

Epidemiological models

Model SIS

Symbol	Importance	Value	Units	
S	population susceptible to infection	199990	ind	Variab
I	infested population	10	ind	
a	rate of recovery from illness	0.018	time ⁻¹	Param
r	rate of spread infection	$1.5 \cdot 10^{-6}$	time ⁻¹	

Differential equations:

$$S'(t) = a \cdot I(t) - r \cdot S(t) \cdot I(t)$$

$$I'(t) = r \cdot S(t) \cdot I(t) - a \cdot I(t)$$

Determination of character of an epidemy

$$\rho = 0.18 / 0.0000015 = 120\,000 < S(0) = 199990 \text{ i.e. epidemic isn't spreading}$$

To stop spreading the infection healing rate should be changed to make $I'=0$. I. e.:

$$0 = r \cdot S(t) \cdot I(t) - a \cdot I(t)$$

$$a \cdot I(t) = r \cdot S(t) \cdot I(t)$$

$$a = r \cdot S(t)$$

$$a = 0.0000015 \cdot 199990 = 0.3, \text{ while current is } 0.18$$

Population constantly susceptible means that $S'=0$.

$$r \cdot S(t) \cdot I(t) - a \cdot I(t) = 0$$

$$S(t) = a/r = \rho = 0.18 / 0.0000015 = 120\,000$$

Figure 1: constant number of healthy people

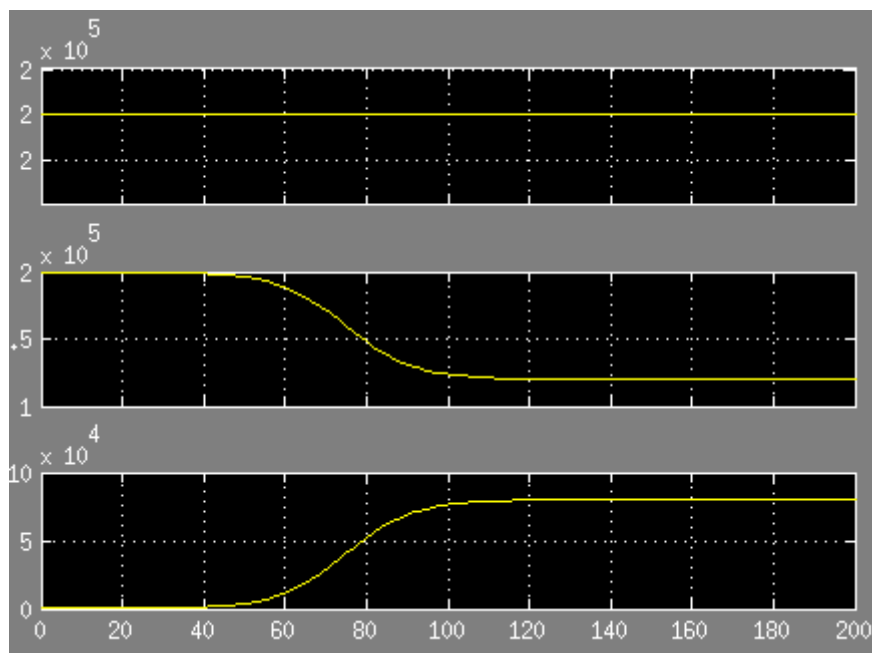


Figure 2: Recovery of population and stopping the epedemy.

