Modelling and simulation

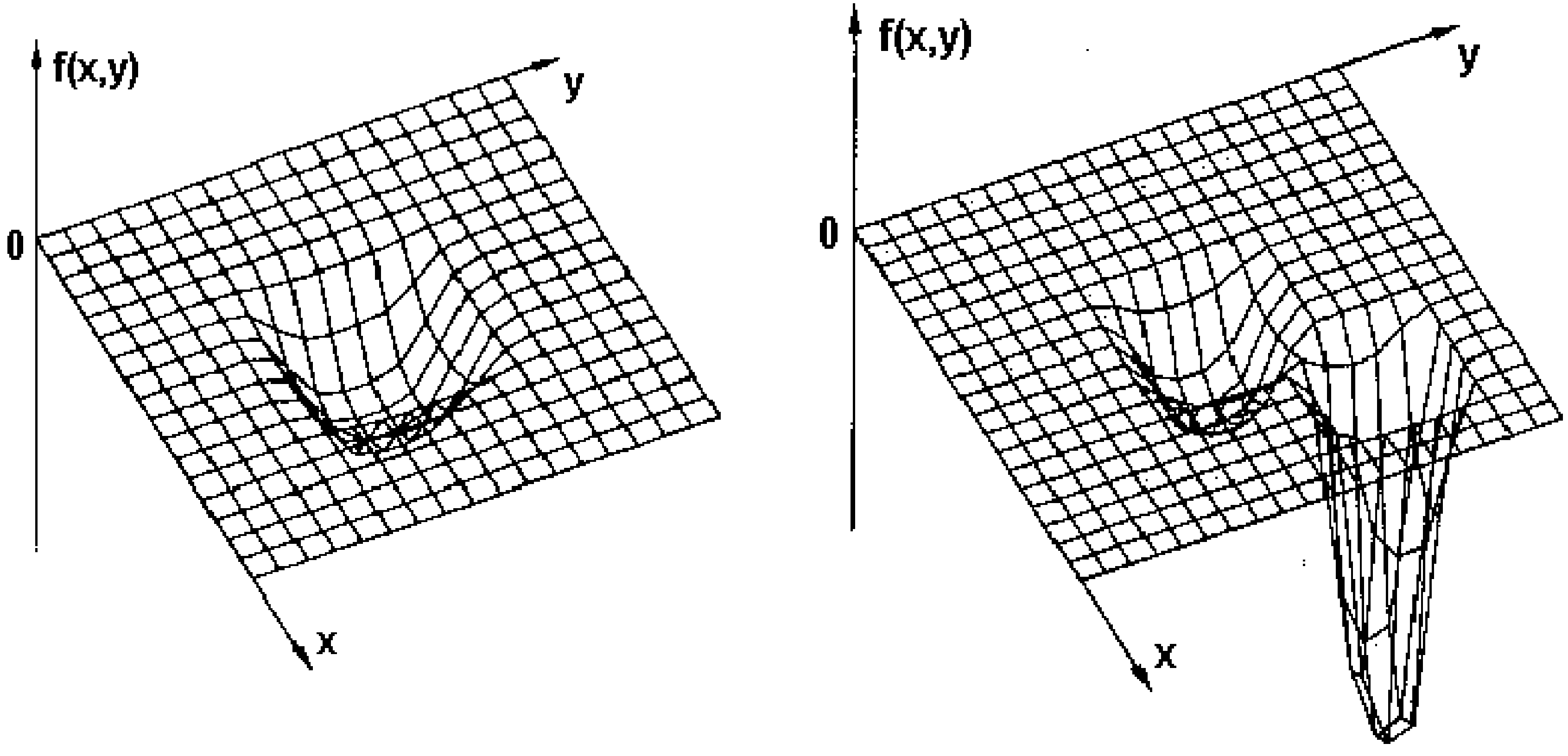
Lecture: Michel Kana

Practice: Daniela Müllerová

Identification of model parameters

- The measured data from physical experimentation represent the observed behavior of system $C(t)$.
- A computer model provides predictive value of the output vector, the simulated behavior of system $Y_1(x, t)$.
- The aim of the parameter identification is to find the unique value of the parameter for which the difference between simulated and observed behavior is a minimum.
- This difference is called the objective function $f(x, t) = Y_1(x, t) - C(t)$.

