McGill University

INSY 672-075: Healthcare Delivery Analytics

Unmoderated feedback extraction for Universities

Presented to:
Prof. Yichuan Ding

Arham Anwar
261137773
Michelle Barabasz
261152119

1 Derive conditions under which the disease will spread, i.e., $\frac{dI}{dt} > 0$.

To determine the conditions under which the disease will spread in the SEIR model, we first examine the differential equation for the infectious compartment $\frac{dI}{dt}$. According to the given equation 3:

$$\frac{dI}{dt} = \theta E - \gamma I$$

For the disease to spread, the rate of change of the infectious compartment $\frac{dI}{dt}$ must be greater than zero, which implies:

$$\theta E - \gamma I > 0$$

Solving this inequality for E gives us:

$$\theta E > \gamma I$$

Condition 1

$$E > \frac{\theta}{\gamma I}$$

This inequality tells us that for the disease to spread, the number of exposed individuals (who are not yet infectious) must be greater than the ratio of the rate at which infectious individuals recover or are removed (γ) to the rate at which exposed individuals become infectious (θ). This condition indicates that the addition of new infectious individuals (from the exposed category) must outpace the recovery/removal of currently infectious individuals.

Additionally, E is affected by the rate at which susceptible individuals become exposed (βIS), and the rate at which exposed individuals transition to the infectious state (θE). For E to be sufficiently large to satisfy the above condition, it indicates that a significant number of susceptibles must be transitioning to the exposed category, and the transition from exposed to infectious must not be so rapid that E is depleted faster than it is replenished. This dynamic ensures a continuous supply of individuals moving from the exposed to the infectious category, thereby allowing the disease to spread.

We can get the rest of the possible conditions/expressions by differentiating other available equations with respect to t to get the other possible expressions of $\frac{dI}{dt}$.

Condition 2

Let's differentiate equation 1 with respect to t:

$$\frac{dS}{dt} = -\beta IS$$

We have:

$$\frac{d^2S}{dt^2} = -\beta I\frac{dS}{dt} - \beta S\frac{dI}{dt}$$

By rearranging we will have:

$$\frac{dI}{dt} = \frac{1}{\beta S}\left(-\beta I\frac{dS}{dt} - \frac{d^2S}{dt^2}\right)$$

$\frac{dI}{dt} > 0$ implies:

$$\frac{1}{\beta S}\left(-\beta I\frac{dS}{dt}-\frac{d^2S}{dt^2}\right)>0$$

When $\frac{dI}{dt}>0$ in Condition 2, it signifies that the rate of change of the infectious compartment is increasing concerning the susceptible population. This suggests that the infection is spreading because the number of newly infected individuals is outpacing the rate at which susceptible individuals are being removed from the susceptible pool or transitioning to the exposed state. In simpler terms, more susceptible individuals are becoming infected, contributing to the propagation of the disease.

Condition 3

$$\frac{dE}{dt}=\beta IS-\theta E$$

Differentiating with respect to t to get $\frac{dI}{dt}$ expression:

$$\frac{d^2E}{dt^2}=-\beta I\frac{dS}{dt}-\beta S\frac{dI}{dt}-\theta\frac{dE}{dt}$$

Rearranging:

$$\frac{dI}{dt}=\frac{1}{\beta S}\left(\beta I\frac{dS}{dt}+\theta\frac{dE}{dt}+\frac{d^2E}{dt^2}\right)$$

For $\frac{dI}{dt}>0$:

$$\frac{1}{\beta S}\left(\beta I\frac{dS}{dt}+\theta\frac{dE}{dt}+\frac{d^2E}{dt^2}\right)>0$$

If $\frac{dI}{dt}>0$ in Condition 3, it implies that the rate of transition from the exposed to the infectious state is contributing to the spread of the disease. This indicates that there is a sufficient influx of individuals from the exposed category into the infectious category, augmenting the pool of infectious individuals. Essentially, the condition highlights the importance of the exposed population in fueling the spread of the disease, emphasizing the significance of managing and monitoring this transition phase to control the outbreak.

