

Practice 6

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● Definition equation model

Model SIR

$$\frac{dS(t)}{dt} = -r \cdot S(t) \cdot I(t)$$

$$\frac{dI(t)}{dt} = r \cdot S(t) \cdot I(t) - a \cdot I(t)$$

$$\frac{dR(t)}{dt} = a \cdot I(t)$$

$$S(t) + I(t) + R(t) = S_0 + I_0 + R_0 = \text{const.}$$

Model SIR with Vaccination

$$\frac{dS(t)}{dt} = -r \cdot S(t) \cdot I(t) - \delta$$

$$\frac{dI(t)}{dt} = r \cdot S(t) \cdot I(t) - a \cdot I(t)$$

$$\frac{dR(t)}{dt} = a \cdot I(t)$$

$$\frac{dV(t)}{dt} = \delta$$

$$S(t) + I(t) + R(t) + V(t) = S_0 + I_0 + R_0 + V_0 = N = \text{const.}$$

● Table of all state variables of the model with columns

$S(t)$	$I(t)$	$R(t)$	$V(t)$
Healthy population	Sick-Infectious population	Resistant population	Vaccinated population
$S(0) = 99$	$I(0) = 1$	$R(0) = 0$	$V(0) = 0$
individuals	individuals	individuals	individuals

● Table of all model parameters:

r	a	δ	N
Rate of spread infection	Rate of treatment	Vaccinate rate	The whole number of population
0.01018	0.5	5	100
\.	\.	\.	individuals

● Practice 6_1

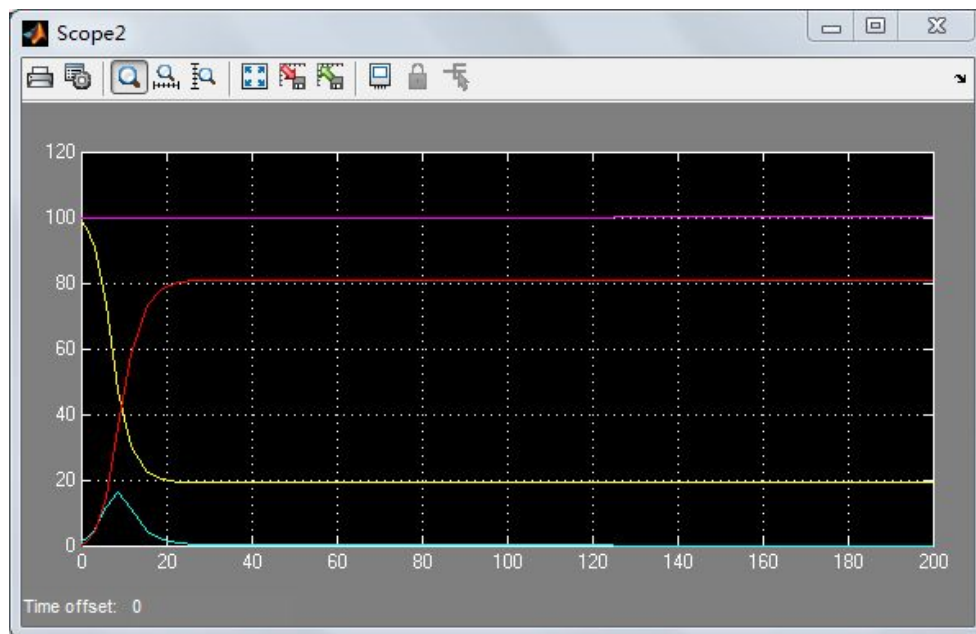
Analytical Determination

1) whether an epidemic breaks out or doesn't break out

$$\rho = \frac{a}{r} = \frac{0.5}{0.01018} = 49.12 < S(0)$$

Epidemic is spreading.

2) Inhabitants remain capable of working



According to the graph, the number of population remain healthy is 19.

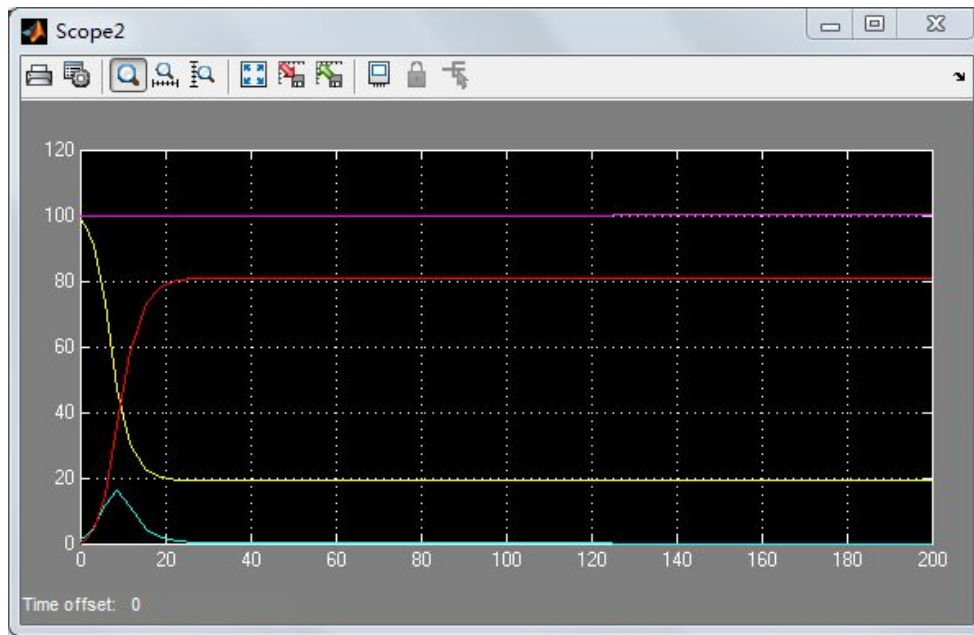
3) the speed of treatment while the size population of patients is constant.

