CSM
Fall 2017

Electrical Engineering 16A
Worksheet 0-2

1. **Solutions of linear equations**

Meta: Prereq: Students should be comfortable solving a three-variable system of equations using GE with the forward/backward elimination method. Additionally, they should know how to convert a solution with a free variable from equations describing the solution set into vector notation.
Description: Simple mechanical gaussian elimination problem + some insight about free variables

(a) Consider the following set of linear equations:

$$2x+3y+5z=0$$
$$-1x-4y-10z=0$$
$$x-2y-8z=0$$

Place these equations into a matrix, and row reduce the matrix.

Meta: Note to mentors: When you do Gaussian Elimination – start by making $a_{2,1}$ = 0 using some multiple of $a_{1,1}$. Next, make $a_{3,1}$ = 0 using some multiple of $a_{1,1}$. Next, make $a_{3,2}$ = 0 by using some multiple of $a_{2,2}$. In this last step, when you use row 2's pivot to subtract out row 3, the first element of row 3 will not be affected (it will remain 0). This is because in the previous steps, we got rid of the first element of row 2 as well. This is what I like to call the zig zag method of doing Gaussian Elimination. (Elena: I call this the 'staircase', and I this is the Gaussian Elimination method 16A currently teaches. I think it is officially called forward/backward elimination.) Start at the top left, move down the column. Then start again at the top of the second column and move down.

Solution:

$$\begin{bmatrix} 2 & 3 & 5 \\ -1 & -4 & -10 \\ 1 & -2 & -8 \end{bmatrix}$$

$$R_2 = R_2 + \frac{1}{2}R_1$$
$$R_3 = R_3 - \frac{1}{2}R_1$$

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & -2.5 & -7.5 \\ 0 & -3.5 & -10.5 \end{bmatrix}$$

Make the numbers nicer by dividing row 2 by -2.5, and multiplying row 3 by -2. This is always a good thing to do if you realize your numbers are getting messy! (Also, feel free to keep all the numbers as non-fractional values by finding the least common multiple of the two numbers you are trying to cancel out.)

$$R_2 = \frac{1}{-2.5}R_2$$

$$R_3 = -2R_3$$

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & 1 & 3 \\ 0 & 7 & 21 \end{bmatrix}$$

$$R_3 = R_3 - 7R_2$$

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

(b) Convert the row reduced matrix back into equation form.

Solution:

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x + 3y + 5z = 0$$

$$0x + 1y + 3z = 0$$

$$0x + 0y + 0z = 0$$

(c) Intuitively, what does the last equation from the previous part tell us?

Solution: It tells us that there are infinite solutions to the equations. $0x + 0y + 0z = 0$ is satisfied by **any** x, y, z.

Meta: If students are confused at this point about why we can infer this, their confusion is well justified. Suppose that there were **4** equations in 3 variables – 3 of them were linearly independent, and the fourth one was $0x + 0y + 0z = 0$, then the system still has just 1 solution. The last equation is never *used* in some sense. Feel free to talk about this with students. Present it as: what if you had 4 equations, you wrote them in matrix form, got pivots in all rows except for one where you got a row of all 0s – are there still infinite solutions? The answer is no.

(d) Now that we've established that this system has infinite solutions, does every possible combination of $x, y, z \in \mathbb{R}$ solve these equations

Meta: This is supposed to be a quick part

Solution: No. $x = 1, y = 1, z = 1$ doesn't work, for instance.

(e) What is the general form (in the form of a constant vector multiplied by a variable t) of the infinite solutions to the system?

Meta: Explain why z is the free variable. (Because it is the one that doesn't have a pivot in the corresponding column). Also explain what "general form" means if students are confused.

Solution: z is a free variable. If $z = t$, then

$$y = -3z = -3t$$

$$2x + 3y + 5z = 0 \implies 2x - 9t + 5t = 0 \implies 2x = 4t \implies x = 2t$$

The general solution is then $t \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$. What this means is that any multiple of the vector $\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$ will satisfy the equations. Try it!

2. **First Proof**

Meta: Prereq: Knowing what linear independence and dependence are
Description: A very simple and basic proof about linear independence.

Prove that a subset of a finite linear independent set of vectors is linearly independent.

