Modelling and simulation

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Discrete Malthus model

- Discrete Malthus equations

$$x_{n+1} = R \cdot x_n$$

1. variant

model does not work for R = 0

$$x_{n+1} - x_n = R \cdot x_n$$

2. variant

x_{n+1} =(1 + R).x_n

in the next generation are all generations of xn plus offspring

$$x_{n+1} = x_n \cdot e^R$$

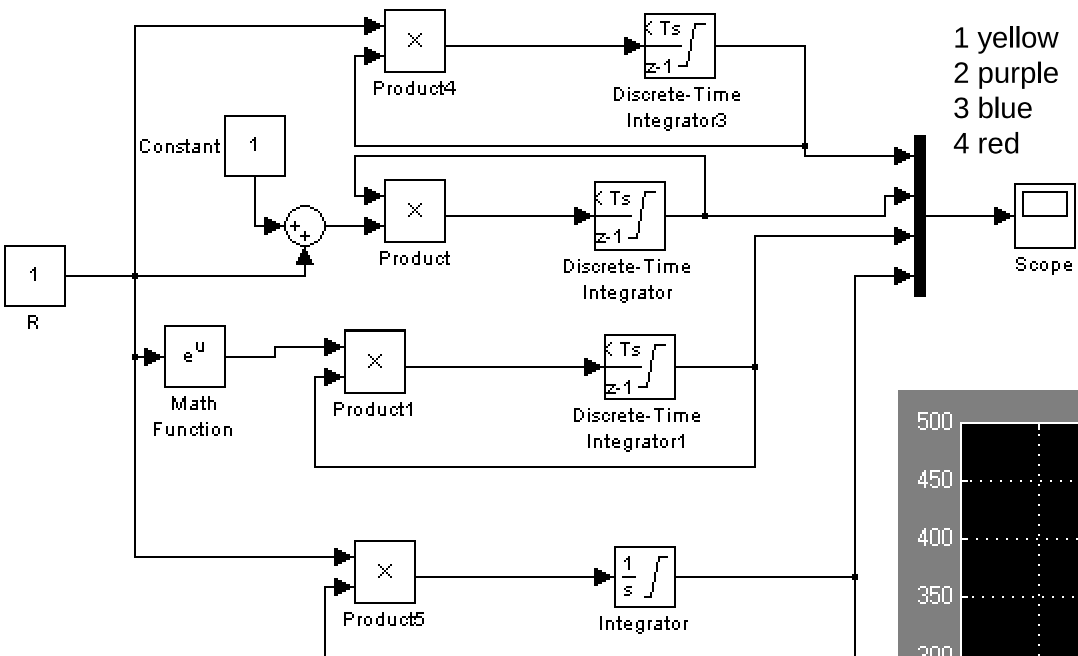
3. variant

operates correctly for R = 0, a small value of R , and negative values of R

Practice 1 – assignment

- In Simulink implement three variants of discrete forms of Malthus model
- Try different values of the parameter R in all variants, compare the results and answer questions
  - What is the different in variants?
  - Which is closer to the original continuous equation?

