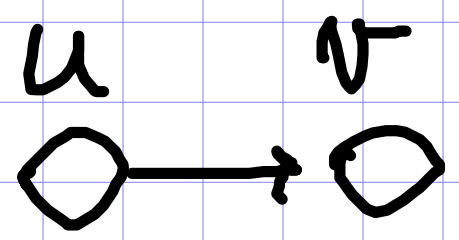
GRAFI:

- Alcune proprietà
- OBIETTIVO:
  visita o esplorazione dei grafi

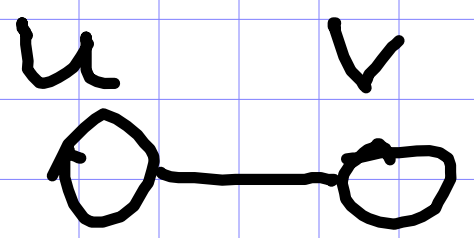
GRAFO

$G = (V, E)$

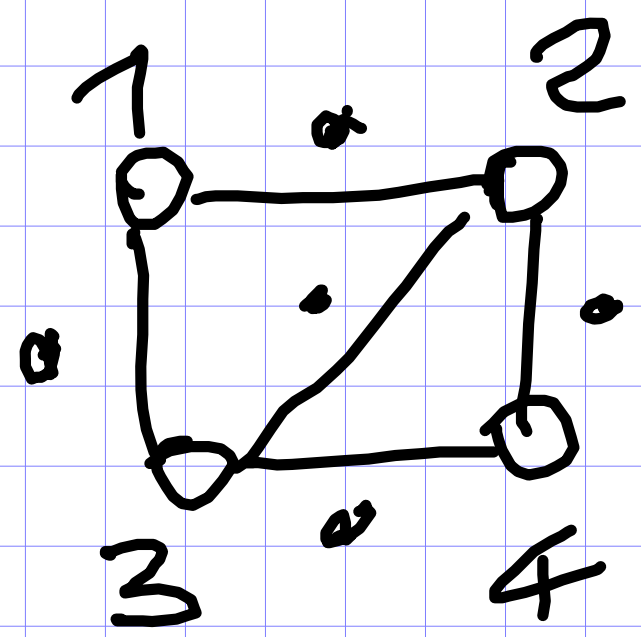
↓ coppie di vertici



- coppia ordinata → arco orientato
