Modelling and simulation

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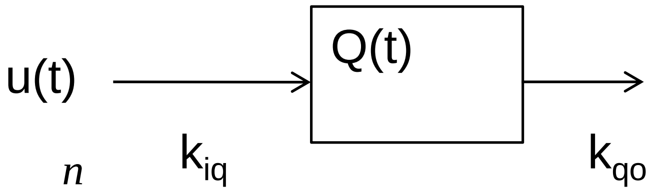
Compartmental models

- Compartment - certain zone of system (physiological area, space)
- Input of compartment - getting of substance
- Output of compartment - movement substance outside the compartment

Compartmental models

- 1 - compartmental models



$$Q'(t) = \sum_{i=1}^{n} k_{iq} u_i(t) - \sum_{j=1}^{n} k_{qj} Q(t)$$

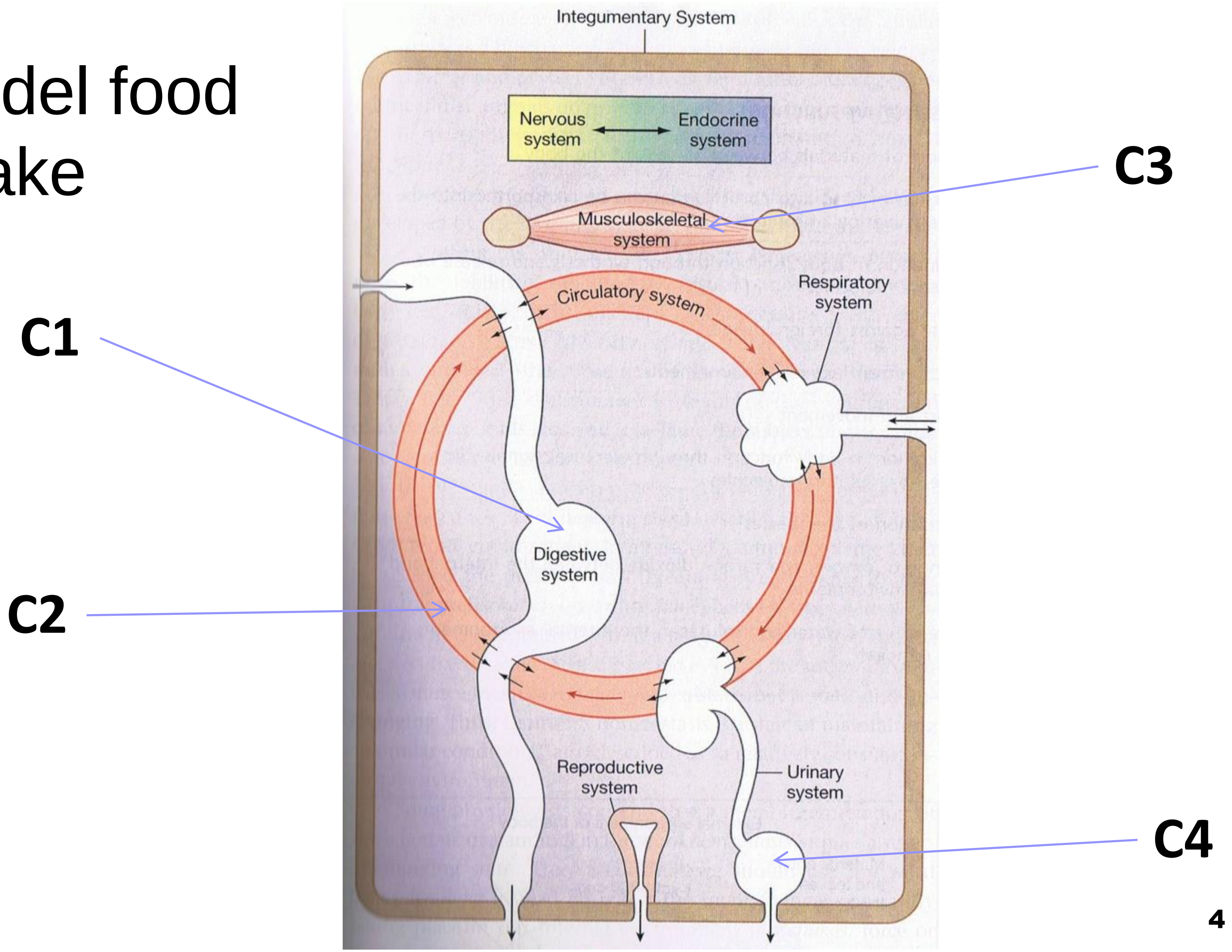
+ sum of inputs **-** sum of outputs

- More - compartmental models

$$Q_k'(t) = \sum_{i=1}^{n} k_{iq(k)} u_i(t) + \sum_{\substack{j=1 \\ j\neq k}}^{K} k_{jk} Q_j(t) - \sum_{\substack{j=1 \\ j\neq k}}^{K} k_{kj} Q_k(t) - \sum_{i=1}^{m} k_{q(k)i} Q_k(t)$$

