**Simple Model of Cardiovascular System**

**Project Work – Laboratory Exercises in Modeling and Simulating Physiological Processes**

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*The problem statement and the models in this project work are a summary from*

1. *J. J. Bazel, F. Kappel, D. Schneditz, H. T. Tran, ‘Cardiovascular and Respiratory Systems – Modeling, Analysis, and Control’, ISBN-13: 978-0-898716-17-7, 2007*
2. *J. Keener, J. Sneyd, ‘Mathematical Physiology’, ISBN: 0-387-98381-3, 1998*

**Step 1 – Defining the System**

The goal of this project is to describe the overall reaction of the cardiovascular system under a constant workload over a period of time of 15 minutes. We are interested only in the time behavior of the heart rate and the time course of the blood pressure.

**Step 2 – Identifying System Components**

We are not interested in the blood flow in the arterial tree and do not distinguish between individual vessels in the venous and arterial part of the systemic or pulmonary circuit. Instead, we lump them together into four single compartments: **Venous Systemic Compartment** and **Arterial Systemic Compartment; Venous Pulmonary Compartment** and **Arterial Pulmonary Compartment**. We assume that those compartments are compliant and only characterized by the blood pressure, which determines the blood volume.

The venous systemic and the arterial systemic compartments on one side; the venous pulmonary and arterial pulmonary compartments on the other side; are connected by capillaries, arterioles and venules, which we lump together into two single compartments: **Systemic Peripheral Compartment** and **Pulmonary Peripheral Compartment**. We assume that both peripheral compartments are only characterized by the resistance of their walls to blood flow.

The venous systemic and the arterial pulmonary compartments are connected by the **Right Ventricle Compartment**. We assume that the right atrium is part of the venous systemic compartment. The venous pulmonary and the arterial systemic compartments are connected by the **Left Ventricle Compartment**. We assume that the left atrium is part of the venous pulmonary compartment.

**Step 3 – Modeling the System with Equations**

Since the venous systemic, arterial pulmonary, venous pulmonary and arterial systemic compartments are compliance compartments, we have the relation

[1] where is the blood volume, is the blood pressure and is the compliance constant of the given compartment.

The left and right ventricle compartments are characterized by the blood flow generated by the ventricle, i.e. the cardiac output, which is given by

[2] where is the heart rate and is the stroke volume, i.e., the volume of blood ejected by one beat of the ventricle.

The blood flow F through the pulmonary and systemic peripheral compartments depends on the blood pressure in the adjacent compartments and the resistance R against blood in the peripheral regions.

[3] where a stands for arterial and v stands for venous.

If we use the subscripts l and r to characterize the left heart and the right heart, we obtain the following so-called Gordins **equations for the four compliance compartments**: [4]

Arterial Systemic Compartment:   
Venous Systemic Compartment:   
Arterial Peripheral Compartment:   
Venous Peripheral Compartment:

For the systemic and pulmonary peripheral compartments, we have: [5]

Systemic Peripheral Compartment:   
Pulmonary Peripheral Compartment:

To model the **filling process**, we consider following equation giving the variation of the ventricle volume at time t, where is the total resistance to the inflow into the ventrical; is the venous filling pressure at time t; is the pressure in the ventricle at time t: . The initial value is the end-systolic ventricle volume   
Further we assume that is constant during diastole and is equal . We assume that for the relaxed ventricle we have the volume-pressure relation where is the compliance constant. By integrating the differential equation above, we obtain:   
Let denote the duration of the filling process after Bazett, where is a constant in the range of 0.0387-0.0516; then we have , the end-diastolic volume.

The stroke volume is given by . The stroke volume can be also modeled by the formula of Frank-Starling: where is the contractility of the ventricle and is the arterial pressure against which the ventricle has to eject. By combining all linear equations obtained, we have: [6]

Where and

Equations [2], [4], [5], [6] together give a nonlinear system of ordinary differential equations, usually called **Grodins’ model of the mechanical part of the cardiovascular system**: [7]

Arterial Systemic Compartment:   
Venous Systemic Compartment:   
Arterial Pulmonary Compartment:   
Venous Pulmonary Compartment:   
Total Blood Volume is constant:

In order to model the reaction of the cardiovascular system under an ergonomic workload, we have to include the regulation of the heart rate. We assume that the contractilities of the ventricles vary according to the changes in heart rate in the same direction. Thus we model **the variation of ventricle contractilities** by the second-order differential equations: [8] where are positive constants.

The **local metabolic control** in tissues is achieved in increasing or decreasing the diameter of arterioles, which results in a change of vessel resistance to blood supply with oxygen. Thus we use Peskin model, where is the peripheral resistance, is a positive constant and is the concentration of oxygen in the venous blood in the capillary region:   
The metabolic rate in the tissue is satisfied both by oxygen supply and anaerobic energy flow provided by anaerobic biochemical reactions as described by the equation following, where is the oxygen concentration in the arterial blood, which is assumed to be constant:   
The biochemical energy flow in assumed to depend on the rate of change of venous oxygen level, thus: where is a positive constant. By differentiating the equations in this section, we obtain the following differential equation: [9]

Now we include a constant ergonomic workload imposed on a test person and rewrite the resulting metabolic rate as: [10]

where is the metabolic rate in the systemic tissue region corresponding to zero workload and is a positive constant.

