Bridging the Gap: Precise Static Analysis of WebAssembly in a JavaScript World

ANONYMOUS AUTHOR(S)

WebAssembly is a low-level bytecode that runs alongside JavaScript to enable near-native performance on the web. Static analysis of WebAssembly is essential to debloat binaries or to specialize them to a JavaScript client. However, current static analysis techniques for WebAssembly are hampered by a *closed-world assumption*, where the WebAssembly module is analyzed in isolation from its host JavaScript client. This approach ignores the rich type information present in the JavaScript host, leading to significant imprecision, particularly in resolving indirect function calls, which are prevalent in code compiled from high-level languages.

This paper bridges the gap between the JavaScript and WebAssembly worlds. We present a multi-language static analysis that leverages information from compiler-generated JavaScript wrappers to improve the precision of a WebAssembly binary analysis. Our key insight is that the interoperation patterns between JavaScript and WebAssembly reveal higher-level type information that is lost during compilation to WebAssembly. We formalize this insight in a refinement type system for WebAssembly that distinguishes pointers from numeric types, two concepts conflated in WebAssembly's i32 type. We develop a tool, WasmBridge, that infers these refinement types by generating and solving constraints derived from both the WebAssembly binary and its JavaScript wrapper. We evaluate WasmBridge on a suite of real-world applications compiled from Rust to WebAssembly. Our approach reduces the number of edges in the generated call graph by an average of 5% compared to state-of-the-art static-analysis tools. This precision gain occurs at indirect call sites, where we reduce targets by an average of 17% at 35.63% of all such sites and discover new monomorphic call sites. Furthermore, we demonstrate the practical impact of this precision on a downstream analysis, showing that WasmBridge enables a dead-code analysis to identify up to 20% more dead functions than a state-of-the-art industry tool. Our work shows that even a lightweight refinement, informed by the multi-language context, can substantially improve the precision and utility of static analysis for WebAssembly.

CCS Concepts: • **Software and its engineering → Automated static analysis**.

Additional Key Words and Phrases: WebAssembly, JavaScript, Interoperation, Multi-Language Static Analysis, Static Analysis, Constraints, Debloating, Dead Code Elimination

1 INTRODUCTION

Program analysis is essential to understanding the functionality and behavior of a binary program when its source code is unavailable. It has been used extensively to detect malware and vulnerabilities in binary programs [23, 37], to reverse engineer binaries [24] and to optimize binaries outside the compiler toolchain [2]. However, program analysis of binaries is also very challenging since a substantial amount of information is lost during compilation. Compilers erase types, variable and function names, and all high-level structure present in the source to create the most performant binary possible. This loss of information, coupled with the under-specified semantics of low-level languages and very large instruction sets, makes writing sound and precise program analyses for binaries difficult [3].

The problem of binary analysis is now also on the Web! With the introduction of WebAssembly (Wasm) in 2017 [20], compilers have been able to compile languages like C, C++, Rust, etc. to WebAssembly, a portable, low-level bytecode format that has been designed for computationally intensive tasks in the browser. These WebAssembly binaries interoperate with a JavaScript client on browsers and in Node.js applications. WebAssembly has a formally specified semantics [42], a relatively small instruction set, structured control flow and low-level integer and floating point types for data (i32, i64, f32, f64) which makes analyzing WebAssembly binaries easier than x86 binaries. However, Lehmann et al. [29] find that static analysis of WebAssembly is still challenging, not only because it is a low-level binary format in which precise data-flow and pointer analysis is

hindered by the lack of structure and type information, but also because WebAssembly interoperates with JavaScript, a highly dynamic language that is notoriously difficult to statically analyze [21]. In the face of the latter hurdle, current state-of-the-art static analysis tools make a *closed-world assumption*, that is, they analyze a WebAssembly binary in isolation from its JavaScript client and make worst-case assumptions about client behavior. This leads to a significant loss in precision.

For a more precise analysis, Lehmann et al. [29] recommend analyzing the JavaScript client of a binary along with the WebAssembly binary and Thalakottur et al. [40] found that analyzing a JavaScript client could lead to better dead-code elimination. This opens up the world of multi-language static analysis. A naive approach to multi-language analysis is to translate one language to the other so that the analysis problem unfolds to an analysis of just one language [26]. For WebAssembly and JavaScript, this would either mean compiling JavaScript to WebAssembly and analyzing both WebAssembly programs, or translating WebAssembly to JavaScript and analyzing a combined JavaScript program. Setting aside the complexity of compiling a language as dynamic as JavaScript to WebAssembly, both scenarios seem unsatisfactory since the former brings with it the aforementioned loss of precision that comes with analyzing low-level code with weak types and the latter brings with it the loss of soundness that comes with statically analyzing a dynamic language. Another approach is to translate both to a lower level, common intermediate language. In prior work Liang et al. [30] explored using LLVM for multi-language analysis. However, since LLVM is lower-level than WebAssembly, there would be further loss of precision [29]. A fourth approach involves implementing analyses for multiple languages in a single framework [6, 31, 36]. This approach can be expensive, since values from one language have to be translated across the language boundary to represent a value in the other language. Notice that each aforementioned technique for multi-language analysis either reframes the analysis problem as a single-language analysis or is concerned with combining several single-language analyses together. None consider the effect of the interoperation itself on the analysis.

In this paper, we explore the effect of interoperation on WebAssembly's low-level type system and develop a novel multi-language analysis approach to improving the precision of a binary analysis. Let us consider the type systems of the languages in question. WebAssembly has a low-level type system of 32- and 64-bit integer types (i32, i64) and 32- and 64-bit floating point types (f32, f64). JavaScript has rich value types: booleans, numbers, strings, arrays, structs and so on. We make the observation that when a WebAssembly function is called with a JavaScript value, *the WebAssembly code should operate on and respect the type and structure of the JavaScript value.* Particularly, JavaScript *lowers* its complex types to WebAssembly by placing them in WebAssembly memory. We augment the original WebAssembly type system with types to describe pointers to this memory and recover a higher-level refinement type system for WebAssembly. Current state-of-the-art static analyses perform a call-graph analysis by assuming that any function with a matching signature could be a target for an indirect call. By performing a call-graph analysis over our interoperation-inspired refinement type system, rather than the original WebAssembly type system, we would be able to perform a more precise analysis of a WebAssembly binary. Our hypothesis is that in working with a refined type system over WebAssembly, *we are able to be more precise in our estimation of indirect call targets simply by virtue of a richer type system.* Our change to the type system is not extensive: we only introduce a small refinement for one of the base WebAssembly types. Specifically, our refinement type system refines only i32 types to either a singleton pointer type $\mathsf{ptr}(l, n)$ or a singleton number $\mathsf{num}(n)$. This paper simply tests if even a small addition of information from an interoperation scenario can improve the precision of a binary analysis. We also restrict ourselves to the initial version of WebAssembly, WebAssembly 1.0.

We perform a call-graph analysis over the refinement type system, through our tool called WasmBridge, and improve precision at an average of 35% of indirect call sites, reduce the number

of edges in the call graph by up to 5% and discover a small number of call sites to be monomorphic, which can enable additional compiler optimizations. We compare against state-of-the-art academic static analysis frameworks like Wassail [38] and Wasma [4], where we show a reduction in edges of up to 5%, and against industry tools like Wasm-Opt [43], where we show a reduction in edges of up to 69%. We also estimate the effects of our analysis on dead-code elimination where WasmBridge finds up to 20% more functions to be dead than an industry tool, Wasm-Metadce[43] does.

1.1 Overview

We summarize the contributions of this work as follows:

(1) We present the first multi-language static analysis of WebAssembly and JavaScript.
(2) We develop a refinement type system for WebAssembly that refines the WebAssembly value type i32 to distinguish between numbers and pointers to the WebAssembly memory.
(3) We develop a novel technique for binary analysis that considers the information present in the interoperation of the binary with a higher-level language. We generate and solve constraints to discover refinement types with an intra-procedural analysis.
(4) We show that our approach, WasmBridge, leads to a reduction in the number of estimated targets at an average of 35.63% of indirect call sites, with an average reduction of upto 17%.
(5) We evaluate WasmBridge against current state-of-the-art static analyses for WebAssembly and show an edge reduction of 5% for those tools.
(6) We estimate the impact of the increased precision of our analysis on dead-code elimination and find up to 20% more dead functions than a state-of-the-art industry tool.

The remainder of this paper is organized as follows. Section 2 provides background on the interoperation between WebAssembly and JavaScript and introduces the intuition behind our approach. Section 3, 4 and 5 delve into the specifics of our approach. We describe our evaluation against real-world subjects in Section 6. Section 7 and 8 discuss related and future work respectively. WasmBridge is anonymously available https://anonymous.4open.science/r/wasmbridge-eval-2D7B/.

2 BACKGROUND AND MAIN IDEAS

2.1 Interoperation Through a Wrapper

We explain the interoperation between WebAssembly and JavaScript using the example shown in Figure 1, where we show a Rust library being compiled to WebAssembly and its associated compiler-generated JavaScript wrapper file. The library functions are then called from a JavaScript client. We use different colors for each language for clarity.

2.1.1 The Library Developer Perspective. Let us consider the Rust library shown in Figure 1. The library consists of operations over numbers and arrays. num_add adds two numbers together, num_mul multiplies two numbers together, arr_add returns the sum of the elements of an array and arr_mul returns the result of multiplying all the elements of an array together. The library also has two functions, indirect_call_num and indirect_call_arr that are used to indirectly call one of the number or array operations based on some user-specified choice. A Rust library developer who wants to compile this library to WebAssembly annotates the functions in the library that they want to expose to a JavaScript client with a special tag, #[wasm_bindgen][1]. We refer to functions that are exposed to the JavaScript client as *exported functions*. The reader can see in Figure 1 that all the library functions are exported. The JavaScript client is meant to call these exported

[1]wasm-bindgen is a popular tool that enables Rust-JavaScript interoperation by generating bindings and glue code for JavaScript clients looking to use a Rust library through WebAssembly. We discuss this glue code in detail shortly.

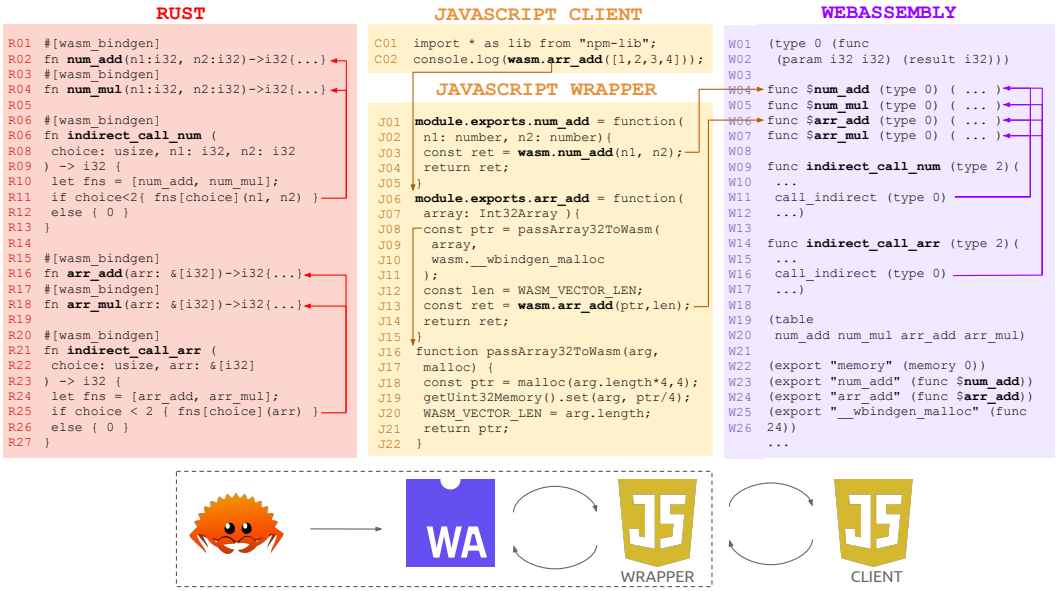


Fig. 1. A Rust Library compiled to WebAssembly along with its associated JavaScript wrapper.

functions with JavaScript objects. The translation from JavaScript values to Rust values is handled by wasm_bindgen and is not something that the Rust library developer has to worry about. For exported Rust functions, the compiler produces a WebAssembly binary along with a JavaScript wrapper file. For libraries being used by Node.js applications, these files (without the Rust source) are bundled in an NPM[2] package. A JavaScript client imports this NPM package to use the library from JavaScript. In Figure 1, the Rust library is bundled in the NPM package "npm-lib".

2.1.2 The JavaScript Client Perspective. A JavaScript client imports the library NPM package as an object lib (line C01). It then calls an exported function with JavaScript objects. On line C02, we see a call to arr_add with an array. The client does not translate this JavaScript object into a WebAssembly value. In fact, they do not interact with WebAssembly directly at all!

2.1.3 The JavaScript Wrapper. In real-world applications, JavaScript and WebAssembly interoperate through a wrapper: a JavaScript file generated by the compiler alongside the binary. This wrapper is used to *lower* a JavaScript object to WebAssembly before a call to a WebAssembly function and to *lift* a WebAssembly value to a JavaScript object after a call to a WebAssembly function. The code that does the lifting and lowering is often referred to as glue code. A WebAssembly module exposes certain functions to JavaScript through exports (lines W22-W26). The wrapper file contains JavaScript functions that have glue code around calls to exported WebAssembly functions.

To understand the glue code that mediates the interoperation between WebAssembly and JavaScript, let us go back to the example in Figure 1. In function num_add (line J01), we see no glue code before the call to num_add. This is because the JavaScript engine internally translates a JavaScript number to a WebAssembly i32 value. However, we see that in function arr_add (line J06), the Int32Array object passed into the function is passed to function passArray32ToWasm along with another exported WebAssembly function, __wbindgen_malloc. In the body of the function, we first see a call to __wbindgen_malloc with the size of the array in bytes. __wbindgen_malloc allocates a block of WebAssembly memory of the required size and returns a pointer to this block.

[2]NPM is the canonical package manager for Node.js applications.

Then a call to the function getUint32Memory accesses the WebAssembly memory[3] and updates the block at the pointer with the contents of the array. The pointer and the array length, both WebAssembly i32 values, are then passed in as arguments to arr_add (line J13). Since the WebAssembly type system is fairly low-level, this is an expected pattern. JavaScript objects that cannot be passed to Wasm directly are stored in memory and the pointer to memory is passed to Wasm instead.

2.2 Call-Graph Analysis Over the WebAssembly Binary

A call-graph analysis over the source code of the Rust library will resolve the targets for the indirect calls at R11 and R25 as num_add, num_mul and arr_add, arr_mul respectively. Unfortunately, a call-graph analysis over the corresponding WebAssembly binary produced by the compiler is not as precise. Indirect calls in WebAssembly occur using a call_indirect instruction and are mediated through a WebAssembly table that contains a list of functions (line W19). WebAssembly is a stack machine and instructions push and pop values from the stack. At runtime, the call_indirect instruction pops an i32 value from the stack which it uses to index into the table, thus determining which function to call. The precision of a call-graph analysis depends on how indirect calls are handled. Current state-of-the-art analysis tools handle indirect calls in one of two ways:

(1) *Naive Analysis:* The target of an indirect call could be any function in the WebAssembly function table. Industry tools like Wasm-Opt and Wasm-Metadce take this approach.
(2) *Type-based Analysis:* The syntax of a call_indirect instruction contains a function type annotation (line W11). The WebAssembly type system guarantees that the target of the indirect call matches this function type. Hence, state-of-the-art static analysis tools like Wassail and Wasma restrict the set of possible targets to be the functions in the WebAssembly table whose type matches the type annotation.

Unfortunately, for the example in Figure 1, both strategies yield the same result. The indirect calls at W11 and W16 have the same type annotation: type0, a function type that takes two i32s as arguments and returns an i32. The number and array add and multiply operations all compile down to have the same function type type0 and the WebAssembly table contains functions for all of these operations. Hence, a static analysis over this binary would resolve all four operations to be potential targets for both indirect calls. With a closed-world (WebAssembly only) analysis, there is no way to distinguish num_add from arr_add since they both have the same type. However, the JavaScript num_add and arr_add functions do have different types. We can see that num_add takes two numbers and passes them to num_add as two i32s, whereas arr_add takes an Int32Array and then calls arr_add with two i32s that actually represent, respectively, a pointer to the array allocated in WebAssembly memory and the length of that array. If our analysis included an analysis of the JavaScript wrapper, we could differentiate between these two Wasm functions more precisely!

2.3 Refining the WebAssembly Type System With Pointers

WebAssembly does not distinguish between numbers and pointers. The WebAssembly memory is only indexed by i32 values, or 32-bit integers, while i32 values are also used as conditionals for if-then-else expressions and as indices into the function table. Pointers to the WebAssembly memory can be distinguished in the JavaScript wrapper from regular numbers by virtue of being passed into WebAssembly through a different pattern, and so, we can refine i32 to two new types, ptr and num, both subtypes of i32. However, we must propagate these refined types through the WebAssembly code to have any hope of improving the precision at an indirect call site. To this end, we modify the WebAssembly typing rules, change the type system to be a refinement type system [9, 16, 19, 35] and type the WebAssembly memory to be able to track reads and writes to

[3]Note that the WebAssembly memory is also exported to JavaScript in W22.

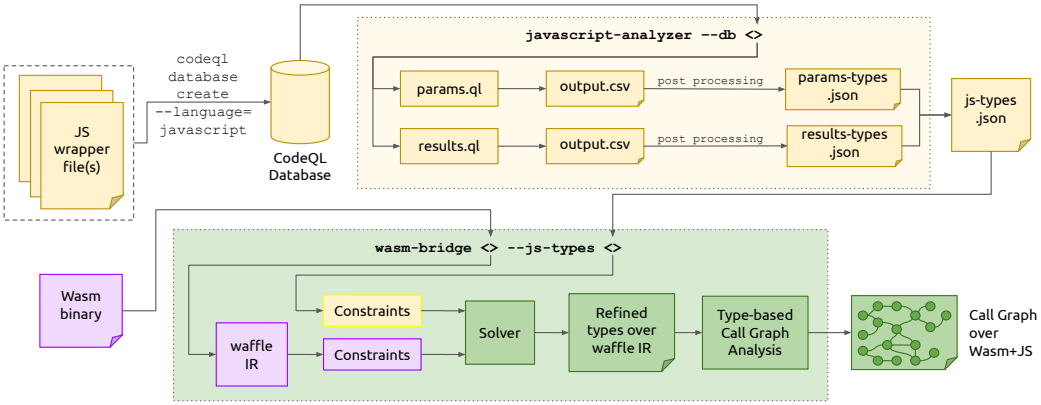


Fig. 2. System Diagram for Multi-Language Static Analysis for WebAssembly and JavaScript.

memory. The changes to the type system are described in Section 3. The changes we make to the type system are not extensive. We only refine i32's and do not refine the other base WebAssembly types, i64, f32 and f64. We also do not incorporate any of the rich JavaScript types for data allocated in the WebAssembly memory into the type system, leaving that to future work. Our hypothesis is that in working with even a simple set of refined types for WebAssembly, *we are able to be more precise in our estimation of indirect call targets simply by virtue of richer type information.*

Figure 2 illustrates how we discover these richer refinement types for WebAssembly. In order to type the arguments and returns for calls to WebAssembly exported functions in the JavaScript wrapper, we write a CodeQL [1] analysis that infers a refined type by checking for specific patterns of accessing the WebAssembly memory. CodeQL is a framework for writing static analyses with datalog-like queries. We then pass these types into our tool WasmBridge that encodes them as constraints. We also generate constraints over the WebAssembly binary under inspection. This allows us to discover refined types — for instance, if a WebAssembly instruction loads from or stores to an i32 then that type should be refined to a pointer. The constraints are used to propagate and discover the refinement types over the WebAssembly binary. Instead of working over the binary directly, we generate constraints over waffle [7], an intermediate representation for WebAssembly that uses Static Single Assignment Form [10]. We then solve the constraints and apply a type-based call-graph analysis over the refined type system. We explain the details of this process in Section 3, 4 and 5. Note that we only generate and solve constraints over WebAssembly 1.0 and support analysis over JavaScript wrapper files generated by the Rust compiler.

