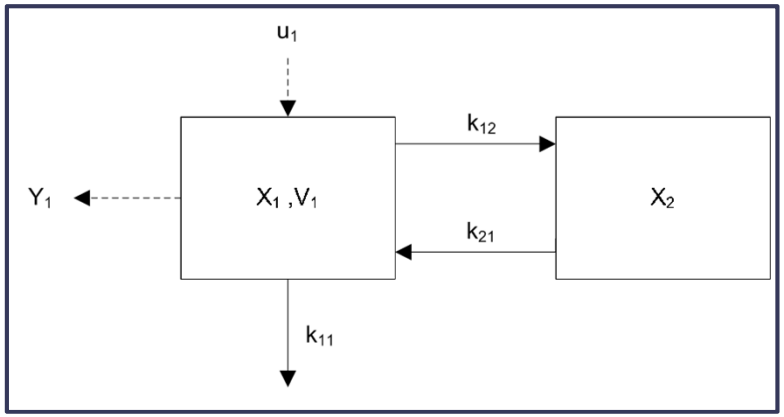
CVIČENÍ MODELOVÁNÍ A SIMULACE

Cvičení 6 - LS 2014 – Michel Kana

Co uděláme ve dnešním cvičení?

1. **Maticový popis kompartmentové modely**
2. **Přenosová funkce kompartmentové modely**
3. **Shrnuti**

Maticový popis 2-Kompartmentové modely

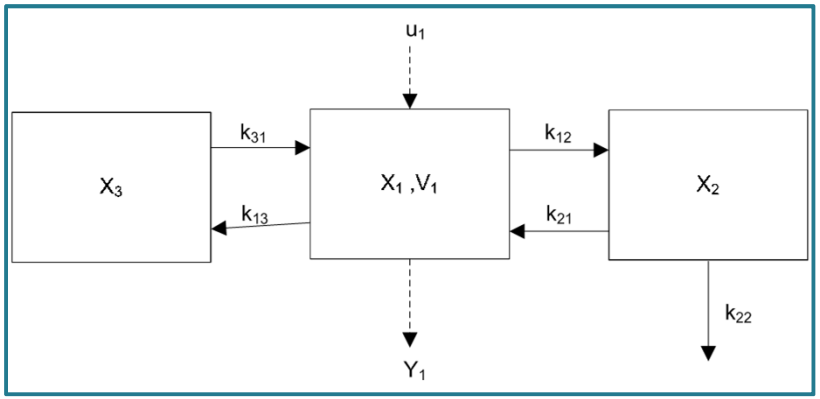


$$\dot{X}_1 = (-k_{11} - k_{12}) \cdot X_1 + k_{21} \cdot X_2 + u_1$$
$$\dot{X}_2 = k_{12} \cdot X_1 + (-k_{21}) \cdot X_2 + 0$$
$$Y_1 = \frac{1}{V_1} \cdot X_1 + 0 \cdot X_2$$
$$Y_2 = 0 \cdot X_1 + 0 \cdot X_2$$

$$\dot{X} = A.X + B.U$$
$$Y = C.X$$

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$
$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$
$$U = \begin{bmatrix} u_1 \\ 0 \end{bmatrix}$$
$$A = \begin{bmatrix} (-k_{11} - k_{12}) & k_{21} \\ k_{12} & -k_{21} \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} \frac{1}{V_1} & 0 \\ 0 & 0 \end{bmatrix}$$