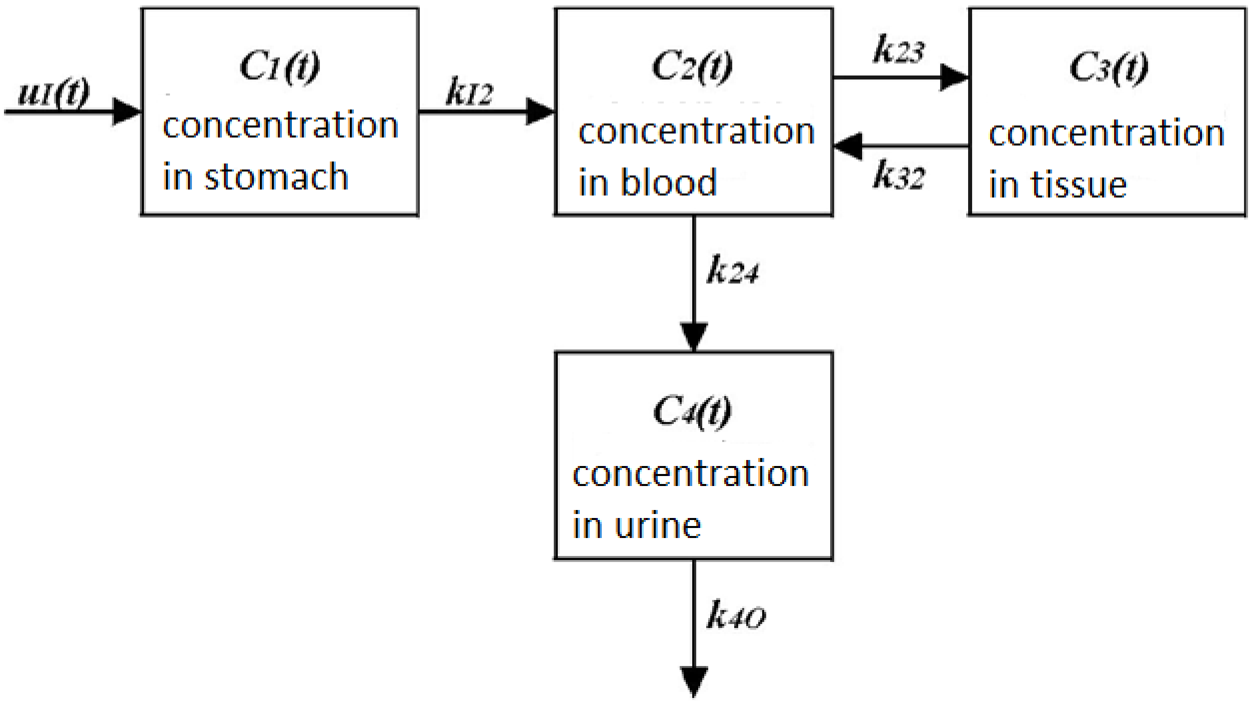
Practice 1 – assignment

- Model food intake



Practice 1 – assignment

- Model food intake (glucose)