- coppia non ordinata → arco non orientato



$u \neq v$

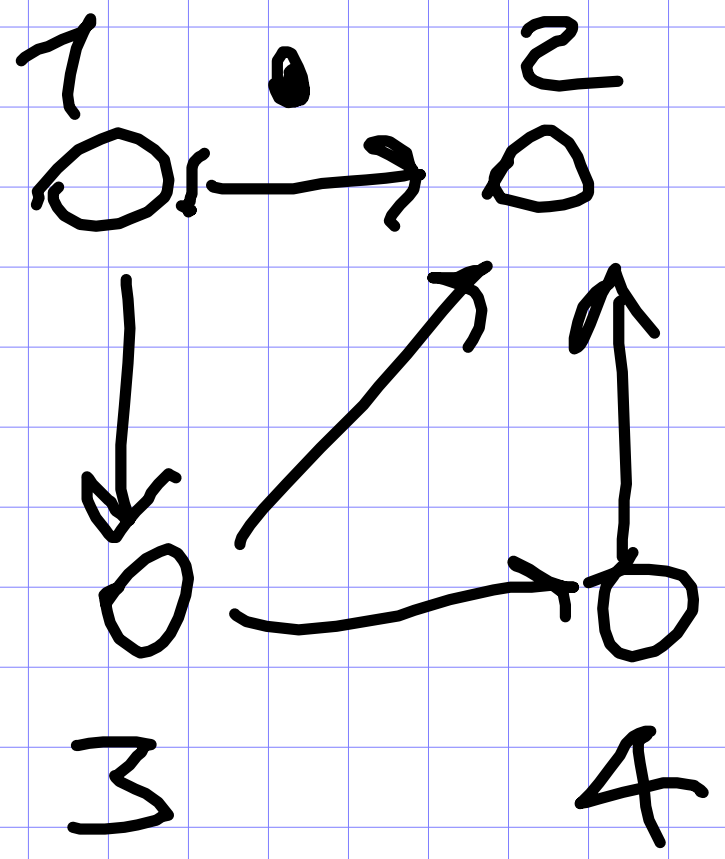


$$V = \{1, 2, 3, 4\}$$

$$E = \{(1,2), (1,3), (2,3), (2,4), (3,4)\}$$

$$E = \{(2,1); (1,3); (3,2); (4,2); (3,4)\}$$

ESEMPIO GRAFO NON ORIENTATO



$G \quad V = \{1, 2, 3, 4\}$

