Laboratory activity 1: Position state–space control of a DC servomotor

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1 Activity goal

This laboratory activity is articulated in two parts: in the first part, some improvements to the position PID–control system designed in the previous laboratory activity are introduced. We focus on the implementation of (1) an anti–windup scheme to reduce the large overshoot occurring in the step response when the controller output saturates, and (2) a friction-plus-inertia feedforward compensator that enhances both the accuracy and speed of response of the conventional feedback controller.

The second part of the activity is devoted to the design of a continuous–time position control system by using state–space techniques. Both *nominal* and *robust* tracking designs are considered. Nominal tracking is performed by exploiting a gain-adjustment scheme; on the other hand, robust tracking is achieved by either exploiting an integral action (for robust tracking of constant set–points, or perfect rejection of constant load disturbances), or by resorting to the internal model principle (for robust tracking and perfect rejection of more general, possibly time–varying signals).

LAB 1 CHALLENGE: the only things you need to write down and submit are described in the last subsection of this document. The DEADLINE for this challenge (that counts as a presence test for remote attendance and potential extra point in the final grade) is APRIL 10, 2025 before midnight. An online repository will be opened on the elearning website, one submission for group.

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2 Laboratory assignments: numerical simulations

2.1 Part 1: position PID–control improvements

1. Reconsider the position PID–controller for the Quanser SRV–02 servomotor designed in the previous laboratory activity with the nominal motor parameters, with *the same specifications*. Implement the anti–windup scheme described in the Handout. The anti–windup gain $K_W = 1/T_W$ should be chosen large enough to maintain the integrator input small under any error condition. As a first trial, consider to choose $T_W = t_{s,5\%}/5$, where $t_{s,5\%}$ is the settling time specification used for the design of the PID controller.

2. Validate the effectiveness of the anti–windup mechanism implemented in point 1 in simulation, using an accurate Simulink model of the experimental system. For the test, use a step reference input with amplitude equal to $360°$. Adjust the value of K_W to obtain a satisfactory response (if needed). Compare the overshoot in the step response obtained by enabling or disabling the anti–windup scheme.

3. Reconsider the position PID–controller for the Quanser SRV–02 servomotor designed in the previous laboratory activity using the nominal motor parameters (nominal design). Implement the feedforward action described in the handout (see Fig. 3 of the Handout), using the equivalent inertia J_{eq}, the viscous friction coefficient B_{eq} and the static friction torque τ_{sf} estimated in the previous laboratory activity.

4. Validate the effectiveness of the feedforward compensation implemented in point 3 in simulation, using an accurate Simulink model of the experimental system. Set the parameters J_{eq}, B_{eq} and τ_{sf} of the motor model equal to the values estimated in the previous laboratory activity. For the test, consider a periodic acceleration reference signal with main period defined as follows

$$a_l^*(t) \triangleq \frac{d\omega_l^*(t)}{dt} = \begin{cases} 900\,\mathrm{rpm/s} & \text{if} \quad 0\,\mathrm{s} \le t < 0.5\,\mathrm{s} \\ 0\,\mathrm{rpm/s} & \text{if} \quad 0.5\,\mathrm{s} \le t < 1\,\mathrm{s} \\ -900\,\mathrm{rpm/s} & \text{if} \quad 1\,\mathrm{s} \le t < 2\,\mathrm{s} \\ 0\,\mathrm{rpm/s} & \text{if} \quad 2\,\mathrm{s} \le t < 2.5\,\mathrm{s} \\ 900\,\mathrm{rpm/s} & \text{if} \quad 2.5\,\mathrm{s} \le t < 3\,\mathrm{s} \end{cases} \tag{1}$$

   which defines a trapezoidal speed profile. The speed and position references are obtained by integration of the acceleration profile, i.e.

$$\omega_l^*(t) = \int_0^t a_l^*(\tau)\,d\tau\,, \qquad \vartheta_l^*(t) = \int_0^t \omega_l^*(\tau)\,d\tau \tag{2}$$

   The final acceleration, speed and position reference profiles are shown in Fig. 1.