Block diagram of the system model of food intake

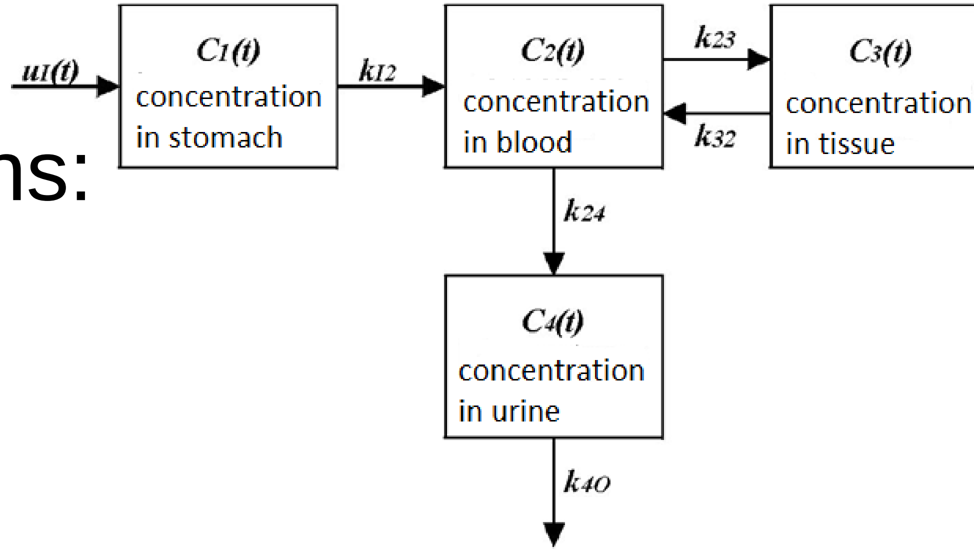
Practice 1 – assignment

- Model food intake
  - Compose differential equations of the model based on the schema model.
  - Convert the external description of system on the internal (state) in which the output is concentration of the substance in the compartment C4.
  - Determine the concentration of a substance (glucose) in individual compartments using simulation
  - Modify the model so that the rate of decline in blood concentration was dependent on the concentration in the blood

$$k_{24}(C_2(t)) = \begin{cases} 0 & \text{if} \quad C_2(t) \leq C_{2m} \\ k_{24} & \text{if} \quad C_2(t) > C_{2m} \end{cases}$$

Practice 1 – solution

- Differential equations:



Block diagram of the system model of food intake

$$C_1'(t) = u - k_{12}C_1(t)$$

$$C_2'(t) = k_{32}C_3(t) + k_{12}C_1(t) - k_{23}C_2(t) - k_{24}C_2(t)$$

$$C_3'(t) = k_{23}C_2(t) - k_{32}C_3(t)$$

$$C_4'(t) = k_{24}C_2(t) - k_{40}C_4(t)$$

Practice 1 – solution

- Internal state space description:

$$C_1'(t) = u - k_{12}C_1(t)$$

$$C_2'(t) = k_{32}C_3(t) + k_{12}C_1(t) - k_{23}C_2(t) - k_{24}C_2(t)$$

$$C_3'(t) = k_{23}C_2(t) - k_{32}C_3(t)$$

$$C_4'(t) = k_{24}C_2(t) + k_{40}C_4(t)$$

$$\boxed{y = C_4}$$

output, we want to know the concentration

Practice 1 – solution

- Internal state space description:

$$\boxed{\dot{x}(t) = A(t)x(t) + B(t)u(t)}$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

$$C_1'(t) = u - k_{12}C_1(t)$$
$$C_2'(t) = k_{32}C_3(t) + k_{12}C_1(t) - k_{23}C_2(t) - k_{24}C_2(t)$$
$$C_3'(t) = k_{23}C_2(t) - k_{32}C_3(t)$$
$$C_4'(t) = k_{24}C_2(t) + k_{40}C_4(t)$$
$$y = C_4$$

$$\begin{bmatrix} C_1' \\ C_2' \\ C_3' \\ C_4' \end{bmatrix} = \begin{bmatrix} -k_{12} & 0 & 0 & 0 \\ k_{12} & -k_{32} - k_{24} & k_{32} & 0 \\ 0 & k_{23} & -k_{32} & 0 \\ 0 & k_{24} & 0 & -k_{40} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

  - $A(t)$ is the system matrix
  - $B(t)$ is the control (input) matrix

Practice 1 – solution

- Internal state space description:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

$$\boxed{y(t) = C(t)x(t) + D(t)u(t)}$$

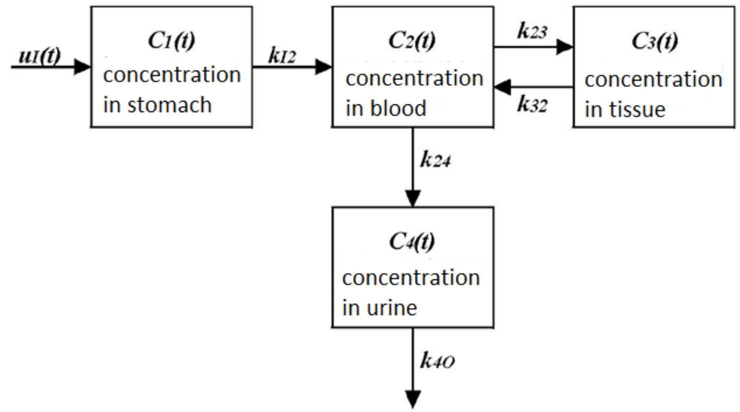
$$[y] = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} + 0u$$