3 A REFINEMENT TYPE SYSTEM FOR WEBASSEMBLY

3.1 Background on WebAssembly Instructions and Typing

Below we discuss a subset of WebAssembly instructions and their typing. For a description of the complete instruction set and module typing, please refer to Haas et. al.[20] and the WebAssembly 1.0 specification [42]. Instructions in WebAssembly operate on an implicit value stack by popping argument values and pushing computed results. As shown in Figure 3, the instruction typing judgment in WebAssembly has the form $C^w \vdash e^* : \tau_w^m \rightarrow \tau_w'^n$ which says that in a context C^w, the one-or-more instructions e^* expect a sequence of m values of types τ_w^i (for $i \in \{1, \ldots, m\}$) at the top of the stack and replace these with a sequence of n values of types $\tau_w'^j$ (for $j \in \{1, \ldots, n\}$) at the top of the stack. Note that here we use subscripts and superscripts w for grammars of original WebAssembly elements, while later we will use r for our refinement-typed WebAssembly.

$$C^w ::= \{\text{func } tf_w{}^*,\ \text{local } \tau_w{}^*,\ \text{global } \tau_w{}^*,\ \text{table } n^?,\ \text{memory } n^?,\ \text{label } (\tau_w^*)^*,\ \text{return } (\tau_w^*)^?\}$$

Typing Instructions With the Original WebAssembly Value Types. $\boxed{C^w \vdash e^* : tf_w}$

$$\tau_w ::= \text{i32} \mid \text{i64} \mid \text{f32} \mid \text{f64}$$
$$tf_w ::= \tau_w^* \to \tau_w^*$$

$$\frac{}{C^w \vdash \tau_w.\mathbf{const}\ c : \epsilon \to \tau_w}\textsc{Constant} \qquad \frac{}{C^w \vdash \tau_w.binop : \tau_w\tau_w \to \tau_w}\textsc{Binary Ops}$$

$$\frac{tf_w = \tau_w^m \to \tau_w^n \qquad C^w, \text{label}(\tau_w^n) \vdash e^* : tf_w}{C^w \vdash \mathbf{block}\ tf_w\ e^*\ \mathbf{end} : tf_w}\textsc{Block} \qquad \frac{C^w_{\text{label}}(i) = \tau_w^m}{C^w \vdash \mathbf{br}\ i : \tau_w^*\ \tau_w^m \to \tau_w^*}\textsc{Break}$$

$$\frac{C^w_{\text{func}}(i) = tf_w}{C^w \vdash \mathbf{call}\ i : tf_w}\textsc{Call} \qquad \frac{C^w_{\text{memory}} = n \qquad 2^a \le (|tp|<)^?|\tau_w| \qquad (tp\_sz)^? = \epsilon \vee \tau_w = \mathbf{im}}{C^w \vdash \tau_w.\mathbf{load}(tp\_sx)^?\ a\ o : \text{i32} \to \tau_w}\textsc{Load}$$

Fig. 3. Typing of a subset of WebAssembly Instructions in the Original Type System.

3.1.1 Values and Arithmetic Instructions. WebAssembly has four value types: i32, i64, f32, f64. They represent 32- and 64-bit integers and 32- and 64-bit floating-point numbers. Values in WebAssembly are numeric constants tagged with the appropriate value type, e.g., the 32-bit integer 42 is represented as i32.const 42. The i32.const 42 instruction pops nothing from the stack and pushes the i32.const 42 value onto the stack. This is reflected in the typing rule for constants, as shown in Figure 3. The figure also contains the typing rule for binary operations, which expect two values of the type they are annotated with on the stack and produce a value for the same type. For example, i32.add expects two i32's on the stack and pushes a i32 onto the stack.

3.1.2 Control Constructs and Breaks. WebAssembly has control constructs such as blocks and loops and provides structured control flow with break instructions that are annotated with an index: br i. Here, i is a de-bruijn index [11] out of n labels that are associated with n enclosing control constructs. The target construct at i determines how the br instruction behaves —if it is a block, control jumps to the *end* of the block and if it is a loop, control jumps to the *start* of a loop. This is evident in the typing of the block and br instructions. Blocks are annotated with a signature tf_w that expects τ_w^m types on the top of the stack at the start of the block, and expects τ_w^n types on the top of the stack at the end of a block. The sequence of instructions in a block is type checked under a label that states that this enclosing block expects τ_w^n values on the stack. A br instruction finds the label associated with the i^{th} enclosing target control construct and before jumping to the end or start of the target, ensures it has the right types on the stack. If the target construct is a block, it would expect τ_w^n on the top of the stack.

3.1.3 Loads and Stores to Linear Memory. Loads and stores to WebAssembly memory are done with load and store instructions that are annotated with the type of data loaded from memory and stored in memory respectively. For example, i64.load loads a i64 value from memory. WebAssembly memory is only indexed by i32 values and so the i64.load expects an i32 value on the stack and pushes an i64 value to the stack.

3.1.4 Function Calls. Functions in WebAssembly are referenced using an immediate index into the function section of a module. Hence, a call instruction is annotated with a index i, identifying the function to be called. This index is used to look up the function type of the called function in the context C^w. The function type specifies the types at the top of the stack before and after the call.

3.2 Refinement Type System

We present a refinement over the original WebAssembly (version 1.0) type system in Figure 4. Here, R is analogous to store context S in the WebAssembly type system. We discuss each change to the type system below:

(1) *Refinement Types, τ_r*: We introduce refinements to each base WebAssembly value type τ_w. For example, if the top of the stack is typed to be τ_w in WebAssembly, it is ascribed the refinement type $\tau_w(n)$ in our type system. A refinement type such as i32(42), in a more traditional presentation of refinement types, might be written as $\{n : \beta \mid \beta <: \text{i32} \wedge n = 42\}$. For types i32($n$) and f32($n$), $n \in \{\mathbb{N}_{32} \cup \top\}$, while for i64($n$) and f64($n$), $n \in \{\mathbb{N}_{64} \cup \top\}$. Floating point numbers are represented by their bits.
(2) *Pointers and Numbers*: Since i32s are used both as pointers into the WebAssembly memory and numbers, we add two new types, ptr(n) and num(n), where ptr and num are subtypes of i32. The types i32(n), ptr(n), and num(n) form a lattice, the subtyping relation and meet and join operations for which are discussed in Section 3.4. Refinments of the other WebAssembly value types, i64(n), f32(n), and f64(n), are not refined further since they are not used to index memory.
(3) *Memory Typing, Ψ*: Unlike the original WebAssembly type system, we type the flat contiguous array of bytes that serves as the WebAssembly linear memory. Since we hope to propagate types from JavaScript to indirect call sites, and since memory operations are ubiquitous in WebAssembly, we need to be able to recover the (refined) types of values that are stored in memory at a specific address. A memory typing Ψ is a mapping from pointer addresses a, which are 32-bit integers, to refinement types τ_r.
(4) *Function Types*: Function types tf_w in WebAssembly are used to encode the types of functions, blocks, loops, and calls. They specify the (input and output) base WebAssembly types τ_w for all of these constructs. In our refinement type system, a function type specifies not only the lists of (input and output) refinement types τ_r, but also the shape of the (input and output) memory typing Ψ. We must track how the memory typing changes over the course of execution since WebAssembly programs perform strong updates that can change the type of data stored in memory. For instance, a function that expects a pointer at location a, because it loads from a, might later store an f32 at location a.
(5) *Label Typing*: In WebAssembly, labels of control constructs contain the shape of the stack expected when there is a jump to the control construct. In our refinement type system, we must specify not only the expected refined types τ_r on the stack but also the expected memory typing Ψ for the control construct in the label.

Next, we describe the refinement typing rules for a subset of WebAssembly instructions (shown in Figure 4). Refinement typing rules for the entire instruction set can be found in Appendix A.

3.2.1 Arithmetic Instructions. We restrict the refinement typing of arithmetic instructions to only allow certain combinations of i32, ptr and num. For instance, in the Binary Ops rule in Figure 4, the result type is computed using the function binop which takes two input refinement types and computes a result refinement type, but only considers certain pairs of input types valid for each specific binary instruction. The instruction will fail to type check if a invalid pair of input types is provided for a given binary operation. We discuss a few specific binary operations below.

- *Comparison*: Comparison of i32s and all its subtypes result in a num. All combinations of i32, ptr and num can be compared.
- *Addition*: WebAssembly allows addition of two i32s, either of which, or the result of which, can be used to index into the WebAssembly memory. We do not allow addition of two

$$R ::= \{\text{inst } C^r{}^*,\ \text{tab } n^*,\ \text{mem } \Psi^*\}$$
$$C^r ::= \{\text{func } tf_r{}^*,\ \text{local } \tau_r{}^*,\ \text{global } \tau_r{}^*,\ \text{table } n^?,\ \text{memory } n^?,\ \text{label } (\tau_r^*, \Psi)^*,\ \text{return } (\tau_r^*, \Psi)^?\}$$

Typing WebAssembly Instructions with Refinement Types $\boxed{R; C^r \vdash e_r{}^* : tf_r}$

$$\tau_{r_{32}} ::= \text{i32}(n) \mid \text{ptr}(n) \mid \text{num}(n) \mid \text{f32}(n)$$
$$\tau_r ::= \tau_{r_{32}} \mid \text{i64}(n) \mid \text{f64}(n)$$
$$\tau ::= \text{i32} \mid \text{ptr} \mid \text{num} \mid \text{i64} \mid \text{f32} \mid \text{f64}$$
$$\Psi ::= \{(\text{ptr}(n) \mapsto \tau_{r_{32}})^*\}$$
$$tf_r ::= \tau_r^*, \Psi \to \tau_r^*, \Psi'$$

$$\frac{}{R; C^r \vdash \tau.\mathbf{const}\ c : \tau(c)}\ \textsc{Constant} \qquad \frac{\tau_r^3 = \text{binop}(\tau_r^1, \tau_r^2) \qquad \tau_r^3 <: \tau(\top)}{R; C^r \vdash \tau.binop : \tau_r^1 \tau_r^2 \to \tau_r^3}\ \textsc{Binary Ops}$$

$$\frac{tf_r = \tau_r^m, \Psi_{\text{pre}} \to \tau_r^n, \Psi_{\text{post}} \qquad R_\Psi = \Psi_{\text{pre}} \qquad R; C^r, \text{label}(\tau_r^n, \Psi_{\text{post}}) \vdash e^* : \tau_r^m, \Psi_{\text{pre}} \to (\tau_r')^n, \Psi'_{\text{post}} \qquad (\tau_r' <: \tau_r)^n \qquad \Psi'_{\text{post}} <: \Psi_{\text{post}}}{R; C^r \vdash \mathbf{block}\ tf_r\ e^*\ \mathbf{end} : tf_r}\ \textsc{Block}$$

$$\frac{C^r_{\text{label}}(i) = (\tau_r')^m, \Psi' \qquad (\tau_r <: \tau_r')^m \qquad R_\Psi <: \Psi'}{R; C^r \vdash \mathbf{br}\ i : \tau_r^*\ \tau_r^m \to \tau_r^*}\ \textsc{Break}$$

$$\frac{C^r_{\text{func}}(i) = (\tau_r')^m, \Psi'_{\text{pre}} \to (\tau_r')^n, \Psi'_{\text{post}} \qquad tf_r = \tau_r^m, \Psi_{\text{pre}} \to \tau_r^n, \Psi_{\text{post}} \qquad R_\Psi = \Psi_{\text{pre}} \qquad (\tau_r <: \tau_r')^m \qquad (\tau_r' <: \tau_r)^n \qquad \Psi_{\text{pre}} <: \Psi'_{\text{pre}} \qquad \Psi'_{\text{post}} <: \Psi_{\text{post}}}{R; C^r \vdash \mathbf{call}\ i\ tf_r : \tau_r^m, \Psi_{\text{pre}} \to \tau_r^n}\ \textsc{Call}$$

$$\frac{C^r_{\text{memory}} = n \qquad \tau_r = \text{load\_and\_extend}(c + o, \tau, (tp\_sx)^?, R_\Psi) \qquad 2^a \le (|tp| <)^? |\tau_r| \qquad (tp\_sz)^? = \epsilon \vee \tau_r = \mathbf{im}}{\Psi; C^r \vdash \tau.\mathbf{load}(tp\_sx)^?\ a\ o : \text{ptr}(c) \to \tau_r}\ \textsc{Load}$$

Fig. 4. Typing of a subset of WebAssembly Instructions in the Refinement Type System.

pointers, but allow addition of every other combination of i32, ptr and num. Moreover, addition of a ptr and a num yields a ptr.

- *Subtraction*: We do not allow subtraction of ptr from a num, but we do allow all other i32, ptr and num combinations for the subtraction operation. Subtracting a ptr from a ptr yields a num (an offset).

3.2.2 Control Constructs and Breaks. The Block rule is annotated with a refined function type tf_f which specifies the stack shape and memory typing expected before the block instruction and at the end of the block instruction. Since the end of a block is the meet of several breaks out of the block, we ensure that the shape of the stack and memory after type checking the instructions in the block should be subtypes of the types expected on the top of the stack and of the expected memory typing. This is necessary since it is valid in WebAssembly for one control path to the end of the block to return a ptr and another to return a i32, since they would both be integers in the WebAssembly type system. br instructions are typed similarly.

3.2.3 Loads from Linear Memory. While in WebAssembly, i32s are used to index into memory, our refinement type system requires that only values of ptr type be used to index into memory. We modify the typing rule for the Load instruction to expect a $\text{ptr}(n)$ on the stack, as shown in Figure 4. The load instruction is optionally annotated with a packed type and size, tp_sx, which is used to pack and sign extend the data stored in memory. Addtionally, our memory typing Ψ only stores types of 32-bit size, $\tau_{r_{32}}$. If the load instruction expects a type of size 64 bits, we have to load the sequence of bytes from address $n + o$, where o is the offset provided to the load instruction, and address $n + o + 4$ as a 64-bit value as directed by the τ annotation on the load instruction which

specifies the type of data we want to load. All this is done by the load_and_extend function, in Appendix A.

3.2.4 Function Calls. Typing a call instruction is more complex than in WebAssembly since our refinement type system has subtyping and since it keeps track of the expected memory typing at various points in the program. We change the WebAssembly **call** i instruction to **call** i tf_r where tf_r specifies the stack shape and memory typing expected before and after the call. When typing the call instruction, we need to ensure that the stack shape and memory typing before the call are subtypes of the stack shape and memory typing expected by the function being called, and that the stack shape and memory typing when the function returns are subtypes of the stack shape and memory typing expected after the call by the callee.

3.3 Discovering Refinement Types using Type and Memory Constraints

In this section, we explain how we discover refinement types for WebAssembly code. WebAssembly's type validation algorithm, described in the specification [42], performs a single forward pass over the instructions in a function body to validate that WebAssembly function. Unfortunately, a forward-only analysis isn't possible when we wish to discover refinement types. Instead, we first generate constraints over a WebAssembly function in order to later discover refinement types for each function, instruction, and local and global variables via constraint solving. Accumulating constraints is necessary because it is often not immediately obvious if an i32 value is a pointer or a number. For example, consider the WebAssembly function in Figure 6. At the start of the function it is not immediately obvious if either parameter is a pointer. Line 8 has a memory load operation from the addition of the two parameter, so we can infer that one of the parameters must be a pointer —a careful reader will remember that we do not allow the addition of two pointers in our type system. However, we do not know which one is a pointer. It is only at line 17, where the first parameter is multiplied by 42, that we know that the first parameter is a pointer! However, we would now like to reflect this type back up to the function parameter and check that every instruction that uses this parameter does so is in accordance with its type. Hence, a forward analysis by itself would be insufficient since it would only propagate type information forwards, when we need the analysis to also propagate types backwards.

Another point of interest is that pointers in WebAssembly don't often have a concrete memory address that can be statically known. Our refinement type system expects the pointer type to have a concrete address a as its value — written $\text{ptr}(a)$ — and for memory typing Ψ to be a mapping from concrete addresses a to values. Note that if the address is unknown, we would have to resort to $\top$ as the address value, which reduces precision. Thus, when discovering refinement types, we introduce *symbolic* pointer types $\text{ptr}(l, n)$ that have a symbolic base address l and a symbolic offset n. With symbolic pointers in hand, we define a grammar for symbolic refinement types $\hat{\tau}_r$, given in Figure 5. We also introduce symbolic memory typing to be a map from symbolic pointers $\text{ptr}(l, n)$ to symbolic refinement types $\hat{\tau}_r$. We generate constraints over WebAssembly functions and solve them to get *symbolic* refinement types $\hat{\tau}_r$ for every stack slot in the value stack, and local and global variables, and *symbolic* memory typing Σ. Our call-graph analysis is over the symbolic refinement type system. We will later use the symbolic refinement types we discover for WebAssembly code e^* to guide the construction and typing of a concrete-refinement-typed WebAssembly program $e{*}_r$ for which we prove type safety in Section 3.5.