   Compare the tracking error obtained by enabling or disabling the feedforward compensation.

2.2 Part 2: position state–space control design

For the state–space design methods considered in this section, assume that the plant state is fully accessible. Indeed, this assumption is not entirely true, since the optical encoder provides only the (load) position measure. However, the (load) velocity can be determined by differentiating the

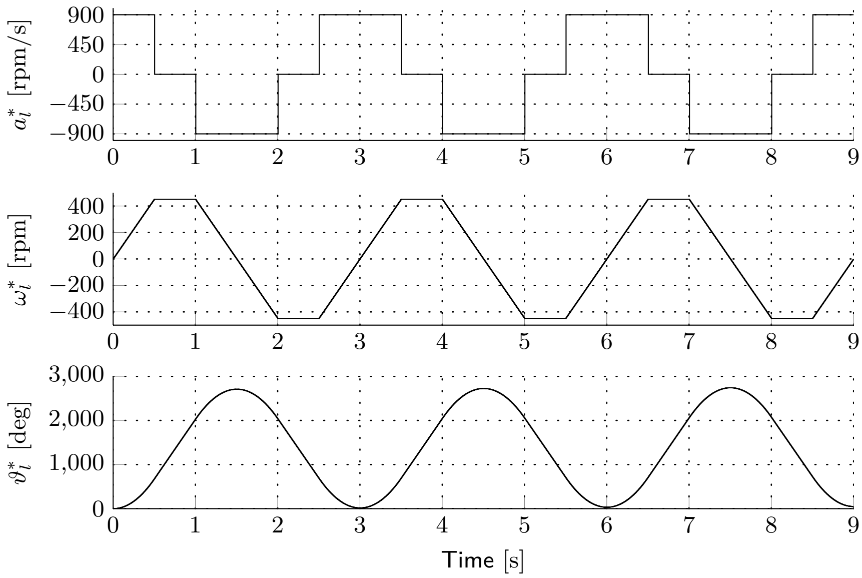


Figure 1: Reference signals template for the feedforward compensation test.

position measure with a suitable high–pass filter ("real derivative"). The filter cut–off frequency and roll–off must be chosen to avoid excessive amplification of the encoder quantization noise at high frequency. Two possible choices are the continuous–time filter

$$H_1(s) \;=\; \frac{\omega_c^2\, s}{s^2 + 2\delta\omega_c s + \omega_c^2} \qquad \text{with} \qquad \omega_c = 2\pi\, 50\,, \quad \delta = 1/\sqrt{2} \tag{3}$$

or the discrete–time filter

$$H_2(z) \;=\; \frac{1 - z^{-N}}{N\, T_s} \qquad \text{with} \qquad N = 10 \tag{4}$$

assuming that $T_s = 1\,\mathrm{ms}$. After this, proceed performing the following tasks:

1. Design a position state–space controller for the Quanser SRV–02 servomotor that guarantees the perfect tracking of constant position set–points in the nominal case (nominal perfect tracking). For the design, use the state–space model introduced in the Handout, with the parameters J_{eq} and B_{eq} as estimated in the previous laboratory activity, and the controller structure that uses the gains N_x, N_u or N_r.

   The control design specifications are

   – *nominal* perfect steady state tracking of step position (load side) references

   – step response (load side) with settling time $t_{s,5\%} \leq 0.15\,\mathrm{s}$ and overshoot $M_p \leq 10\%$

   For the design of the state feedback matrix $\boldsymbol{K} \in \mathbb{R}^{1\times 2}$, use either the `place` or `acker` routines of the Control System Toolbox (CST). To decide the location of the the closed–loop eigenvalues (namely the eigenvalues of $\boldsymbol{A} - \boldsymbol{BK}$), assume that the dominant closed–loop dynamics can be approximated with that of a second order system (dominant pole approximation approach) of the type

$$T(s) \;=\; \frac{\omega_n^2}{s^2 \,+\, 2\delta\omega_n s \,+\, \omega_n^2}\,, \qquad 0 \leq \delta < 1 \tag{5}$$

whose eigenvalues (poles) are equal to

$$\lambda_{1,2} \;=\; -\delta\omega_n \;\pm\; j\,\omega_n\sqrt{1-\delta^2} \tag{6}$$

