Corso di Laurea in Informatica
2021/2022
FISICA GENERALE
Cinematica del Moto Piano

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Moto nel Piano

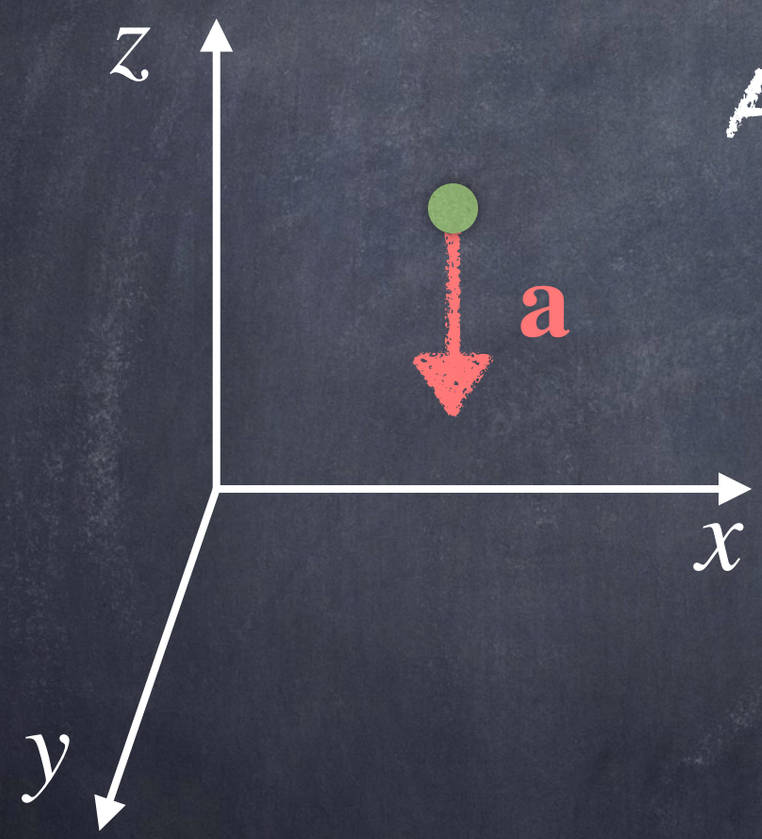
Traiettoria: insieme di posizioni che il corpo assume al trascorrere del tempo

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$

$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

Caduta dei Gravi

Scorsa Lezione



Accelerazione di Gravita'

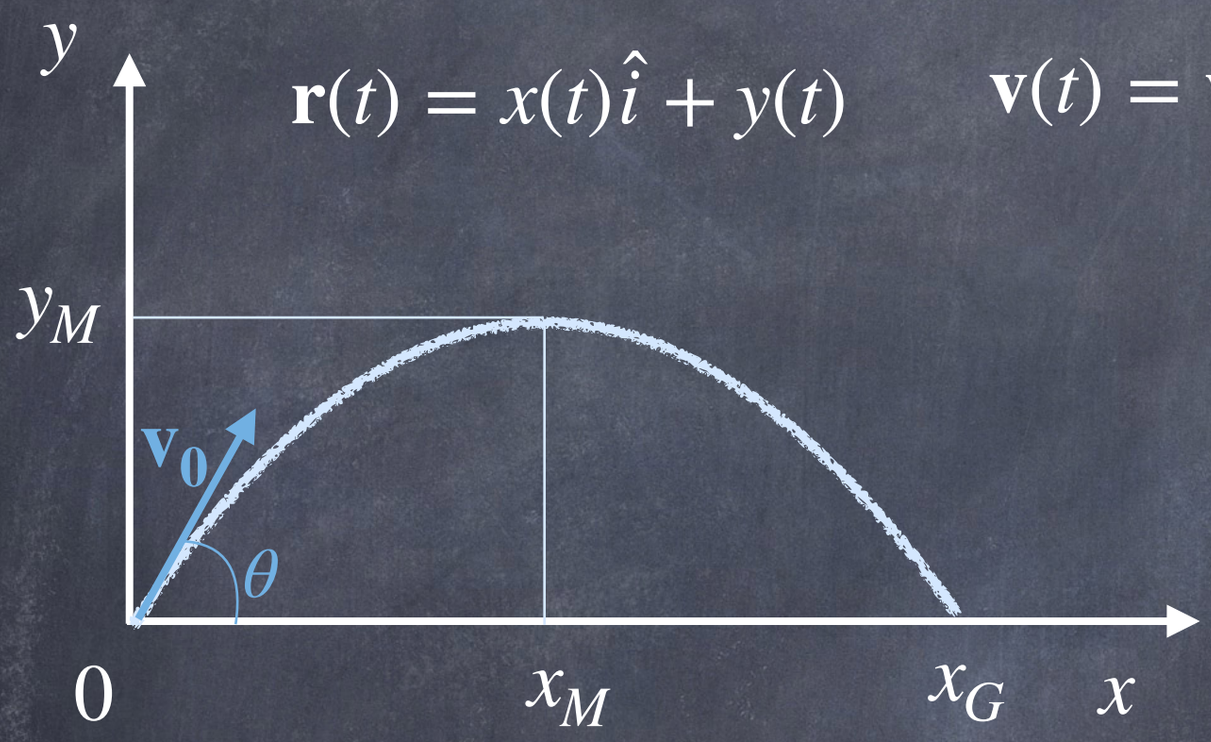
$$\mathbf{a} = -g\hat{k}$$

$$g = 9.81\ \frac{m}{s^2}$$

$$z(t) = z_0 - \frac{1}{2}gt^2$$

Moto dei Proiettili

$$\mathbf{r}(t) = x(t)\hat{i} + y(t) \qquad \mathbf{v}(t) = \mathbf{v_0} + \mathbf{a}\, t$$



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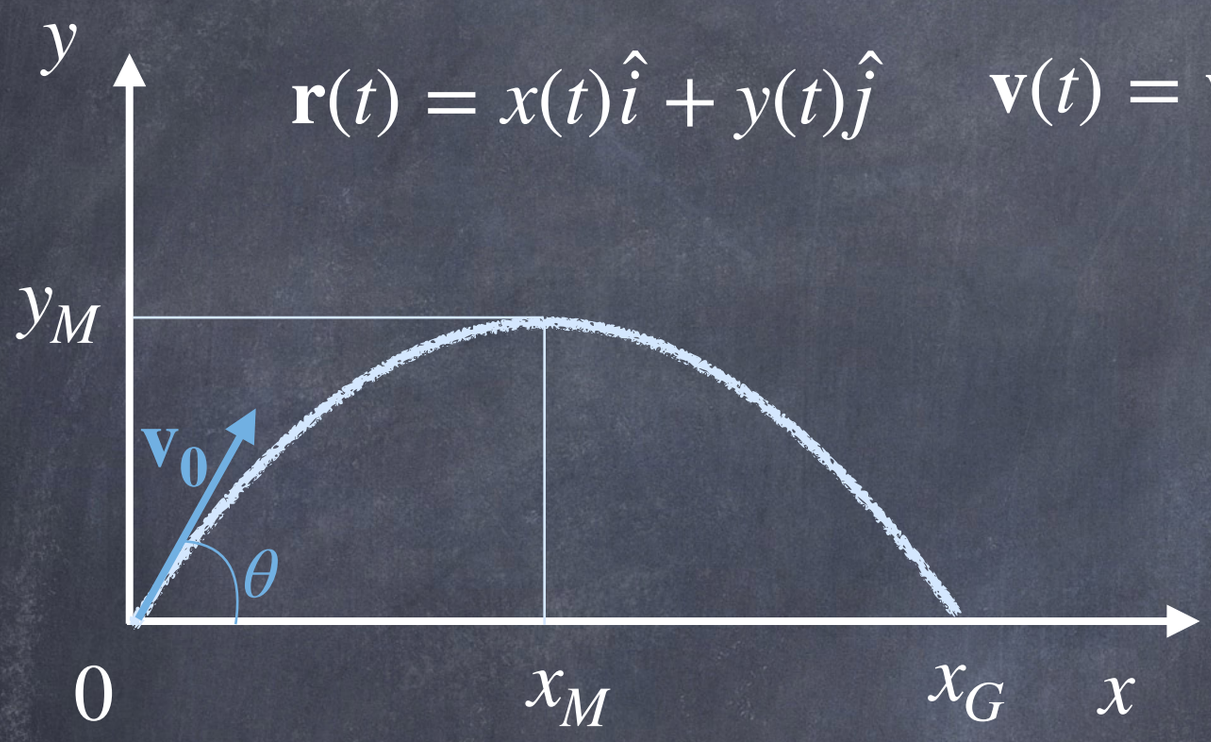
$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j} \qquad \mathbf{v}(t) = \mathbf{v_0} + \mathbf{a}\,t \qquad v_0 = |\mathbf{v_0}|$$



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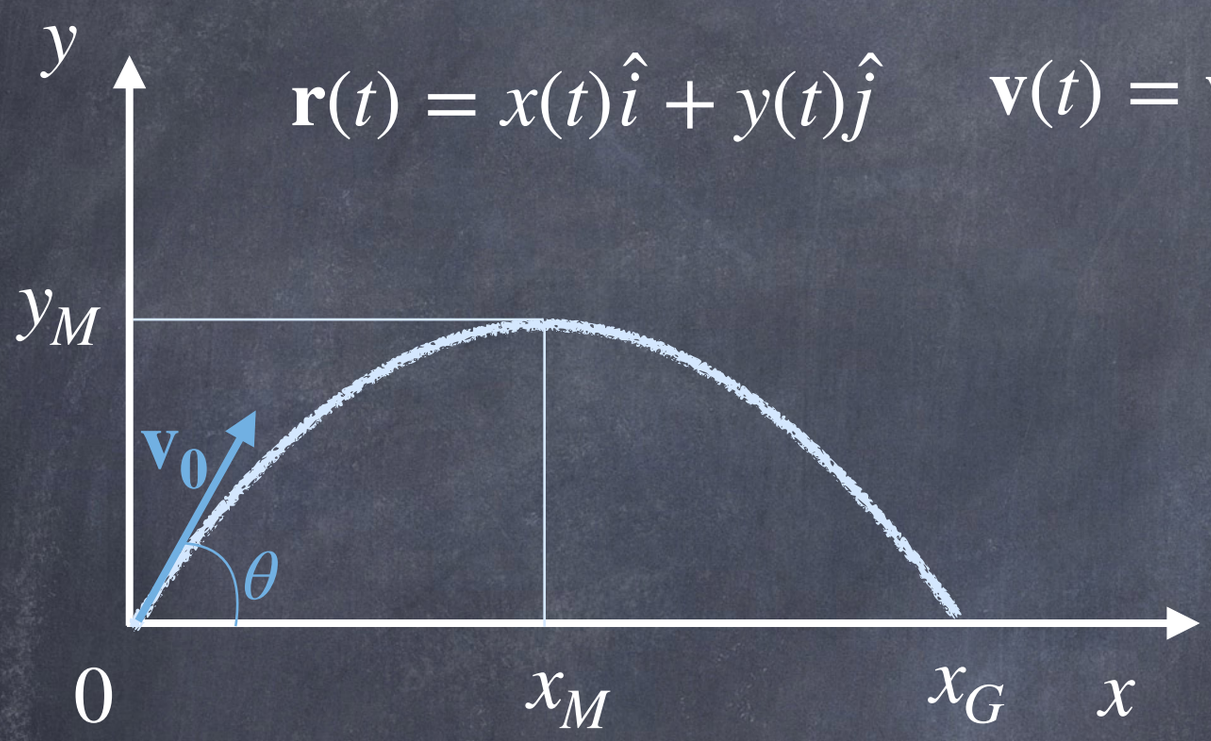

$$\begin{cases} v_{0_x} = v_0 \cos\theta \\ v_{0_y} = v_0 \sin\theta \end{cases}$$

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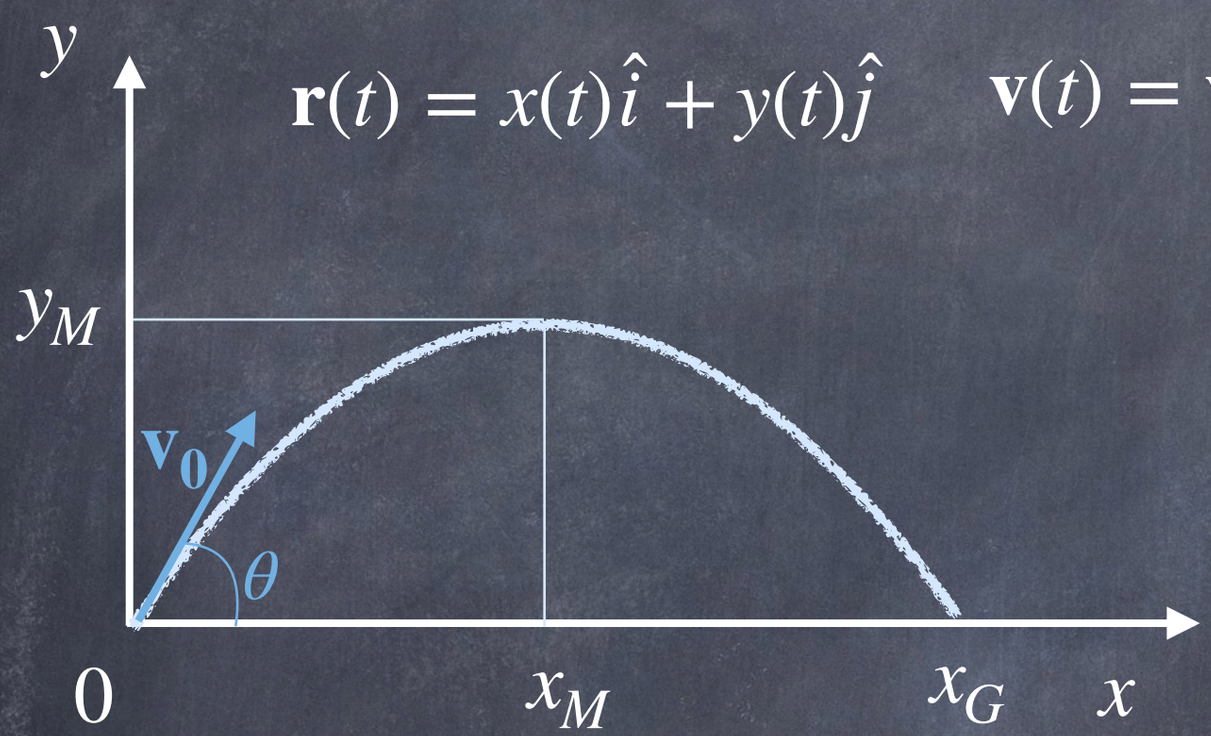

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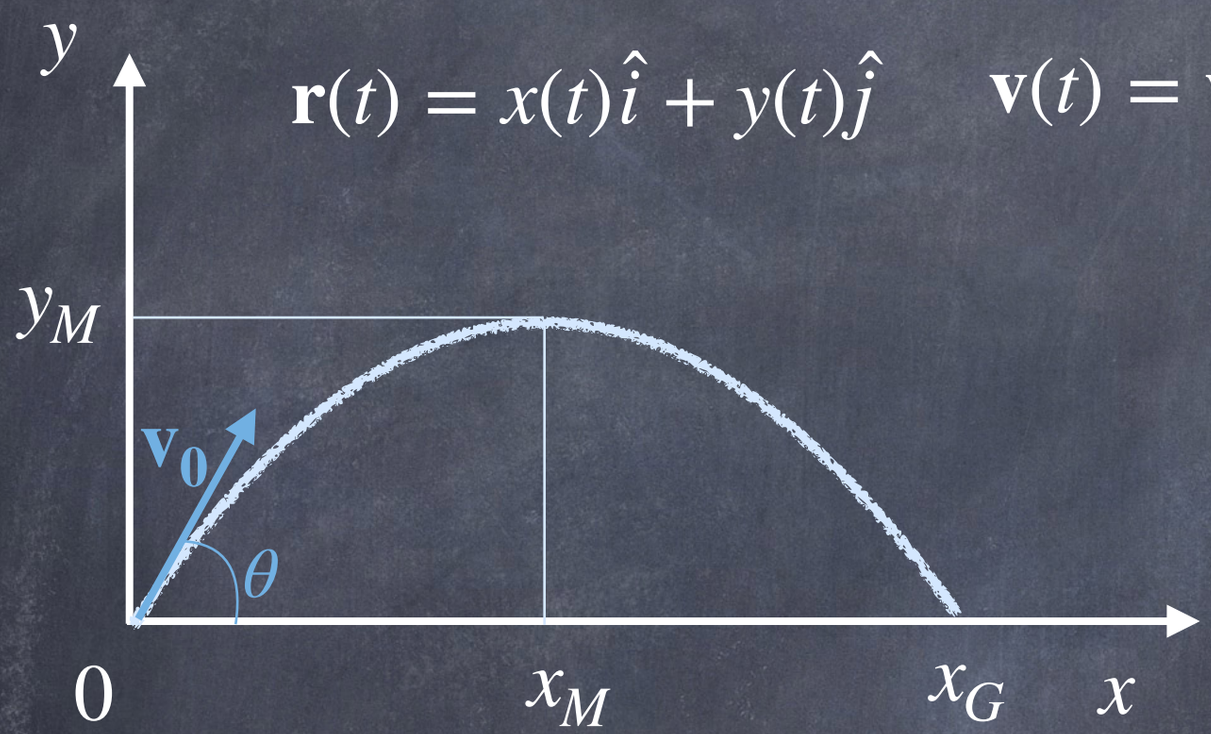

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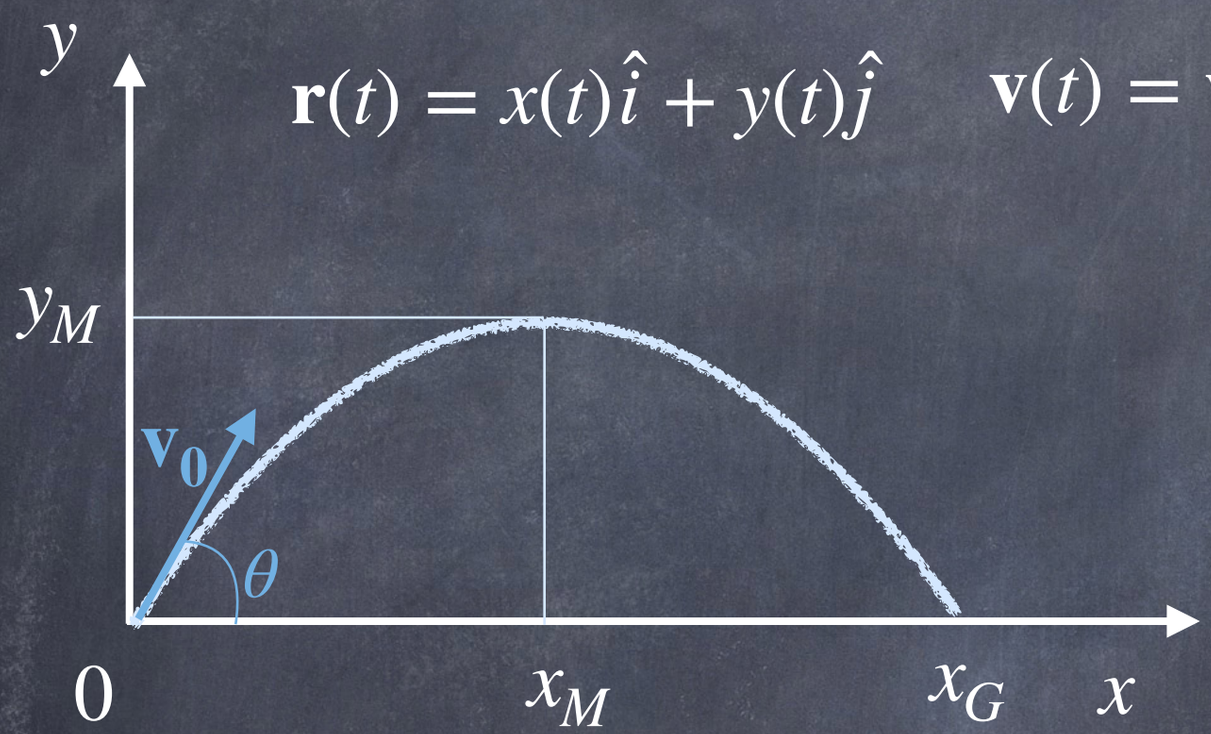

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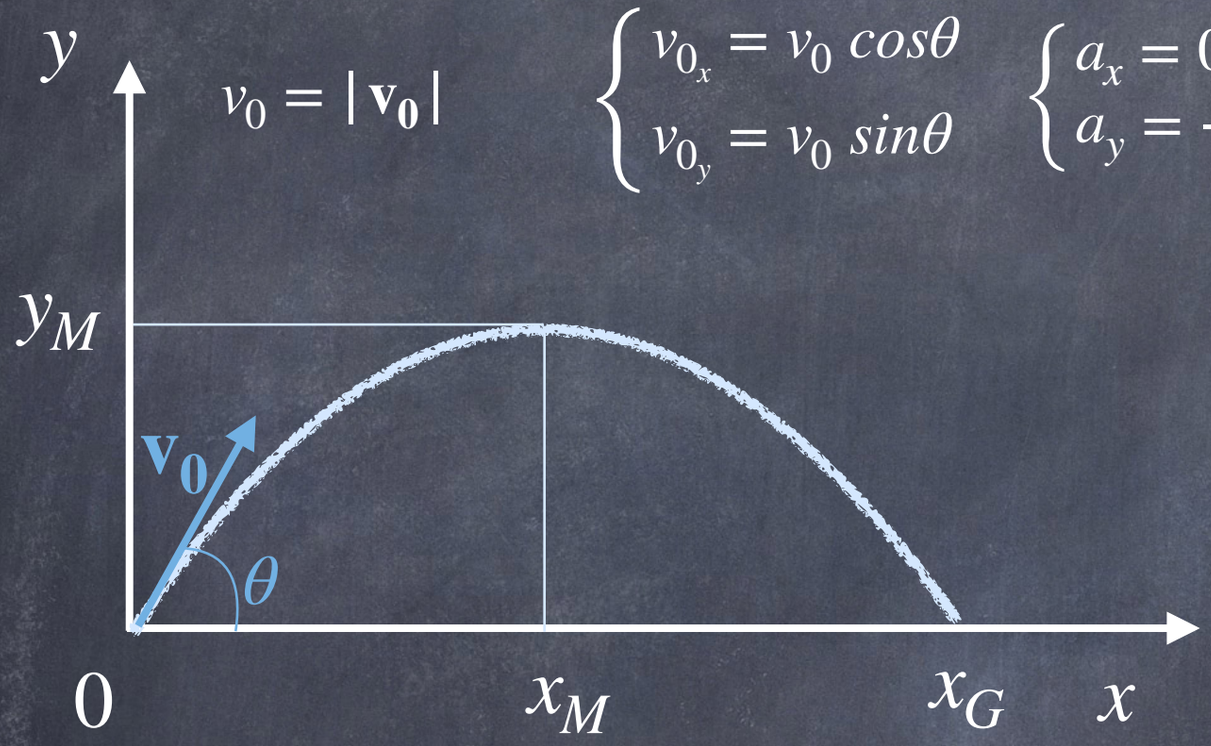

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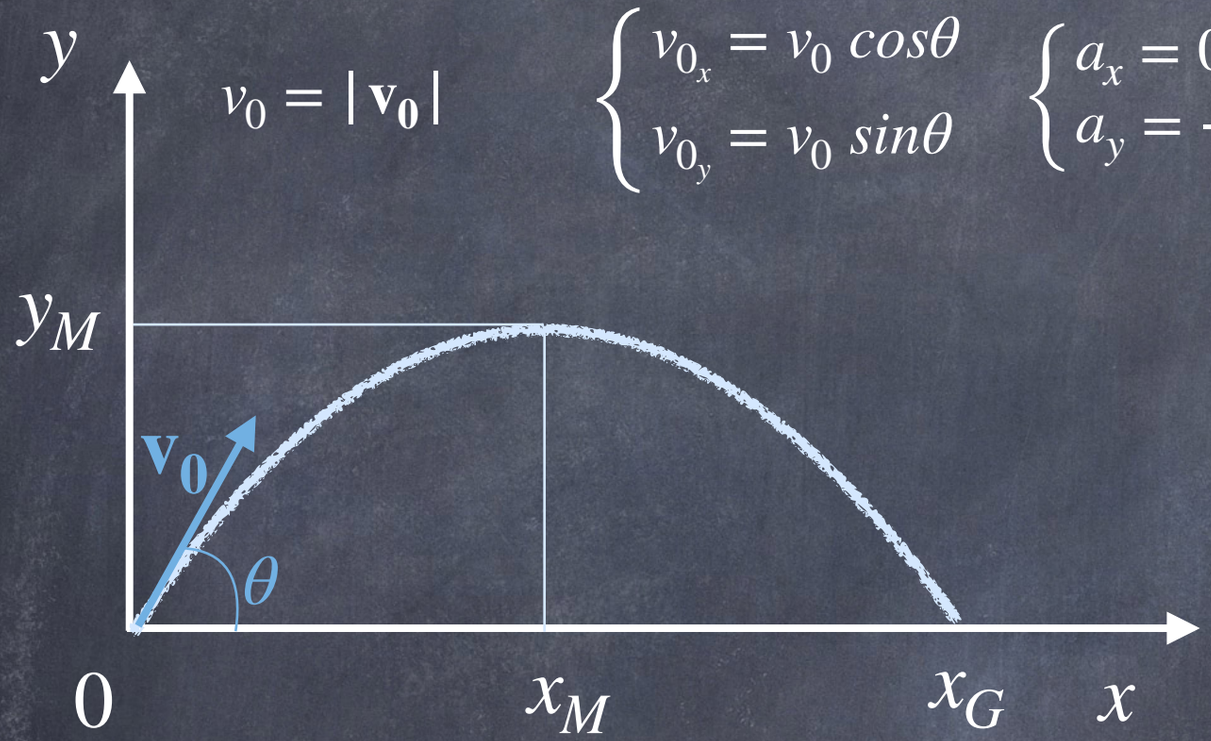

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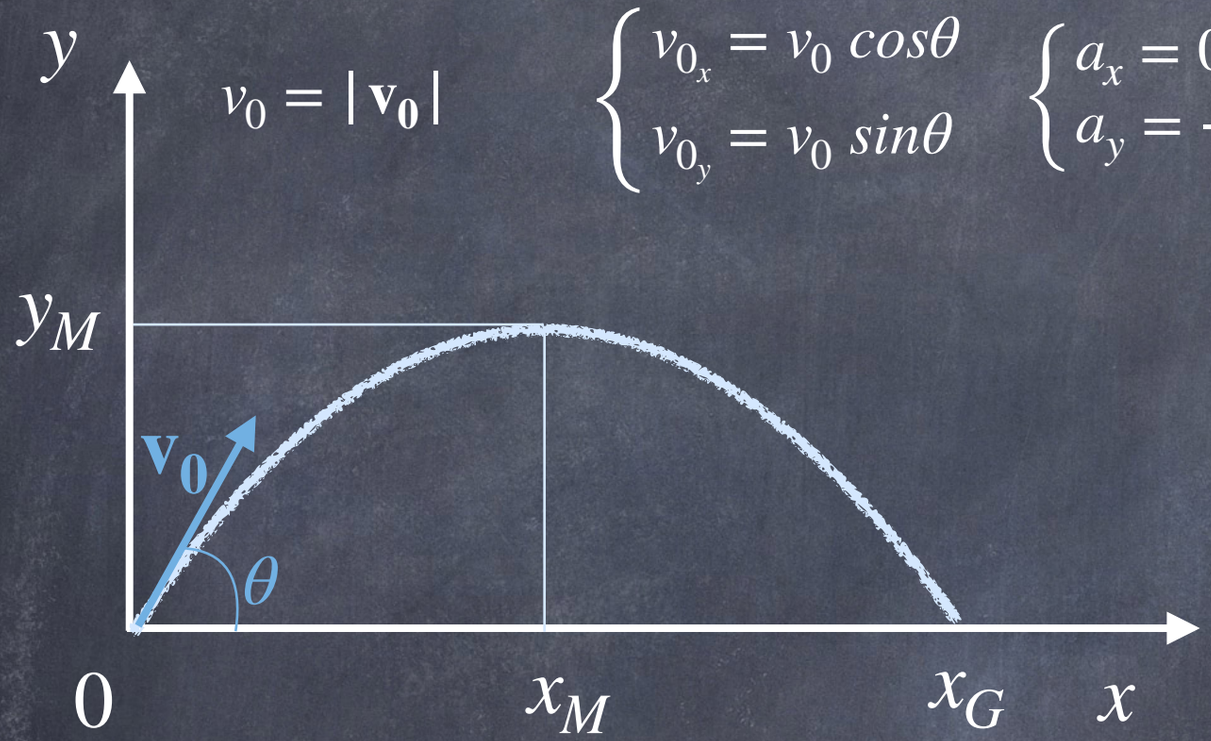

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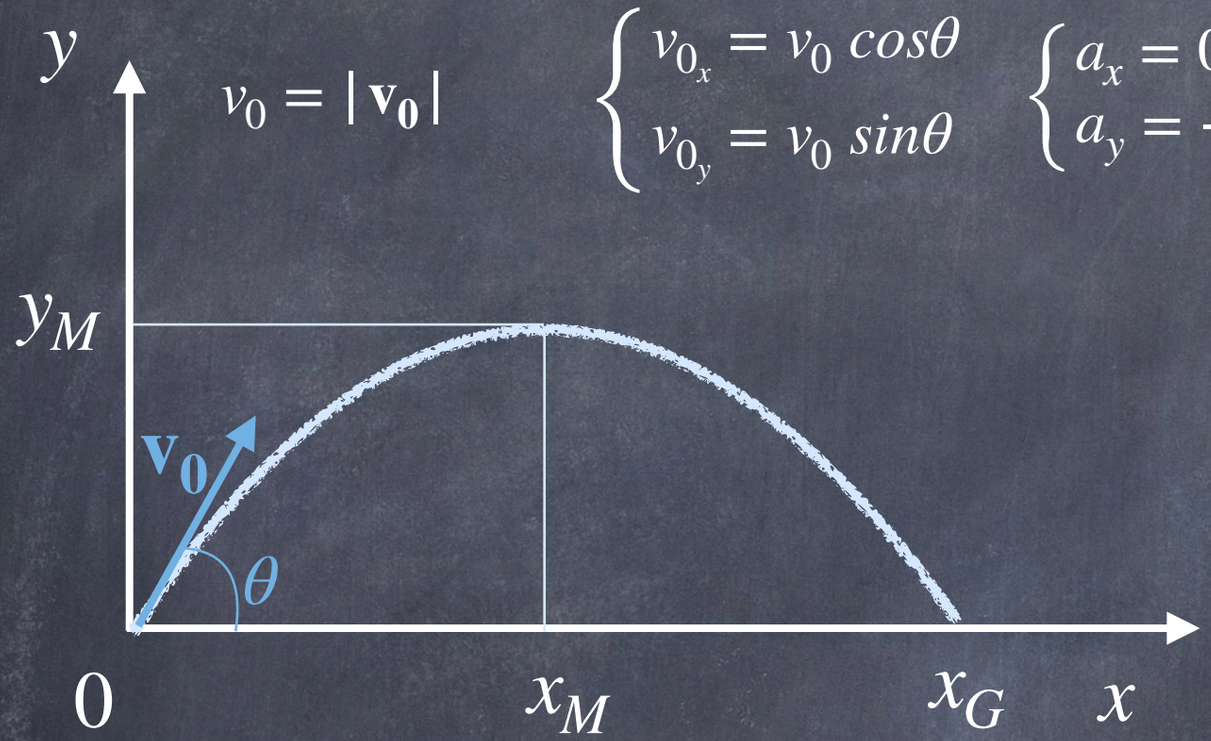

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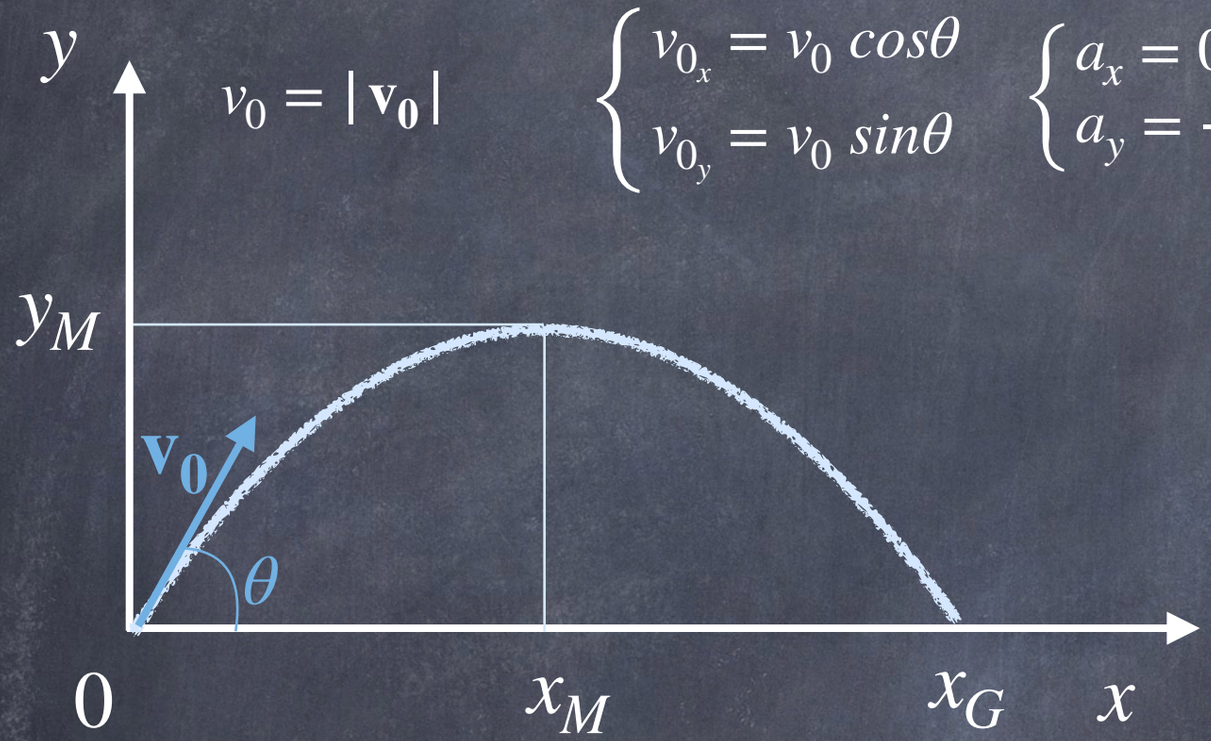

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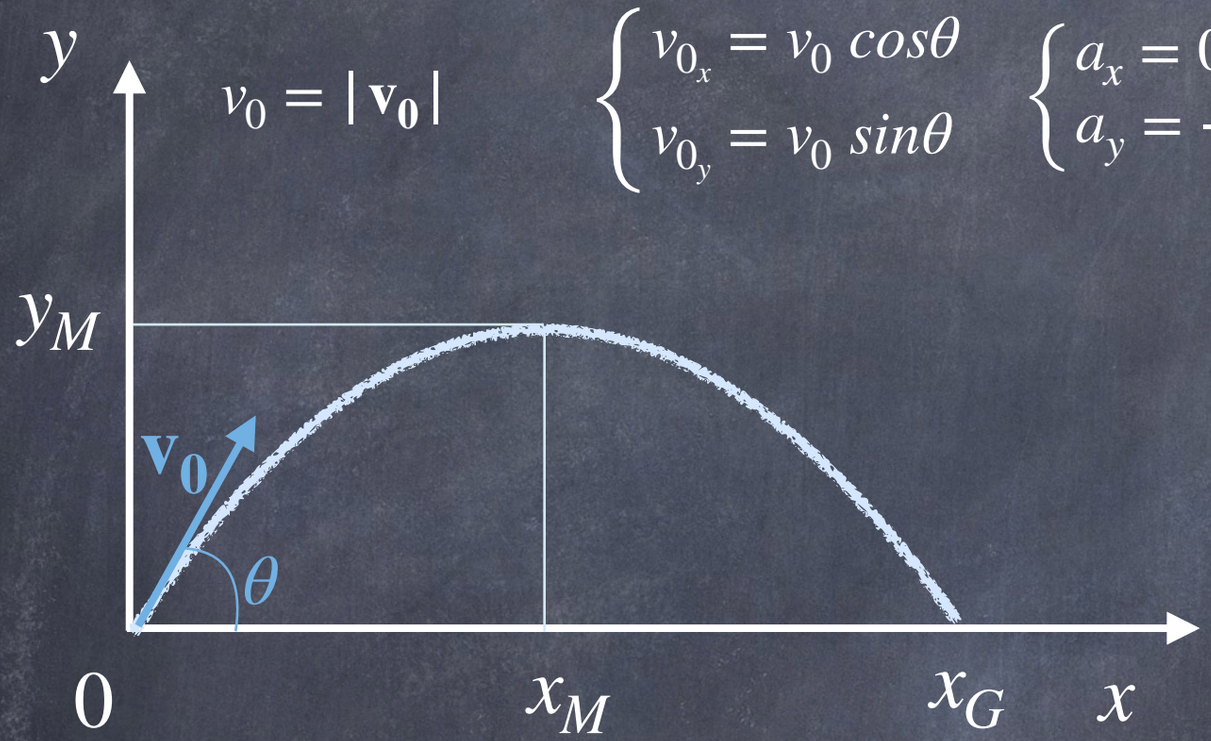

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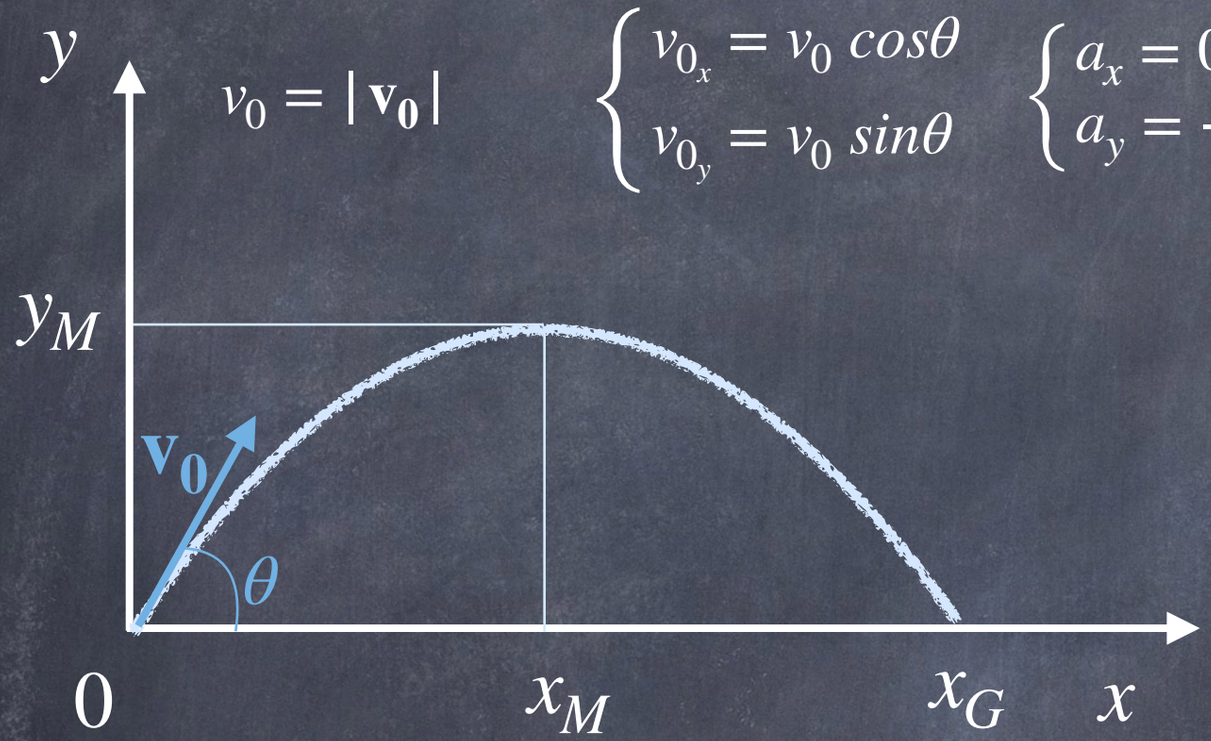

