PHYS 512 Assignment 8

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Problem 1

$$\frac{f(t+dt,x)-f(t-dt,x)}{2dt} = -v\frac{f(t,x+dx)-f(t,x-dx)}{2dx}$$

Let:

$$f(x,t) = \xi^t e^{ikx}$$

$$\frac{(\xi^{t+dt}-\xi^{t-dt})e^{ikx}}{2dt} = -v\frac{(e^{ik(x+dx)}-e^{ik(x-dx)})\xi^t}{2dx}$$
$$\frac{(\xi^{dt}-\xi^{-dt})\xi^t e^{ikx}}{2dt} = -v\frac{(e^{ikdx}-e^{-ikdx})e^{ikx}\xi^t}{2dx}$$
$$\frac{(\xi^{dt}-\xi^{-dt})}{2dt} = iv\frac{sin(kdx)}{dx}$$
$$(\xi^{dt}-\xi^{-dt}) = \alpha 2isin(kdx)$$
$$\xi^{-dt}(\xi^{2dt}-1) = \alpha 2isin(kdx)$$
$$\xi^{2dt}-1 = \xi^{dt}\alpha 2isin(kdx)$$

This gives the solution:

$$\xi^{dt} = -i\alpha sin(kdx) \pm \sqrt{1-(\alpha sin(kdx)^2}$$

$$|\xi^{dt}|^2 = \alpha^2 sin^2(kdx) + 1 - \alpha^2 sin^2(kdx) = 1$$

Therefore, for the above to be true, the term under the square root has to be real:

$$1-\alpha^2 sin^2(kdx) >= 0$$
$$1 >= \alpha sin(kdx)$$

Since, $sin(kdx)$ can have the max value of 1, α also needs to be 1 or less than 1, to satisfy the above inequality, i.e. when the CFL condition is satisfied.

Problem 2

Part A

Using Gauss' law in 2D, instead of using a gaussian surface to enclose charge, enclose in gaussian line, which is a circle for our 2D point charge.

$$\int E\cdot ds = \frac{q_{enc}}{\epsilon_0}$$
$$E2\pi r = \frac{q_{enc}}{\epsilon_0}$$
$$E = \frac{q_{enc}}{2\pi\epsilon_0 r}$$
$$V = \int Edr = \frac{q_{enc}}{2\pi\epsilon_0}ln(r)$$

- potential at (1,0) = average of neighbours
- ignore ϵ_0, set ρ = V - average of neighbours

- rescale $\rho[0,0] = 1$, V[0,0]=1

$$/omega_0 = /gammaB_0$$