Hand Strength

Table and Graph of Hand Strengths

```
preferred =
 {28, 27, 45, 20, 40, 47, 28, 54, 52, 21, 53, 52, 49, 45, 39, 26, 25, 32, 30, 32}
```

```
{28, 27, 45, 20, 40, 47, 28, 54, 52, 21, 53, 52, 49, 45, 39, 26, 25, 32, 30, 32}
```

```
nonpreferred =
 {24, 26, 43, 22, 40, 45, 26, 46, 46, 22, 47, 47, 41, 44, 33, 20, 27, 30, 29, 29}
```

```
{24, 26, 43, 22, 40, 45, 26, 46, 46, 22, 47, 47, 41, 44, 33, 20, 27, 30, 29, 29}
```

```
list1 = Transpose[{preferred, nonpreferred}]
```

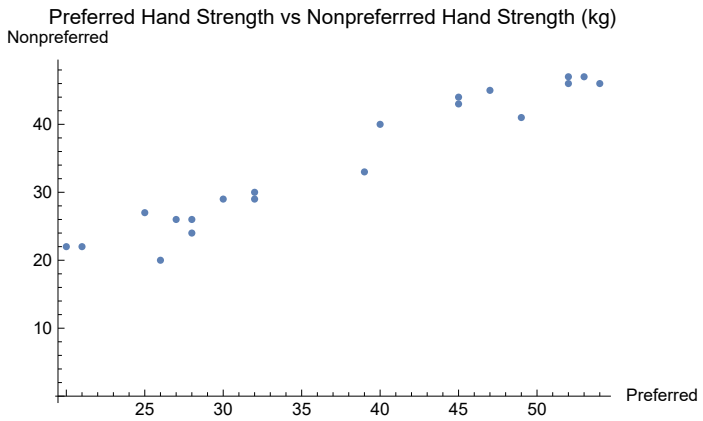
```
{{28, 24}, {27, 26}, {45, 43}, {20, 22}, {40, 40}, {47, 45},
 {28, 26}, {54, 46}, {52, 46}, {21, 22}, {53, 47}, {52, 47}, {49, 41},
 {45, 44}, {39, 33}, {26, 20}, {25, 27}, {32, 30}, {30, 29}, {32, 29}}
```

```
Text[Grid[
  Prepend[list1, {"Preferred Hand Strength (kg)", "Nonpreferred Hand Strength (kg)"}],
  Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {1, 1}]]
```

Preferred Hand Strength (kg)	Nonpreferred Hand Strength (kg)
28	24
27	26
45	43
20	22
40	40
47	45
28	26
54	46
52	46
21	22
53	47
52	47
49	41
45	44
39	33
26	20
25	27
32	30
30	29
32	29

```
Graph1 = ListPlot[list1,
   PlotLabel → "Preferred Hand Strength vs Nonpreferrred Hand Strength (kg)",
   AxesLabel → {"Preferred", "Nonpreferred"}];
```

```
Show[Graph1]
```



Summary Points

First, I ordered the points in order from least to greatest by x coordinate value.

```
list1 = Sort[list1]
```

```
{{20, 22}, {21, 22}, {25, 27}, {26, 20}, {27, 26}, {28, 24},
 {28, 26}, {30, 29}, {32, 29}, {32, 30}, {39, 33}, {40, 40}, {45, 43},
 {45, 44}, {47, 45}, {49, 41}, {52, 46}, {52, 47}, {53, 47}, {54, 46}}
```

Next, I divided the 20 points into 3 groups; the first group has the first 7 points, the second group has the next 6 points, and the third group has the last 6 points. For the first group, I took the x coordinates and found the median then took the y coordinates and found their median. Then I put the x and y coordinate together to get the first summary point. I repeated this for the second and third groups to find the second and third summary points.

```
group1 = Table[{list1[[q]][[1]], list1[[q]][[2]]}, {q, 1, 7}]
```

```
{{20, 22}, {21, 22}, {25, 27}, {26, 20}, {27, 26}, {28, 24}, {28, 26}}
```

```
group2 = Table[{list1[[q]][[1]], list1[[q]][[2]]}, {q, 8, 13}]
```

```
{{30, 29}, {32, 29}, {32, 30}, {39, 33}, {40, 40}, {45, 43}}
```

```
group3 = Table[{list1[[q]][[1]], list1[[q]][[2]]}, {q, 14, 20}]
```

```
{{45, 44}, {47, 45}, {49, 41}, {52, 46}, {52, 47}, {53, 47}, {54, 46}}
```