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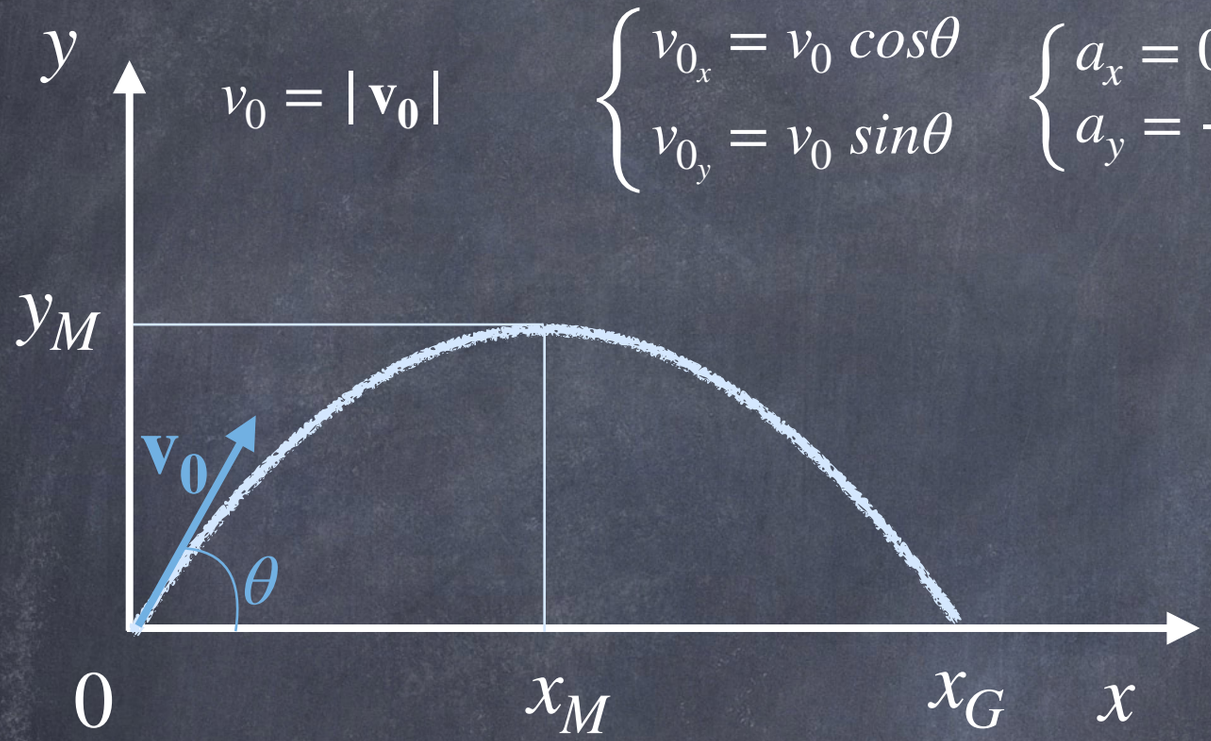

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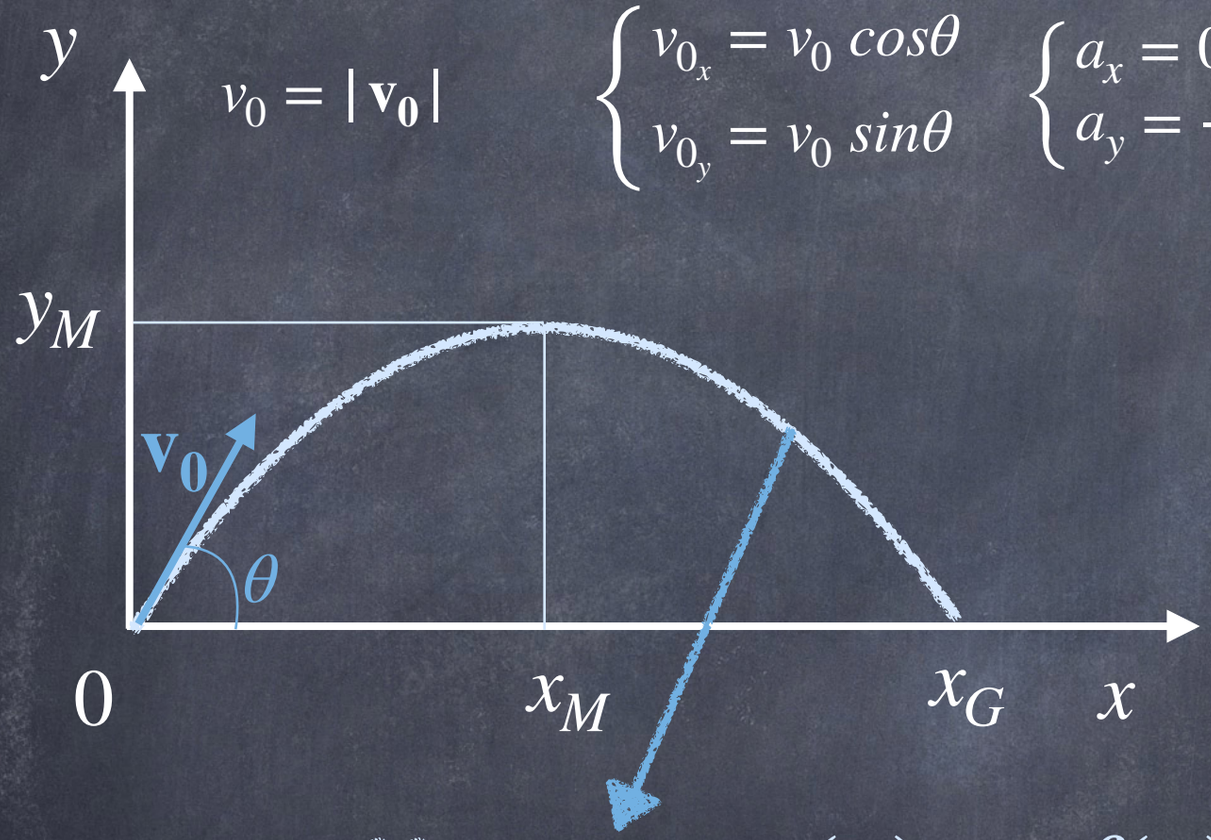

Traiettoria: $y(x) = f(x)$

Moto dei Proiettili

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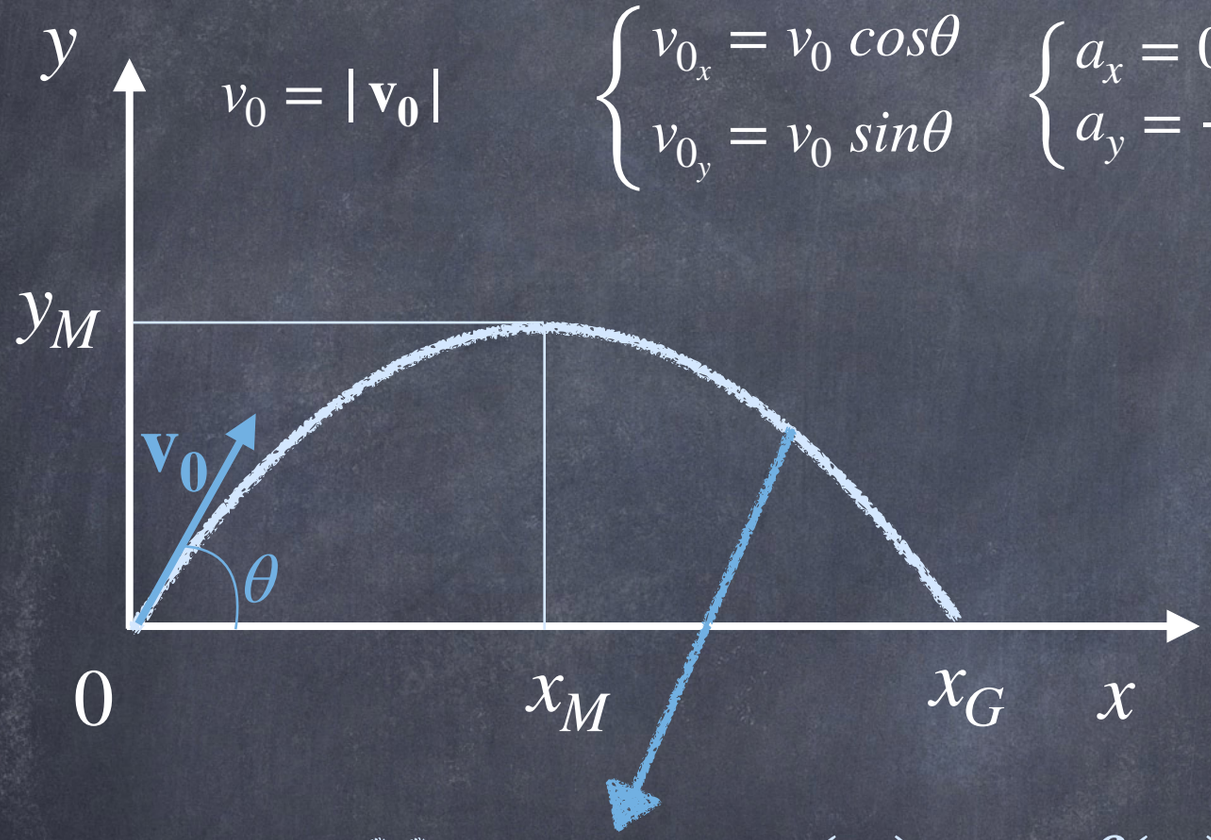

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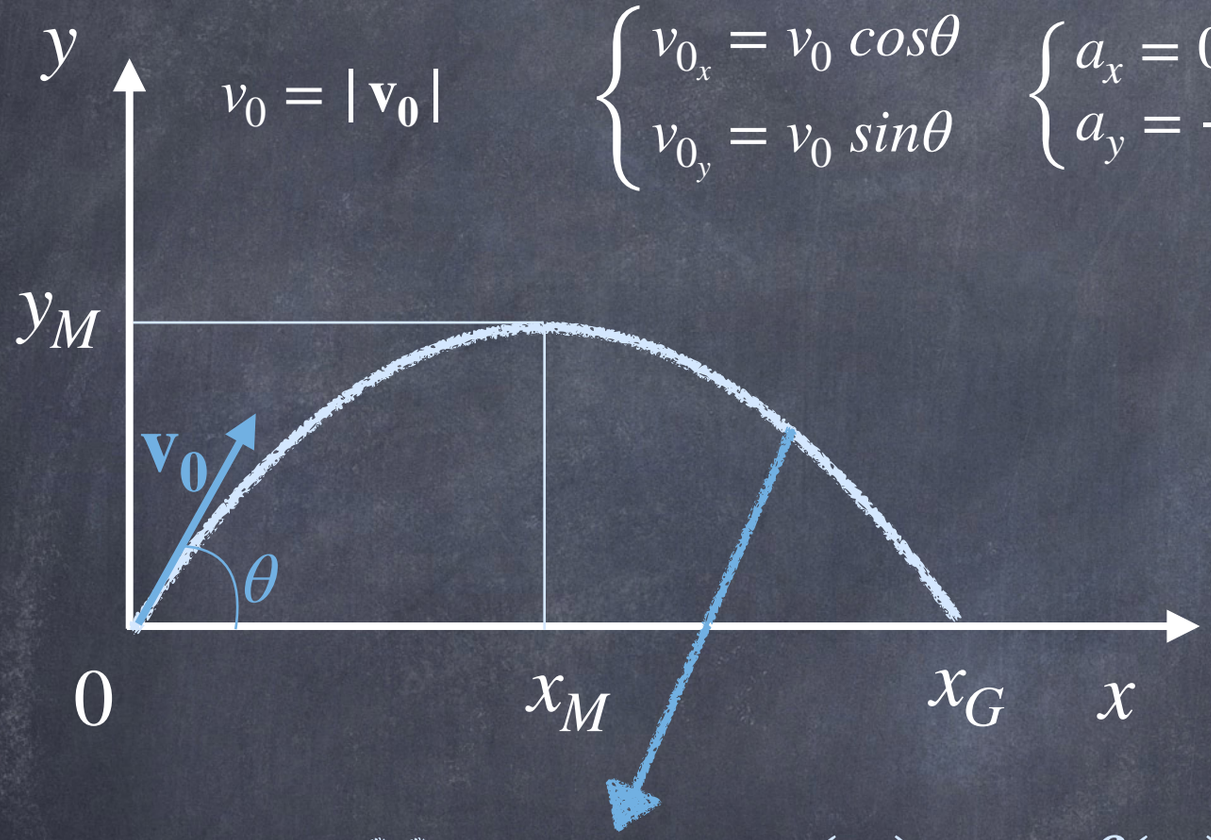

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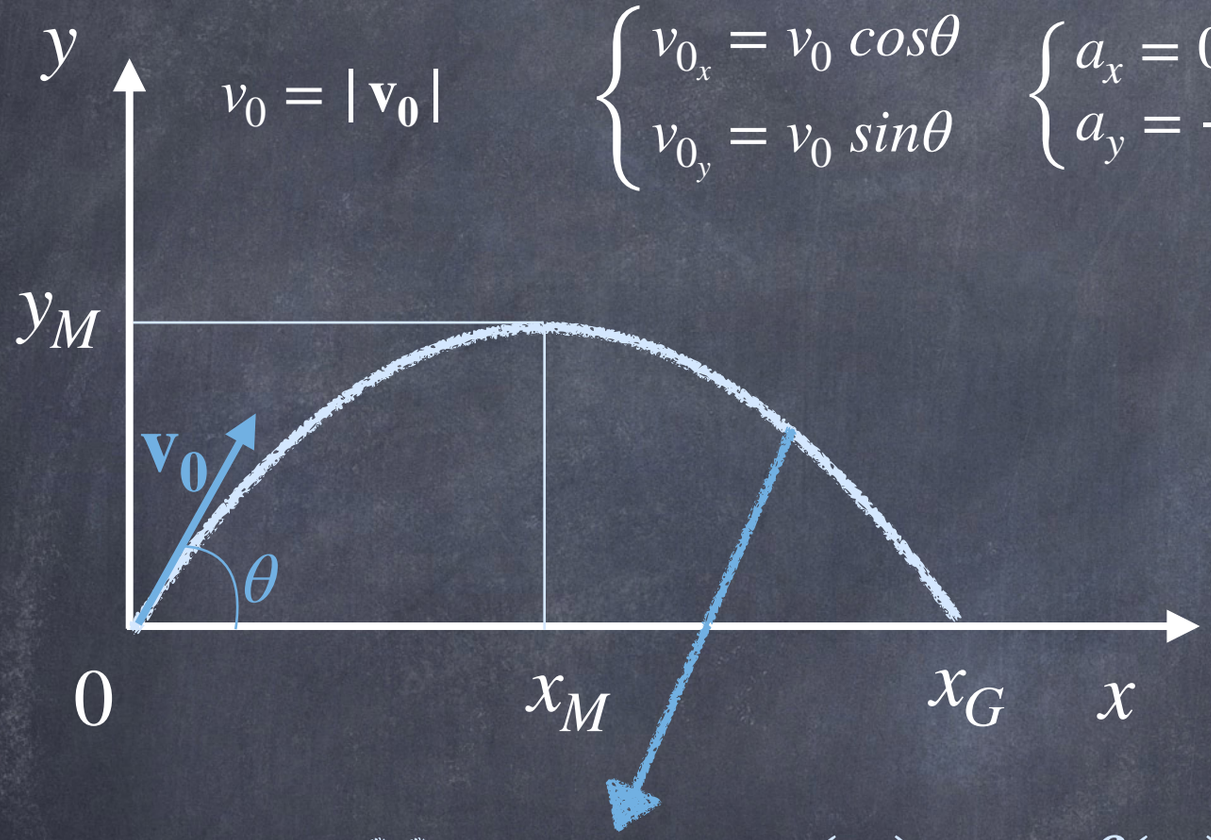
Moto dei Proiettili



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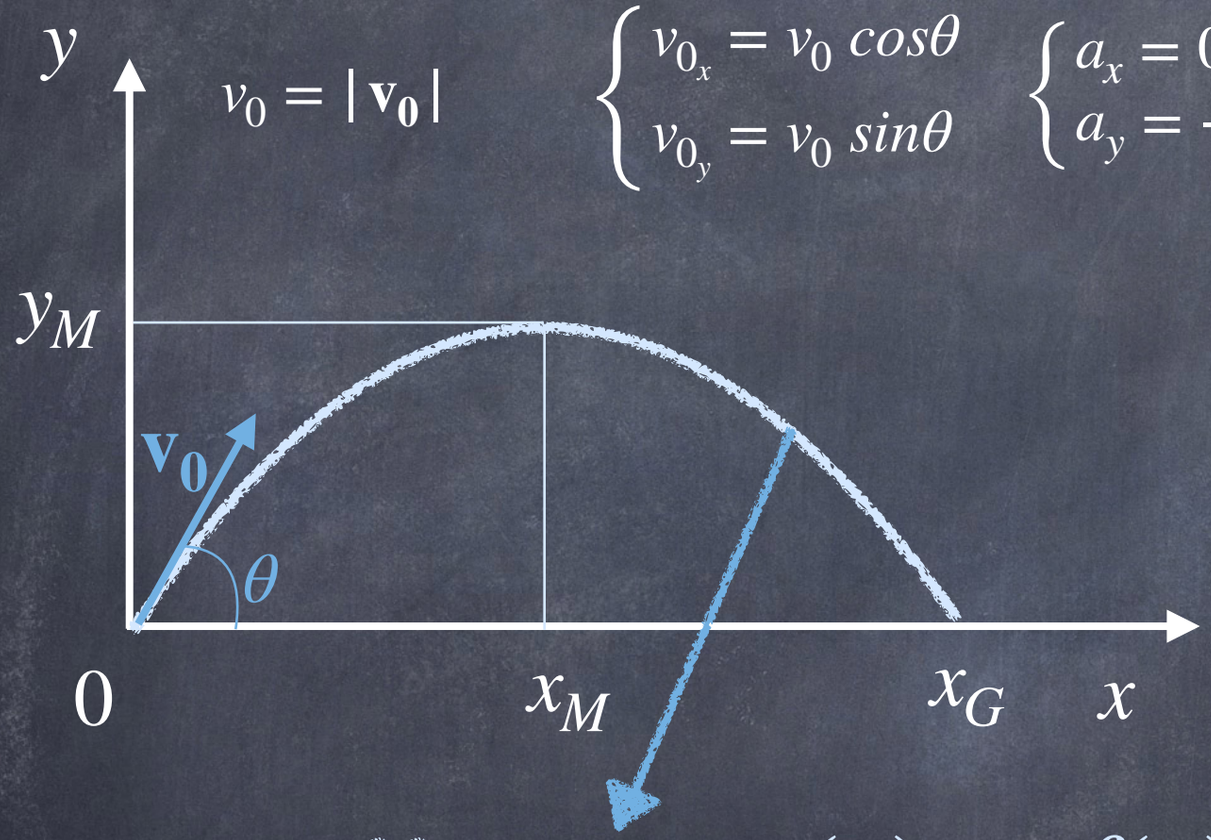

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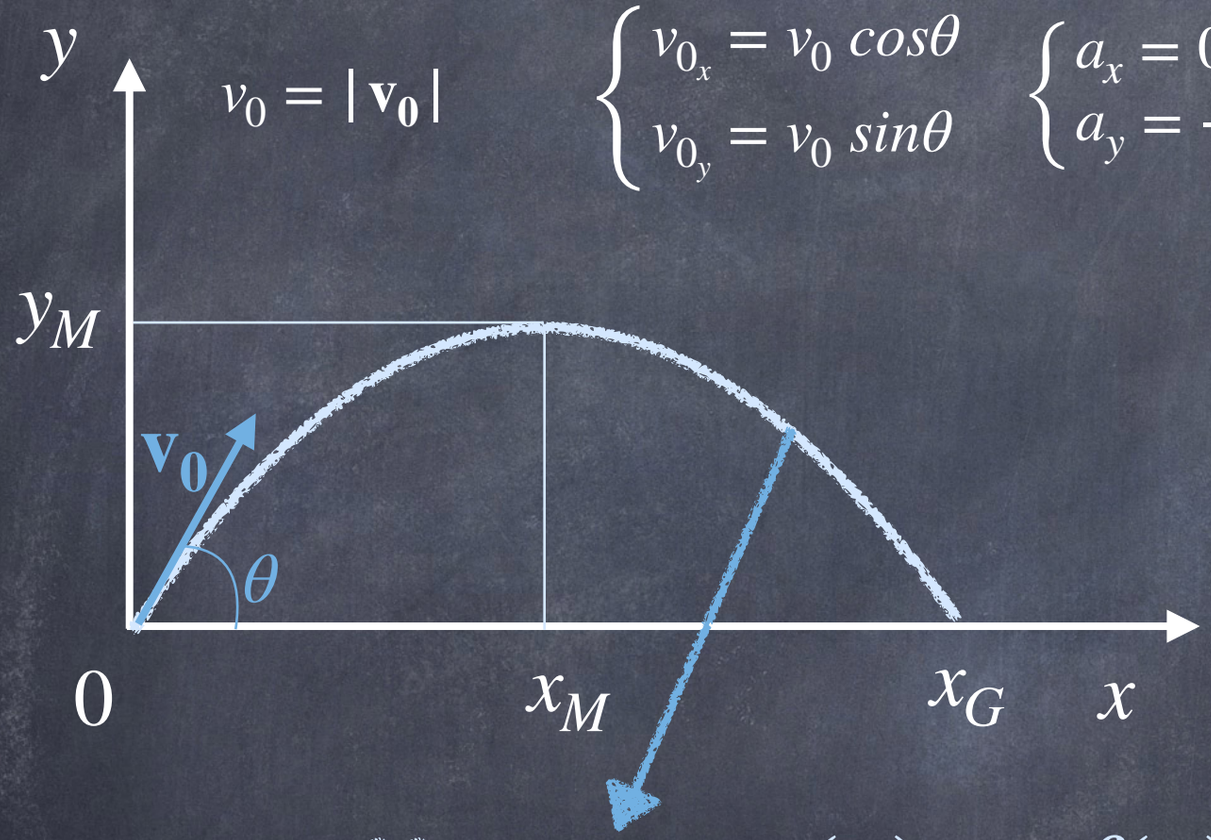

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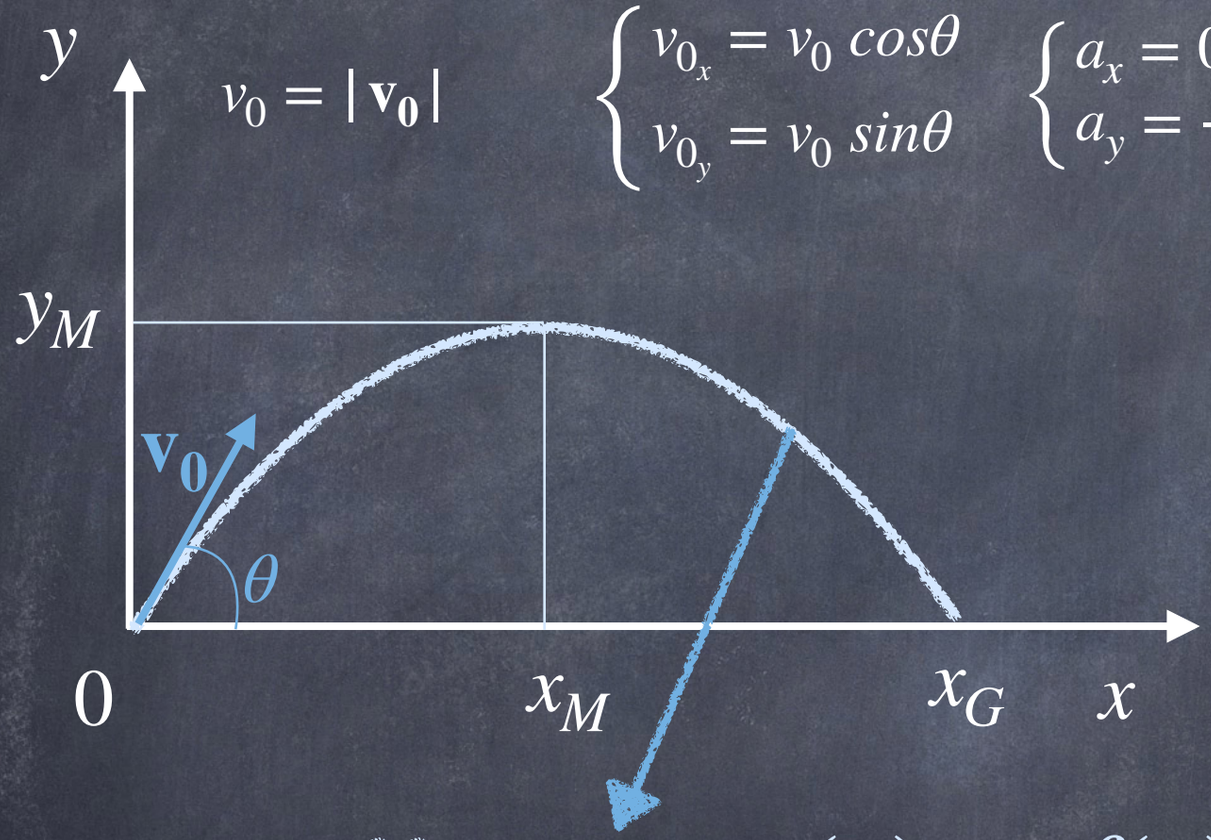

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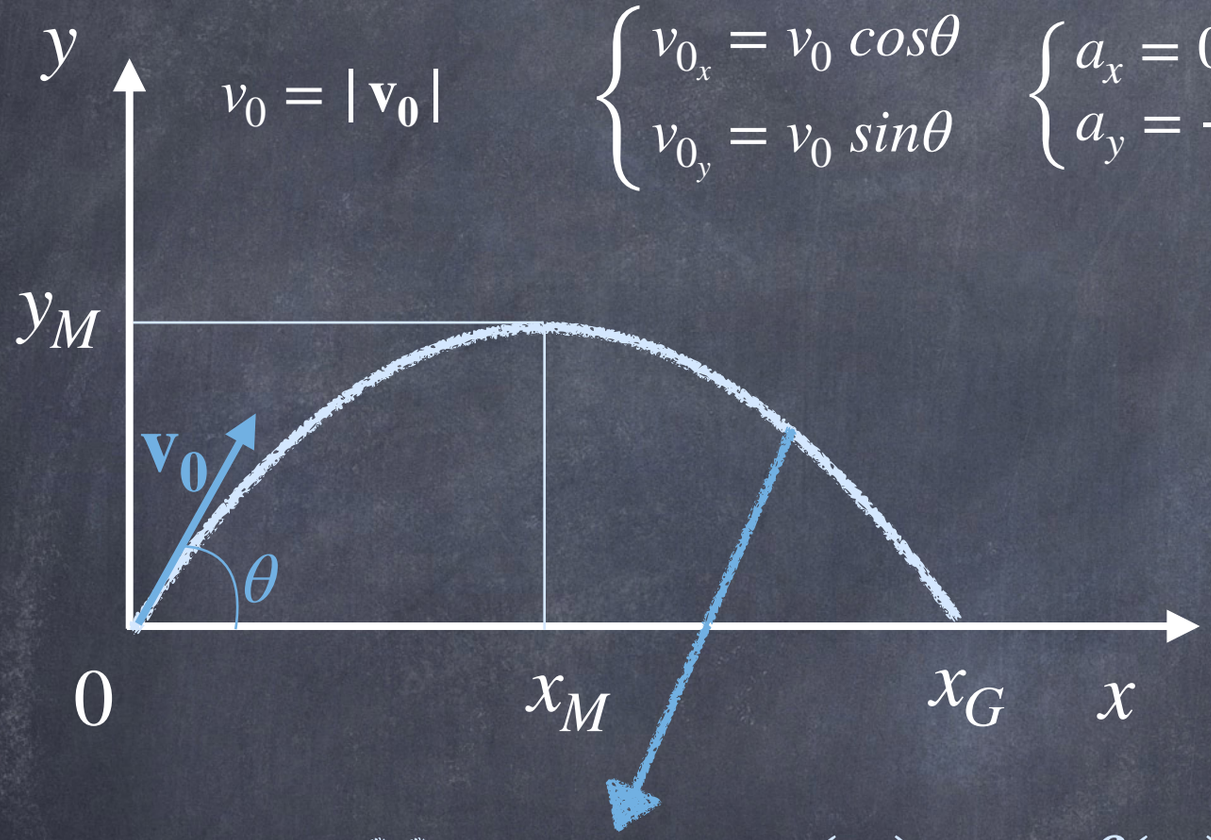

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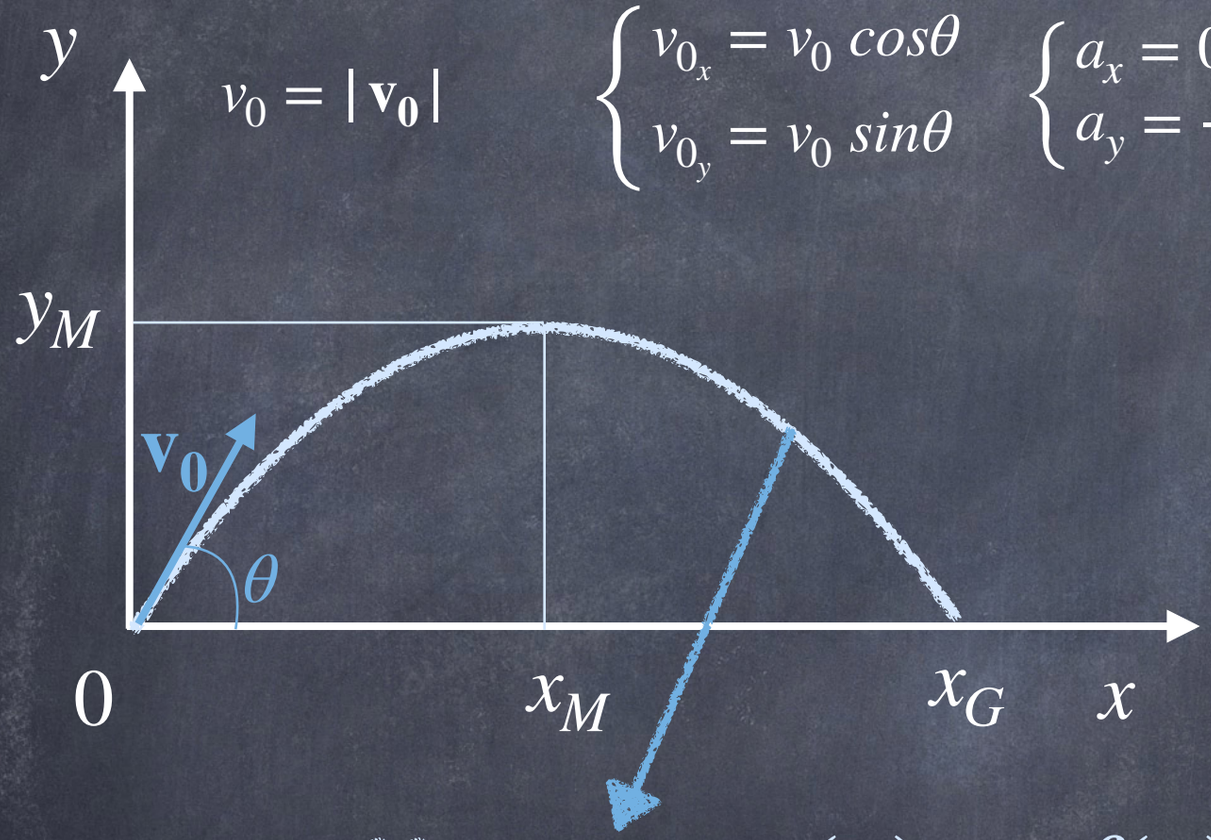
$\tan\theta$

Moto dei Proiettili



$$v_0 = |\mathbf{v_0}|$$

$$\begin{cases} v_{0_x} = v_0 \cos\theta \\ v_{0_y} = v_0 \sin\theta \end{cases} \quad \begin{cases} a_x = 0 \\ a_y = -g \end{cases} \quad \begin{cases} v_x(t) = v_0 \cos\theta \\ v_y(t) = v_0 \sin\theta - gt \end{cases}$$

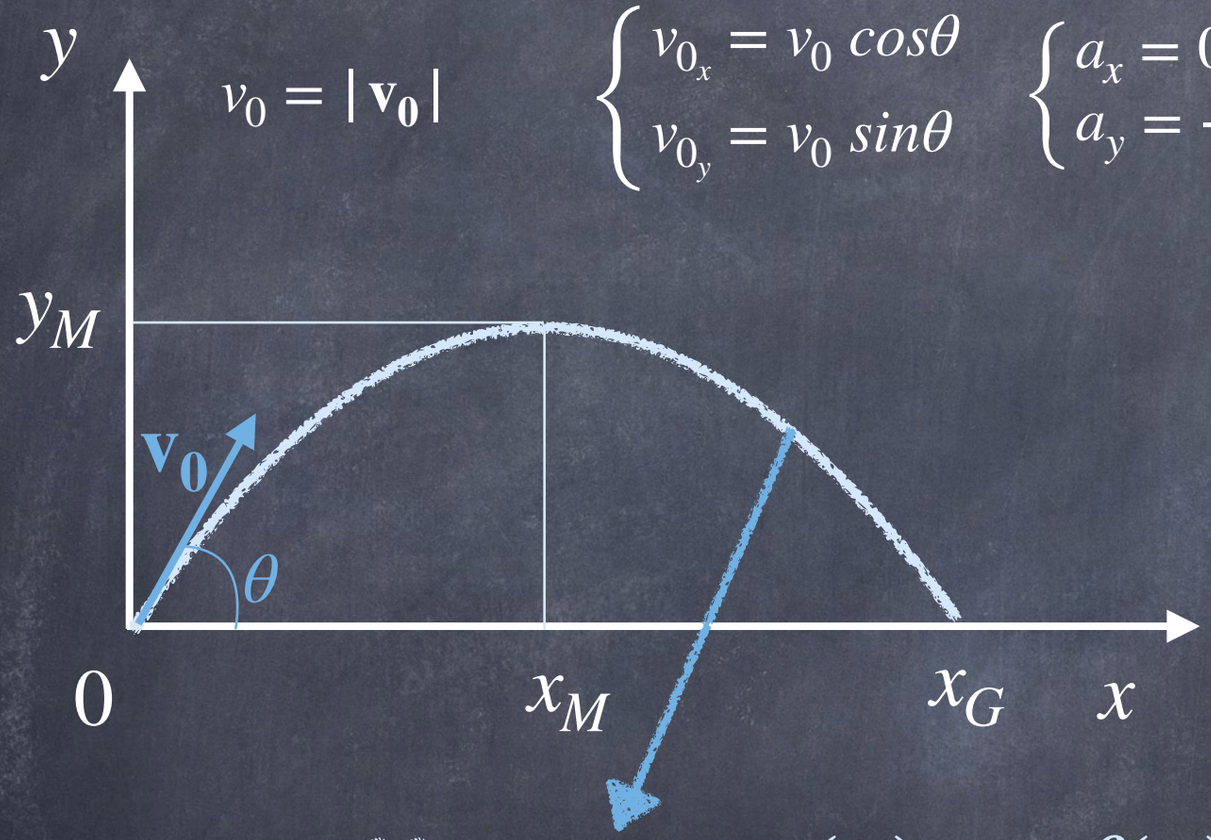
$$x_0 = 0 \quad y_0 = 0 \quad t_0 = 0$$

$$\begin{cases} x(t) = v_0 \cos\theta \; t \\ y(t) = v_0 \sin\theta \; t - \frac{1}{2} g t^2 \end{cases}$$

Traiettoria: $y(x) = f(x)$

$$\begin{cases} t = \dfrac{x}{v_0 \cos\theta} \\ y = \tan\theta \; x - \dfrac{g}{2 v_0^2 \cos^2\theta} \; x^2 \end{cases}$$

Moto dei Proiettili



$$v_0 = |\mathbf{v_0}|$$

$$\begin{cases} v_{0_x} = v_0 \cos\theta \\ v_{0_y} = v_0 \sin\theta \end{cases} \quad \begin{cases} a_x = 0 \\ a_y = -g \end{cases} \quad \begin{cases} v_x(t) = v_0 \cos\theta \\ v_y(t) = v_0 \sin\theta - gt \end{cases}$$

$$x_0 = 0 \quad y_0 = 0 \quad t_0 = 0$$

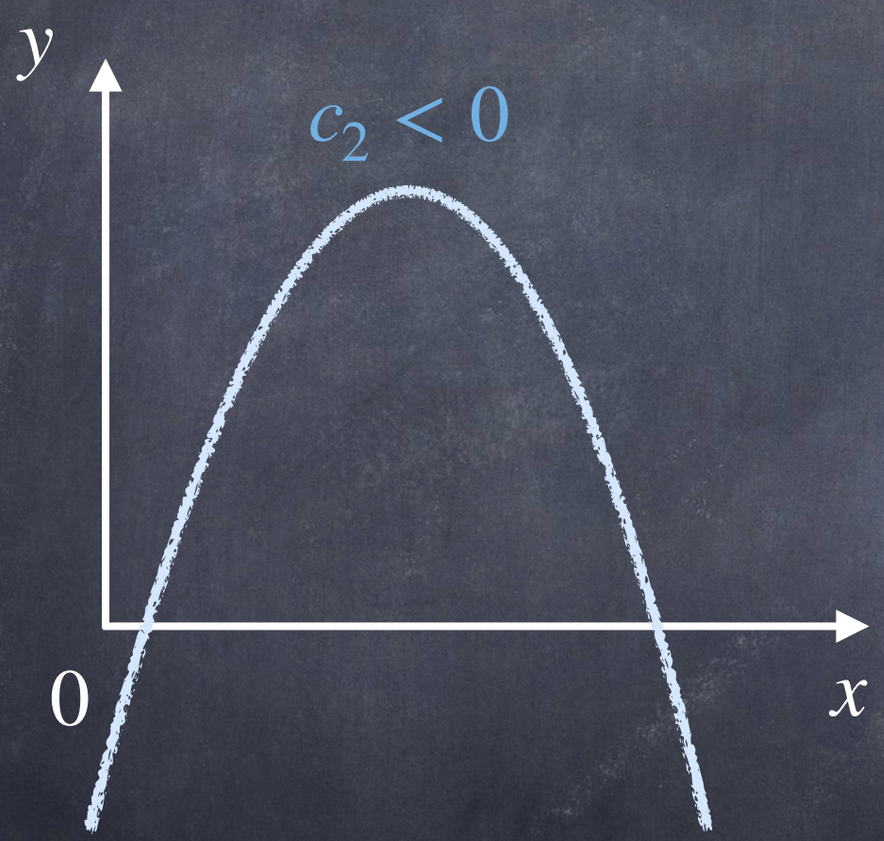
$$\begin{cases} x(t) = v_0 \cos\theta \, t \\ y(t) = v_0 \sin\theta \, t - \frac{1}{2} g t^2 \end{cases}$$