- $C(t)$, $D(t)$ are output matrix

$$C_1'(t) = u - k_{12}C_1(t)$$
$$C_2'(t) = k_{32}C_3(t) + k_{12}C_1(t) - k_{23}C_2(t) - k_{24}C_2(t)$$
$$C_3'(t) = k_{23}C_2(t) - k_{32}C_3(t)$$
$$C_4'(t) = k_{24}C_2(t) + k_{40}C_4(t)$$
$$y = C_4$$

Practice 1 – solution

- Diferenciální rovnice:



Block diagram of the system model of food intake

$$C_1'(t) = u - k_{12}C_1(t)$$

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$$C_3'(t) = k_{23}C_2(t) - k_{32}C_3(t)$$

$$C_4'(t) = k_{24}C_2(t) - k_{40}C_4(t)$$

u = 1 mg/l/hour
k_{12} = 1 hour^{-1}
k_{23} = 0.8 hour^{-1}
k_{24} = 0.6 hour^{-1}
k_{32} = 0.3 hour^{-1}
k_{4o} = 0.6 hour^{-1}

C1(0)=10 mg/l
C2(0)=C3(0)=C4(0)=0 mg/l

Practice 1 – solution

- Diferenciální rovnice:

$$C_1'(t) = u - k_{12}C_1(t)$$

$$C_2'(t) = k_{32}C_3(t) + k_{12}C_1(t) - k_{23}C_2(t) - k_{24}C_2(t)$$

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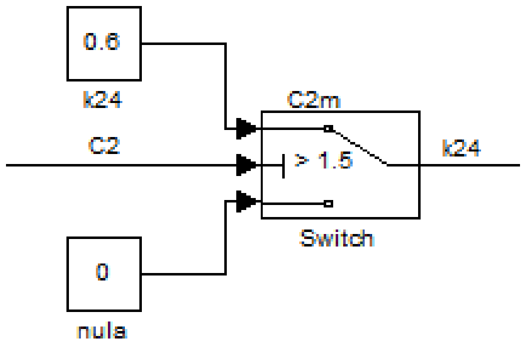
k_{24} = 0.6 hour^{-1}
k_{32} = 0.3 hour^{-1}
k_{4o} = 0.6 hour^{-1}

C1(0) = 10 mg/l
C2(0)=C3(0)=C4(0)=0 mg/l
C_{2m} = 1.5 mg/l

$$k_{24}(C_2(t)) = \begin{cases} 0 & \text{if} \quad C_2(t) \le C_{2m} \\ k_{24} & \text{if} \quad C_2(t) > C_{2m} \end{cases}$$



Practice 2 - assignment

- Zombies are spreading in the city.
- An individual may be either live person dead person, zombie, or definitely dead zombie
- 300 000 people and 1 zombie are in the city.
- Every year 20 000 new residents is coming to town.
- 6% of people per day are dying of natural causes.

Practice 2 - assignment

- Zombie are becoming
  - Daily 1/7 of all living people
  - Daily 4.5% of dead people
- Some zombie (1% per day) are closed in quarantine by people, where they can not meet people. Zombie finally die after a 30 day stay in quarantine (they never can become a zombie).

Practice 2 - assignment

- Every day, every 68th zombie escapes from quarantine.
- Some zombie (2% per day) are destroyed by people, thus zombies finally die (they can not become a zombie).

Practice 2 - assignment

- Solve the task using compartments
- Determine how many zombie finally dies for 6 days.
- Write the state equation of system with two outputs:
  - Number of dead people
  - Number of dead zombie
- Verify the accuracy of equations of state by simulation- use State-space block.

Practice 2 – desired output

- Model file *. mdl with correctly described blocks
- Short paper in *. pdf containing
  - Definition equation model
  - Block diagram of the model
  - State equations of model
  - Table of all the parameters of the original model with columns: symbol, importance, value, unit
  - Table of all state variables of the model with columns: symbol, meaning the initial value, unit
  - Graphical output of simulations with the labeled number of dead zombie after 6 hours
  - Graphical output of verifying the correctness of equations