

# An Adjoint Separation Logic for the Wasm Call Stack

Work in Progress!

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# Context: Formally Specifying ABIs

**Application Binary Interface (ABI)**

The run-time contract for using a particular API

– Swift

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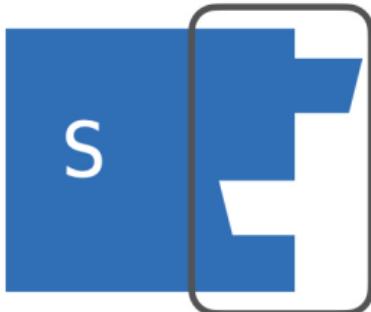
– Swift

S



This Type  $\tau$

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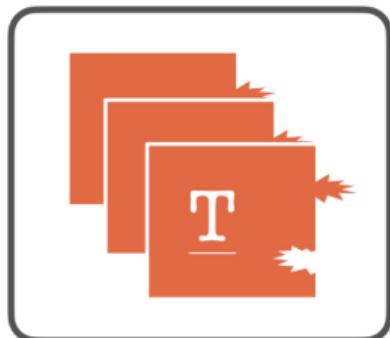


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Is *Realistically Realized [Benton06]*

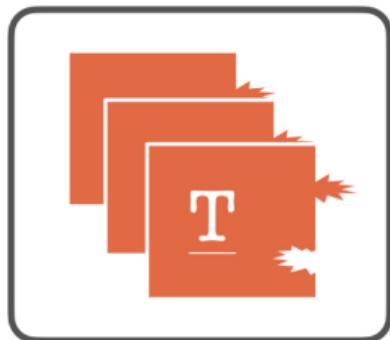
By These Target Programs

$[\![\tau]\!] = \{ e \mid \dots \}$

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## Application Binary Interface (ABI)

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## Our Approach

$e$  is ABI compliant with  $\tau$  if

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OOPSLA24

## Realistic Realizability: Specifying ABIs You Can Count On

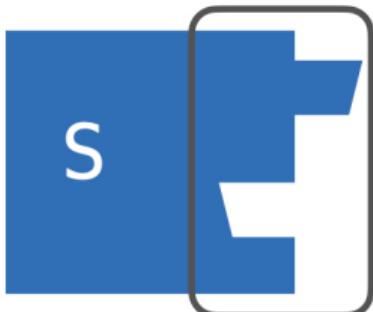
ANDREW WAGNER, Northeastern University, USA

ZACHARY EISBACH, Northeastern University, USA

AMAL AHMED, Northeastern University, USA

The Application Binary Interface (ABI) for a language defines the interoperability rules for its target platforms, including data layout and calling conventions, such that compliance with the rules ensures "safe" execution and perhaps certain resource usage guarantees. These rules are relied upon by compilers, libraries, and foreign-function interfaces. Unfortunately, ABIs are typically specified in prose, and while type systems for source languages allow one to check ABI-compatibility, they do not provide quantitative guarantees about the safety of the generated code.

# Context: Formally Specifying ABIs

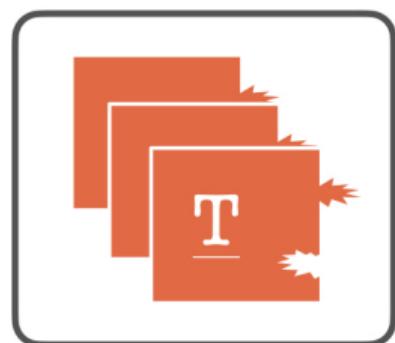


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## Application Binary Interface (ABI)

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$$[\tau] = \{ e \mid \dots \}$$

Target-level separation logic predicate

## Our Approach

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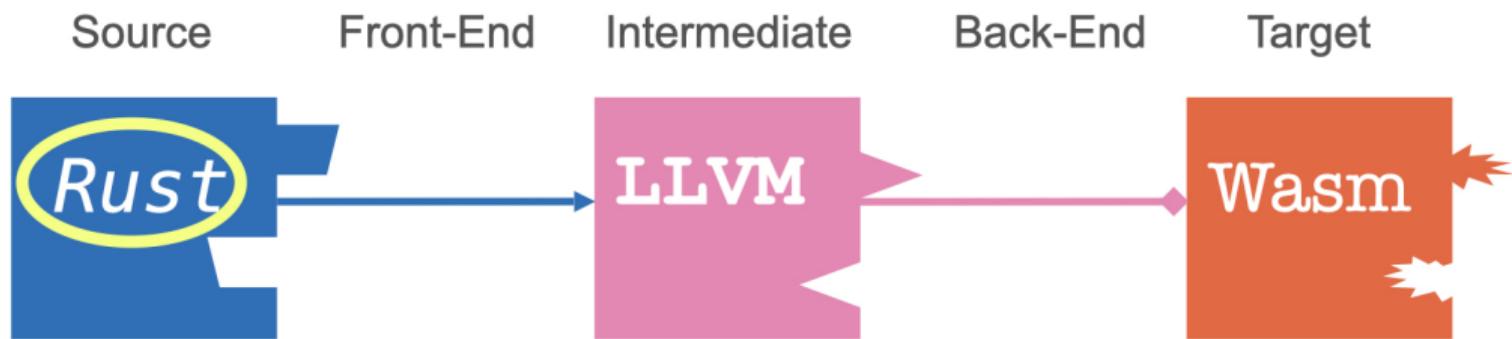
The Application Binary Interface (ABI) for a language defines the interoperability rules for its target platforms, including data layout and calling conventions, such that compliance with the rules ensures ‘safe’ execution and perhaps certain resource usage guarantees. These rules are relied upon by compilers, libraries, and foreign-function interfaces. Unfortunately, ABIs are typically specified in prose, and while type systems for source languages allow one to check ABI-compliance at compile time, handwritten annotations are often necessary to express the rules precisely. We propose a formal approach to specifying ABIs that allows one to reason about them in a compositional manner. Our approach is based on a separation logic predicate that takes a target program and an ABI as inputs and returns a value indicating whether the program is ABI-compliant. This predicate can be used to verify that a program satisfies the rules defined in the ABI, and it can also be used to generate test cases for the ABI. Our approach is based on a separation logic predicate that takes a target program and an ABI as inputs and returns a value indicating whether the program is ABI-compliant. This predicate can be used to verify that a program satisfies the rules defined in the ABI, and it can also be used to generate test cases for the ABI.

# Toward a Rust ABI for Wasm



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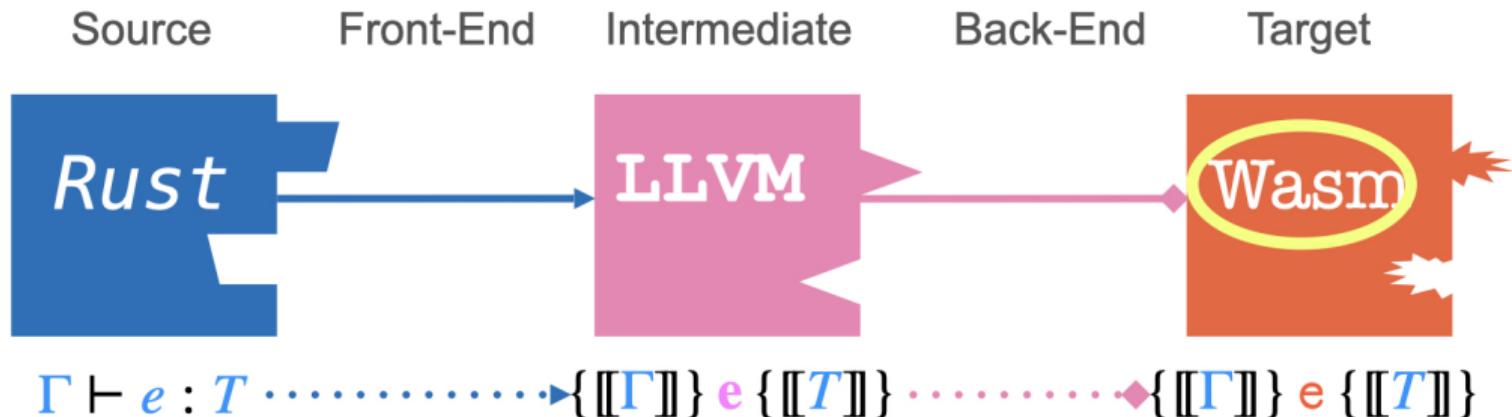
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# Toward a Rust ABI for Wasm



We need:

1. A **semantics** for borrowing
2. To support *independent updates* to the **front-end** and **back-end**
3. A **separation logic** for Wasm

# State of the Art: IrisWasm

PLDI23

## Iris-Wasm: Robust and Modular Verification of WebAssembly Programs

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AÏNA LINN GEORGES\*, Aarhus University, Denmark  
MAXIME LEGOUPIIL†, Aarhus University, Denmark  
CONRAD WATT, University of Cambridge, UK  
JEAN PICHON-PHARABOD, Aarhus University, Denmark  
PHILIPPA GARDNER‡, Imperial College London, UK  
LARS BIRKEDAL‡, Aarhus University, Denmark

WebAssembly makes it possible to run C/C++ applications on the web. A WebAssembly program is expressed as a collection of higher-order functions, which are linked together through a system of explicit imports and exports using a higher-order modular programming. We present Iris-Wasm, a mechanized verification framework for WebAssembly. It provides a specification of Wasm 1.0 mechanized in Coq and the Iris framework. Iris-Wasm allows one to specify and verify individual modules separately, and then compose them into larger programs.



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## Iris-MSWasm: Elucidating and Mechanising the Security Invariants of Memory-Safe WebAssembly

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JUNE ROUSSEAU, Aarhus University, Denmark  
AÏNA LINN GEORGES, MPI-SWS, Germany  
JEAN PICHON-PHARABOD, Aarhus University, Denmark  
LARS BIRKEDAL, Aarhus University, Denmark



WebAssembly offers coarse-grained encapsulation guarantees via its module system, but does not support fine-grained sharing of its linear memory. MSWasm is a recent proposal which extends WebAssembly with fine-grained memory safety guarantees. In this paper, we mechanise the security invariants of MSWasm, and provide a formal proof of their correctness.

# IrisWasm: Sample wp Rules

$$\frac{\text{wp\_binop} \quad \llbracket t.\text{binop} \rrbracket(c_1, c_2) = c * \triangleright \Phi(\text{immV}[t.\text{const } c]) * \xleftarrow{\text{Fr}} F}{\text{wp } [t.\text{const } c_1; t.\text{const } c_2; t.\text{binop } \textit{binop}] \left\{ w, \Phi(w) * \xleftarrow{\text{Fr}} F \right\}}$$

$$\frac{\text{wp\_call} \quad (F.\text{inst.funcs}[i] = \text{addr}) * \xleftarrow{\text{Fr}} F * \triangleright \left( \xleftarrow{\text{Fr}} F \multimap \text{wp } [\text{invoke } \text{addr}] \{w, \Phi(w)\} \right)}{\text{wp } [\text{call } i] \{w, \Phi(w)\}}$$

$$\frac{\text{wp\_invoke\_native} \quad |vs| = |ts_1| * cl = \{(inst; ts); es\}_{(ts_1 \rightarrow ts_2)}^{\text{NativeCl}} * F' = \{\text{locs} := vs \text{++} \text{zeros}(ts); \text{inst} := inst\} * \\ i \xrightarrow{\text{wf}} cl * \xleftarrow{\text{Fr}} F * \triangleright \left[ \begin{array}{c} (i \xrightarrow{\text{wf}} cl * \xleftarrow{\text{Fr}} F) \multimap \\ \text{wp } [\text{local}_{|ts_2|}\{F'\} (\text{block } ([] \rightarrow ts_2) es) \text{ end}] \{w, \Phi(w)\} \end{array} \right]}{\text{wp } (vs \text{++} \text{invoke } i) \{w, \Phi(w)\}}$$