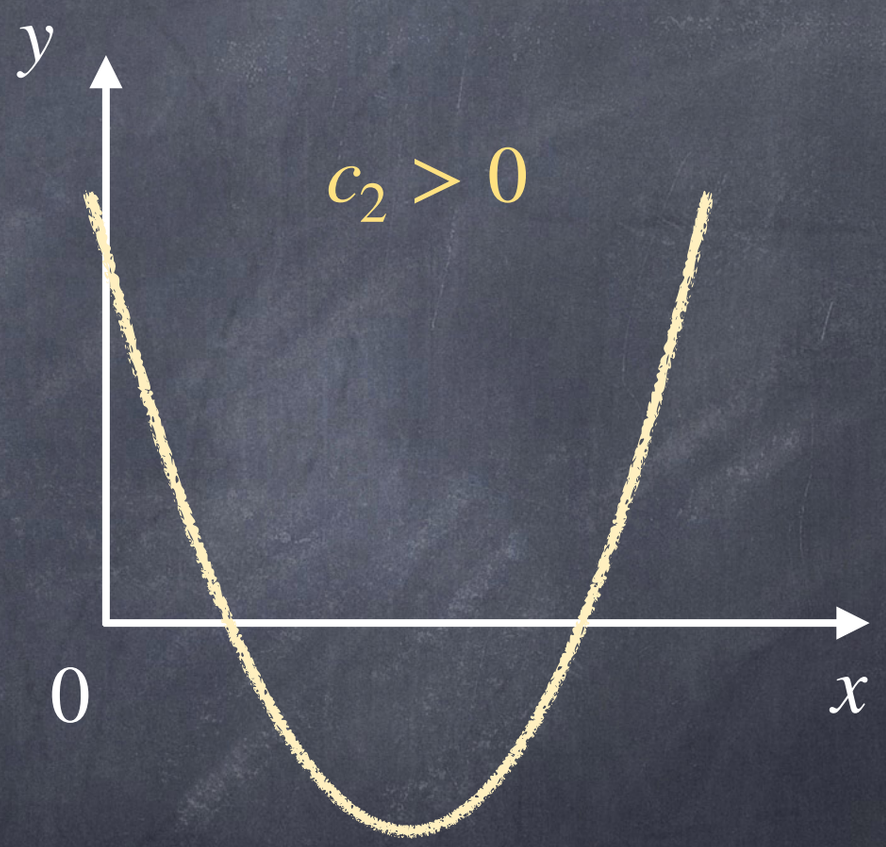
Traiettoria: $y(x) = f(x)$

$$\begin{cases} t = \dfrac{x}{v_0 \cos\theta} \\ y = \tan\theta \, x - \dfrac{g}{2 v_0^2 \cos^2\theta} x^2 \end{cases} \longrightarrow \text{Moto Parabolico}$$

Richiamo di Matematica

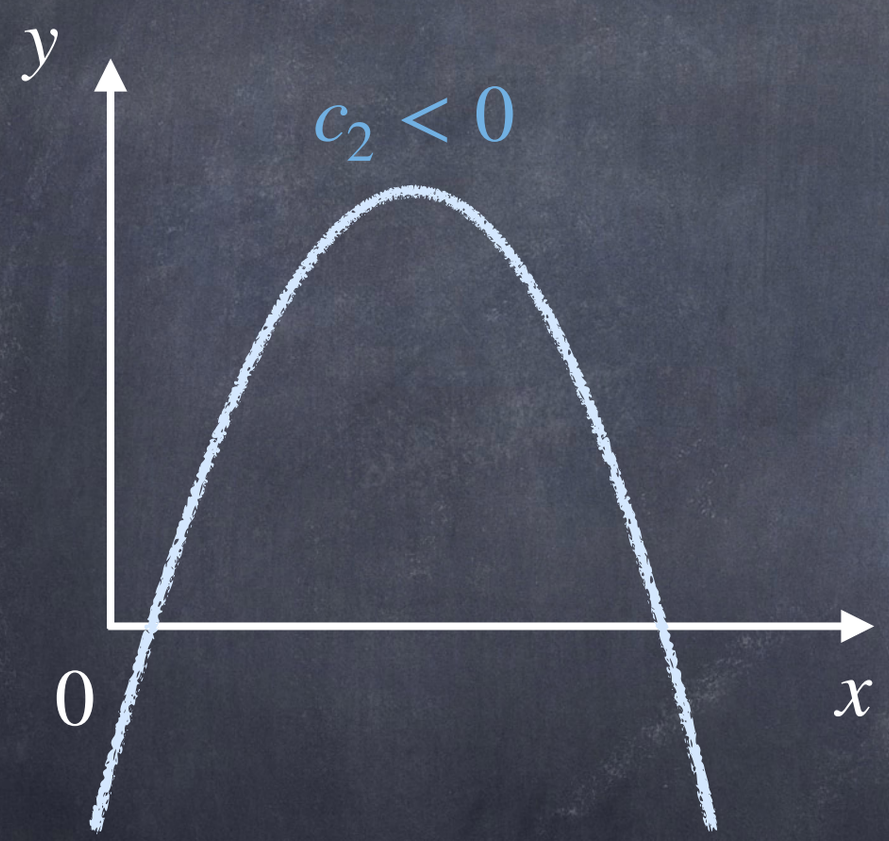
Equazione Parabola: $y = c_0 + c_1 x + c_2 x^2$

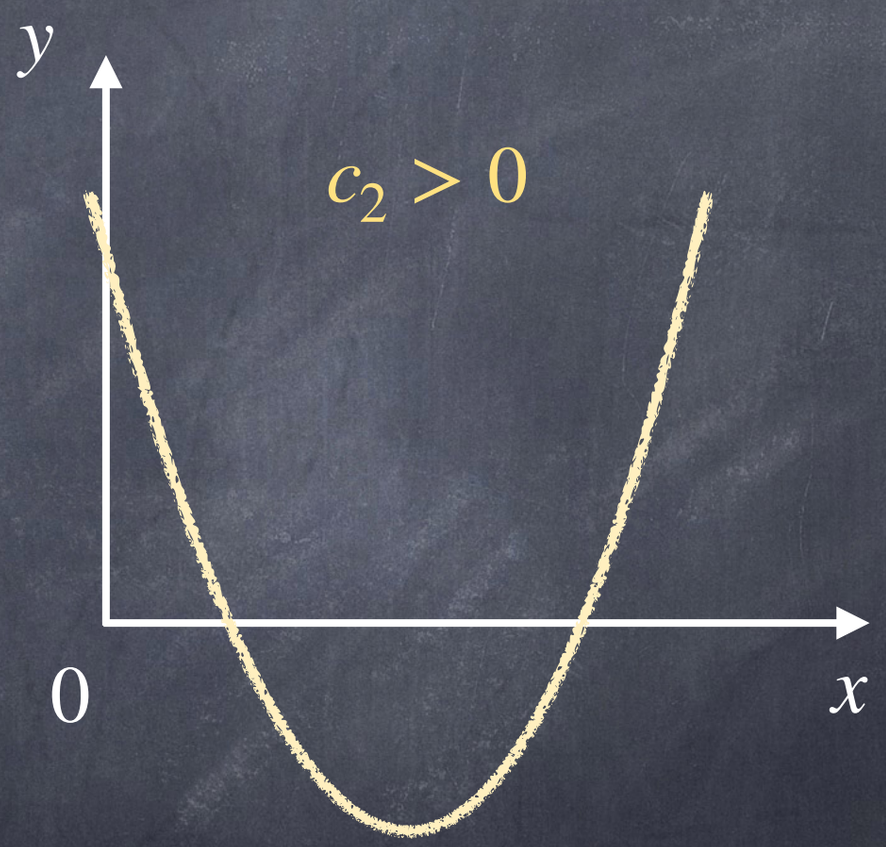




Richiamo di Matematica

Equazione Parabola: $y = c_0 + c_1 x + c_2 x^2$

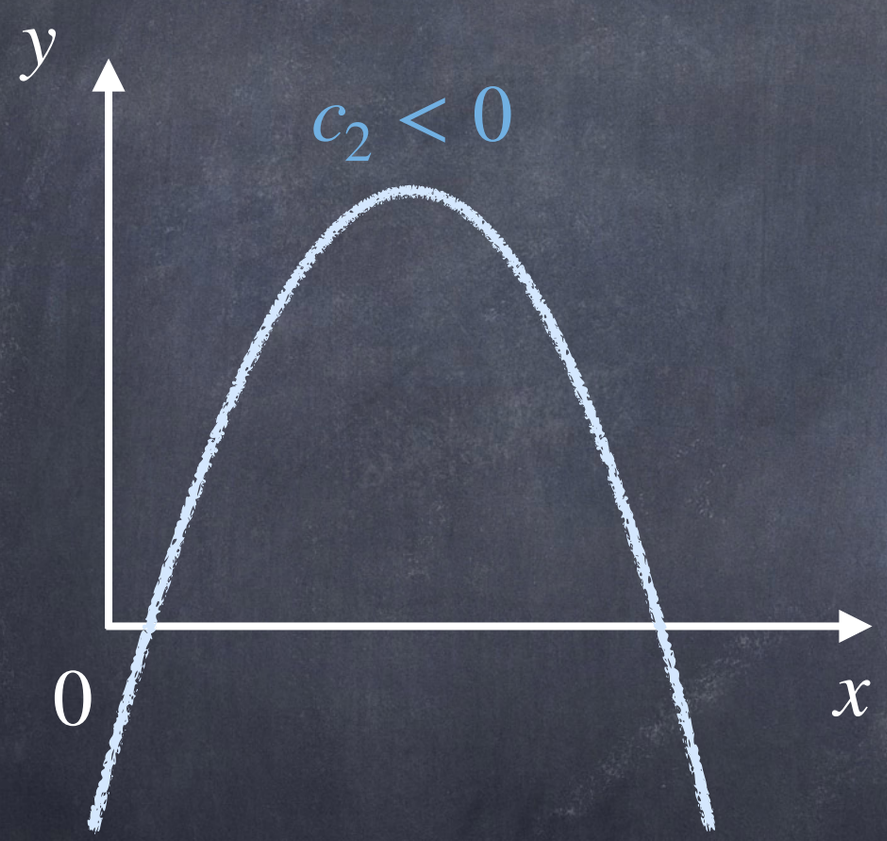


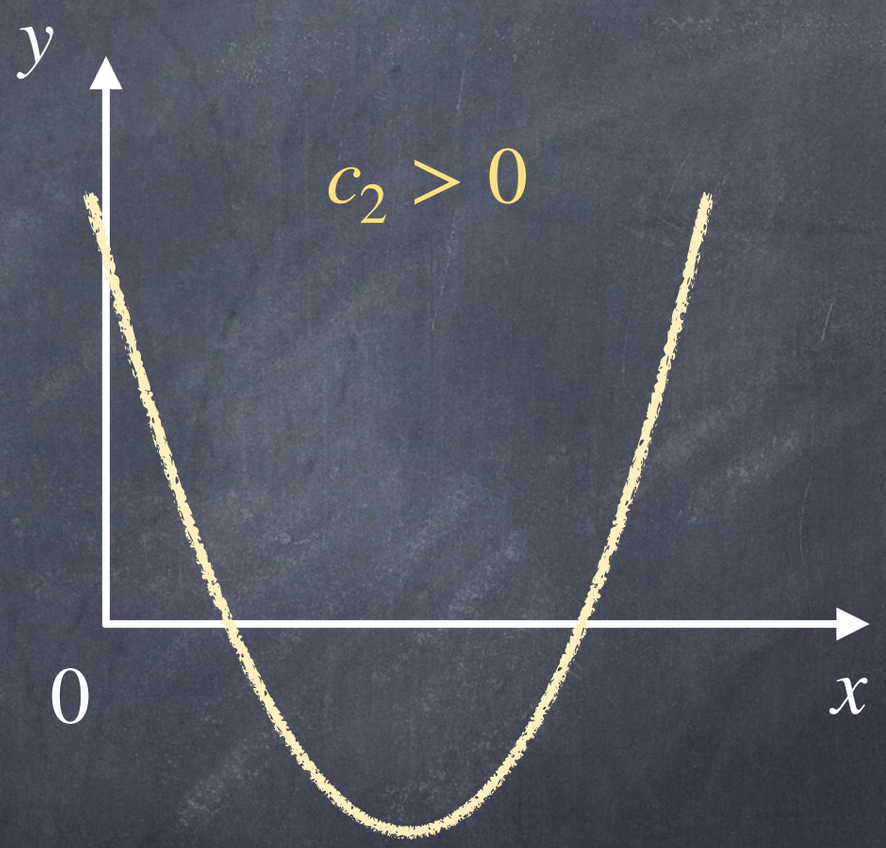


$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$

Richiamo di Matematica

Equazione Parabola: $y = c_0 + c_1 x + c_2 x^2$

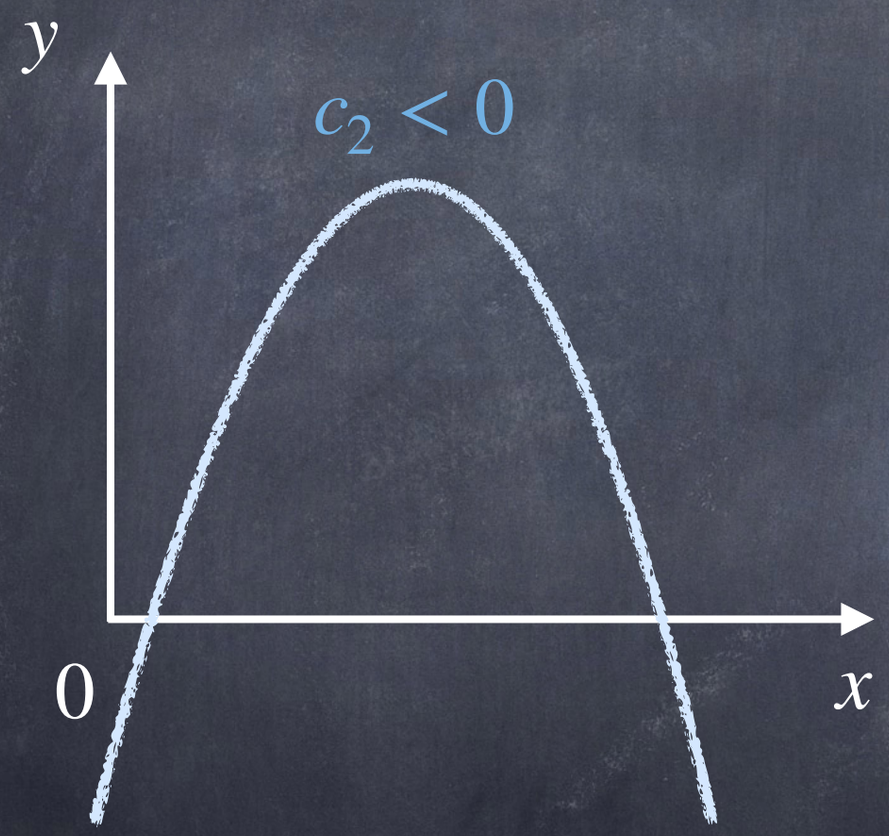




$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2 \qquad c_2 = -\frac{g}{2v_0^2\, cos^2\theta} < 0 \longrightarrow y_{Max}$$

Richiamo di Matematica

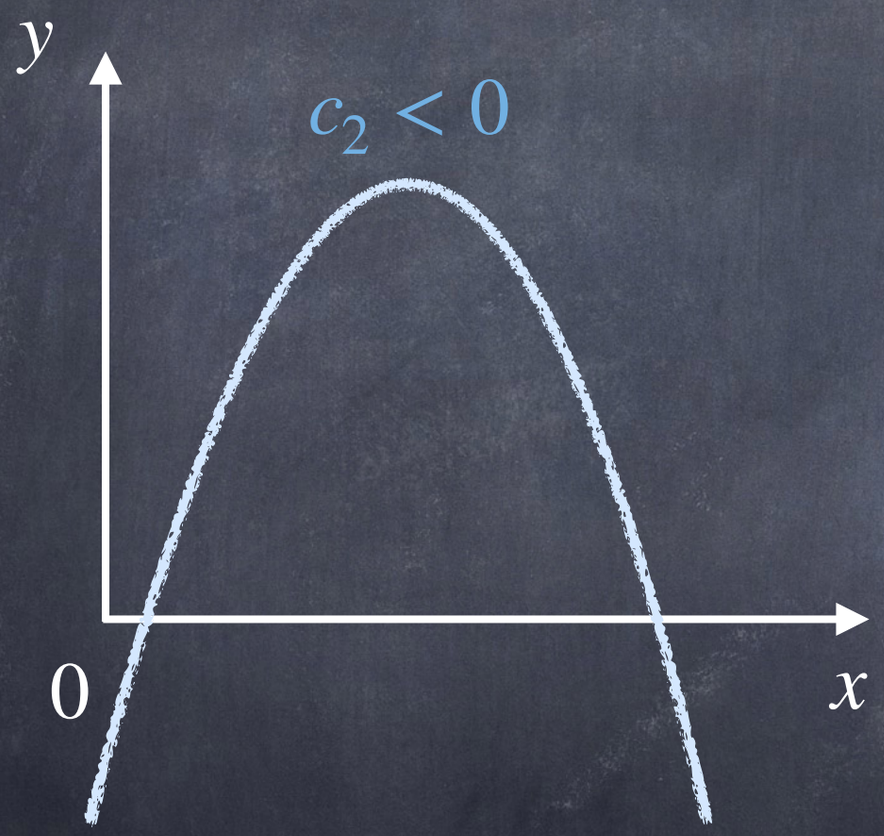
Equazione Parabola: $y = c_0 + c_1 x + c_2 x^2$



Minimo o Massimo: $\dfrac{dy}{dx} = 0$

Richiamo di Matematica

Equazione Parabola: $y = c_0 + c_1 x + c_2 x^2$

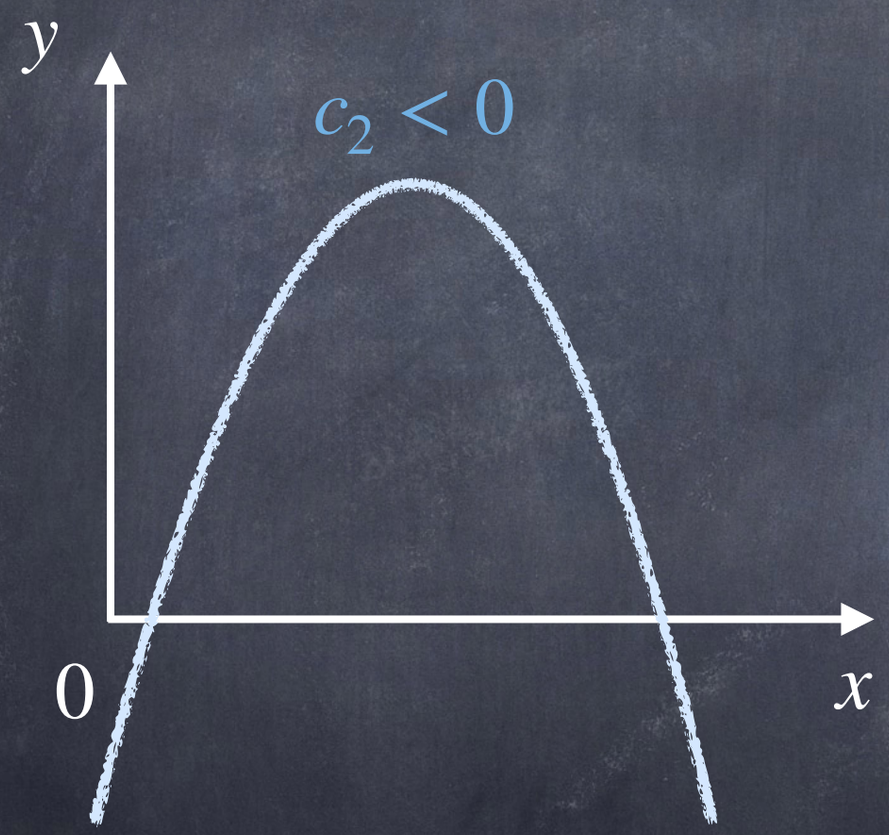


Minimo o Massimo: $\dfrac{dy}{dx} = 0$

Derivate: $\dfrac{d\,k}{dx} = 0$

Richiamo di Matematica

Equazione Parabola: $y = c_0 + c_1 x + c_2 x^2$



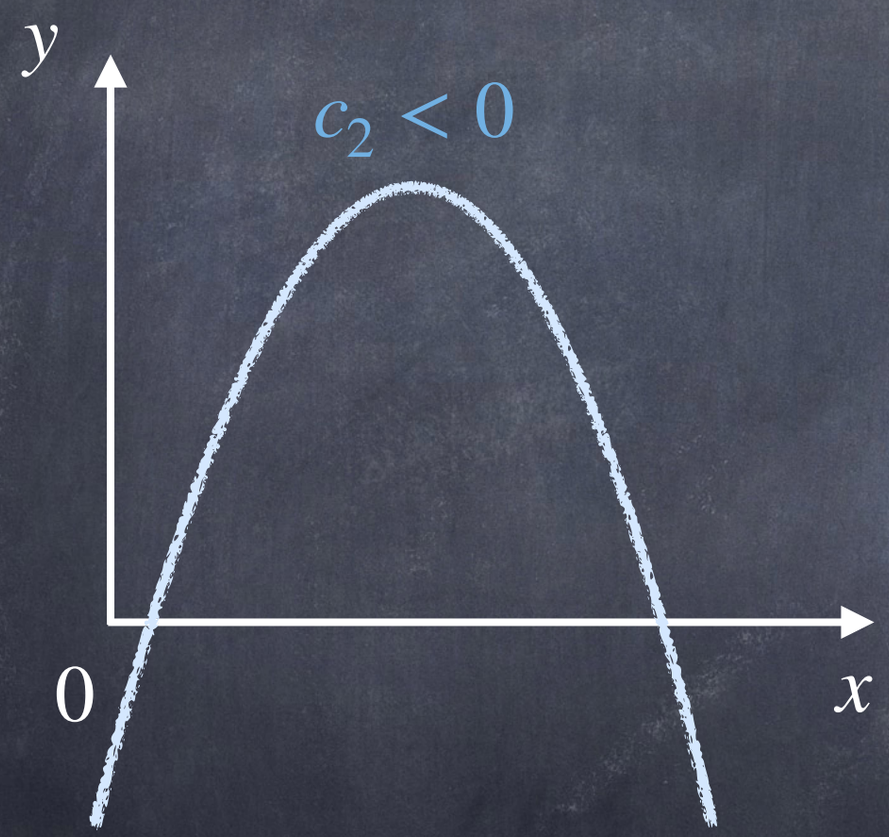
Minimo o Massimo: $\dfrac{dy}{dx} = 0$

Derivate: $\dfrac{d\,k}{dx} = 0$

$\dfrac{d\,(kx)}{dx} = k$

Richiamo di Matematica

Equazione Parabola: $y = c_0 + c_1 x + c_2 x^2$



Minimo o Massimo: $\dfrac{dy}{dx} = 0$

Derivate:

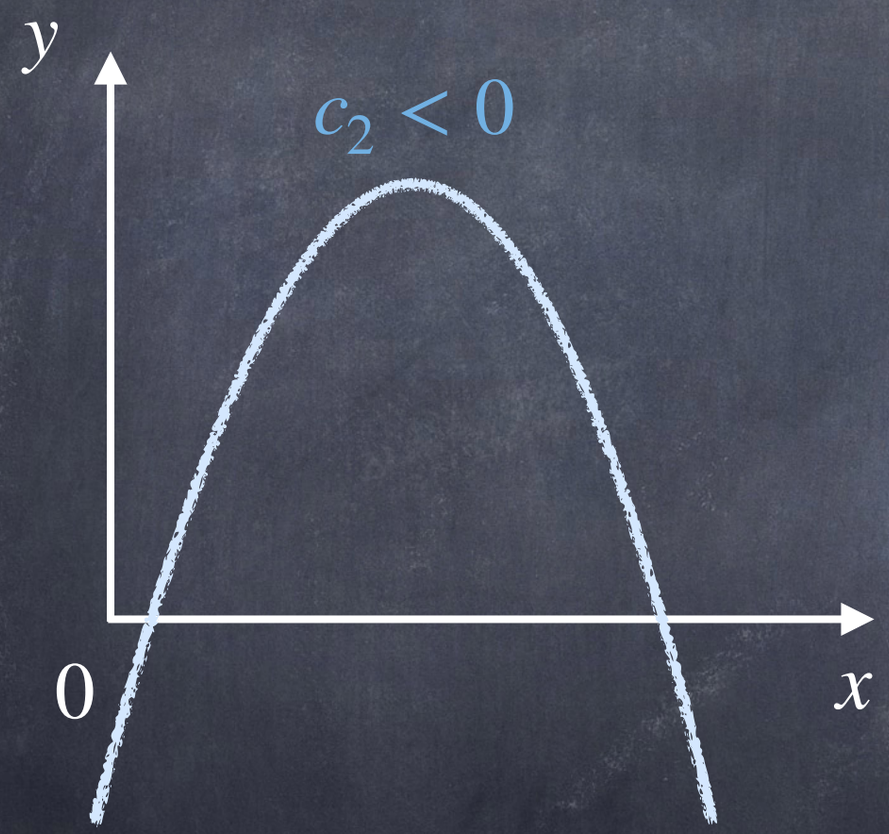
$$\frac{d\,k}{dx} = 0$$

$$\frac{d\,(kx)}{dx} = k$$

$$\frac{d\,(kx^2)}{dx} = 2kx$$

Richiamo di Matematica

Equazione Parabola: $y = c_0 + c_1 x + c_2 x^2$



Minimo o Massimo: $\dfrac{dy}{dx} = 0$

Derivate:

$$\frac{d\,k}{dx} = 0$$

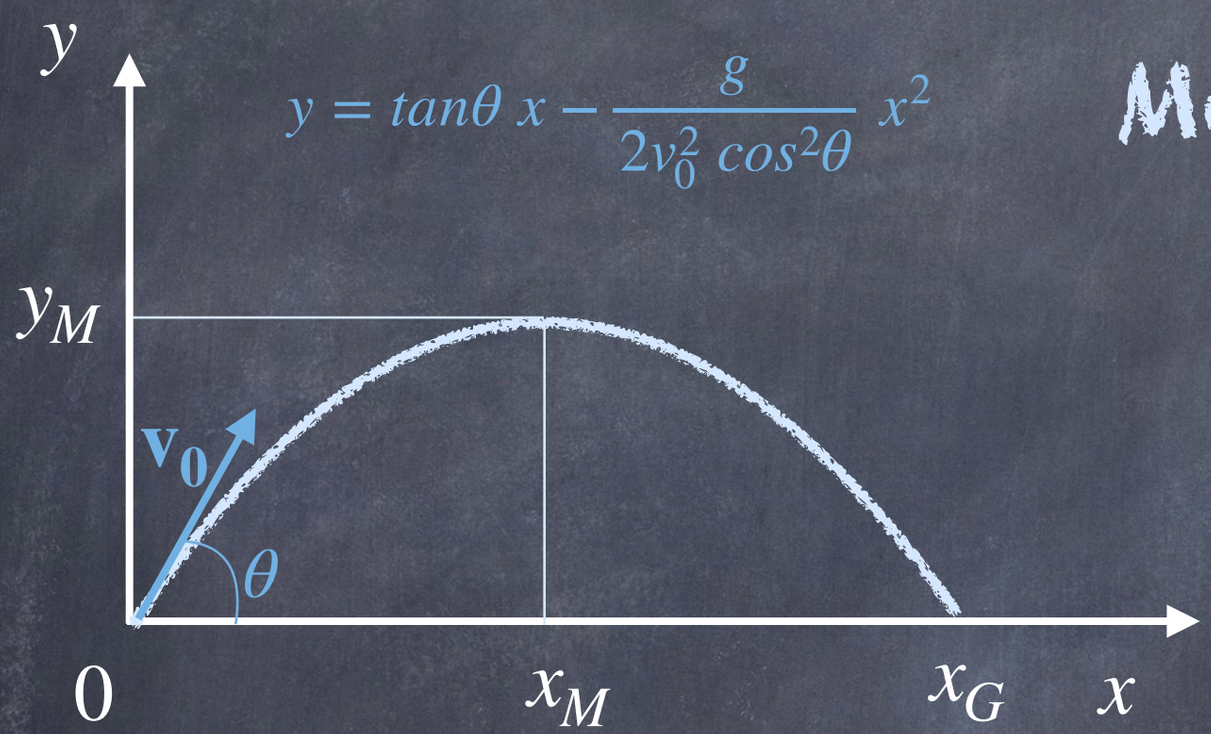
$$\frac{d\,(kx)}{dx} = k$$

$$\frac{d\,(kx^2)}{dx} = 2kx$$

$$\frac{d\,(x^n)}{dx} = n\,x^{n-1}$$

Moto dei Proiettili

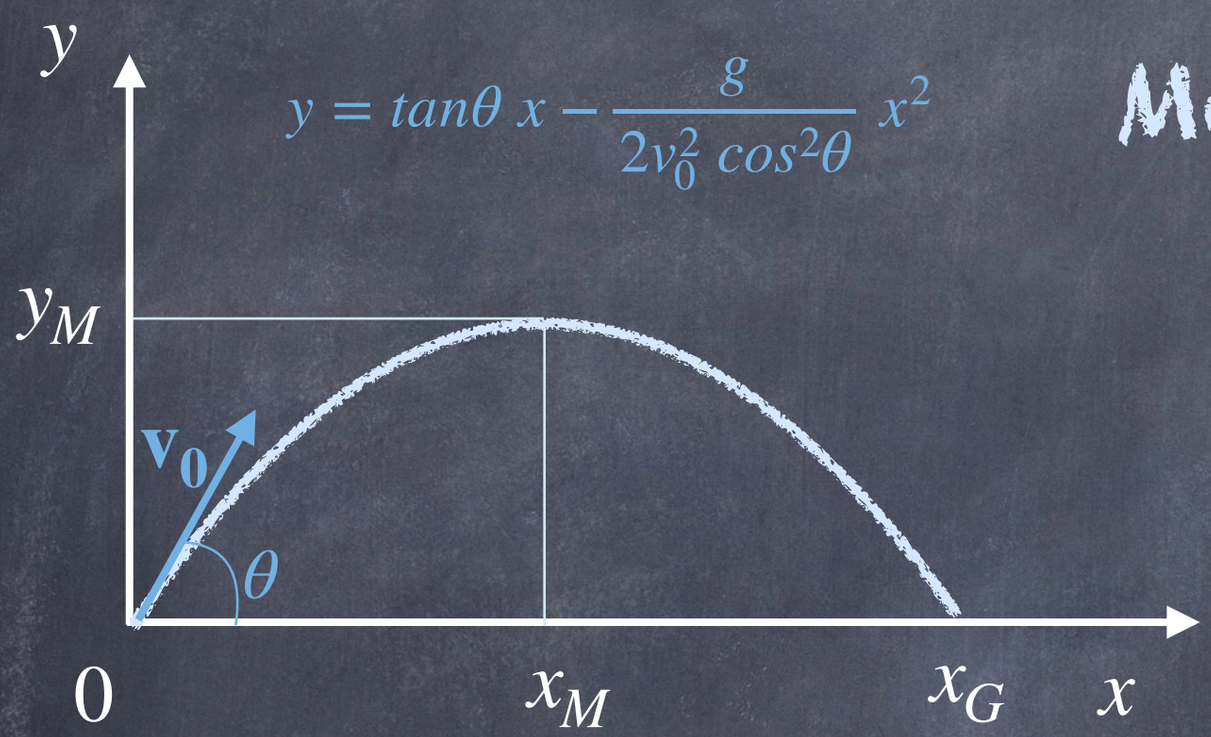
$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$



Massima Altezza: y_M

Moto dei Proiettili

$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$



Massima Altezza: y_M

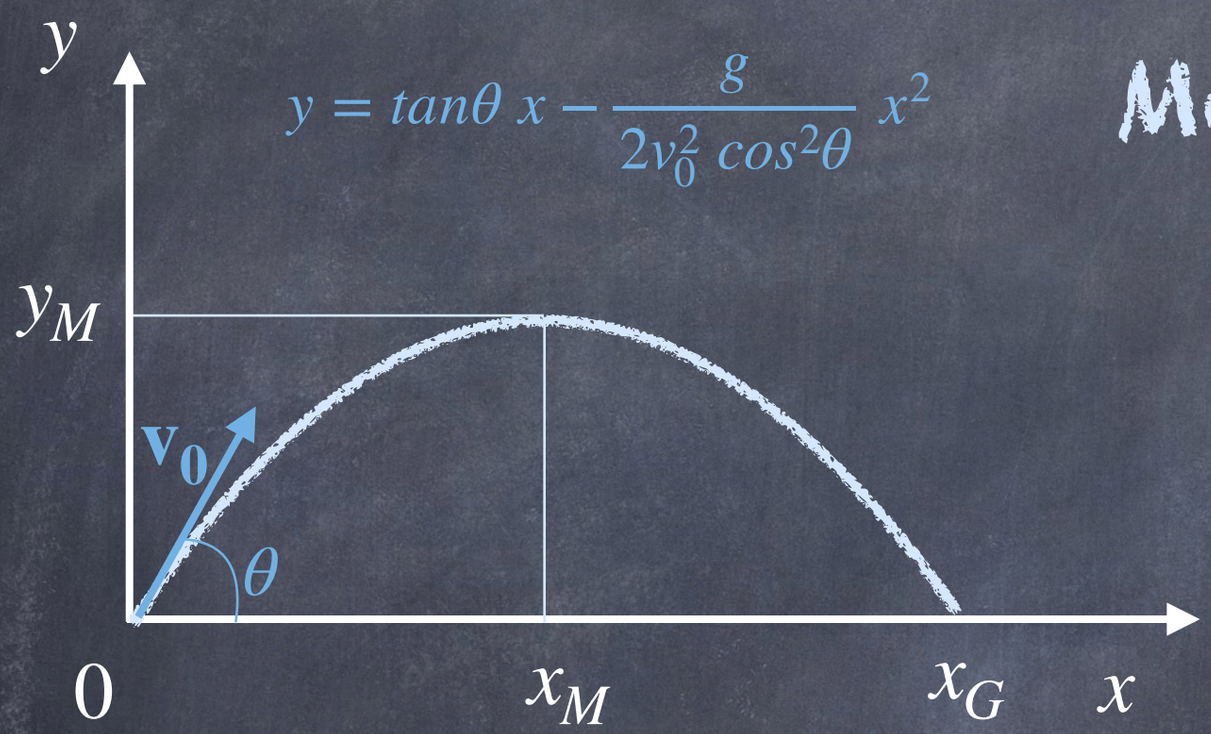
Derivata Nulla

$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

Moto dei Proiettili

$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$



Massima Altezza: y_M

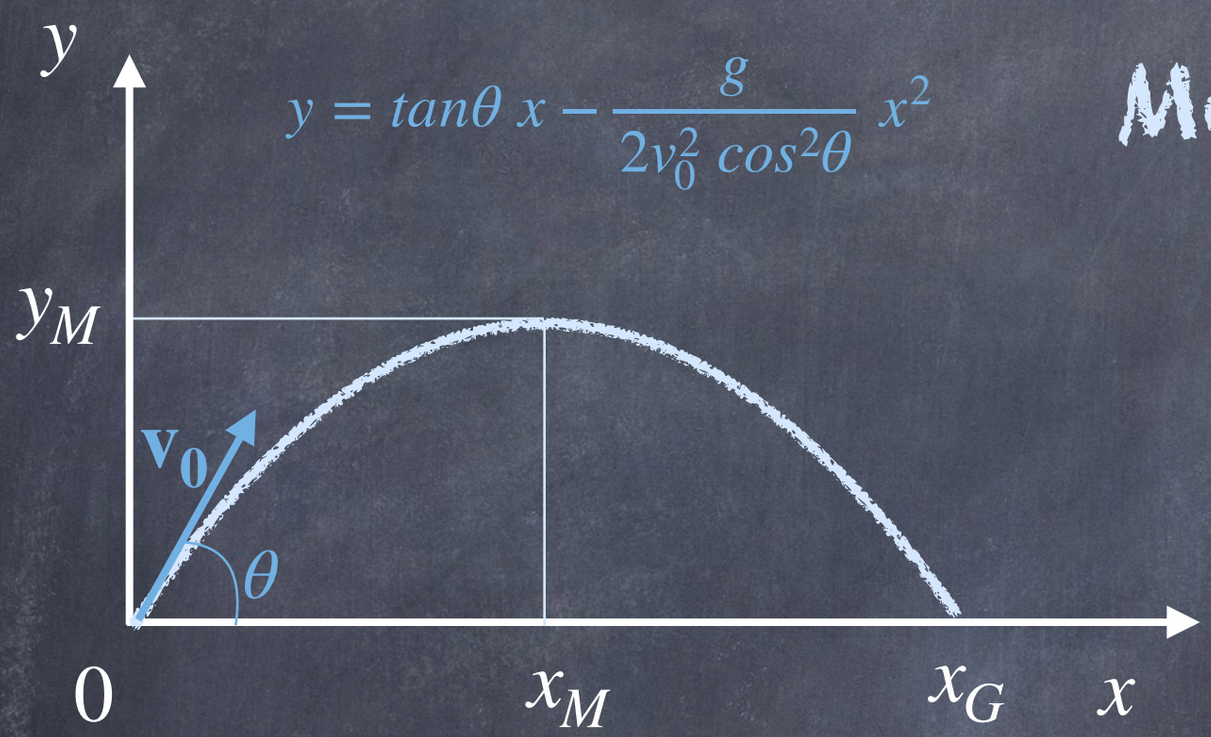
Derivata Nulla

$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$\frac{dy(x_M)}{dx} = 0$$

Moto dei Proiettili



$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$

Massima Altezza: y_M

Derivata Nulla

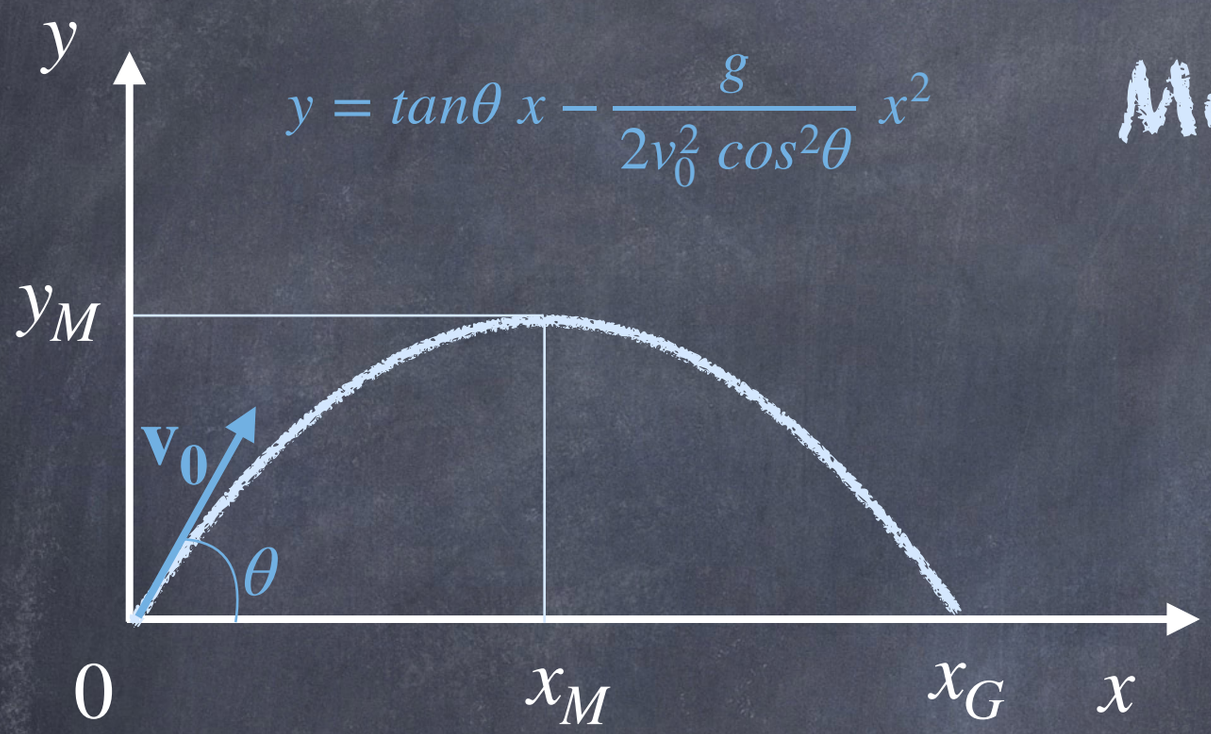
$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$\frac{dy(x_M)}{dx} = 0 \qquad tan\theta = \frac{g}{v_0^2\, cos^2\theta}\, x_M$$