Meta: This is probably pretty early for when students will see proof. Very carefully introduce general proving techniques. Take the question, write down what is given in mathematical notation, and write out what needs to be proven in mathematical notation. A proof is essentially going from the 'given' to the 'to prove'.
Another note is that remember to assume that students have not taken CS70. Assume that they do not know proof techniques such as proof by contradiction, direct proof, induction, etc. This question is a proof by contradiction, so introduce it as such.
Proof by contradiction is not taught in 16A, so it is a good idea to go over the general structure of a proof in this format - assuming the negation of the statement you are trying to prove, and then using reductions to show an impossible scenario/contradiction. Final note: explain the 'without loss of generality' in the 'To Prove' section. Why

Solution: **Given:** $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are linearly independent. This, by definition of linear independence, means that if there exist $\alpha_1, \alpha_2, \ldots, \alpha_n$, such that:

$$\alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 + \ldots + \alpha_n\vec{v}_n = 0$$

then

$$\alpha_1 = \alpha_2 = \ldots = \alpha_n = 0$$

In other words, the only solution to the above αs is that the αs are all 0.
To Prove: $\beta_1\vec{v}_1 + \beta_2\vec{v}_2 + \ldots + \beta_k\vec{v}_k = 0 \implies \beta_1 = \beta_2 = \ldots = \beta_k = 0$.
Note that $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$ are a subset of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$.

Assume that $\beta_1\vec{v}_1 + \beta_2\vec{v}_2 + \ldots + \beta_k\vec{v}_k = 0$ is true but not $\beta_1 = \beta_2 = \ldots = \beta_k = 0$.

Consider $\beta_1\vec{v}_1 + \beta_2\vec{v}_2 + \ldots + \beta_k\vec{v}_k + 0\vec{v}_{k+1} + 0\vec{v}_{k+2} + \ldots + 0\vec{v}_n$. If $\beta_1\vec{v}_1 + \beta_2\vec{v}_2 + \ldots + \beta_k\vec{v}_k = 0$ then

$$\beta_1\vec{v}_1 + \beta_2\vec{v}_2 + \ldots + \beta_k\vec{v}_k + 0\vec{v}_{k+1} + 0\vec{v}_{k+2} + \ldots + 0\vec{v}_n = 0$$

However, since we assumed that not all $\beta_1, \beta_2, \ldots, \beta_k$ are 0, this means that the set $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ is not linearly independent, which is a contradiction because it is given that the set is linearly independent. Therefore, $\beta_1 = \beta_2 = \ldots = \beta_k = 0$ must have been true.

3. **Inverses!**

Meta: **prereqs** for this problem include being comfortable with Gaussian Elimination, comfortable with the idea of an inverse in general (use scalar numbers like 5 and one-fifth, for example, and then extrapolate to matrices), comfortable with inverses *not* existing. Some examples of inverses not existing:

- the number zero
- functions that are not one-to-one (sine, x^2, etc.)

(a) Find the inverse of:

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

Meta: Students may not know the shortcut formula and may instead use row reduction of this matrix alongside an identity matrix. They usually are eager to learn the shortcut, but then you may have to derive it for them so it makes sense.

If you don't have time to derive it, and they don't care about the shortcut, instead of the below answer, solve with GE - augment the matrix with the identity and row reduce.

Solution: We know that for 2×2 matrices, we can simply use the formula for the inverse. The inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{(1)(5)-(3)(2)}\begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$$

(b) Find the inverse of:

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 6 & 8 \end{bmatrix}$$

using Gaussian Elimination.

Meta: Students have been taught to augment the matrix they are trying to find the inverse of with the identity matrix, and then perform row operations until the original matrix looks like the identity. However, they may not know the intuition behind this. Be sure to show how this is valid.

If they ask about using cofactor expansion, tell them to use this method instead.

Solution: To find the inverse of a matrix, start with the equation

$$\mathbf{A} = \mathbf{IA}$$

Do row operations on the left hand side of this equation, to end up with

$$\mathbf{I} = \mathbf{A}^{-1}\mathbf{A}$$

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 6 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{A}$$

$$R_2 = R_2 - 2R_1$$

$$and$$

$$R_3 = R_3 - 3R_1.$$

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & -2 & -4 \\ 0 & -3 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}\mathbf{A}$$

$$R_2 = -0.5R_2$$

$$\begin{bmatrix}1 & 3 & 5\\0 & 1 & 2\\0 & -3 & -7\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\1 & \frac{-1}{2} & 0\\-3 & 0 & 1\end{bmatrix}\mathbf{A}$$