The symbolic refinement types and memory typing, as well as the constraints that are generated over WebAssembly instructions can be found in Figure 5. We associate every stack slot in the value stack and local and global variables, with a unique abstract refinement type variable α and every memory state with a unique abstract memory variable ς. We generate constraints over WAFFLE [7],

Refinement Type System

Symbolic Number	n		
Symbolic Location	l		
Abstract Refinement Type	α		
Symbolic Refinement Type	$\hat{\tau}_r$	::=	$\mathsf{i32}(n) \mid \mathsf{ptr}(l, n) \mid \mathsf{num}(n) \mid \mathsf{i64}(n) \mid \mathsf{f32}(n) \mid \mathsf{f64}(n)$
Type Tag	τ	::=	$\mathsf{i32} \mid \mathsf{ptr} \mid \mathsf{num} \mid \mathsf{i64} \mid \mathsf{f32} \mid \mathsf{f64}$
Original Value Types	τ_w	::=	$\mathsf{i32} \mid \mathsf{i64} \mid \mathsf{f32} \mid \mathsf{f64}$
Symbolic Function Type	tf_α	::=	$\alpha^* \rightarrow \alpha^*$
Symbolic Global Type	tg_α	::=	$\mathsf{mut}^?\ \alpha^*$
Abstract Memory Typing	ς		
Symbolic Memory Typing	Σ	::=	$\cdot \mid \Sigma, (l, n) \mapsto \hat{\tau}_r$

WebAssembly Syntax

Packed Type	tp	::=	$\mathsf{i8} \mid \mathsf{i16} \mid \mathsf{i32}$
Global Type	tg_w	::=	$\mathsf{mut}^?\tau_w$
Function Type	tf_w	::=	$\tau_w^* \rightarrow \tau_w^*$
Function	f	::=	ex^* **func** tf_w **local** $\tau_w{}^*\ e^*$ | ex^* **func** $tf_w\ im$
Global	$glob$	::=	ex^* **global** $tg_w\ e^*$ | ex^* **global** $tg_w\ im$
Table	tab	::=	ex^* **table** $n\ i^*$ | ex^* **table** $n\ import$
Memory	mem	::=	ex^* **memory** n | ex^* **memory** $n\ im$
Import	im	::=	**import** "name" "name"
Export	ex	::=	**export** "name"
Module	m	::=	**module** $f^*\ glob^*\ tab^*\ mem^?$
Instructions	e	::=	**unreachable** | **nop** | **drop** | **select** | **block** $tf_w\ e^*$ **end** | **loop** $tf_w\ e^*$ **end** | **if** $tf_w\ e^*$ **else** e^* **end** | **br** i | **br_if** i | **br_table** i^+ | **return** | **call** | **call_indirect** tf_w | **get_local** i | **set_local** i | **tee_local** i | **get_global** i | **set_global** i | τ_w.**load** $(tp\_sx)^?$ **a o** | τ_w.**store** $tp^?$ **a o** | **current_memory** | **grow_memory** | τ_w.**const** c | $\tau_w.unop_{\tau_w}$ | $\tau_w.binop_{\tau_w}$ | $\tau_w.testop_{\tau_w}$ | $\tau_w.relop_{\tau_w}$ | $\tau_w.cvtop\ \tau_w\_sx^?$
	$unop_{iN}$	::=	**clz** | **ctz** | **popcnt**
	$unop_{fN}$	::=	**neg** | **abs** | **ceil** | **floor** | **trunc** | **nearest** | **sqrt**
	$binop_{iN}$	::=	**add** | **sub** | **mul** | **div**_sx | **rem**_sx | **and** | **or** | **xor** | **shl** | **shr**_sx | **rotl** | **rtor**
	$binop_{fN}$	::=	**add** | **sub** | **mul** | **div** | **min** | **max** | **copysign**
	$testop_{iN}$	::=	**eqz**
	$relop_{iN}$	::=	**eq** | **ne** | **lt**_sx | **gt**_sx | **le**_sx | **ge**_sx
	$relop_{fN}$	::=	**eq** | **ne** | **lt** | **gt** | **le** | **ge**
	$cvtop$	::=	**convert** | **reinterpret**
	sx	::=	**s** | **u**

Constraint System

Operations	op	::=	$unop_{iN} \mid unop_{fN} \mid cvtop \mid binop_{iN} \mid binop_{fN} \mid testop_{iN} \mid relop_{iN} \mid relop_{fN}$
Type Constraint	t	::=	$\alpha \doteq \hat{\tau}_r \mid \alpha <: \tau \mid \alpha \doteq \alpha \mid l \neq l \mid \alpha \doteq \bigsqcup \alpha^+ \mid \alpha \doteq \varsigma[\alpha]_{(tp\_sx^?,\ a,\ o)} \mid \alpha \doteq op((\alpha^?)^+)$
Memory Constraint	m	::=	$\varsigma \doteq \Sigma \mid \varsigma \doteq \varsigma \mid \varsigma \doteq \bigsqcup \varsigma^+ \mid \varsigma \doteq \varsigma[\alpha \mapsto \alpha]_{(tp^?,\ a,\ o)}$
Constraint Set	S	::=	$\cdot \mid S, t \mid S, m$

Fig. 5. Syntax for Symbolic Refinement Types and Constraints Generated Over WebAssembly.

	WebAssembly Function	*Value Stack*	*Generated Constraints*
1	`(func $foo`		
2	`(param i32 i32)`		$\alpha_0 <: \text{i32} \wedge \alpha_1 <: \text{i32} \wedge \varsigma \doteq []$
3	`(result i32)`		$\alpha_5 <: \text{i32}$
4	`(local i32 i32 i32)`		$\alpha_2 <: \text{i32} \wedge \alpha_3 <: \text{i32} \wedge \alpha_4 <: \text{i32}$
5	`local.get 0`	$[\alpha_0]$	
6	`local.get 1`	$[\alpha_0\alpha_1]$	
7	`i32.add`	$[\alpha_6]$	$\alpha_6 \doteq \text{i32.add}(\alpha_0, \alpha_1, \_) \wedge \alpha_0 \doteq \text{i32.add}(\_, \alpha_1, \alpha_6) \wedge \alpha_1 \doteq \text{i32.add}(\alpha_0, \_, \alpha_6)$
8	`i32.load`	$[\alpha_7]$	$\alpha_6 <: \text{ptr} \wedge \alpha_7 <: \text{i32} \wedge \alpha_7 \doteq \varsigma[\alpha_6]$
9	`local.set 2`	$[]$	$\alpha_2 \doteq \alpha_7$
10	`local.get 1`	$[\alpha_1]$	
11	`i32.const 4`	$[\alpha_1\alpha_8]$	$\alpha_8 \doteq \text{i32}(4)$
12	`i32.add`	$[\alpha_9]$	$\alpha_9 \doteq \text{i32.add}(\alpha_1, \alpha_8, \_) \wedge \alpha_1 \doteq \text{i32.add}(\_, \alpha_8, \alpha_9) \wedge \alpha_8 \doteq \text{i32.add}(\alpha_1, \_, \alpha_9)$
13	`i32.load`	$[\alpha_{10}]$	$\alpha_9 <: \text{ptr} \wedge \alpha_{10} <: \text{i32} \wedge \alpha_{10} \doteq \varsigma[\alpha_9]$
14	`local.set 3`	$[]$	$\alpha_3 \doteq \alpha_{10}$
15	`local.get 0`	$[\alpha_0]$	
16	`i32.const 42`	$[\alpha_0\alpha_{11}]$	$\alpha_{11} \doteq \text{i32}(42)$
17	`i32.mul`	$[\alpha_{12}]$	$\alpha_{12} \doteq \text{i32.mul}(\alpha_0,\alpha_{11},\_) \wedge \alpha_0 \doteq \text{i32.mul}(\_,\alpha_{11},\alpha_{12}) \wedge \alpha_{11} \doteq \text{i32.mul}(\alpha_0,\_,\alpha_{12})$
18	`local.set 4`	$[]$	$\alpha_4 \doteq \alpha_{12}$
19	`local.get 2`	$[\alpha_2]$	
20	`i32.eqz`	$[\alpha_{13}]$	$\alpha_{13} \doteq \text{i32.eqz}(\alpha_2)$
21	`(if (param) (result i32)`		
22	`(then`		
23	`local.get 3)`	$[\alpha_3]$	
24	`(else`		
25	`local.get 4)`	$[\alpha_4]$	$\alpha_{14} \doteq \sqcup\alpha_3\alpha_4$
26	`))`	$[\alpha_{14}]$	$\alpha_{14} \doteq \alpha_5$

High-Level Pseudocode for WebAssembly:

```
int foo (x, y) {
  a = *(x + y);
  b = *(y + 4);
  c = x * 42;
  if a == 0 { b }
  else { c }
}
```

Fig. 6. WebAssembly function with generated constraints.

an SSA IR over WebAssembly. For each instruction, we generate *type constraints* over αs and *memory constraints* over ςs. The following type constraints can be generated:

(1) $\alpha \doteq \hat{\tau}_r$, equates the type of an α to some $\hat{\tau}_r$,
(2) $\alpha <: \tau$, constrains the type of α with a subtyping relation,
(3) $\alpha \doteq \alpha'$, equates two αs,
(4) $l \neq l$, says that two symbolic locations are not equal and so do not alias,
(5) $\alpha \doteq \bigsqcup(\alpha')^+$, the type of an α is the join over several α's,
(6) $\alpha \doteq \varsigma[\alpha']_{(tp\_sx^?,\ a,\ o)}$, the type of a α is read from some memory state described by ς, at some pointer described by α', with an optional sign-extended static width tp, an alignment a and offset o.
(7) $\alpha \doteq op((\alpha^?)^+)$, performs some arithmetic operation op over several α's.

Similarly, the memory constraints that can be generated are as follows:

(1) $\varsigma \doteq \Sigma$, equates an abstract ς with a symbolic memory typing Σ,
(2) $\varsigma \doteq \varsigma'$, equates two ςs,
(3) $\varsigma \doteq \bigsqcup(\varsigma')^+$, performs a join over several ςs.
(4) $\varsigma \doteq \varsigma'[\alpha \mapsto \alpha']_{(tp^?,\ a,\ o)}$, copies over the memory state described by ς' and performs a memory update on it, with an optional static width tp, an alignment a and offset o.

We show the generated constraints for an example WebAssembly function in Figure 6. A subset of the constraint-generation rules for WebAssembly instructions are described in Figure 7, with all

$$C^{\alpha} ::= \{\text{func } tf_w{}^*,\ \text{local } \alpha^*,\ \text{global } tg_\alpha{}^*,\ \text{table } n^?,\ \text{memory } n^?,\ \text{label } (\alpha^*, \varsigma)^*,\ \text{return } (\alpha^*, \varsigma)^?\}$$

Constraint Generation for Instructions $\boxed{S; \varsigma; C^{\alpha} \vdash e : \alpha^* \to \alpha^*; S'; \varsigma'}$

$$\frac{\alpha \text{ fresh} \qquad S' = S :: [\alpha \doteq \tau_w(c)]}{S; \varsigma; C^{\alpha} \vdash \tau_w.\mathbf{const}\ c : \epsilon \to \alpha; S'; \varsigma}\textsc{Constant}$$

$$\frac{\alpha_1\ \alpha_2 \in dom(S) \qquad \alpha_3 \text{ fresh} \qquad S' = S :: [\alpha_1 \doteq binop(\_, \alpha_2, \alpha_3) \wedge \alpha_2 \doteq binop(\alpha_1, \_, \alpha_3) \wedge \alpha_3 \doteq binop(\alpha_1, \alpha_2, \_)]}{S; \varsigma; C^{\alpha} \vdash \tau_w.binop : \alpha_1\ \alpha_2 \to \alpha_3; S'; \varsigma}\textsc{Binary Ops}$$

$$\frac{tf_w = \tau_w^n \to \tau_w^m \qquad \alpha^n \in dom(S) \qquad \alpha^m, \varsigma' \text{ fresh} \qquad S; \varsigma; C, \text{label}(\alpha^m, \varsigma') \vdash e^* : \alpha^n \to (\alpha')^m; S'; \varsigma'' \qquad S'' = S' :: [(\alpha \doteq \alpha')^m \wedge \varsigma' \doteq \varsigma'']}{S; \varsigma; C^{\alpha} \vdash \mathbf{block}\ tf_w\ e^*\ \mathbf{end} : \alpha^n \to \alpha^m; S''; \varsigma'}\textsc{Block}$$

$$\frac{\alpha^*\alpha^n \in dom(S) \qquad C^{\alpha}_{\text{label}}(i) = \alpha'_n, \varsigma' \qquad S' = S :: [\varsigma \doteq \sqcup\varsigma\varsigma' \wedge (\alpha \doteq \sqcup\alpha\alpha')^n]}{S; \varsigma; C^{\alpha} \vdash \mathbf{br}\ i : \alpha^*\alpha^n \to \alpha^*; S'; \varsigma}\textsc{Break}$$

$$\frac{C^{\alpha}_{\text{func}} = \tau_w^m \to \tau_w^n \qquad \alpha^n \in dom(S) \qquad S' = S :: [(\alpha <: \tau_w)^m \wedge (\alpha' <: \tau_w)^n]}{S; \varsigma; C^{\alpha} \vdash \mathbf{call} : \alpha^m \to (\alpha')^n; S'; \varsigma}\textsc{Call}$$

$$\frac{\alpha_1 \in dom(S) \qquad S' = S :: [\alpha_1 <: \text{ptr} \wedge \alpha_2 \doteq \varsigma[\alpha_1]_{(tp\_sx^?,\ a,\ o)} \wedge \alpha_2 <: \tau_w]}{S; \varsigma; C^{\alpha} \vdash \tau_w.\mathbf{load}(tp\_sx)^?\ a\ o : \alpha_1 \to \alpha_2; S'; \varsigma}\textsc{Load}$$

Fig. 7. Constraint-Generation Rules for a Subset of WebAssembly Instructions.

the rules in Appendix D. We discuss the interesting constraint-generation cases below which will help to understand the constraints generated in Figure 6.

3.3.1 Arithmetic Instructions. Let us consider the example of i32.add, which expects two values on the stack, of types α_1 and α_2 and produces a single value of type α_3, i.e., $\alpha_3 = \alpha_1 + \alpha_2$. We know that the only disallowed case for this operation is the addition of two pointers. This has interesting implications for constraint solving, since if $\alpha_1 = \text{ptr}(l, n_1)$ and $\alpha_2 = \text{i32}(n_2)$, α_2 can be refined to be $\text{num}(n_2)$, since the alternative is not permitted. In fact, the types of each argument and result can affect each other. If $\alpha_1 = \text{num}(n_1)$, $\alpha_2 = \text{i32}(n_2)$ and $\alpha_3 = \text{ptr}(l, n_3)$, α_2 is refined to be ptr. Meanwhile, if $\alpha_3 = \text{num}(n_3)$, α_2 is refined to num. We leave discovering the appropriate refinements to constraint solving and at the time of constraint generation, generate constraints to tie each α to each other α, with the relevant arithmetic instruction. For i32.add, we generate the constraints, $\alpha_1 \doteq \text{i32.add}(\_, \alpha_2, \alpha_3)$, $\alpha_2 \doteq \text{i32.add}(\alpha_1, \_, \alpha_3)$ and $\alpha_3 \doteq \text{i32.add}(\alpha_1, \alpha_2, \_)$. The underscore in the constraint holds the place of the α current being refined. A similar strategy is followed for all arithmetic instructions.

3.3.2 Memory Instructions. The i64.store instruction expects a pointer type α_{ptr} on the stack and some data type α_{data}. α_{ptr} is constrained to be a pointer by generating a subtyping constraint $\alpha_{\text{ptr}} <: \text{ptr}$, while the data is constrained to be a subtype of the type expected by the store instruction, $\alpha_{\text{data}} <: \text{i64}$. The abstract memory typing represented by ς, an input to the rule, is updated to record a mapping from $\alpha_{\text{ptr}} \mapsto \alpha_{\text{data}}$, and is copied into a new ς', which is returned by the rule. The constraint on the abstract memory typing is $\varsigma' = \varsigma[\alpha_{\text{ptr}} \mapsto \alpha_{\text{data}}]$. The i64.load instruction is similarly constrained to expect a pointer on the stack and a subtype of i64 as the result. Here, the load constraint is represented as $\alpha_{\text{data}} \doteq \varsigma[\alpha_{\text{ptr}}]$. Similar constraints are generated for all τ_w.load and τ_w.store instructions.

3.3.3 Blocks, Loops. Let us consider the sequence of instructions, block $tf_w\ e^*$ end. Let the block expect n values on the stack and return m values. m α_1's are freshly generated and added to the context with a label. This is done so that break instructions out of blocks know the number of values required on the output stack of the target block. We generate constraints for the body of the block, with this new context, which gives us m α_2s. The m α_1 and α_2s are now equated with equality constraints, and m α_1s are returned by the instruction on the result stack. A similar scenario arises for loop instructions. For loops, the n input αs are added to the context instead of the m result αs that are added to the context for blocks. This is because break instructions in loops restart the loop and have to know the number of values required on the input stack of the loop.

3.3.4 Break Instructions. br instructions are used to break out of blocks and restart loops. In the case of blocks, the values on the stack at the br instruction are returned by the target block. Since our analysis is flow-insensitive, we join all possible result stacks for a block. The context carries the expected value stack at the target block, so we generate join constraints for the αs on the stack and the αs expected by the target block. In the case of loops, the values on the stack at the br instruction are used to restart the loop. Constraint generation is the same regardless of whetherthe target of a br instruction is a loop or a block. Note that generating join constraints in this fashion embeds the looping structure of the program into the constraints. For example, consider the following sequence of instructions: loop $tf_w\ e^*$ end. If $tf_w = \mathsf{i32} \rightarrow ()$, we would generate constraints for the loop inputs as $\alpha_0 <: \mathsf{i32}$. We would then generate constraints for the body of the loop with $\mathsf{label}(\alpha_0)$ added to the context. If we came across the instruction br 0, in the loop body, with α_1 on the stack, we would generate the constraint $\alpha_0 \doteq \bigsqcup \alpha_0\alpha_1$.

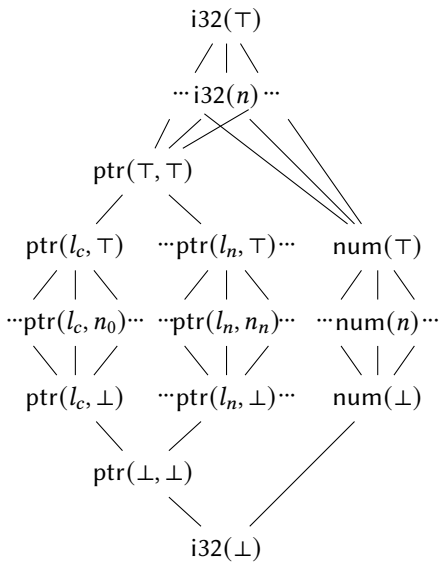
3.4 The Sub-Lattice for Integers

We define the integer sub-lattice in Figure 8. Here, we only present a few cases of the meet and join operations over the lattice in the figure. The entire definition can be found in Appendix C.

3.4.1 Join of two pointers. The canonical example for joins in static analysis is the if-then-else expression. Let us suppose that the then branch of such an expression returns $\mathsf{ptr}(l_1, n_1)$, and the else branch returns $\mathsf{ptr}(l_2, n_2)$. What pointer does the if-then-else expression return? Since it could be either of the two, we do not presume to know what the pointer could be and instead return $\mathsf{ptr}(\top, \top)$. Instead, if the then branch returned $\mathsf{ptr}(l, n_1)$ and the else branch returned $\mathsf{ptr}(l, n_2)$, we say that the if-then-else expression returns $\mathsf{ptr}(l, n_1 \sqcap n_2)$.