For the determination of the natural frequency ω_n and the damping factor δ that satisfy the design specifications, remember that the following relations hold (see previous laboratory activity):

$$t_{s,5\%} \;=\; \frac{3}{\delta\omega_n}\,, \qquad\qquad \delta \;=\; \frac{\log(1/M_p)}{\sqrt{\pi^2+\log^2(1/M_p)}} \tag{7}$$

2. Validate the design of point 1 in simulation, using an accurate Simulink model of the experimental system. A possible implementation of the Simulink model is shown in Fig. 2. Note that the load position ϑ_l (*first* state component) is directly measured by the encoder, while the load speed ω_l (*second* state component) is obtained by filtering the position measurement with a high–pass filter ("real" derivative) such as (3) or (4).

   For the simulations, consider either an "ideal" situation with no static friction, or a "real" situation with a static friction torque as estimated in the previous laboratory activity.

   Perform the tests with three different position set–points, e.g. $40°$, $70°$ and $120°$.

3. Design a position state–space controller for the Quanser SRV–02 servomotor that guarantees the perfect steady state *robust* tracking of *constant* position set–points and perfect rejection of *constant* load disturbances, by using the integral action approach with the extended state-space pole allocation.

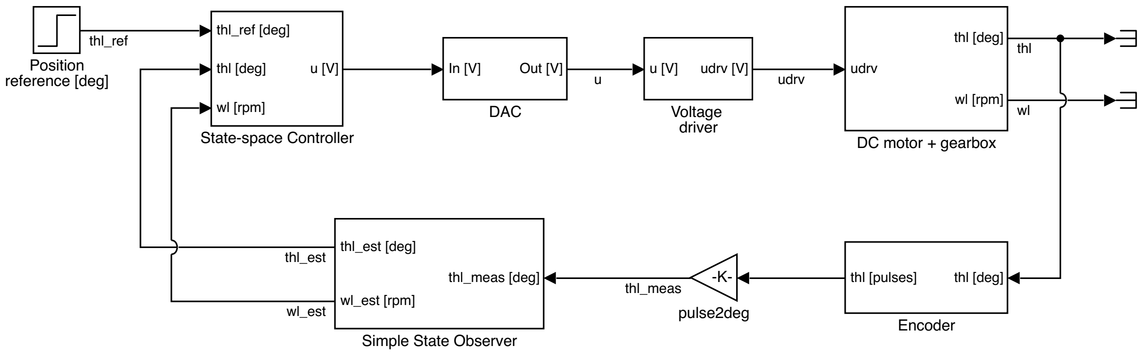
   The method requires to place three closed–loop eigenvalues $\lambda_{c,\{1,2,3\}}$ (namely the eigenvalues of $\boldsymbol{A}_e - \boldsymbol{B}_e\boldsymbol{K}_e$). Their position can be still decided according to the dominant pole approximation criterion, assuming that the closed–loop system should behave similarly to second order system (5). Let σ and $\pm\omega_d$ be the real and imaginary parts of the eigenvalues in (6). Then, consider the following eigenevalue placements for the design of the closed–loop system

   – $\lambda_{c,\{1,2\}} = \sigma \pm j\,\omega_d, \quad \lambda_{c,3} = \sigma$
   – $\lambda_{c,\{1,2,3\}} = \sigma$
   – $\lambda_{c,\{1,2\}} = 2\,\sigma \pm j\,\omega_d, \quad \lambda_{c,3} = 2\,\sigma$
   – $\lambda_{c,\{1,2\}} = 2\,\sigma \pm j\,\omega_d, \quad \lambda_{c,3} = 3\,\sigma$