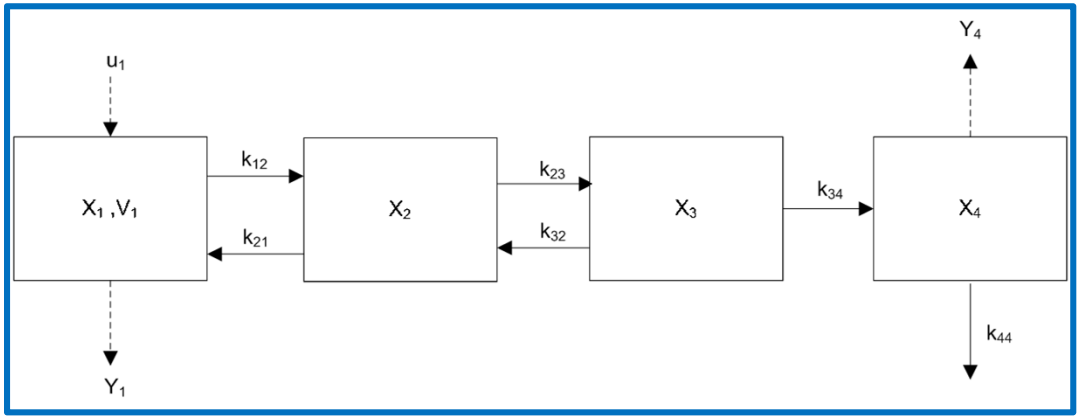
Maticový popis 3-Kompartmentové modely



$$\dot{X}_1 = (-k_{12} - k_{13}) \cdot X_1 + k_{21} \cdot X_2 + k_{31} \cdot X_3 + u_1$$
$$\dot{X}_2 = k_{12} \cdot X_1 + (-k_{21} - k_{22}) \cdot X_2 + 0.X_3 + 0$$
$$\dot{X}_3 = k_{13} \cdot X_1 + 0.X_2 + (-k_{31}) \cdot X_3 + 0$$
$$Y_1 = \frac{1}{V_1} \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3$$
$$Y_2 = 0 \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3$$
$$Y_3 = 0 \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$
$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}$$
$$U = \begin{bmatrix} u_1 \\ 0 \\ 0 \end{bmatrix}$$
$$A = \begin{bmatrix} -k_{12} - k_{13} & k_{21} & k_{31} \\ k_{12} & -k_{21} - k_{22} & 0 \\ k_{13} & 0 & -k_{31} \end{bmatrix}$$
$$\mathrm{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} \frac{1}{V_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Maticový popis 4-Kompartmentové modely



$$
\begin{aligned}
\dot{X}_1 &= (-k_{12}) \cdot X_1 + k_{21} \cdot X_2 + 0 \cdot X_3 + 0 \cdot X_4 + u_1 \\
\dot{X}_2 &= k_{12} \cdot X_1 + (-k_{21} - k_{23}) \cdot X_2 + k_{32} \cdot X_3 + 0 \cdot X_4 + 0 \\
\dot{X}_3 &= 0 \cdot X_1 + k_{23} \cdot X_2 + (-k_{32} - k_{34}) \cdot X_3 + 0 \cdot X_4 + 0 \\
\dot{X}_4 &= 0 \cdot X_1 + 0 \cdot X_2 + k_{34} \cdot X_3 + (-k_{44}) \cdot X_4 + 0
\end{aligned}
$$

$$
\begin{aligned}
Y_1 &= \frac{1}{V_1} \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3 + 0 \cdot X_4 \\
Y_2 &= 0 \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3 + 0 \cdot X_4 \\
Y_3 &= 0 \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3 + 0 \cdot X_4 \\
Y_4 &= 0 \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3 + 1 \cdot X_4
\end{aligned}
$$

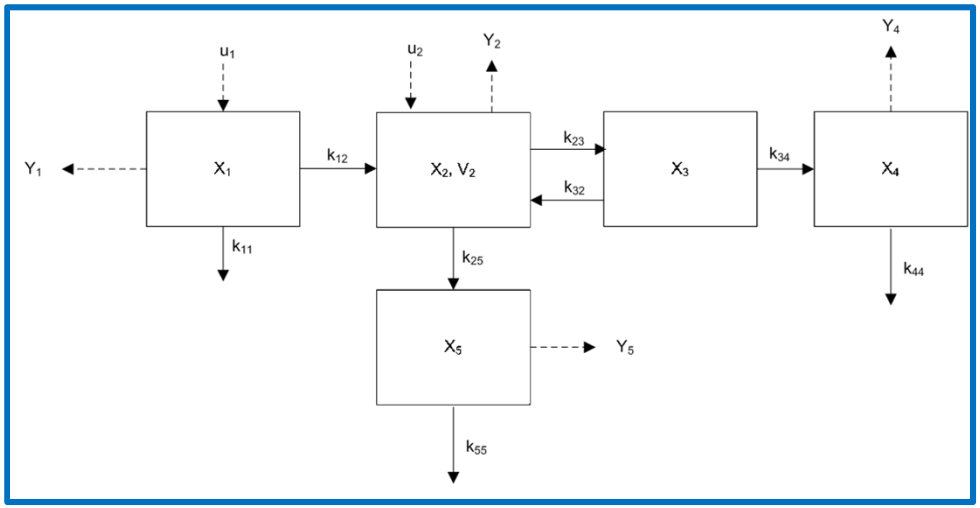
$$
X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \quad Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} \quad U = \begin{bmatrix} u_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
A = \begin{bmatrix} -k_{12} & k_{21} & 0 & 0 \\ k_{12} & (-k_{21} - k_{23}) & k_{32} & 0 \\ 0 & k_{23} & (-k_{32} - k_{34}) & 0 \\ 0 & 0 & k_{34} & -k_{44} \end{bmatrix}
$$

