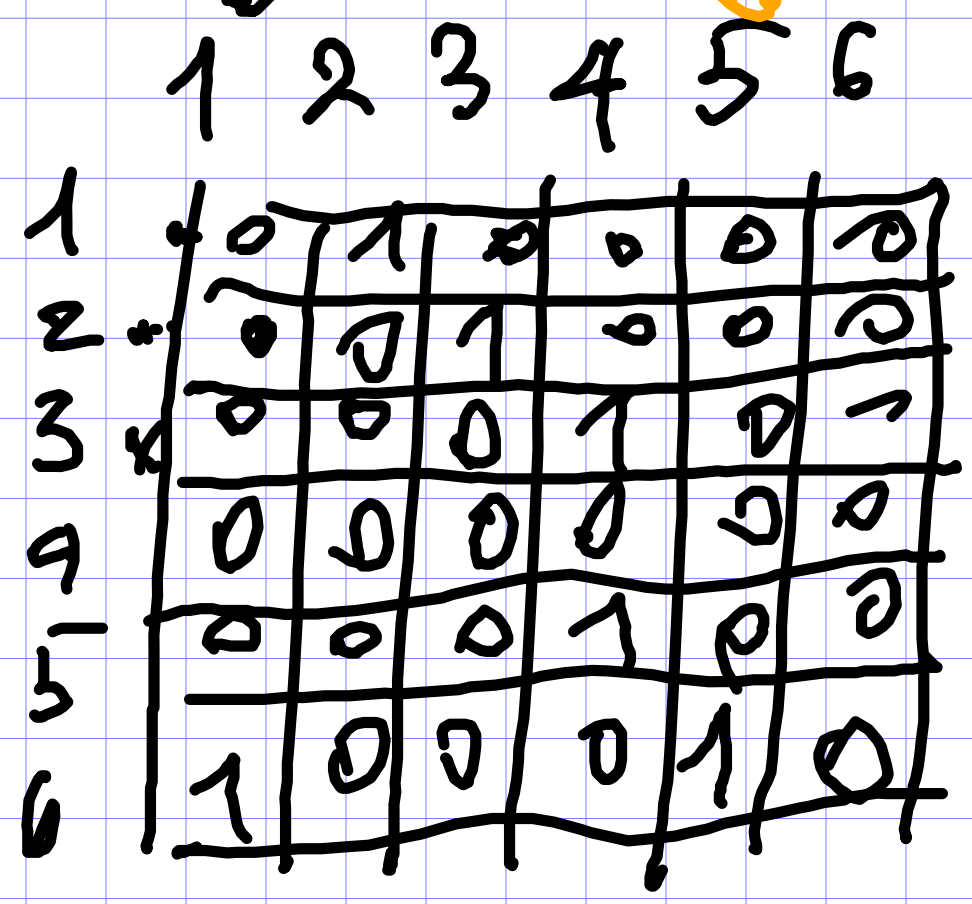
$(1\,2)(2\,3) = 1\,3$

n

$(1,1)\,(1,3)\,(1,3)$

$j = 1$





Square (G, G^2)

for $r \leftarrow 1$ to n do

for $c \leftarrow 1$ to n do

{ $G^2[r, c] \leftarrow \emptyset$

$\Rightarrow$ for $k \leftarrow 1$ to n do

if $G(r, k)$ AND $G(k, c) == 1$ then

$G^2[r, c] \leftarrow G^2[r, c] \vee 1$

}

· while

$G = n \times \underline{n}$

$G = \underline{n} \times n$

Confronto:

$O(n^3)$

$n = |V|$

G^2 $(i,j) \in E^2 \iff$ i ed j $\exists$ cammino di lunghezza ≤ 2 in G da i a j

Dato $G = (V, E)$ $i \to w \to j$

$G^* = (V, E^*)$ $(i,j) \in E^* \iff$ i ed j $\exists$ un cammino di lunghezza 1 o un cammino di lunghezza 2 in G

$E^* = E^2 \cup E$

Calcolo G^x

$$G^* = G \cup G^2$$

$$G^*[i,j] = 1 \iff \left.\begin{array}{l} G[i,j] = 1 \\ \text{OR} \quad G^2[i,j] = 1 \end{array}\right\} \in E \cup E^2$$

Complessità $O(|V|^3)$

$G^* = G \cup G^2$:

$G^* = (V, E^*)$ ove $E^* = E \cup E^2$

Scrivere algo per calcolare G^1

```
{ G^2 ← SQUARE(G)
  for r ← 1 to n ob
      for c ← 1 to n ob
          G*[r,c] = G[r,c] V G^2[r,c]
}
```

Ster (G, G^*)

$\bar{G} = \underline{n} \times \bar{n}$

for $r \leftarrow 1$ to n do

for $c \leftarrow 1$ to n do

{ $G^*[r, c] \leftarrow G[r, c]$

$\Rightarrow$ for $k \leftarrow 1$ to n do

if $G(r, k)$ .AND. $G[k, c] == 1$ then

$G^*[r, c] \leftarrow G^*[r, c] \vee 1$

}

Conferma: è $O(n^3)$

$n = |V|$

Proporre piccole modifiche di SQUARE per ottenere G^3

Calcolare $G^3 = (V, E^3)$

$(i,j) \in E^3 \iff$ $\exists$ cammino da i a j di lunghezza 3

$G^3 = G \cdot G \cdot G$

$= (G \cdot G) \cdot G$

$= G(G \cdot G)$