

Model Predator-Prey

Differential equations:

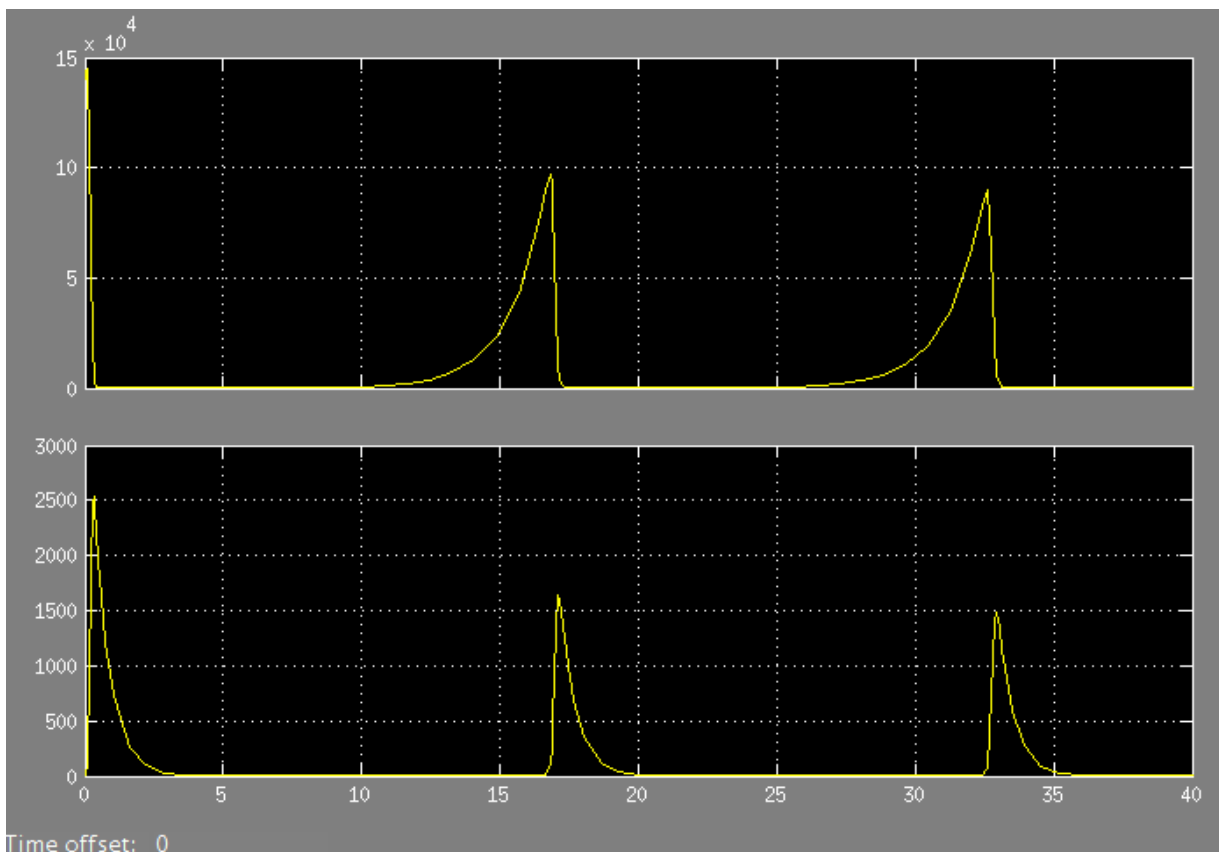
$dX/dt = K_1 \cdot X(t) - K_2 \cdot X(t) \cdot Y(t) = f_1$ - prey population growth function.

$dY/dt = K_2 \cdot K_3 \cdot X(t) \cdot Y(t) - K_4 \cdot Y(t) = f_2$ - predator population growth function.

Symbol	Importance	Value	Units	
X - pigeons population size	key variable	140000 at the beginning	individuals	Variables
Y - predators population size	key variable	10 at the beginning	individuals	
K_1 - pigeon birth rate	growth factor of pigeons	0.75	ind/p.a.	Parameters
K_2 - predator's efficacy	limiting factor for both predators' and preys' growth	0.01	-	
K_3 - predator's conv. coeff	description how much nestling can predators bear from the food	0.02	ind/p.a.	
K_4 - predator's mortality	death rate, second limiting factor of predators population size	1.8	ind/p.a.	

In absence of predators size of population of prey in this model won't be limited by any factor and would grow much faster.

This model tends to dynamic equilibrium, which could be seen in output. In case of grows of predators population of prey decrease and consequently cause lowering predators' population size. If all the pigeons die, predators will extinct.



$dX/dt = K_1 \cdot X(t) - K_2 \cdot X(t) \cdot Y(t) = f_1$ - prey population growth function.

$dY/dt = K_2 \cdot K_3 \cdot X(t) \cdot Y(t) - K_4 \cdot Y(t) = f_2$ - predator population growth function.

$$f_1 = 0.75 \cdot X - 0.01 \cdot X \cdot Y$$

$$f_2 = 0.0002 \cdot X \cdot Y - 1.8 \cdot Y$$

Equilibrium points: (0, 0), (0, 75), (9000, 0)

Jakobian matrix: $A = [df_1/dX, df_1/dY; df_2/dX, df_2/dY] =$

$$= [K_1 - K_2 \cdot Y(t), -K_2 \cdot X(t); K_2 \cdot K_3 \cdot Y(t), K_2 \cdot K_3 \cdot X(t) - K_4]$$

$$\det |A - \lambda \cdot I| = |K_1 - K_2 \cdot Y(t) - \lambda, -K_2 \cdot X(t); K_2 \cdot K_3 \cdot Y(t), K_2 \cdot K_3 \cdot X(t) - K_4 - \lambda| = 0$$

for point (0, 0)

$$|0.75 - \lambda, 0; 0, -1.8 - \lambda| = (0.75 - \lambda)(-1.8 - \lambda) = \lambda^2 + 10.5\lambda - 13.5 = 0 \quad (1.158, -11.658)$$

for point (0, 75)

$$|\lambda, 0; 0.015, -1.8 - \lambda| = \lambda \cdot (-1.8 - \lambda) - (0.015 \cdot 0) = -\lambda^2 - 1.8\lambda = 0 \quad (0, -1.8)$$

for point (9000, 0)

$$|0.75 - \lambda, -90; 0, -\lambda| = -\lambda \cdot (0.75 - \lambda) - (-90 \cdot 0) = \lambda^2 - 0.75\lambda = 0 \quad (0, 0.75)$$