$$
\mathrm{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
C = \begin{bmatrix} \frac{1}{V_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Maticový popis 5-Kompartmentové modely



$$\dot{X}_1 = (-k_{11} - k_{12}) \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3 + 0 \cdot X_4 + 0 \cdot X_5 + u_1$$
$$\dot{X}_2 = k_{12} \cdot X_1 + (-k_{23} - k_{25}) \cdot X_2 + k_{32} \cdot X_3 + 0 \cdot X_4 + 0 \cdot X_5 + u_2$$
$$\dot{X}_3 = 0 \cdot X_1 + k_{23} X_2 + (-k_{32} - k_{34}) \cdot X_3 + 0 \cdot X_4 + 0 \cdot X_5 + 0$$
$$\dot{X}_4 = 0 \cdot X_1 + 0 \cdot X_2 + k_{34} \cdot X_3 + (-k_{44}) \cdot X_4 + 0 \cdot X_5 + 0$$
$$\dot{X}_5 = 0 \cdot X_1 + k_{25} X_2 + 0 \cdot X_3 + 0 \cdot X_4 + (-k_{55}) \cdot X_5 + 0$$
$$Y_1 = 0 \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3 + 0 \cdot X_4 + 0 \cdot X_5$$
$$Y_2 = 0 \cdot X_1 + \frac{1}{V_2} \cdot X_2 + 0 \cdot X_3 + 0 \cdot X_4 + 0 \cdot X_5$$
$$Y_3 = 0 \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3 + 0 \cdot X_4 + 0 \cdot X_5$$
$$Y_4 = 0 \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3 + 1 \cdot X_4 + 0 \cdot X_5$$
$$Y_5 = 0 \cdot X_1 + 0 \cdot X_2 + 0 \cdot X_3 + 0 \cdot X_4 + 1 \cdot X_5$$

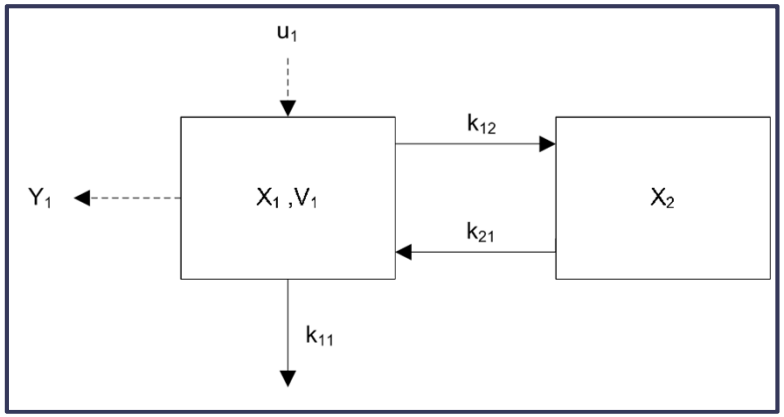
$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} \quad Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix} \quad U = \begin{bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -k_{11} - k_{12} & 0 & 0 & 0 & 0 \\ k_{12} & -k_{23} - k_{25} & k_{32} & 0 & 0 \\ 0 & k_{23} & -k_{32} - k_{34} & 0 & 0 \\ 0 & 0 & k_{34} & -k_{44} & 0 \\ 0 & k_{25} & 0 & 0 & -k_{55} \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{V_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Kompartmentové modely jako LTI systém