$E = \{ (1,2); (1,3); (3,2); (3,4); (4,2) \}$

GRAFO ORIENTATO

Grafi modellano molte situazioni

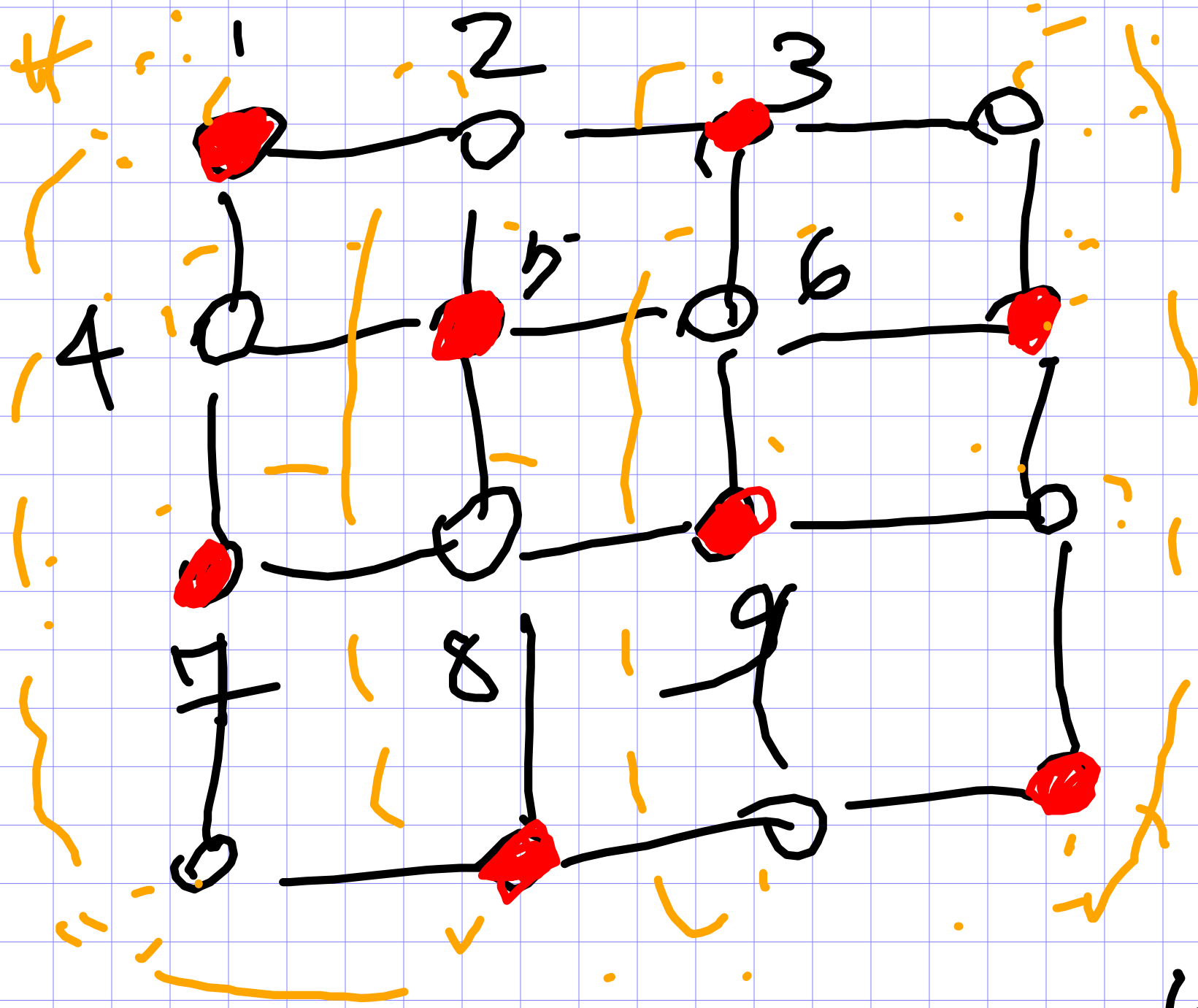
rete stradale

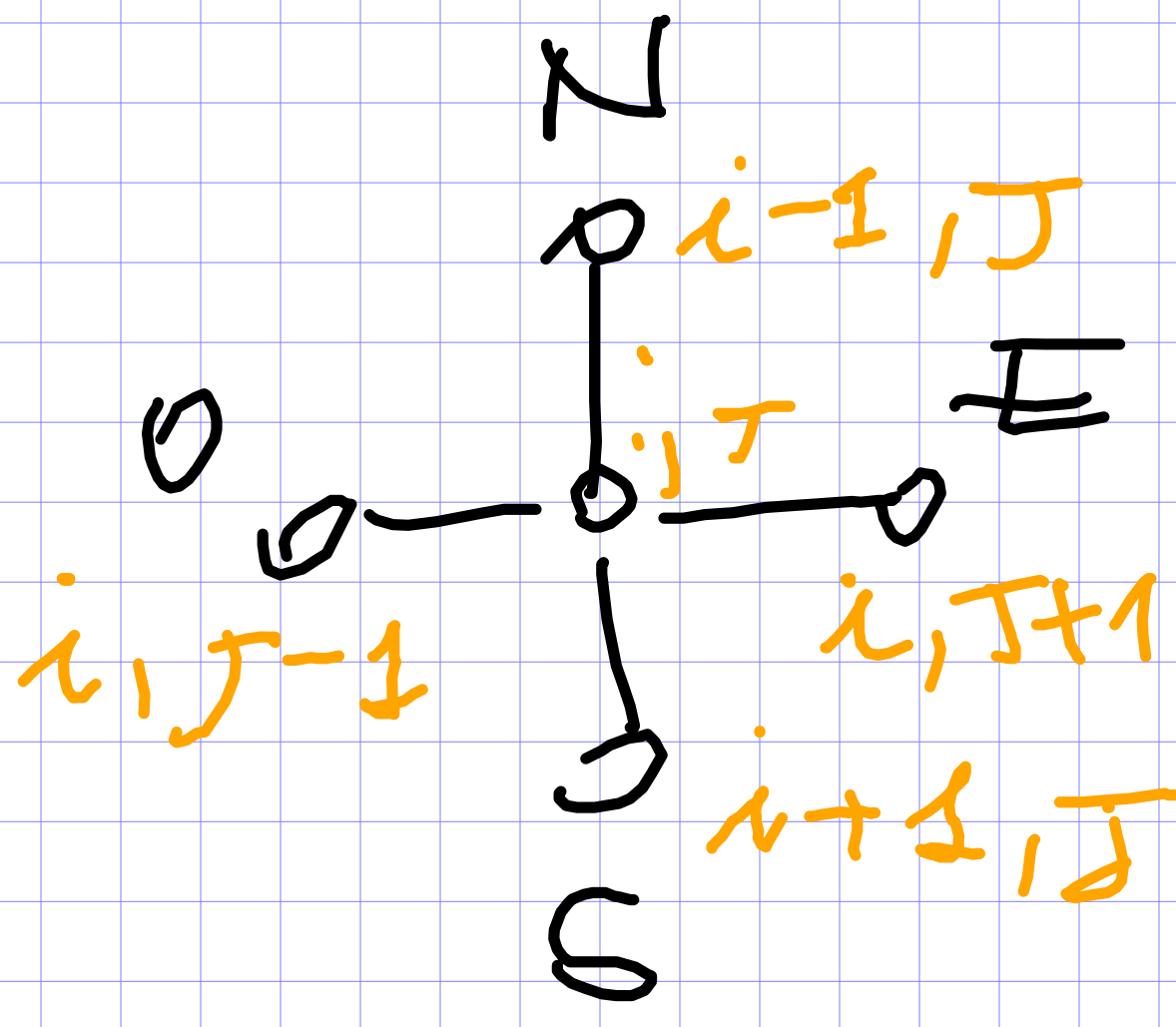
rete sociale

GRAFI con topologie particolari

- → alberi
- → mesh, griglie