Moto dei Proiettili

$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$



Massima Altezza: y_M

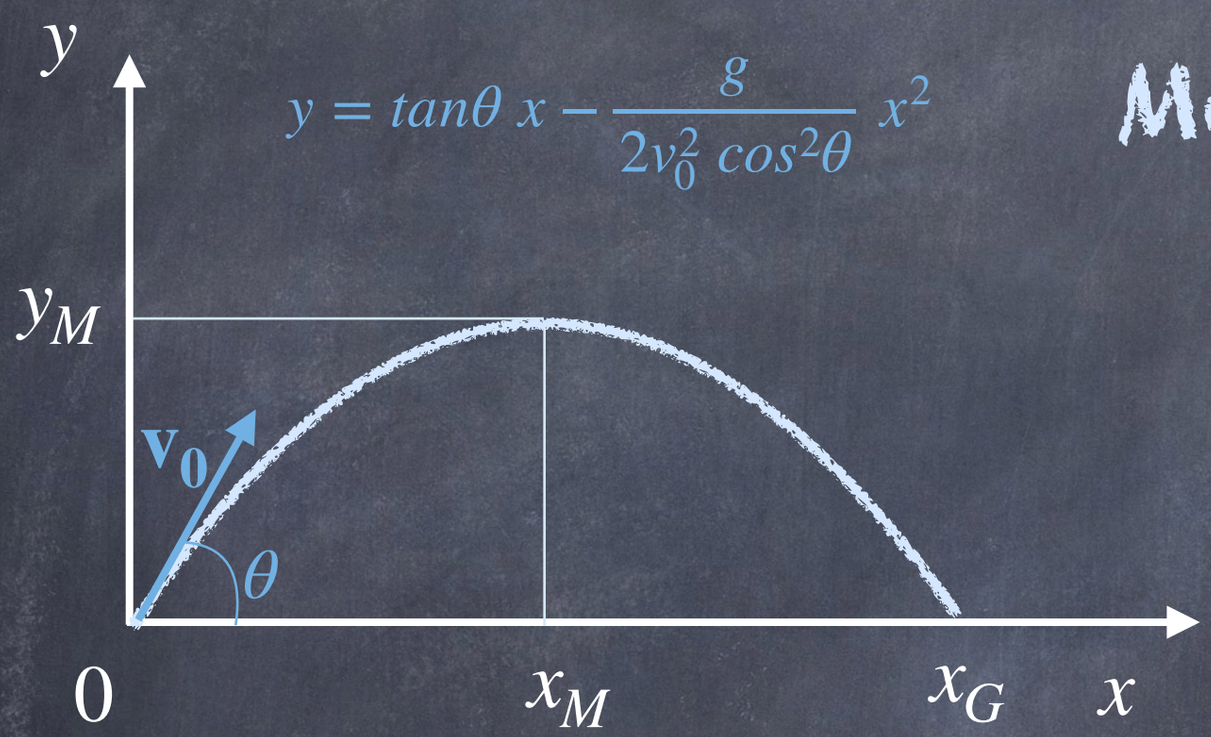
Derivata Nulla

$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$\frac{dy(x_M)}{dx} = 0 \qquad tan\theta = \frac{g}{v_0^2\, cos^2\theta}\, x_M \qquad x_M = \frac{sin\theta}{cos\theta}\frac{v_0^2\, cos^2\theta}{g}$$

Moto dei Proiettili



$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$

Massima Altezza: y_M

Derivata Nulla

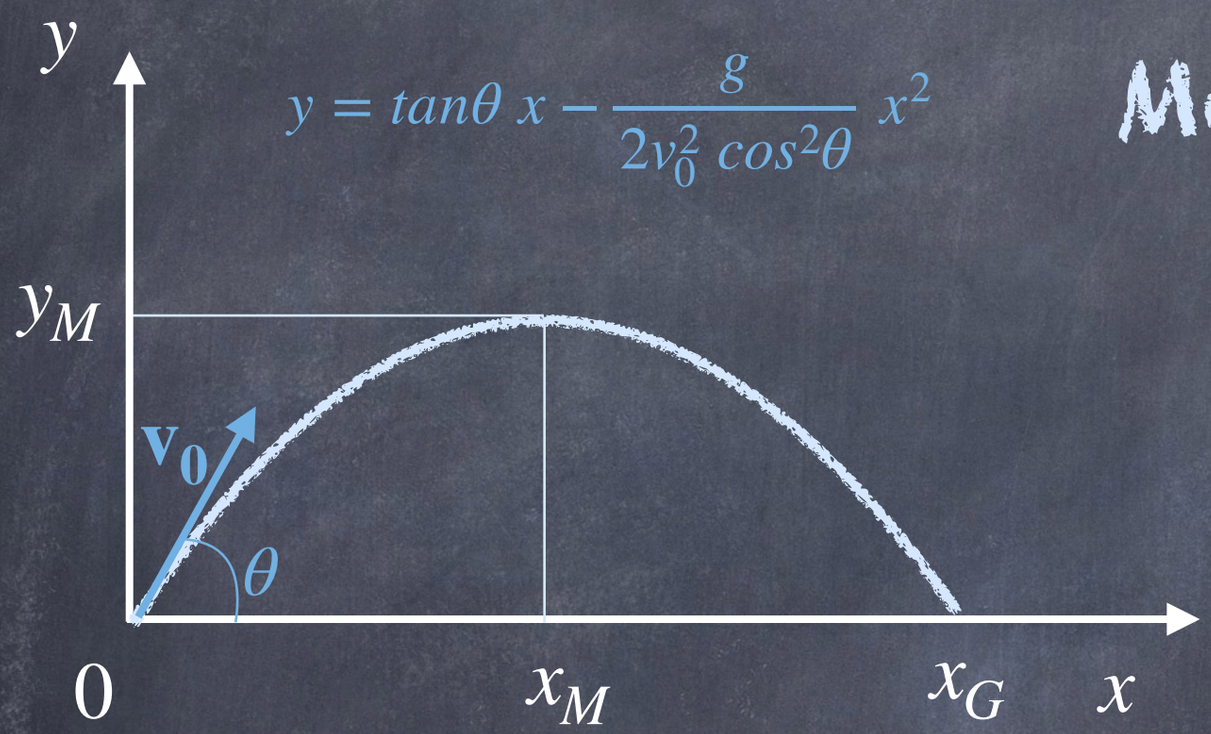
$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$\frac{dy(x_M)}{dx} = 0 \qquad tan\theta = \frac{g}{v_0^2\, cos^2\theta}\, x_M \qquad x_M = \frac{sin\theta}{\cancel{cos\theta}} \frac{v_0^2\, cos^{\cancel{2}}\theta}{g} \qquad x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta$$

Moto dei Proiettili

$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$



Massima Altezza: y_M

Derivata Nulla

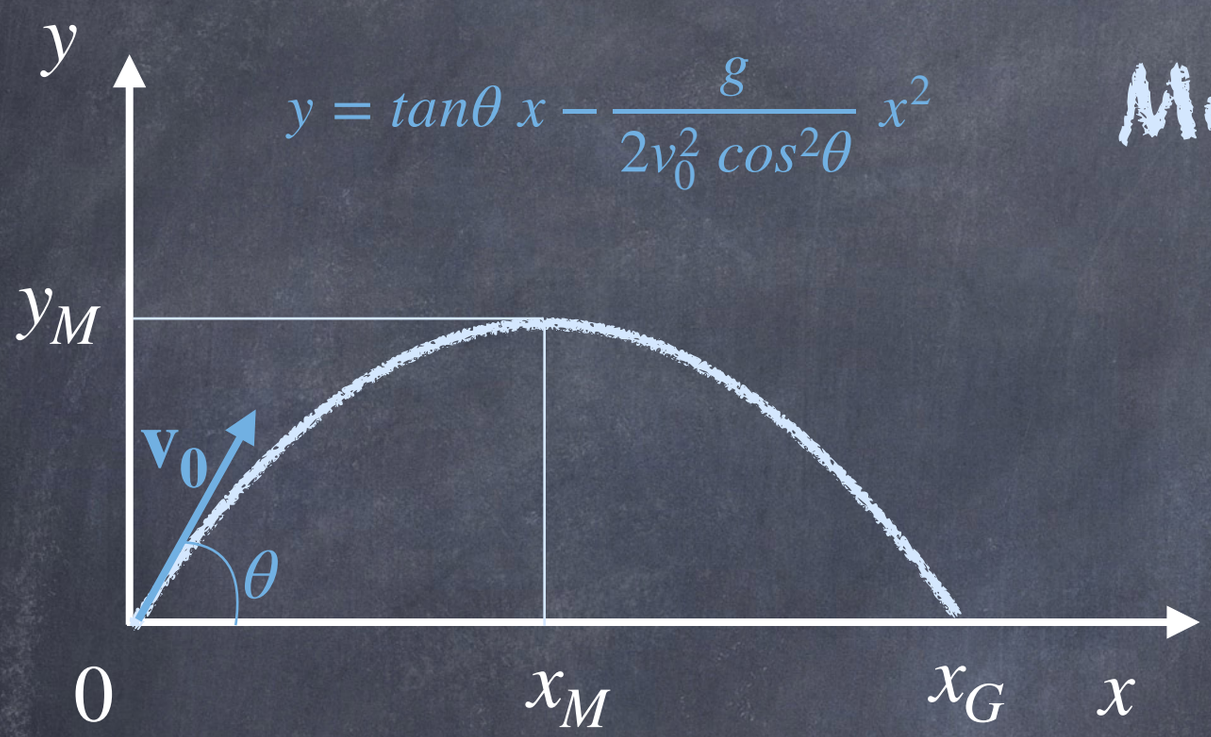
$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$\frac{dy(x_M)}{dx} = 0 \qquad tan\theta = \frac{g}{v_0^2\, cos^2\theta}\, x_M \qquad x_M = \frac{sin\theta}{\cancel{cos\theta}} \frac{v_0^2\, cos^{\cancel{2}}\theta}{g} \qquad x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta$$

$$y_M = y(x_M)$$

Moto dei Proiettili



$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$

Massima Altezza: y_M

Derivata Nulla

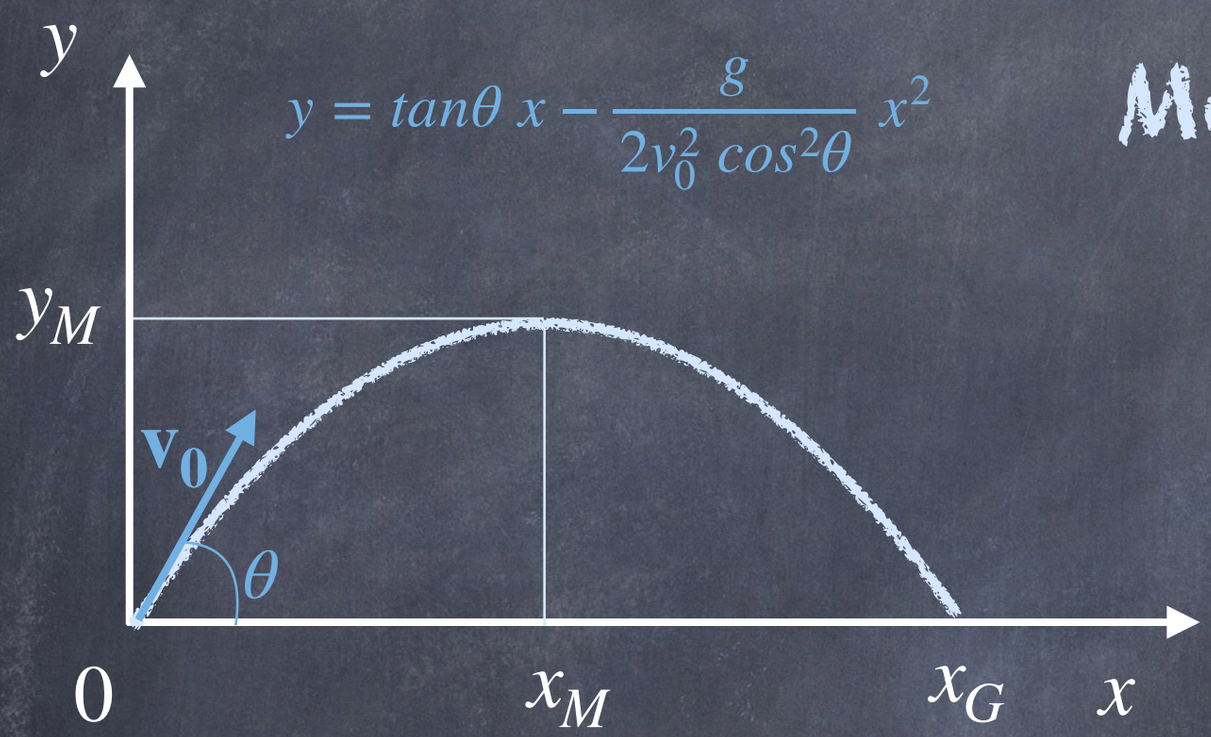
$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$\frac{dy(x_M)}{dx} = 0 \qquad tan\theta = \frac{g}{v_0^2\, cos^2\theta}\, x_M \qquad x_M = \frac{sin\theta}{\cancel{cos\theta}}\frac{v_0^2\, cos^{\cancel{2}}\theta}{g} \qquad x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta$$

$$y_M = y(x_M) \qquad y_M = \frac{sin\theta}{cos\theta}\left(\frac{v_0^2}{g}\, sin\theta\, cos\theta\right) - \frac{g}{2v_0^2\, cos^2\theta}\left(\frac{v_0^2}{g}\, sin\theta\, cos\theta\right)^2$$

Moto dei Proiettili



$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$

Massima Altezza: y_M

Derivata Nulla

$$\frac{dy(x_M)}{dx} = 0$$

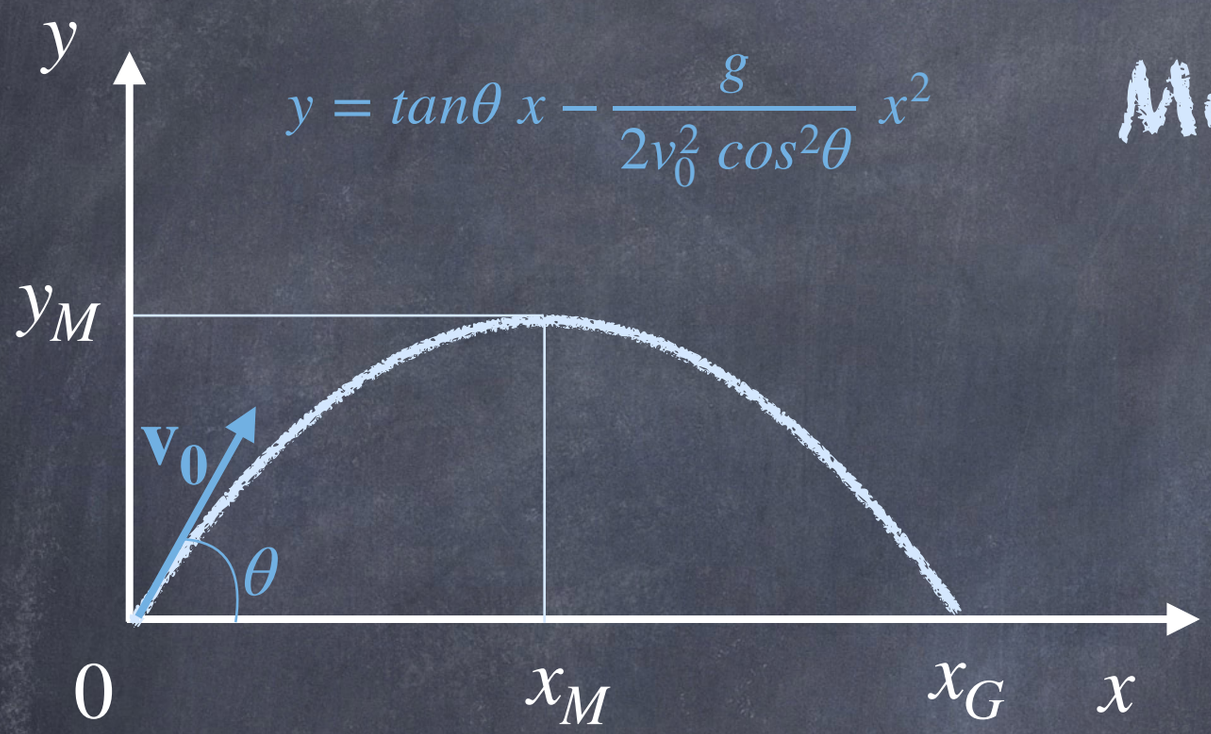
$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$\frac{dy(x_M)}{dx} = 0 \qquad tan\theta = \frac{g}{v_0^2\, cos^2\theta}\, x_M \qquad x_M = \frac{sin\theta}{\cancel{cos\theta}}\frac{v_0^2\, cos^{\cancel{2}}\theta}{g} \qquad x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta$$

$$y_M = y(x_M) \qquad y_M = \frac{sin\theta}{\cancel{cos\theta}}\left(\frac{v_0^2}{g}\, sin\theta\, \cancel{cos\theta}\right) - \frac{g}{2v_0^2\, cos^2\theta}\left(\frac{v_0^2}{g}\, sin\theta\, cos\theta\right)^2$$

Moto dei Proiettili

$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$



Massima Altezza: y_M

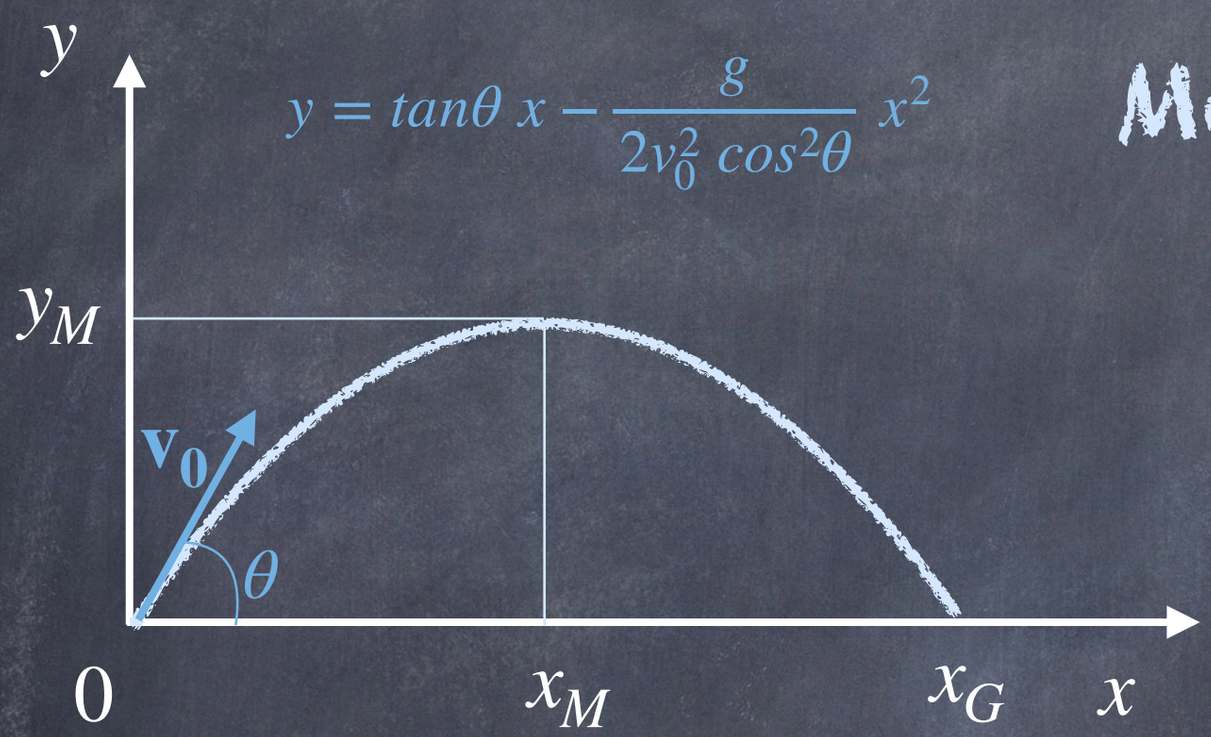
Derivata Nulla

$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$y_M = y(x_M) \qquad y_M = \frac{v_0^2}{g}\, sin^2\theta - \frac{g}{2v_0^2\, cos^2\theta}\left(\frac{(v_0^2)^2}{g^2}\, sin^2\theta\, cos^2\theta\right)$$

Moto dei Proiettili



$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$

Massima Altezza: y_M

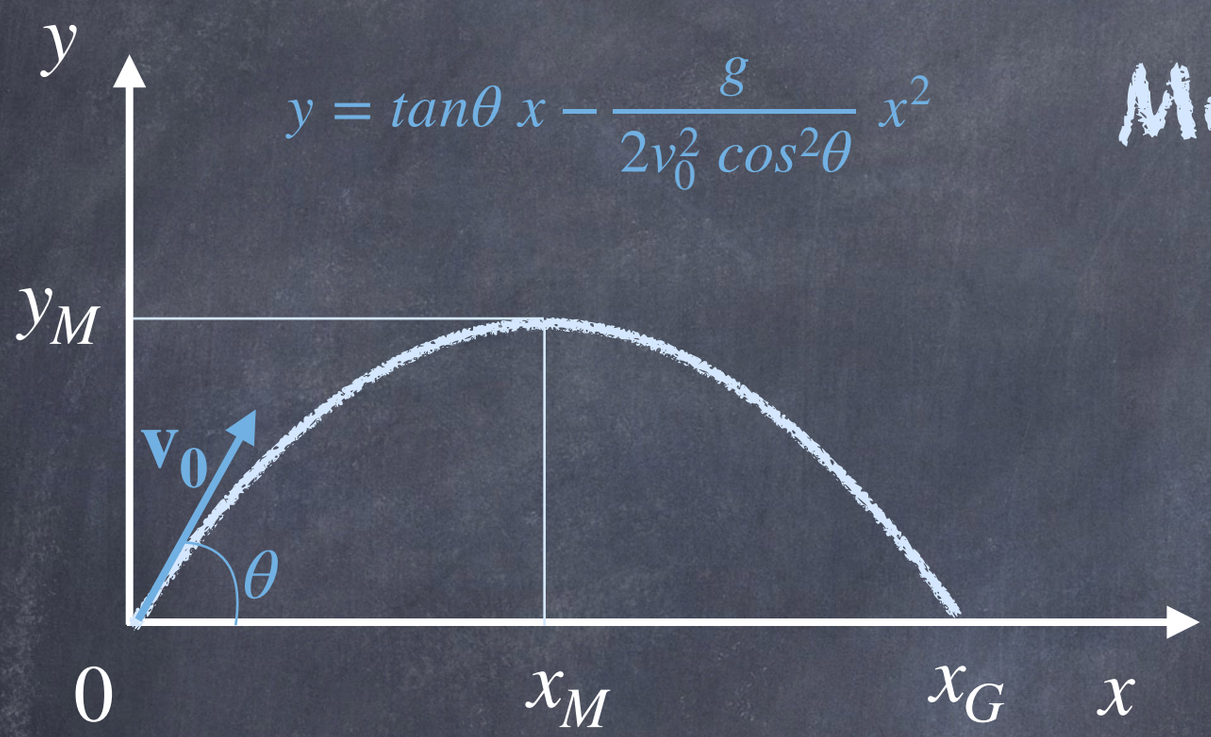
Derivata Nulla

$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$y_M = y(x_M) \qquad y_M = \frac{v_0^2}{g}\, sin^2\theta - \frac{\cancel{g}}{2\cancel{v_0^2}\, \cancel{cos^2\theta}} \left(\frac{(v_0^2)^{\cancel{2}}}{g^{\cancel{2}}}\, sin^2\theta\, \cancel{cos^2\theta} \right)$$

Moto dei Proiettili



$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$

Massima Altezza: y_M

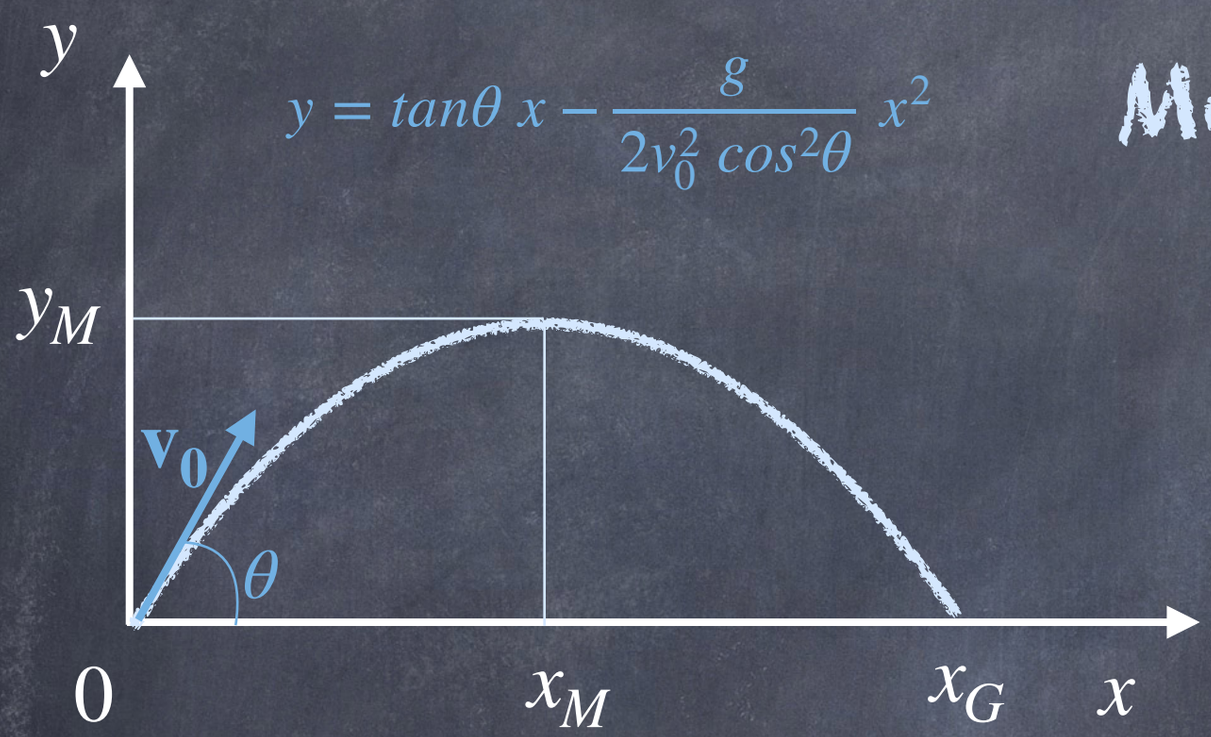
Derivata Nulla

$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$y_M = y(x_M) \qquad y_M = \frac{v_0^2}{g}\, sin^2\theta - \frac{\cancel{g}}{2\cancel{v_0^2}\, \cancel{cos^2\theta}} \left(\frac{(v_0^2)^{\cancel{2}}}{g^{\cancel{2}}}\, sin^2\theta\, \cancel{cos^2\theta} \right) = \frac{v_0^2}{g}\, sin^2\theta - \frac{v_0^2}{2g}\, sin^2\theta$$

Moto dei Proiettili



$$y = tan\theta\, x - \frac{g}{2v_0^2\, cos^2\theta}\, x^2$$

Massima Altezza: y_M

Derivata Nulla

$$\frac{dy(x_M)}{dx} = 0$$

$$\frac{dy(x)}{dx} = tan\theta - \frac{g}{v_0^2\, cos^2\theta}\, x$$

$$y_M = y(x_M)$$

$$y_M = \frac{v_0^2}{g}\, sin^2\theta - \frac{\cancel{g}}{2\cancel{v_0^2}\, \cancel{cos^2\theta}} \left(\frac{(v_0^2)^{\cancel{2}}}{g^{\cancel{2}}}\, sin^2\theta\, \cancel{cos^2\theta} \right) = \frac{v_0^2}{g}\, sin^2\theta - \frac{v_0^2}{2g}\, sin^2\theta$$

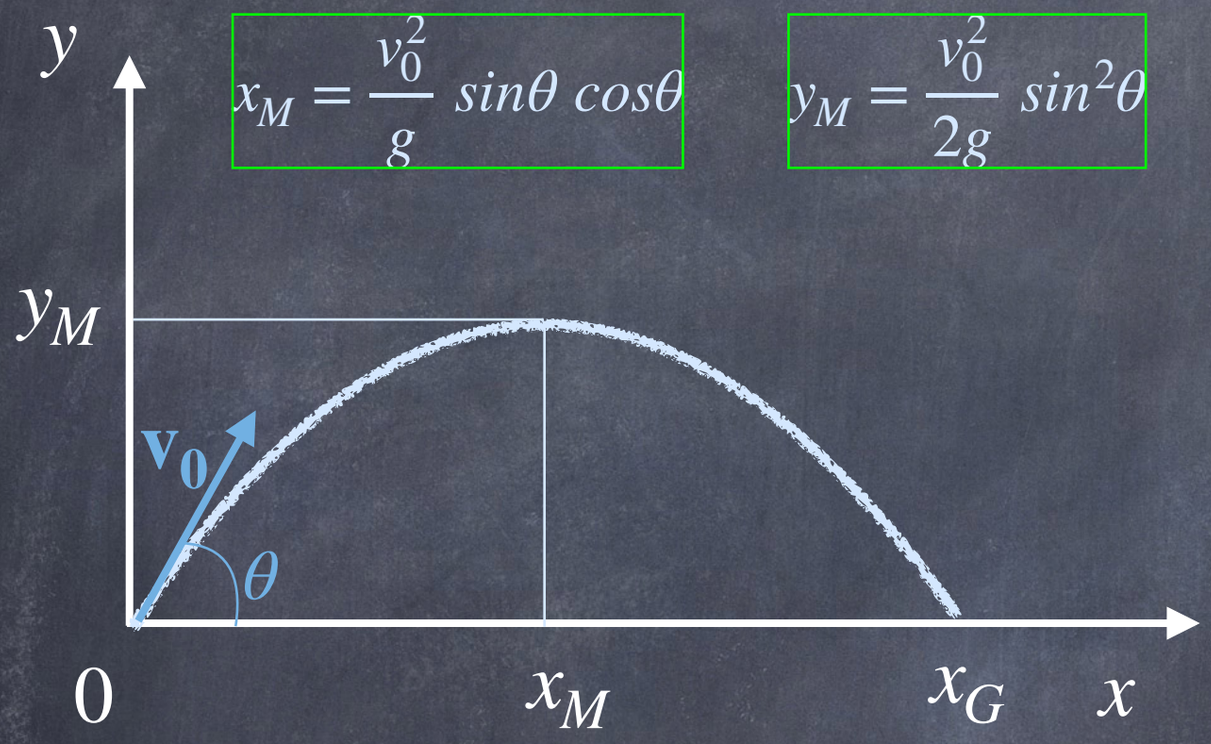
$$\boxed{y_M = \frac{v_0^2}{2g}\, sin^2\theta}$$

Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta$$

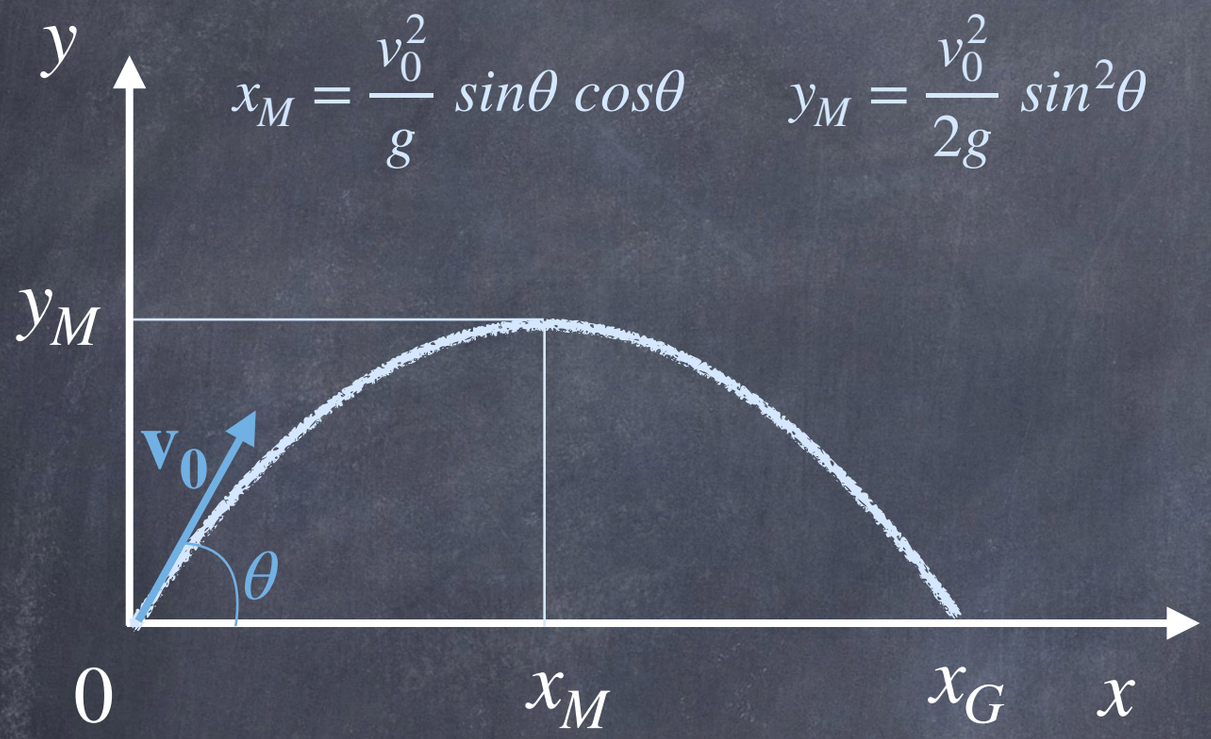
$$y_M = \frac{v_0^2}{2g}\, sin^2\theta$$

Gittata: x_G



Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$



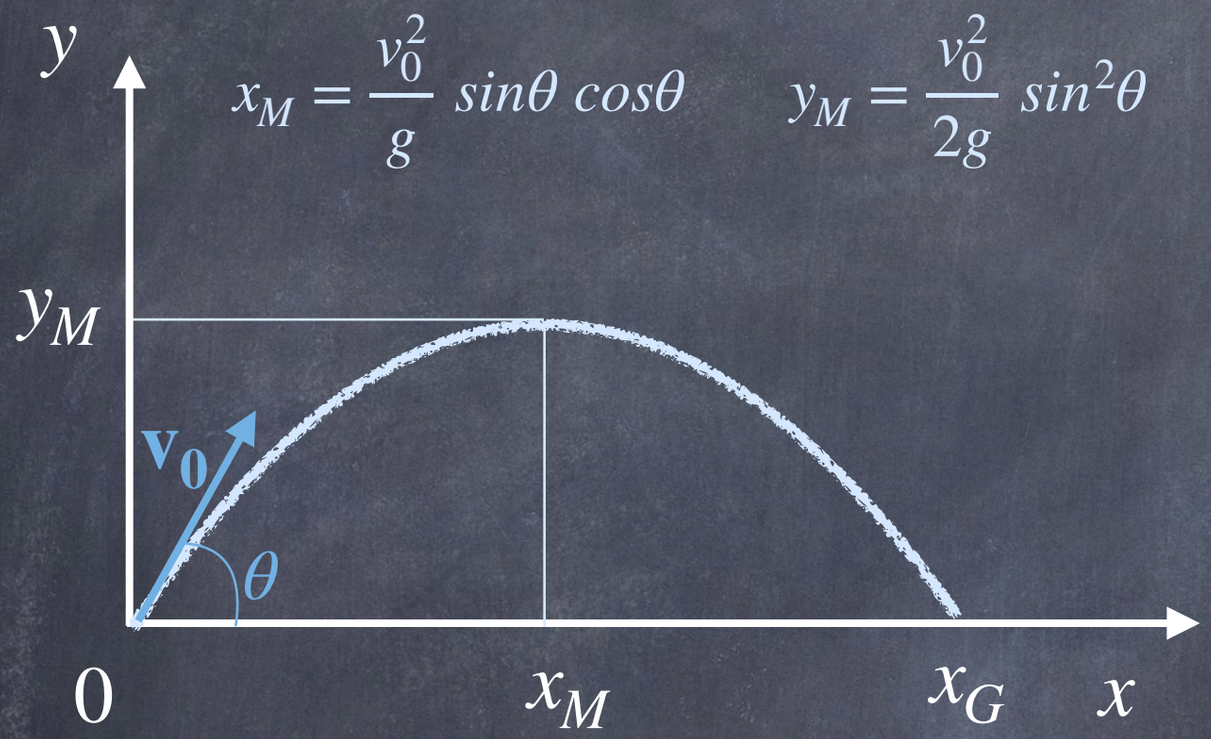
Gittata: x_G

Parabola Simmetrica risposte al Massimo:

$$x_G = 2\, x_M \quad = \frac{2v_0^2}{g}\, sin\theta\, cos\theta$$

Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$



Gittata: x_G

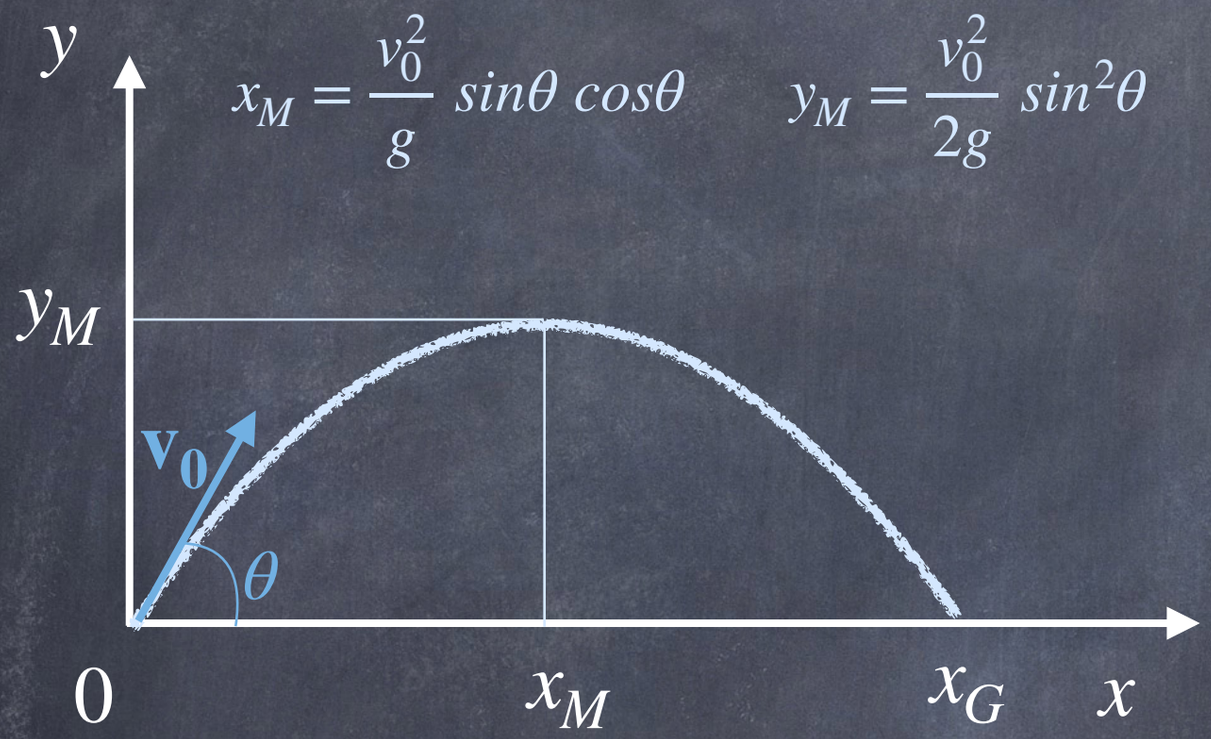
Parabola Simmetrica risposte al Massimo:

$$x_G = 2\, x_M \quad = \frac{2v_0^2}{g}\, sin\theta\, cos\theta$$

x_G dipende solo da θ

Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$



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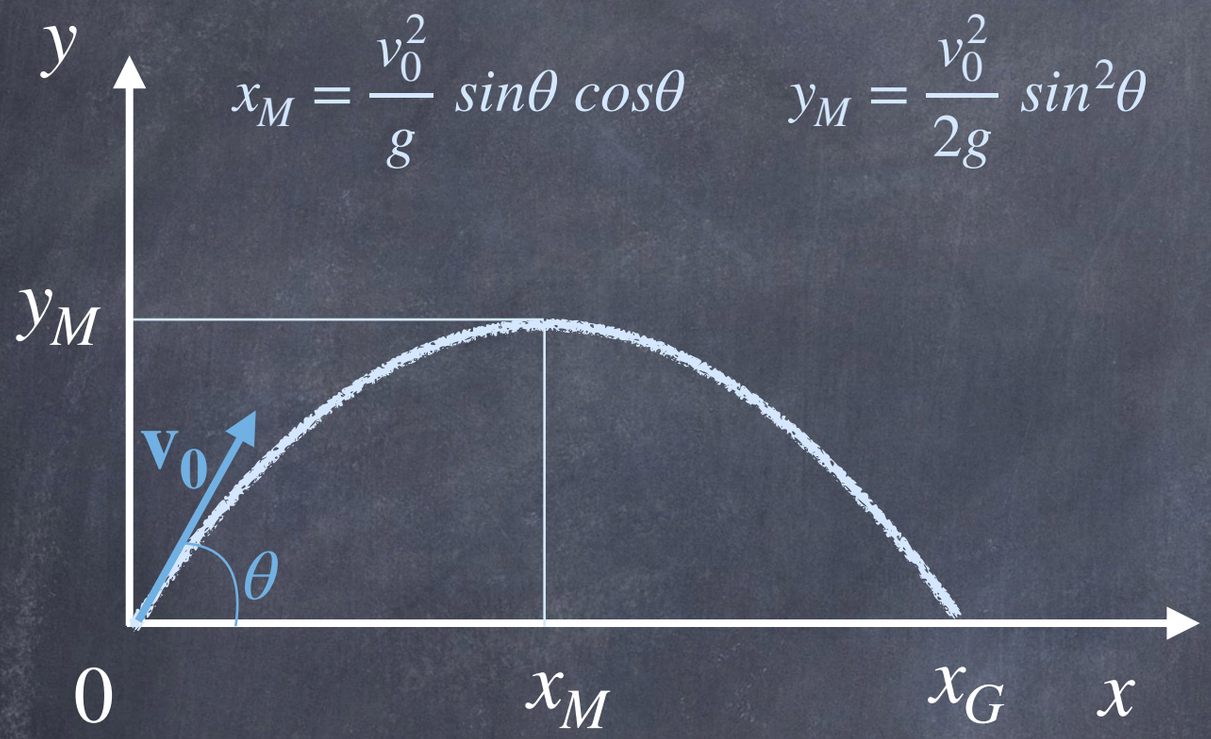
Gittata Massima: $\dfrac{dx_G}{d\theta} = 0$

Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$



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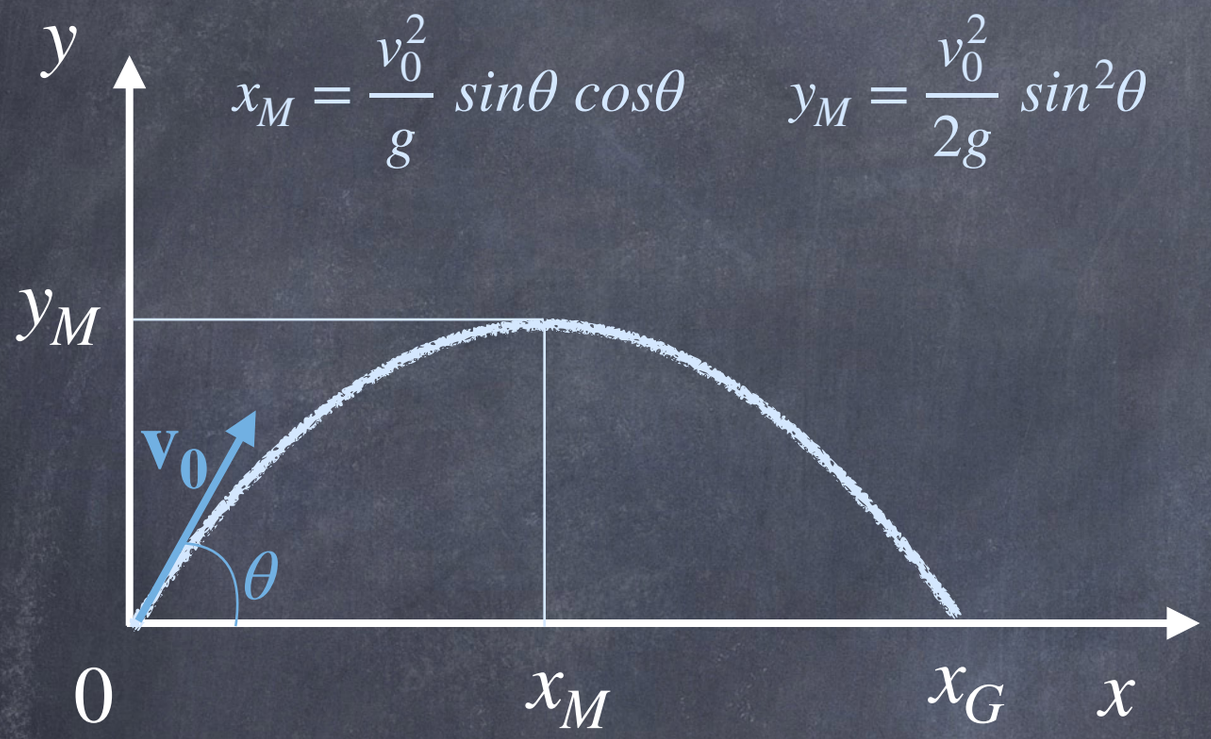
Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$



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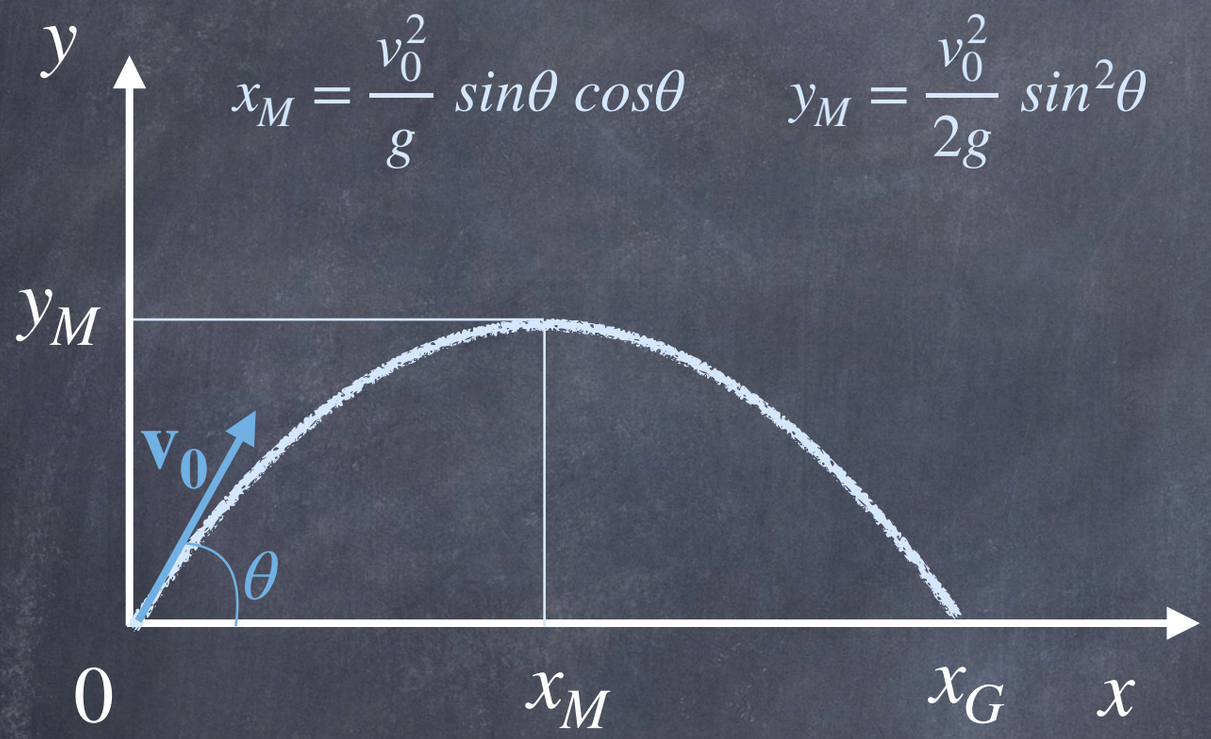
Gittata Massima: $\dfrac{dx_G}{d\theta} = 0$

$$\frac{dx_G}{d\theta} = \frac{2v_0^2}{g}\,(cos^2\theta - sin^2\theta) \;\; = 0$$

Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$



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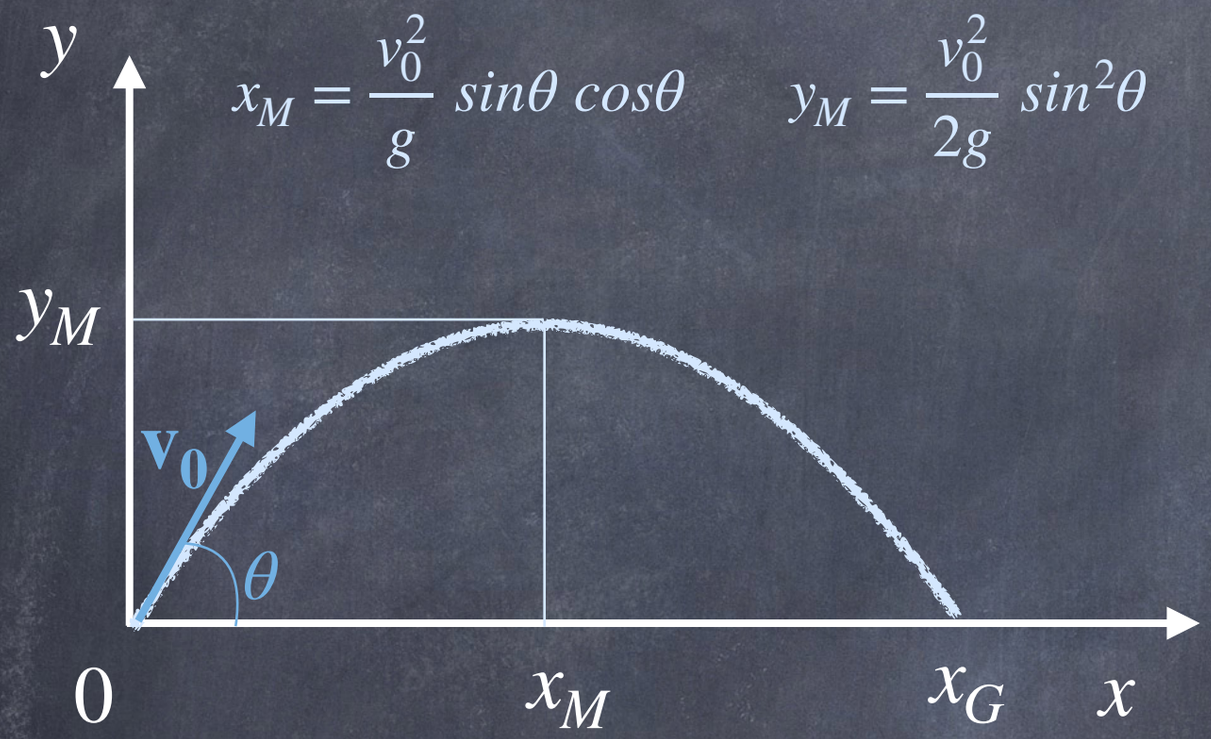
Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$



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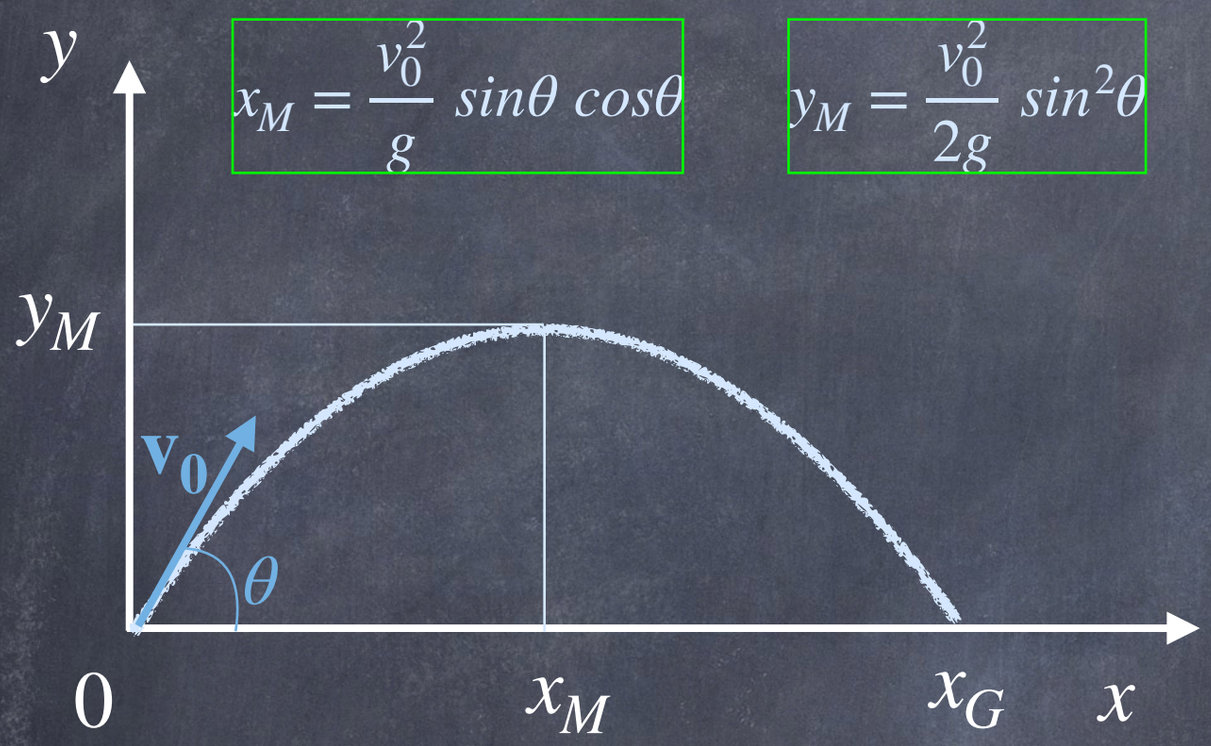
Gittata Massima: $\dfrac{dx_G}{d\theta} = 0$

$$\frac{dx_G}{d\theta} = \frac{2v_0^2}{g}\,(cos^2\theta - sin^2\theta) \;= 0 \longrightarrow cos^2\theta = sin^2\theta \longrightarrow \theta_{G_{Max}} = 45^\circ$$

$$x_M = \frac{v_0^2}{g}$$

Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$



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$$x_G = \frac{2v_0^2}{g}\, sin\theta\, cos\theta$$

$$2\, sin\theta\, cos\theta = sin(2\theta)$$

$$x_G = \frac{v_0^2}{g}\, sin(2\theta)$$

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$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$

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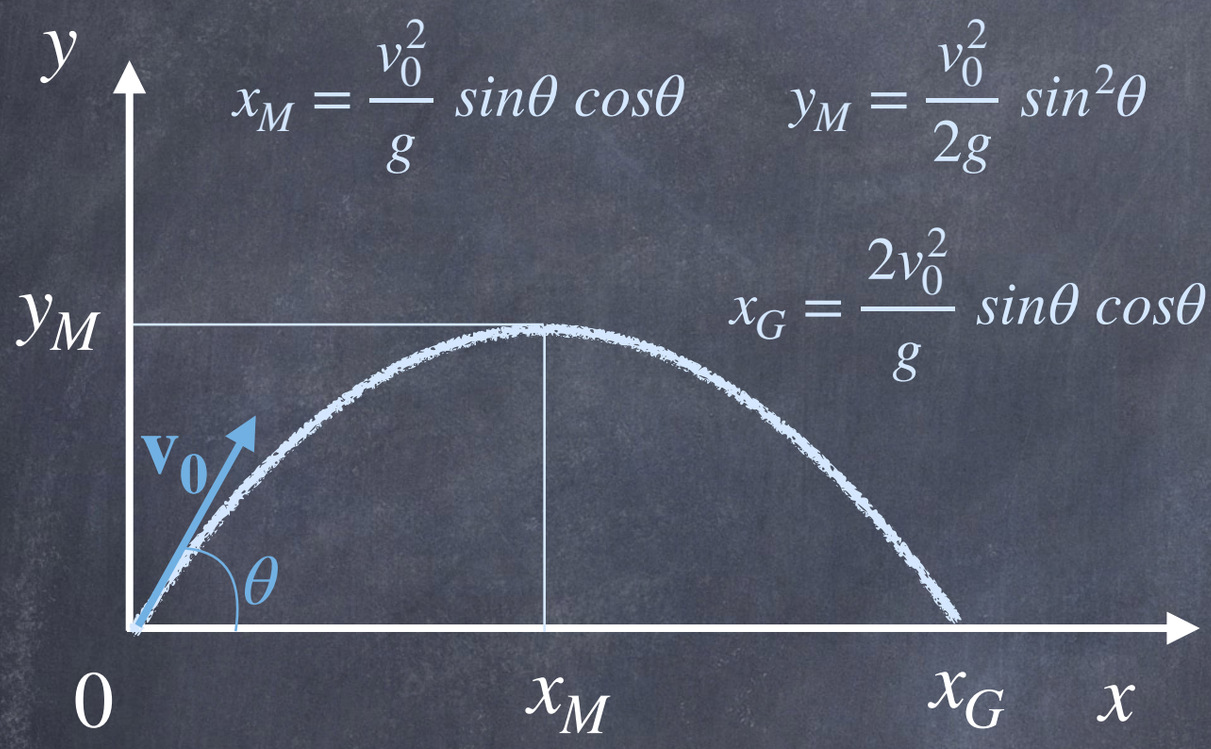

Tempo di Volo: t_G

Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$

Tempo di Volo: t_G

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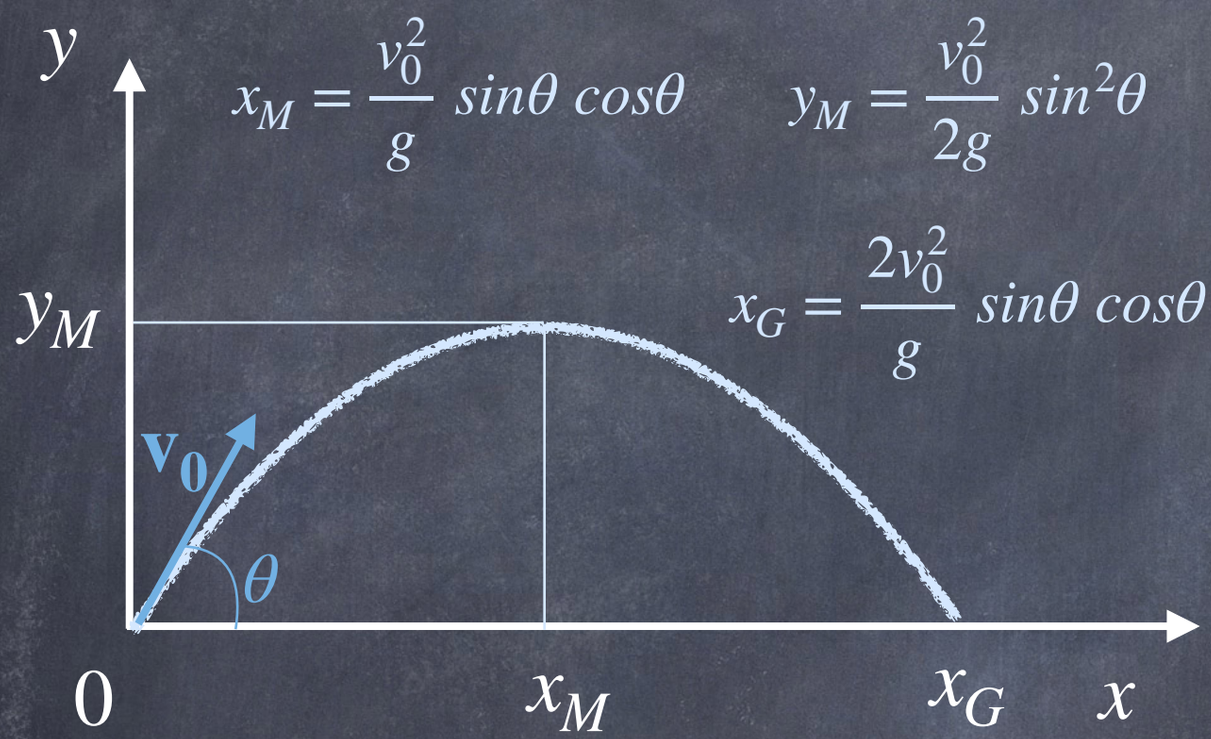


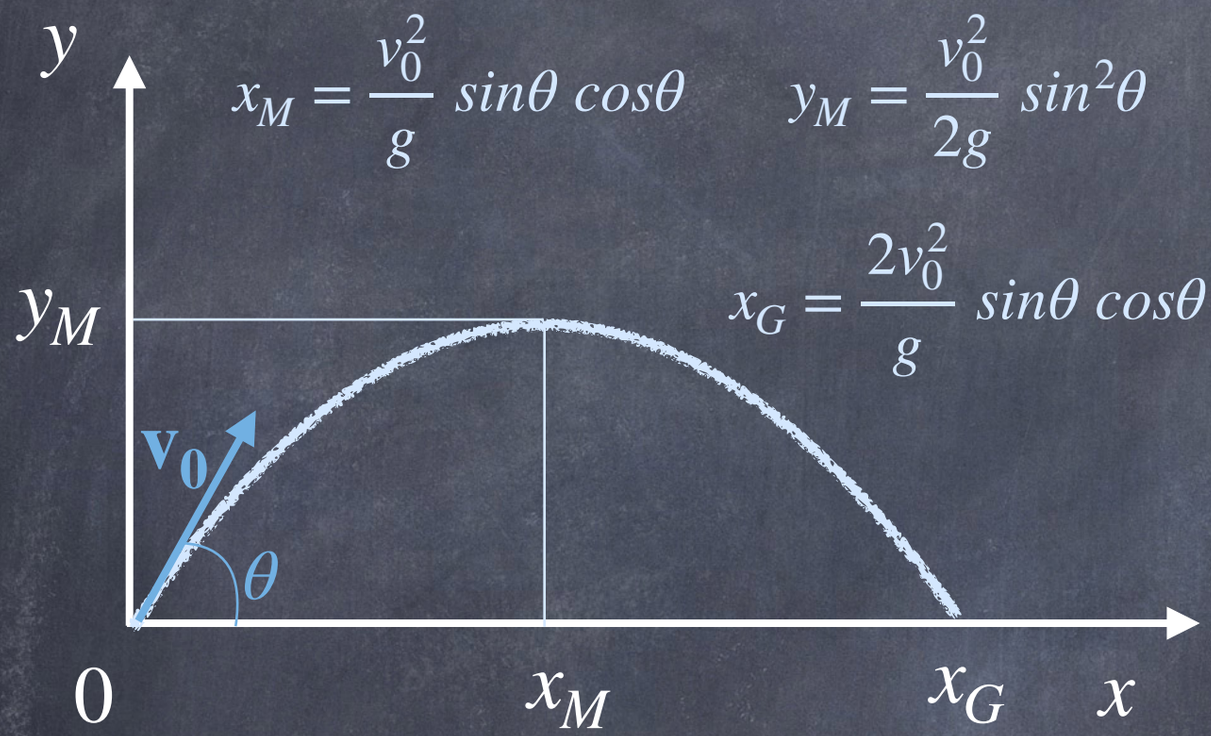
Moto lungo asse x $\qquad x(t) = v_0\, cos\theta\, t$

Moto dei Proiettili

$$x_M = \frac{v_0^2}{g}\, sin\theta\, cos\theta \qquad y_M = \frac{v_0^2}{2g}\, sin^2\theta$$

Tempo di Volo: t_G

$$x_G = \frac{2v_0^2}{g}\, sin\theta\, cos\theta$$



Moto lungo asse x

$$x(t) = v_0\, cos\theta\, t$$

$$x_G = v_0\, cos\theta\, t_G$$

Moto dei Proiettili

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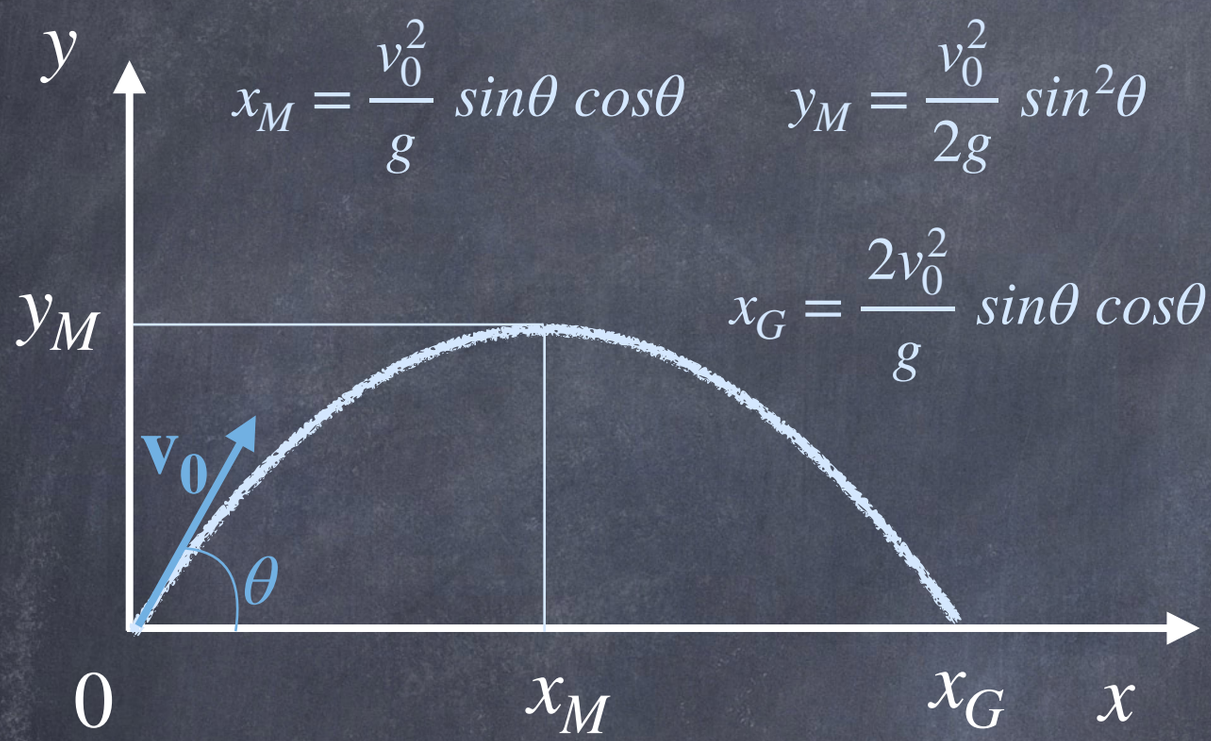


Tempo di Volo: t_G

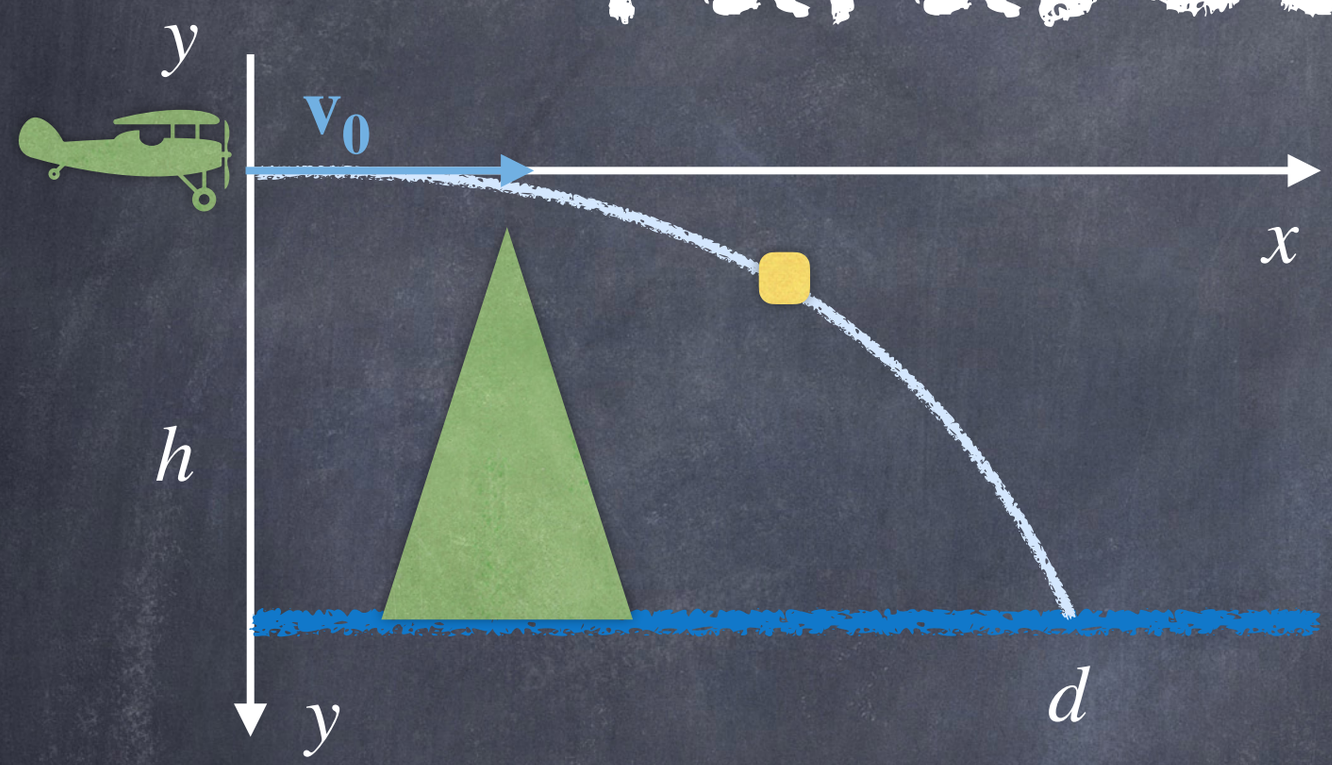
Moto lungo asse x

$$x(t) = v_0\, cos\theta\, t$$

$$x_G = v_0\, cos\theta\, t_G$$

$$\boxed{t_G = \frac{x_G}{v_0\, cos\theta} = \frac{2v_0\, sin\theta}{g}}$$

Esercizio Moto Parabolico 2



- Aereo $v_0 = 200\ km/h$
- Quota $h = 600\ m$

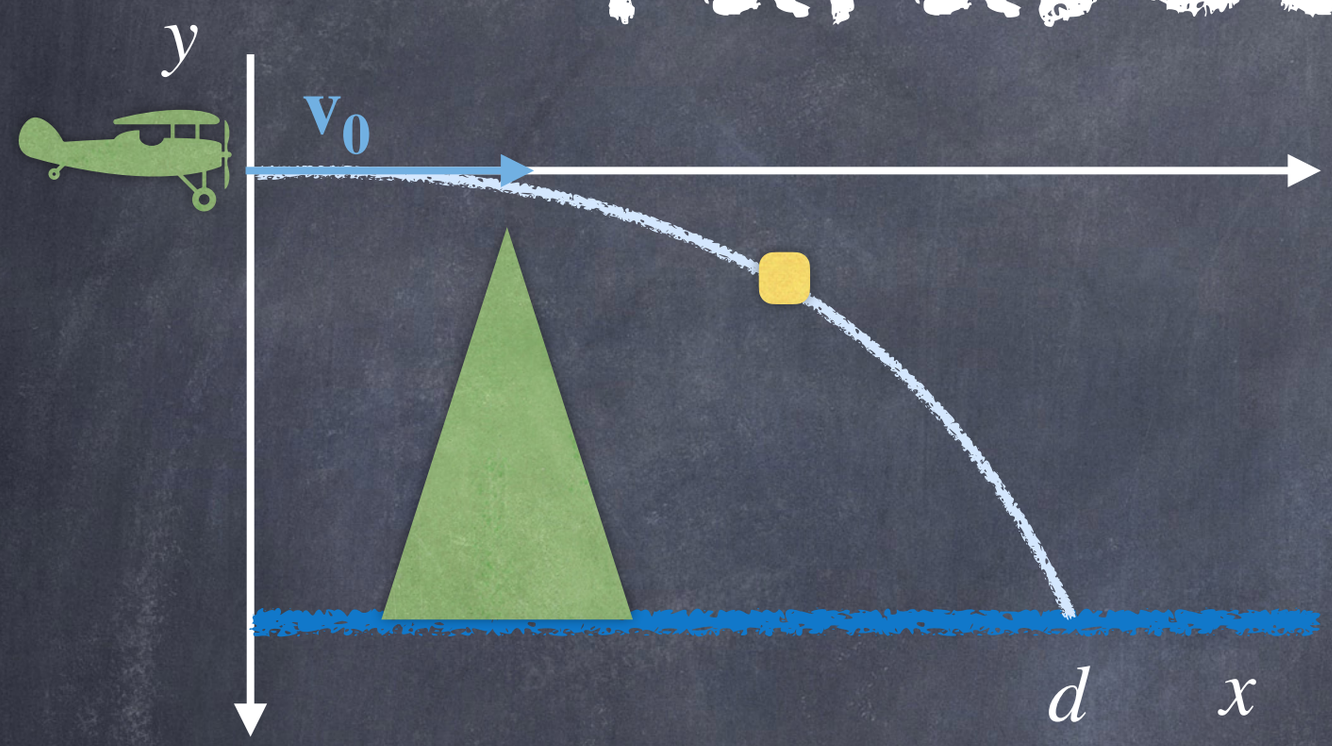
Trovare:

- Distanza Rilascio d

$$v_0 = 200 \frac{100\ m}{3600\ s} = 55.6\ m/s$$

$$\begin{cases} x(t) = v_0\, t \\ y(t) = \frac{1}{2} g t^2 \end{cases} \qquad \begin{cases} d = v_0\, t \\ h = \frac{1}{2} g t^2 \end{cases} \qquad \begin{cases} t = \frac{d}{v_0} \\ h = \frac{1}{2} g \left(\frac{d}{v_0}\right)^2 \end{cases} \longrightarrow \quad d^2 = v_0^2 \frac{2h}{g}$$

Esercizio Moto Parabolico 2



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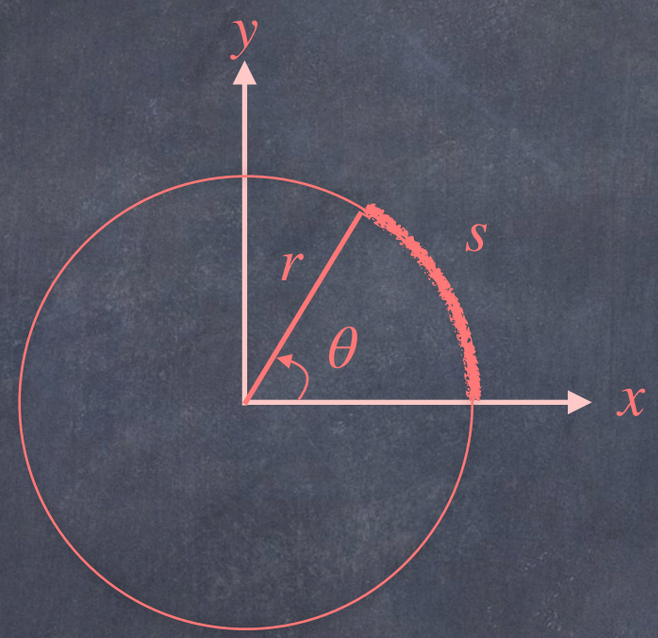
$$d = v_0\sqrt{\frac{2h}{g}} \qquad = 55.6\sqrt{\frac{2\cdot 600}{9.81}} \simeq 614\ m$$