Practice 1– solution



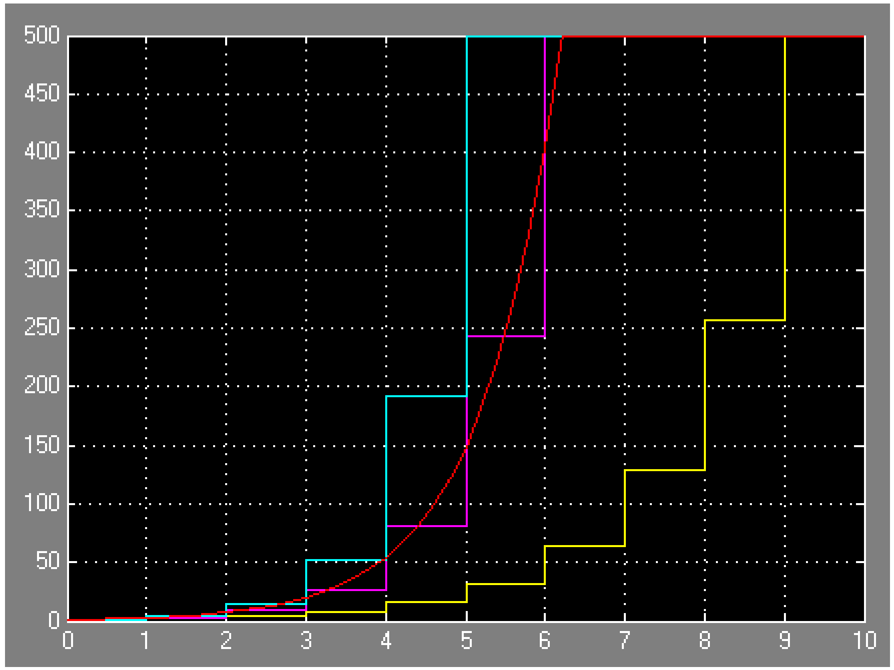
1 yellow
2 purple
3 blue
4 red

(1) $x_{n+1} = R.x_n$

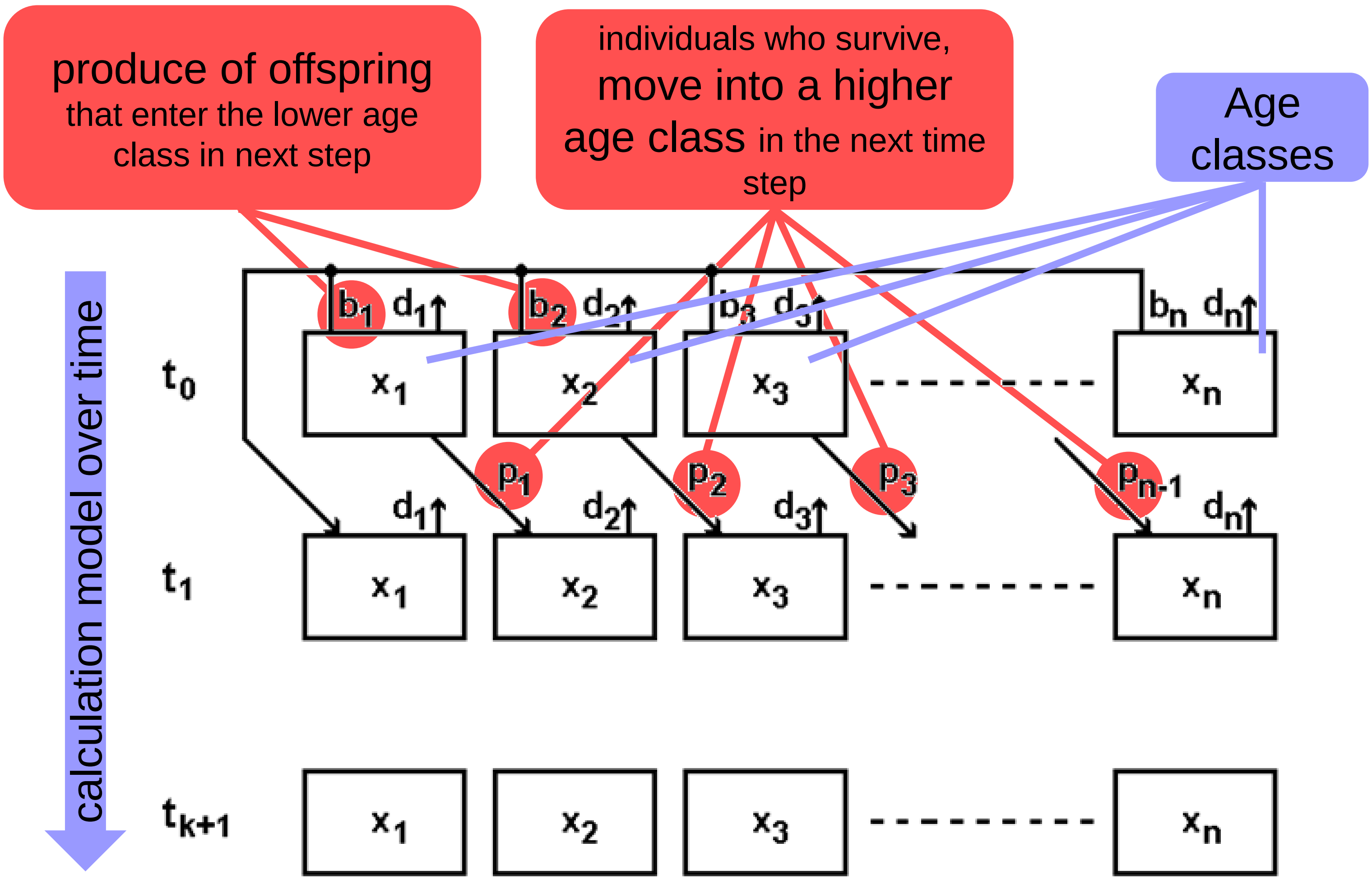
(2) $x_{n+1} = (1 + R).x_n$

(3) $x_{n+1} = x_n.exp(R)$

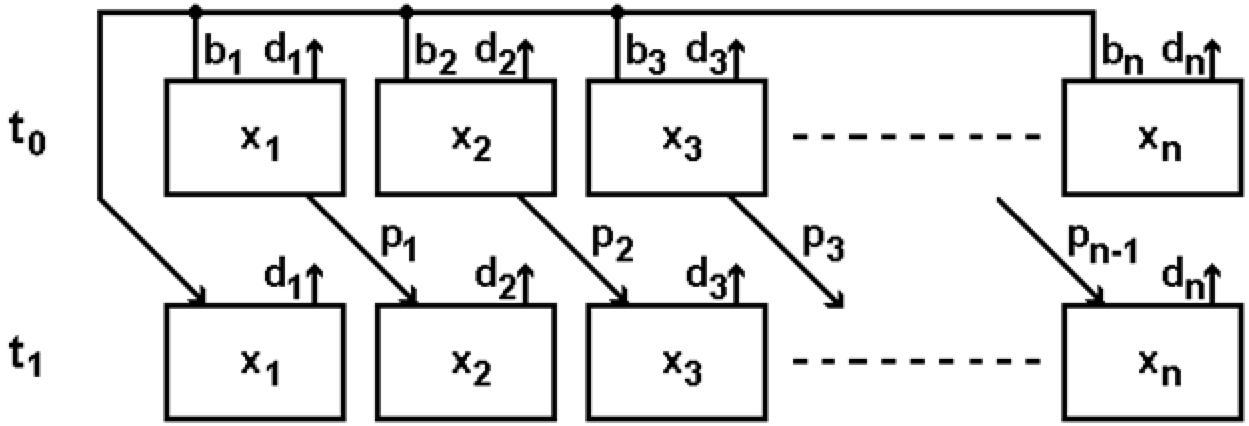
(4) $x'(t) = r.x(t)$



Leslie multi-class model



Leslie multi-class model



relative natality b_i in each **age class** x_i per unit of time (is the average number of offspring per individual in age class x_i for the entire duration of this category);

relative mortality d_i in each **age class** (determined by the average number of deaths in each age category x_i considering the number of individuals in this category for the duration - range - this category);

(probability) survival $p_i = 1 – d_i$; probability that an individual that enters into this category will survive and advance to the next category

