PHYS 512 Assignment 2

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Problem 1

$$
\begin{aligned}
E_z &= \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2} \\
&= \frac{1}{4\pi\epsilon_0}\int \frac{r^2 sin\theta d\theta d\phi(z - rcos\theta)}{(r^2 + z^2 - 2rzcos\theta)^{3/2}}
\end{aligned}
\tag{1}
$$

Set u = $cos\theta$, u = cos(π) = -1, cos(0) = 1:

$$
E_z = \frac{1}{4\pi\epsilon_0}\int_{-1}^{1} \frac{z - ru}{(r^2 + Z^2 - 2rzu)^{3/2}} du \tag{2}
$$

Use Eq. 2, in integral solver which uses legendre coefficients. Run **ps2_Problem1.py**

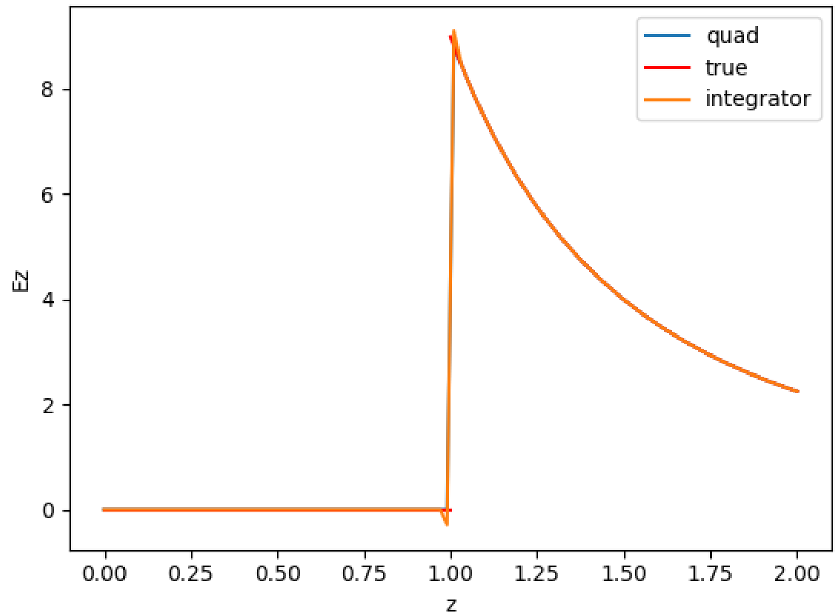

Figure 1: Output from **ps2_Problem1.py**

Both quad and the legendre integrator behave mostly the same. There's a singularity at z=R, but the integrator is a little spikier near it.

Problem 2

Run **ps2_Problem2.py** script.

The integrate_adaptive function uses a smaller amount of function evaluations than the class version, while keeping the same error.

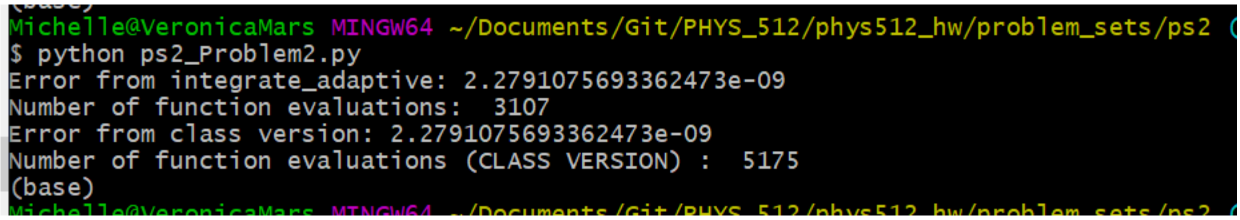

Figure 2: Output from **ps2_Problem2.py**

Problem 3

Run **ps2_**$Problem3.py.np.frexp(x), where$ **x = mantissa * 2**^{exponent}**, outputs mantissa, exponent**

$$\begin{aligned} log_2(x) &= log_2(mantissa * 2^{exp}) \\ &= log_2(mantissa) + log_2(2^{exp}) \\ &= log_2(mantissa) + exp \end{aligned} \tag{3}$$

Since we have a cheb fit for a $log_2(x)$ **function, to find log(x) we do the following to calculate log(x). And test our natural log function by comparing the true log function and our cheb fit mylog function.**

$$\begin{aligned} log(x) &= log_2(x)log(2) \\ log_2(e) &= \frac{1}{log(2)} \\ log(x) &= \frac{log_2(x)}{log_2 e} \end{aligned} \tag{4}$$

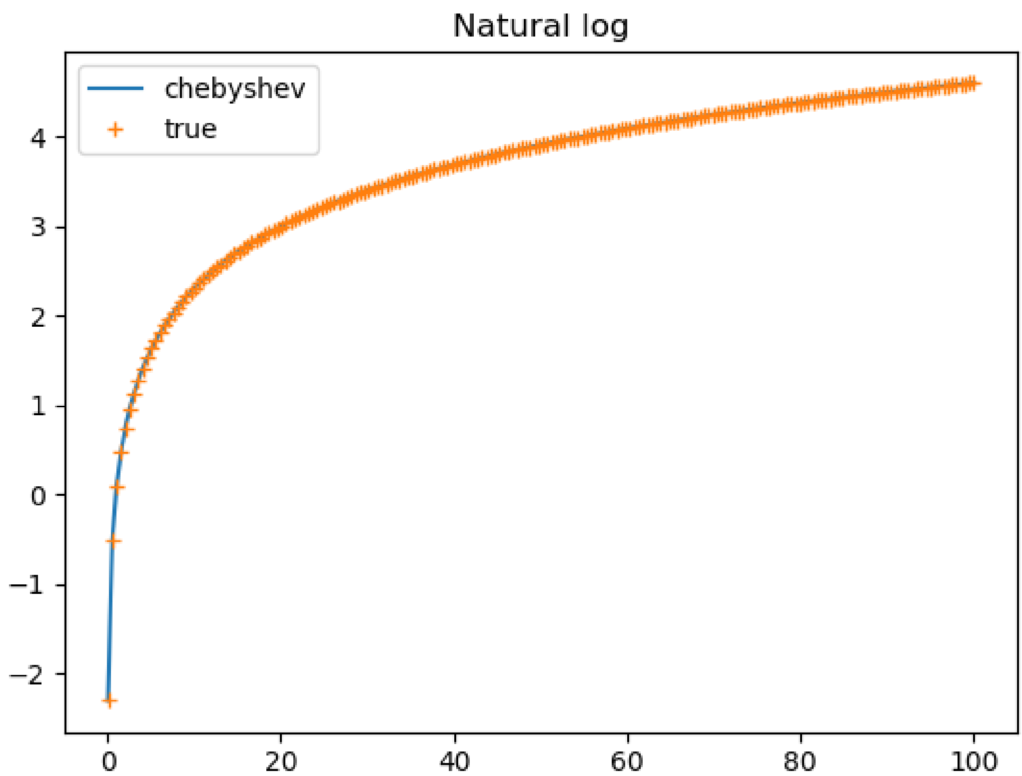

Figure 3: Output from **ps2_Problem2.py**