To find each summary point, I took the median of the x values and y values in each group.

```
sum1 = {Median[Table[list1[[q]][[1]], {q, 1, 7}]],
  Median[Table[list1[[q]][[2]], {q, 1, 7}]]}
```

```
{26, 24}
```

```
sum2 = {Median[Table[list1[[q]][[1]], {q, 8, 13}]],
  Median[Table[list1[[q]][[2]], {q, 8, 13}]]}
```

$$\left\{\frac{71}{2}, \frac{63}{2}\right\}$$

```
sum3 = {Median[Table[list1[[q]][[1]], {q, 14, 20}]],
  Median[Table[list1[[q]][[2]], {q, 14, 20}]]}
```

```
{52, 46}
```

```
summarypoints = {sum1, sum2, sum3}
```

$$\left\{\{26, 24\}, \left\{\frac{71}{2}, \frac{63}{2}\right\}, \{52, 46\}\right\}$$

Equating the Median Median Line

To find the slope, I found the slope of the line between the first and third summary points.

```
slope = (sum3[[2]] - sum1[[2]]) / (sum3[[1]] - sum1[[1]])
```

$$\frac{11}{13}$$

Then, I found the intercept of the line between summary points 1 and 3.

```
intercept1 = sum1[[2]] - slope (sum1[[1]])
```

2

Then, I found the intercept of the line that goes through summary point 2 and is parallel to the line between summary points 1 and 3.

```
intercept2 = sum2[[2]] - slope (sum2[[1]])
```

$\frac{19}{13}$

Then, I found the distance between the intercepts and divided it by 3. Then I subtracted that answer from the original intercept between summary points 1 and 3 to get the intercept of the median median line. The slope is the same slope between summary points 1 and 3.

```
(intercept1 - intercept2) / 3
```

$\frac{7}{39}$

```
intercept1 - %
```

$\frac{71}{39}$

```
linemed = slope (x) + %
```

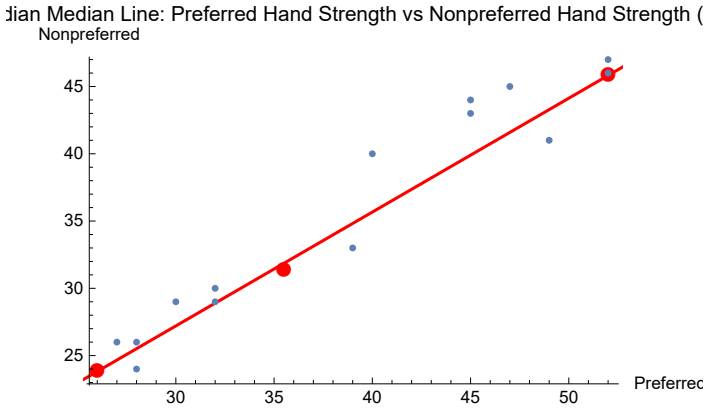
$\frac{71}{39} + \frac{11\,x}{13}$

Graph of Median Median Line and Summary Points

```
Graphmed = Plot [linemed, {x, 0 , 60}, PlotStyle → {Red}];
```

```
Graphsum = ListPlot[summarypoints, PlotLabel →
    "Median Median Line: Preferred Hand Strength vs Nonpreferred Hand Strength (kg)",
   AxesLabel → {"Preferred", "Nonpreferred"}, PlotStyle → {Red},
   PlotMarkers → {Automatic, Medium}];
```

```
Show[Graphsum, Graphmed, Graph1]
```



Median Median Line Residuals

$$f[x\_] := \frac{71}{39} + \frac{11\,x}{13}$$

preferred

{28, 27, 45, 20, 40, 47, 28, 54, 52, 21, 53, 52, 49, 45, 39, 26, 25, 32, 30, 32}

nonpreferred

{24, 26, 43, 22, 40, 45, 26, 46, 46, 22, 47, 47, 41, 44, 33, 20, 27, 30, 29, 29}

I plugged all the x-values into the function for the median median line to get the y values for each x value on the median median line.