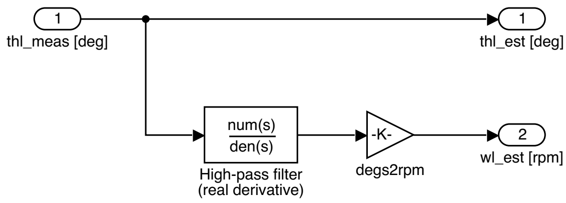
4. Validate the design of point 3 in simulation, using an accurate Simulink model of the experimental system. Of the several design alternatives produced in that point, consider only that showing the best response to a $40°$ step reference, assuming zero static friction torque.

   Test the controller either in an "ideal" situation with no static friction, or in a "real" situation with a static friction torque as estimated in the previous laboratory activity. Perform the tests with three different position set–points, e.g. $40°$, $70°$ and $120°$.

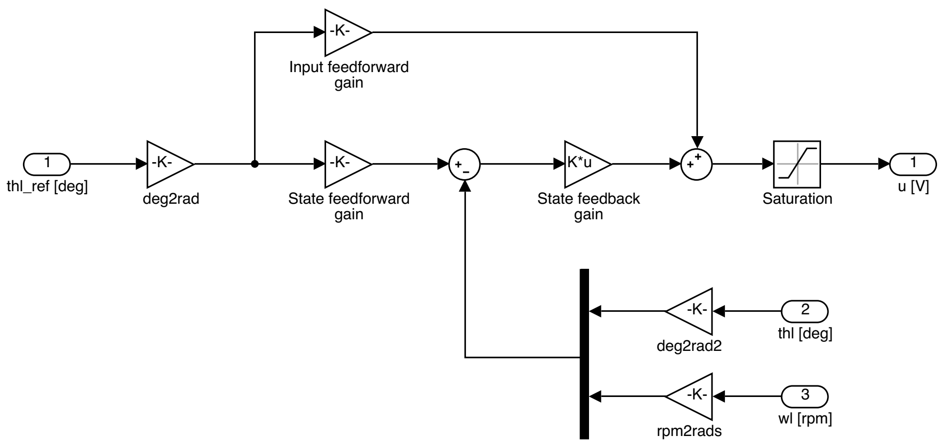
5. Design a position state–space controller for the Quanser SRV–02 servomotor that guarantees the perfect steady state *robust* tracking of a *sinusoidal* position reference of a specified period T_r, and perfect rejection of *constant* load disturbances, using the error–space approach described in the Handout (and lectures).



(a) Position state–space control loop (top model)



(b) "Simple" state observer



(c) State–space controller (for nominal tracking)

Figure 2: Detailed Simulink model of the position state–space control system (for the nominal tracking case).

Do the design for the following choices of the period T_r:

$$T_{r,1} = 0.15\,\mathrm{s}\,, \qquad T_{r,2} = 0.25\,\mathrm{s}\,, \qquad T_{r,3} = 0.5\,\mathrm{s}\,, \qquad T_{r,4} = 1\,\mathrm{s} \tag{8}$$

The design with the error–space approach requires to place five closed–loop eigenvalues $\lambda_{c,\{1,2,3,4,5\}}$ (namely the eigenvalues of $\boldsymbol{A}_z - \boldsymbol{B}_z\boldsymbol{K}_z$). Consider the following tentative choice for the eigenvalues placement

$$\lambda_{c,\{1,2\}} = \omega_n\, e^{j\,(-\pi \pm \pi/4)}\,, \qquad \lambda_{c,\{3,4\}} = \omega_n\, e^{j\,(-\pi \pm \pi/6)}\,, \qquad \lambda_{c,5} = -\omega_n \tag{9}$$

where ω_n is the natural frequency of the dominant poles approximation (5). Adapt this to the simulation response in order to obtain a satisfactory response.

6. Validate the design of point 5 in simulation, using an accurate Simulink model of the experimental system. Consider the controller designed to track a sinusoidal reference input with period $T_r = T_{r,3} = 0.5\,\mathrm{s}$.