- → grafi bipartiti
- → anelli grafici

MESH



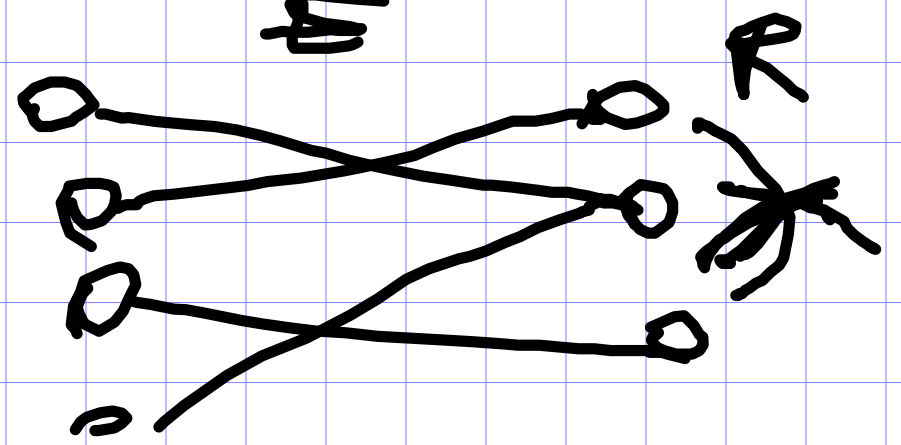


LATTICE

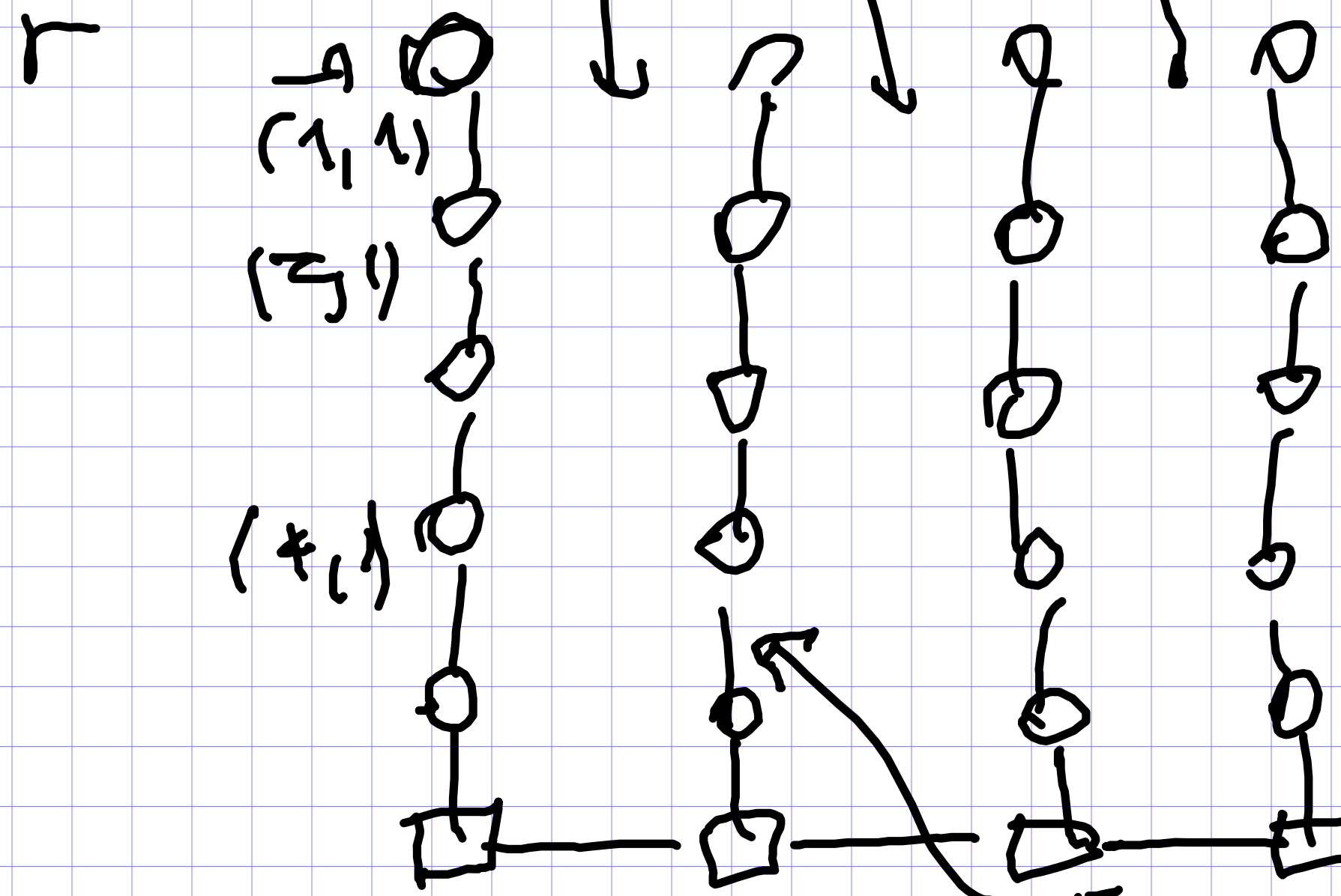
$V = [L \cup R]$

$\phi = L \cap R$

BIPARTITE



AISLE GRAPH = modello frutteto

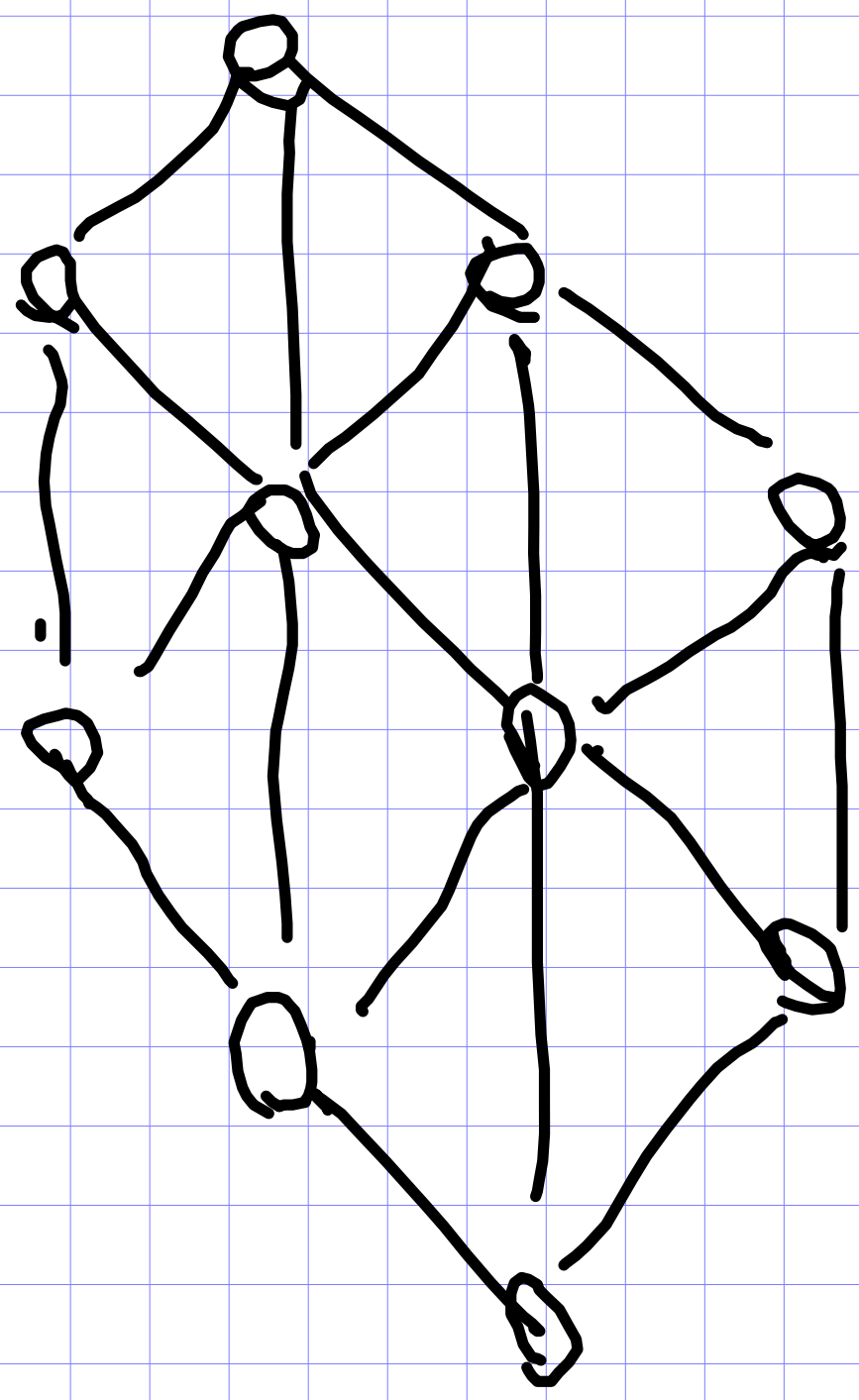


(i, j)

