MODELLING AND SIMULATION PRACTICES

Practice 3 - SS 2014 – Daniela Müllerová

What do we do in today's practice?

1. **Summary of the previous practice**
2. **Population model with age structure**
3. **Two species populations models of predator – prey**
4. **Summary**

Summary of the previous practice

[Population models]
Model changes in number of inhabitants due to the interaction of organisms with the environment, with individuals of their own kind, and with other types of organisms.
Malthus model: growth is unlimited
Logistic model: capacity of the environment of the study population
Logistic model with delay: mean time to reach reproductive
Discrete models: the number of individuals in the next generation is determined by functions only the number of individuals in the previous generation.

[Simulink]
Simulation of dynamic systems using block diagram

Population models with age structure

- The population is a group of individuals of a particular species.
- The population is divided according to age groups $n+1$
- $X_{i,t}$ represents the number of individuals in the age group i at time t.
- The age group 0 determines the number of offspring.
- Age group n determines the number of the oldest individuals.
- b_i represents fertility (the average fraction of births) for an individual in age class
- p_i an aging (average fraction of survival) in the age class i.

$$A \cdot X_t = X_{t+1}$$

⇩

$$\begin{bmatrix} b_0 & b_1 & b_2 & \cdots & b_{n-1} & b_n \\ p_0 & 0 & 0 & & 0 & 0 \\ 0 & p_1 & 0 & & 0 & 0 \\ 0 & 0 & p_2 & & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & \cdots & p_{n-1} & 0 \end{bmatrix} . \begin{bmatrix} x_{0,t} \\ x_{1,t} \\ x_{2,t} \\ x_{3,t} \\ \vdots \\ x_{n,t} \end{bmatrix} = \begin{bmatrix} x_{0,t+1} = \sum b_i . x_{i,t} \\ x_{1,t+1} = p_0 . x_{0,t} \\ x_{2,t+1} = p_1 . x_{1,t} \\ x_{3,t+1} = p_2 . x_{2,t} \\ \vdots \\ x_{n,t+1} = p_{n-1} . x_{n-1,t} \end{bmatrix}$$

⇩

$$A^t \cdot X_0 = X_t$$

- A is Leslie matrix
- Eigenvalue of A marked λ represents the asymptotic growth population in the a stable age distribution : $\mathrm{A} \cdot v = \lambda \cdot v$
- The corresponding eigenvector v represents a stable age distribution, the proportion of individuals of each age in the population. As soon as a stable age distribution was achieved, the population is undergoing exponential growth in the ratio λ.

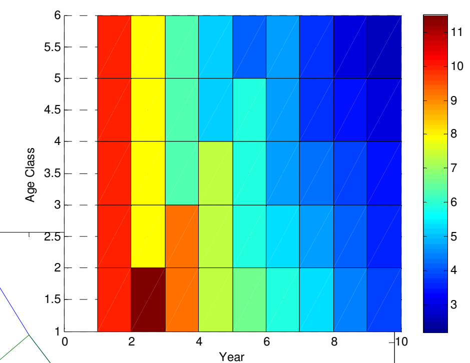
Population models with age structure

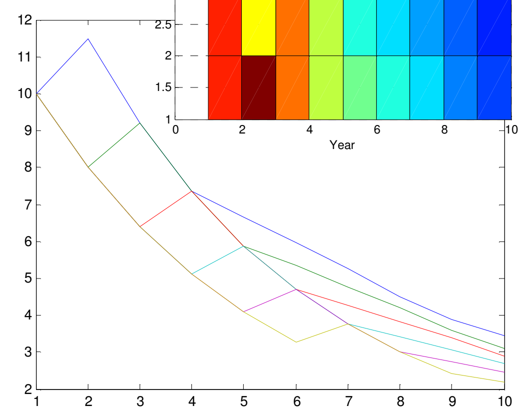
- 6 age groups.
- 10 individuals in each age group at time 0.
- The age group 0 and 1 are not fertile.
- In the age group 2-4 is 0.35 fertility of offspring per individual.
- In the age group 5 is a 0.1 fertility of offspring per individual.
- In each age group (except for group 5) 80% of individuals will survive.