listyexp = Map[f, preferred]

$$\left\{\frac{995}{39}, \frac{74}{3}, \frac{1556}{39}, \frac{731}{39}, \frac{107}{3}, \frac{1622}{39}, \frac{995}{39}, \frac{1853}{39}, \frac{1787}{39}, \frac{764}{39}, \frac{140}{3}, \frac{1787}{39}, \frac{1688}{39}, \frac{1556}{39}, \frac{1358}{39}, \frac{929}{39}, \frac{896}{39}, \frac{1127}{39}, \frac{1061}{39}, \frac{1127}{39}\right\}$$

To get the residuals, I subtracted the new y values calculated above from the original y values from the original dataset.

listred = nonpreferred – listyexp

$$\left\{-\frac{59}{39}, \frac{4}{3}, \frac{121}{39}, \frac{127}{39}, \frac{13}{3}, \frac{133}{39}, \frac{19}{39}, -\frac{59}{39}, \frac{7}{39}, \frac{94}{39}, \frac{1}{3}, \frac{46}{39}, -\frac{89}{39}, \frac{160}{39}, -\frac{71}{39}, -\frac{149}{39}, \frac{157}{39}, \frac{43}{39}, \frac{70}{39}, \frac{4}{39}\right\}$$

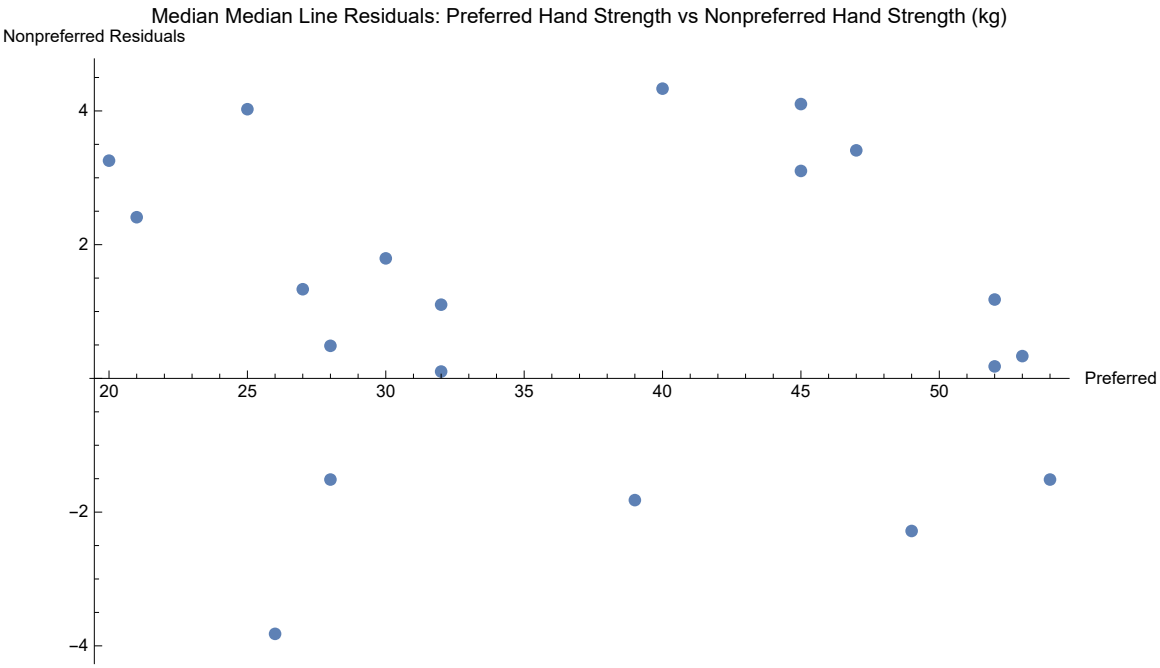
residuals = Transpose[{preferred, listred}]

$$\left\{\left\{28, -\frac{59}{39}\right\}, \left\{27, \frac{4}{3}\right\}, \left\{45, \frac{121}{39}\right\}, \left\{20, \frac{127}{39}\right\}, \left\{40, \frac{13}{3}\right\}, \left\{47, \frac{133}{39}\right\}, \left\{28, \frac{19}{39}\right\}, \left\{54, -\frac{59}{39}\right\}, \left\{52, \frac{7}{39}\right\}, \left\{21, \frac{94}{39}\right\}, \left\{53, \frac{1}{3}\right\}, \left\{52, \frac{46}{39}\right\}, \left\{49, -\frac{89}{39}\right\}, \left\{45, \frac{160}{39}\right\}, \left\{39, -\frac{71}{39}\right\}, \left\{26, -\frac{149}{39}\right\}, \left\{25, \frac{157}{39}\right\}, \left\{32, \frac{43}{39}\right\}, \left\{30, \frac{70}{39}\right\}, \left\{32, \frac{4}{39}\right\}\right\}$$