3.4.2 Meet of two pointers. The canonical example for meet operations in static analysis is values during several loop iterations. Say, at loop iteration n, the type of a stack slot is $\mathsf{ptr}(l_1, n_1)$, a pointer with a symbolic address, and at loop iteration $n+1$, its type is $\mathsf{ptr}(l_c, n_0)$, a constant pointer. Since it is impossible for a stack slot to have a symbolic and constant pointer, we return $\mathsf{ptr}(\bot, \bot)$. On the other hand, say that at loop iteration n, the type of a stack slot is $\mathsf{ptr}(l_1, n_1)$ and at loop iteration $n+1$, its type is $\mathsf{ptr}(l_2, n_2)$. This means that the type at this stack slot is both $\mathsf{ptr}(l_1, n_1)$ and $\mathsf{ptr}(l_2, n_2)$. We equate these two pointers to a third pointer $\mathsf{ptr}(l_3, 0)$, where l_3 is a fresh symbolic location and $\mathsf{ptr}(l_1, n_1) = \mathsf{ptr}(l_3, 0) \wedge \mathsf{ptr}(l_2, n_2) = \mathsf{ptr}(l_3, 0)$. Pointers can be written as polynomials (since the offset is usually added into the base address) and we get that $l_1 + n_1 = l_3$ or that, $l_1 = l_3 - n_1$ and $l_2 = l_3 - n_2$. We say that the type of this stack slot is $\mathsf{ptr}(l_3, 0)$.

3.4.3 Join and Meet of a pointer and a number. The join of a pointer with a number results in an $\mathsf{i32}(\top)$, except in the case of constant pointers $\mathsf{ptr}(l_c, n_0)$. For a constant pointer, we know that the base address l_c equates to 0 and so the operation $\mathsf{ptr}(l_c, n_0) \sqcup \mathsf{num}(n_1)$, produces $\mathsf{i32}(n_0 \sqcup n_1)$ as its result. The meet of a pointer and number unequivocally results in a $\mathsf{i32}(\bot)$, as per our relation definition shown in Figure 8.



$$
\begin{array}{lcll}
\mathsf{i32}(n_1) & \sqcup & \mathsf{i32}(n_1) & = \mathsf{i32}(n_1 \sqcup n_2) \\
\mathsf{i32}(n_1) & \sqcup & \mathsf{ptr}(l_c, n_0) & = \mathsf{i32}(n_1 \sqcup n_2) \\
\mathsf{i32}(n_1) & \sqcup & \mathsf{ptr}(l_2, n_2) & = \mathsf{i32}(\top) \\
\mathsf{i32}(n_1) & \sqcup & \mathsf{num}(n_2) & = \mathsf{i32}(n_1 \sqcup n_2) \\
\mathsf{ptr}(l_1, n_1) & \sqcup & \mathsf{ptr}(l_1, n_2) & = \mathsf{ptr}(l_1, n_1 \sqcup n_2) \\
\mathsf{ptr}(l_1, n_1) & \sqcup & \mathsf{ptr}(l_2, n_2) & = \mathsf{ptr}(\top, \top) \\
\mathsf{num}(n_1) & \sqcup & \mathsf{num}(n_2) & = \mathsf{num}(n_1 \sqcup n_2) \\
\mathsf{num}(n_1) & \sqcup & \mathsf{ptr}(l_c, n_0) & = \mathsf{i32}(n_1 \sqcup n_0) \\
\mathsf{num}(n_1) & \sqcup & \mathsf{ptr}(l_2, n_2) & = \mathsf{i32}(\top) \\
 & & & \\
\mathsf{i32}(n_1) & \sqcap & \mathsf{i32}(n_1) & = \mathsf{i32}(n_1 \sqcap n_2) \\
\mathsf{i32}(n_1) & \sqcap & \mathsf{ptr}(l_c, n_0) & = \mathsf{ptr}(l_c, n_1 \sqcap n_0) \\
\mathsf{i32}(n_1) & \sqcap & \mathsf{ptr}(l_2, n_2) & = \mathsf{ptr}(l_c, n_1) \sqcap \mathsf{ptr}(l_2, n_2) \\
\mathsf{i32}(n_1) & \sqcap & \mathsf{num}(n_2) & = \mathsf{num}(n_1) \sqcup \mathsf{num}(n_2) \\
\mathsf{ptr}(l_1, n_1) & \sqcap & \mathsf{ptr}(l_c, n_0) & = \mathsf{ptr}(\bot, \bot) \\
\mathsf{ptr}(l_1, n_1) & \sqcap & \mathsf{ptr}(l_1, n_2) & = \mathsf{ptr}(l_1, n_1 \sqcap n_2) \\
\mathsf{ptr}(l_1, n_1) & \sqcap & \mathsf{ptr}(l_2, n_2) & = \mathsf{ptr}(l_3, 0) \\
\multicolumn{4}{l}{\text{where } l_3 \text{ fresh} \wedge l_1 = l_3 - n_1 \wedge l_2 = l_3 - n_2} \\
\mathsf{num}(n_1) & \sqcap & \mathsf{num}(n_2) & = \mathsf{num}(n_1 \sqcap n_2) \\
\mathsf{num}(n_1) & \sqcap & \mathsf{ptr}(l_c, n_0) & = \mathsf{i32}(\bot) \\
\mathsf{num}(n_1) & \sqcap & \mathsf{ptr}(l_1, n_1) & = \mathsf{i32}(\bot)
\end{array}
$$

Fig. 8. The i32 Sub-Lattice With a Subset of Meet and Join Operations Defined.

3.4.4 Join and meet of a i32 *with a pointer.* If the then branch of a if-then-else expression returned $\mathsf{i32}(n_1)$ and the else branch returned a constant pointer $\mathsf{ptr}(l_c, n_0)$, the if-then-else expression would return $\mathsf{i32}(n_0 \sqcup n_1)$. If the else branch returned a pointer with a symbolic address $\mathsf{ptr}(l_2, n_2)$ instead, the if-then-else expression would return $\mathsf{i32}(\top)$, since the base address l_1 is unknown. For the meet operation between i32's and ptr's, let us imagine that the type of a stack slot at loop iteration n is $\mathsf{i32}(n_1)$ and that at loop iteration $n+1$, the type of the stack slot is a constant pointer $\mathsf{ptr}(l_c, n_0)$. The type of the stack slot is then both these types and so, $\mathsf{ptr}(l_c, n_0 \sqcap n_1)$. If instead, at loop iteration $n+1$, the type of the stack slot is a symbolic pointer $\mathsf{ptr}(l_2, n_2)$, the type of the stack slot would be $\mathsf{ptr}(l_c, n_1) \sqcap \mathsf{ptr}(l_2, n_2)$, which results in $\mathsf{ptr}(\bot, \bot)$.

3.4.5 Join and meet of a i32 *with a number.* If the then branch of a if-then-else expression returned a $\mathsf{i32}(n_0)$ and the else branch returned $\mathsf{num}(n_1)$, the if-then-else expression would return $\mathsf{i32}(n_0 \sqcup n_1)$. For the meet operation between i32's and num's, let us imagine that the type of a stack slot at loop iteration n is $\mathsf{i32}(n_0)$ and that at loop iteration $n+1$, the type of the stack slot is a constant pointer $\mathsf{num}(n_1)$. The type of the stack slot is then both these types and so, $\mathsf{num}(n_0 \sqcap n_1)$.

Note that we never explicitly join or meet locations separate from their offsets. When the meet operation is performed over two pointers with the same symbolic base address l, the result is a pointer with that base address l and the join of their offsets. When the meet operation is performed on two pointers with different symbolic locations, we produce a pointer with a $\top$ location and $\top$ offset. This is analogous to $\mathsf{ptr}(\top)$, since we never construct $\mathsf{ptr}(\top, n)$. Similarly, for join operations, we never construct $\mathsf{ptr}(\bot, n)$ and $\mathsf{ptr}(\bot, \bot)$ is analogous to $\mathsf{ptr}(\bot)$. However, in both cases, we do construct $\mathsf{ptr}(l, \top)$ and $\mathsf{ptr}(l, \bot)$.

3.5 Refining WebAssembly and Type Safety for the Refinement Type System

In this section, we show two results. First, we explain how, given a WebAssembly program, potentially in a JavaScript context, we can discover symbolic refinement types and use them to arrive at a

concrete-refinement-typed WebAssembly program. Second, we prove type safety for our refinement type system using Progress and Preservation.

From WebAssembly to Refinement-Typed WebAssembly. We've seen how to generate constraints over a given WebAssembly function to discover *symbolic* refinement types rt. However, our typing rules are over concrete refinement types τ_r and to show that a WebAssembly program is refinement-typed we need to use the symbolic refinement types and symbolic memory typing we have discovered to guide the annotation of the WebAssembly program with refinement types so that type-checks using the refinement type system. In order to obtain the concrete refinement types for a module, we first generate constraints for a given function. During constraint generation, we map each expression at every point of the program to the αs and ςs it generates on the stack. We elide this in Figure 7 for conciseness. Section 5 describes how we solve constraints over αs and ςs to get a least solution ρ that maps αs to symbolic refinement types $\hat{\tau}_r$ and ςs to symbolic memory typing Σ. However, these refinement types are still symbolic. After constraint solving, we posit that there exists a mapping η from symbolic addresses to concrete addresses for every Σ, where, for a Σ_{pre} before and Σ_{post} after an instruction, $\eta_{\text{post}} \supseteq \eta_{\text{pre}}$. Hence, for every α and ς, we recover a τ_r and Ψ. Since Ψ is only typed with 32-bit types, we split each 64-bit sized type $\texttt{i64}(n)$ or $\texttt{f64}(n)$, into two i32 values from the high and low 32 bits of n. We now make several transformations to the WebAssembly function, in order to type check it:

- Instructions tagged with a WebAssembly type annotation τ_w are transformed to have a refined type tag τ. E.g. i32.const 42 $\rightsquigarrow$ num.const 42, where the latter expects a num on the stack rather than a i32.
- Instructions annotated with WebAssembly function type annotations tf_w are instead annotated with concrete refinement function types tf_r that include Ψ_{pre} and Ψ_{post} memory typing before and after the instruction.
- Function types are also transformed to have concrete refinement function types tf_r.
- The call instruction is typed to have a tf_r annotation (shown in Figure 4) to denote the shape of the stack before and after the call.

Type Safety for the Refinement Type System. The operational semantics of this transformed WebAssembly program ${e_r}^*$ remains largely unchanged from the original WebAssembly operational semantics, but there are a few changes. For example, for the binary operation rule, the type annotation on the $\tau.binop$ instruction no longer tells us the types expected on the WebAssembly stack. Instead of, $(\tau_w.\text{const } c_1)(\tau_w.\text{const } c_2)\ \tau_w.binop \hookrightarrow \tau_w.\text{const}(\text{binop } (c_1, c_2))$, in the refinement type system, the operational semantics of binop steps as follows, $(\tau_1.\text{const } c_1)\ (\tau_2.\text{const } c_2)\ \tau_3.binop \hookrightarrow binop\ ((\tau_1.\text{const } c_1), (\tau_2.\text{const } c_2))$.

We now prove type safety as the standard progress and preservation theorems.

THEOREM 1. ***Progress**: If* $R; C^r \vdash_i {e_r}^* : \tau_r^1 \cdots \tau_r^m \rightarrow \tau_r^* \wedge (v : \tau_r)^m \wedge \vdash_i s : R$ *then* ${e_r}^* = v^*$ *or* ${e_r}^* = \text{trap}$ *or* $s; {v_r}^*; {e_r}^* \hookrightarrow s'; {v'}^*; {e_r}'^*$

THEOREM 2. ***Preservation**: If* $R; C^r \vdash_i {e_r}^* : \tau_r^1 \cdots \tau_r^m \rightarrow \tau_r^* \wedge \vdash_i (v : \tau_r)^m \wedge \vdash_i s : R \wedge s, v^*, {e_r}^* \hookrightarrow s', v'^*, {e_r}'^*$ *then* $\exists\ R'$ *such that* $dom(R_\Psi) \subseteq dom(R'_\Psi) \wedge \vdash_i s' : R' \wedge R'; C^r \vdash_i {e_r}'^* : \tau_r^*$

Proofs of the above theorems are very similar to the WebAssembly progress and preservation proofs except for minor changes to account for the refined types. The main interesting aspect of our refinement type system is that, unlike WebAssembly, we also type the linear memory, keeping track of the memory typing Ψ in the context R. We present the proof of progress and preservation for the load instruction in Appendix B.

4 GENERATING CONSTRAINTS OVER THE JAVASCRIPT WRAPPER

We've seen how to recover refinement types over WebAssembly, but we can also type the parameters passed into WebAssembly exported function calls and the results of these calls to have refinement types. We can analyze the glue code before and after a call to an exported WebAssembly function to type exported functions. Specifically, we want to type arguments as pointers when a JavaScript object is *lowered* to WebAssembly by fetching a pointer from malloc and writing the object's contents to memory at that pointer. We also want to be able to type the data being put into the WebAssembly memory. Similarly, we want to type results as pointers when a WebAssembly value is *lifted* to JavaScript by reading data out of WebAssembly memory, into a JavaScript object. We also want to type the data that is expected to be at that WebAssembly memory location.

To this end, we write a CodeQL analysis to infer refinement types for exported WebAssembly function calls. CodeQL [1] is a declarative semantic code analysis framework maintained by GitHub, in which analysis writers can write datalog-like queries for languages like JavaScript, Python, etc. It is particularly helpful in analyzing code to check for specific patterns. The CodeQL analysis consists of two parts: an analysis to identify calls to the exported `malloc` function and accesses to WebAssembly memory, and a type inference engine that infers a type for a given JavaScript expression. The former is a series of rules that check for a sequence of patterns through data-flow and the latter is a type-inference engine. We rely on type annotations provided by the wrapper file, annotated TypeScript library functions and typing through dataflow expressions to infer a type. For example, to infer the type for a if-then-else expression, we type the then and else branches. We also infer the type of an expression, by trying to infer the type of its parent or, more generally, its predecessor (an expression whose data flows into the current expression) or successor (an expression into which the current expression's data flows).

The inferred types for each argument and result of exported WebAssembly functions are passed into WasmBridge which generates constraints from them, as shown in Figure 2. Generating constraints from the types is fairly straightforward. If an argument or result is typed to be a pointer with that memory block containing a i64, we generate the subtype constraints $\alpha_1 \doteq \mathsf{ptr}(l_1, 0)$, $\alpha_2 <: \mathsf{i64}$ and the memory constraint $\varsigma_1 \doteq \varsigma_0[\alpha_1 \mapsto \alpha_2]$. Note that since we do not analyze the JavaScript client, we do not have any data to associate with these types. If a function has two arguments that have a pointer type and we assume that a call to malloc gives us fresh, disjoint labels, we can generate a constraint that encodes that the two pointers' symbolic base addresses are not equal to each other. For example, if func \$foo (α_1, α_2) where $\alpha_1 \doteq \mathsf{ptr}(l_1, 0)$ and $\alpha_2 \doteq \mathsf{ptr}(l_2, 0)$, we generate the constraint $l_1 \neq l_2$ since our analysis over the JavaScript wrapper tells us that α_1 and α_2 came from two separate malloc calls. We solve these constraints along with the constraints generated over the associated WebAssembly binary to obtain refinement types for each WebAssembly function, stack slot and local variable.

5 CONSTRAINT SOLVING

We solve a set of constraints generated over a WebAssembly function and its associated JavaScript glue code using Algorithm 1. At the end of constraint generation, we have a set S of constraints over a function body. We create a mapping *Constraint*, that maps each α in S to the type constraints on α and similarly, maps each ς in S to the memory constraints on ς. We resolve the equality constraints for α's and ς's when creating this map. We also record dependencies between α's and ς's using the influence vector *infl*. This is used to recompute the constraints of a certain α or ς when its dependencies have changed.

We pass the *Constraint* and *infl* maps as inputs to the algorithm and compute a least solution for every α and ς in S. The constraints on α's are solved to get a refinement type, τ_r, and the constraints

Algorithm 1 Algorithm for solving constraints described in Figure 5

Require: $Constraint = [\alpha \mapsto t^+; \varsigma \mapsto m^+]$, $infl = [\alpha \mapsto \{\alpha^*, \varsigma^*\}; \varsigma \mapsto \{\alpha^*, \varsigma^*\}]$
Ensure: The least Solution $\rho = [\alpha \mapsto \hat{\tau}_r; \varsigma \mapsto \Sigma]$

