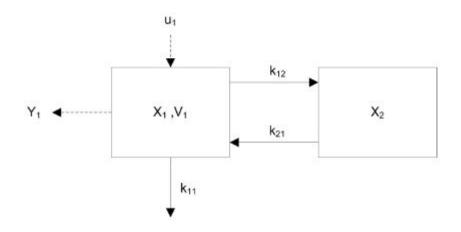
# **Practice 10**

# Nian.Liu

### • Block diagram of the model



#### • Differential equations of the model

$$\dot{X}_{1} = u_{1} + k_{21} \cdot X_{2} - X_{1} \cdot (k_{12} + k_{11})$$

$$\dot{X}_{2} = k_{12} \cdot X_{1} - K_{21} \cdot X_{2}$$

$$Y_{1} = \frac{X_{1}}{V_{1}}$$

### • Matrix A,B,C

$$A = \begin{bmatrix} -k_{12} - k_{11} & k_{21} \\ k_{12} & -k_{21} \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} \frac{1}{V_1} & 0 \\ 0 & 0 \end{bmatrix}$$

# • Sensitivity equation

$$\dot{\lambda} = A \cdot \lambda + H \cdot X$$
$$\eta = C \cdot \lambda + V \cdot X$$

#### • Sensitivity Matrix

**k11:** 
$$H_1 = \frac{\partial A}{\partial k_{11}} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$
  $V_1 = \frac{\partial C}{\partial k_{11}} = \begin{bmatrix} 0 & 0 \end{bmatrix}$ 

$$\dot{\lambda}_{1} = -(k_{11} + k_{12}) \cdot \lambda_{1} + k_{21} \cdot \lambda_{2} - x_{1}$$

$$\dot{\lambda}_{2} = k_{12} \cdot \lambda_{1} - k_{21} \cdot \lambda_{2}$$

$$\eta = \frac{\lambda_{1}}{V_{1}}$$

$$\begin{aligned} \mathbf{k12:} \, H_2 &= \frac{\partial A}{\partial k_{12}} = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \quad V_2 = \frac{\partial C}{\partial k_{12}} = \begin{bmatrix} 0 & 0 \end{bmatrix} \\ \lambda_1^{\bullet} &= -(k_{11} + k_{12}) \cdot \lambda_1 + k_{21} \cdot \lambda_2 - x_1 \\ \lambda_2^{\bullet} &= k_{12} \cdot \lambda_1 - k_{21} \cdot \lambda_2 + x_1 \\ \eta &= \frac{\lambda_1}{V} \end{aligned}$$

$$\mathbf{k21:} \ H_3 = \frac{\partial A}{\partial k_{21}} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \quad V_3 = \begin{bmatrix} 0 & 0 \end{bmatrix}$$
 
$$\dot{\lambda}_1 = -(k_{11} + k_{12}) \cdot \lambda_1 + k_{21} \cdot \lambda_2 + x_2$$
 
$$\dot{\lambda}_2 = k_{12} \cdot \lambda_1 - k_{21} \cdot \lambda_2 - x_2$$
 
$$\eta = \frac{\lambda_1}{V_1}$$

$$\mathbf{V1:} H_{4} = \frac{\partial A}{\partial V_{1}} = \begin{bmatrix} 0 \end{bmatrix} \quad V_{4} = \frac{\partial A}{\partial V_{1}} = \begin{bmatrix} -\frac{1}{V_{1}^{2}} & 0 \\ 0 & 0 \end{bmatrix}$$

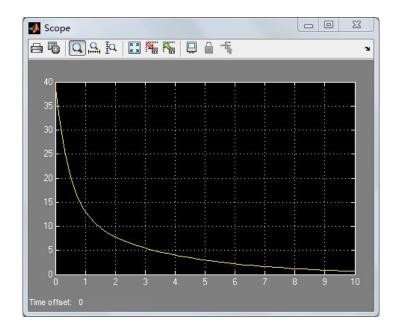
$$\dot{\lambda}_{1} = -(k_{11} + k_{12}) \cdot \lambda_{1} + k_{21} \cdot \lambda_{2}$$

$$\dot{\lambda}_{2} = k_{12} \cdot \lambda_{1} - k_{21} \cdot \lambda_{2}$$

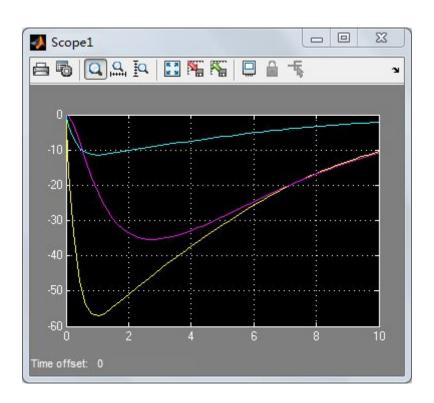
$$\eta = \frac{\lambda_{1}}{V_{1}} - \frac{x_{1}}{V_{1}^{2}}$$

### • Graphical output of the simulation

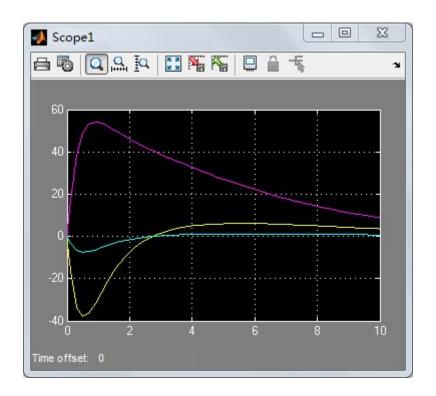
**Y1:** 



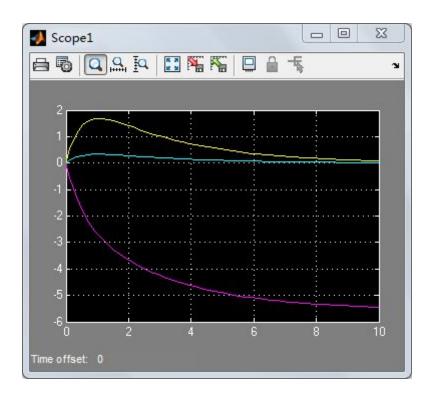
# K11:



# K12:



# **K21:**



# V1:

