Modelling and simulation

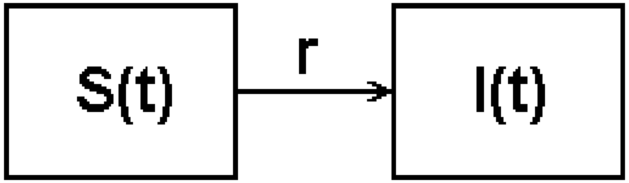
Lecture: Michel Kana

Practice: Daniela Müllerová

Epidemiological models

- Basic epidemiological models
  - SI
  - SIS
  - SIR
  - SIR with vaccination

model SI

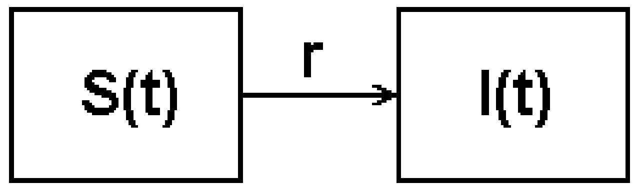


- Population is divided into healthy (S - suscetible) and sick - infectious (I - infective) individuals. Patients don't die or don't become health, remain constant potential source of infection.

- The disease is transmitted by contact between the healthy and the sick.

- The total population size is unchanged.

Cvičení 7 – model SI



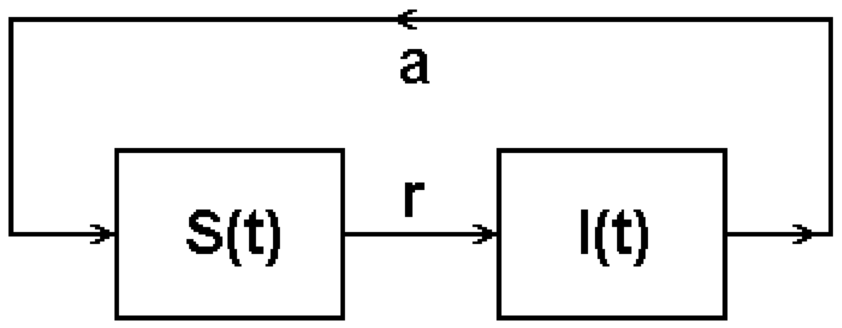
$$S'(t) = -r \cdot S(t) \cdot I(t)$$

$$I'(t) = r \cdot S(t) \cdot I(t)$$

$$S(t) + I(t) = S_0 + I_0 = N = \text{const.}$$

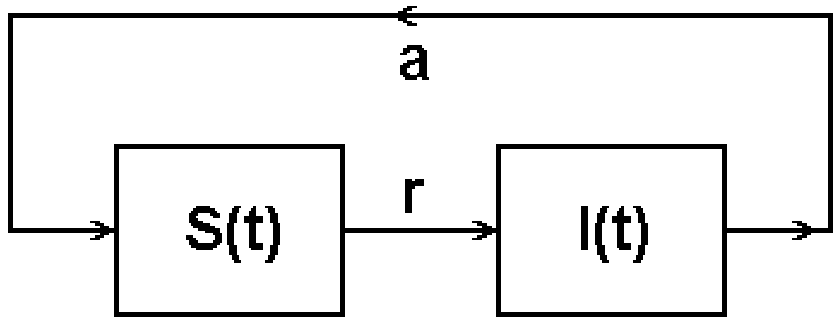
S = population susceptible to infection
I = infected population
r = rate of spread infection [time^{-1}]

model SIS



- Population is divided into healthy (S - suscetible) and sick - infectious (I - infective) individuals. Patients don't die, but in contrast to model SI they become health.
- The disease is transmitted by contact between the healthy and the sick.
- The total population size is unchanged.