$Worklist \leftarrow dom(Constraint)$
for all $\alpha \in dom(Constraint)$ **do**
  if $\exists\ \alpha \doteq \hat{\tau}_r \in Constraint[\alpha]$ **then** $\rho[\alpha] \doteq \hat{\tau}_r$ **else** $\rho[\alpha] = \top$
for all $\varsigma \in dom(Constraint)$ **do** $\rho[\varsigma] = \cdot$
while $Worklist \neq \emptyset$ **do**
  $v := Worklist.\text{pop}()$
  $Old_v := \rho[v]$
  if $v = \alpha$ **then** $\text{EVAL}_\alpha(Constraint, \rho, \alpha)$
  if $v = \varsigma$ **then** $\text{EVAL}_\varsigma(Constraint, \rho, \varsigma)$
  if $Old_v \neq \rho[v]$ **then** $Worklist.\text{append}(infl[v])$
procedure $\text{EVAL}_\alpha(Constraint, \rho, \alpha)$
  $\hat{\tau}_r^\alpha := \rho[\alpha]$
  for all $t \in Constraint[\alpha]$ **do**
    if $t = \alpha <: \tau$ **then** $\hat{\tau}_r^\alpha := \hat{\tau}_r^\alpha \sqcap \tau(\top)$
    if $t = \alpha \doteq op(\alpha'_1, \ldots, \alpha'_n)$ **then** $\hat{\tau}_r^\alpha := \hat{\tau}_r^\alpha \sqcap eval_{op}(\rho[\alpha'_1], \ldots, \rho[\alpha'_n])$
    if $t = \alpha \doteq \bigsqcup \alpha'_1 \ldots \alpha'_m$ **then** $\hat{\tau}_r^\alpha := \hat{\tau}_r^\alpha \sqcap (\rho[\alpha'_1] \sqcup \ldots \sqcup \rho[\alpha'_m])$
    if $t = \alpha \doteq \varsigma[\alpha']$ **then** $\Sigma := \rho[\varsigma] \wedge \hat{\tau}_r^{\text{ptr}} \doteq \rho[\alpha']$
      if $\hat{\tau}_r^{\text{ptr}} = \text{ptr}(l, n) \wedge (l, n) \mapsto \hat{\tau}_r \in \Sigma$ **then** $\hat{\tau}_r^{(l,n)} := \hat{\tau}_r$ **else** $\hat{\tau}_r^{(l,n)} := \top$
      if $\exists\ \text{ptr}(\top, \top) \mapsto \hat{\tau}_r \in \Sigma$ **then** $\hat{\tau}_r^{(\top,\top)} := \hat{\tau}_r$ **else** $\hat{\tau}_r^{(\top,\top)} := \bot$
      if $\exists\ \text{ptr}(l, \top) \mapsto \hat{\tau}_r \in \Sigma$ **then** $\hat{\tau}_r^{(l,\top)} := \hat{\tau}_r$ **else** $\hat{\tau}_r^{(l,\top)} := \bot$
      if $\exists\ \text{ptr}(l', n') \mapsto \hat{\tau}_r \in \Sigma$ **then**
        if $\nexists\ l \neq l' \in Constraint[\alpha']$ **then** $\hat{\tau}_r^{(l',n')} := \hat{\tau}_r$
        else if $\exists\ l \neq l' \in Constraint[\alpha'] \wedge n' = \top$ **then** $\hat{\tau}_r^{(l',n')} := \hat{\tau}_r$
        else $\hat{\tau}_r^{(l',n')} := \bot$
      $\hat{\tau}_r^\alpha := \hat{\tau}_r^\alpha \sqcap \hat{\tau}_r^{(l,n)} \sqcup \hat{\tau}_r^{(\top,\top)} \sqcup \hat{\tau}_r^{(l,\top)} \sqcup \hat{\tau}_r^{(l',n')}$
  $\rho[\alpha] \doteq \hat{\tau}_r^\alpha$
procedure $\text{EVAL}_\varsigma(Constraint, \rho, \varsigma)$
  $\Sigma \doteq \rho[\varsigma]$
  for all $m \in Constraint[\varsigma]$ **do**
    if $m = \varsigma \doteq \Sigma'$ **then** $\Sigma := \Sigma' \sqcap \Sigma$
    if $m = \varsigma \doteq \bigsqcup \varsigma'_1 \ldots \varsigma'_m$ **then** $\Sigma := (\rho[\varsigma'_1] \sqcup \ldots \sqcup \rho[\varsigma'_m]) \sqcap \Sigma$
    if $m = \varsigma \doteq \varsigma'[\alpha_1 \mapsto \alpha_2]$ **then** $\Sigma' := \rho[\varsigma'] \wedge \hat{\tau}_r^{\text{ptr}} := \rho[\alpha_1] \wedge \hat{\tau}_r^{\text{data}} := \rho[\alpha_2]$
      if $\hat{\tau}_r^{\text{ptr}} = \text{ptr}(l, n)$ **then** $\Sigma := \Sigma \sqcap \Sigma'[(l, n) \mapsto \hat{\tau}_r^{\text{data}}]$ **else** $\Sigma := \Sigma \sqcap \Sigma'$
      if $\exists\ \text{ptr}(\top, \top) \mapsto \hat{\tau}'_r \in \Sigma$ **then** $\Sigma := \Sigma[(\top, \top) \mapsto \hat{\tau}'_r \sqcup \hat{\tau}_r^{\text{data}}]$
      if $\exists\ \text{ptr}(l, \top) \mapsto \hat{\tau}'_r \in \Sigma$ **then** $\Sigma := \Sigma[(l, \top) \mapsto \hat{\tau}'_r \sqcup \hat{\tau}_r^{\text{data}}]$
      if $\exists\ \text{ptr}(l', n') \mapsto \hat{\tau}'_r \in \Sigma$ **then**
        if $\nexists\ l \neq l' \in Constraint[\alpha']$ **then** $\Sigma := \Sigma[(l', o') \mapsto \hat{\tau}'_r \sqcup \hat{\tau}_r^{\text{data}}]$
        else if $\exists\ l \neq l' \in Constraint[\alpha'] \wedge n' = \top$ **then** $\Sigma := \Sigma[(l', o') \mapsto \hat{\tau}'_r \sqcup \hat{\tau}_r^{\text{data}}]$
  $\rho[\varsigma] \doteq \Sigma$

Application	Size (KB)	Functions	Exported Functions	Imported Functions	Direct Calls	Indirect Calls	Functions in Table
webcam	28.61	125	6	33	270	79	58
leaflet-rs	20.16	110	8	45	228	39	39
wasm-rsa	369.00	785	25	5	6058	107	237
blake3	34.41	81	14	1	206	26	21
epqueue	23.26	123	17	9	471	25	29
serde	13.61	44	0	1	67	25	31
snowpack-ts	35.11	107	4	37	236	55	35
warp	162.09	283	10	14	1856	112	141
wasm-pack	13.78	43	1	6	69	19	17
source-map	48.53	68	10	1	395	1	18

Table 1. Description of Subject Applications

on ς's are solved to get a memory state, Σ. We obtain the least solution using a standard worklist algorithm. If there exists a $\alpha \doteq \tau_r$ in the constraints for α, we initialize the solution for the α to be τ_r. Otherwise, we initialize the solution to be the $\top$ type. We initialize the solution of all ς's to be an empty memory state. We initialize the worklist with all α's and ς's in the domain of the *Constraint* map, and continue solving till the worklist is empty. If the solution of an α or ς is found to be different from its solution in the previous iteration, we add all α's and ς's that depend on it into the worklist. We briefly discuss solving type and memory constraints below.

5.0.1 Solving Type Constraints. To solve the type constraints for a given α we iterate over each type constraint and do a meet operation between the currently computed type for the α and the type computed with a specific constraint. The solution of most type constraints is standard: we simply evaluate the arithmetic operations or perform joins as dictated by the join constraints. However, the memory load constraint is more complicated in the face of possible aliasing between pointers. Let us suppose that we are trying to load from a memory state, Σ, at $\text{ptr}(l, n)$. If Σ contains $\text{ptr}(\top, \top)$, $\text{ptr}(l, \top)$ or $\text{ptr}(l', n')$ where there does not exist a $l \neq l'$ constraint, we make the conservative assumption that all these pointers might alias with $\text{ptr}(l, n)$ and join the data pointed to by these potential aliased pointers with τ_r where $\Sigma[(l, n) \mapsto \tau_r]$.

5.0.2 Solving Memory Constraints. Solving the memory constraints for a given ς is similar to solving type constraints. Here too, when evaluating a memory store constraint $\Sigma[\text{ptr}(l, n) \mapsto \tau_r]$, we have to be conservative in the face of aliasing and join the data at a potential alias site, i.e., $\text{ptr}(\top, \top)$, $\text{ptr}(l, \top)$ or $\text{ptr}(l', n')$, with the type currently being stored in the memory state, τ_r.

6 EVALUATION ON REAL-WORLD SUBJECT APPLICATIONS

We evaluate a call graph analysis over the refinement type system on ten real-world WebAssembly/-JavaScript applications that have been compiled from Rust. Our approach only supports generating constraints over WebAssembly 1.0, the core WebAssembly language introduced in 2017, and only supports an analysis of JavaScript wrappers generated by the Rust compiler. We discuss extending our analysis to include other compilers and extensions to WebAssembly in Section 8. We pick subject applications from two datasets [29] [40] and identify 29 NPM packages that have been compiled from Rust. Of these, we find 10 libraries which satisfy our criteria of only using WebAssembly 1.0 features. All the packages are widely used—wasm-rsa implements RSA cryptographic primitives; epqueue is a priority queue data structure; blake3 is a high-performance cryptographic hash libary; source-map is a library for parsing and generating JavaScript source maps; warp is a popular Rust web server framework; webcam provides WebAssembly bindings for webcam streaming;

Applications	WebAssembly			Refined Type System						WebAssembly + JavaScript					
	$\|E\|$	$\|U\|$	$\|M\|$	$\|E\|$	$\|U\|$	$\|M\|$	**% $\|E\|$ Red.**	**% Call Sites w/ Red.**	**Avg Red.**	$\|E\|$	$\|U\|$	$\|M\|$	**% $\|E\|$ Red.**	**% Call Sites w/ Red.**	**Avg Red.**
webcam	526	7	4	499	7	5	**9.28**	**50.00**	**16.64**	499	7	5	**9.28**	50.00	16.64
leaflet-rs	264	9	3	229	11	6	**17.78**	**62.86**	**36.74**	229	11	6	**17.78**	62.86	36.74
wasm-rsa	3896	8	2	3696	9	2	**6.15**	**35.35**	**12.41**	3696	9	2	**6.15**	35.35	12.41
blake3	202	5	13	195	16	14	**5.03**	**19.23**	**13.00**	195	16	14	**5.03**	19.23	13.00
epqueue	287	14	3	284	14	5	**1.51**	**15.00**	**9.50**	284	14	5	**1.51**	15.00	9.50
serde	96	43	3	90	43	5	**12.50**	**45.00**	**23.12**	90	43	5	**12.50**	45.00	23.12
snowpack-ts	352	45	4	338	45	21	**7.37**	**51.92**	**30.88**	338	45	21	**7.37**	51.92	30.88
warp	1832	9	3	1707	11	3	**8.73**	**34.86**	**13.04**	1707	11	3	**8.73**	34.86	13.04
wasm-pack	88	35	9	83	35	9	**11.11**	**42.11**	**14.04**	83	35	9	**11.11**	42.11	14.04
source-map	126	7	0	126	7	0	**0.00**	**0.00**	**0.00**	126	7	0	**0.00**	0.00	0.00
Averages	766.9	18.2	4.4	724.7	19.8	7	7.946	35.633	16.937	724.7	19.8	7	7.946	35.633	16.937

Table 2. Comparing callgraphs generated over the original WebAssembly type system, the refinement type system, and the combined WebAssembly and JavaScript analysis. $|E|$ denotes the number of edges in the callgraph, $|M|$ denotes the number of monomorphic calls, and $|U|$ denotes the number of unreachable nodes. Avg Red. is average reduction at call sites.

leaflet-rs is wrapper for a JavaScript library to be used in Rust; serde enables serialization between WebAssembly and JavaScript; snowpack-ts is a template for TypeScript and Rust WebAssembly projects; and wasm-pack refers to wasm-pack-plugin, a webpack plugin for Rust. Table 1 shows the characteristics of the subject applications, including the size of the binary and the number of direct and indirect call sites in the binary. We run WasmBridge over the chosen subject applications in order to evaluate the precision of the refinement types, the analysis over JavaScript and how WasmBridge compares to other state-of-the-art call graph analysis tools.

RQ1: How much more precise is the call graph over the refinement type system when compared to the original WebAssembly type system?

In order to determine the increase in precision afforded by the refinement type system, we compare the same type-based call graph analysis on the refinement type system and the original WebAssembly type system. A type-based call graph analysis restricts the set of targets at an indirect call sites with the type annotation at the call_indirect instruction as described in Section 2.2. The results of this evaluation are shown in the first two rows of Table 2. The call graph over the refinement type system is more precise than the call graph over the original WebAssembly type system:

(1) *Edge Reduction*: We reduce the total number of call graph edges by up to 17.50% (leaflet-rs) compared to the baseline WebAssembly analysis. This reduction occurs at indirect call sites where precision matters most; we observe a reduction at 62.9% of indirect call sites.
(2) *Call Site Level Improvements*: The precision gains are more apparent at indirect call sites. For all indirect call sites, we achieve an average edge reduction ranging from 9.5% to 36.74% per call site and an average reduction of 16.9%.
(3) *Monomorphic Call site Discovery*: Our refinement type system enables us to identify several additional monomorphic call sites—indirect calls that resolve to exactly one target. In snowpack-ts, we discover 17 additional monomorphic call sites (from 4 to 21 call sites); leaflet-rs has 3 additional monomorphic call sites (from 3 to 6); webcam-stream has 1 additional monomorphic call sites (from 4 to 5); epqueue has 2 additional monomorphic call sites (from 3 to 5); serde-bindgen has 2 additional monomorphic call sites (from 3 to 5) and blake3 shows 1 additional monomorphic call site (from 13 to 14). These discoveries enable subsequent compiler optimizations and improve the precision of downstream analyses.

Applications	WebAssembly Type System	**Refined Type System**	Wasm+JS Analysis			
	Time (s)	**Time (s)**	CodeQL DB Creation	CodeQL Query Execution	Constraint Solving	**Total Time**
webcam	0.005	**0.053**	4.80	8.33	0.06	**13.20**
leaflet-rs	0.005	**0.024**	5.36	7.81	0.03	**13.19**
wasm-rsa	0.032	**1.330**	5.14	7.78	1.44	**14.36**
blake3	0.005	**0.045**	4.93	7.68	0.05	**12.66**
epqueue	0.007	**0.042**	4.29	7.82	0.08	**12.18**
serde	0.004	**0.027**	4.82	8.49	0.03	**13.34**
snowpack-ts	0.007	**0.052**	5.58	7.38	0.06	**13.01**
warp	0.023	**1.184**	4.54	7.97	1.32	**13.83**
wasm-pack	0.004	**0.025**	4.46	6.90	0.03	**11.39**
source-map	0.010	**0.189**	5.04	7.28	0.20	**12.52**

Table 3. Runtime performance comparison (in seconds) for analysis over the original WebAssembly type system, the refinement type system, and the combined Wasm+JS analysis for each subject application.

(4) *Unreachable Function Detection*: The refinement type information also improves our ability to identify unreachable functions. Blake3 shows the most dramatic improvement, with unreachable nodes increasing from 5 to 16—indicating that there are functions in the WebAssembly binary that are never called in practice.

Notably, source-map shows no improvement in our analysis because it is a highly optimized binary with only one indirect call. This result is expected in the case of call graph analysis—applications with simple control flow gain little from the refinement type analysis. However, the richer types might improve precision in other analyses.

RQ2: How much more precise is the call graph over WebAssembly and JavaScript when compared to WebAssembly alone?

We compare call graphs generated over the refinement type system with constraints generated over WebAssembly alone versus constraints generated over WebAssembly and its associated JavaScript wrapper in the second and third row of Table 2. We find no change in the precision at indirect call sites and no reduction in the number of edges. This is not too surprising since the refinements to the type system are not discovered through the JavaScript wrapper alone. WebAssembly instructions also provide these refinements, such as the load instruction which expects a ptr on the stack rather than just an i32. The one constraint that is unique to the JavaScript wrapper is one that specifies the disjointedness of symbolic base addresses of pointers that have come from different calls to malloc ($l_1 \neq l_2$). It seems as though the imprecision of pointer analysis leads to this constraint not improving precision at call sites.

Does this mean that the CodeQL analysis over the JavaScript wrapper is unnecessary? Perhaps for the current simple lightweight analysis. However, this analysis framework can be extended in future work to have richer typing of the WebAssembly memory. For example, in the case of an UInt32Array, the constraints that are generated for the glue code that lowers this UInt32Array array into WebAssembly memory only specifies that a pointer is passed into a WebAssembly exported function and that the data in memory is i32. Future work could add UInt32Array and other rich JavaScript types to the typing of WebAssembly memory and extend the CodeQL analysis to include the JavaScript client to specify the actual data placed into WebAssembly memory. We discuss future work in Section 8 and the possible impact of including an analysis of the client in RQ5. Note that regardless of a CodeQL analysis, the interoperation of the JavaScript wrapper informs the design of the refinement types and so is essential to the analysis.

Applications	WasmBridge			Wasm-Opt				Wasma				Wassail			
	$\|N\|$	$\|E\|$	$\|U\|$	$\|N\|$	$\|E\|$	$\|U\|$	**Edge Diff**	$\|N\|$	$\|E\|$	$\|U\|$	**Edge Diff**	$\|N\|$	$\|E\|$	$\|U\|$	**Edge Diff**
webcam	125	499	7	125	1903	0	**73.78%**	125	526	7	**5.13%**	125	526	7	**5.13%**
leaflet-rs	110	229	11	110	845	0	**72.90%**	110	264	9	**13.26%**	110	264	9	**13.26%**
wasm-rsa	785	3696	9	785	9501	0	**61.10%**	785	3896	8	**5.13%**	785	3896	8	**5.13%**
blake3	81	195	16	81	269	0	**27.51%**	78	202	2	**3.47%**	81	202	5	**3.47%**
epqueue	123	284	14	123	602	0	**52.82%**	119	287	10	**1.05%**	123	287	14	**1.05%**
serde	44	90	43	44	405	43	**77.78%**	-	-	-	-	44	96	43	**6.25%**
snowpack-ts	107	338	45	107	846	0	**60.05%**	107	352	45	**3.98%**	107	352	45	**3.98%**
warp	283	1707	11	283	5537	0	**69.15%**	280	1832	6	**6.82%**	283	1832	9	**6.82%**
wasm-pack	43	83	35	43	153	35	**45.75%**	43	88	35	**5.68%**	43	88	35	**5.68%**
source-map	68	126	7	68	133	0	**5.26%**	62	126	1	**0.00%**	68	126	7	**0.00%**
Average	**176.90**	**724.70**	**19.80**	176.90	2019.40	7.80	54.61%	189.89	841.44	13.67	4.94%	176.90	766.90	18.2	5.08%

Table 4. Call Graph Comparison across different WebAssembly analysis tools. Edge difference shows the % change in callgraph edges compared to our WasmBridge analysis. Negative values indicate the tool produces fewer edges than WasmBridge. $|N|$ denotes the number of nodes (functions) in the callgraph, $|E|$ denotes the number of edges (calls), and $|U|$ denotes the number of unreachable nodes in the callgraph.

RQ3: What is the cost of the analysis over WebAssembly and JavaScript?

Table 3 shows the runtime overhead of each component of our multi-language analysis and how it compares to the analysis over constraints generated from WebAssembly alone versus the call graph analysis over the original WebAssembly type system. The latter is our baseline. The call graph analysis over the refinement type system over WebAssembly alone shows a maximum of a 1.33 second increase over the baseline. The multi-language analysis, however, is dominated by the time it takes to create a CodeQL database and run the analysis (or query) over the database. Even so, the total time of the multi-language analysis does not exceed 14.36s in the worst case and constraint generation and solving is relatively inexpensive.

RQ4: How does our approach compare to other state-of-the-art call graph analysis tools?

We compare our approach with other state-of-the-art call graph analysis tools, Wasm-Opt[43], Wassail[39] and Wasma[4]. Several tools were excluded from our evaluation: Wassilly [32] timed out on each subject application while generating a call graph, Sturdy [22] does not generate a call graph but rather a control flow graph, and Wasmati [5] does not support WebAssembly functions that return more than one result (multi-value proposal). Table 4 compares the different analysis tools and we discuss their call graph analyses below.

- *Wasm-Opt* performs a naive call graph analysis. At every indirect call site, it assumes that all the functions in the function table may be called. This results in a call graph with up to 3.8× more edges than our approach (e.g., 1,903 edges vs. 504 for webcam-stream). While this is the most sound analysis possible, the resulting call graph is very imprecise.
- *Wasma* performs a closed-world type-based call graph analysis. At every indirect call site, it considers all the functions in the table with the same function type as the type annotation at the call site to be a potential target of this call site. As expected from the results of RQ1 and 2, we see a reduction in the number of edges compared to Wasma. Note that Wasma removes functions that do not have any incoming or outgoing edges, which leads to a reduction in the number of nodes for subjects like epqueue. Our analysis does not remove these functions. Additionally, Wasma fails to run on one subject.

Metric		blake3			wasm-rsa	
		Baseline	Client 1	Client 2	Baseline	Client 1
Binary Before DCE	Functions	81	81	81	785	785
	Edges	269	269	269	9501	9501
Wasm-Metadce	Functions	81	63	65	785	605
	Edges	269	221	228	9501	8840
	Reduction%	**0.0**	**22.22**	**19.75**	**0.00**	**22.93**
WasmBridge	Functions	65	47	49	776	596
	Edges	151	103	110	3652	2991
	Reduction%	**19.75**	**41.98**	**39.51**	**1.15**	**24.08**

Table 5. Comparison of Dead Code Elimination performed by Wasm-Metadce and WasmBridge. Reduction% shows the reduction in the number of functions after dead-code elimination.