$r(i, j)$ → proprietà reticle

$w(e)$

$e = [(i, j), (i+1, j)]$



$|V| = n$

$|E| = m$

$|E| \leq n\,\frac{(n-1)}{2}$ grafo non orientato

$|E| \leq n(n-1)$ grafo orientato

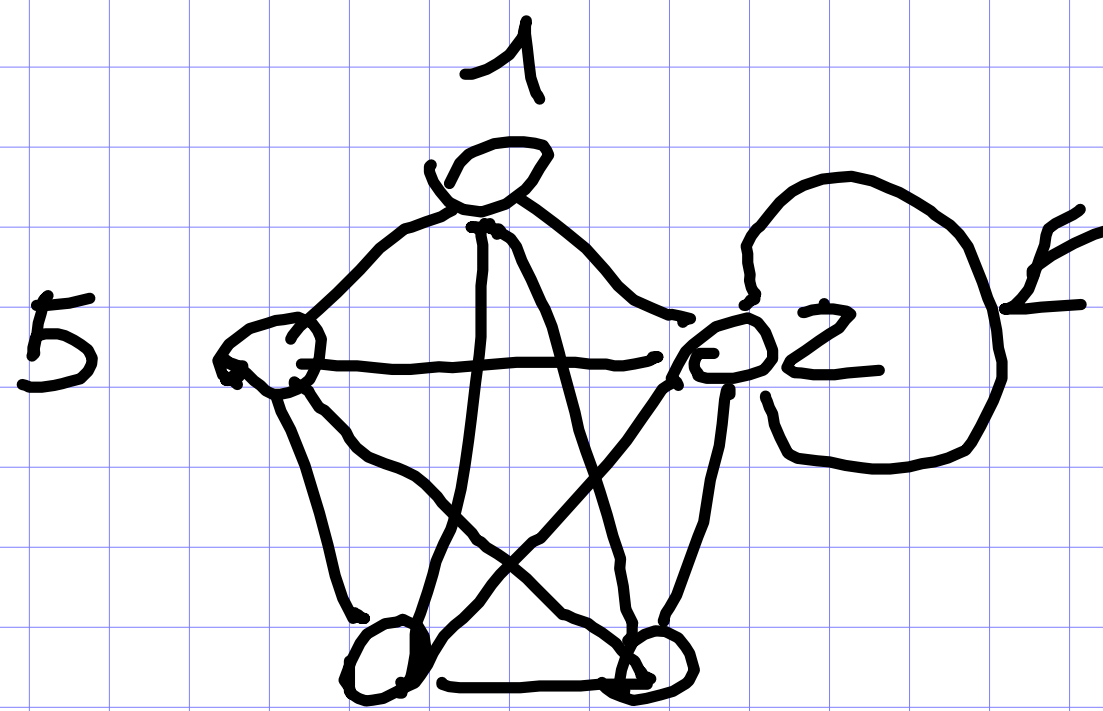
$\#$ coppie ordinate di nodi

non orientato

$G = K_n \quad |V| = n$

$|E| = \frac{n(n-1)}{2}$ ←

K_5



$\exists$ ciclo

$K_{1,n}$

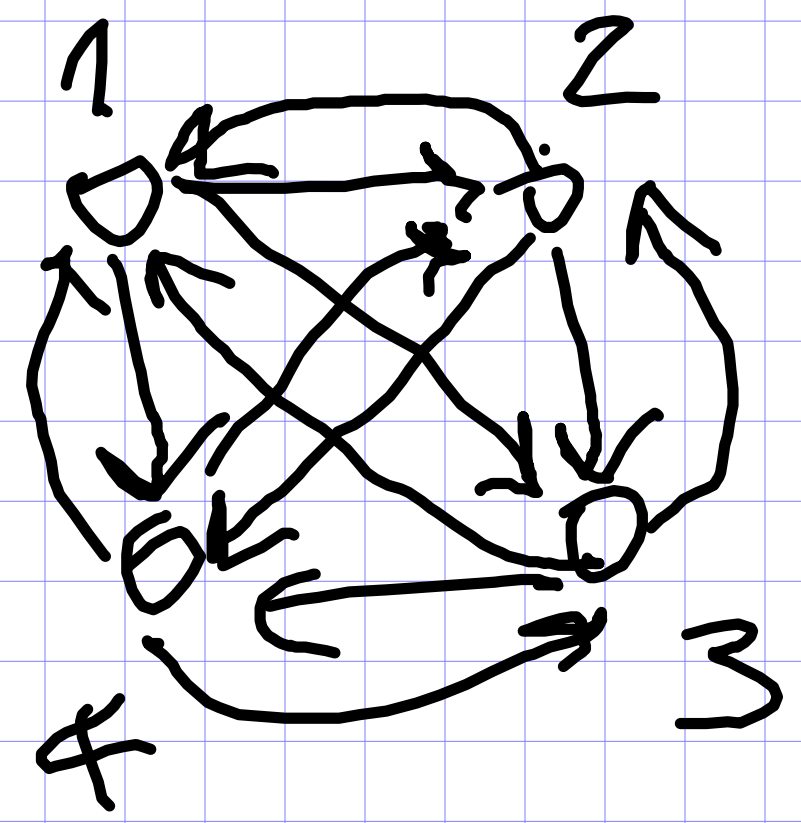
$n-1$

$\vec{K_n}$

$n = 4$

archi uscenti
da $u = (u, *)$

$n = 12$



$\delta^-(u) =$
$|V| - 1$

archi
entranti in
u
$(*, u)$

$\forall u \in V \quad \delta^+(u) = |V| - 1 = 3$

$m = |E| = |V|(|V| - 1) = n(n-1)$

n m

$m = \Theta(n)$ sparso

$m = \Theta(n^2)$ denso

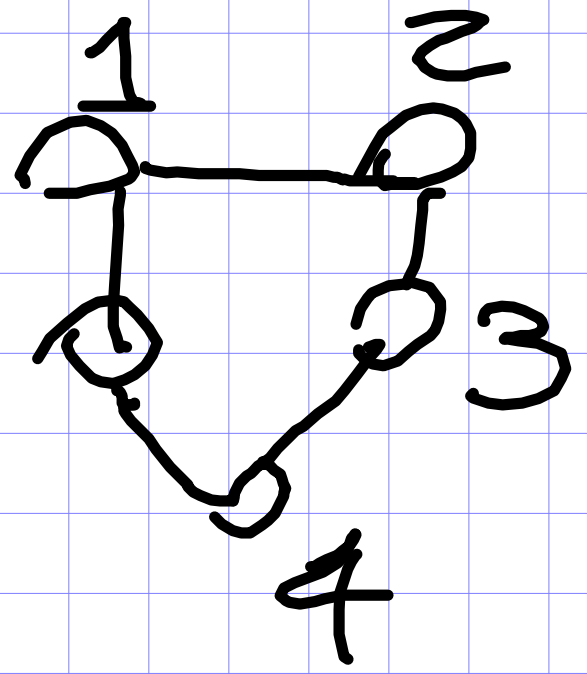
$n + m$

$G = (V, E)$ non orientato $(5,2) = (5,1,2)$

G è <u>connesso</u> se