   Test the controller either in an "ideal" situation with no static friction, or in a "real" situation with a static friction torque as estimated in the previous laboratory activity. For the tests, use three different sinusoidal reference inputs, with same period T_r of $0.5\,\mathrm{s}$, and amplitudes A_r equal to $30°$, $60°$ and $90°$.

7. Repeat the test of point 6 with a sinusoidal reference input of amplitude $A_r = 40°$ and period $T_r = 0.1$ (but with the control designed to track a reference sinusoid with period $T_r = 0.5\,\mathrm{s}$). Compare the tracking error with that obtained in the previous case.

8. Repeat the test of point 6 with a step reference input of amplitude $40°$. Is the controller capable of robustly tracking the step reference input, with zero steady state tracking error ? Motivate your answer.

9. (**Optional**) Repeat the design of point 5 by using the extended–state estimator method described in the Handout.

   For the design of the extended estimator, it is required to place two "controller" eigenvalues $\lambda_{c,\{1,2\}}$ (namely, the eigenvalues of $\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}$) and five "estimator" eigenvalues $\lambda_{e,\{1,2,3,4,5\}}$ (namely, the eigenvalues of $\boldsymbol{A}_e - \boldsymbol{L}_e\boldsymbol{C}_e$). Consider the following tentative choice for the controller and estimator eigenvalues

$$\lambda_{c,\{1,2\}} = -\delta\omega_n \pm j\,\omega_n\sqrt{1-\delta^2} \tag{10}$$

$$\lambda_{e,\{1,2\}} = 2\,\omega_n\, e^{j\,(-\pi \pm \pi/3)}\,, \qquad \lambda_{e,\{3,4\}} = 2\,\omega_n\, e^{j\,(-\pi \pm \pi/6)}\,, \qquad \lambda_{e,5} = -2\,\omega_n \tag{11}$$

   where δ and ω_n are the parameters of the dominant poles approximation (5).

   The extended estimator gain matrix $\boldsymbol{L}_e$ can be designed by resorting to the usual routines for the eigenvalues placement, namely `acker` and `place` of the Control System Toolbox (CST). However, these routines are designed to solve the controller eigenvalues placement problem, namely they compute the state–feedback matrix $\boldsymbol{K}$ such that the closed–loop system matrix $\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}$ has all its eigenvalues at the specified locations, provided that the pair $(\boldsymbol{A}, \boldsymbol{B})$ is reachable.

Nevertheless, the observer eigenvalues placement problem, which consists of computing the observer gain matrix $\boldsymbol{L}_e$ that places the eigenvalues of the matrix $\boldsymbol{A}_e - \boldsymbol{L}_e\boldsymbol{C}_e$ (which governs the estimation error dynamics) to the desired locations, can be solved as a controller eigenvalues placement problem for the dual system $(\boldsymbol{A}_e^T, \boldsymbol{C}_e^T)$ (*duality theorem*), simply by computing the controller gain matrix $\boldsymbol{K}_e = \boldsymbol{L}_e^T$ that places the eigenvalues of $\boldsymbol{A}_e^T - \boldsymbol{C}_e^T\boldsymbol{K}_e$ at the desired observer eigenvalues locations.

Regarding the Simulink implementation of the extended observer, note that $\hat{\Sigma}_e$ is a state–space model with two inputs, namely u and e. Therefore, the estimator can be implemented as the regular continuous–time state–space model (in Library Browser, select: *Continuous* $\rightarrow$ *State–Space*):

$$\hat{\Sigma}_e = (\boldsymbol{A}_e - \boldsymbol{L}_e\boldsymbol{C}_e,\, [\boldsymbol{B}_e,\, \boldsymbol{L}_e]\,,\, \boldsymbol{I},\, \boldsymbol{0}) \tag{12}$$

with inputs u and e and output $\boldsymbol{y} \equiv \hat{\boldsymbol{x}}_e$. Use *Signal Routing* $\rightarrow$ *Mux* to stack the two input signals u and e into a single input vector, and *Signal Routing* $\rightarrow$ *Demux* to separate the two components $\hat{\boldsymbol{x}}_\rho$ and $\hat{\boldsymbol{x}}'$ of the estimator output vector.