Graph and Sum of Median Median Line Residuals

```
Graphred = ListPlot[residuals,
   PlotLabel → "Median Median Line Residuals: Preferred Hand Strength vs Nonpreferred
      Hand Strength (kg)", AxesLabel → {"Preferred", "Nonpreferred Residuals"}];
```

```
Show[Graphred]
```



I found the total of the residuals by adding the list of residuals from before (just the y values).

```
Total [listred]
```

$$\frac{788}{39}$$

Equating the Least Squares Line

To find the equation of the Least Squares line, you must find B, C, D, E and n, where B is the sum of the products of each x value squared, C is the sum of all x values, D is the sum of the products of each x value multiplied by their corresponding y value, E is the sum of all y values, and n is the number of

points.

```
n = Length[list1]
```

20

```
b = Total[(Table[list1[[q]][[1]], {q, 1, 20}])^2]
```

30 301

```
c = Total[Table[list1[[q]][[1]], {q, 1, 20}]]
```

745

```
d = Total[Table[list1[[q]][[1]] * list1[[q]][[2]], {q, 1, 20}]]
```

27 694

```
Es = Total[Table[list1[[q]][[2]], {q, 1, 20}]]
```

687

$$\left(\frac{d * n - Es * c}{b * n - c^2}\right) * x + \frac{Es * b - d * c}{n * b - c^2}$$

$$\frac{3931}{1085} + \frac{179\,x}{217}$$

$$\text{linesquares} = \frac{3931}{1085} + \frac{179\,x}{217}$$

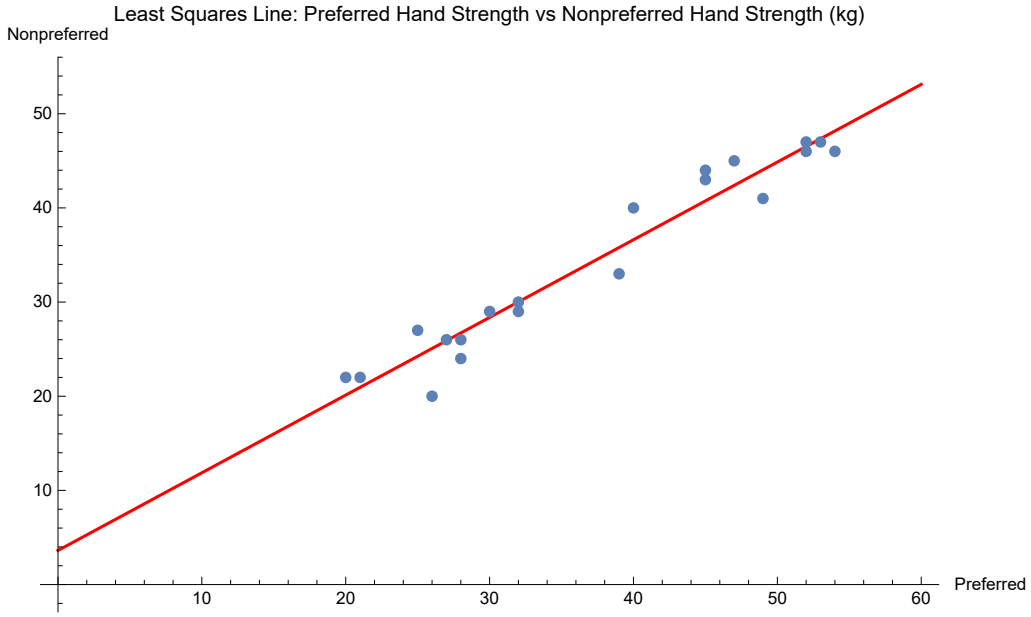
$$\frac{3931}{1085} + \frac{179\,x}{217}$$

This is the equation of the least squares line.