model SIS



$$S'(t) = -r \cdot S(t) \cdot I(t) + a \cdot I(t)$$

$$I'(t) = r \cdot S(t) \cdot I(t) - a \cdot I(t)$$

$$S(t) + I(t) = S_0 + I_0 = N = \text{const.}$$

Character of the development of the epidemic

S = population susceptible to infection
I = infected population
r = rate of spread infection [time^{-1}]
a = rate of treatment [time^{-1}]

$$\rho = \frac{a}{r}$$

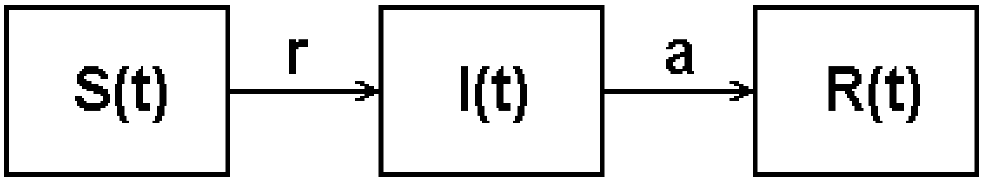
$S(0) > \rho$ epidemic is spreading

$S(0) < \rho$ epidemic isn‘t spreading

Practice 1 – assignment

- City with 200 000 people is facing an epidemic of influenza.
- The virus is spread by contact healthy person with a sick. The disease doesn't have an incubation period. The virus mutates constantly, so the healed individual does not acquire immunity and may be re-infected.
- The infection is spreading by speed of 1.5 * 10-6 per day.
- Rate of treatment is 0.18 per day.
  - Analytically determine whether an epidemic breaks out.
  - Analytically determine how many inhabitants remain capable of working (healthy but at risk).
  - At what value it is necessary to adjust the parameters of the speed of treatment, to avoid an increase the patient population?
  - Verify the calculations using simulations.

model SIR



- Population divided into healthy (S - suscetible), sick - infectious (I - infective) and resistant (R - removed) of an individual. Resistant is either healed with immunity or dead.
- The disease is transmitted by contact between the healthy and the sick.
- The total population size is unchanged.

model SIR

$$\boxed{S(t)} \xrightarrow{r} \boxed{I(t)} \xrightarrow{a} \boxed{R(t)}$$

$$S'(t) = -r \cdot S(t) \cdot I(t)$$

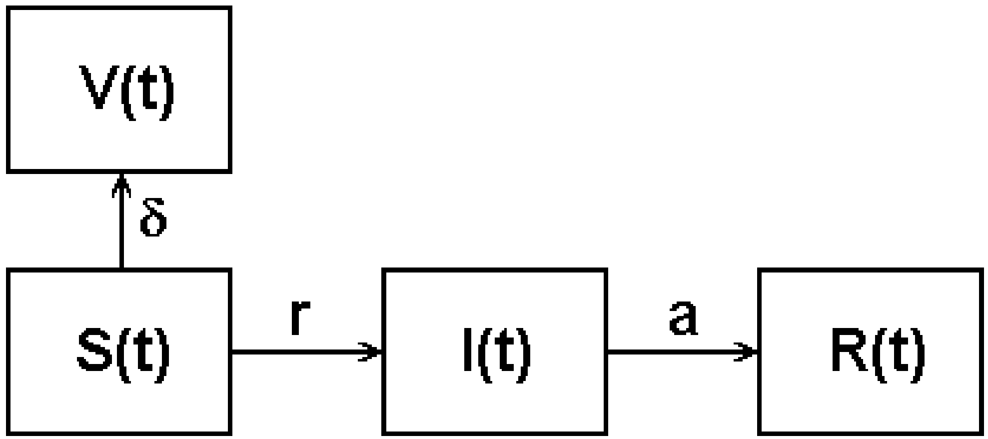
$$I'(t) = r \cdot S(t) \cdot I(t) - a \cdot I(t)$$

