Modelling and simulation

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Malthus model

- The population is expressed by its size (number of individuals, X)
- Difference decrease and increase in the population is stable in time
- Effect of environment isn´t changed at the time
- The birth rate is marked by symbol $r \approx \rho$

$$\boxed{\dot{x}(t) = r\,x(t)}$$

Analytical solution

$$x(t) = x_0 \mathrm{e}^{rt}$$

Practice 1– assignment

- Model city's population
  - at time t = 8 years has 39 individuals
  - at time t = 12 years has 60 individuals

  - implement in Simulink model with exponential population growth
  - analytically determine the coefficient (measure) population growth
  - analytically determine the population size at time t = 20 years
  - verify the calculation with simulation

Practice 1– solution

- Determination of the growth coefficient *r*

based on: $$x(t) = x_0 e^{rt}$$

substituting: $$39 = x_0 e^{r8}$$

$$60 = x_0 e^{r12}$$

mathematically adjust : $$\frac{39}{e^{r8}} = \frac{60}{e^{r12}}$$

next: $$r = \frac{1}{4} \ln \frac{60}{39} \approx 0.1076$$

Příklad 1 – solution

- Determination of the population size at time t = 20 years
  - calculating the population at time t = 0

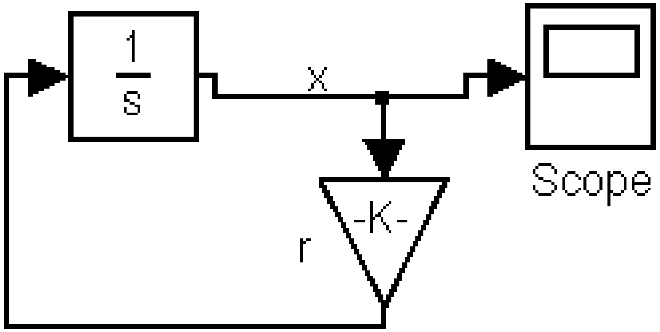
    based on: $x(t) = x_0 e^{rt}$

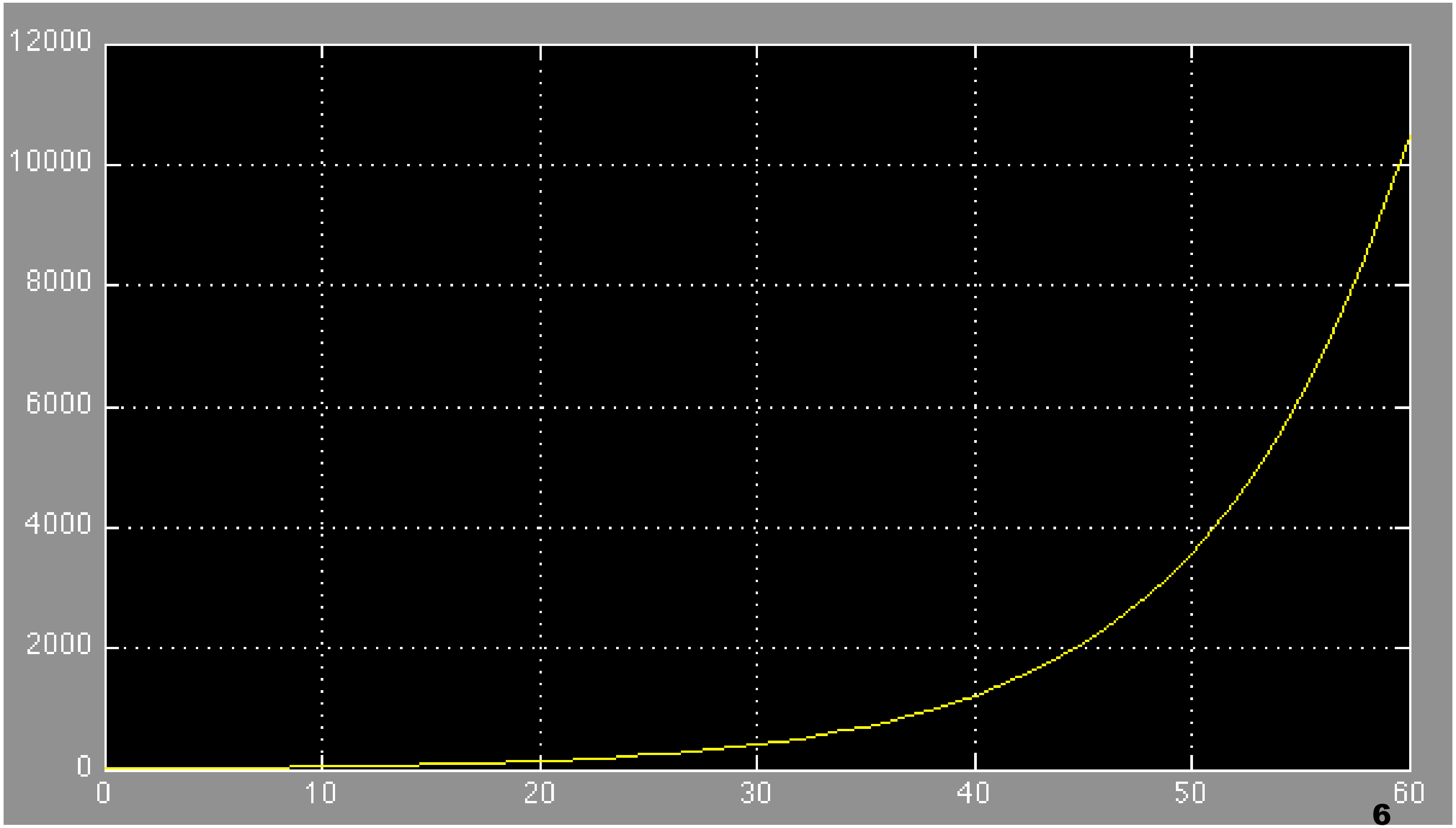
    substituting: $39 = x_0 e^{0,1076 \cdot 8}$