$$\frac{\text{wp\_local\_bind} \quad \xleftarrow{\text{Fr}} F * \left( \xleftarrow{\text{Fr}} F_1 \multimap \text{wp } es \left\{ w, \exists F'_1, \xleftarrow{\text{Fr}} F'_1 * \left( \xleftarrow{\text{Fr}} F \multimap \text{wp } [\text{local}_n\{F'_1\} w \text{ end}] \{w', \Phi(w')\} \right) \right\} \right)}{\text{wp } [\text{local}_n\{F_1\} es \text{ end}] \{w', \Phi(w')\}}$$

What do all of these rules have in common?

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What do all of these rules have in common?

Explicit threading of the monolithic “frame resource”  $\xrightarrow{\text{FR}} F$

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$$\begin{array}{c} \Gamma \vdash e : T \\ \Downarrow \\ \{\llbracket \Gamma \rrbracket\} \ e^* \ \{\llbracket T \rrbracket\} \end{array}$$

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$$\begin{array}{c} \Gamma \vdash e : T \quad (\Gamma = \overline{x : T_x}) \\ \Downarrow \\ \{F \star \dots\} \ e^* \ \{\llbracket T \rrbracket\} \quad (F = \overline{\$x \mapsto v_x}) \end{array}$$

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$$\frac{\begin{array}{c} \otimes| \\ \Gamma_1 \vdash e_1 : T_1 \quad \Gamma_2 \vdash e_2 : T_2 \end{array}}{\Gamma_1, \Gamma_2 \vdash (e_1, e_2) : T_1 \otimes T_2}$$

↓

$$\{F_1 \star F_2 \star \dots\} \ e_1^*; e_2^* \ \{\llbracket T_1 \rrbracket \star \llbracket T_2 \rrbracket\}$$

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Local points-to:  $\$x \xrightarrow{\text{loc}} v$

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Rules are small footprint:

WP-PLUS

$$\Phi(\text{i32.const}(n_1 + n_2))$$

---

wp (i32.const  $n_1$ ; i32.const  $n_2$ ; i32.add) { $\Phi$ }

WP-LOC-GET

$$\$x \xrightarrow{\text{loc}} v \star \Phi(v)$$

---

wp (local.get  $\$x$ ) { $\Phi$ }

WP-LOC-SET

$$\$x \xrightarrow{\text{loc}} v' \star (\$x \xrightarrow{\text{loc}} v \multimap \Phi(\epsilon))$$

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WP-FRAME-BIND†

$$F \multimap \text{wp } (e^*) \ \{v^n. \Phi(v^n)\}$$

---

$$\text{wp } (\text{frame}_n F e^*) \ \{\Phi\}$$

# Problem 1: Popped Frames Not Encapsulated

---

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Binding under a frame moves the postcondition under a pop:

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$$\frac{F \rightarrow \text{wp } (e^*) \{v^n. \downarrow \Phi(v^n)\}}{\text{wp } (\text{frame}_n F e^*) \{\Phi\}}$$

## Problem 2: Suspended Frames Not Encapsulated

---

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Binding under a frame moves the continuation under a push:

$$\frac{\text{WP-FRAME-BIND}}{\uparrow(F \multimap \text{wp}(e^*) \{v^n. \downarrow \Phi(v^n)\})} \\ \text{wp}(\text{frame}_n F e^*) \{\Phi\}}$$

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$$\frac{\Downarrow \text{-ADJ}}{\frac{\downarrow P \vdash Q}{\overline{P \vdash \uparrow Q}}}$$

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$$\begin{array}{c} \Downarrow\text{-ADJ} \\ \frac{}{\Downarrow P \vdash Q} \\ \hline \hline P \vdash \uparrow Q \end{array}$$

$$\begin{array}{c} \uparrow\downarrow\text{-UNIT} \\ \frac{}{P \vdash \uparrow\downarrow P} \end{array}$$

$$\begin{array}{c} \downarrow\uparrow\text{-COUNIT} \\ \frac{}{\downarrow\uparrow P \vdash P} \end{array}$$