Leslie multi-class model

- The number of individuals in the first age group x1 at time t +1 is the sum of the offspring all age classes in time t

$$x_1(t_{k+1}) = \sum_{i=1}^{n} b_i x_i(t_k)$$

- Number of individuals in the second age class is

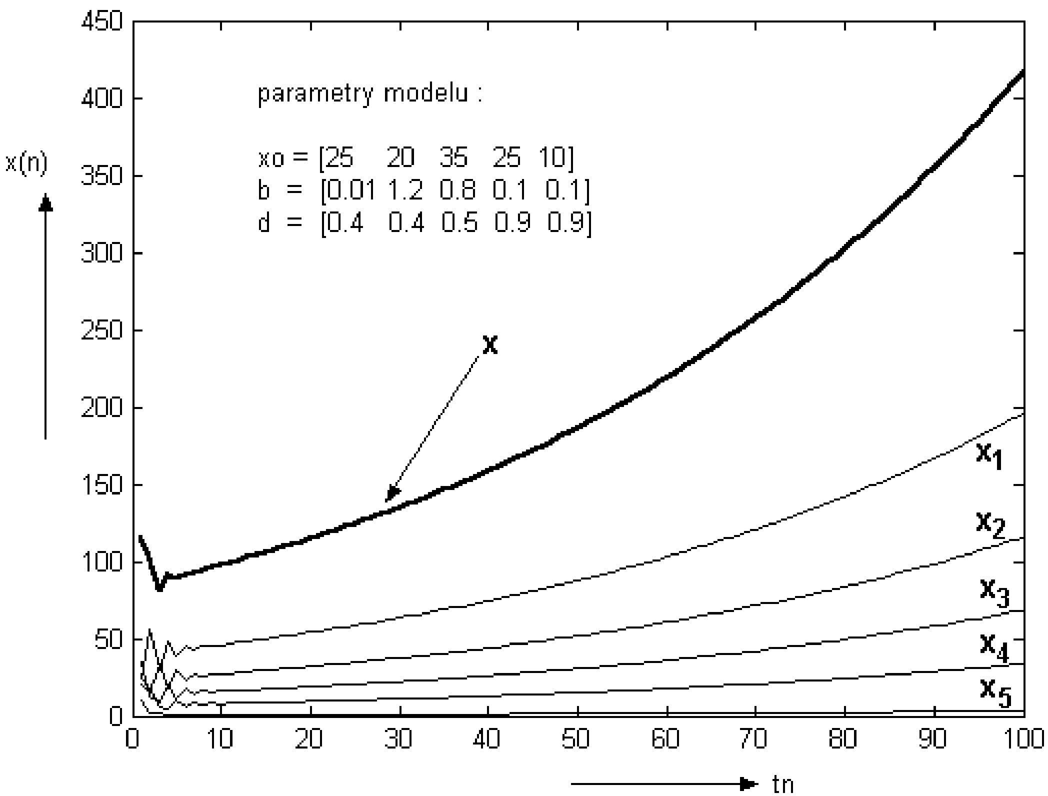
$$x_2(t_{k+1}) = p_1 \cdot x_1(t_k)$$

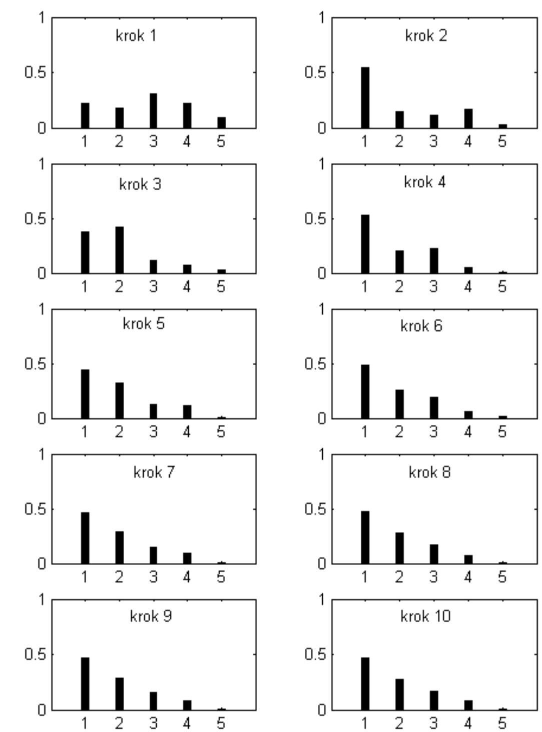
- in general i-th age class (2 ≤ i ≤ n)