$$R_1 = R_1 - 3R_2$$

$$\textit{and}$$

$$R_3 = R_3 + 3R_2.$$

$$\begin{bmatrix}1 & 0 & -1\\0 & 1 & 2\\0 & 0 & -1\end{bmatrix} = \begin{bmatrix}-2 & \frac{3}{2} & 0\\1 & \frac{-1}{2} & 0\\0 & \frac{-3}{2} & 1\end{bmatrix}\mathbf{A}$$

$$R_3 = -1R_3$$

$$\begin{bmatrix}1 & 0 & -1\\0 & 1 & 2\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}-2 & \frac{3}{2} & 0\\1 & \frac{-1}{2} & 0\\0 & \frac{3}{2} & -1\end{bmatrix}\mathbf{A}$$

$$R_1 = R_1 + R_3$$

$$\textit{and}$$

$$R_2 = R_2 - 2R_3$$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}-2 & 3 & -1\\1 & \frac{-7}{2} & 2\\0 & \frac{3}{2} & -1\end{bmatrix}\mathbf{A}$$

Now this equation looks like $\mathbf{I} = \mathbf{A}^{-1}\mathbf{A}$

$$\begin{bmatrix}-2 & 3 & -1\\1 & -3.5 & 2\\0 & 1.5 & -1\end{bmatrix}$$

A rotation matrix is a matrix that takes a vector and rotates it by some number of degrees. That matrix looks like:

$$\begin{bmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{bmatrix}$$

for some angle θ. For example, if we had a rotation matrix with $\theta = 45^\circ$, and we multiplied it with the vector [.5, .5], what would you expect?

Meta: Go over a couple of examples of what this matrix does to a vector. Consider the x-axis and the y-axis for instance. If you are good at drawing on the board, it is SUPER helpful for students to see tranformations in action.
Also, rotation/reflection matrices are a cool example of a type of matrix where you can find an inverse without using cofactors/GE/etc., so make sure to let the students know that so they can build intuition for transformations.

(c) find the inverse of this matrix:

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

Solution:

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

Note that you don't have to actually do the normal math for finding inverses (and are probably pretty sad if you did). Instead think of what the inverse of a rotation matrix should probably do. What is the inverse of rotating 30 degrees? Rotating -30 (or 330) degrees! So by plugging -30 or 330 into the general form of the rotation matrix you can get the correct answer

(d) i. Will a rotation matrix always have an inverse? Why or why not?

ii. Consider a matrix that mirrors a vector across the x-axis. Will it always have an inverse?

iii. Consider a matrix flattens a vector on to the x-axis (so for example $[3, 5]^\top$ becomes $[3, 0]^\top$). Will it have an inverse?

Solution: i) Yes, you can always rotate in the opposite direction.
ii) Yes, you can always invert back (In fact, the inverse would be itself)
iii) No, since the transformation loses information, there probably is NOT an inverse.

4. Invertibility and equations

Meta: Prereq: Understanding of how to convert equations to matrix form, and knowing what inverses are and figuring our invertibility using gaussian elimination.
Description: Problem shows that pattern matching = failure. Basically, non-invertibility doesn't always mean no solutions.

(a) Consider the following system of equations

$$2x - 2y = -6$$
$$x - y + z = 1$$
$$3y - 2z = -5$$

Write these equations in matrix form. Then, write an expression for the solution to the equations using inverses, but don't compute the inverse.

Solution:
$$\begin{bmatrix} 2 & -2 & 0 \\ 1 & -1 & 1 \\ 0 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \\ -5 \end{bmatrix}$$

This can be rewritten as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -2 & 0 \\ 1 & -1 & 1 \\ 0 & 3 & -2 \end{bmatrix}^{-1} \begin{bmatrix} -6 \\ 1 \\ -5 \end{bmatrix}$$

(b) Let the system of equations be $\mathbf{A}\vec{x} = \vec{y}$. What does it mean if $\mathbf{A}$ is not invertible?
Hint: The solution to the previous part.

Solution: If $\mathbf{A}$ is not invertible, then the system cannot be solved uniquely. We may have infinite solutions or no solutions. **Make sure you read through the entire problem. We talk more in depth about what invertibility means in later parts!**

Meta: Mentors – at this point, get your students to nod. It's important that they think this is always the case, because we're going to trick them soon :-)
Also, don't read the "make sure..." part in section. This is mainly so that when students are looking over the solutions online, they don't look at this part and think this is always true. The whole point of this problem is to show that sometimes $\mathbf{A}$ can be non-invertible, but the system can still have solutions. Lastly, you can tell the students that this is why iPython can't find the solution to a system with infinite solutions when you use numpy.linalg.solve - instead of using GE, it inverts your A matrix and computes $A^-1 * b$. This will save them many ipython headaches in the future.