$\forall u \in V \quad \forall z \in V$

$\exists$ u ~~cammino~~ z



non

path

sequenza di vertici tale che ogni coppia di vertici consecutivi è legata da un arco

$G = (V, E)$ connesso $\Rightarrow$ $E \geq |V| - 1$

$G = (V, E)$ connesso tale che

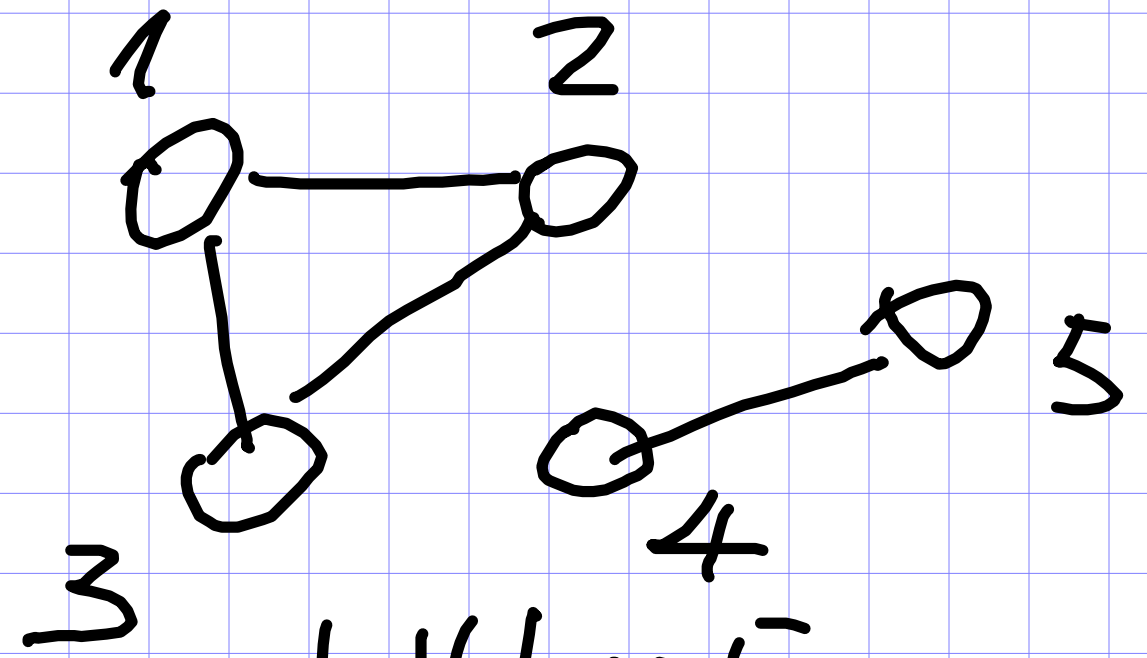
$$|E| = |V| - 1$$

$\Rightarrow$ G è un albero T.

T è un grafo connesso con il minimo numero di archi.

$G = (V, E)$ $\quad |E| = |V| - 1$

$\not\Rightarrow$ G non è un albero



$|V| = 5$

$|E| = 4$

$T - \{e\}$ $\Rightarrow$ grafo si disconnette

Dato un grafo connesso G, sfruttare G modo di trovare un albero che copra tutti i vertici di G.

$$\delta(v) = \#\text{ archi incidenti a } v$$

Proprietà

$$\sum_{v \in V} \delta(v) = 2|E|$$

Come memorizzo un grafo?