$$\dot{X} = A.X + B.U$$
$$Y = C.X$$

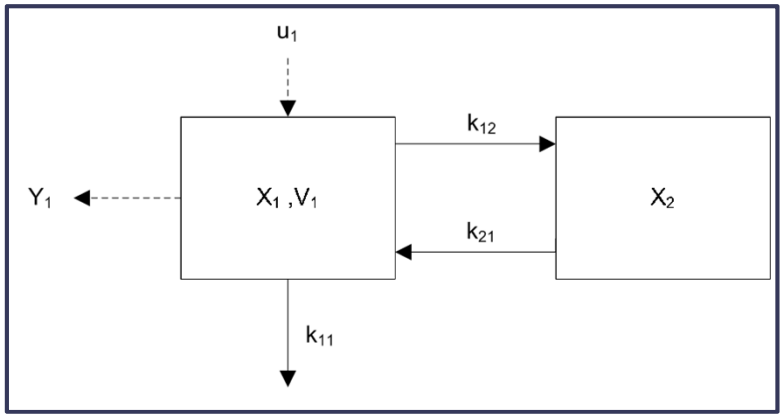
lineární t-invariantní dynamický systém

LTI systém

$$\frac{\boldsymbol{L\{Y\}}}{\boldsymbol{L\{U\}}} = C.(s.I - A)^{-1}.B$$

Přenosová funkce je závislost mezi výstupem a vstupem LTI systému

Přenosová funkce 2-Kompartmentové modely



$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad U = \begin{bmatrix} u_1 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} (-k_{11} - k_{12}) & k_{21} \\ k_{12} & -k_{21} \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{1}{V_1} & 0 \\ 0 & 0 \end{bmatrix}$$

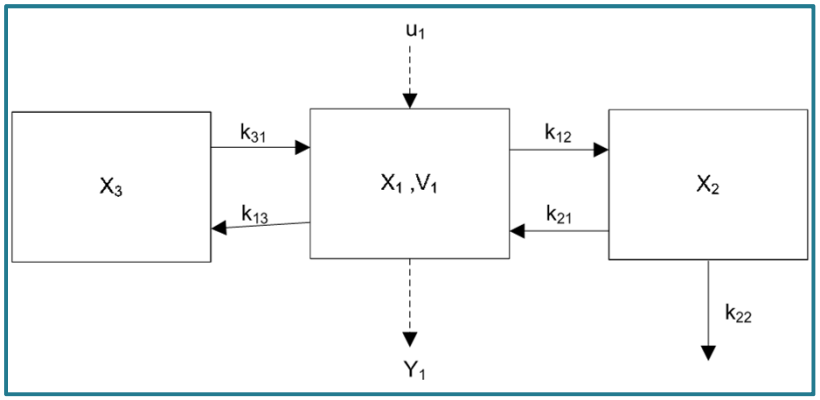
$$\frac{L\{Y\}}{L\{U\}} = C \cdot (s.I - A)^{-1} \cdot \mathrm{B}$$

$$\begin{bmatrix} \frac{L\{Y_1\}}{L\{u_1\}} & \frac{L\{Y_1\}}{L\{u_2\}} \\ \frac{L\{Y_2\}}{L\{u_1\}} & \frac{L\{Y_2\}}{L\{u_2\}} \end{bmatrix} = \begin{bmatrix} \frac{1}{V_1} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} (s + k_{11} + k_{12}) & k_{21} \\ k_{12} & s + k_{21} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

```
syms s k11 k12 k21 v1;
A=[-k11-k12 k21; k12 -k21];
B=[1 0; 0 0];
C=[1/v1 0; 0 0];
I=eye(2);
TF=C*inv(s*I-A)*B;
Collect(TF(1,1),s);
```

$$\frac{L\{Y_1\}}{L\{u_1\}} = \frac{\frac{1}{V_1}s + \frac{1}{V_1}k_{21}}{s^2 + (k_{11} + k_{12} + k_{21})s + k_{21}k_{11}}$$