(c) Consider the matrix
$$\begin{bmatrix} -1 & 1 \\ -2 & 1 \\ 1 & -3 \end{bmatrix}$$

Is it invertible?

Solution: We don't actually need to do gaussian elimination on this matrix to check whether it is invertible. It is not. An R^{NxM} matrix, where $N \neq M$ is **never** invertible. Why? Because when we do gaussian elimination, for invertible matrix, we must get 3 pivots since we have 3 rows. But we cannot get 3 pivots because we have only 2 columns.

Meta: Be careful here. The solution is worded as if it this is obvious. This might be obvious to you having taken the class and studied linear algebra, but this is not immediately obvious to students! If students are confused by this, and please check if they are, then do guassian elimination, and show that you cannot possible get 3 pivots.

(d) Does the system of equations that is represented by the following have any solutions?

$$\underbrace{\begin{bmatrix} -1 & 1 \\ -2 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{B}\vec{x}} = \underbrace{\begin{bmatrix} 5 \\ 9 \\ -7 \end{bmatrix}}_{\vec{y}}$$

Solution: From part (b), we want to say that the system doesn't have any solutions. We saw in the previous part that $\mathbf{B}$ is not invertible. But... let's convert this system to actual equations.

$$-1x_1 + x_2 = 5$$
$$-2x_1 + x_2 = 9$$
$$1x_1 - 3x_2 = -7$$

These are 3 equations in 2 variables. Surely, they could have a solution. If we solve them, we get $(x_1 = -4, x_2 = 1)$ as a solution. How could this happen? **B** was not invertible!

The invertibility test actually only holds well for the case when $\mathbf{B}_{NxN}\vec{x}_{Nx1} = \vec{y}_{Nx1}$ are the dimensions of the matrices and vectors in question. In this case, **B** being non-invertible, means that gaussian elimination gives you a row of zeros.

- If you have infinite solutions, then in your augmented matrix, you will have a row of 0s and the corresponding element from $\vec{y}$ will also be 0. (Why does this mean infinite solutions?)
- If you have no solutions, then in your augmented matrix, you will have a row of 0s, but the corresponding element from $\vec{y}$ will be non-zero. (Why does this mean zero solutions?)

However, if you have an equation of the form $\mathbf{B}_{MxN}\vec{x}_{Nx1} = \vec{y}_{Mx1}$, then you have M equations in N variables.

- If $M > N$ (like in this example), then you have more equations than variables.
  - This could certainly have a solution if the equations are linearly dependent. For instance, $x + y = 1, 2x + 2y = 2, x - y = 4$ definitely has a solution. The first 2 equations are linearly dependent, so we can remove one of them. Notice how Gaussian Elimination would help you realize this and find the one solution!
  - You could also have no solution, if the equations are all linearly independent. $x + y = 3, x - y = -1, x + 2y = 0$ does not have any solutions. Notice how Gaussian Elimination would help you realize there are no solutions! *If you don't see it, try it out. Do you get a row of 0s on the left part of the augmented matrix, but not a corresponding zero element?*
  - You could also have infinite solutions. Consider $x + y = 1, 2x + 2y = 2, 3x + 3y = 3$. Gaussian elimination helps here too!
- If $M < N$, then you have more variables than equations.
  - This could have many solutions. Consider 1 equation: $x + y = 3$.
  - This could have no solutions. Consider $x + y + z = 1$ and $x + y + z = 2$
  - This cannot possibly have just one solution. You have more variables than you have equations!

  Notice though that Gaussian Elimination would help you realize all 3 of these cases.

  Big take away: Do not pattern match. If you hear it once "no inverse means no solutions", don't pattern match. That holds in a *particular* case. Remember that at the end, these are all equations, and use your intuition about equations.

Meta: A few notes

i. let students solve the equations themselves, don't just give them $-4, 1$
ii. There is a lot of 'answer' in here, but that does not mean that you as a mentor need to do all the talking. This material is very discussion-y, so discuss with them. Ask what happens in the case of $M > N$, and $M < N$. Let them come up with examples and ideas.

5. Are you linear?

(a) Consider a matrix $\mathbf{S}$ that transforms a vector $\vec{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to $\vec{y} = \begin{bmatrix} a-b-c \\ a-b-c \\ a-b+c \end{bmatrix}$. Note that a, b, c can take on any values in $\mathbb{R}$. In other words, $\mathbf{S}\vec{x} = \vec{y}$. Is this transformation linear?