We assume that the **baroreceptor loop** is modeled by a feedback law , which controls the heart rate: [11]

**Step 4 – Building the Simulink Block Diagrams**

Scope

Scope

H

,

Baroreceptor Loop Subsystem [11]

Total Blood Volume Subsystem [7]

H

Scope

Scope

H

Scope

Scope

Workload Subsystem [10]

Pulmonary Peripheral Subsystem [5]

Scope

Scope

Scope

Systemic Peripheral Subsystem [9]

Scope

Scope

Venous Pulmonary Subsystem [7]

Arterial Pulmonary Subsystem [7]

Arterial Systemic Subsystem [7]

Venous Systemic Subsystem [7]

Left Ventricle Subsystem [2, 6, 8]

Right Ventricle Subsystem [2, 6, 8]

As homework students should model the subsystems illustrated above using Simulink block diagrams, with respect of inputs /outputs from/to other subsystems. The scopes are part of subsystems and should multiplex and display the required signals. We count max 3 students per group.

Work Group 1 – Arterial Systemic Subsystem  
Work Group 2 – Venous Systemic Subsystem  
Work Group 3 – Arterial Pulmonary Subsystem  
Work Group 4 – Venous Pulmonary Subsystem  
Work Group 5 – Pulmonary and Systemic Peripheral Subsystems  
Work Group 6 – Right Ventricle Subsystem  
Work Group 7 – Left Ventricle Subsystem  
Work Group 8 – Baroreceptor Loop Subsystem  
Work Group 9 - Workload, Total Blood Volume Subsystems and Model Integration

**Step 5 – Running the Simulation**

The following table gives a summary of the parameters (constants) of our model and their values.

|  |  |  |  |
| --- | --- | --- | --- |
| **Parameter** | **Meaning** | **Value** | **Unit** |
|  | Compliance of the arterial systemic compartment | 0.0101 | Liter/mmHg |
|  | Compliance of the venous systemic compartment | 0.6425 | Liter/mmHg |
|  | Compliance of the arterial pulmonary compartment | 0.0357 | Liter/mmHg |
|  | Compliance of the venous pulmonary compartment | 0.1391 | Liter/mmHg |
|  | Compliance of the relaxed left ventricle | 0.0220 | Liter/mmHg |
|  | Compliance of the relaxed right ventricle | 0.0443 | Liter/mmHg |
|  | Resistance in the pulmonary peripheral region | 0.32 | mmHg min/Liter |
|  | Coefficient in Bazett’s formula | 0.0516 | 1 |
|  | Oxygen concentration in arterial systemic blood | 0.2 | 1 |
|  | Oxygen concentration in venous systemic blood | 0.0001 | 1 |
|  | Constant in the formula for biochemical energy flow | 15.959 | Liter |
|  | Constant exercise workload | 75 | Watt |
|  | Constant in the formula for peripheral resistance | 254 | mmHg min/Liter |
|  | Metabolic rate in the systemic peripheral region at zero workload | 0.35 | Liter/min |
|  | Coefficient in the formula for metabolic rate | 0.0011 | Liter/(min Watt) |
|  | Coefficient for contractilities of the left ventricle | 31.592 |  |
|  | Coefficient for contractilities of the right ventricle | 28.342 |  |
|  | Coefficient for contractilities of the left ventricle | 25.065 | mmHg/min |
|  | Coefficient for contractilities of the right ventricle | 1.416 | mmHg/min |
|  | Coefficient for contractilities of the left ventricle | -1.332 |  |
|  | Coefficient for contractilities of the right ventricle | -2.045 |  |

**Step 5 – Validating the Model**

The simulation results should be validated against following estimated values for the state variables of our model:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Variable** | **Meaning** | **Initial Value** | **Expected Simulation Value Range** | **Unit** |
|  | Pressure in the arterial systemic compartment | 105.5 | [80 .. 130] | mmHg |
|  | Pressure in the venous systemic compartment | 4 | [1 .. 20] | mmHg |
|  | Pressure in the arterial pulmonary compartment | 15 |  | mmHg |
|  | Pressure in the venous pulmonary compartment | 6 |  | mmHg |
|  | Contractility of the left ventricle | 61 | Around 61 | mmHg |
|  | Contractility of the right ventricle | 4 | Around 4 | mmHg |
|  | Resistance in the systemic peripheral region | 9.5 | Can reach 18 | mmHg min/Liter |
|  | Heart rate |  | [60 .. 180] |  |
|  | Stroke volume of the left ventricle |  |  | Liter |
|  | Stroke volume of the right ventricle |  |  | Liter |
|  | End-diastolic volume of the left ventricle |  | [0.102 .. 0.135] | Liter |
|  | End-diastolic volume of the right ventricle |  |  | Liter |
|  | End-systolic volume of the left ventricle |  | [0.031 .. 0.065] | Liter |
|  | End-systolic volume of the right ventricle |  |  | Liter |
|  | Total Blood Volume |  | [4.5 .. 5.5] | Liter |
|  | Cardiac Output of the left ventricle |  | Approx. 5 | Liter/min |
|  | Cardiac Output of the right ventricle |  |  | Liter/min |

Students should make own research for usual physiological values for healthy adults, in order to validate their models.