$$R'(t) = a \cdot I(t)$$

$$S(t) + I(t) + R(t) = S_0 + I_0 + R_0 = N = \text{const.}$$

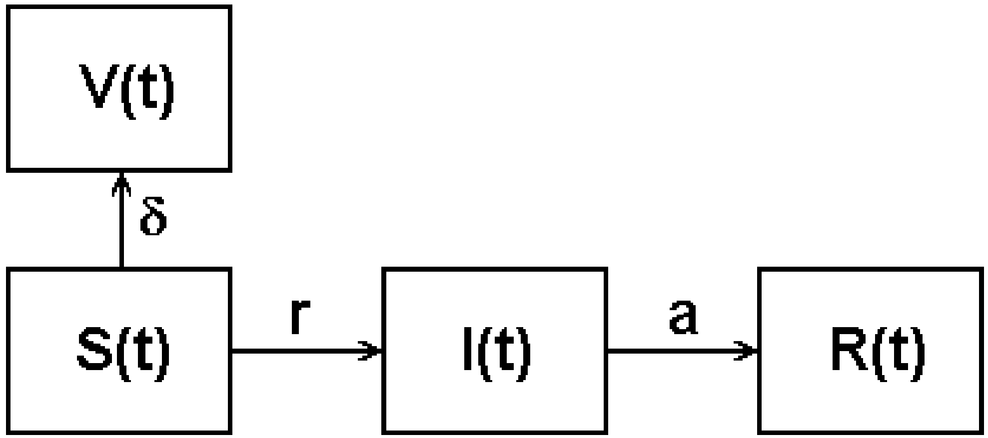
S = population susceptible to infection
I = infected population
r = rate of spread infection [time^{-1}]
a = rate of treatment [time^{-1}]

model SIR with vaccination



- Population divided into healthy (S), sick - infectious (I), resistant (R) and vaccinated (V - vaccinate) individuals. Resistant is either healed with immunity or dead.
- Vaccination may be a healthy (S) and the rate of vaccination is constant until there is someone to vaccinate (S> 0).
- The disease is transmitted by contact between the healthy and the sick.
- The total population size is unchanged.

model SIR with vaccination



$$S'(t) = -r \cdot S(t) \cdot I(t) - \delta$$

$$R'(t) = a \cdot I(t)$$

$$I'(t) = r \cdot S(t) \cdot I(t) - a \cdot I(t)$$

$$V'(t) = \delta$$

$$S(t) + I(t) + R(t) + V(t) = S_0 + I_0 + R_0 + V_0 = N = \text{const.}$$

Practice 2 - assignment

- Farmer is owner of herd beef cattle with 100 cows. A neighbor said that one of his cows on pasture showed signs of disease (blisters, drooling).
- Farmer inspected the herd immediately but did not find infected cows.
- What are the prospects farmer's herd, if we know that one of the cows infected with foot-and-mouth disease? We don‘t know which cow and we can not isolate it.
- The foot-and-mouth disease ends in two days with culling or death of an animal, it is 100% fatal.
- How would the situation change if it was possible and farmer began with vaccination immediately?

Practice 2 - assignment

- What are the prospects of the farmer's herd?
  - determine how many cows survives epidemic using simulation

- How would the situation change if it was possible and farmer began with vaccination immediately?
  - Determine how many cows survives epidemic using simulation, when it will begin instantly with vaccination rate of 5 cows a day. It will last until all healthy cows are vaccinated.
  - How much will vaccination cost in total, if the cost of vaccination is CZK 840 per cow?

- We also need:
  - Value rate of transmission of infection is 0.01018 when the time unit is day (r).
  - Each day of farming is deleted half sick cows ($a = 0.5\ day^{-1}$).

Practice 1 – desired output

- Model file *. mdl with correctly described blocks
- Short paper in *. pdf containing
  - Definition equation model
  - Table of all the parameters of the original model with columns: symbol, importance, value, unit
  - Table of all state variables of the model with columns: symbol, meaning the initial value, unit
  - Analytical determination
    - whether an epidemic breaks out or doesn‘t break out
    - how many inhabitants remain capable of working (healthy but at risk).
    - the speed of treatment while the size population of patients is a constant.
  - Graphical representation of the simulation results of three variant
    - epidemic is spreading; epidemic isn't spreading; patient population is constant

Practice 2 – desired output

- Model file *. mdl with correctly described blocks
- Short paper in *. pdf containing
  - Definition equation model
  - Table of all the parameters of the original model with columns: symbol, importance, value, unit
  - Table of all state variables of the model with columns: symbol, meaning the initial value, unit
  - Graphical representation of the simulation results of the two variant of the model (with and without vaccination) with marked desired value
  - Calculate the cost of vaccination and its result
  - A brief conclusion summarizing all the values
    - How many cows survive epidemic if we do not vaccinate?
    - How many cows will survive the epidemic when a farmer starts vaccination?
    - How much will vaccination cost in total?