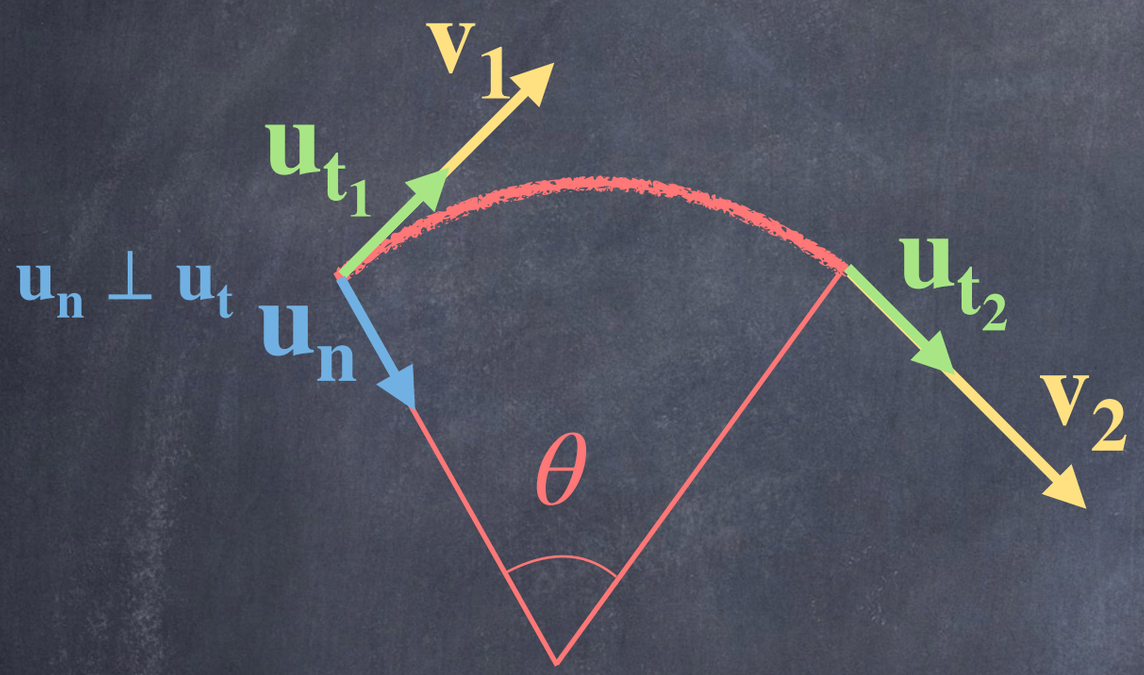
Richiamo di Matematica



Lunghezza Arco $s(\theta) = r\,\theta$

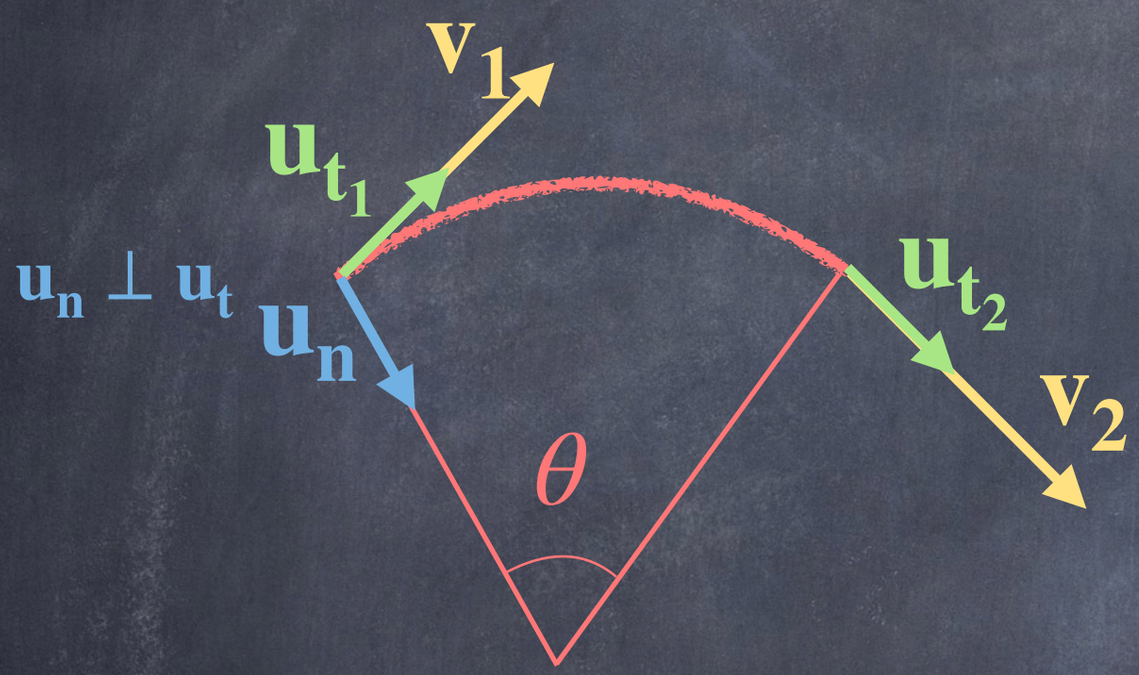
Circonferenza $s(2\pi) = r\,2\pi$

Accelerazione nel Moto Piano



$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

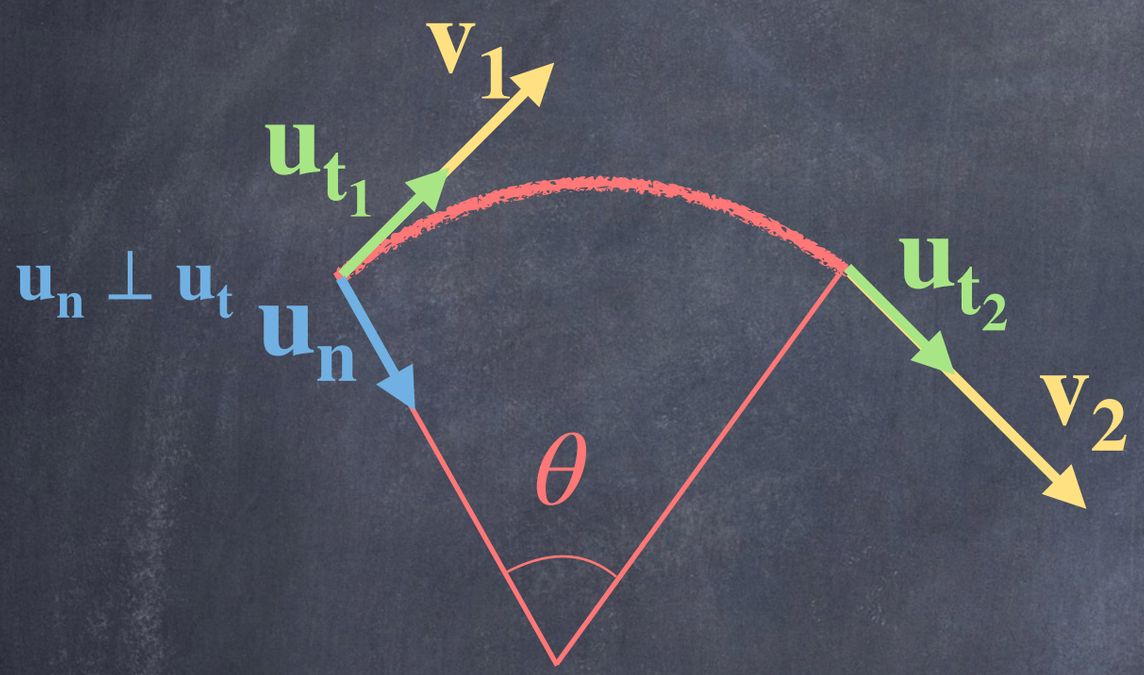
Accelerazione nel Moto Piano



$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

$$v = |\mathbf{v}| \qquad \mathbf{v} = v\,\mathbf{u}_t$$

Accelerazione nel Moto Piano



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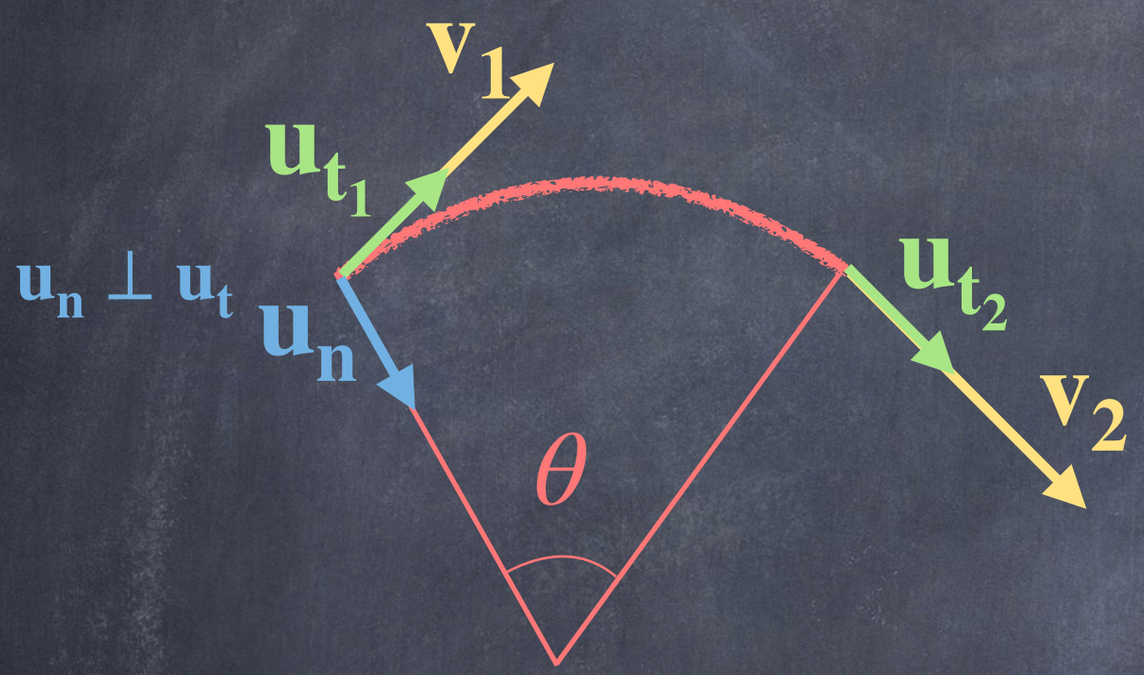
Accelerazione nel Moto Piano



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$$\frac{d\mathbf{v}}{dt} = \boxed{\frac{dv}{dt}\,\mathbf{u}_t} + \boxed{v\,\frac{d\mathbf{u}_t}{dt}}$$

Variazione Modulo Velocita'

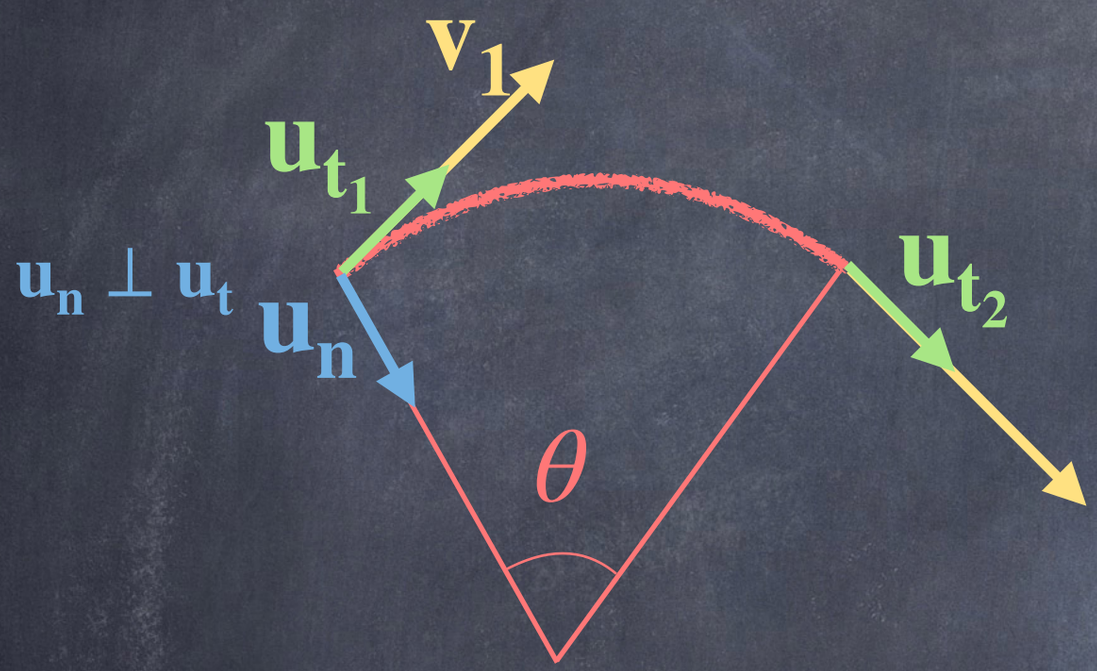
Variazione Direzione Velocita'

Accelerazione nel Moto Piano

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

$$\mathbf{v} = v\ \mathbf{u_t}$$

$$v = |\mathbf{v}|$$



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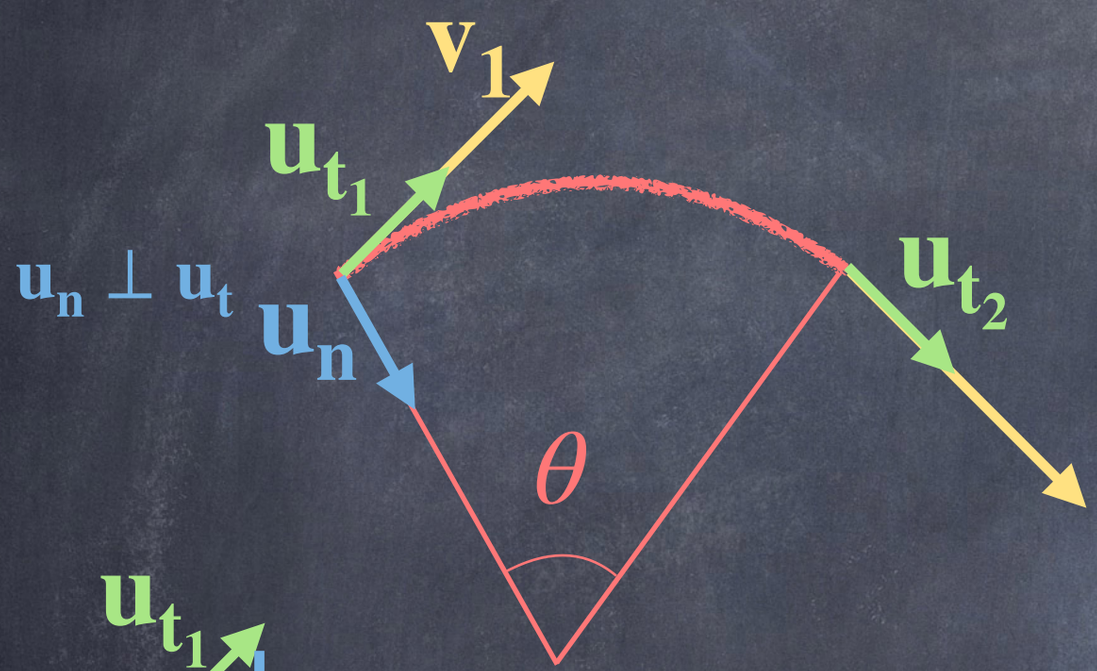

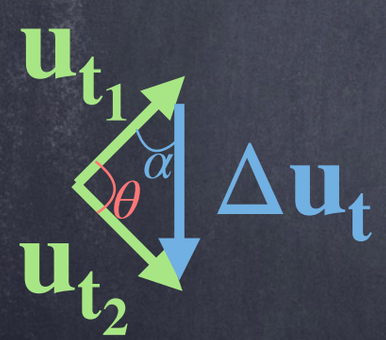

$$\alpha = \frac{1}{2}(180^\circ - \theta)$$

Accelerazione nel Moto Piano

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

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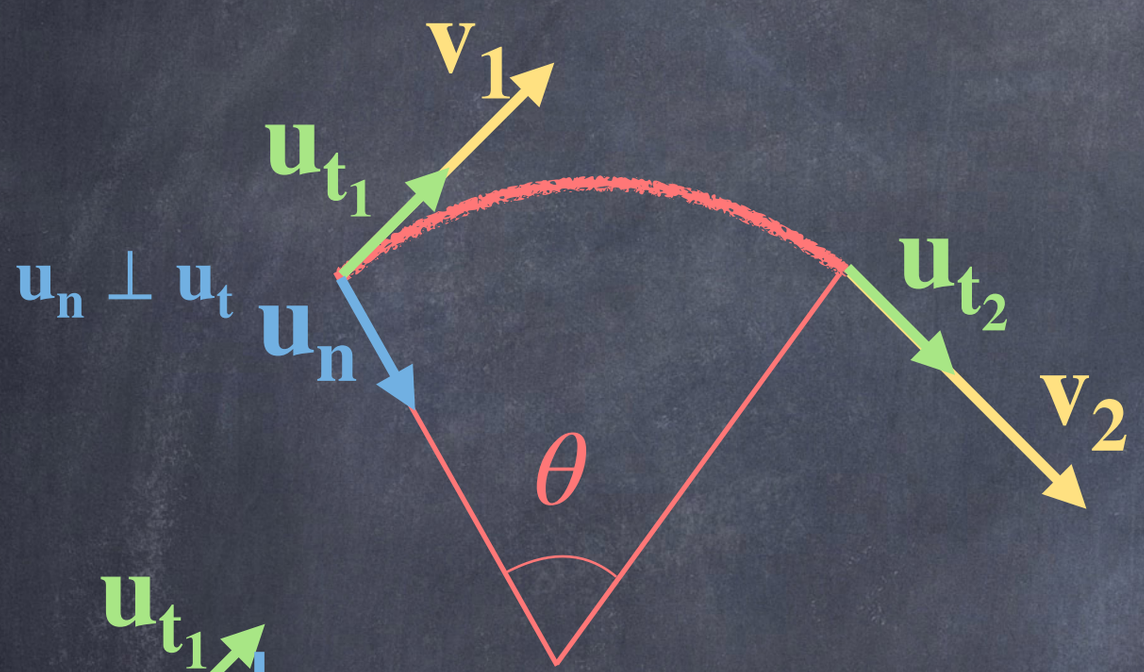

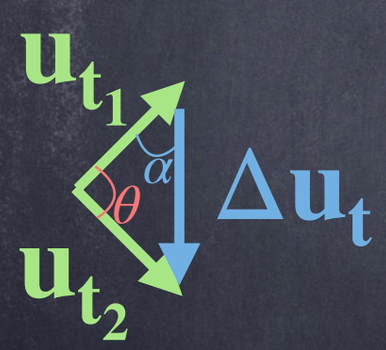

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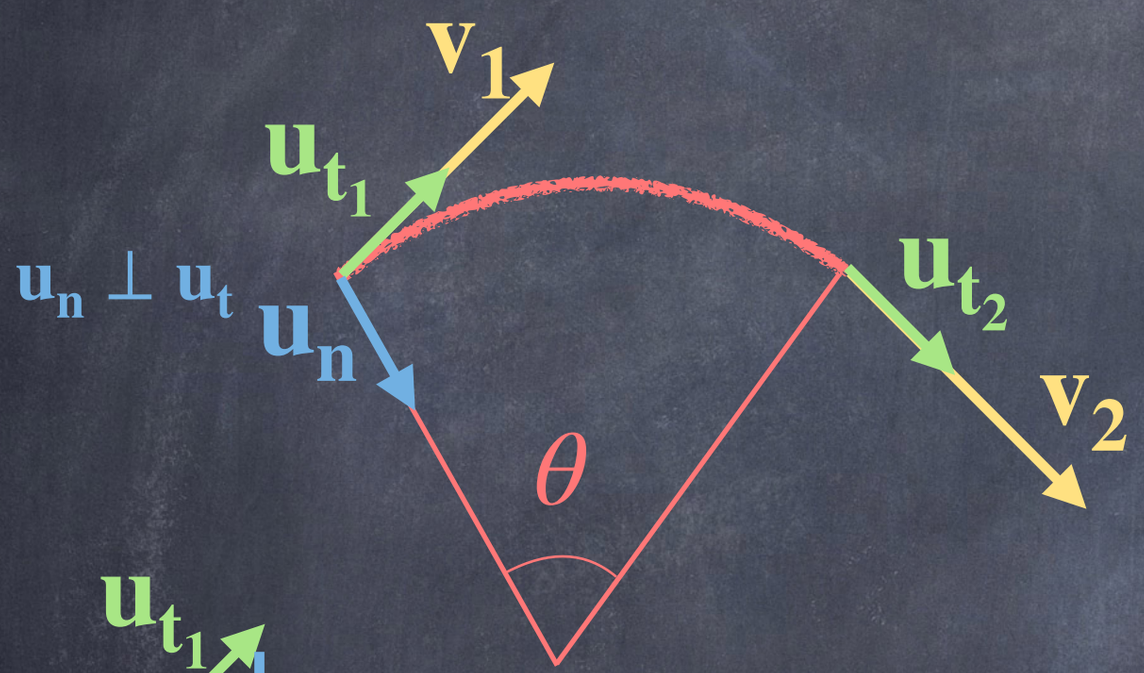

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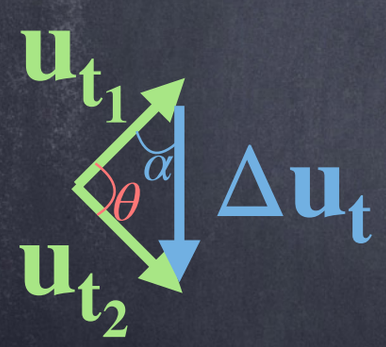

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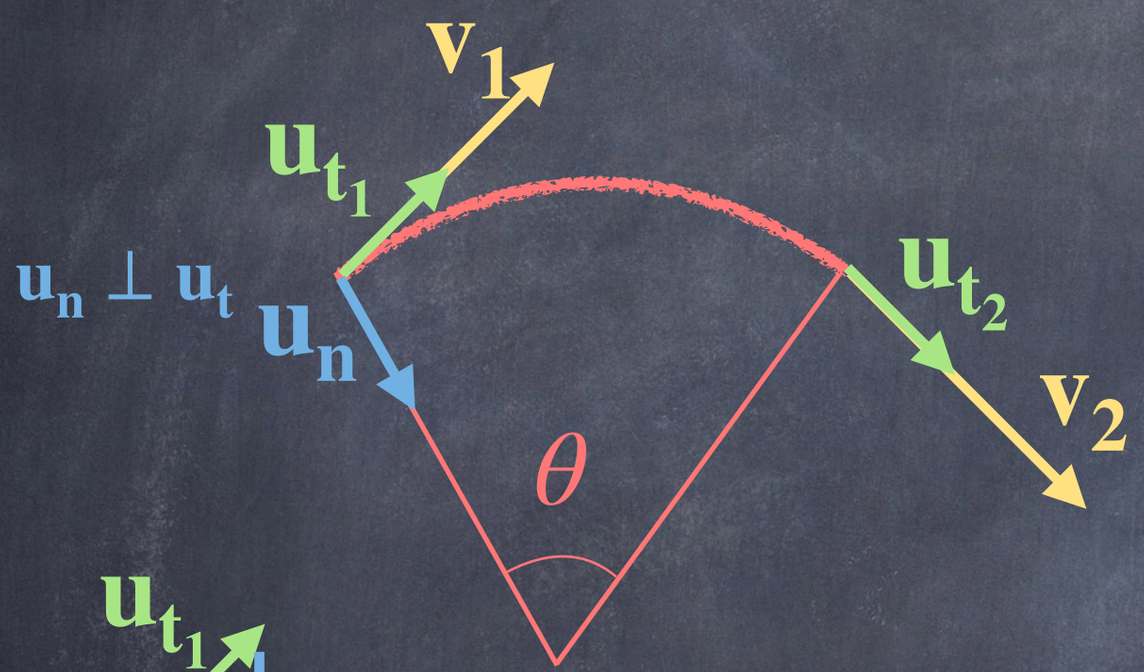

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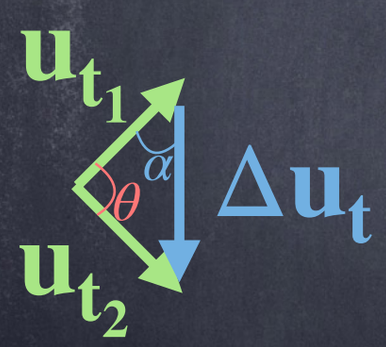

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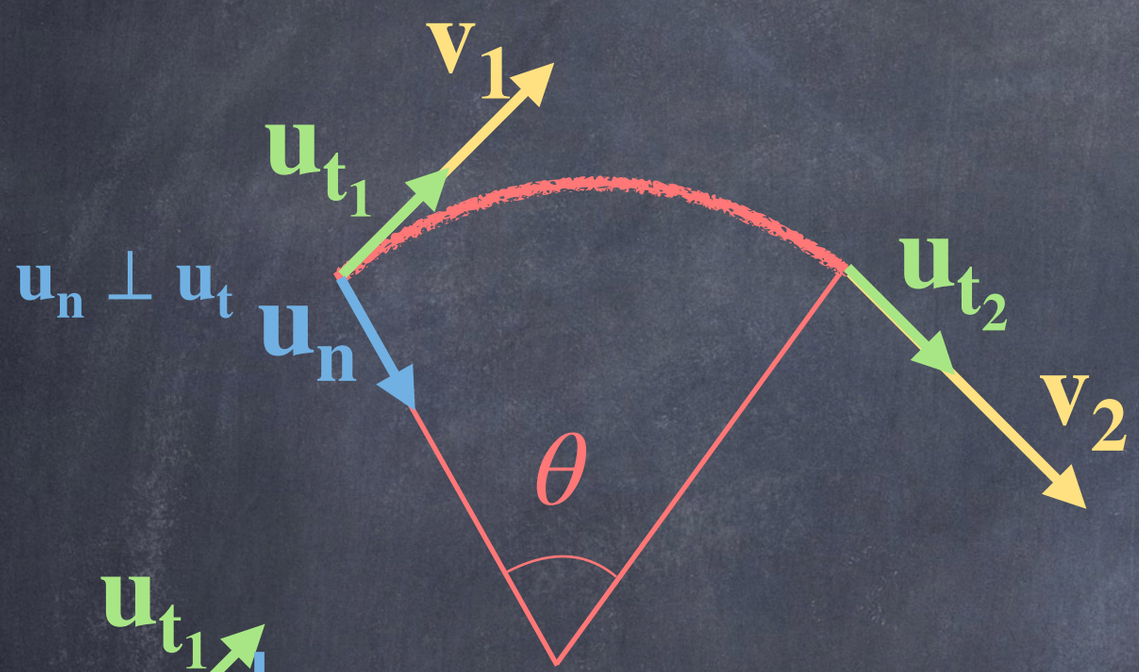

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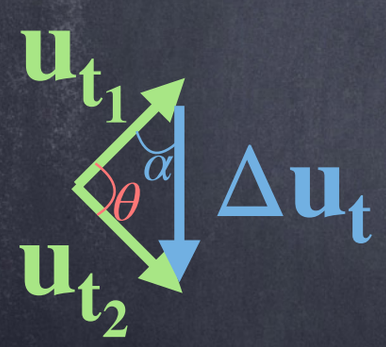
$$\frac{d\mathbf{u_t}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{u_t}}{\Delta t} \qquad \Delta t \to 0$$

$$\theta \to 0 \qquad \alpha \to 90^\circ \qquad d\mathbf{u_t} \,//\, \mathbf{u_n}$$



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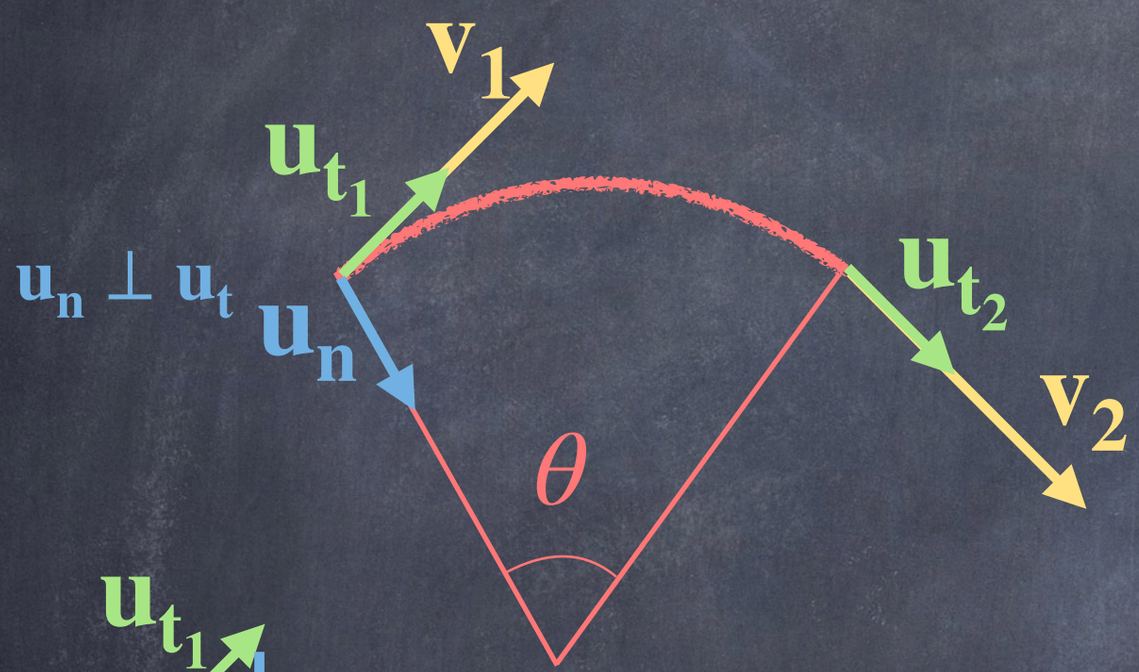

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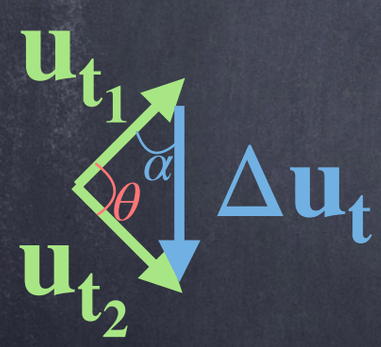
$$\frac{d\mathbf{u_t}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{u_t}}{\Delta t} \qquad \Delta t \to 0$$

$$\theta \to 0 \qquad \alpha \to 90^\circ \qquad d\mathbf{u_t} \,//\, \mathbf{u_n}$$



$$d\mathbf{u_t} = d\theta\ \mathbf{u_n}$$

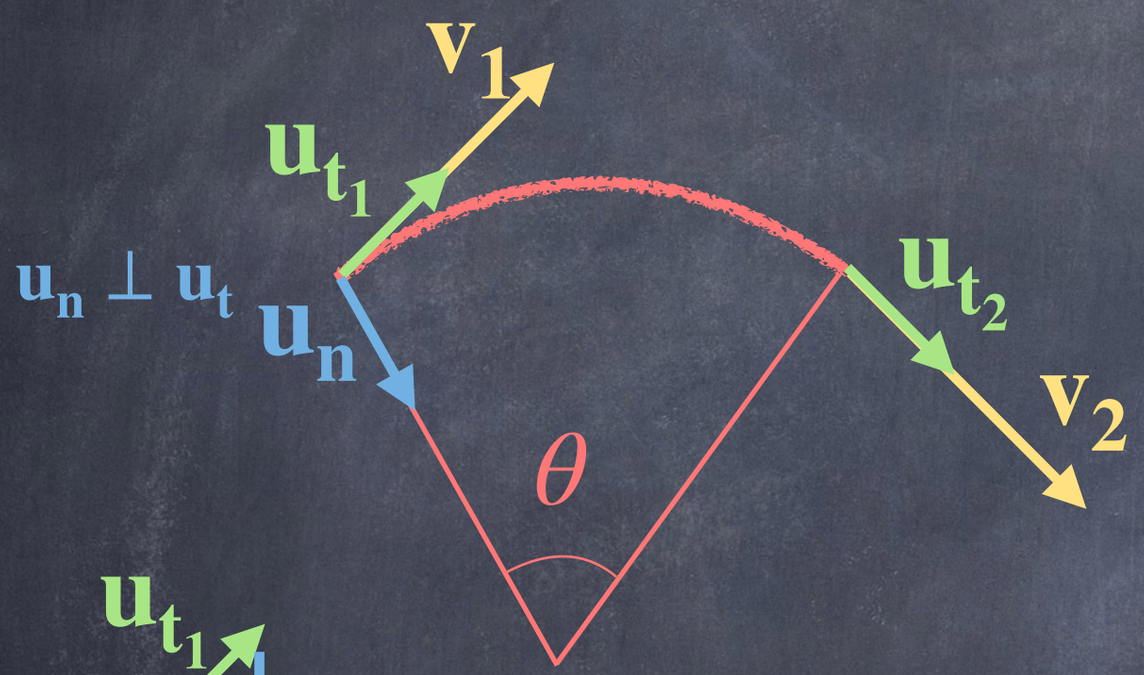
$$\alpha = \frac{1}{2}(180^\circ - \theta)$$

Accelerazione nel Moto Piano

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

$$\mathbf{v} = v\ \mathbf{u_t}$$

$$v = |\mathbf{v}|$$



$$\frac{d\mathbf{v}}{dt} = \frac{dv}{dt}\,\mathbf{u_t} + v\,\frac{d\mathbf{u_t}}{dt}$$

$$\frac{d\mathbf{u_t}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{u_t}}{\Delta t} \qquad \Delta t \to 0$$

$$\theta \to 0 \qquad \alpha \to 90^\circ \qquad d\mathbf{u_t} \,//\, \mathbf{u_n}$$

$$\Delta \mathbf{u_t} \longrightarrow d\mathbf{u_t} = d\theta\ \mathbf{u_n} \longrightarrow \frac{d\mathbf{u_t}}{dt} = \frac{d\theta}{dt}\mathbf{u_n}$$

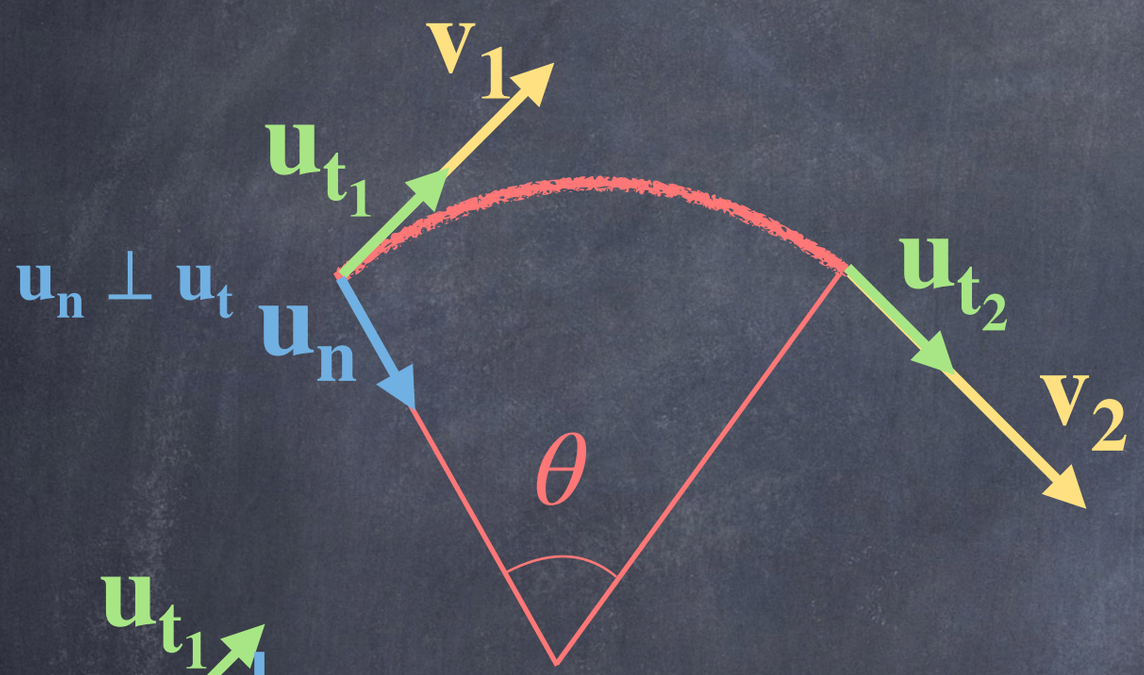
$$\alpha = \frac{1}{2}(180^\circ - \theta)$$

Accelerazione nel Moto Piano

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

$$\mathbf{v} = v\,\mathbf{u_t}$$

$$v = |\mathbf{v}|$$



$$\frac{d\mathbf{v}}{dt} = \frac{dv}{dt}\,\mathbf{u_t} + v\,\frac{d\mathbf{u_t}}{dt}$$

$$\frac{d\mathbf{u_t}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{u_t}}{\Delta t} \qquad \Delta t \to 0$$

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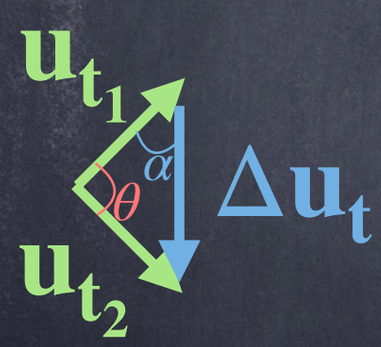


$$\Delta\mathbf{u_t} \longrightarrow d\mathbf{u_t} = d\theta\,\mathbf{u_n} \longrightarrow \frac{d\mathbf{u_t}}{dt} = \frac{d\theta}{dt}\mathbf{u_n}$$

$$\alpha = \frac{1}{2}(180^\circ - \theta)$$

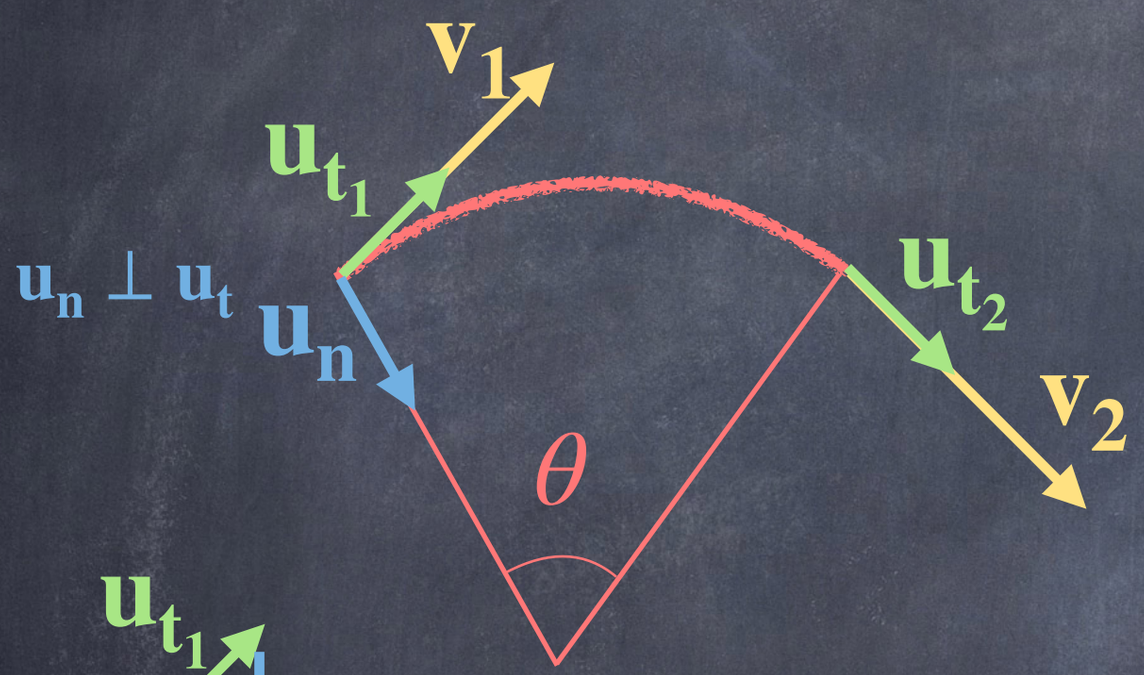
$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + v\,\frac{d\theta}{dt}\,\mathbf{u_n}$$

Accelerazione nel Moto Piano

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

$$\mathbf{v} = v\,\mathbf{u_t}$$

$$v = |\mathbf{v}|$$



$$\frac{d\mathbf{v}}{dt} = \frac{dv}{dt}\,\mathbf{u_t} + v\,\frac{d\mathbf{u_t}}{dt}$$

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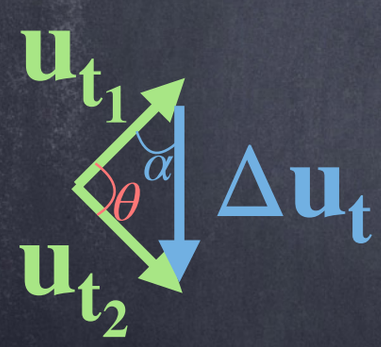
$$\theta \to 0 \qquad \alpha \to 90^\circ \qquad d\mathbf{u_t} \,//\, \mathbf{u_n}$$



$$\Delta \mathbf{u_t} \longrightarrow d\mathbf{u_t} = d\theta\,\mathbf{u_n} \longrightarrow \frac{d\mathbf{u_t}}{dt} = \frac{d\theta}{dt}\mathbf{u_n}$$

$$\alpha = \frac{1}{2}(180^\circ - \theta)$$

$$\mathbf{a} = \boxed{\frac{dv}{dt}\,\mathbf{u_t}} + \boxed{v\,\frac{d\theta}{dt}\,\mathbf{u_n}}$$