$$r \cdot S \cdot I - a \cdot I = 0$$

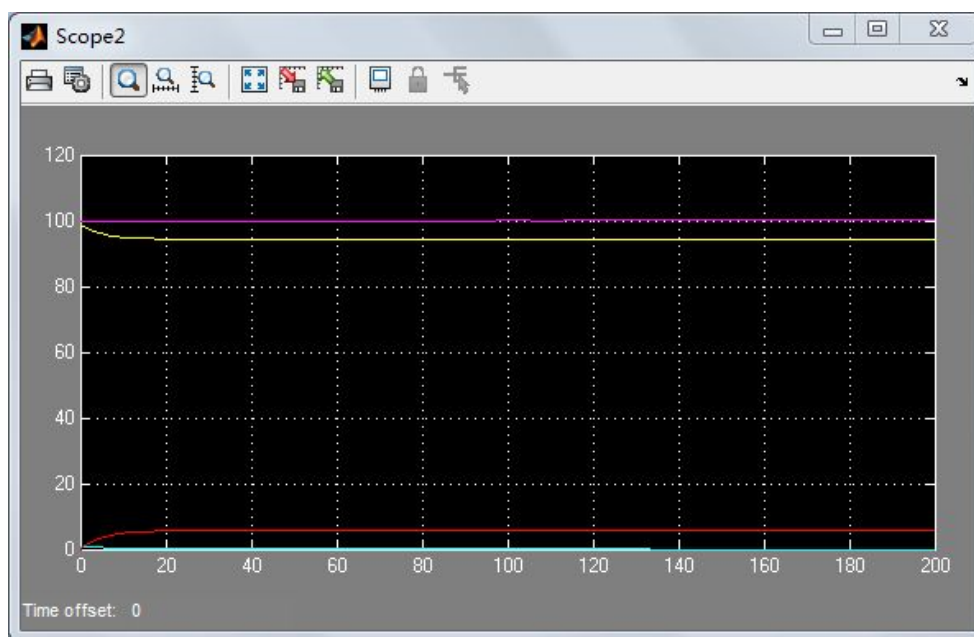
$$a = 1.00782$$

Graphical Representation

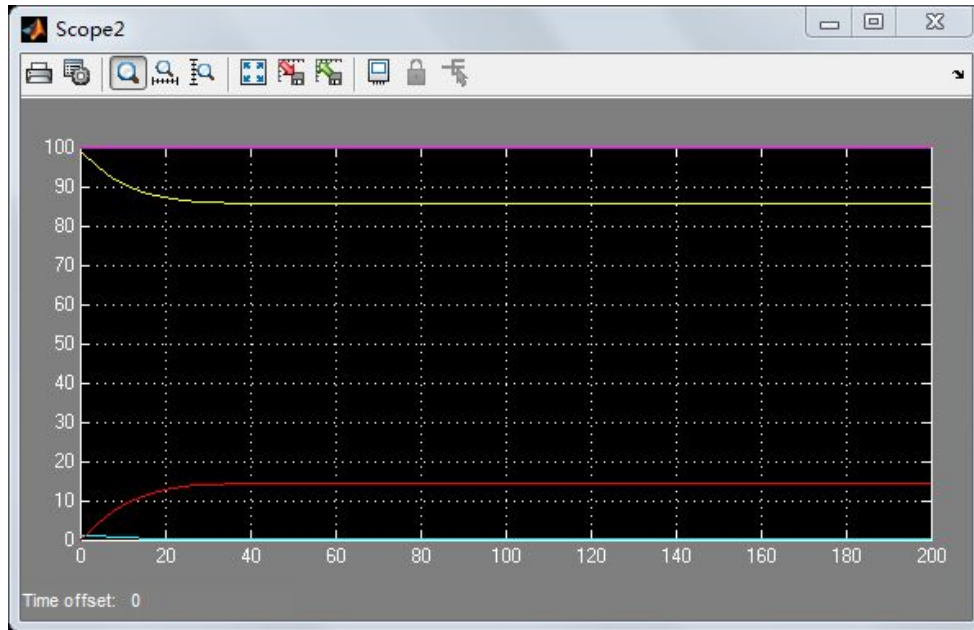
Epidemic is spreading.