$$x_i(t_{k+1}) = p_{i-1} \cdot x_{i-1}(t_k)$$

Leslie multi-class model

- Leslie model has two basic features:
  - after several damped oscillations at the beginning of the simulation follows the **exponential growth** of the total number of individuals in the population;
  - relative age distribution converges to a **stable state**.





Practice 2 – 1. part - assignment

- With Leslie multi-class population model, verify that is really currently needed reform of the pension system in the Czech Republic
  - Model implement in Simulink according to the presented data
  - As the model outputs follow the total population size and the ratio of pensioners to people of working age (ie how many pensioners accounts per one worker)
  - Do simulation from 1.1.2010 to at least 200 years ahead
  - From simulations determine in what period (time) will be the highest ratio of pensioners to workers, its maximum value and in what value approximately converges

Practice 2 – 2. part - assignment

- Without reform, we have two conditions .The current average gross monthly wage (CZK 23,951) will increase by 1.7 % per year and the current fair value of the average pension will be maintained (CZK 10,396) in the inflation rate of 1.4 % per year. What percentage of gross wage is deducted for pension at the beginning of the critical period? We expect to pay -as -you -go system.

$$y_n = y_0 \cdot \left(1 + \frac{p}{100}\right)^n$$

- How do all these calculations change if every second couple aged 15 to 29 had a child more? (Increase relative natality of 0.25)

Practice 2 – Necessary data

Age class	Number of children per 1,000 individuals of the class for the year*
0-14	0
15-29	26,67422
30-44	23,029775
45-59	0,0380419
60-74	0
75-90	0
90+	0

* - Source data of the Czech Statistical Office, Demographic Yearbook 2009
http://www.czso.cz/csu/2010edicniplan.nsf/p/4019-10

Practice 2 – Necessary data

Age class	Number of individuals to 31. 12. 2009 *
0-14	1494370
15-29	2075652
30-44	2474329
45-59	2142379
60-74	1626214
75-89	665283
90+	28586

* - Source data of the Czech Statistical Office, Demographic Yearbook 2009
http://www.czso.cz/csu/2010edicniplan.nsf/p/4019-10

Practice 2 – Necessary data

Age class	The average annual death rate per 1,000 individuals of the class*
0-14	0,822164
15-29	0,465078
30-44	1,16375
45-59	5,48891
60-74	21,34731
75-89	102,3956
90+	neuvedeno

* - Source data of the Czech Statistical Office, Demographic Yearbook 2009
http://www.czso.cz/csu/2010edicniplan.nsf/p/4019-10

Practice 2 – Necessary data

- For pensioners consider all individuals aged 60 years and older
- For worker consider all the individual classes 45-59, 30-45 and 15-29 half the individuals
- Critical period = period in which the ratio of pensioners to workers is the highest

Practice 2– desired output

- Model file *. mdl with correctly described blocks
- Short paper in *. pdf containing
  - Definition equation model
  - Table of all the parameters of the original model with columns: symbol, importance, value, unit
  - Table of all state variables of the model with columns: symbol, meaning the initial value, unit
  - Table of all model parameters with a modified birth rate with columns: symbol, importance, value, unit
  - Graphical representation of the results of both simulations and clearly marked ratios of pensioners and workers, times when there is a maximum limit values of these ratios

Practice 2– desired output

- Model file *. mdl with correctly described blocks
- Short paper in *. pdf containing (continuation)
  - Calculations of necessary critical pension insurance for both variants of the model (s birth and modified without it)
  - Comparison of the behavior of the model from a structural perspective and overall size of the population declared by essential features Leslie multi-class model