

PHYS 512 Assignment 2

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Problem 1

$$\begin{aligned} E_z &= \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \int \frac{r^2 \sin\theta d\theta d\phi (z - r\cos\theta)}{(r^2 + z^2 - 2rz\cos\theta)^{3/2}} \end{aligned} \quad (1)$$

Set $u = \cos\theta$, $u = \cos(\pi) = -1$, $\cos(0) = 1$:

$$E_z = \frac{1}{4\pi\epsilon_0} \int_{-1}^1 \frac{z - ru}{(r^2 + Z^2 - 2rzu)^{3/2}} du \quad (2)$$

Use Eq. 2, in integral solver which uses legendre coefficients. Run **ps2_Problem1.py**

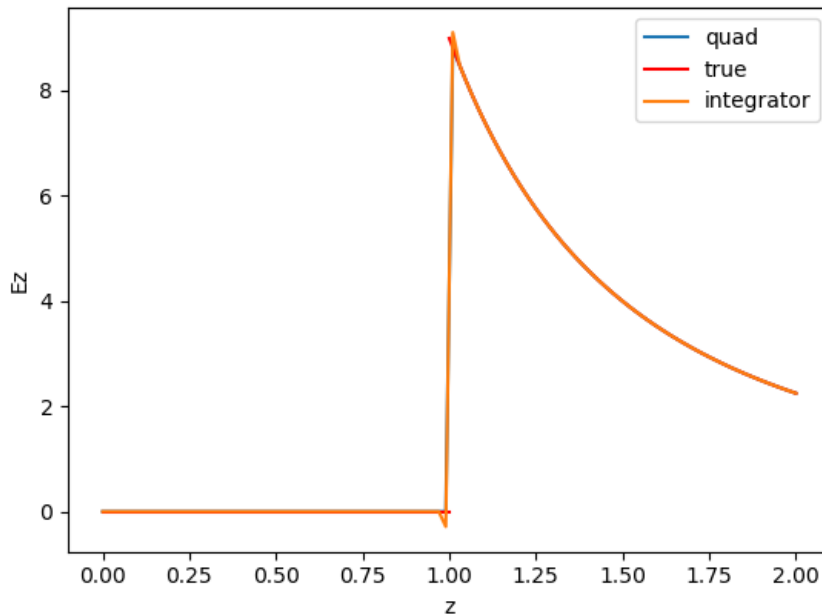


Figure 1: Output from **ps2_Problem1.py**

Both quad and the legendre integrator behave mostly the same. There's a singularity at $z=R$, but the integrator is a little spikier near it.

Problem 2

Run **ps2_Problem2.py** script.

The `integrate_adaptive` function uses a smaller amount of function evaluations than the class version, while keeping the same error.

```

Michelle@VeronicaMars MINGW64 ~/Documents/Git/PHYS_512/phys512_hw/problem_sets/ps2
$ python ps2_Problem2.py
Error from integrate_adaptive: 2.2791075693362473e-09
Number of function evaluations: 3107
Error from class version: 2.2791075693362473e-09
Number of function evaluations (CLASS VERSION) : 5175
(base)
Michelle@VeronicaMars MINGW64 ~/Documents/Git/PHYS_512/phys512_hw/problem_sets/ps2

```

Figure 2: Output from `ps2_Problem2.py`

Problem 3

Run `ps2_Problem3.py.np.frexp(x)`, where $x = \text{mantissa} * 2^{\text{exponent}}$, outputs mantissa, exponent

$$\begin{aligned}
 \log_2(x) &= \log_2(\text{mantissa} * 2^{\text{exp}}) \\
 &= \log_2(\text{mantissa}) + \log_2(2^{\text{exp}}) \\
 &= \log_2(\text{mantissa}) + \text{exp}
 \end{aligned} \tag{3}$$

Since we have a cheb fit for a $\log_2(x)$ function, to find $\log(x)$ we do the following to calculate $\log(x)$. And test our natural log function by comparing the true log function and our cheb fit mylog function.

$$\begin{aligned}
 \log(x) &= \log_2(x) \log(2) \\
 \log_2(e) &= \frac{1}{\log(2)} \\
 \log(x) &= \frac{\log_2(x)}{\log_2 e}
 \end{aligned} \tag{4}$$

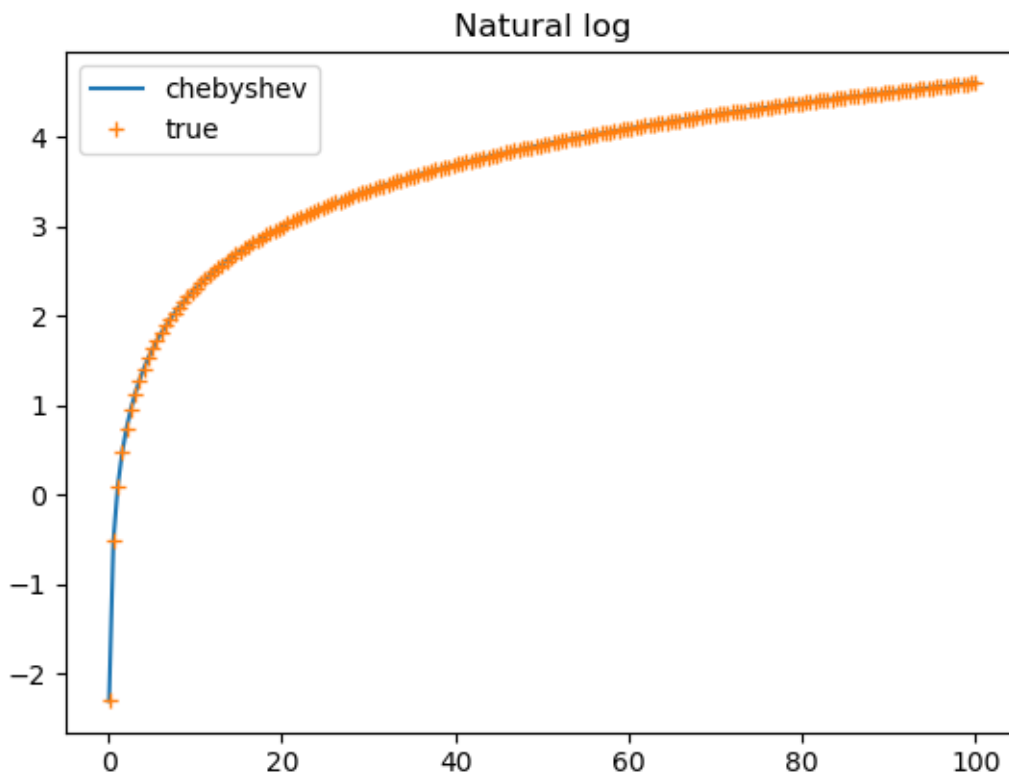


Figure 3: Output from `ps2_Problem3.py`