Epidemic is not spreading.



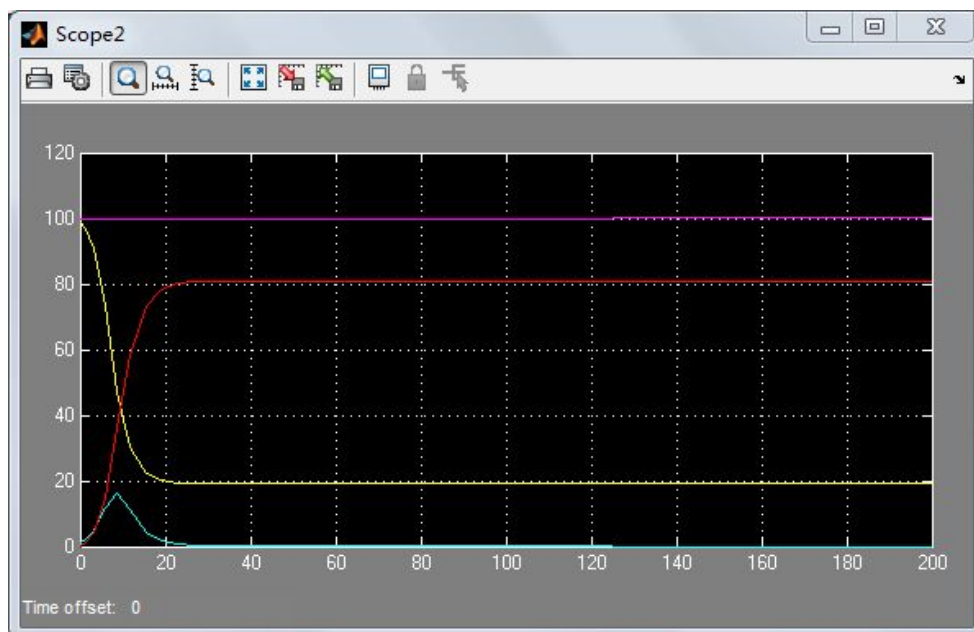
Patient population is constant.



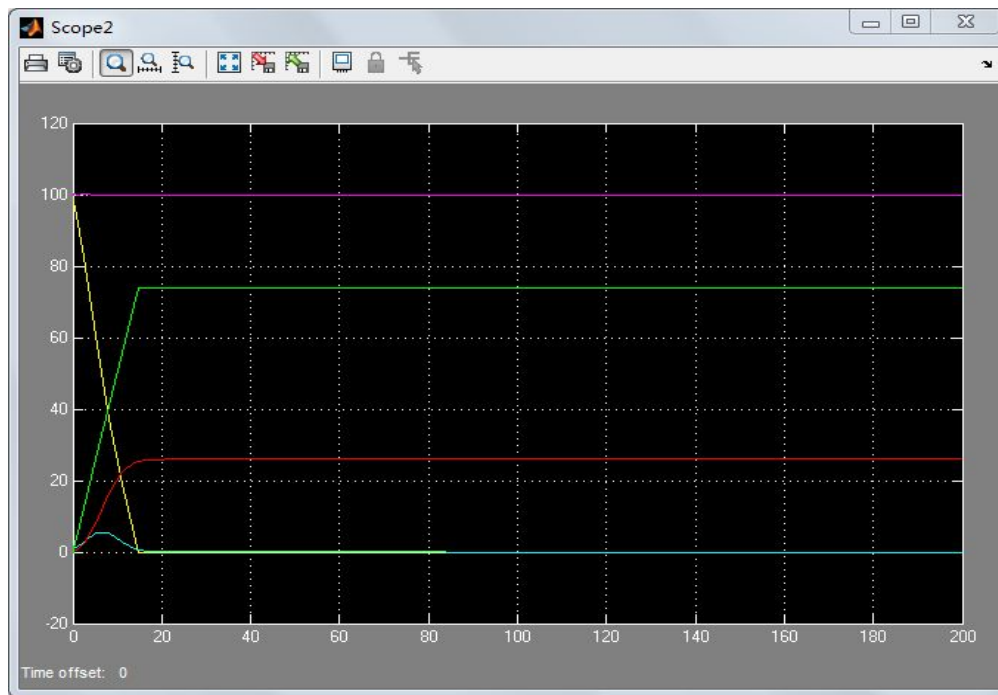
- Practice 6_2

Graphical representation of the simulation results

Without Vaccination



With Vaccination



A Brief Conclusion Summarize the Value

1)How many cows survive epidemic if we do not vaccinate?

19

2)How many cows will survive the epidemic when a farmer starts vaccination?

5

3)How much will vaccination cost in total?

The cost: $74 \times 840 = 62\,160$ CZK