Condition 4

$$\frac{d^2R}{dt^2}=\gamma\frac{dI}{dt}$$

Which implies for $\frac{dI}{dt}>0$:

$$\frac{1}{\gamma}\frac{d^2R}{dt^2}>0$$

When $\frac{dI}{dt}>0$, according to Condition 4, it suggests that the acceleration of individuals moving from the infectious to the removed state is increasing. This acceleration is directly influenced by the rate of recovery or removal (γ). Therefore, a positive $\frac{dI}{dt}$ indicates that the rate of recovery or removal is not only active but accelerating, which could signify effective intervention measures or natural immune responses at play. In essence, the condition implies that the removal of infectious individuals is gaining momentum, potentially mitigating the spread of the disease.

Summary of Possible Conditions:

- **Condition 1**: Indicates that the addition of new infectious individuals from the exposed category must outpace the recovery/removal of currently infectious individuals.

- **Condition 2**: Signifies that more susceptible individuals are becoming infected, contributing to the propagation of the disease.
- **Condition 3**: Highlights the importance of the exposed population in fueling the spread of the disease, emphasizing the significance of managing and monitoring this transition phase to control the outbreak.
- **Condition 4**: Suggests that the acceleration of individuals moving from the infectious to the removed state is increasing, potentially mitigating the spread of the disease.

In conclusion, the conditions for the spread of the disease in the SEIR model are exhaustively derived from the full system of differential equations and the parameters involved. It is necessary for the transmission rate (β) to be greater than zero to facilitate new exposures, while the susceptible population (S) must also be greater than zero to provide individuals who can become exposed. Additionally, a positive exposed population (E) is required for a pool of individuals who can transition to the infectious state, along with a positive rate of transition from exposed to infectious (θ) to ensure that exposed individuals eventually become infectious. The infectious population (I) must also be greater than zero for the infection to continue spreading. Although the recovery rate (γ) does not directly affect the spread to new individuals, its relative size to θ influences the duration of the infectious period, with a lower γ relative to θ suggesting a longer infectious period and thus increasing the potential for disease spread. Thus, the comprehensive conditions for disease spread encompass ensuring positive values for β, S, E, θ, and I, while recognizing the interplay between γ and θ in determining the duration of infectiousness.

2 Describe the possible equilibrium states (S*,E*,I*,R*).

Equilibrium States:

Equilibrium states occur when the derivatives with respect to time are zero (i.e., $\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$).

Let's find the equilibrium states:

Equilibrium 1 (S, E, I^*, R^*): When $\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$, we have:

$$S^* = \frac{\gamma}{\beta}$$
$$E^* = 0$$
$$I^* = 0$$
$$R^* = N - S^*, \text{ where } N \text{ represents the total population size.}$$

Stability: If $\frac{\gamma}{\beta} < 1$, this equilibrium is stable. If $\frac{\gamma}{\beta} > 1$, this equilibrium is unstable.

Equilibrium 2 (S, E, I^*, R^*): When $\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$, we have:

$$S^* = 0$$
$$E^* = 0$$
$$I^* = \frac{\gamma}{\theta}$$
$$R^* = N - I^*$$

Stability: If $\frac{\gamma}{\theta} < 1$, this equilibrium is stable. If $\frac{\gamma}{\theta} > 1$, this equilibrium is unstable.

Interpretation: Equilibrium 1 represents a situation where no one is infected initially, and the disease does not spread. Equilibrium 2 represents a situation where the infection has spread, and some individuals are infected. New infections essentially balance out recoveries!

In summary, the SEIR model allows us to analyze the dynamics of an epidemic, considering both exposed and infected individuals. The stability of equilibrium states depends on the relative values of the model parameters.

3 Revise the code SIRmodel and adapt it to the SEIR model

Please see attached R file

4 Competition for Parameter Tuning

Please see attached R file