Tangente

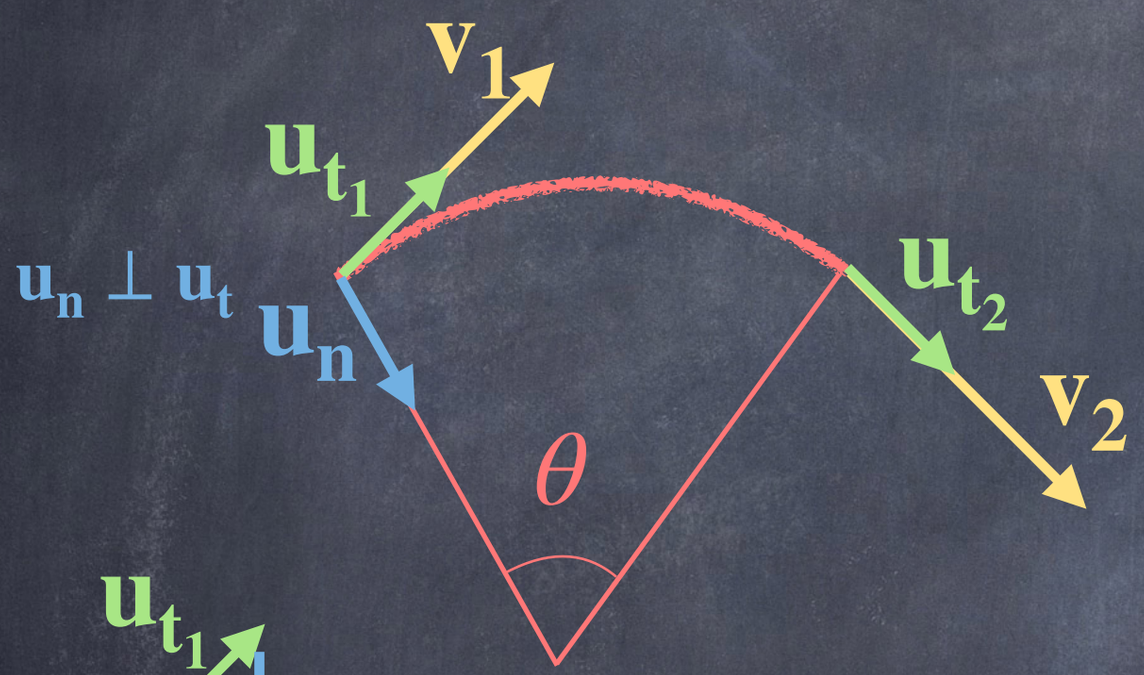
Normale all Tangente

Accelerazione nel Moto Piano

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

$$\mathbf{v} = v\,\mathbf{u_t}$$

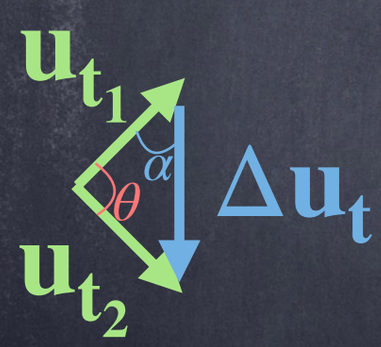
$$v = |\mathbf{v}|$$



$$\frac{d\mathbf{v}}{dt} = \frac{dv}{dt}\,\mathbf{u_t} + v\,\frac{d\mathbf{u_t}}{dt}$$

$$\frac{d\mathbf{u_t}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{u_t}}{\Delta t} \qquad \Delta t \to 0$$

$$\theta \to 0 \qquad \alpha \to 90^\circ \qquad d\mathbf{u_t} \,//\, \mathbf{u_n}$$



$$\Delta \mathbf{u_t} \longrightarrow d\mathbf{u_t} = d\theta\,\mathbf{u_n} \longrightarrow \frac{d\mathbf{u_t}}{dt} = \frac{d\theta}{dt}\mathbf{u_n}$$

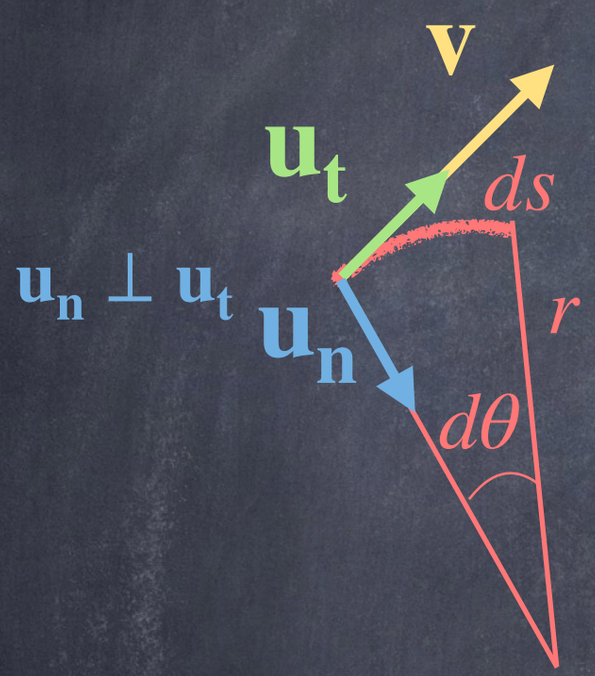
$$\alpha = \frac{1}{2}(180^\circ - \theta)$$

$$\mathbf{a} = \boxed{\frac{dv}{dt}\,\mathbf{u_t}} + \boxed{v\,\frac{d\theta}{dt}\,\mathbf{u_n}} \qquad \mathbf{a} = \mathbf{a_t} + \mathbf{a_n}$$

Tangente

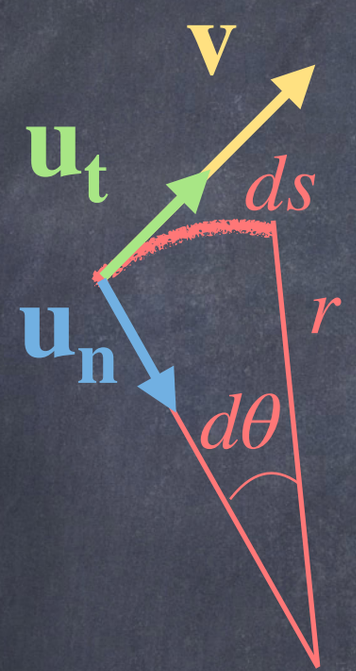
Normale all Tangente

Accelerazione nel Moto Piano



$$\boxed{\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + v\,\frac{d\theta}{dt}\,\mathbf{u_n}}$$

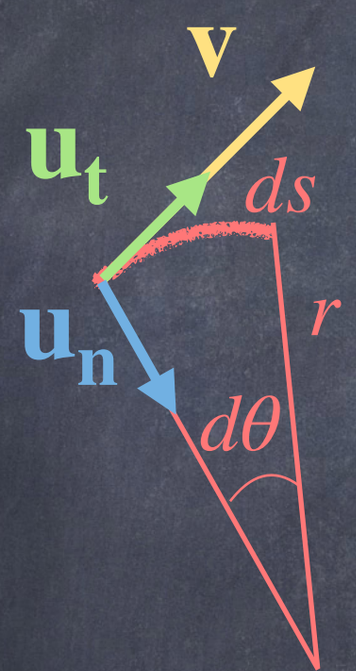
Accelerazione nel Moto Piano



$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + v\,\frac{d\theta}{dt}\,\mathbf{u_n}$$

$$ds = r\,d\theta \qquad d\theta = \frac{ds}{r} \qquad \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt} = \frac{v}{r}$$

Accelerazione nel Moto Piano

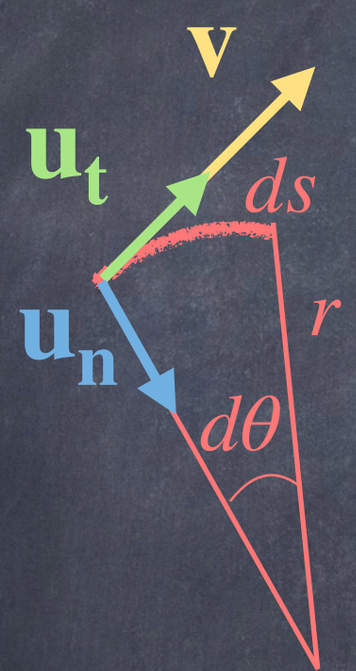


$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + v\,\frac{d\theta}{dt}\,\mathbf{u_n}$$

$$ds = r\,d\theta \qquad d\theta = \frac{ds}{r} \qquad \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt} = \frac{v}{r}$$

$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + \frac{v}{r}\,\frac{ds}{dt}\,\mathbf{u_n}$$

Accelerazione nel Moto Piano

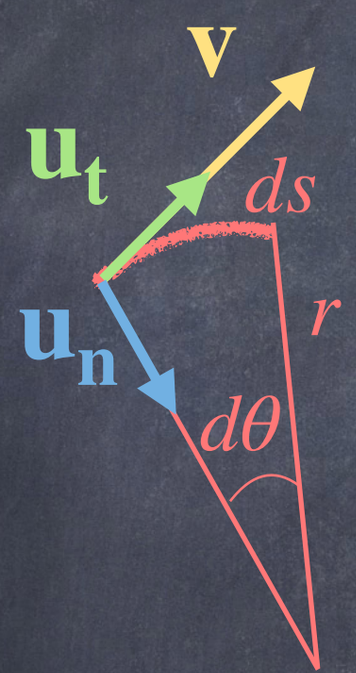


$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + v\,\frac{d\theta}{dt}\,\mathbf{u_n}$$

$$ds = r\,d\theta \qquad d\theta = \frac{ds}{r} \qquad \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt} = \frac{v}{r}$$

$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + \frac{v}{r}\,\frac{ds}{dt}\,\mathbf{u_n} \qquad \boxed{\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + \frac{v^2}{r}\,\mathbf{u_n}}$$

Accelerazione nel Moto Piano



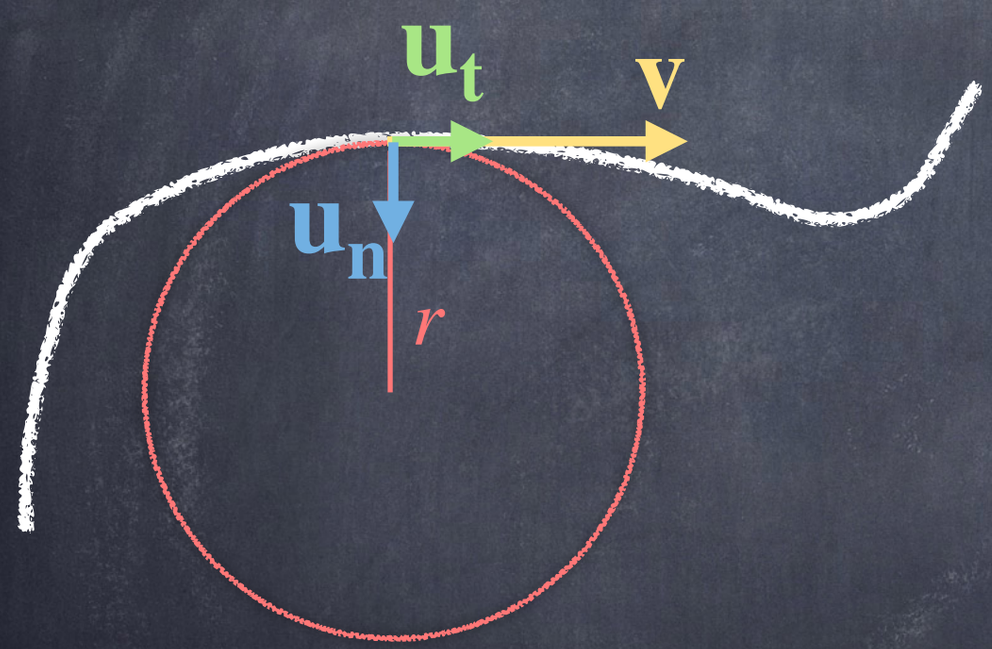
$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + v\,\frac{d\theta}{dt}\,\mathbf{u_n}$$

$$ds = r\,d\theta \qquad d\theta = \frac{ds}{r} \qquad \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt} = \frac{v}{r}$$

$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + \frac{v}{r}\,\frac{ds}{dt}\,\mathbf{u_n} \quad = \frac{dv}{dt}\,\mathbf{u_t} + \frac{v^2}{r}\,\mathbf{u_n}$$

$$\mathbf{a} = \mathbf{a_t} + \mathbf{a_n} \qquad \boxed{\mathbf{a_t} = \frac{dv}{dt}\,\mathbf{u_t}} \qquad \boxed{\mathbf{a_n} = \frac{v^2}{r}\,\mathbf{u_n}}$$

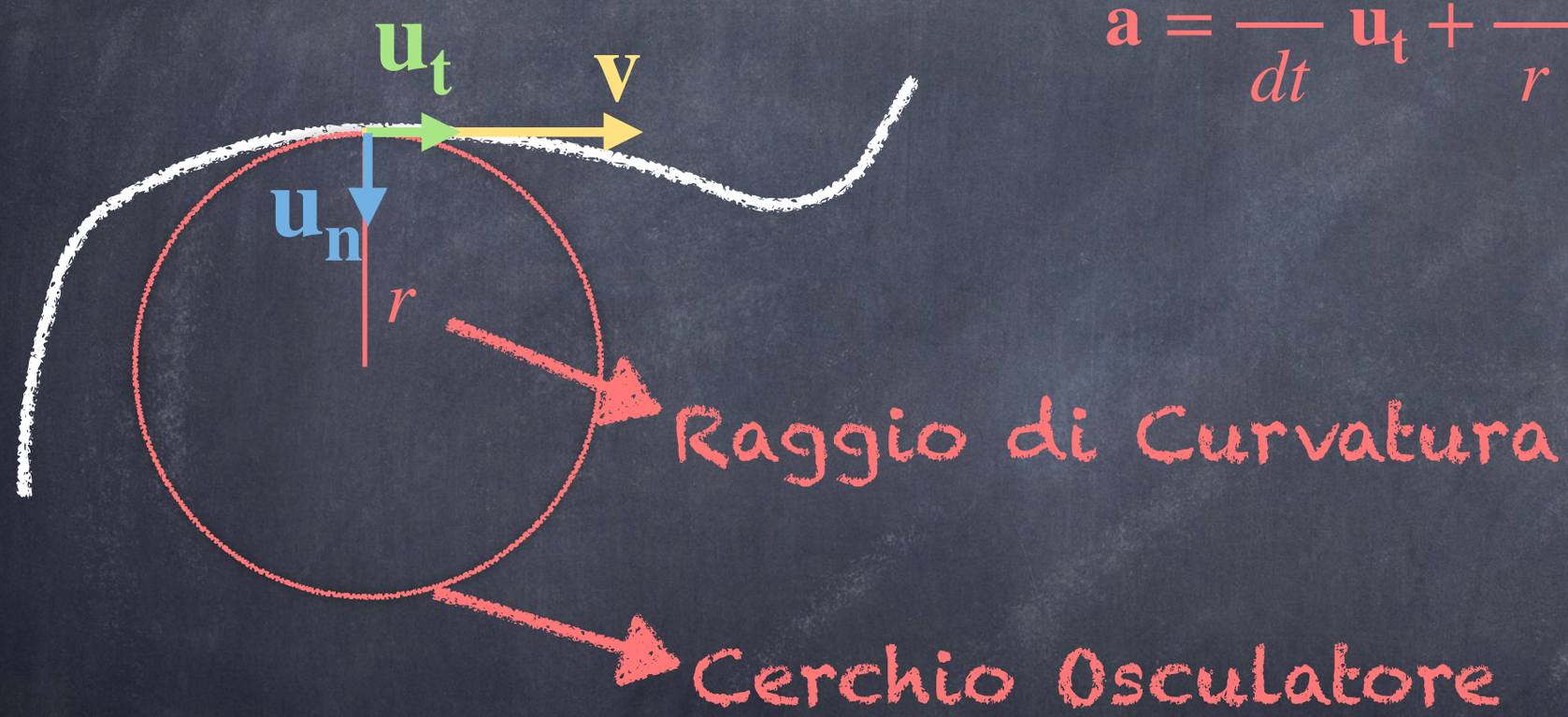
Accelerazione nel Moto Piano



$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + \frac{v^2}{r}\,\mathbf{u_n}$$

Accelerazione nel Moto Piano

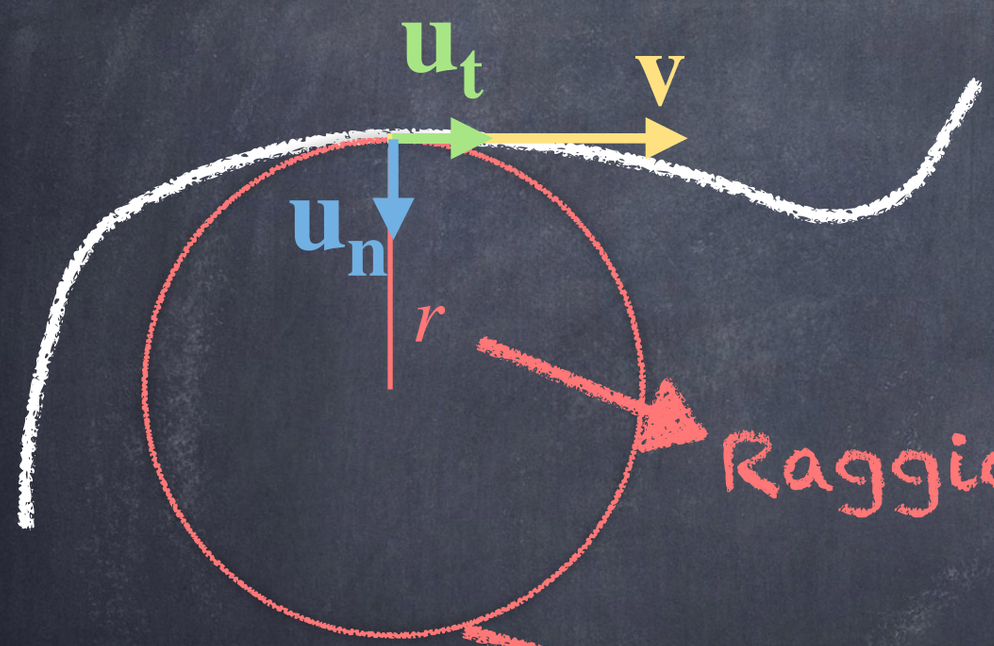
$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + \frac{v^2}{r}\,\mathbf{u_n}$$



Accelerazione nel Moto Piano

$$\mathbf{a} = \frac{dv}{dt}\,\mathbf{u_t} + \boxed{\frac{v^2}{r}\,\mathbf{u_n}}$$

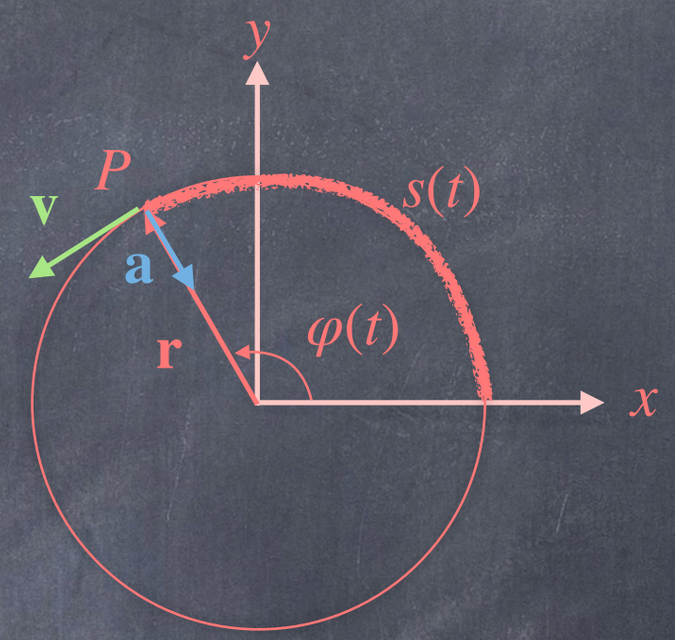
Accelerazione Centripeta



Raggio di Curvatura

Cerchio Osculatore

Moto Circolare



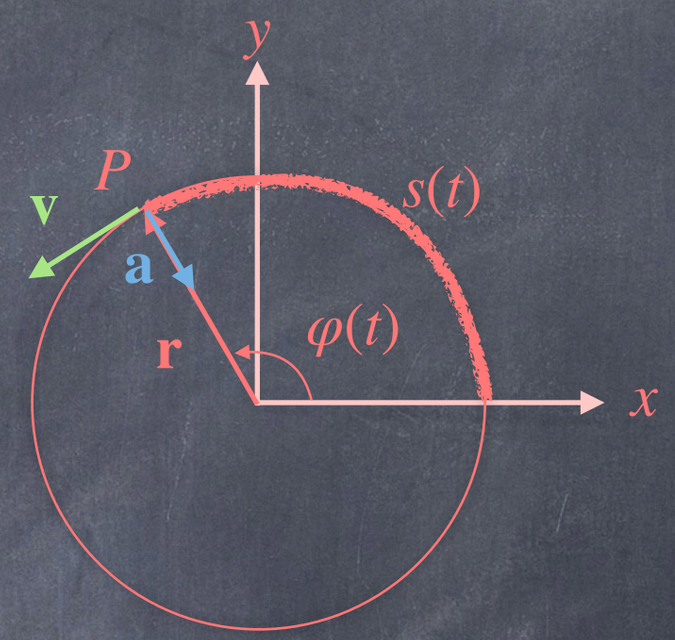
$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$|\,\mathbf{r}(t)\,| = r$$

$$|\,\mathbf{v}(t)\,| = v_s(t)$$

Moto Circolare

$$s(t) = r\,\varphi(t)$$

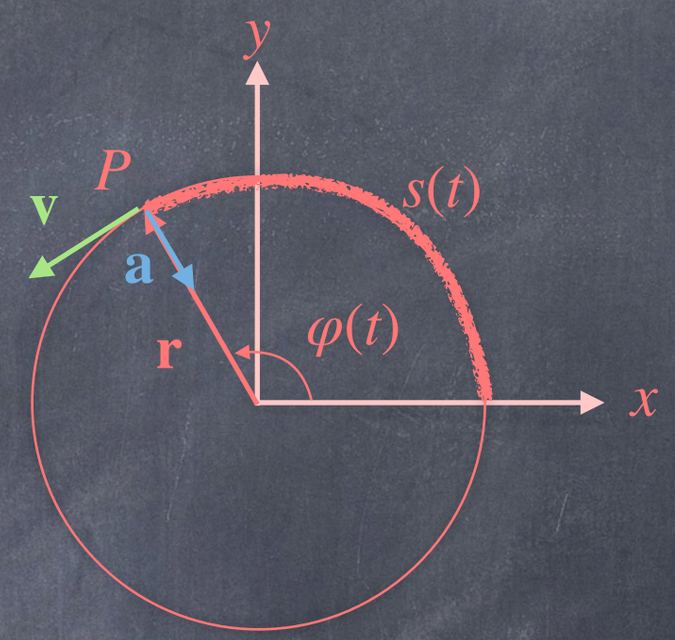


$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$|\,\mathbf{r}(t)\,| = r$$

Moto Circolare

$$s(t) = r\,\varphi(t)$$



Velocita' Scalare $\quad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t)$

Velocita' Angolare $\quad \omega(t) = \dot{\varphi}(t) = \dfrac{d\varphi}{dt}$

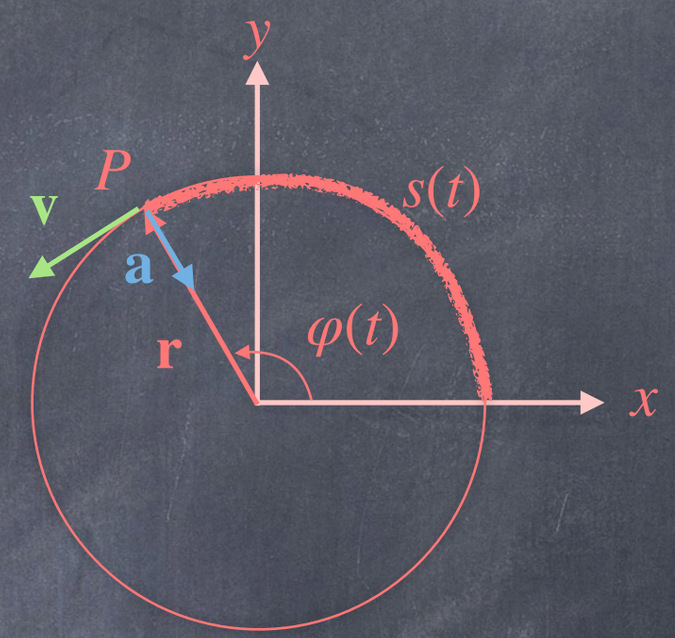
$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$|\,\mathbf{r}(t)\,| = r$$

$$|\,\mathbf{v}(t)\,| = v_s(t)$$

Moto Circolare

$$s(t) = r\,\varphi(t)$$



Velocita' Scalare $\quad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t)$

$$v_s(t) = r\,\omega(t)$$

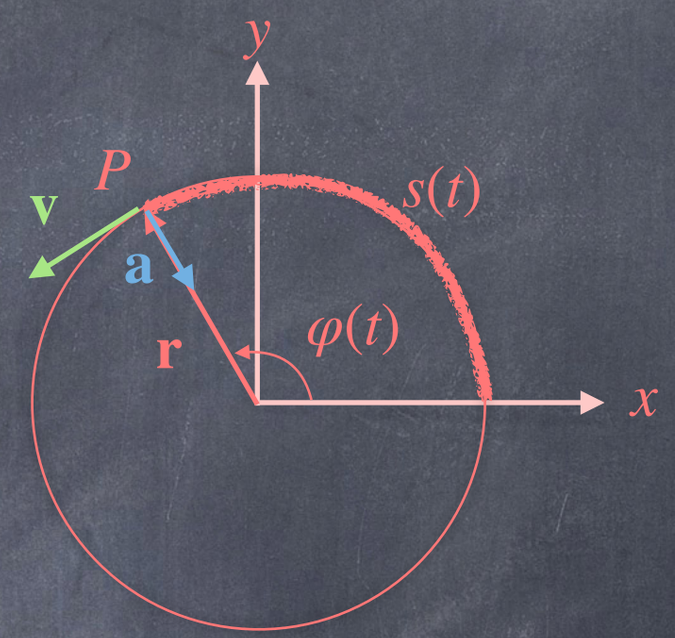
Velocita' Angolare $\quad \omega(t) = \dot{\varphi}(t) = \dfrac{d\varphi}{dt}$

$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$|\,\mathbf{r}(t)\,| = r$$

$$|\,\mathbf{v}(t)\,| = v_s(t)$$

Moto Circolare



$$s(t) = r\,\varphi(t)$$

Velocita' Scalare $\quad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t)$

$$v_s(t) = r\,\omega(t)$$

Velocita' Angolare $\quad \omega(t) = \dot{\varphi}(t) = \dfrac{d\varphi}{dt}$

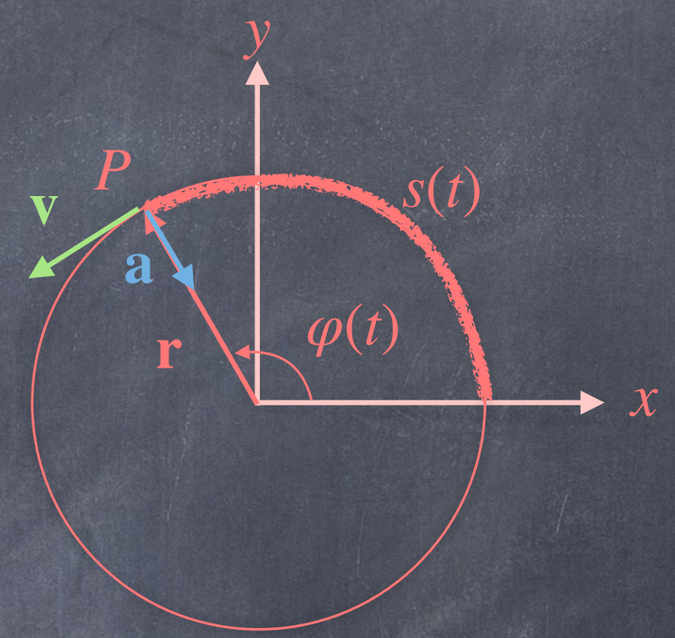
$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$\mathbf{v}(t) = v_s(t)\,\hat{\mathbf{u}}(t)$$

$$|\,\mathbf{r}(t)\,| = r$$

$$|\,\mathbf{v}(t)\,| = v_s(t)$$

Moto Circolare



$$s(t) = r\,\varphi(t)$$

Velocita' Scalare $\quad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t)$

$$v_s(t) = r\,\omega(t)$$

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$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$\mathbf{v}(t) = v_s(t)\,\hat{\mathbf{u}}(t)$$

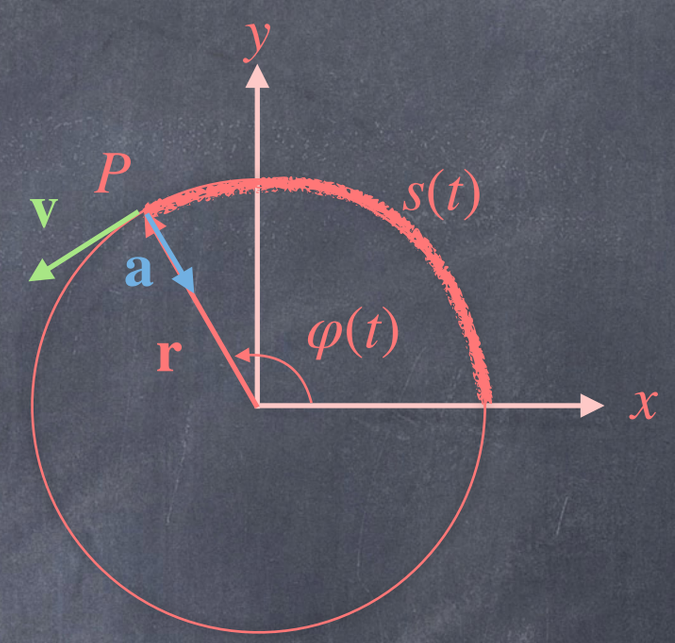
$$|\,\mathbf{r}(t)\,| = r$$

$$|\,\mathbf{v}(t)\,| = v_s(t)$$

$$\mathbf{a}(t) = \frac{dv(t)}{dt}\,\mathbf{u_t} + \frac{v(t)^2}{r}\,\mathbf{u_n}$$

Moto Circolare Uniforme

$v_s = cost$ $\omega = cost$ $v_s = r\,\omega$



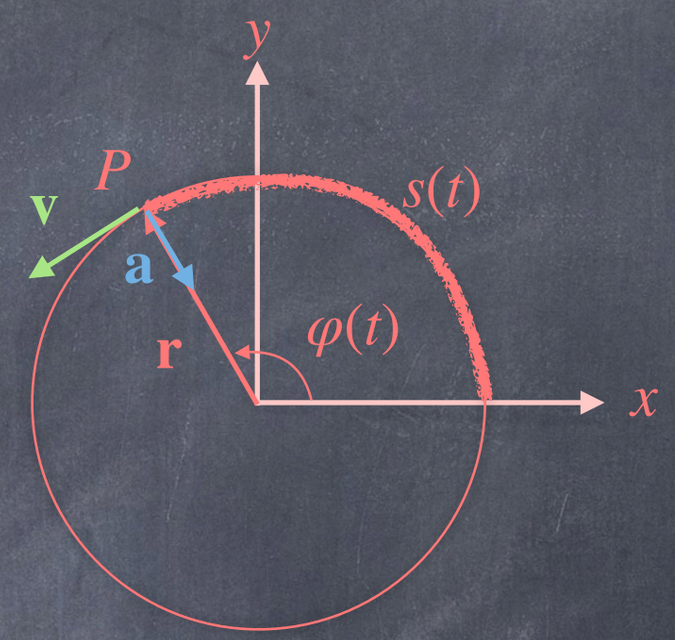
$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$$

Moto Circolare Uniforme

$v_s = cost$ $\qquad$ $\omega = cost$ $\qquad$ $v_s = r\,\omega$

$s(t) = s_0 + v_s \cdot t$ $\qquad$ $\phi(t) = \varphi_0 + \omega t$

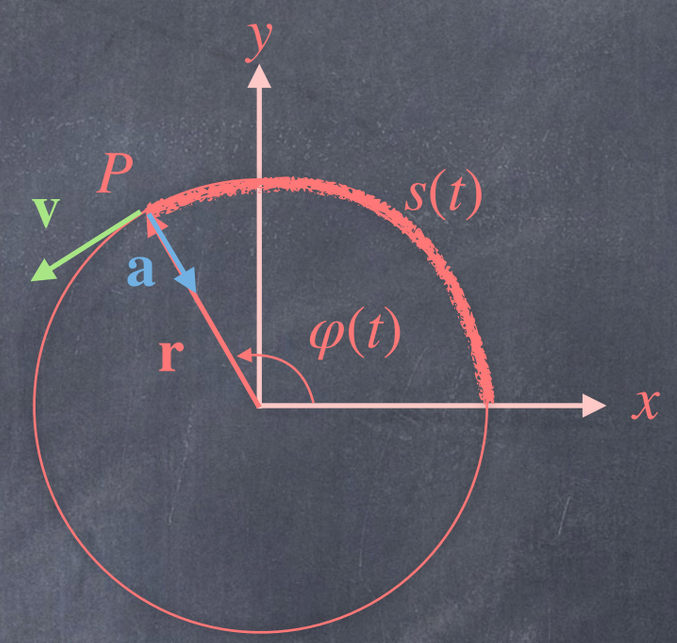


$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r$ $\qquad$ $|\mathbf{v}(t)| = v_s$

Moto Circolare Uniforme

$v_s = cost$ $\qquad$ $\omega = cost$ $\qquad$ $v_s = r\,\omega$



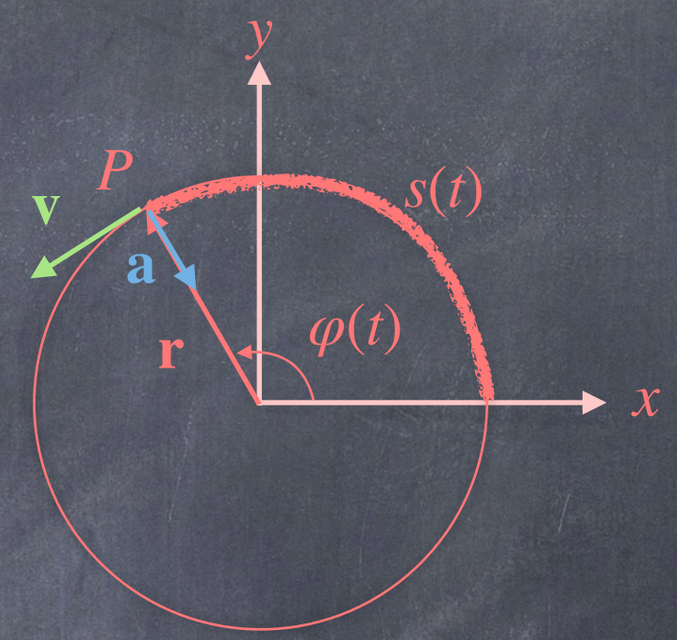
$s(t) = s_0 + v_s \cdot t$ $\qquad$ $\phi(t) = \varphi_0 + \omega t$

$\mathbf{v}(t) = v_s\,\hat{\mathbf{u}}(t)$

$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r$ $\qquad$ $|\mathbf{v}(t)| = v_s$

Moto Circolare Uniforme



$v_s = cost$ $\qquad$ $\omega = cost$ $\qquad$ $v_s = r\,\omega$

$$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t$$

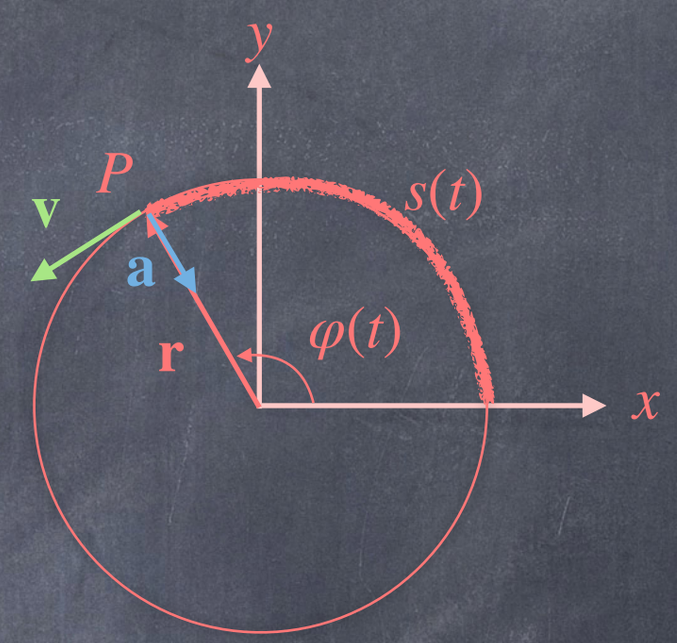
$$\mathbf{v}(t) = v_s\,\hat{\mathbf{u}}(t)$$

$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$$

$$\mathbf{a}(t) = \frac{dv_s}{dt}\,\mathbf{u_t} + \frac{v_s^2}{r}\,\mathbf{u_n}$$

Moto Circolare Uniforme



$v_s = cost$ $\qquad$ $\omega = cost$ $\qquad$ $v_s = r\,\omega$

$$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t$$

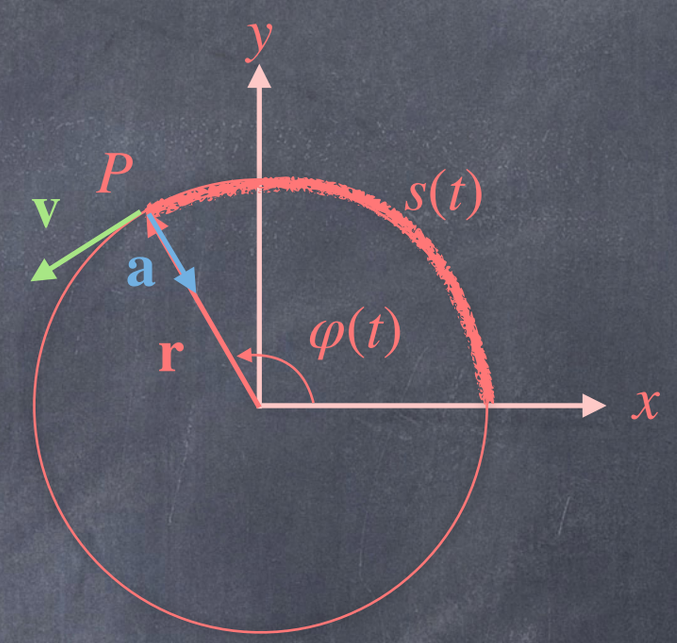
$$\mathbf{v}(t) = v_s\,\hat{\mathbf{u}}(t)$$