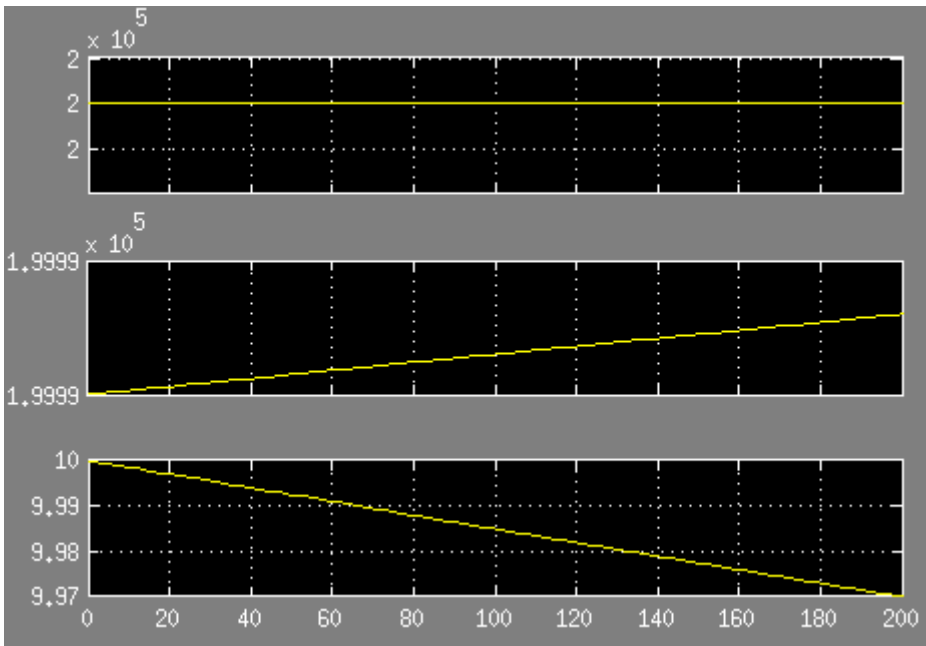
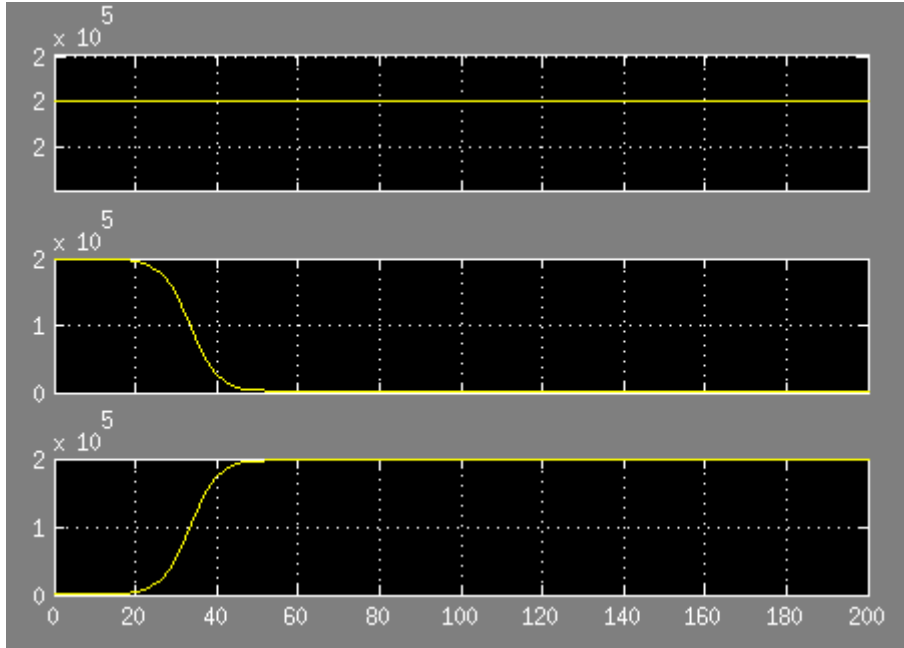


Figure3: Epidemic break out.



Model SIR & SIRV

Symbol	Importance	Value	Units	
S	population susceptible to infection	99	ind	Variables
I	infested population	1	ind	
R	died individuals	0	ind	
V	vaccinated individuals	0	ind	Parameters
a	virus death rate	0.5	time ⁻¹	
r	rate of spread infection	0.01018	time ⁻¹	
N	number of vaccination	5	time ⁻¹	