Meta: Prereq: Knowledge of transformations and linearity (definitions, examples, and proofs).

Description: Please note that this might be the first time students are thinking of matrices as transformations. Let this settle in. The fact that a matrix is essentially a function that takes one vector and makes it a different vector. This is no different from a real-valued function like $f(x) = x^2$, except the only difference is that x is a vector, and f is a matrix. It might also be useful to show simple (non-matrix) examples of linear and non-linear transformations. A simple example of a non-linear transformation is something that squares each component of the vector. A simple example of a linear transformation is the 0 transformation.

Solution: To prove whether a transformation is linear, we must check whether it preserves scalar multiplication, addition and the zero vector.

Scalar multiplication

Let $\alpha \in \mathbb{R}$. Is $\mathbf{S}(\alpha\vec{x}) = \alpha\vec{y}$?

$\mathbf{S}\begin{bmatrix} \alpha a \\ \alpha b \\ \alpha c \end{bmatrix} = \alpha \begin{bmatrix} a-b-c \\ a-b-c \\ a-b+c \end{bmatrix}$. Try it!

Addition

Is $\mathbf{S}(\vec{x}_1 + \vec{x}_1) = \mathbf{S}\vec{x}_1 + \mathbf{S}\vec{x}_2$?

Let $\vec{x}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$ and $\vec{x}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$. Then $\mathbf{S}(\vec{x}_1 + \vec{x}_2) = \mathbf{S}\vec{x}_1 + \mathbf{S}\vec{x}_2$. Try it out!

Zero vector

Is $\mathbf{S} \cdot \vec{0} = \vec{0}$? Yes.

This proves that $\mathbf{S}$ is indeed a linear transformation.

Meta: At the end of this, get the students to ask you "well... but... since matrix-vector multiplication is linear, of course every matrix is a linear operator!!" This should be the next question they ask. Bonus: If every matrix is a linear operator, can we also say that every linear operator is a matrix?

(b) Now let's consider another matrix $\mathbf{Q}$ which takes a vector $\vec{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to $\begin{bmatrix} a+5 \\ b \\ c \end{bmatrix}$. Is this matrix a linear operator?

Meta: There might be a couple of students who immediately see the answer that the zero vector won't be preserved, but try to make sure they don't just blurt out the answer. Let everyone realize this by themselves.

Solution: Let's try the preservation of the zero vector first. Is $\mathbf{Q} \cdot \vec{0} = \vec{0}$? Nope, it is $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$. This matrix is not a linear operator! Notice that even though matrix-vector multiplication is generally linear

(c) Let's dive deeper. Write out the matrix $\mathbf{S}$ and $\mathbf{Q}$. Are they invertible?

Solution: $\mathbf{S} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. This matrix is not invertible, but it was still linear!

Writing out the matrix for $\mathbf{Q}$ is actually a trick question. There is no easy way to do this. In fact, you cannot write it just using numbers. Try it out. Let the first row of $\mathbf{S}$ be some $\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \end{bmatrix}$. Consider the first equation in the matrix vector multiplication: $\alpha_1 a + \alpha_2 b + \alpha_3 c = a + 5$. Using α_is from $\mathbb{R}$, there are no α_is that satisfy this equation. Since it isn't possible to write such a matrix, the invertibility question is invalid.

Meta: Make sure students try writing out $\mathbf{Q}$ and realize it isn't possible. Also, don't present the next part in section. Let students see it if they want online.

Solution: **Disclaimer: Read the following with caution. It abuses notation, and matrices in EE16A are not typically seen in the way that they are presented below (with their rows being functions of vectors).** You cannot simply write $\mathbf{Q}$ without considering the context in which it is being used. Say that the context is multiplication with a vector.

$$\mathbf{Q} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a+5 \\ b \\ c \end{bmatrix}$$

In this case,

$$\mathbf{Q} = \begin{bmatrix} 1 & \frac{5}{b} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Notice how it is impossible to write the matrix out entirely using just numbers and that we need to use either a, b or c inside the matrix itself. Finally, note that the first row of this matrix could be written in other ways too. It could be $\begin{bmatrix} 1 + \frac{5}{a} & 0 & 0 \end{bmatrix}$, or $\begin{bmatrix} 1 & \frac{3}{b} & \frac{2}{c} \end{bmatrix}$ too. But the essential idea is that a **non-linear transformation matrix cannot be expressed using just scalars**.

So it is invertible? We can't say.