Objective function



Practice 1 – assignment

- Determine analytically (using the least squares method) constant speed and starting point of car. The car started at noon that was in Všenory at 12.41, in Všetaty at 13.23 and in Všehrdy at 13.43.
  - Distance from Všenory to Všetaty is 46 km
  - Distance from Všetaty to Všehrd 29 km
  - How far from Všenory car was, when car started a journey?

Practice 1 – solution

- Všenory: time t_1 = 41 min, distance s_1 = 0 km
- Všetaty: time t_2 = 83 min, distance s_2 = 46 km
- Všehrdy: time t_3 = 103 min, distance s_3 = 75 km
- Initial point: time t_0 = 0 min, distance s_0 = ? Km

- Speed: v = ? km/min

Practice 1 – solution

$$s(t) = s_0 + v \cdot t$$

- Objective function

$$S(s_0, v) = \sum_j \left(s_{sim\,j} - s_{data\,j}\right)^2$$

$$S(s_0, v) = \left(s_0 + v \cdot t_1 - s_1\right)^2 + \left(s_0 + v \cdot t_2 - s_2\right)^2 + \left(s_0 + v \cdot t_3 - s_3\right)^2$$

$$S(s_0, v) = \left(s_0 + 41v\right)^2 + \left(s_0 + 83v - 46\right)^2 + \left(s_0 + 103v - 75\right)^2$$

Practice 1 – solution

$$\frac{\partial S(s_0,v)}{\partial s_0} = 2\left(s_0 + 41v\right) + 2\left(s_0 + 83v - 46\right) + 2\left(s_0 + 103v - 75\right)$$

$$\frac{\partial S(s_0,v)}{\partial s_0} = 2s_0 + 82v + 2s_0 + 166v - 92 + 2s_0 + 206v - 150 = \underline{6s_0 + 454v - 242}$$

$$\frac{\partial S(s_0,v)}{\partial v} = 2\left(s_0 + 41v\right)\cdot 41 + 2\left(s_0 + 83v - 46\right)\cdot 83 + 2\left(s_0 + 103v - 75\right)\cdot 103$$

$$\frac{\partial S(s_0,v)}{\partial v} = 82s_0 + 3362v + 166s_0 + 13778v - 7636 + 206s_0 + 21218v - 15450$$

$$\frac{\partial S(s_0,v)}{\partial v} = \underline{454s_0 + 38358v - 23086}$$

Practice 1 – solution

- look for minimum – extreme of objective function
  - put the derivative equal to zero

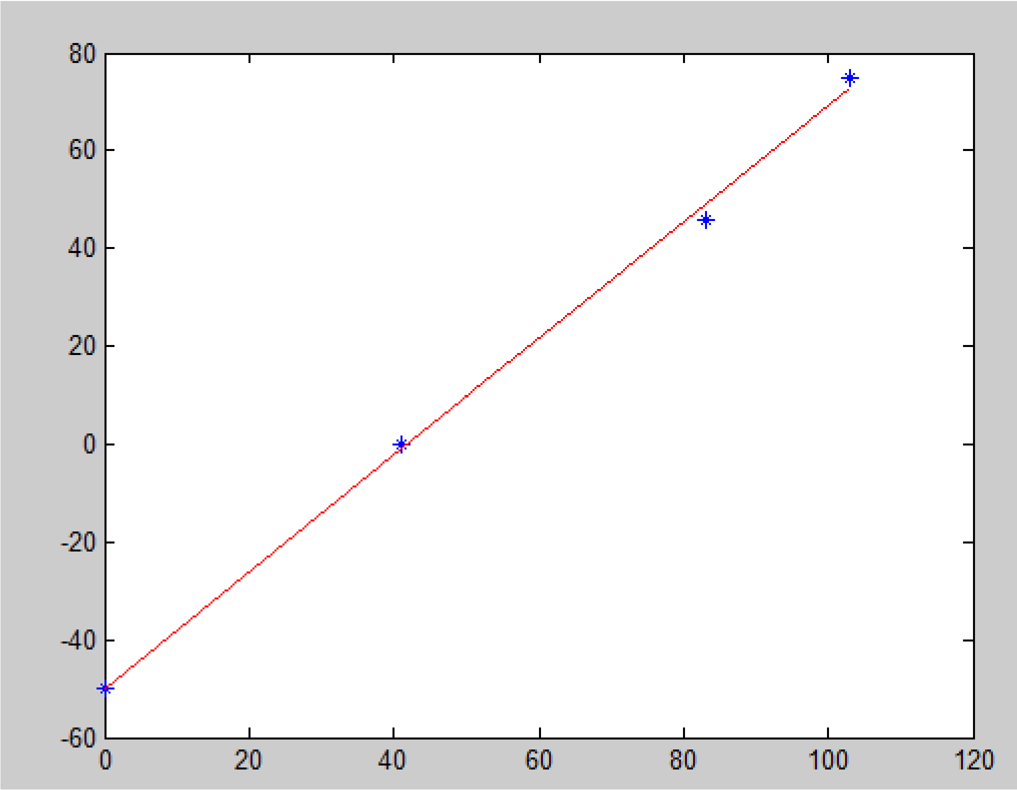
$$\frac{\partial S(s_0, v)}{\partial s_0} = 6s_0 + 454v - 242 = 0 \qquad \frac{\partial S(s_0, v)}{\partial v} = 454s_0 + 38358v - 23086 = 0$$