10. (**Optional**) Repeat the tests of points 6, 7 and 8 with the controller designed in point 9.

3 Laboratory assignments

In the following, you will be required to validate your designs with either the actual motor in the laboratory (LAB1 activity). As in the previous assignment, you will need to modify and adapt your simulink files:

– replace the model of the DC servomotor with the blocks of the Simulink Desktop Real–Time (SLDRT) toolbox that allow to communicate with the experimental device, as you did in the previous experience.

  In addition to send the voltage command to the DAC (using the *Analog Output* block) and read the pulse count from the encoder (using the *Encoder Input* block), consider also to implement a block for sensing the motor current. Follow the instructions provided in the introductory guide to the experimental setup to implement such block.

– configure the simulation parameters of the new Simulink model to perform a "real–time simulation". For the purpose, consult the instructions provided in the introductory guide to the experimental setup.

  In particular, choose a *fixed–step ode3* solver, with step size equal to $T_s = 1\,\mathrm{ms}$ (controller sampling time).

3.1 Part 1: position PID–control improvements

1. Validate the anti–windup scheme designed in point 1 of Sec. 2.1 on the "experimental" setup, as described in Assignment 0, where the model of the motor is substituted with the Quanser Motor Model provided in the course site. Perform the experimental tests by adopting the same methodology used for simulations, namely by following the procedure described in step 2 of Sec. 2.1.

2. Validate the feedforward compensator designed in point 3 of Sec. 2.1 on the experimental setup. Perform the experimental tests by adopting the same methodology used for simulations, namely by following the procedure described in step 4 of Sec. 2.1.

3.2 Part 2: position state–space control design

1. Validate the position state–space controller designed in point 1 of Sec. 2.2 (for nominal perfect tracking of constant position set–points) on the experimental setup.

   Perform the tests with three different position set–points, e.g. $40°$, $70°$ and $120°$. Verify that the controller is unable to guarantee perfect steady state tracking, because of the presence of friction and parameter uncertainties.

2. Validate the position state–space controller designed in point 3 of Sec. 2.2 (for robust perfect tracking of constant position set–points, using the integral action) on the experimental setup.

   Perform the tests with three different position set–points, e.g. $40°$, $70°$ and $120°$. Verify that the controller is capable of tracking the constant position set–points with zero steady state error, thanks to presence of the integral action.

3. Validate the position state–space controller designed in point 5 of Sec. 2.2 (for robust perfect tracking of a specified class of position reference inputs, using the internal model principle) on the experimental setup.

   Consider the design for tracking a sinusoidal reference input with period $T_r = 0.5\,\mathrm{s}$ (and rejecting constant input disturbances).

   Perform the experimental tests by using three different sinusoidal reference inputs, with same period $T_r = 0.5\,\mathrm{s}$, and amplitudes A_r equal to $30°$, $60°$ and $90°$. Verify that the controller is capable of tracking the input sinusoid with zero steady state tracking error, even in presence of the static friction torque (constant input torque disturbance).

3.3 LAB 1 CHALLENGE

Design, using one or a combinations of the techniques presented in the course, a control system for the QUANSER SRV-02 MOTOR (real or black-box model) such that:

- **It ensures asymptotic tracking of step references;**
- **It ensures an overshoot $M_p \leq 10\%$ for a 70deg step reference;**
- **It attains the minimum raise time t_r you are able to achieve.**

A pdf file (2 pages max.) including:

- **a brief description of the control structure and strategy;**
- **the parameters and values needed to implement the controller;**
- **a plot of the response to the 70 degree step (with real motor or black box);**
- **the corresponding best value of t_r you attained (specifying if it is for the black-box model or the actual motor);**

should be uploaded in an online repository that will be opened on the elearning website.