Differential equations for model without vaccination

$$S'(t) = -r \cdot S(t) \cdot I(t)$$

$$I'(t) = r \cdot S(t) \cdot I(t) - a \cdot I(t)$$

$$R'(t) = a \cdot I(t)$$

Differential equations for model with vaccination

$$S'(t) = -r \cdot S(t) \cdot I(t) - N$$

$$I'(t) = r \cdot S(t) \cdot I(t) - a \cdot I(t)$$

$$R'(t) = a \cdot I(t)$$

$$V'(t) = N$$

Figure 4: illustration of a herd prospect without vaccination

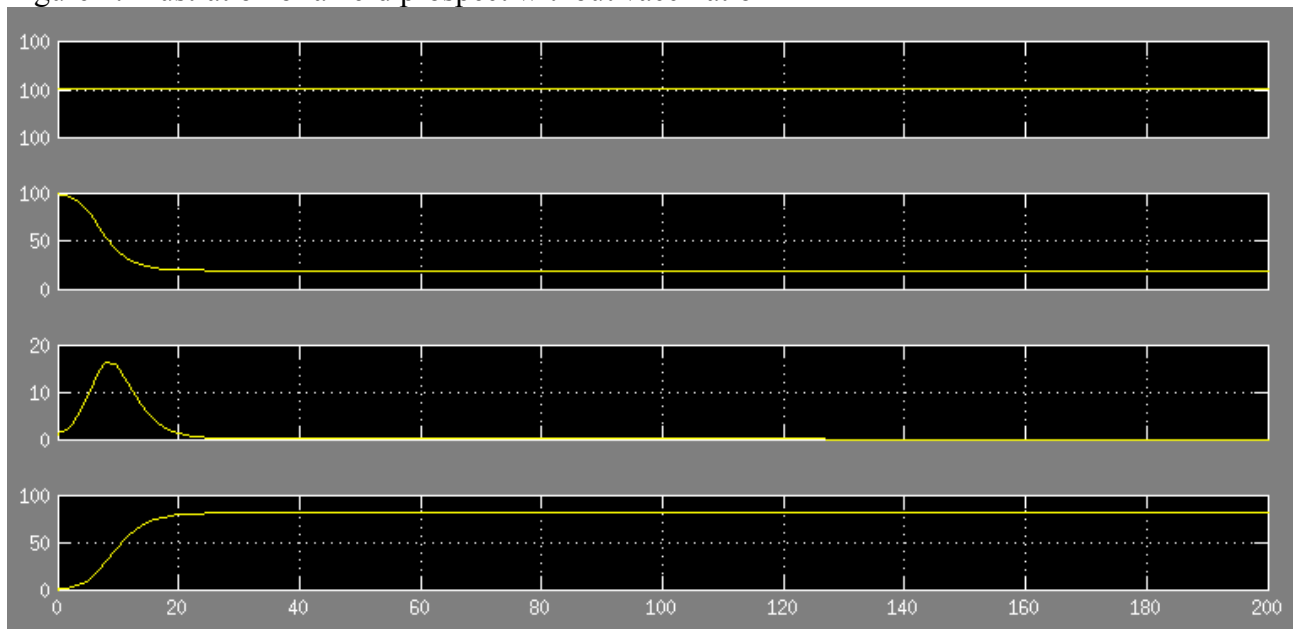
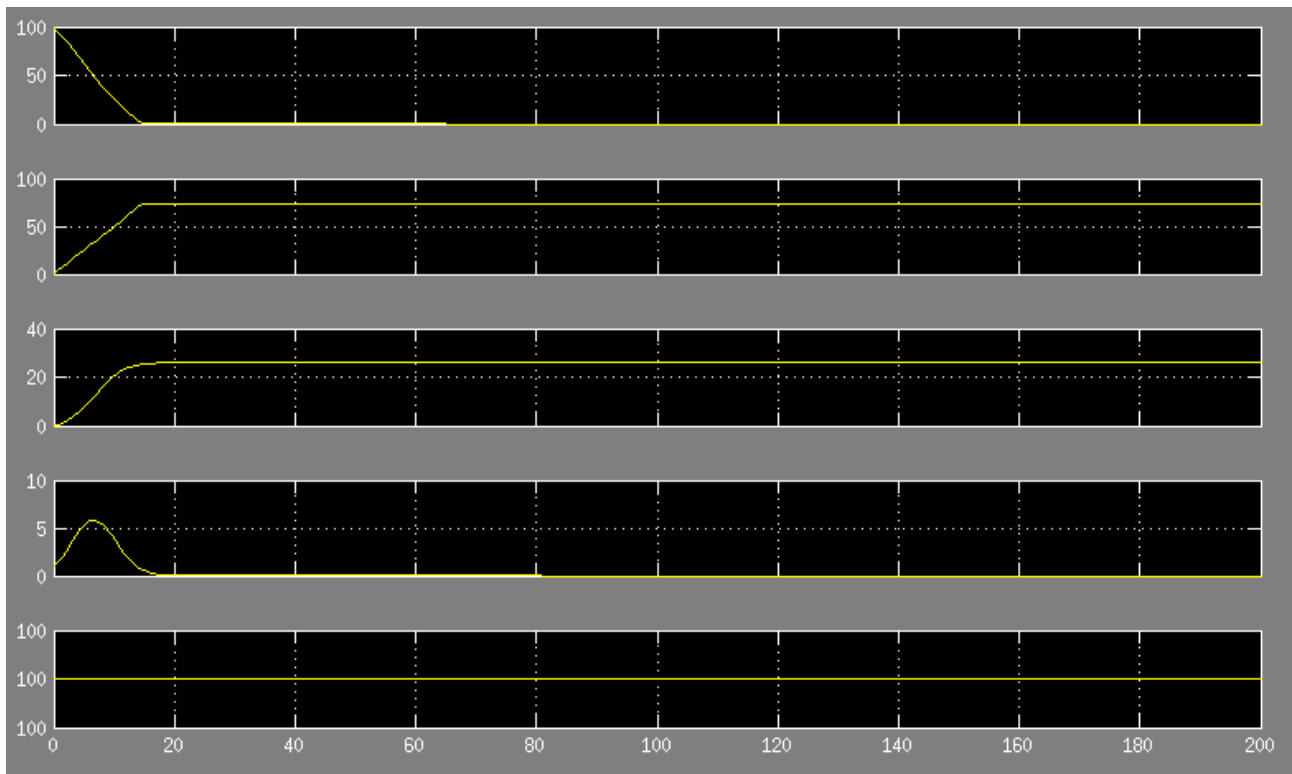


Figure 5: illustration of a herd prospect with vaccination



If farmer don't use vaccination only 19 cows will survive.

If he starts vaccination he will save 74 cows and spend $74 \cdot 840 = 62160$ CZK.