Graph of Least Squares Line

```
Graphsquares = Plot[linesquares, {x, 0, 60}, PlotLabel →
    "Least Squares Line: Preferred Hand Strength vs Nonpreferred Hand Strength (kg)",
   AxesLabel → {"Preferred", "Nonpreferred"}, PlotStyle → {Red}];

Show[Graphsquares, Graph1]
```



Least Squares Line Residuals

$$g[x\_] := \left(\frac{d * n - Es * c}{b * n - c\text{^}2}\right) * x + \frac{Es * b - d * c}{n * b - c\text{^}2}$$

preferred

{28, 27, 45, 20, 40, 47, 28, 54, 52, 21, 53, 52, 49, 45, 39, 26, 25, 32, 30, 32}

nonpreferred

{24, 26, 43, 22, 40, 45, 26, 46, 46, 22, 47, 47, 41, 44, 33, 20, 27, 30, 29, 29}

I plugged all the x-values into the function for the least squares line line to get the y values for each x value on the least squares line.

listyexpsq = Map[g, preferred]

$$\left\{\frac{28\,991}{1085}, \frac{28\,096}{1085}, \frac{1426}{35}, \frac{21\,831}{1085}, \frac{39\,731}{1085}, \frac{45\,996}{1085}, \frac{28\,991}{1085}, \frac{52\,261}{1085}, \frac{50\,471}{1085}, \frac{22\,726}{1085}, \frac{7338}{155}, \frac{50\,471}{1085}, \frac{47\,786}{1085}, \frac{1426}{35}, \frac{5548}{155}, \frac{27\,201}{1085}, \frac{3758}{155}, \frac{4653}{155}, \frac{30\,781}{1085}, \frac{4653}{155}\right\}$$

listredsq = nonpreferred - listyexpsq

$$\left\{-\frac{2951}{1085}, \frac{114}{1085}, \frac{79}{35}, \frac{2039}{1085}, \frac{3669}{1085}, \frac{2829}{1085}, -\frac{781}{1085}, -\frac{2351}{1085}, -\frac{561}{1085}, \frac{1144}{1085}, -\frac{53}{155}, \frac{524}{1085}, -\frac{3301}{1085}, \frac{114}{35}, -\frac{433}{155}, -\frac{5501}{1085}, \frac{427}{155}, -\frac{3}{155}, \frac{684}{1085}, -\frac{158}{155}\right\}$$

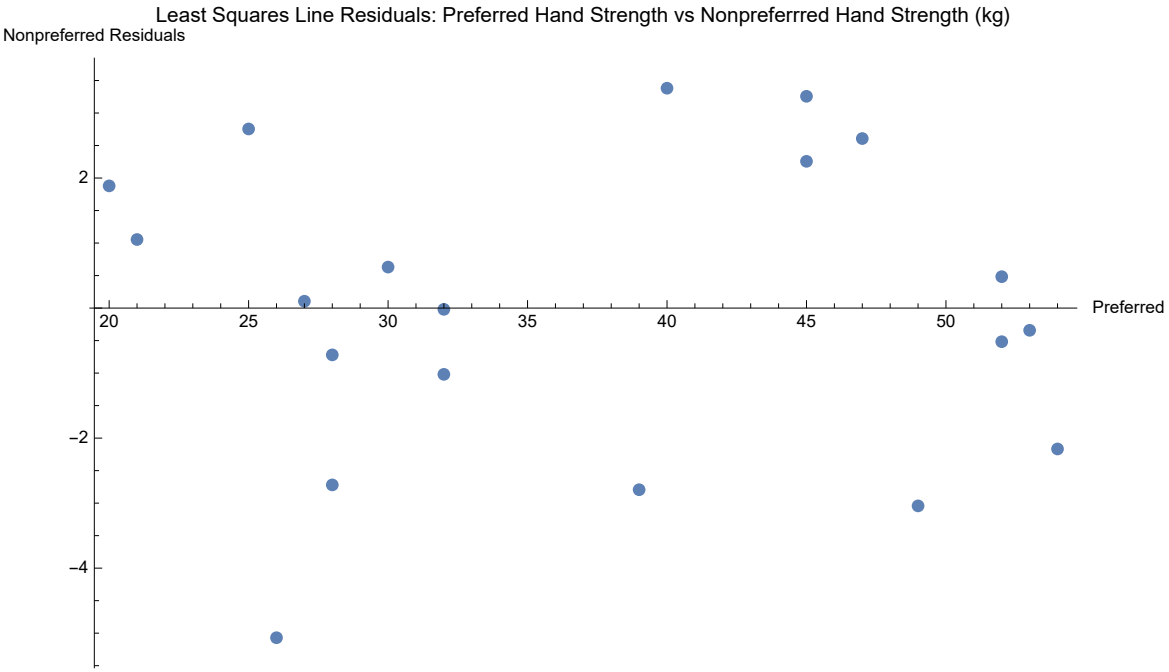
residualssq = Transpose[{preferred, listredsq}]