$$A = \begin{bmatrix} 0 & 0 & 0.35 & 0.35 & 0.35 & 0.10 \\ 0.8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0 \end{bmatrix} X_0 = \begin{bmatrix} 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \end{bmatrix}$$

$$A^t \cdot X_0 = X_t$$

```
A = [ 0.00 0.00 0.35 0.35 0.35 0.10; 0.8 0 0 0 0 0; 0 0.8 0 0 0 0;
0 0 0.8 0 0 0; 0 0 0 0.8 0 0; 0 0 0 0 0.8 0]
X = [10;10;10;10;10;10]
A^10 * X
Population = X
for x=1:10, X= A * X; Population = [Population X], end
surf(Population)
view(0,90)
colormap(jet)
colorbar
xlabel('Year')
ylabel('Age Class')
plot(Population')
plot(sum(Population))
[v,lambda] = eig(A)
```





Population models with age structure

$$A = \begin{bmatrix} 0 & 4 & 20 & 60 \\ 0.05 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} 500 \\ 50 \\ 6 \\ 3 \end{bmatrix}$$

Population 1

$$\begin{bmatrix} 0 & 1 & 1.5 & 1.2 \\ .8 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \\ 0 & 0 & .25 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 45 \\ 18 \\ 11 \\ 4 \end{bmatrix}$$

Population 2

$$\begin{bmatrix} b_0 & b_1 & b_2 & \cdots & b_{n-1} & b_n \\ p_0 & 0 & 0 & & 0 & 0 \\ 0 & p_1 & 0 & & 0 & 0 \\ 0 & 0 & p_2 & & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & \cdots & p_{n-1} & 0 \end{bmatrix} . \begin{bmatrix} x_{0,t} \\ x_{1,t} \\ x_{2,t} \\ x_{3,t} \\ \vdots \\ x_{n,t} \end{bmatrix} = \begin{bmatrix} x_{0,t+1} = \sum b_i . x_{i,t} \\ x_{1,t+1} = p_0 . x_{0,t} \\ x_{2,t+1} = p_1 . x_{1,t} \\ x_{3,t+1} = p_2 . x_{2,t} \\ \vdots \\ x_{n,t+1} = p_{n-1} . x_{n-1,t} \end{bmatrix}$$

$$A^t \cdot X_0 = X_t$$

- Calculate the stable age distribution vector v and continuous relative exponential growth λ.

Two species populations models of predator - prey

- One population prospers at the expense of the other.
- $X(t)$ represents the number of prey in time t.
- $Y(t)$ represents the number of predators in time t.
- k_1 represents the relative fertility prey.
- $k_1 \cdot X(t)$ represents the number of prey that were born during the time interval $\langle t-1 \cdots t \rangle$.
- k_2 represents the probability that predator will kill prey when prey and predator are meeting.
- $k_2 \cdot X(t) \cdot Y(t)$ represents the number of prey caught by predators during the time interval $\langle t-1 \cdots t \rangle$.
- k_3 represents the conversion efficiency of the biomass of prey to predator biomass.
- $k_3 \cdot k_2 \cdot X(t) \cdot Y(t)$ represents the number of births of predators during the time interval $\langle t-1 \cdots t \rangle$.
- k_4 represents the relative mortality of predators.
- $k_4 \cdot Y(t)$ represents the decrease in the population of predators during the time interval $\langle t-1 \cdots t \rangle$.

$$\frac{dX(t)}{dt} = k_1 \cdot X(t) - k_2 \cdot X(t) \cdot Y(t)$$
$$\frac{dY(t)}{dt} = k_3 \cdot k_2 \cdot X(t) \cdot Y(t) - k_4 \cdot Y(t)$$