- *Wassail* performs a closed-world type-based call graph analysis like Wasma but like us, does not remove functions that do not have any incoming or outgoing edges.

RQ5: What is the effect of the increased precision of our approach on a downstream analysis like dead-code elimination?

To demonstrate the potential impact of our analysis technique on downstream analyses, we evaluate how improved call graph precision affects dead-code elimination. For two subject applications, blake3 and wasm-rsa, the dataset we obtain them from has mock JavaScript clients for these subjects. The JavaScript clients call a subset of exported functions from the JavaScript wrapper of the WebAssembly binary. Through manual inspection, we determine the set of WebAssembly exported functions that are called by the mock client. Ideally this would be done by an analysis of the client that also specifies the values being passed to WebAssembly. We then pass this information to Wasm-Metadce, an industry state-of-the-art WebAssembly dead-code elimination tool, which, given a WebAssembly binary and a list of exported functions that are called by a client, performs dead-code elimination and produces a smaller binary that has been specialized to that client. We also use the list of exported functions called by the client to perform dead-code elimination based on our call graph analysis. We then compare the number of functions that have been removed by Wasm-Metadce versus WasmBridge in Table 5. As a baseline, we pass in all the exported functions to both tools, to measure dead code elimination for a client that uses all exported functions.

- *Function Reduction*: WasmBridge identifies more functions to be dead than Wasm-Metadce, across both subjects and their clients. For blake3, we achieve function reductions of 19.75%, 41.98%, and 39.51% compared to metadce's 0%, 22.22%, and 19.75% respectively. The reduction difference is not as high in wasm-rsa, but we still outperform Wasm-Metadce.
- *Edge Reduction*: WasmBridge continues to produce more precise call graphs with fewer edges, which can enable further optimizations and analyses.
- *Baseline*: Even in the baseline case, where a mock client uses all exported functions of a binary, our analysis identifies dead code that Wasm-Metadce misses entirely (19.75% reduction vs. 0% for blake3).

7 RELATED WORK

Inferring Richer Types for Binary Analysis. Lehmann et al. [28] present SnowWhite, a learning-based approach that recovers high-level function types for WebAssembly functions using DWARF information produced at compile time. Zhao et al. [45] present WasmHint, which infers 'Rust-like' function types for WebAssembly functions using semantic learning in combination

with program slicing. We do not compare against these approaches since they infer much more complicated types like structs and arrays and don't provide an estimation of the soundness of their type systems or how the inferred types affect program analysis. Both systems infer a ptr type. In fact, the former finds that pointers are the most commonly inferred type and the latter method has trouble distinguishing pointers from strings.

In general, inferring higher-level types for binaries is a popular research direction. The use of deep learning to infer types has risen in popularity in the past few years. Priyanga et al. [33] study the rise of Machine Learning in binary code analysis and find that it is used extensively to identify the start and end of functions and statements like conditionals. The disassembly of x86 binaries is a much harder task than for WebAssembly binaries. Zheng Leong et al. [8] present Eklavya, a RNN-based engine that recovers function types from x86 code. However, there is still recent work that attempts to reconstruct higher-level type information for binaries using traditional static analysis techniques. Lee et al. [25] recover high-level types for x86 binaries by inferring types based on how code operates on data in the binary, similar to our inference algorithm over WebAssembly. ElWazeer et al. [12] use a "best-effort" flow and context-insensitive pointer analysis to recover type and variable information from x86 binaries.

Multi-Language Static Analysis. We summarize different approaches to multi-language analysis in Section 1 and now explain the specifics of each work. Lee et al. [26] analyze JNI interoperation between Java and C by generating semantic summaries for C functions called by Java code. These summaries are translated into Java and calls to C functions are replaced with their Java summaries. Liang et al. [30] use LLVM IR as a common intermediate from multiple languages and detect bugs using symbolic execution coupled with Z3. Fornaia et al. [14] translates C, C++ and Java code into LLVM. Roth[36] reuses existing single-language analyses and combines them to support multiple languages by developing architecture that can be used to combine analyses. Monat et al. [31] implement an extensive C-Python analysis framework that relies on abstract interpretation and multi-language transfer functions to convert C values to Python and vice-versa. Buro et al. [6] provide a general, theoretical framework combining abstract interpretations of different languages. Furr and Foster [17] also perform multilingual type inference for C code using client OCaml and Java types for checking the type-safety of FFI calls between C and OCaml and C and Java. This work is the most similar to our approach, but we show how multilingual type inference can also be extended to low-level bytecode to improve analysis precision.

WebAssembly Program Analysis. There are several static analysis frameworks for WebAssembly. We compare against Wassail[38], Wasma[4], Wasm-Opt[43] and Wasm-Metadce[43]. The latter two are part of the Binaryen toolchain. Wassilly [32] and Sturdy [22] improve the precision of indirect calls by relying on a better value analysis to determine the index into the function table. They are both abstract interpretation frameworks. We do not compare against them because the former times out on all our subjects and the latter does not provide us with a call graph. Wasmati [5] uses code property graphs to statically detect vulnerabilities in WebAssembly binaries and use a type-based call graph analysis. Wasabi [27] is a dynamic analysis framework for WebAssembly.

Refinement Type Systems. Refinement types have long been used to enhance type systems with logical predicates that constrain the set of values described by a type [15, 16, 34, 41, 44], allowing programmers to express more precise program properties and enabling program verification. Xi and Pfenning [44] use indexed types to capture invariants such as array bounds and memory layout, enabling verification of low-level code. Liquid Types [34, 41] use refinement types with predicate inference, enabling automatic verification of properties such as null safety, ordering and sortedness,

and various data-structure invariants. RefinedC [15] brings refinement types to C, using Coq to verify memory safety, control-flow integrity, and pointer aliasing in C programs.

The most closely related work is WasmPrecheck [18], a richer type system for WebAssembly that uses *indexed types* to express static constraints that enable safe removal of dynamic checks for type and memory safety. WasmPrecheck supports general constraints on index terms, while we support only singleton refinements and refining i32 to ptr(a) or num(n). Another point of comparison is that both WasmPrecheck and our work shows how to embed a Wasm module into a refined type system. However, Geller et al. give only a naive embedding — they show that a Wasm module can be (automatically) embedded into WashPrecheck by replacing all type annotations in the Wasm module with indexed types that have no constraints. While this requires no additional effort, it does not enhance the performance of the program. Meanwhile, we develop a novel binary analysis that considers information from interoperation with JavaScript and show how to generate and solve constraints to discover refinement types that in turn lead to improved static analysis.

8 FUTURE WORK

Supporting WebAssembly 2.0. Since 2017, WebAssembly has had one version update with the introduction of vector instructions to support 128-bit wide SIMD functionality; bulk memory instructions to support copying and initializing regions of memory; multi-value results to support returning more than one value by functions, blocks and instructions; reference types to support opaque first-class references to functions or pointers; non-trapping conversions to allow seamless conversions from float to integer types; and sign-extension instructions that allow directly extending the width of signed integer values. We already support multi-value results and do not anticipate trouble with extending our refinement type system to include these new instructions.

JavaScript Wrapper Analysis. Currently, our analysis only supports a CodeQL analysis of JavaScript wrappers generated by `wasm-bindgen`. However, this is not the only option to support an FFI between WebAssembly and JavaScript. C/C++ library developers typically use Emscripten to compile code to WebAssembly and JavaScript. Emscripten provides several interoperation mechanisms between the two languages [13]:

(1) `ccall` or `cwrap` to call C functions from JavaScript with types other than structs.
(2) `WebIDL` to call compiled C++ classes. `WebIDL` does not support standalone functions and only supports a subset of C++ types.
(3) `Embind` to call compiled C++ classes. `Embind` supports a greater subset of C++ classes.

We expect to be able to extend our analysis to include wrapper files generated by these methods. For example, the glue code generated by `ccall` and `cwrap` does the same lowering and lifting done by the glue code generated by `wasm-bindgen`.

Improvements to Precision. Our analysis discovers that refinement types are flow-insensitive, context-insensitive and intra-procedural. Additionally, the domain of values is fairly simple —a flat lattice of natural numbers. Considering the relative simplicity of our analysis and that we outperform state-of-the-art tools, we plan to employ several traditional techniques to increase precision of the analysis. For example, we plan to change the domain of values to be the power-set lattice over natural numbers. We also plan to make our analysis flow and context sensitive. We also plan to extend our CodeQL analysis to the JavaScript client to determine two key pieces of information: the values being passed to WebAssembly or being stored into the WebAssembly linear memory and the subset of exported functions being used by the client. The former would increase the precision of the analysis and latter would be useful for a dead-code elimination analysis that specializes a WebAssembly binary to a JavaScript client.

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Appendices

A INSTRUCTION TYPING FOR THE REFINEMENT TYPE SYSTEM

$$C^r ::= \{\text{func } tf_r^*,\ \text{local } \tau_r^*,\ \text{global } \tau_r^*,\ \text{table } n^?,\ \text{memory } n^?,\ \text{label } (\tau_r^*, \Psi)^*,\ \text{return } (\tau_r^*, \Psi)^?\}$$

Typing WebAssembly Instructions with Refinement Types $\boxed{R; C^r \vdash e_r^* : tf_r}$

$$\frac{R = \{\text{inst } C^{r*}, \text{tab } n^*, \text{mem } \Psi^*\} \qquad (R \vdash inst : C^r)^* \qquad (n \le |cl|^*)^* \qquad \vdash s.\text{mem} : R_\Psi}{\vdash \{\text{inst } inst^*, \text{tab } (cl^*)^*, \text{mem } (b^*)^*\} : R}$$

$$\frac{\forall a \in 0...\text{size}(s.\text{mem}) \qquad s.\text{mem}(a) = R_\Psi(a)}{\vdash s.\text{mem} : R_\Psi}$$

$$\frac{\begin{array}{c}\varsigma \text{ fresh} \qquad \varsigma; \cdot; C^\alpha \vdash ex^* \ \textbf{func}\ tf_w\ \textbf{local}\ \tau_w^\ell\ e^* : \alpha^m \to \alpha^n; S'; \varsigma' \\ \rho = \text{solve}(S') \\ \tau_r^m = \rho[\alpha^m] \qquad \tau_r^n = \rho[\alpha^n] \qquad \Sigma_{pre} = \rho[C] \qquad \Sigma_{post} = \rho[C'] \\ \exists \Psi_{\text{pre}}, \Psi_{\text{post}}.\ \exists \eta_{\text{pre}}, \eta_{\text{post}}. \\ \text{Bij}(\eta_{\text{pre}}) \wedge \text{Bij}(\eta_{\text{post}}) \wedge \eta_{\text{pre}}(\Sigma_{\text{pre}}) = \Psi_{\text{pre}} \wedge \eta_{\text{post}}(\Sigma_{\text{post}}) = \Psi_{\text{post}} \wedge \eta_{\text{post}} \supseteq \eta_{\text{pre}} \\ tf_r = \tau_r^m, \Psi_{\text{pre}} \to \tau_r^n, \Psi_{\text{post}}\end{array}}{C^\alpha \vdash ex^* \ \textbf{func}\ tf_w\ \textbf{local}\ \tau_w^\ell\ e^* : ex^*\ tf_r; \rho}$$

$$\frac{\begin{array}{c}\alpha_w^g \text{ fresh} \\ C^\alpha = \{\textbf{func}\ tf_w^*, \textbf{global}\ \alpha_w^g, \textbf{table}\ n^?, \textbf{memory}\ n^?\} \\ (C^\alpha \vdash ex^* \ \textbf{func}\ tf_w\ \textbf{local}\ \tau_w^l\ e^* : ex^*\ tf_r; \rho)^* \\ C^r = \{\textbf{func}\ tf_r^*, \textbf{global}\ \tau_r^g, \textbf{table}\ n^?, \textbf{memory}\ n^?\} \\ (\textbf{module}\ f^*\ \textbf{global}\ \tau_w^g\ \textbf{tab}^?\ \textbf{mem}^?) \overset{\rho}{\rightsquigarrow} (\textbf{module}\ f^*\ \textbf{global}\ \tau_r^g\ \textbf{tab}^?\ \textbf{mem}^?) \\ (R; C^r \vdash f : ex^*\ tf_r)^*\end{array}}{R \vdash \textbf{module}\ f^*\ \textbf{global}\ \tau_w^g\ \textbf{tab}^?\ \textbf{mem}^?}\ \textsc{Module}$$

$$\frac{\begin{array}{c}tf_r = \tau_r^m, \Psi_{\text{pre}} \to \tau_r^n, \Psi_{\text{post}} \\ R, C^r, \textbf{local}(\tau_r^m, \tau_r^\ell),\ \textbf{label}(\tau_r^n, \Psi_{\text{post}}),\ \textbf{return}(\tau_r^n, \Psi_{\text{post}}) \vdash e^* : \epsilon \to \tau_r^n\end{array}}{R; C^r \vdash ex^* \textbf{func}\ tf_r\ \textbf{local}\ \tau_r^\ell\ e^* : ex^*\ tf_r}\ \textsc{Function}$$

$$\frac{\begin{array}{c}tf_r = \tau_r^m, \Psi_{\text{pre}} \to \tau_r^n, \Psi_{\text{post}} \qquad R_\Psi = \Psi_{\text{pre}} \\ R; C^r, \text{label}(\tau_r^n, \Psi_{\text{post}}) \vdash e^* : \tau_r^m, \Psi_{\text{pre}} \to (\tau_r')^n, \Psi'_{\text{post}} \\ (\tau_r' <: \tau_r)^n \qquad \Psi'_{\text{post}} <: \Psi_{\text{post}}\end{array}}{R; C^r \vdash \textbf{block}\ tf_r\ e^*\ \textbf{end} : tf_r}\ \textsc{Block}$$

$$\frac{\begin{array}{c}tf_r = \tau_r^m, \Psi_{\text{pre}} \to \tau_r^n, \Psi_{\text{post}} \qquad R_\Psi = \Psi_{\text{pre}} \\ R; C^r, \text{label}(\tau_r^m, \Psi_{\text{pre}}) \vdash e^* : \tau_r^m, \Psi_{\text{pre}} \to (\tau_r')^n, \Psi'_{\text{post}} \\ (\tau_r' <: \tau_r)^n \qquad \Psi'_{\text{post}} <: \Psi_{\text{post}}\end{array}}{R; C^r \vdash \textbf{loop}\ tf_r\ e^*\ \textbf{end} : tf_r}\ \textsc{Loop}$$

Fig. 9. Typing Rules for the Refinement Type System

def load_and_extend($a, \tau, tp\_sz^?, \Psi$) :=
  let $\tau_r(n_1) = \Psi(a)$
  let $\tau_r(n_2) = \Psi(a-4)$
  if tp given then let $N = |tp|$ else let $N = |\tau|$
  if $wasm\_type(\tau_r(n_1)) == \tau$ then let $\tau_r(n) = \tau_r(n_1)$
  if $|\tau_r(n_1)| < N$ then let $\tau_r(n) = \tau(\text{val}(\text{bits}(n_2)\ \text{bits}(n_1)))$
  if $|\tau_r(n_1)| > N$ then let $\tau_r(n) = \tau(\text{val}(\text{bits}(n_1)_{0..N}))$
  if $(\tau_r(n_1) = (\text{i32}(n_1) \vee \text{ptr}(n_1) \vee \text{num}(n_1)) \wedge \tau = \text{f32})$ then let $\tau_r(n) = \text{f32}(bits(n_1 \text{ as f32}))$
  if $(\tau_r(n_1) = \text{f32}(n_1) \wedge \tau = (\text{i32} \vee \text{ptr} \vee \text{num}))$ then let $\tau_r(n) = \text{i32}(\text{f32}(n_1) \text{ as i32})$
  if tp given
    then return $\tau_r(\text{extend\_sx}_{\text{N},|\tau_w|}(n))$
    else return $\tau_r(n)$

$$\frac{C^r_{\text{memory}} = n \qquad \tau^1_r <: \text{ptr}(c) \qquad \tau^2_r = \text{load\_and\_extend}(c+o, \tau, (tp\_sx)^?, R_\Psi) \qquad 2^a \le (|tp| <)^? |\tau^2_r| \qquad (tp\_sz)^? = \epsilon \vee \tau^2_r = \textbf{im}}{R; C^r \vdash \tau.\textbf{load}(tp\_sx)^?\ a\ o : \tau^1_r \to \tau^2_r}\ \textsc{Load}$$

def wrap_and_store($p, \tau_r(n), \tau, tp^?, R$) :=
  if tp then let $N = |tp|$ else let $N = |\tau|$
  if $|\tau| == 64$ then $R_\Psi[p \mapsto \text{i32}(\text{val}(\text{bits}(n)_{0..32})) \wedge (p-4) \mapsto \text{i32}(\text{val}(\text{bits}(n)_{32..64}))]$
  if $|\tau_r(n)| > N$ then
    $R_\Psi[p \mapsto \tau_r(\text{wrap}_{|\tau|,N}(n))]$
    else $R_\Psi[p \mapsto \tau_r(n)]$

$$\frac{C^r_{\text{memory}} = n \qquad 2^a \le (|tp| <)^? |\tau^2_r| \qquad tp^? = \epsilon \vee \tau^2_r = \textbf{im} \qquad \tau^1_r <: \text{ptr32}(c) \qquad \text{wrap\_and\_store}(c+o, \tau^2_r, \tau_w, tp^?, R) \qquad \tau^2_r <: \tau(\top)}{R, C^r \vdash \tau.\textbf{store}\ tp^?\ a\ o : \tau^1_r\ \tau^2_r \to \epsilon}\ \textsc{Store}$$

$$\frac{C^r_{\text{memory}} = n \qquad \text{num32}(n_1) <: \tau^1_r \qquad \text{num32}(n_2) <: \tau^2_r}{R; C^r \vdash \textbf{grow\_memory} : \tau^1_r \to \tau^2_r}\ \textsc{Grow-memory}$$

$$\frac{C^r_{\text{memory}} = n \qquad \text{num32}(n_1) <: \tau_r \qquad |R_\Psi| = n}{R; C^r \vdash \textbf{current\_memory} : \epsilon \to \tau_r}\ \textsc{Current-memory}$$

$$\frac{C^r_{\text{func}}(i) = (\tau'_r)^m, \Psi'_{\text{pre}} \to (\tau'_r)^n, \Psi'_{\text{post}} \qquad tf_r = \tau^m_r, \Psi_{\text{pre}} \to \tau^n_r, \Psi_{\text{post}} \qquad (\tau_r <: \tau'_r)^m \qquad (\tau'_r <: \tau_r)^n \qquad \Psi'_{\text{pre}} <: \Psi_{\text{pre}} \qquad \Psi_{\text{post}} <: \Psi'_{\text{post}}}{R; C^r \vdash \textbf{call}\ i\ tf_r : tf_r}\ \textsc{Call}$$

$$\frac{tf_r = \tau^m_r, \Psi_{\text{pre}} \to \tau^n_r, \Psi_{\text{post}} \qquad C^r_{\text{table}} = n \qquad \text{num32}(c) <: \tau^1_r}{R; C^r \vdash \textbf{call\_indirect}\ tf_r : \tau^m_r, \tau^1_r \to \tau^n_r, \Psi_{\text{post}}}\ \textsc{Call-Indirect}$$