    $x_0 = 16.4775$
  - calculating the population at time t = 20

    substituting: $x(20) = 16,4775\, e^{0,1076 \cdot 20} \approx 141$

Practice 1– solution





Logistic model

- Real growth can´t be unlimited
- The logistic model or the Verhulst model is a slight modification of Malthus model with the second parameter.
- parameter K is the capacity of the environment of the study population.

$$x'(t) = \rho\left(1 - \frac{x(t)}{K}\right).x(t)$$

Analytical solution

$$x(t) = \frac{K x_0 e^{\rho t}}{K + x_0 (e^{\rho t} - 1)}$$

Logistic model- modification

- LM with variable parameters

$$x'(t) = \rho(t).\left(1 - \frac{x(t)}{K(t)}\right).x(t)$$

- LM with harvesting

$$x'(t) = \rho.\left(1 - \frac{x(t)}{K}\right).x(t) - cx(t)$$

- LM with delay

- Hutchinson's equation
- Analytical solution does not exist

$$x'(t) = \rho.x(t).\left[1 - \frac{x(t-\tau)}{K}\right]$$

Practice 2– 1. part - assignment

- Logistic model of forest biomass
  - capacity of the environment $K = 54 * 10^4$ individuals
  - initial biomass is equal to ¼ of the total capacity
  - growth rate $\rho = 0{,}71$ za rok

  - analytically specify how large the population after the first year
  - analytically specify for how long the population reaches half capacity of the environment
  - create a logistic model of forest biomass in Simulink
  - verify the calculations by simulation

Practice 2– 2. part - assignment

- Logistic model of forest biomass
  - implement to the model time delay τ, τ is 2 months

    (block variable time delay)
  - create a subsystem, which is a variable parameter ρ,

    $\rho = \mathrm{arctg}(1/t) + 1$ (block Trigonometric function and Clock)
  - implement to the model capture c, c is 10% the size of the population

Practice 2– desired output

- Model file *. mdl with correctly described blocks
- Short paper in *. pdf containing
  - The differential equation model
  - analytically calculation of population size after the first year
  - analytically calculation of time when the population reaches half capacity of the environment
  - Table of all model parameters with columns: symbol, importance, value, unit
  - Table of all state variables of the model with columns: symbol, meaning the initial value, unit

Practice 2 – desired output

- Short paper in *. pdf containing
  - Simulation output according to set parameters
  - Simulation output according to set parameters with time delay
  - Simulation output according to set parameters with variable parameter ρ
  - Simulation output according to set parameters with capture c