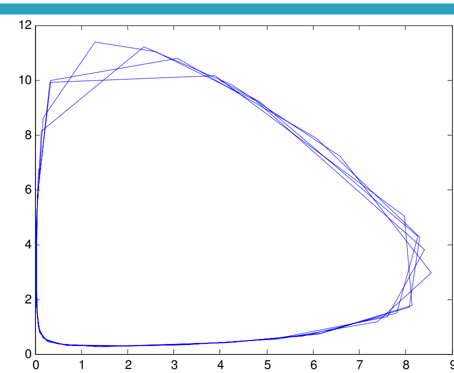
Equation of model of Lotka – Volterra

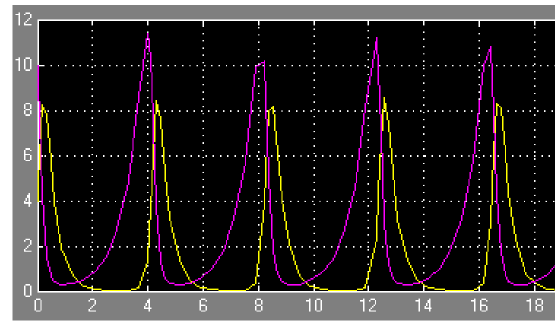
Two species populations models of predator - prey

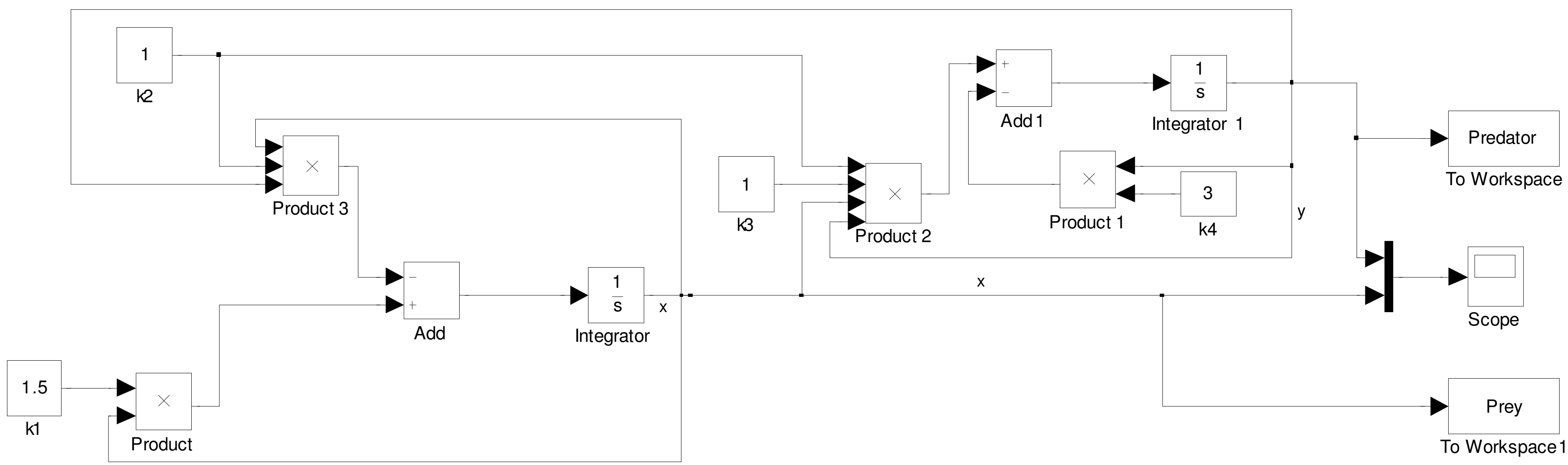
$$\frac{dX(t)}{dt} = k_1 \cdot X(t) - k_2 \cdot X(t) \cdot Y(t)$$
$$\frac{dY(t)}{dt} = k_3 \cdot k_2 \cdot X(t) \cdot Y(t) - k_4 \cdot Y(t)$$

In Matlab

plot(Predator.signals.values,Prey.signals.values)







Summary of today's practice

[Population models]
Population models with age structure
Two species populations models of predator - prey: *Lotka – Volterra*

[What is next?]
Two species populations models of predator - prey with delay, Kolmogorov model
Two species populations models with competition
Two species populations models with cooperation
Epidemiology models.