Fig. 10. Typing Rules for Refinement Type System continued.

$$\frac{}{R; C^r \vdash \tau.\mathbf{const}\ c : \epsilon \to \tau(c)}\textsc{Const} \qquad \frac{\tau_r^2 = \mathrm{unop}(\tau_r^1)}{R; C^r \vdash \tau.\mathbf{unop} : \tau_r^1 \to \tau_r^2}\textsc{Unop}$$

$$\frac{\tau_r^3 = \mathrm{binop}(\tau_r^1, \tau_r^2) \qquad \tau_r^3 <: \tau(\top)}{R; C^r \vdash \tau.\mathbf{binop} : \tau_r^1\ \tau_r^2 \to \tau_r^3}\textsc{Binop} \qquad \frac{\tau_r^2 = \mathrm{testop}(\tau_r^1) \qquad \tau_r^2 <: \mathrm{num}(\top)}{R; C^r \vdash \tau.\mathbf{testop} : \tau_r^1 \to \tau_r^2}\textsc{Testop}$$

$$\frac{\tau_r^3 = \mathrm{relop}(\tau_r^1, \tau_r^2) \qquad \tau_r^3 <: \mathrm{num}(\top)}{R; C^r \vdash \tau.\mathbf{relop} : \tau_r^1\ \tau_r^2 \to \tau_r^3}\textsc{Relop} \qquad \frac{}{R; C^r \vdash \mathbf{unreachable} : \tau_r^n \to \tau_r^m}\textsc{Unreachable}$$

$$\frac{}{R; C^r \vdash \mathbf{nop} : \epsilon \to \epsilon}\textsc{Nop} \qquad \frac{}{R; C^r \vdash \mathbf{drop} : \tau_r \to \epsilon}\textsc{Drop}$$

$$\frac{}{R; C^r \vdash \mathbf{select} : \tau_r^1\ \tau_r^2\ \mathrm{num32}(n) \to \tau_r^3}\textsc{Select}$$

$$\frac{\begin{array}{c} tf_r = \tau_r^m, \Psi_{\mathrm{pre}} \to \tau_r^n, \Psi_{\mathrm{post}} \qquad R_\Psi = \Psi_{\mathrm{pre}} \\ R;, C^r, \mathrm{label}(\tau_r^n) \vdash {e_1}^* : \tau_r^m, \Psi_{\mathrm{pre}} \to (\tau_r')^n, \Psi'_{\mathrm{post}} \\ R; C^r, \mathrm{label}(\tau_r^n) \vdash {e_2}^* : \tau_r^m, \Psi_{\mathrm{pre}} \to (\hat{\tau}_r'')^n, \Psi''_{\mathrm{post}} \\ (\tau_r' <: \tau_r)^n \qquad (\hat{\tau}_r'' <: \tau_r)^n \qquad \Psi'_{\mathrm{post}} <: \Psi_{\mathrm{post}} \qquad \Psi''_{\mathrm{post}} <: \Psi_{\mathrm{post}} \end{array}}{R; C^r \vdash \mathbf{if}\ tf_r\ \mathbf{then}\ {e_1}^*\ \mathbf{else}\ {e_2}^*\ \mathbf{end} : tf_r}\textsc{If-Else}$$

$$\frac{C^r_{\mathrm{label}}(i) = (\tau_r')^m, \Psi' \qquad (\tau_r <: \tau_r')^m \qquad R_\Psi <: \Psi'}{R; C^r \vdash \mathbf{br}\ i : \tau_r^*\ \tau_r^m \to \tau_r^*}\textsc{Br}$$

$$\frac{C^r_{\mathrm{label}}(i) = (\tau_r')^m, \Psi' \qquad \mathrm{num32}(n) <: \tau_r^1 \qquad (\tau_r <: \tau_r')^m \qquad R_\Psi <: \Psi'}{R; C^r \vdash \mathbf{br\_if}\ i : \tau_r^m\ \tau_r^1 \to \tau_r^m}\textsc{Br-if} \qquad \frac{C^r_{\mathrm{label}}(i) = ((\tau_r')^m, \Psi)^+ \qquad ((\tau_r <: \tau_r')^m)^+ \qquad (R_\Psi <: \Psi)^+}{R; C^r \vdash \mathbf{br\_table}\ i^+ : \tau_r^*\ \tau_r^m\ \tau_r^1 \to \tau_r^*}\textsc{Br-table}$$

$$\frac{C^r_{\mathrm{label}}(i) = (\tau_r')^m, \Psi \qquad (\tau_r <: \tau_r')^m \qquad R_\Psi <: \Psi}{R; C^r \vdash \mathbf{return} : \tau_r^*\ \tau_r^m \to \tau_r^*}\textsc{Return} \qquad \frac{C^r_{\mathrm{local}}(i) = \tau_r}{R; C^r \vdash \mathbf{get\_local}\ i : \epsilon \to \tau_r}\textsc{Get-local}$$

$$\frac{C^r_{\mathrm{local}}(i) = \tau_r^2 \qquad \tau_r^1 <: \tau_r^2}{R; C^r \vdash \mathbf{set\_local}\ i : \tau_r^1 \to \epsilon}\textsc{Set-local} \qquad \frac{C^r_{\mathrm{local}}(i) = \tau_r^2 \qquad \tau_r^1 <: \tau_r^2}{R; C^r \vdash \mathbf{tee\_local}\ i : \tau_r^1 \to \tau_r^1}\textsc{Tee-local}$$

$$\frac{C^r_{\mathrm{global}}(i) = \tau_r}{R; C^r \vdash \mathbf{get\_global}\ i : \epsilon \to \tau_r}\textsc{Get-global} \qquad \frac{C^r_{\mathrm{global}}(i) = \tau_r^2 \qquad \tau_r^1 <: \tau_r^2}{R; C^r \vdash \mathbf{set\_global}\ i : \tau_r^1 \to \epsilon}\textsc{Set-global}$$

$$\frac{}{R; C^r \vdash \epsilon : \epsilon \to \epsilon}\textsc{EmptyStack} \qquad \frac{R; C^r \vdash {e_1}^* : \tau_r^a \to \tau_r^b \qquad R; C^r \vdash e_2 : \tau_r^b \to \tau_r^c}{R; C^r \vdash {e_1}^*\ e_2 : \tau_r^a \to \tau_r^c}\textsc{Sequencing}$$

$$\frac{R; C^r \vdash e^* : \tau_r^b \to \tau_r^c}{R; C^r \vdash e^* : \tau_r^a\ \tau_r^b \to \tau_r^a\ \tau_r^c}\textsc{TopOfStack} \qquad \frac{}{R; C^r \vdash \mathrm{trap} : tf_r}\textsc{Trap}$$

Fig. 11. Typing Rules for the Refinement Type System continued.

B TYPE SAFETY PROOF

Theorem 3. ***Progress:*** *If* $R; C^r \vdash_i \hat{e}^* : \tau_r^1 \cdots \tau_r^m \rightarrow \tau_r^* \wedge (v : \tau_r)^m \wedge \vdash_i s : R$ *then* $e^* = v^*$ *or* $e^* = \text{trap}$ *or* $s; v^*; e^* \hookrightarrow s'; v'^*; e'^*$

Proof. Proof by induction of the deriviation of $R; C^r \vdash_i \hat{e}^* : \tau_r^1 \cdots \tau_r^m \rightarrow \tau_r^*$
Case Load:

$$\frac{\begin{array}{ll} C^r_{\text{memory}} = n & \tau_r^1 <: \text{ptr}(c) \qquad \tau_r^2 = \text{load\_and\_extend}(c+o, \tau_w, (tp\_sx)^?, R_\Psi) \\ & 2^a \leq (|tp| <)^? |\tau_r^2| \qquad (tp\_sz)^? = \epsilon \vee \tau_r^2 = \textbf{im} \end{array}}{R; C^r \vdash \tau.\textbf{load}(tp\_sx)^?\ a\ o : \tau_r^1 \rightarrow \tau_r^2}\ \textsc{Load}$$

The load instruction has to take a step since only end instructions signify the end of a function. We now case on the shape of a well typed stack. The only possible case is that the load instruction has a $\text{ptr}(a)$ on the stack, since this is a premise of our typing rule. The operational semantics of the load instruction matches this stack shape. If the side conditions are not met, it steps to a trap. □

Theorem 4. ***Preservation:*** *If* $R; C^r \vdash_i \hat{e}^* : \tau_r^1 \cdots \tau_r^m \rightarrow \tau_r^* \wedge \vdash_i (v : \tau_r)^m \wedge \vdash_i s : R \wedge s, v^*, e^* \hookrightarrow s', v'^*, e'^*$ *then* $\exists\, R'$ *such that* $dom(R_\Psi) \subseteq dom(R'_\Psi) \wedge \vdash_i s' : R' \wedge R'; C^r \vdash_i e'^* : \tau_r^*$

Proof. Proof by induction of the deriviation of $R; C^r \vdash_i \hat{e}^* : \tau_r^1 \cdots \tau_r^m \rightarrow \tau_r^*$
Case Load:

$$\frac{\begin{array}{ll} C^r_{\text{memory}} = n & \tau_r^1 <: \text{ptr}(c) \qquad \tau_r^2 = \text{load\_and\_extend}(c+o, \tau_w, (tp\_sx)^?, R_\Psi) \\ & 2^a \leq (|tp| <)^? |\tau_r^2| \qquad (tp\_sz)^? = \epsilon \vee \tau_r^2 = \textbf{im} \end{array}}{R; C^r \vdash \tau.\textbf{load}(tp\_sx)^?\ a\ o : \tau_r^1 \rightarrow \tau_r^2}\ \textsc{Load}$$

From our typing rule for the load instruction we have that it expects τ_r^1 on the stack and pushes τ_r^2 on the stack. From the operational semantics of the load instruction we know that there are three possible cases:

Case 1: $s;\ \text{ptr.const}(c);\ \tau.\text{load}\ a\ o \hookrightarrow \tau.\text{const}(b*)$ if $s_{\text{mem}}(i, c+o, |\tau|) = b^*$

From $\vdash_i s : R$, we know that $\vdash s_{\text{mem}} : R_\Psi$, from which we know that $s_{\text{mem}}(c+o) : R_\Psi(c+o)$. Since our memory typing R_Ψ maps addresses to types of size 32, we have to now case on the data expected after the load instruction. This is annotated on the load instruction itself as τ.

Case 1.1: $\tau.\text{load}\ a\ o \wedge R_\Psi[(c+o) \mapsto \tau_{r_{32}}(n)]$

This is the case where the data in memory is of the type that the load expects to produce on the stack. We now have to show that $b^* : \tau(n)$. From the typing rule, we have that load expects τ_r^2. On inspecting the load_and_extend function, we see that for the case where the data in memory is the same type as is expected on the stack, $\tau_r^2 = \tau_r(n_1)$ where $\tau_r(n_1) = \Psi(c+o)$. Since $s_{\text{mem}}(c+o) : R_\Psi(c+o)$, we know that $s_{\text{mem}}(c+o) : \tau_r^2$ and since $s_{\text{mem}}(c+o) = b^*$, $b^* : \tau_r^2$.

Case 1.2: $\text{i64.load}\ a\ o \wedge R_\Psi[(c+o) \mapsto \tau_{r_{32}}(n_1), (c+o-4) \mapsto \tau_{r_{32}}(n_2)]$

In this case, the load expects a i64 on the stack and the memory has $\tau_{r_{32}}(n_1)$ at $(c+o)$ and $\tau_{r_{32}}(n_2)$ at $(c+o-4)$. We now have to show that $b^* : \text{i64}(n)$. Note that b^* is the byte sequence from $s_{\text{mem}}[c+o : 8]$, as specified in the detailed Wasm specification [42]. From the typing rule, we know that load expects τ_r^2 on the stack. On inspecting the load_and_extend function, we see that this case is the one that matches $|\tau_r(n_1)| < N$, where $N = |\tau| = 64$. We see then that $\tau_r^2 = \tau(\text{val}(\text{bit}(n_1), \text{bit}(n_2)))$. n_1 and n_2 are the values in memory at $c+o$ and $c+o-4$. Since $s_{\text{mem}}(c+o) : R_\Psi(c+o) \wedge s_{\text{mem}}(c+o-4) : R_\Psi(c+o-4)$, we know that $s_{\text{mem}}(c+o:8) : \tau_r^2$ and since $s_{\text{mem}}(c+o) = b^*$, $b^* : \tau_r^2$.

Case 1.3: f64.load $a\ o \wedge R_\Psi[(c+o) \mapsto \tau_{r_{32}}(n_1), (c+o-4) \mapsto \tau_{r_{32}}(n_2)]$

In this case, the load expects a f64 on the stack and the memory has $\tau_{r_{32}}(n_1)$ at $(c+o)$ and $\tau_{r_{32}}(n_2)$ at $(c+o-4)$. We now have to show that $b^* : \text{f64}(n)$. The proof proceeds exactly as the case above.

Case 1.4: ptr.load $a\ o \wedge R_\Psi[(c+o) \mapsto \text{f32}(n)]$
Case 1.5: num.load $a\ o \wedge R_\Psi[(c+o) \mapsto \text{f32}(n)]$
Case 1.6: i32.load $a\ o \wedge R_\Psi[(c+o) \mapsto \text{f32}(n)]$

In this case, the load expects a i32 on the stack and Ψ has $\text{f32}(n)$ at $c+o$. We now have to show that $b^* : \text{f32}(n)$. From the typing rule, we have that load expects τ_r^2. On inspecting the load_and_extend function, we see that for this case, $\tau_r^2 = \text{f32}(\text{bits}(n_1 \text{ as f32}))$ where $\tau_r(n_1) = \Psi(c+o)$. Since $s_{\text{mem}}(c+o) : R_\Psi(c+o)$, we know that $s_{\text{mem}}(c+o) : \tau_r^2$ and since $s_{\text{mem}}(c+o) = b^*$, $b^* : \tau_r^2$.

Case 1.7: f32.load $a\ o \wedge R_\Psi[(c+o) \mapsto \text{ptr}(n)]$
Case 1.8: f32.load $a\ o \wedge R_\Psi[(c+o) \mapsto \text{num}(n)]$
Case 1.9: f32.load $a\ o \wedge R_\Psi[(c+o) \mapsto \text{i32}(n)]$

In this case, the load expects a f32 on the stack and Ψ has $\text{i32}(n)$ at $c+o$. We now have to show that $b^* : \text{i32}(n)$. From the typing rule, we have that load expects τ_r^2. On inspecting the load_and_extend function, we see that for this case, $\tau_r^2 = \text{i32}(\text{f32}(n_1) \text{ as i32})$ where $\tau_r(n_1) = \Psi(c+o)$. Since $s_{\text{mem}}(c+o) : R_\Psi(c+o)$, we know that $s_{\text{mem}}(c+o) : \tau_r^2$ and since $s_{\text{mem}}(c+o) = b^*$, $b^* : \tau_r^2$.

Case 2: $s;\ \text{ptr.const}(c);\ \tau.\text{load}\ tp\_sx\ a\ o \hookrightarrow \tau.\text{const}(b*)$ if $s_{\text{mem}}(i, c+o, |tp|) = b^*$

From $\vdash_i s : R$, we know that $\vdash s_{\text{mem}} : R_\Psi$, from which we know that $s_{\text{mem}}(c+o) : R_\Psi(c+o)$. Since our memory typing R_Ψ maps addresses to types of size 32, we have to now case on the data expected after the load instruction. The proof proceeds as above except with a small change introduced by the *tp_sx* annotation on the load. In the WebAssembly operational semantics [42], when a load has this packed type annotation, the bits are read from memory upto $|tp|$ and the value read from memory is size extended with the $extend\_sx_{N,|\tau|}$ function. Both of these cases are handled in the load_and_extend function, in case $|\tau_r(n_1)| > N$ and size extending function $\text{extend}\_sx_{N,|\tau|}$.

Case 3: $s;\ \text{ptr.const}(c);\ \tau.\text{load}\ (tp\_sx)^?\ a\ o \hookrightarrow \text{trap}$ otherwise

The trap instruction is well typed. □

C MEET AND JOIN OPERATIONS OVER THE INTEGER SUB-LATTICE

$\sqcup$	$\text{i32}(\top)$	$\text{i32}(n)$	$\text{ptr}(\top,\top)$	$\text{ptr}(l_0,n_0)$	$\text{ptr}(l_1,n_1)$	$\text{ptr}(\bot,\bot)$	$\text{num}(\top)$	$\text{num}(n)$	$\text{num}(\bot)$	$\text{i32}(\bot)$
$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$
$\text{i32}(n')$	$\text{i32}(\top)$	$\text{i32}(n \sqcup n')$	$\text{i32}(n')$	$\text{i32}(n' \sqcup n_0)$	$\text{i32}(\top)$	$\text{i32}(n')$	$\text{i32}(\top)$	$\text{i32}(n' \sqcup n)$	$\text{i32}(n')$	$\text{i32}(n')$
$\text{ptr}(\top,\top)$	$\text{i32}(\top)$	$\text{i32}(n)$	$\text{ptr}(\top,\top)$	$\text{ptr}(\top,\top)$	$\text{ptr}(\top,\top)$	$\text{ptr}(\top,\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{ptr}(\top,\top)$
$\text{ptr}(l_0,n_0')$	$\text{i32}(\top)$	$\text{i32}(n \sqcup n_0')$	$\text{ptr}(\top,\top)$	$\text{ptr}(l_0,n_0' \sqcup n_0)$	$\text{ptr}(\top,\top)$	$\text{ptr}(l_0,n_0')$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(n_0')$	$\text{ptr}(l_0,n_0')$
$\text{ptr}(l_1,n_1')$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{ptr}(\top,\top)$	$\text{ptr}(\top,\top)$	$\text{ptr}(l_1,n_1 \sqcup n_1')$	$\text{ptr}(l_1,n_1')$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{ptr}(l_1,n_1')$
$\text{ptr}(l_2,n_2')$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{ptr}(\top,\top)$	$\text{ptr}(\top,\top)$	$\text{ptr}(\top,\top)$	$\text{ptr}(l_2,n_2')$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{ptr}(l_2,n_2')$
$\text{ptr}(\bot,\bot)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{ptr}(\top,\top)$	$\text{ptr}(l_0,n_0)$	$\text{ptr}(l_1,n_1)$	$\text{ptr}(\bot,\bot)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{ptr}(\bot,\bot)$
$\text{num}(\top)$	$\text{i32}(\top)$	$\text{i32}(n)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{num}(\top)$	$\text{num}(\top)$	$\text{num}(\top)$	$\text{num}(\top)$
$\text{num}(n')$	$\text{i32}(\top)$	$\text{i32}(n \sqcup n')$	$\text{i32}(\top)$	$\text{i32}(n_0 \sqcup n')$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{num}(n')$	$\text{num}(n' \sqcup n)$	$\text{num}(n')$	$\text{num}(n')$
$\text{num}(\bot)$	$\text{i32}(\top)$	$\text{i32}(n)$	$\text{i32}(\top)$	$\text{i32}(n_0)$	$\text{i32}(\top)$	$\text{i32}(\top)$	$\text{num}(\top)$	$\text{num}(n)$	$\text{num}(\bot)$	$\text{num}(\bot)$
$\text{i32}(\bot)$	$\text{i32}(\top)$	$\text{i32}(n)$	$\text{ptr}(\top,\top)$	$\text{ptr}(l_0,n_0)$	$\text{ptr}(l_1,n_1)$	$\text{ptr}(\bot,\bot)$	$\text{num}(\top)$	$\text{num}(n)$	$\text{num}(\bot)$	$\text{i32}(\bot)$

Table 6. Join $\sqcup$ opertation for the i32 sub-lattice.