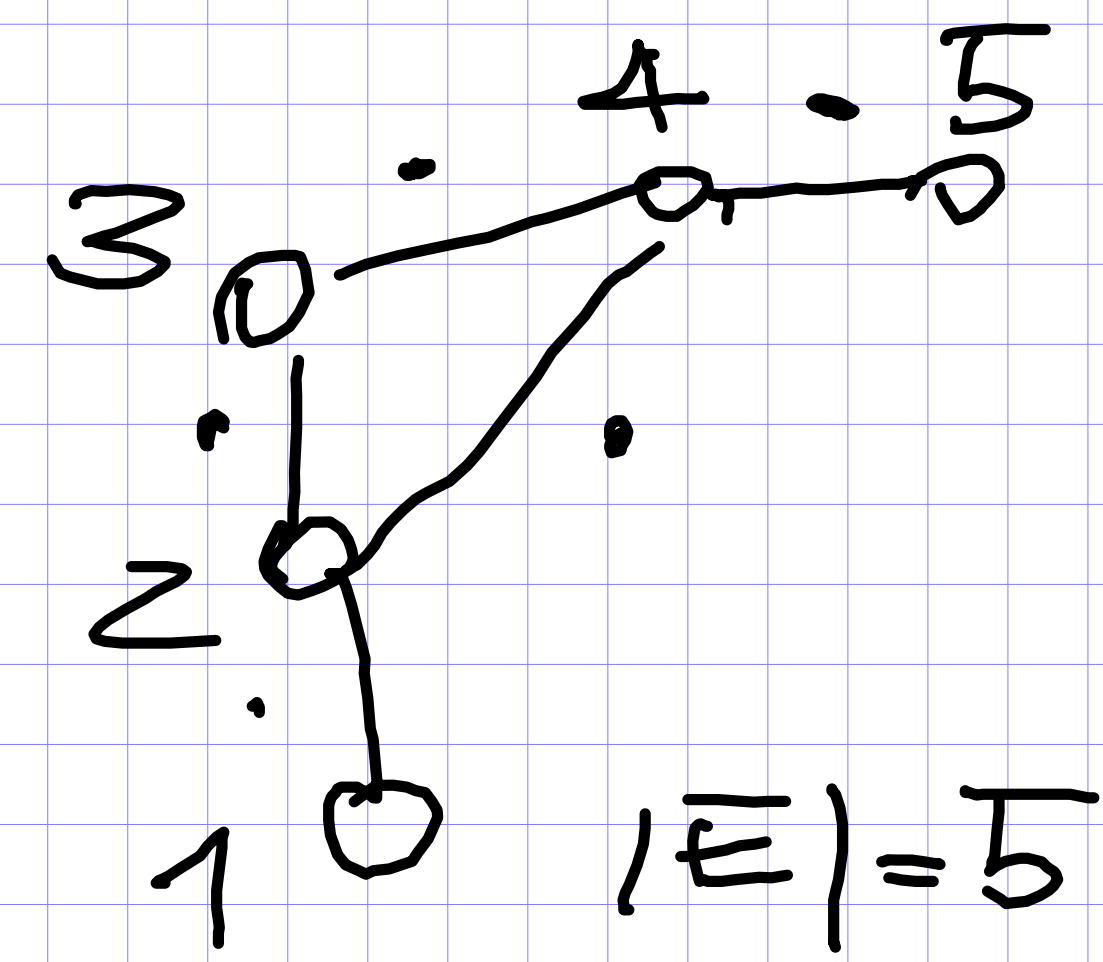
$G = (V, E)$

$|V| \times |V| = n^2$

MATRICE DELLE ADIACENZE

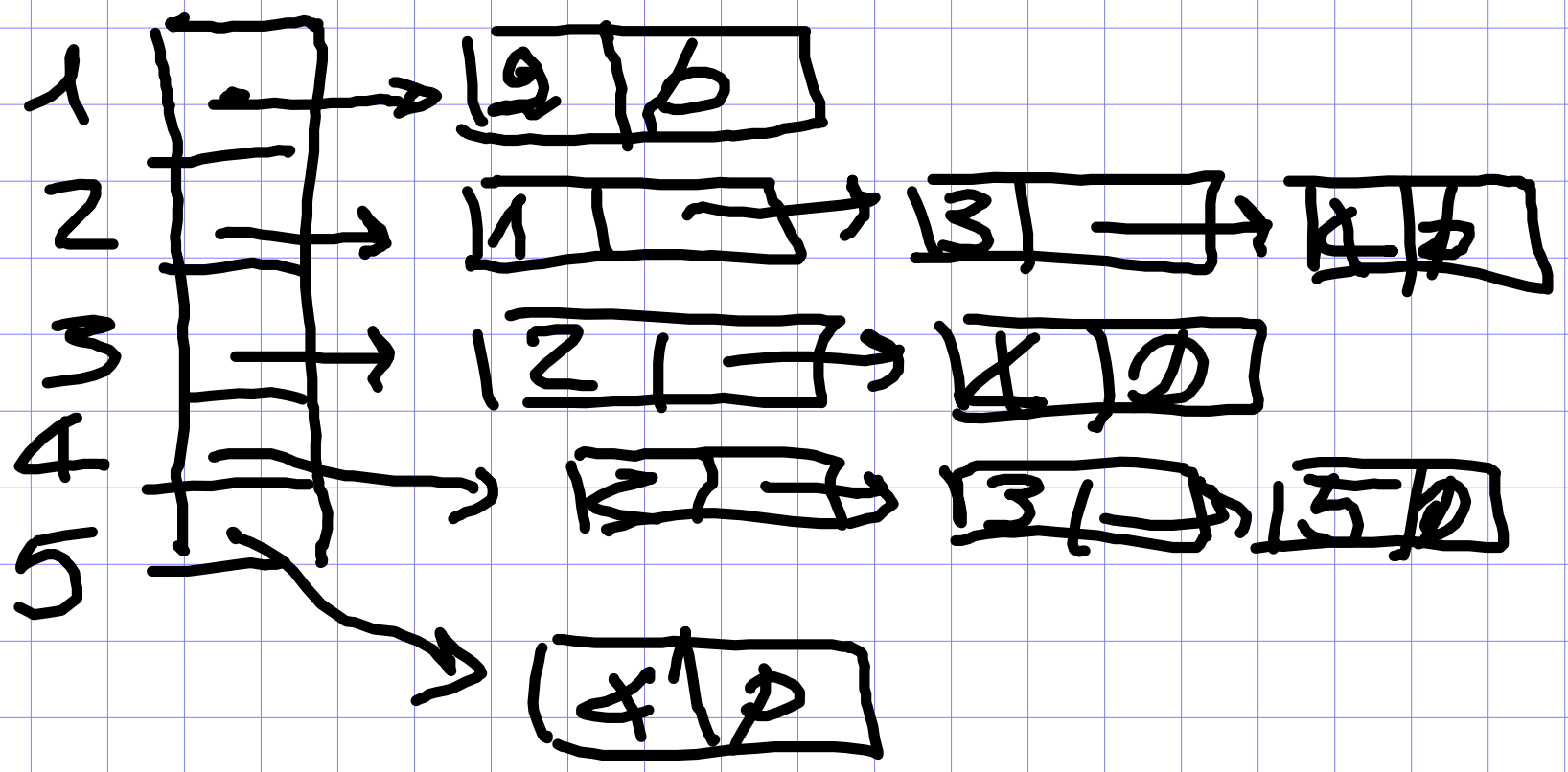
G

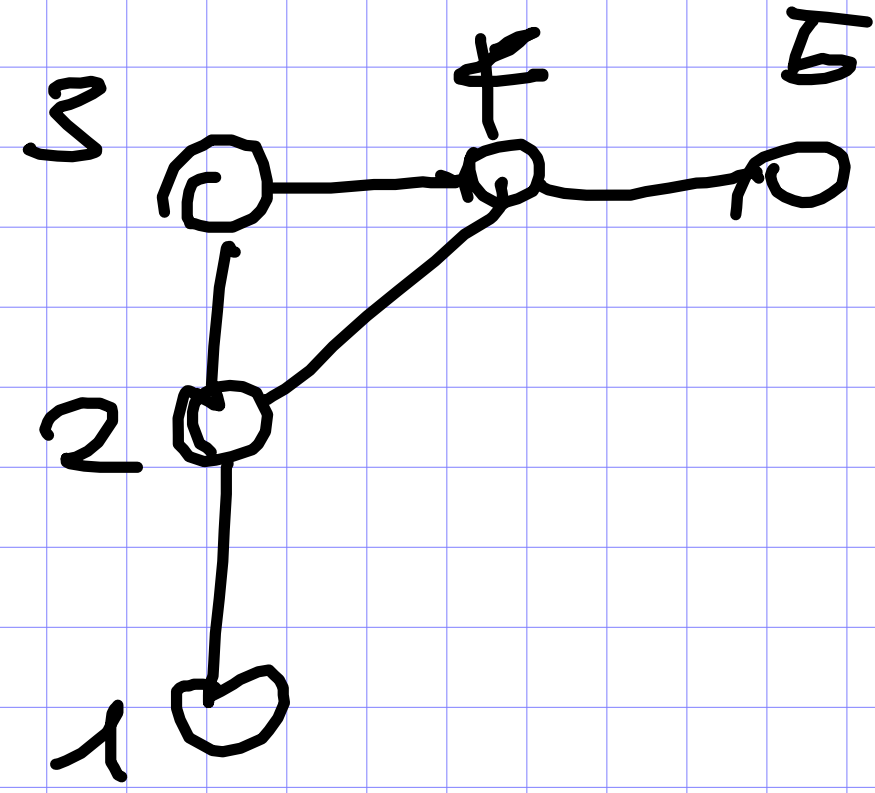
	1	2	3	4	5 = |V|
1	0	1	0	0	0
2	1	0	1	1	0
3	0	1	0	1	0
4	0	1	1	0	1
|V|=5	0	0	0	1	0



$|E| = 5$

lista delle adiacenze





$\forall v : Adj(v)$