$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$$

$$\mathbf{a}(t) = \cancel{\frac{dv_s}{dt}}\,\mathbf{u_t} + \frac{v_s^2}{r}\,\mathbf{u_n}$$

Moto Circolare Uniforme



$v_s = cost$ $\qquad$ $\omega = cost$ $\qquad$ $v_s = r\,\omega$

$s(t) = s_0 + v_s \cdot t$ $\qquad$ $\phi(t) = \varphi_0 + \omega t$

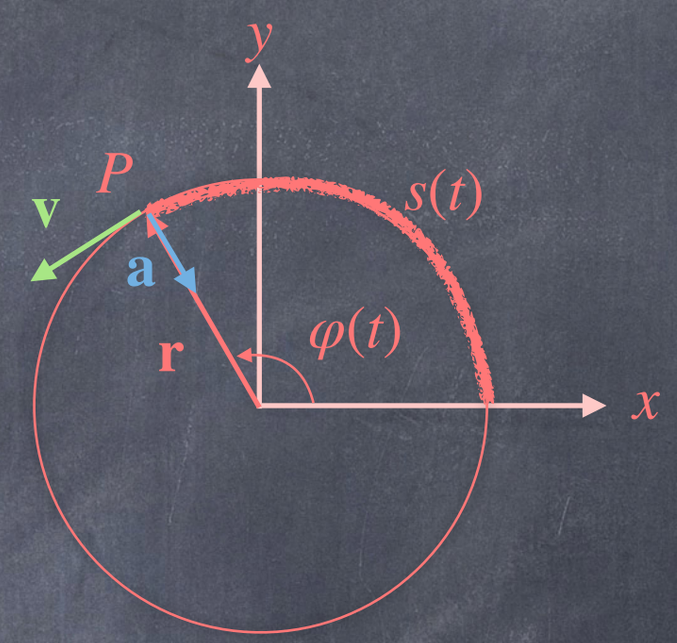
$\mathbf{v}(t) = v_s\,\hat{\mathbf{u}}(t)$

$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r$ $\qquad$ $|\mathbf{v}(t)| = v_s$

$$\mathbf{a}(t) = \cancel{\frac{dv_s}{dt}}\,\mathbf{u_t} + \frac{v_s^2}{r}\,\mathbf{u_n} \qquad \mathbf{a}(t) = \frac{v_s^2}{r}\,\mathbf{u_n}$$

Moto Circolare Uniforme



$v_s = cost$ $\qquad$ $\omega = cost$ $\qquad$ $v_s = r\,\omega$

$s(t) = s_0 + v_s \cdot t$ $\qquad$ $\phi(t) = \varphi_0 + \omega t$

$\mathbf{v}(t) = v_s\,\hat{\mathbf{u}}(t)$

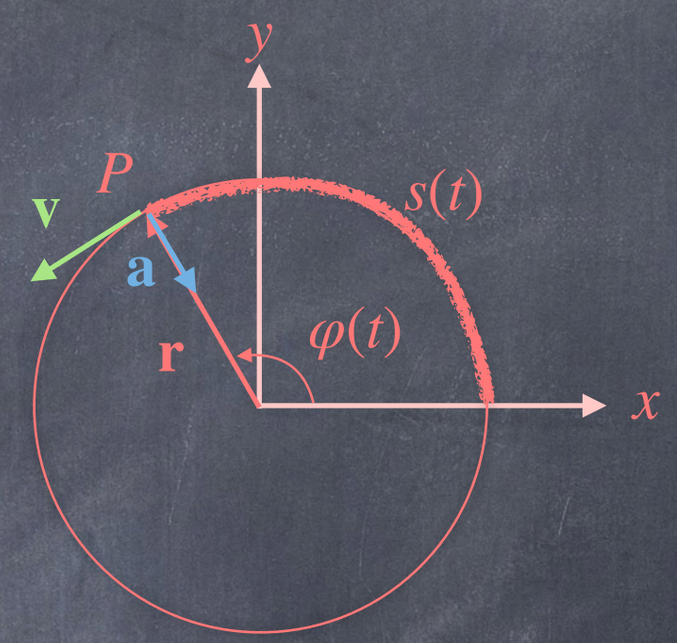
$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r$ $\qquad$ $|\mathbf{v}(t)| = v_s$

$\mathbf{a}(t) = \cancel{\frac{dv_s}{dt}}\,\mathbf{u_t} + \frac{v_s^2}{r}\,\mathbf{u_n}$ $\qquad$ $\mathbf{a}(t) = \frac{v_s^2}{r}\,\mathbf{u_n}$

Solo Accelerazione Centripeta

Moto Circolare Uniforme



$v_s = cost \qquad \omega = cost \qquad v_s = r\,\omega$

$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t$

$\mathbf{v}(t) = v_s\,\hat{\mathbf{u}}(t)$

$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$

$$\mathbf{a}(t) = \cancel{\frac{dv_s}{dt}}\,\mathbf{u_t} + \frac{v_s^2}{r}\,\mathbf{u_n} \qquad \mathbf{a}(t) = \frac{v_s^2}{r}\,\mathbf{u_n}$$

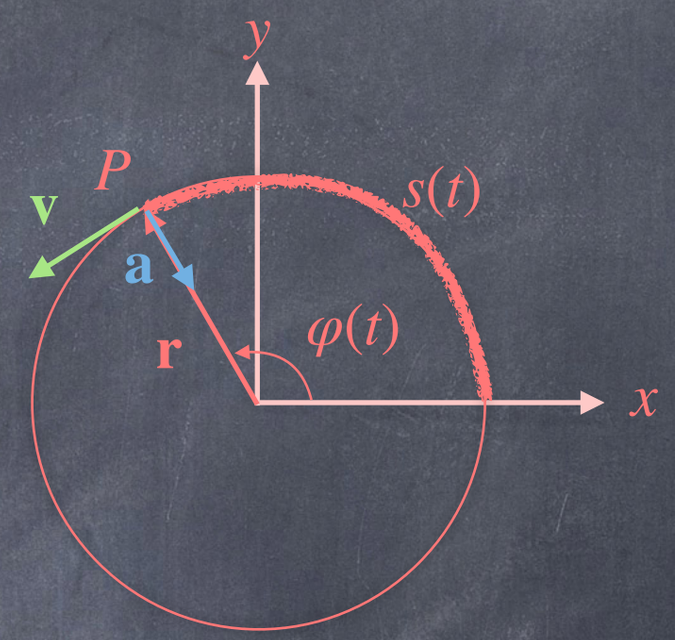
Solo Accelerazione Centripeta

$$|\mathbf{a}(t)| = a = \frac{v_s^2}{r} = cost$$

Moto Circolare Uniforme

$v_s = cost \qquad \omega = cost \qquad v_s = r\,\omega \quad \omega = \frac{v_s}{r}$

$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t$



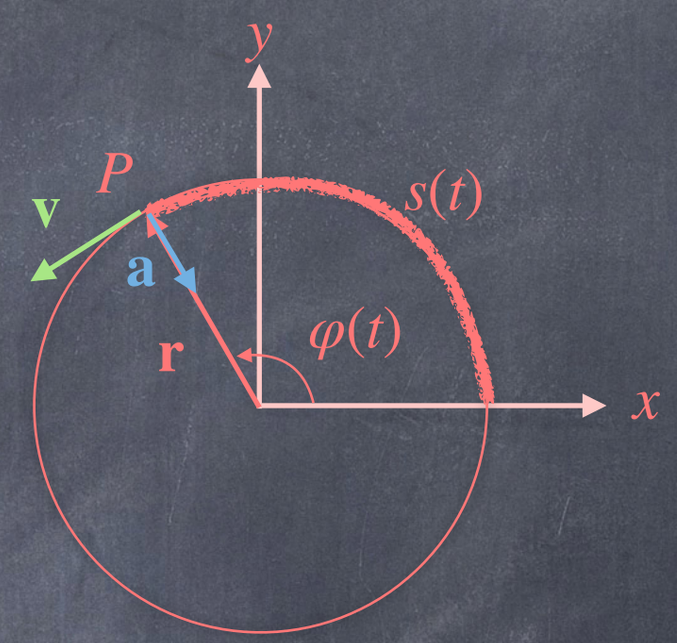
$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$

Moto Circolare Uniforme

$v_s = cost \qquad \omega = cost \qquad v_s = r\,\omega \quad \omega = \dfrac{v_s}{r}$

$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t$



$$\mathbf{a}(t) = \frac{v_s^2}{r}\,\mathbf{u_n} = \frac{\omega^2\, r^2}{r}\,\mathbf{u_n} = \omega^2\, r\,\mathbf{u_n}$$

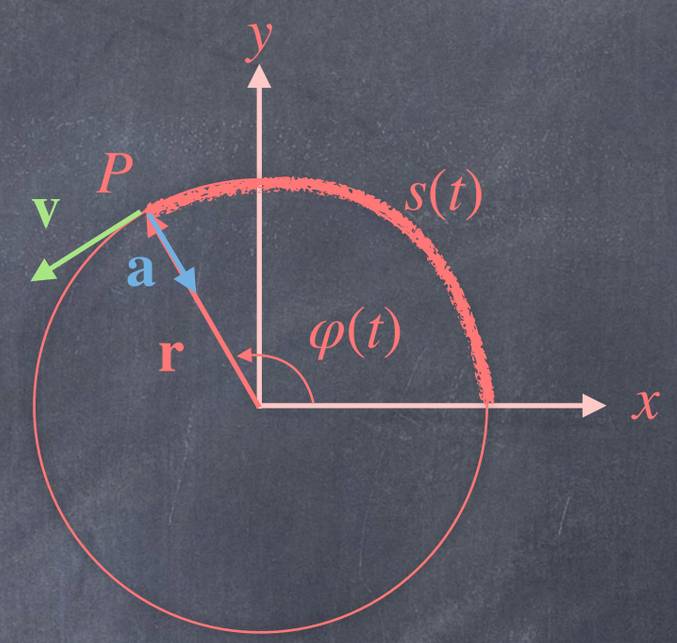
$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$

Moto Circolare Uniforme

$v_s = cost \qquad \omega = cost \qquad v_s = r\,\omega \quad \omega = \dfrac{v_s}{r}$

$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t$



$$\mathbf{a}(t) = \frac{v_s^2}{r}\,\mathbf{u_n} = \frac{\omega^2\, r^2}{r}\,\mathbf{u_n} = \omega^2\, r\,\mathbf{u_n}$$

$a = \omega^2\, r$

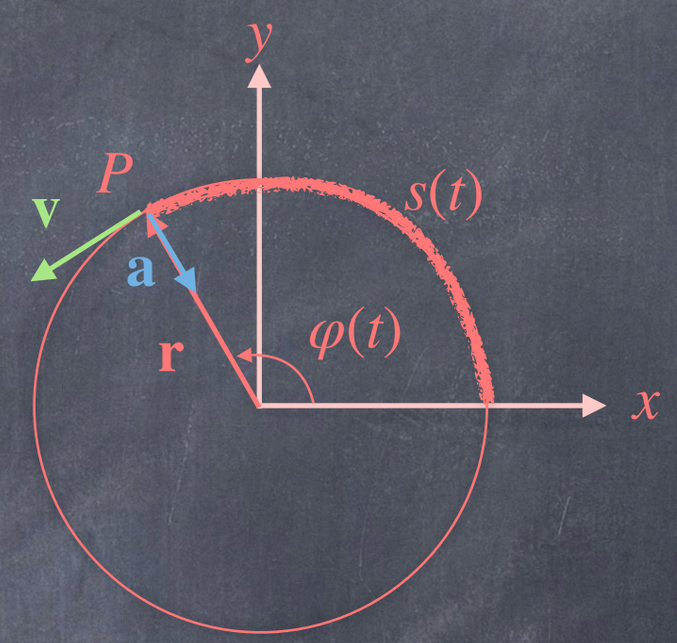
$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$

Moto Circolare Uniforme

$v_s = cost \qquad \omega = cost \qquad v_s = r\,\omega \quad \omega = \dfrac{v_s}{r}$

$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t$



$$\mathbf{a}(t) = \frac{v_s^2}{r}\,\mathbf{u_n} = \frac{\omega^2\, r^2}{r}\,\mathbf{u_n} = \omega^2\, r\,\mathbf{u_n}$$

$$a = \omega^2\, r$$

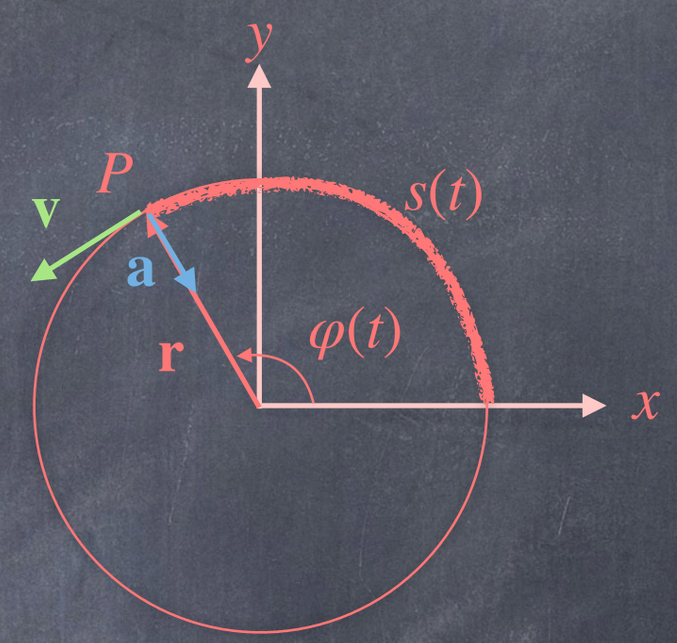
$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$$

Tempo impiegato a percorrere Circonferenza

$$T = \frac{2\pi r}{v_s} = \frac{2\pi}{\omega}$$

Moto Circolare Uniforme



$v_s = cost \qquad \omega = cost \qquad v_s = r\,\omega \quad \omega = \dfrac{v_s}{r}$

$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t$

$$\mathbf{a}(t) = \frac{v_s^2}{r}\,\mathbf{u_n} = \frac{\omega^2\, r^2}{r}\,\mathbf{u_n} = \omega^2\, r\,\mathbf{u_n}$$

$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$$

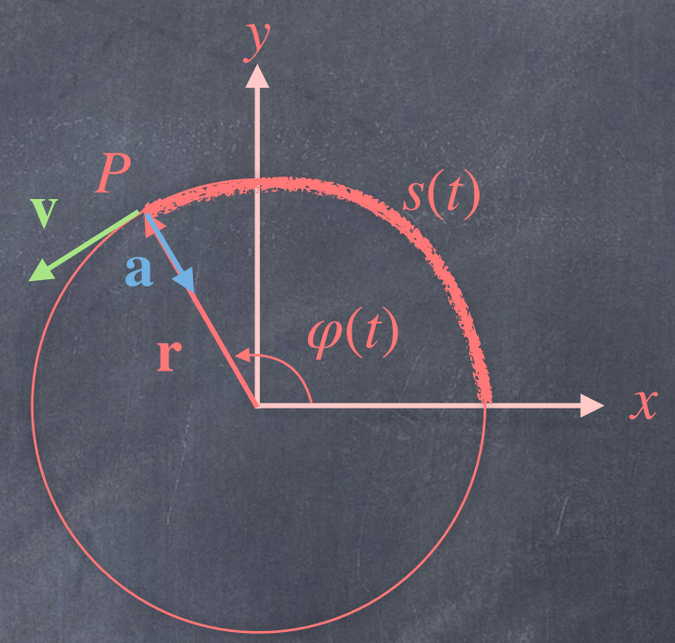
$$a = \omega^2\, r$$

Tempo impiegato a percorrere Circonferenza $\quad T = \dfrac{2\pi r}{v_s} = \dfrac{2\pi}{\omega} \longrightarrow \omega = \dfrac{2\pi}{T}$

T: Periodo $\longrightarrow$ Moto Periodico

Frequenza: $f = \dfrac{1}{T}$

Moto Circolare Uniforme



$v_s = cost \qquad \omega = cost \qquad v_s = r\,\omega$

$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t \qquad \mathbf{a}(t) = \frac{v_s^2}{r}\,\mathbf{u_n}$

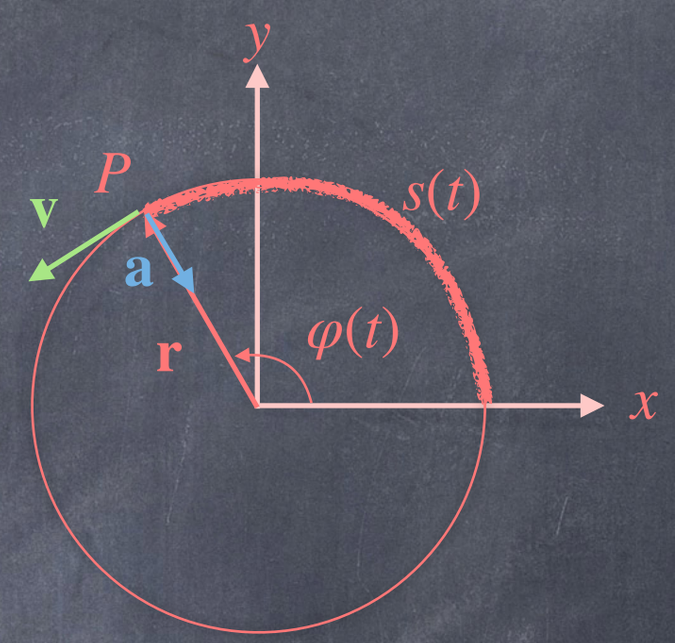
$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$

Componenti Lungo assi x e y

$$\begin{cases} x(t) = r\cos\varphi(t) \\ y(t) = r\sin\varphi(t) \end{cases}$$

Moto Circolare Uniforme



$v_s = cost \qquad \omega = cost \qquad v_s = r\,\omega$

$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t \qquad \mathbf{a}(t) = \frac{v_s^2}{r}\,\mathbf{u_n}$

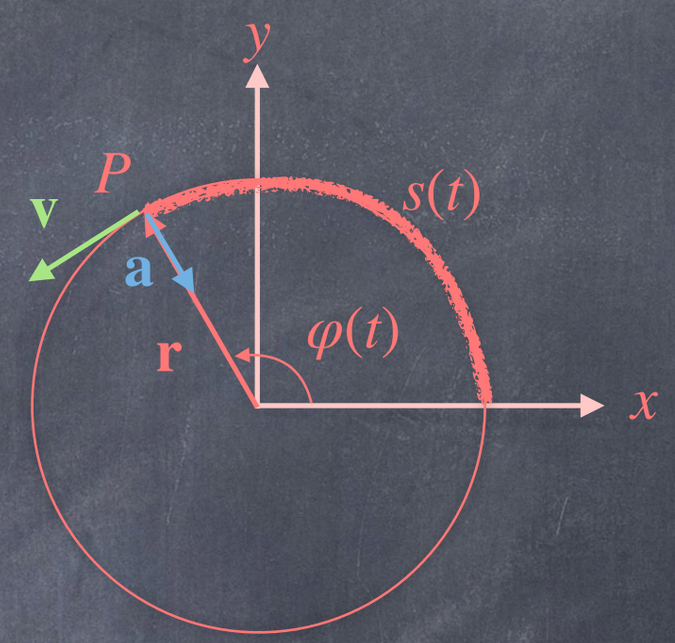
$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$

Componenti Lungo assi x e y

$$\begin{cases} x(t) = r\,cos\,\varphi(t) \\ y(t) = r\,sin\,\varphi(t) \end{cases} \qquad \begin{cases} x(t) = r\,cos(\varphi_0 + \omega t) \\ y(t) = r\,sin(\varphi_0 + \omega t) \end{cases}$$

Moto Circolare Uniforme



$v_s = cost \qquad \omega = cost \qquad v_s = r\,\omega$

$s(t) = s_0 + v_s \cdot t \qquad \phi(t) = \varphi_0 + \omega t \qquad \mathbf{a}(t) = \frac{v_s^2}{r}\,\mathbf{u_n}$

$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

$|\mathbf{r}(t)| = r \qquad |\mathbf{v}(t)| = v_s$

Componenti Lungo assi x e y

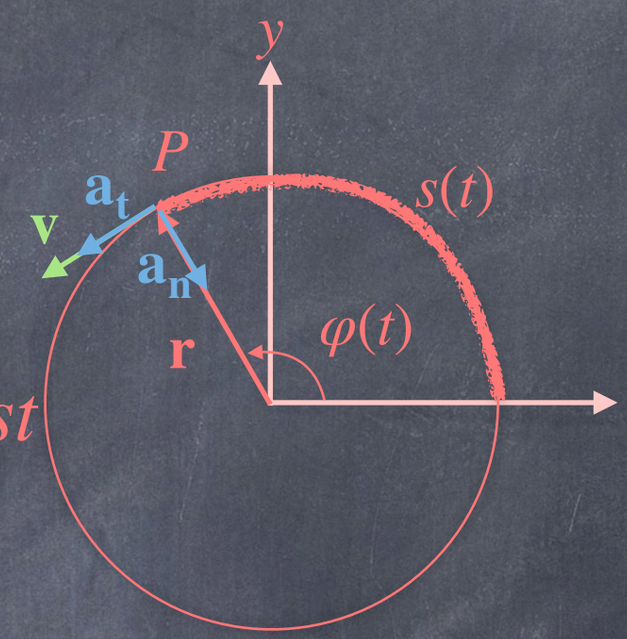
$$\begin{cases} x(t) = r\,cos\,\varphi(t) \\ y(t) = r\,sin\,\varphi(t) \end{cases} \qquad \begin{cases} x(t) = r\,cos(\varphi_0 + \omega t) \\ y(t) = r\,sin(\varphi_0 + \omega t) \end{cases}$$

Moti Armonici

φ_0 Fase Iniziale

Moto Circolare Uniformemente Accelerato

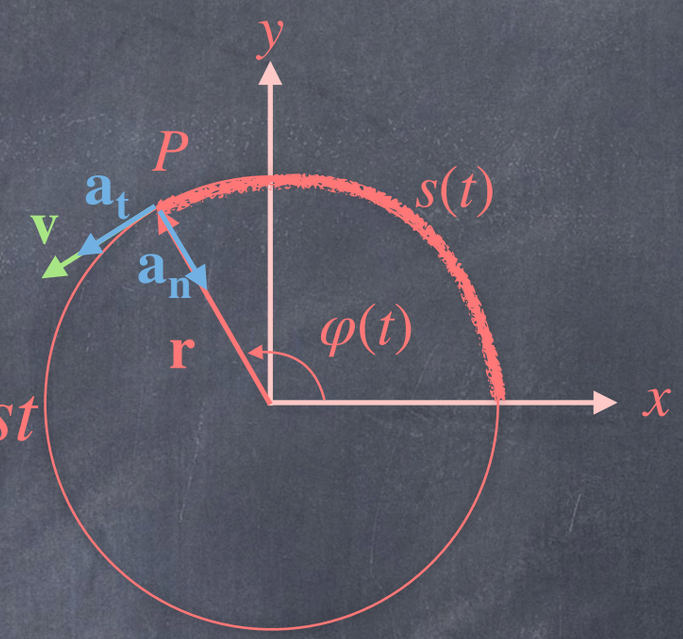
$$s(t) = r\,\varphi(t) \qquad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t) \qquad \omega(t) = \dot{\varphi}(t) = \frac{d\varphi}{dt}$$



$$\mathbf{a}(t) = \frac{dv(t)}{dt}\,\mathbf{u_t} + \frac{v(t)^2}{r}\,\mathbf{u_n} = a_t\,\mathbf{u_t} + a_n\,\mathbf{u_n} \qquad |\mathbf{a_t}| = a_t = cost$$

Moto Circolare Uniformemente Accelerato

$$s(t) = r\,\varphi(t) \qquad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t) \qquad \omega(t) = \dot{\varphi}(t) = \frac{d\varphi}{dt}$$



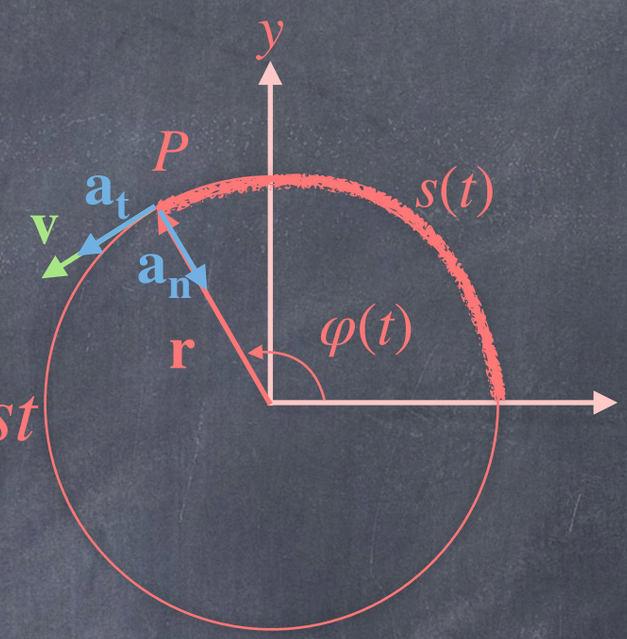
$$\mathbf{a}(t) = \frac{dv(t)}{dt}\,\mathbf{u_t} + \frac{v(t)^2}{r}\,\mathbf{u_n} = a_t\,\mathbf{u_t} + a_n\,\mathbf{u_n} \qquad |\mathbf{a_t}| = a_t = cost$$

Accelerazione Angolare

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} = \frac{1}{r}\frac{dv(t)}{dt} = \frac{a_t}{r}$$

Moto Circolare Uniformemente Accelerato

$$s(t) = r\,\varphi(t) \qquad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t) \qquad \omega(t) = \dot{\varphi}(t) = \frac{d\varphi}{dt}$$



$$\mathbf{a}(t) = \frac{dv(t)}{dt}\,\mathbf{u_t} + \frac{v(t)^2}{r}\,\mathbf{u_n} = a_t\,\mathbf{u_t} + a_n\,\mathbf{u_n} \qquad |\mathbf{a_t}| = a_t = cost$$

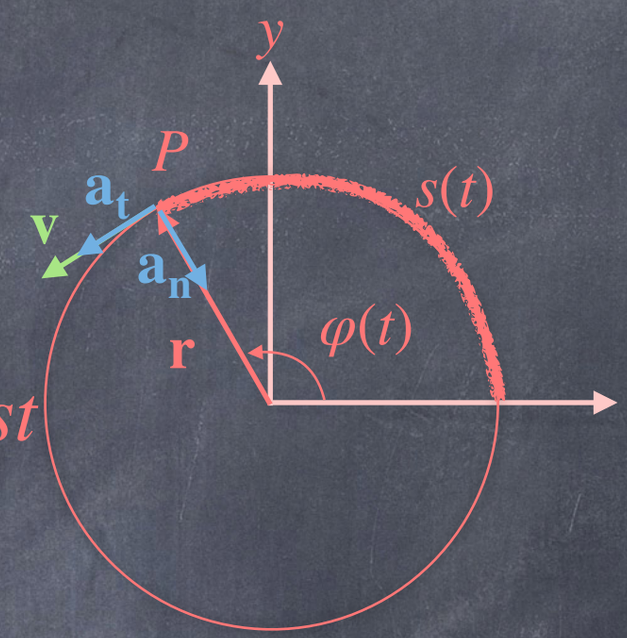
Accelerazione Angolare

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} = \frac{1}{r}\frac{dv(t)}{dt} = \frac{a_t}{r}$$

$$\alpha = cost$$

Moto Circolare Uniformemente Accelerato

$$s(t) = r\,\varphi(t) \qquad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t) \qquad \omega(t) = \dot{\varphi}(t) = \frac{d\varphi}{dt}$$



$$\mathbf{a}(t) = \frac{dv(t)}{dt}\,\mathbf{u_t} + \frac{v(t)^2}{r}\,\mathbf{u_n} = a_t\,\mathbf{u_t} + a_n\,\mathbf{u_n} \qquad |\mathbf{a_t}| = a_t = cost$$

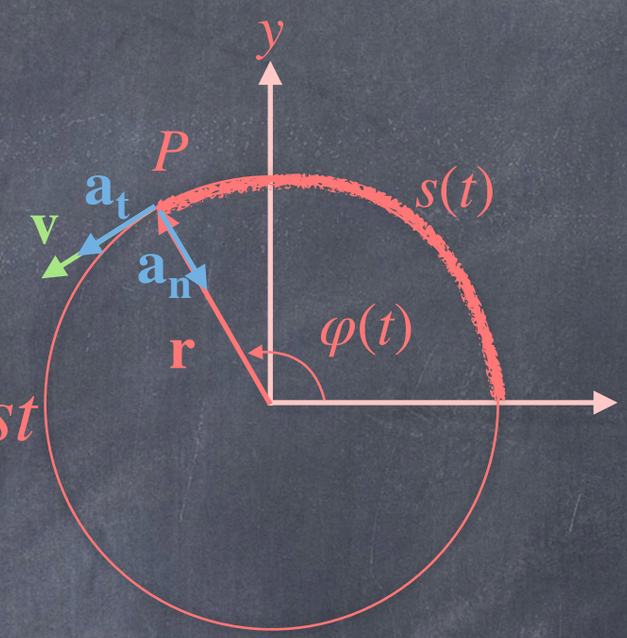
Accelerazione Angolare

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} = \frac{1}{r}\frac{dv(t)}{dt} = \frac{a_t}{r} \qquad \alpha = cost$$

$$\omega(t) = \omega_0 + \int_0^t \alpha(t')\,dt' = \omega_0 + \alpha t$$

Moto Circolare Uniformemente Accelerato

$$s(t) = r\,\varphi(t) \qquad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t) \qquad \omega(t) = \dot{\varphi}(t) = \frac{d\varphi}{dt}$$



$$\mathbf{a}(t) = \frac{dv(t)}{dt}\,\mathbf{u_t} + \frac{v(t)^2}{r}\,\mathbf{u_n} = a_t\,\mathbf{u_t} + a_n\,\mathbf{u_n} \qquad |\mathbf{a_t}| = a_t = cost$$

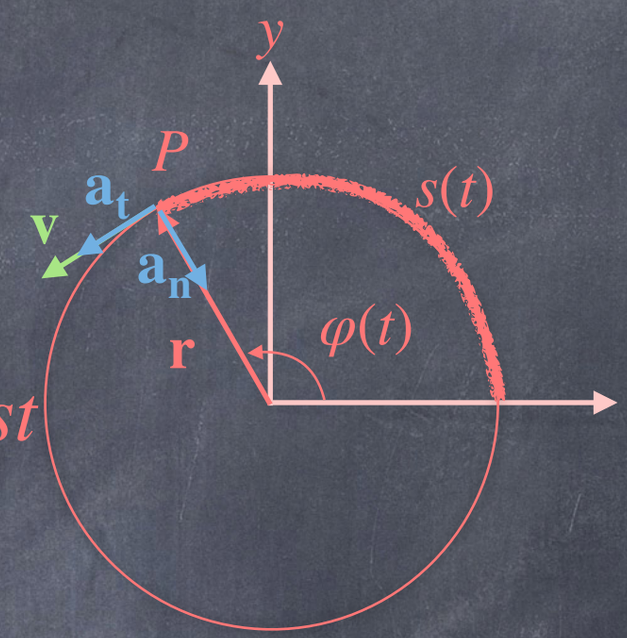
Accelerazione Angolare

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} = \frac{1}{r}\frac{dv(t)}{dt} = \frac{a_t}{r} \qquad \alpha = cost$$

$$\omega(t) = \omega_0 + \int_0^t \alpha(t')\,dt' = \omega_0 + \alpha t \qquad \varphi(t) = \varphi_0 + \int_0^t \omega(t')\,dt' = \varphi_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Moto Circolare Uniformemente Accelerato

$$s(t) = r\,\varphi(t) \qquad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t) \qquad \omega(t) = \dot{\varphi}(t) = \frac{d\varphi}{dt}$$



$$\mathbf{a}(t) = \frac{dv(t)}{dt}\,\mathbf{u_t} + \frac{v(t)^2}{r}\,\mathbf{u_n} \quad = a_t\,\mathbf{u_t} + a_n\,\mathbf{u_n} \qquad |\mathbf{a_t}| = a_t = cost$$

Accelerazione Angolare

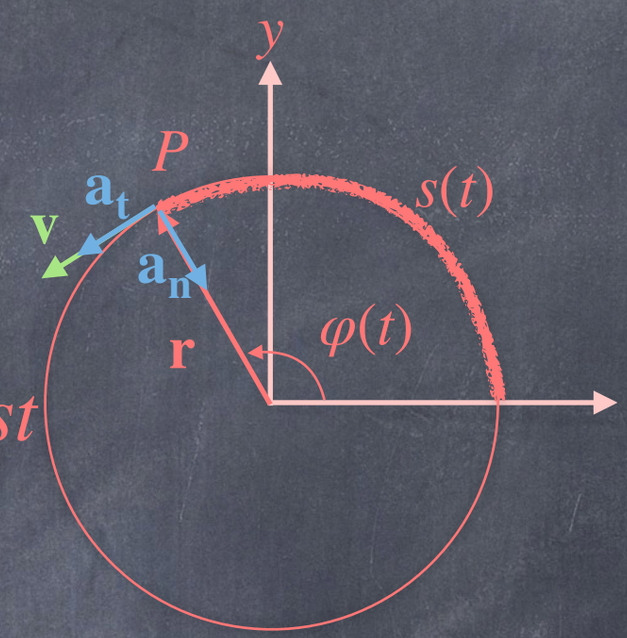
$$\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} = \frac{1}{r}\frac{dv(t)}{dt} = \frac{a_t}{r} \qquad \alpha = cost$$

$$\omega(t) = \omega_0 + \int_0^t \alpha(t')\,dt' \quad = \omega_0 + \alpha t \qquad \varphi(t) = \varphi_0 + \int_0^t \omega(t')\,dt' \quad = \varphi_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$a_n = \frac{v^2}{r} = \omega^2 r \quad = r(\omega_0 + \alpha t)^2$$

Moto Circolare Uniformemente Accelerato

$$s(t) = r\,\varphi(t) \qquad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t) \qquad \omega(t) = \dot{\varphi}(t) = \frac{d\varphi}{dt}$$



$$\mathbf{a}(t) = \frac{dv(t)}{dt}\,\mathbf{u_t} + \frac{v(t)^2}{r}\,\mathbf{u_n} \quad = a_t\,\mathbf{u_t} + a_n\,\mathbf{u_n} \qquad |\mathbf{a_t}| = a_t = cost$$

Accelerazione Angolare

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} = \frac{1}{r}\frac{dv(t)}{dt} = \frac{a_t}{r} \qquad \alpha = cost$$

$$\omega(t) = \omega_0 + \int_0^t \alpha(t')\,dt' \quad = \omega_0 + \alpha t \qquad \varphi(t) = \varphi_0 + \int_0^t \omega(t')\,dt' \quad = \varphi_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

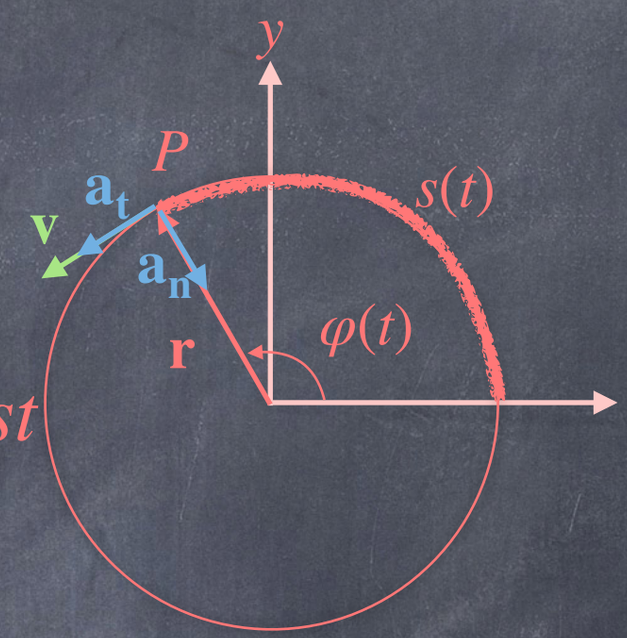
$$a_t = \alpha r$$

$$a_n = \frac{v^2}{r} = \omega^2 r \quad = r(\omega_0 + \alpha t)^2$$

$$\longrightarrow \quad \mathbf{a} = a_t\,\mathbf{u_t} + a_n\,\mathbf{u_n} \quad = \alpha r\,\mathbf{u_t} + \omega^2 r\,\mathbf{u_n}$$

Moto Circolare Uniformemente Accelerato

$$s(t) = r\,\varphi(t) \qquad v_s(t) = \dot{s}(t) = r\,\dot{\varphi}(t) \qquad \omega(t) = \dot{\varphi}(t) = \frac{d\varphi}{dt}$$



$$\mathbf{a}(t) = \frac{dv(t)}{dt}\,\mathbf{u_t} + \frac{v(t)^2}{r}\,\mathbf{u_n} \quad = a_t\,\mathbf{u_t} + a_n\,\mathbf{u_n} \qquad |\mathbf{a_t}| = a_t = cost$$

Accelerazione Angolare

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \frac{1}{r}\frac{dv(t)}{dt} = \frac{a_t}{r} \qquad \alpha = cost$$

$$\omega(t) = \omega_0 + \int_0^t \alpha(t')\,dt' \quad = \omega_0 + \alpha t \qquad \varphi(t) = \varphi_0 + \int_0^t \omega(t')\,dt' \quad = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\mathbf{a} = a_t\,\mathbf{u_t} + a_n\,\mathbf{u_n} \quad = \alpha r\,\mathbf{u_t} + \omega^2 r\,\mathbf{u_n} \longrightarrow \mathbf{a}(t) = r\,\left(\alpha\,\mathbf{u_t} + (\omega_0 + \alpha t)^2\,\mathbf{u_n}\right)$$

Esercizio 1

- Auto F1 $v = 200\ km/h$
- Raggio Curvatura $R = 100\ m$

Calcolare

- Accelerazione Laterale a_n

Esercizio 1

- Auto F1 $v = 200\ km/h$
- Raggio Curvatura $R = 100\ m$

Calcolare

- Accelerazione Laterale a_n

$$v = 200\ km/h = 55.6\ m/s$$

$$a = \frac{v^2}{R} = \frac{55.6^2}{100} \simeq 30.9\ m/s^2 \simeq 3g$$

Esercizio 2

- Giostra $\omega = 0.2\ rad/s$
- Bambino 1 $R_1 = 1.8\ m$
- Bambino 2 $R_2 = 2.5\ m$

Calcolare

- Velocita' v_1, v_2
- Accelerazione a_1, a_2

Esercizio 2

- Giostra $\omega = 0.2\ rad/s$
- Bambino 1 $R_1 = 1.8\ m$
- Bambino 2 $R_2 = 2.5\ m$

Calcolare

- Velocita' v_1, v_2
- Accelerazione a_1, a_2

$v = r\,\omega$ $\quad v_1 = 1.8 \cdot 0.2 = 0.36\ m/s$ $\quad v_2 = 2.5 \cdot 0.2 = 0.50\ m/s$

Esercizio 2

- Giostra $\omega = 0.2\ rad/s$
- Bambino 1 $R_1 = 1.8\ m$
- Bambino 2 $R_2 = 2.5\ m$

Calcolare

- Velocita' v_1, v_2
- Accelerazione a_1, a_2

$v = r\,\omega$ $\quad v_1 = 1.8 \cdot 0.2 = 0.36\ m/s$ $\quad v_2 = 2.5 \cdot 0.2 = 0.50\ m/s$

$a = \omega^2 R$ $\quad a_1 = 0.2^2 \cdot 1.8 = 0.072\ m/s^2$ $\quad a_2 = 0.2^2 \cdot 2.5 = 0.10\ m/s^2$