$$\left\{\left\{28, -\frac{2951}{1085}\right\}, \left\{27, \frac{114}{1085}\right\}, \left\{45, \frac{79}{35}\right\}, \left\{20, \frac{2039}{1085}\right\}, \left\{40, \frac{3669}{1085}\right\}, \left\{47, \frac{2829}{1085}\right\}, \left\{28, -\frac{781}{1085}\right\}, \left\{54, -\frac{2351}{1085}\right\}, \left\{52, -\frac{561}{1085}\right\}, \left\{21, \frac{1144}{1085}\right\}, \left\{53, -\frac{53}{155}\right\}, \left\{52, \frac{524}{1085}\right\}, \left\{49, -\frac{3301}{1085}\right\}, \left\{45, \frac{114}{35}\right\}, \left\{39, -\frac{433}{155}\right\}, \left\{26, -\frac{5501}{1085}\right\}, \left\{25, \frac{427}{155}\right\}, \left\{32, -\frac{3}{155}\right\}, \left\{30, \frac{684}{1085}\right\}, \left\{32, -\frac{158}{155}\right\}\right\}$$

Graph and Sum of Least Squares Line Residuals

```
Graphredsq =
  ListPlot[residualssq, PlotLabel → "Least Squares Line Residuals: Preferred Hand
      Strength vs Nonpreferrred Hand Strength (kg)",
   AxesLabel → {"Preferred", "Nonpreferred Residuals"}];
```

```
Show[Graphredsq]
```



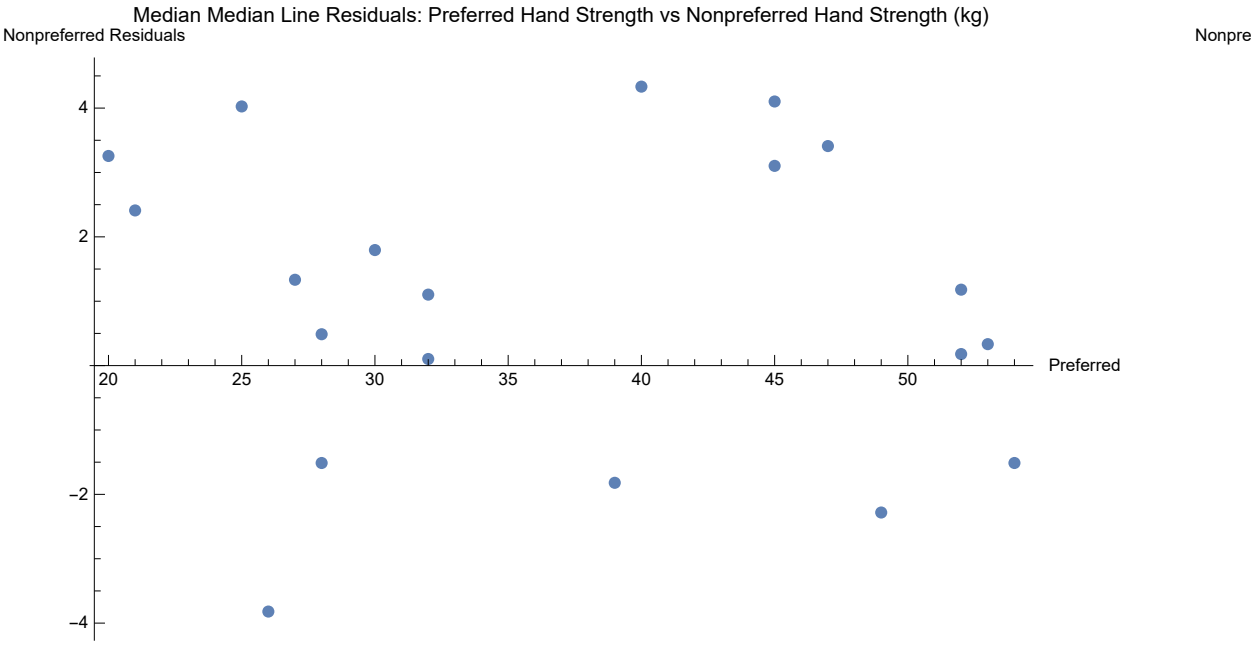
I found the total of the residuals by adding the list of residuals from before (just the y values).

```
Total [listredsq]
```

```
0
```

