Modelation and simulation

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Demonstration examples

- Model freefall body in a vacuum
  - From physics we know:

$$a = g = konst. \qquad v = a \cdot t \qquad s = v \cdot t$$

  - Prefer:

$$\underbrace{a = g = \frac{d\,v(t)}{dt} \qquad v(t) = -\frac{d\,h(t)}{dt}}_{\text{differential equations - the description of the model}}$$

Demonstration examples – freefall

- DE Solutions: $a = g = \dfrac{d\,v(t)}{dt} \qquad v(t) = -\dfrac{d\,h(t)}{dt}$
  - Analytical :

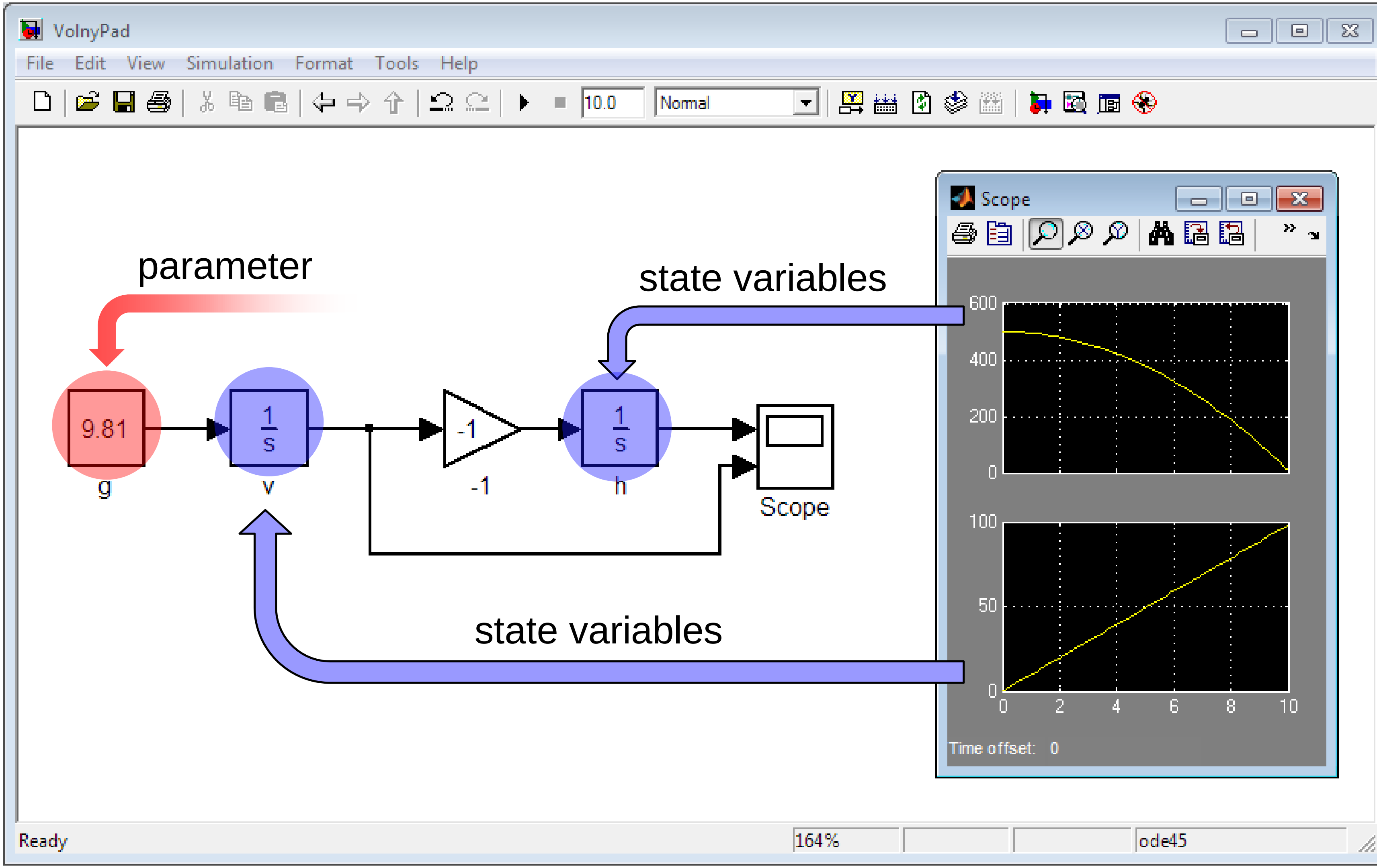
$$v(t) = v_0 + \int_0^t g\,dt = \underline{\underline{v_0 + gt}}$$

$$h(t) = h_0 - \int_0^t v(t)\,dt = h_0 - \int_0^t \left(v_0 + gt\right)dt =$$

$$h_0 - \int_0^t v_0\,dt - \int_0^t gt\,dt = \underline{\underline{h_0 - v_0 t - \frac{gt^2}{2}}}$$

  - Numerical example using Simulink

Demonstration examples – freefall



Practice 1 - task

- The mathematical model freefall skydiver
  - In Simulink implement mathematical model freefall skydiver
  - Analytically determine the relationship of the limit rate of fall, the rate at which a skydiver due to air resistance already not accelerate
  - The parameter (s) to specify the drag so that the speed limit was 180 km / h (with units!)
  - Using simulation to verify its calculation
  - From the simulation results, estimate the time and covered distance than a skydiver reaches 95% of the speed limit

Practice 1 – freefall skydiver

- Gravitational force and the aerodynamic drag force act on skydriver
  - The gravitational force F_g is considered constant

$$\left|F_g\right| = mg$$

  - Size of the resistive force F_o is a function of velocity

$$\left|F_o\right| = \frac{1}{2} CS\rho v^2$$

  - The forces are oppositely oriented, the resultant F is oriented to the ground

$$\left|F\right| = \left|F_g\right| - \left|F_o\right| = mg - \frac{1}{2} CS\rho v^2$$

Practice 1 – freefall skydiver

- 2laws of motion says: $F = am \quad \Rightarrow \quad a = \dfrac{F}{m}$

- therefore

k

$$a(t) = \frac{mg - \frac{1}{2}CS\rho\, v(t)^2}{m} = g - \frac{\frac{1}{2}CS\rho}{m} v(t)^2 = \frac{d\, v(t)}{dt}$$

- therefore

1. DE of model

$$\frac{d\, v(t)}{dt} = g - k\, v(t)^2$$

Practice 1 – freefall skydiver

- Still applies 2. DE:

$$\frac{d\,h(t)}{dt} = -v(t)$$

2. DE of model

- When you reach the speed limit are active forces in equilibrium - magnitude of the velocity does not change. Its derivative is thus zero.
  - applies $$\frac{d\,v(t)}{dt} = 0\ \mathrm{m.s}^{-2}$$

Practice 1– desired output

- Model file *. mdl with correctly described blocks
- Short paper in *. pdf containing
  - The differential equation model
  - General calculation of the parameter of drag, substituting into equation and numerical result with units
  - Table of all model parameters with columns: symbol, importance, value, unit
  - Table of all state variables of the model with columns: symbol, meaning the initial value, unit
  - Graphical representation of the simulation results with marked estimates of time and distance while achieving a 95% limit rate