  - solve the obtained system of linear equations with two unknowns s_0 and v

$$\underline{\underline{v = 1{,}192\,\mathrm{km/min}}} \qquad \underline{\underline{s_0 = -49{,}87\,\mathrm{km}}}$$

Practice 1 – solution

$$\underline{\underline{v = 1{,}192\,\text{km/min}}} \qquad \underline{\underline{s_0 = -49{,}87\,\text{km}}}$$



Practice 2 – assignment

- During the experiment has been poured 0.75 liters of boiling water into a thermos. thermos was sealed and placed in a bath at a constant temperature of 0 ° C. Density of boiling water was 961 $kg.m^{-3}$. Specific heat capacity of water was 4211 $J.kg^{-1}.K^{-1}$.

- Every hour was measured water temperature in a thermos.

- From the presented tables of measured data, determine the parameters of heat exchange of thermos and the initial temperature of the water in it (it may not be 100 ° C due to the non-zero altitude and cool during refilling)

- Estimate at what time the temperature of the water in a thermos was 10 ° C based on the results parameter

Practice 2 – assignment

- Identifikaci parametrů (T_0 a k) proveďte v Matlabu prostřednictvím funkce lsqcurvefit ve vlastním skriptu.
- Draw in one graph simulation results with optimized parameters and the measured data and make comparisons.
- Also draw a graph of the objective function in the neighbourhood of the found minima (function mesh, surf) in Matlab (in the same script).
- Implement the mathematical model thermos in Simulink.
- Analytically make an estimate of time to reach 10 °C.

Practice 2 – assignment

- DE of model: $$\frac{dT(t)}{dt} = \frac{-k}{m \cdot c} T(t)$$
- Parameters:
  - *m* – mass of water in a thermos
  - *c* – specific heat capacity of water
  - *k* – heat exchange of thermos
- Analytical solutin: $$dT(t) = T_0 \cdot e^{\frac{-k}{m \cdot c}t}$$
- Measured data (also in data.mat):

t / hour	1	2	3	4	5	6	7	8	9	10
T / °C	73,94	55,43	44,93	30,39	26,29	17,75	14,23	12,53	10,83	4,47

Practice 2 – assignment

- From the data measured during the repeated experiment specify analytically (using least squares method) initial water temperature (heat exchange parameter copy from the first part)

- Data from repeated experiment:

t / hour	2	4	6
T / °C	33,06	20,91	11,91

Practice 2 – desired output

- Model file *. mdl with correctly described blocks
- M-file with desired script
- Short paper in *. pdf containing
  - Definition equation model
  - Table of all the parameters of the model with columns: symbol, importance, value (in the case of optimizing the value of each resource separately), unit
  - Table of all state variables of the model with columns: symbol, meaning the initial value (in the case of optimizing the value of each resource separately), unit
  - graph of objective function
  - analytical calculation of the estimate of the time at 10 ° C
  - graph comparing the simulation results (with the identified parameters) and measured data, accompanied by a brief commentary

Practice 2 – desired output

- analytical calculation of the initial temperature of the method of least squares with clearly defined outcome and verbal response