$\sqcap$	i32($\top$)	i32(n)	ptr($\top,\top$)	ptr(l_0,n_0)	ptr(l_1,n_1)	ptr($\bot,\bot$)	num($\top$)	num(n)	num($\bot$)	i32($\bot$)
i32($\top$)	i32($\top$)	i32(n)	ptr($\top,\top$)	ptr(l_0,n_0)	ptr(l_1,n_1)	ptr($\bot,\bot$)	num($\top$)	num(n)	num($\bot$)	i32($\bot$)
i32(n')	i32(n')	i32($n \sqcap n'$)	ptr($\top,\top$)	ptr($l_0,n_0 \sqcap n'$)	ptr($\bot,\bot$)	ptr($\bot,\bot$)	num(n')	num($n \sqcap n'$)	num($\bot$)	i32($\bot$)
ptr($\top,\top$)	ptr($\top,\top$)	ptr($\top,\top$)	ptr($\top,\top$)	ptr(l_0,n_0)	ptr(l_1,n_1)	ptr($\bot,\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)
ptr(l_0,n_0')	ptr(l_0,n_0')	ptr($l_0,n_0' \sqcap n$)	ptr(l_0,n_0')	ptr($l_0,n_0 \sqcap n_0'$)	ptr($\bot,\bot$)	ptr($\bot,\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)
ptr(l_1,n_1')	ptr(l_1,n_1')	ptr($\bot,\bot$)	ptr(l_1,n_1')	ptr($\bot,\bot$)	ptr($l_1,n_1 \sqcap n_1'$)	ptr($\bot,\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)
ptr(l_2,n_2')	ptr(l_2,n_2')	ptr($\bot,\bot$)	ptr(l_2,n_2')	ptr($\bot,\bot$)	ptr($l_3,0$)*	ptr($\bot,\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)
ptr($\bot,\bot$)	ptr($\bot,\bot$)	ptr($\bot,\bot$)	ptr($\bot,\bot$)	ptr($\bot,\bot$)	ptr($\bot,\bot$)	ptr($\bot,\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)
num($\top$)	num($\top$)	num($\top$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	num($\top$)	num(n)	num($\bot$)	i32($\bot$)
num(n')	num(n')	num($n \sqcap n'$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	num(n')	num($n \sqcap n'$)	num($\bot$)	i32($\bot$)
num($\bot$)	num($\bot$)	num($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	num($\bot$)	num($\bot$)	num($\bot$)	i32($\bot$)
i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)	i32($\bot$)

*l_3 fresh $\wedge\ l_1 = l_3 - n_1 \wedge l_2 = l_3 - n_2'$

Table 7. Meet $\sqcap$ opertation for the i32 sub-lattice.

D CONSTRAINT GENERATION FOR ALL WEBASSEMBLY INSTRUCTIONS

$$\textit{context}\quad C^{\alpha} ::= \{\text{func } tf^{*},\ \text{local } \alpha^{*},\ \text{global } \alpha^{*},\ \text{table } n^{?},\ \text{memory } n^{?},\ \text{label } (\alpha^{*},\varsigma)^{*},\ \text{return } (\alpha^{*},\varsigma)^{*}\}$$

Constraints for Instructions $\quad \boxed{S;\varsigma;C^{\alpha} \vdash e : \alpha^{*} \to \alpha^{*};S';\varsigma'}$

$$\frac{\alpha \text{ fresh} \qquad S' = S :: [\alpha \doteq \tau_w(c)]}{S;\varsigma;C^{\alpha} \vdash \tau_w.\mathbf{const}\ c : \epsilon \to \alpha;S';\varsigma}\ \textsc{Constant} \qquad \frac{\alpha_1 \in dom(S) \qquad \alpha_2 \text{ fresh} \qquad S' = S :: [\alpha_1 \doteq unop(\alpha_2)]}{S;\varsigma;C^{\alpha} \vdash \tau_w.unop : \alpha_1 \to \alpha_2;S';\varsigma}\ \textsc{Unary-Ops}$$

$$\frac{\alpha_1\ \alpha_2 \in dom(S) \qquad \alpha_3 \text{ fresh} \qquad S' = S :: [\alpha_1 \doteq binop(\_,\alpha_2,\alpha_3) \wedge \alpha_2 \doteq binop(\alpha_1,\_,\alpha_3) \wedge \alpha_3 \doteq binop(\alpha_1,\alpha_2,\_)]}{S;\varsigma;C^{\alpha} \vdash \tau_w.binop : \alpha_1\ \alpha_2 \to \alpha_3;S';\varsigma}\ \textsc{Binary-Ops}$$

$$\frac{\alpha_1\ \alpha_2 \in dom(S) \qquad \alpha_3 \text{ fresh} \qquad S' = S :: [\alpha_1 \doteq testop(\_,\alpha_2,\alpha_3) \wedge \alpha_2 \doteq testop(\alpha_1,\_,\alpha_3) \wedge \alpha_3 \doteq testop(\alpha_1,\alpha_2,\_)]}{S;\varsigma;C^{\alpha} \vdash \tau_w.testop : \alpha_1\ \alpha_2 \to \alpha_3;S';\varsigma}\ \textsc{Test-Ops}$$

$$\frac{\alpha_1\ \alpha_2 \in dom(S) \qquad \alpha_3 \text{ fresh} \qquad S' = S :: [\alpha_1 \doteq relop(\_,\alpha_2,\alpha_3) \wedge \alpha_2 \doteq relop(\alpha_1,\_,\alpha_3) \wedge \alpha_3 \doteq relop(\alpha_1,\alpha_2,\_)]}{S;\varsigma;C^{\alpha} \vdash \tau_w.relop : \alpha_1\ \alpha_2 \to \alpha_3;S';\varsigma}\ \textsc{Rel-Ops}$$

$$\frac{\alpha_1 \in dom(S) \qquad \alpha_2 \text{ fresh} \qquad S' = S :: [\alpha_1 \doteq cvtop\ \tau_w\_sx^{?}(\_,\alpha_2) \wedge \alpha_2 \doteq cvtop\ \tau_w\_sx^{?}(\alpha_1,\_)]}{S;\varsigma;C^{\alpha} \vdash \tau_w.cvtop\ \tau_w\_sx^{?} : \alpha_1 \to \alpha_2;S';\varsigma}\ \textsc{Convert-Ops}$$

$$\frac{\alpha_1 \in dom(S) \qquad S' = S :: [\alpha_1 <: \text{ptr} \wedge \alpha_2 \doteq \varsigma[\alpha_1]_{(tf\_sx^{?},\ a,\ o)} \wedge \alpha_2 <: \tau_w]}{S;\varsigma;C^{\alpha} \vdash \tau_w.\mathbf{load}(tp\_sx)^{?}\ a\ o : \alpha_1 \to \alpha_2;S';\varsigma}\ \textsc{Load}$$

$$\frac{\alpha_1\ \alpha_2 \in dom(S) \qquad S' = S :: [\alpha_1 <: \text{ptr} \wedge \alpha_2 <: \tau_w \wedge \varsigma' \doteq \varsigma[\alpha_1 \mapsto \alpha_2]_{(tp^{?},\ a,\ o)}]}{S;\varsigma;C^{\alpha} \vdash \tau_w.\mathbf{store}\ tp^{?}\ a\ o : \alpha_1\ \alpha_2 \to \epsilon;S';\varsigma'}\ \textsc{Store}$$

Fig. 12. Constraint Generation Rules.

$$\frac{tf_w = \tau_w^m \to \tau_w^n \qquad \alpha^m \in dom(S) \qquad \alpha^n \text{ fresh} \qquad S; \varsigma; C^\alpha, \text{label}(\alpha^n) \vdash e^* : \alpha^m \to (\alpha')^n; S'; \varsigma' \qquad S'' = S' :: [(\alpha \doteq \alpha')^n]}{S; \varsigma; C^\alpha \vdash \textbf{block } tf\, e^* \textbf{ end} : \alpha_1^n \ldots \alpha_n^n \to \alpha_1^m \ldots \alpha_m^m; S''; \varsigma'} \textsc{Block}$$

$$\frac{tf_w = \tau_w^m \to \tau_w^n \qquad \alpha^m \in dom(S) \qquad \alpha^n \text{ fresh} \qquad S; \varsigma; C^\alpha, \text{label}(\alpha^m) \vdash e^* : \alpha^m \to (\alpha')^n; S'; \varsigma' \qquad S'' = S' :: [(\alpha \doteq \alpha')^n]}{S; \varsigma; C^\alpha \vdash \textbf{loop } tf_w\, e^* \textbf{ end} : \alpha^m \to \alpha^n; S''; \varsigma'} \textsc{Loop}$$

$$\frac{\begin{array}{c} tf_w = \tau_w^m \to \tau_w^n \qquad \alpha^m\, \alpha \in dom(S) \qquad \alpha_t^n, \alpha_e^n, \alpha^n \text{ fresh} \qquad S' = S :: [\alpha <: \text{num}] \\ S'; \varsigma; C^\alpha, \text{label}(\alpha_t^n) \vdash {e_t}^* : \alpha_m \to \alpha_t^n; S_e; \varsigma_e \qquad S'; \varsigma; C^\alpha, \text{label}(\alpha_e^n) \vdash {e_e}^* : \alpha_m \to \alpha_e^n; S_t; \varsigma_t \\ S'' = S' :: S_e :: S_t :: [\varsigma' \doteq \bigsqcup \varsigma_e \varsigma_t \wedge (\alpha' \doteq \bigsqcup \alpha_t \alpha_e)^n] \end{array}}{S; \varsigma; C^\alpha \vdash \textbf{if } tf\, {e_t}^* \textbf{ else } {e_e}^* \textbf{ end} : \alpha^m\, \alpha \to \alpha^n; S''; \varsigma'} \textsc{If-Else}$$

$$\frac{\alpha^* \alpha^n \in dom(S) \qquad C^\alpha_{\text{label}}(i) = (\alpha')^n, \varsigma' \qquad S' = S :: [\varsigma \doteq \bigsqcup \varsigma\varsigma' \wedge (\alpha \doteq \bigsqcup \alpha\alpha')^n]}{S; \varsigma; C^\alpha \vdash \textbf{br } i : \alpha^* \alpha_1 \ldots \alpha_n \to \alpha^*; S'; \varsigma} \textsc{Br}$$

$$\frac{\alpha^m \alpha \in dom(S) \qquad C^\alpha_{\text{label}}(i) = (\alpha')^n, \varsigma' \qquad S' = S :: [\varsigma \doteq \bigsqcup \varsigma\varsigma' \wedge \alpha <: \text{num} \wedge (\alpha \doteq \bigsqcup \alpha\alpha')^n]}{S; \varsigma; C^\alpha \vdash \textbf{br\_if } i : \alpha^m \alpha \to \alpha^n; S'; \varsigma} \textsc{Br-If}$$

$$\frac{\alpha^* \alpha^m \alpha \in dom(S) \qquad (C^\alpha_{\text{label}}(i) = (\alpha')^n)^+, \varsigma' \qquad S' = S :: ([\varsigma \doteq \bigsqcup \varsigma\varsigma' \wedge \alpha_{n+1} <: \text{num} \wedge \alpha_1 \doteq \bigsqcup \alpha_1\alpha_1' \wedge \cdots \wedge \alpha_n \doteq \bigsqcup \alpha_n\alpha_n'])^+}{S; \varsigma; C^\alpha \vdash \textbf{br\_table } i^+ : \alpha^* \alpha_1 \ldots \alpha_n \alpha_{n+1} \to \alpha^*; S'; \varsigma} \textsc{Br-Table}$$

$$\frac{\alpha^* \alpha_1 \ldots \alpha_n \in dom(S) \qquad C_{\text{return}}(i) = \alpha_1' \ldots \alpha_n' \qquad S' = S :: [\alpha_1 \doteq \bigsqcup \alpha_1\alpha_1' \wedge \cdots \wedge \alpha_n \doteq \bigsqcup \alpha_n\alpha_n']}{S; \varsigma; C \vdash \textbf{return} : \alpha^* \alpha_1 \ldots \alpha_n \to \alpha^*; S'; \varsigma}$$

$$\frac{C_{\text{func}} = tf \qquad tf = \tau_w^1 \ldots \tau_w^n \to \tau_w^1 \ldots \tau_w^m \qquad \alpha_1 \ldots \alpha_n \in dom(S) \qquad S' = S :: [\alpha_1' <: \tau_1 \wedge \cdots \wedge \alpha_m' <: \tau_m]}{S; \varsigma; C \vdash \textbf{call} : \alpha_1 \ldots \alpha_n \to \alpha_1' \ldots \alpha_m'; S'; \varsigma}$$

$$\frac{C_{\text{table}} = n \qquad tf = \tau_w^1 \ldots \tau_w^n \to \tau_w^1 \ldots \tau_w^m \qquad \alpha_1 \ldots \alpha_n \alpha_{n+1} \in dom(S) \qquad S' = S :: [\alpha_{n+1} <: \text{num} \wedge \alpha_1' <: \tau_1 \wedge \cdots \wedge \alpha_m' <: \tau_m]}{S; \varsigma; C \vdash \textbf{call\_indirect } tf : \alpha_1 \ldots \alpha_n \alpha_{n+1} \to \alpha_1' \ldots \alpha_m'; S'; \varsigma}$$

$$\frac{\alpha \text{ fresh} \qquad S' = S :: [\alpha <: \text{num}]}{S; \varsigma; C^\alpha \vdash \textbf{current\_memory} : \epsilon \to \alpha; S'; \varsigma} \textsc{Current-Memory}$$

$$\frac{\alpha_1 \in dom(S) \qquad \alpha_2 \text{ fresh} \qquad S' = S :: [\alpha_1 <: \text{num} \wedge \alpha_2 <: \text{num}]}{S; \varsigma; C^\alpha \vdash \textbf{grow\_memory} : \alpha_1 \to \alpha_2; S'; \varsigma} \textsc{Grow-Memory}$$

$$\frac{\alpha_1\, \alpha_2; \alpha_3 \in dom(S) \qquad \alpha_4 \text{ fresh} \qquad S' = S :: [\alpha_3 <: \text{num} \wedge \alpha_4 \doteq \bigsqcup \alpha_1\alpha_2]}{S; \varsigma; C^\alpha \vdash \textbf{select} : \alpha_1\, \alpha_2\, \alpha_3 \to \alpha_4; S'; \varsigma} \textsc{Select}$$

Fig. 13. Constraint Generation Rules cont.

$$\frac{\alpha_1^* \in dom(S) \qquad \alpha_2^* \text{ fresh}}{S;\varsigma;C^{\alpha} \vdash \textbf{unreachable} : \alpha_1^* \to \alpha_2^*;S;\varsigma}\textsc{Unreachable} \qquad \frac{}{S;\varsigma;C \vdash \textbf{nop} : \epsilon \to \epsilon;S;\varsigma}\textsc{Nop}$$

$$\frac{\alpha \in dom(S)}{S;\varsigma;C \vdash \textbf{drop} : \alpha \to \epsilon;S;\varsigma}\textsc{Drop} \qquad \frac{\alpha \text{ fresh} \qquad S' = S :: [\alpha \doteq C_{\text{local}}(i)]}{S;\varsigma;C \vdash \textbf{get\_local } i : \epsilon \to \alpha;S';\varsigma}\textsc{Get-Local}$$

$$\frac{\alpha \in dom(S) \qquad C_{\text{local}}(i) = \alpha' \qquad S' = S :: [\alpha' \doteq \alpha]}{S;\varsigma;C \vdash \textbf{set\_local } i : \alpha \to \epsilon;S';\varsigma}\textsc{Set-Local} \qquad \frac{\alpha \in dom(S) \qquad C_{\text{local}}(i) = \alpha' \qquad S' = S :: [\alpha' \doteq \alpha]}{S;\varsigma;C \vdash \textbf{tee\_local } i : \alpha \to \alpha;S';\varsigma}\textsc{Tee-Local}$$

$$\frac{\alpha \text{ fresh} \qquad S' = S :: [\alpha \doteq C_{\text{global}}(i)]}{S;\varsigma;C \vdash \textbf{get\_global } i : \epsilon \to \alpha;S';\varsigma}\textsc{Get-Global} \qquad \frac{\alpha \in dom(S) \qquad C_{\text{global}}(i) = \alpha' \qquad S' = S :: [\alpha' \doteq \alpha]}{S;\varsigma;C \vdash \textbf{set\_global } i : \alpha \to \epsilon;S';\varsigma}\textsc{Set-Global}$$

$$\frac{\begin{array}{c} tf = \tau_w^1 \dots \tau_w^n \to \tau_w^1 \dots \tau_w^m \qquad \tau^f = \alpha_{n_1} \dots \alpha_{n_n} \to \alpha_{m_1} \dots \alpha_{m_m} \\ \alpha_{n_1} \dots \alpha_{n_n}, \alpha_{m_1} \dots \alpha_{m_m}, \alpha_{l_1} \dots \alpha_{l_l} \text{ fresh} \\ S = [\varsigma \doteq \cdot \wedge \alpha_{n_1} <: \tau_w^1 \wedge \dots \wedge \alpha_{n_n} <: \tau_w^n \wedge \alpha_{m_1} <: \tau_w^1 \wedge \dots \wedge \alpha_{m_m} <: \tau_w^m \wedge \alpha_{l_1} <: \tau_w^1 \wedge \dots \wedge \alpha_{l_l} <: \tau_w^l] \\ S;\varsigma;C, \text{local}(\alpha_{n_1}, \dots \alpha_{n_n}, \alpha_{l_1}, \dots \alpha_{l_l}),\ \text{label}(\alpha_{m_1}, \dots \alpha_{m_m}),\ \text{return}(\alpha_{m_1}, \dots \alpha_{m_m}) \vdash e^* : \tau^f;S';\varsigma' \end{array}}{[];\varsigma;C \vdash ex^* \textbf{ func } tf \textbf{ local } \tau_w^1 \dots \tau_w^l \ e^* : \tau^f;S';\varsigma'}$$

Fig. 14. Constraint Generation Rules cont.