$n + Q \cdot m$ ← SPAZIO

↑ grafo non orientato

Vettore delle adiacenze

V

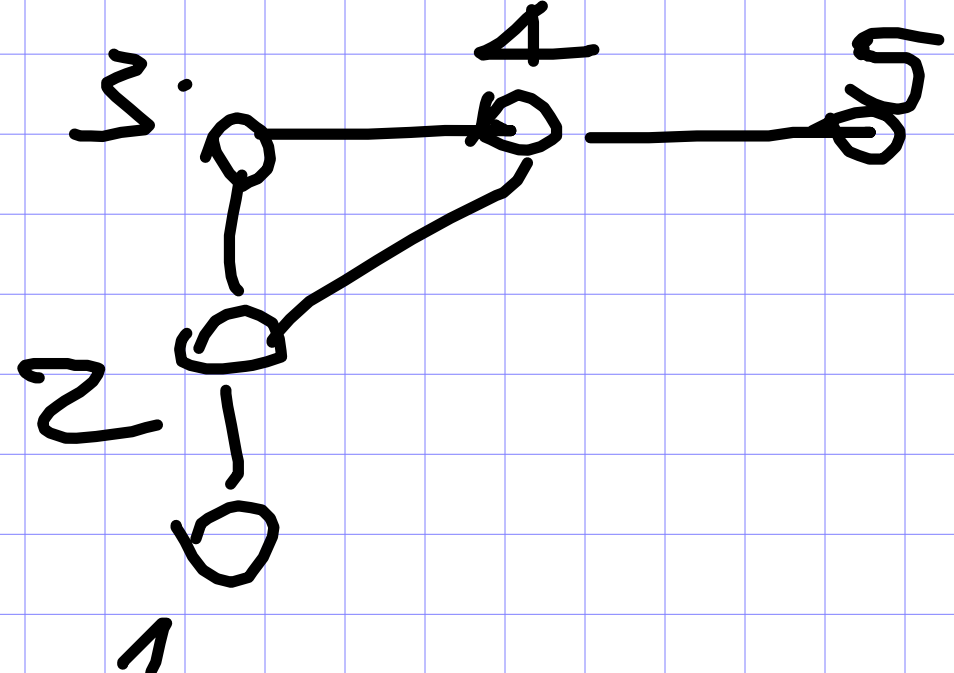
1	1
2	2
3	5
4	7
5	10
6	11

E

2	1
1	2
3	5
4	4
2	5
4	6
2	7
3	8
5	9
4	10
—	11

$V(i) =$ posizione in E in cui è memorizzato il 1° arco uscente da i

$Adj(i) = E(V(i)) \dots E(V(i+1)-1)$



$Adj(4) = \{2, 3, 5\}$