Comparison of Regression Line Residuals

```
GraphicsRow[{Graphred, Graphredsq}]
```



The sum of the residuals for the median median line was 788/39 while the sum of the residuals for the least squares line was 0 meaning the least squares line was the best line of fit for the data.

Questions

1. What would the predicted grip strength in the non-preferred hand be for a student with preferred hand
strength of 37 for each model?

```
f[37]
```

$$\frac{1292}{39}$$

```
g[37]
```

$$\frac{37\,046}{1085}$$

The predicted non-preferred hand strength for someone with a preferred hand strength of 37 kg would be 1292/39 or approximately 33.1282 kg according to the median median line and 37046/1085 or approximately 34.1438 kg according to the least squares line.

2. Why do the points tend to cluster into 2 groups on your listplot?

The points on the original listplot of the original data tend to cluster around 2 areas: around the preferred hand strength of 25-35 kg and around the preferred hand strength of 50-55 kg. The cluster around these points because according to the data, most of the points are in these ranges. There are 8 points between 25-35 kg of preferred hand strength, all within 1-2 kg of each other with 28 kg repeating

twice and there were 4 points between 50-55 kg all within 1 kg of each other, with 52 kg repeating twice. The modes are in these areas. Perhaps that indicates that the most people tend to have a preferred hand strength around 25-35 kg and around 50-55 kg.

3. Discuss interpolation and extrapolation using specific examples in relation to this regression line.

Interpolation is when one determines an estimation of a value, for instance given a plot of points and an x or y coordinate, when the given coordinate falls within the range of the given data. One could use the regression lines to estimate a value. For instance, question 1 asked for an estimation of a non-preferred hand strength when the preferred hand strength is 37 kg. One can estimate the strength by using the regression lines. The data used to create the regression lines had a range that extended from the preferred hand strength of 20 to 54 kg and a non-preferred hand strength 20 to 47 kg. Since a preferred hand strength of 37 falls within the given range, one would use interpolation to estimate the non-preferred hand strength. For the median median line, the value 37 would be substituted for x in the equation of the median median line at the predicted non-preferred hand strength is approx. 33.1282 kg. To estimate it on the least squares line, one would repeat the process but using the equation for the least square line instead and get the answer off approx. 34.1438 kg. This estimation is more accurate to the given data because the sum of that equation's residuals is 0 which means it is an accurate representation of the data.

Extrapolation is when one determines an estimation of a value, for instance given a plot of points and an x or y coordinate, but the given coordinate falls outside of the range of the given data. In this case, one can make an assumption that a linear regression line continues on forever at the same slope and a curved line would follow continue forever as the equation dictates, i.e. not changing polynomial degrees. For instance, for the given data on hand strength, if one were asked to predict the non preferred hand strength of an individual who had a preferred hand strength of 60 kg, then extrapolation would be used because the range of preferred hand strength of 20 to 54 kg, so 60 kg falls out of that range. Since the regression lines equated above are linear (they do not have any terms with powers higher than 1), to solve this, assume that the line continues straight forever. The one would plug 60 into the equation for the median median line and the least squares line.

```
f[60]
```

$$\frac{2051}{39}$$

```
g[60]
```

$$\frac{8233}{155}$$

The estimated non preferred hand strength for the median median line is 2051/39, approx. 52.5897 kg while for the least squares line, it was 8233/155, approx. 53.1161 kg. The latter estimation is more accurate to the given data because the sum of that equation's residuals is 0 which means it is an accurate representation of the data.

```
Clear[preferred, nonpreferred, list1, Graph1, group1, group2, group3, sum1,
 sum2, sum3, summarypoints, slope, intercept1, intercept2, linemed, Graphmed,
 Graphsum, f, g, listyexp, listred , residuals, Graphred, n, b, c, d, Es,
 linesquares, Graphsquares, listyexpsq , listredsq , residualssq , Graphredsq ]
```