## Example: Suspending and Resuming Frames

$$\$x \xleftarrow{\text{loc}} 20 \vdash \text{wp } ((\text{frame}_1 (\text{locals}) \text{i32.const} 25) ; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}$$

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$$\frac{\begin{array}{c} \$x \xrightarrow{\text{loc}} 20 \vdash \text{wp } (\text{frame}_1(\text{locals}) \text{i32.const } 25) \{v'. \text{wp } (v'; \text{local.get } \$x; \text{i32.add}) \{v.v = 45\}\} \\ \hline \end{array}}{\begin{array}{c} \text{---(WP-CTX-BIND)} \\ \hline \\ \$x \xrightarrow{\text{loc}} 20 \vdash \text{wp } ((\text{frame}_1(\text{locals}) \text{i32.const } 25); \text{local.get } \$x; \text{i32.add}) \{v. v = 45\} \end{array}}$$

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$$\frac{\downarrow (\$x \xrightarrow{\text{loc}} 20) \vdash \text{wp} \ (\text{i32.const } 25) \ \{v'. \downarrow \text{wp} \ (v'; \text{local.get } \$x; \text{i32.add}) \ \{v. v = 45\}\} \\ \text{---(\uparrow -ADJ)} \quad \text{---}}{\$x \xrightarrow{\text{loc}} 20 \vdash \uparrow \text{wp} \ (\text{i32.const } 25) \ \{v'. \downarrow \text{wp} \ (v'; \text{local.get } \$x; \text{i32.add}) \ \{v. v = 45\}\}}$$

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$$\frac{\$x \xrightarrow{\text{loc}} 20 \vdash \uparrow \text{wp} \ (\text{i32.const } 25) \ \{v'. \downarrow \text{wp} \ (v'; \text{local.get } \$x; \text{i32.add}) \ \{v. v = 45\}\} \\ \text{---(WP-FRAME-BIND)} \quad \text{---}}{\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp} \ (\text{frame}_1 \ (\text{locals}) \ \text{i32.const } 25) \ \{v'. \text{wp} \ (v'; \text{local.get } \$x; \text{i32.add}) \ \{v. v = 45\}\}}$$

---

$$\frac{\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp} \ (\text{frame}_1 \ (\text{locals}) \ \text{i32.const } 25) \ \{v'. \text{wp} \ (v'; \text{local.get } \$x; \text{i32.add}) \ \{v. v = 45\}\}}{\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp} \ ((\text{frame}_1 \ (\text{locals}) \ \text{i32.const } 25); \text{local.get } \$x; \text{i32.add}) \ \{v. v = 45\}}$$

# Example: Suspending and Resuming Frames

$\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp } (\text{i32.const } 25; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}$

$\rule[-1ex]{0pt}{3ex} (\downarrow\text{-MONO})$

---

$\downarrow (\$x \xrightarrow{\text{loc}} 20) \vdash \downarrow \text{wp } (\text{i32.const } 25; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}$

$\rule[-1ex]{0pt}{3ex} (\text{WP-VAL})$

---

$\downarrow (\$x \xrightarrow{\text{loc}} 20) \vdash \text{wp } (\text{i32.const } 25) \{v'. \downarrow \text{wp } (v'; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}\}$

$\rule[-1ex]{0pt}{3ex} (\uparrow\downarrow\text{-ADJ})$

---

$\$x \xrightarrow{\text{loc}} 20 \vdash \uparrow \text{wp } (\text{i32.const } 25) \{v'. \downarrow \text{wp } (v'; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}\}$

$\rule[-1ex]{0pt}{3ex} (\text{WP-FRAME-BIND})$

---

$\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp } (\text{frame}_1(\text{locals}) \text{i32.const } 25) \{v'. \text{wp } (v'; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}\}$

$\rule[-1ex]{0pt}{3ex} (\text{WP-CTX-BIND})$

---

$\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp } ((\text{frame}_1(\text{locals}) \text{i32.const } 25); \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}$