Přenosová funkce 3-Kompartmentové modely

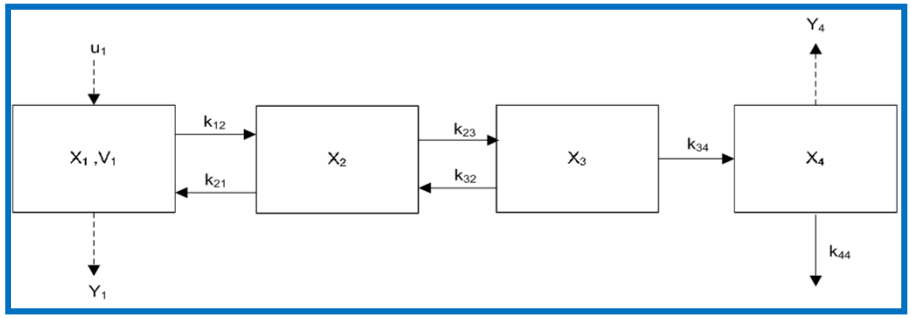


$$\frac{L\{Y\}}{L\{U\}} = C \cdot (s.I - A)^{-1} \cdot \mathrm{B}$$

$$\begin{bmatrix} \frac{L\{Y_1\}}{L\{u_1\}} & \frac{L\{Y_1\}}{L\{u_2\}} & \frac{L\{Y_1\}}{L\{u_3\}} \\ \frac{L\{Y_2\}}{L\{u_1\}} & \frac{L\{Y_2\}}{L\{u_2\}} & \frac{L\{Y_2\}}{L\{u_3\}} \\ \frac{L\{Y_3\}}{L\{u_1\}} & \frac{L\{Y_3\}}{L\{u_2\}} & \frac{L\{Y_3\}}{L\{u_3\}} \end{bmatrix} = \begin{bmatrix} \frac{1}{V_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} -k_{12} - k_{13} & k_{21} & k_{31} \\ k_{12} & -k_{21} - k_{22} & 0 \\ k_{13} & 0 & -k_{31} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\boldsymbol{L\{Y_1\}}}{\boldsymbol{L\{u_1\}}} = \frac{\boldsymbol{\frac{1}{V_1}(s + k_{21} + k_{22})(s + k_{31})}}{\boldsymbol{(s + k_{12} + k_{13})(s + k_{21} + k_{22})(s + k_{31}) - k_{13}k_{31}(s + k_{21} + k_{22}) - k_{12}k_{21}(s + k_{31})}}$$

Přenosová funkce 4-Kompartmentové modely



$$\frac{L\{Y\}}{L\{U\}} = C \cdot (s.I - A)^{-1} \cdot \mathrm{B}$$

$$\begin{bmatrix} \frac{L\{Y_1\}}{L\{u_1\}} & \frac{L\{Y_1\}}{L\{u_2\}} & \frac{L\{Y_1\}}{L\{u_3\}} & \frac{L\{Y_1\}}{L\{u_4\}} \\ \frac{L\{Y_2\}}{L\{u_1\}} & \frac{L\{Y_2\}}{L\{u_2\}} & \frac{L\{Y_2\}}{L\{u_3\}} & \frac{L\{Y_2\}}{L\{u_4\}} \\ \frac{L\{Y_3\}}{L\{u_1\}} & \frac{L\{Y_3\}}{L\{u_2\}} & \frac{L\{Y_3\}}{L\{u_3\}} & \frac{L\{Y_3\}}{L\{u_4\}} \\ \frac{L\{Y_4\}}{L\{u_1\}} & \frac{L\{Y_4\}}{L\{u_2\}} & \frac{L\{Y_4\}}{L\{u_3\}} & \frac{L\{Y_4\}}{L\{u_4\}} \end{bmatrix} = \begin{bmatrix} \frac{1}{V_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -k_{12} & k_{21} & 0 & 0 \\ k_{12} & (-k_{21} - k_{23}) & k_{32} & 0 \\ 0 & k_{23} & (-k_{32} - k_{34}) & 0 \\ 0 & 0 & k_{34} & -k_{44} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{L\{Y_1\}}{L\{u_1\}} = \frac{\frac{1}{v1}(\mathrm{s\hat{}2} + (\mathrm{k34} + \mathrm{k32} + \mathrm{k23} + \mathrm{k21})\mathrm{s} + \mathrm{k21k32} + \mathrm{k21k34} + \mathrm{k23k34})}{\mathrm{s\hat{}3} + (\mathrm{k34} + \mathrm{k12} + \mathrm{k32} + \mathrm{k23} + \mathrm{k21})\mathrm{s\hat{}2} + (\mathrm{k21k34} + \mathrm{k12k34} + \mathrm{k23k34} + \mathrm{k21k32} + \mathrm{k12k32} + \mathrm{k12k23})\mathrm{s} + k12k23k34}$$

$$\frac{L\{Y_4\}}{L\{u_1\}} = \frac{k12 * k23 * k34}{\begin{array}{c} s\hat{}4+(k21+k23+k12+k32+k44+k34)s\hat{}3+ \\ (k21k44+k12k23+k23k44+k32k44+k34k44+k21k32+k12k44+k21k34+k23k34+k12k32+k12k34)s\hat{}2+ \\ (k23k34k44 + k21k32k44 + k21k34k44 + k12k23k34 + k12k34k44 + k12k23k44 + k12k32k44)s + \\ k12k23k34k44 \end{array}}$$

Shrnutí dnešního cvičení

[Modely populací]
Maticový popis kompartmentové modely

[Co bude dál?]
analýza identifikovatelnosti kompartmentové modely.