# Example: Suspending and Resuming Frames

—(WP-CTX-BIND, WP-LOCAL-GET, WP-BINOP, WP-VAL)

---

$\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp} (\text{i32.const } 25; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}$

—( $\downarrow$ -MONO)

---

$\downarrow (\$x \xrightarrow{\text{loc}} 20) \vdash \downarrow \text{wp} (\text{i32.const } 25; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}$

—(WP-VAL)

---

$\downarrow (\$x \xrightarrow{\text{loc}} 20) \vdash \text{wp} (\text{i32.const } 25) \{v'. \downarrow \text{wp} (v'; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}\}$

—( $\uparrow$ -ADJ)

---

$\$x \xrightarrow{\text{loc}} 20 \vdash \uparrow \text{wp} (\text{i32.const } 25) \{v'. \downarrow \text{wp} (v'; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}\}$

—(WP-FRAME-BIND)

---

$\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp} (\text{frame}_1 (\text{locals}) \text{i32.const } 25) \{v'. \text{wp} (v'; \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}\}$

—(WP-CTX-BIND)

---

$\$x \xrightarrow{\text{loc}} 20 \vdash \text{wp} ((\text{frame}_1 (\text{locals}) \text{i32.const } 25); \text{local.get } \$x; \text{i32.add}) \{v. v = 45\}$

# Other Stacks

# Other Stacks

A shadow stack

$$\frac{\text{WP-SPUSH} \quad \uparrow^s \Phi(\epsilon)}{\text{wp } (\$spush) \ \{\Phi\}}$$

$$\frac{\text{WP-SPOP} \quad \downarrow_s \Phi(\epsilon)}{\text{wp } (\$spop) \ \{\Phi\}}$$

$$\frac{\text{WP-SALLOC} \quad \forall n', b^n. \ n' \xrightarrow{\text{stk}} b^n \rightarrow \Phi(\text{i32.const } n')}{\text{wp } (\$salloc } n) \ \{\Phi\}}$$

# Other Stacks

A shadow stack

$$\frac{\text{WP-SPUSH} \quad \uparrow^s \Phi(\epsilon)}{\text{wp } (\$spush) \ \{\Phi\}} \quad \frac{\text{WP-SPOP} \quad \downarrow_s \Phi(\epsilon)}{\text{wp } (\$spop) \ \{\Phi\}} \quad \frac{\text{WP-SALLOC} \quad \forall n', b^n. \ n' \xrightarrow{\text{stk}} b^n \rightarrow \Phi(\text{i32.const } n')}{\text{wp } (\$salloc } n) \ \{\Phi\}}$$

The operand stack?

$\text{top}(20) * \downarrow_o \text{top}(25) * \downarrow_o \downarrow_o \text{top}(10) \approx \text{i32.const } 10; v; \text{i32.const } 25; \text{i32.const } 20$

# Other Stacks

A shadow stack

$$\frac{\text{WP-SPUSH} \quad \uparrow^s \Phi(\epsilon)}{\text{wp } (\$spush) \ \{\Phi\}} \quad \frac{\text{WP-SPOP} \quad \downarrow_s \Phi(\epsilon)}{\text{wp } (\$spop) \ \{\Phi\}} \quad \frac{\text{WP-SALLOC} \quad \forall n', b^n. \ n' \xrightarrow{\text{stk}} b^n \star \Phi(\text{i32.const } n')}{\text{wp } (\$salloc } n) \ \{\Phi\}}$$

The operand stack?

$\text{top}(20) \star \downarrow_o \text{top}(25) \star \downarrow_o \downarrow_o \text{top}(10) \approx \text{i32.const } 10; v; \text{i32.const } 25; \text{i32.const } 20$

Stack switching?

# Summary

$$\frac{\text{WP-FRAME-BIND} \\ \uparrow(F \rightarrow \text{wp } (e^*) \{v^n. \downarrow \Phi(v^n)\})}{\text{wp } (\text{frame}_n F e^*) \{\Phi\}}$$

$$\frac{\downarrow P \vdash Q}{P \vdash \uparrow Q}$$

