Uber Rocket Problem- Leaping Larry's Luge Launcher (Algebra version)

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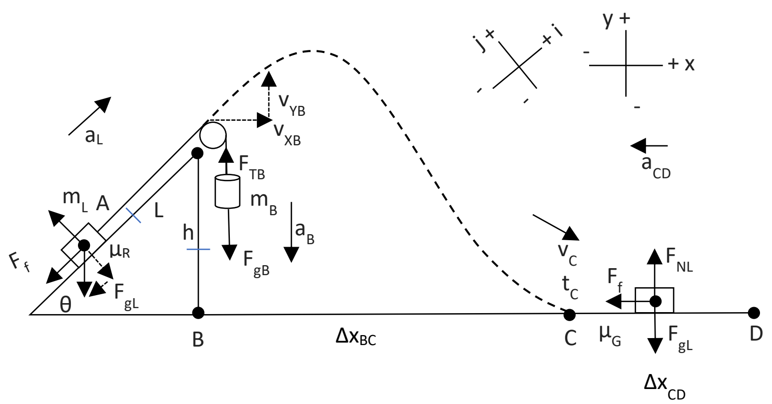
Problem Description

Leaping Larry decided to make a laborious launcher for his luxury luge using a pulley and ramp system (see diagram). His method was to attach one end of a massless stretchless rope to a barrel of rocks and to hold the other end of the rope. He placed the rope over a massless frictionless pulley, and then walked down the ramp far as down possible to point A (where L = h). When he sat in the luge he accelerated up the ramp to point B and then launched off the top at the same angle as the ramp (all while releasing the rope and avoiding the pulley). He flew through the air as a projectile to point C, transitioning all of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any height differences between luge height, barrel height, and size of the pulley, and the diagram is not drawn to scale. Solve for μ_G.

Diagram



Givens and Equations

$m_L = 25$ kg	$\sum F : \vec{F} = m\vec{a}$
$m_B = 38$ kg	$F_f = \mu(F_N)$
$\theta = 37°$	$v_f^{\,2} = v_i^{\,2} + 2a\Delta x$
$\mu_R = 0.20$	$x_f = \frac{1}{2}at^2 + v_i t + x_i$
$h = 8.9$ m	$y = (R)\sin\theta$
$\Delta x_{BC} = 60$ m	$x = (R)\cos\theta$
$V_i = 0$ m/s	$c^2 = a^2 + b^2$
$L = h$	$v_f = v_i + at$
	$F_{T1} = F_{T2}$
	$a_1 = -a_2$

Strategy:

Calculate the acceleration of the luge on the ramp:

List the sum of the forces in the j direction to find F_N.

$$\sum F_j: F_{N_L} - F_{g_L}\cos\theta = m_L a_{L_j}$$

$$\underline{F_N = m_L g(\cos\theta)}$$

Substitute the equation for F_N into the equation for friction.

$$F_f = \mu F_N$$

$$\underline{F_f = \mu(m_L g(\cos\theta))}$$

List the sum of the forces in the i direction and solve for F_{TL}.

$$\sum F_i: F_{TL} - F_f - F_{gL}(\sin\theta) = m_L a_L$$

$$\underline{F_{TL} = \mu(m_L g\cos\theta) + m_L g\sin\theta + m_L(a_L)}$$

List the sum of the forces in the y direction and solve for F_{TB}.

$$\sum F_y: F_{TB} - F_{gB} = m_B a_B$$

$$\underline{F_{TB} = m_B g + m_B a_B}$$

Solve for a_L by setting the equations of F_{TL} and F_{TB} equal to each other and replacing a_B with $-a_L$.

$$F_{T1} = F_{T2} \text{ and } a_1 = -a_2$$

$$m_B g + m_B(-a_L) = \mu(m_L g\cos\theta) + m_L g(\sin\theta) + m_L a_L$$

$$m_B g - \mu(m_L g\cos\theta) - m_L g(\sin\theta) = m_L a_L + m_B(a_L)$$

$$\frac{m_B g - \mu(m_L g\cos\theta) - m_L g(\sin\theta)}{m_B + m_L} = a_L$$

$$\frac{38(9.8) - 0.2(25 * 9.8 * \cos 37) - 25(9.8)(\sin 37)}{25 + 38} = a_L$$

$$\frac{372.4 - 39.1331 - 147.445}{63} = a_L$$

$$\underline{a_L = 2.94956\ \frac{m}{s^2}} \nearrow$$

Find the velocity at point B. The distance traveled on the ramp is equal to the height of the ramp.

$$v_f^{\,2} = v_i^{\,2} + 2a\Delta x$$

$$v_f^{\,2} = 0 + 2(2.94956)(8.9)$$

$$\underline{v_f = 7.24584\frac{m}{s}} \nearrow$$

Use trig to calculate the x and y initial velocities of B.

$$x = (R)\cos\theta$$

$$\underline{v_{xB} = (7.24584)\cos 37\ \frac{m}{s}} \rightarrow$$

$$y = (R)\sin\theta$$

$$\underline{v_{iyB} = (7.24584)\sin 37\ \frac{m}{s}} \uparrow$$

Determine the equations for x dir and y dir using the initial velocities at B and the given height of the ramp.

X dir:

$$x_f = \frac{1}{2}at^2 + v_i t + x_i$$

$$x[t] = \frac{1}{2}(0)t^2 + ((7.24584)(\cos 37))\,t + 0$$

$$\underline{x[t] = ((7.24584)(\cos 37))\,t}$$

Y dir:

$$x_f = \frac{1}{2}at^2 + v_i t + x_i$$

$$y[t] = \frac{1}{2}(-9.8)t^2 + ((7.24584)(\sin 37))\,t + 8.9$$

$$\underline{y[t] = -4.9t^2 + ((7.24584)(\sin 37))\,t + 8.9}$$

Determine the time it takes for the luge to travel from point B to C.

Y dir:

$$y[t] = -4.9t^2 + ((7.24584)(\sin 37))\,t + 8.9$$

$$0 = -4.9t^2 + ((7.24584)(\sin 37))\,t + 8.9\text{, Solver}$$

$$\underline{t = 1.86423\text{ s}}$$

Determine the horizontal distance traveled from point B to C.

$$x[t] = ((7.24584)(\cos 37))\,t$$

$$x[t] = ((7.24584)(\cos 37))\,(1.86423)$$

$$\underline{x = 10.7879\ m}$$

Determine the velocity at point C first by finding the y velocity at C. The initial velocity is the velocity at point B, the acceleration is the acceleration of gravity, and the time was previously found.

$$v_f = v_i + at$$

$$v_y = 7.24584(\sin 37) + (-9.8)(1.86423)$$

$$\underline{v_y = -13.9088\frac{m}{s}} \downarrow$$

Solve for the resultant velocity at C, with the Pythagorean Theorem, using the y velocity and x velocity (which remains constant from point B to C) as the legs of a right triangle.

$$c^2 = a^2 + b^2$$

$$R^2 = (7.24584 * cos37)^2 + (-13.9088)^2$$

$$\underline{R = 15.0646\frac{m}{s}} \searrow$$

Find the x distance from point C to D, using the distance from B to D (given) and the distance from B to C (found above).

$$\Delta X_{BD} - \Delta X_{BC} = \Delta X_{CD}$$

$$60 - 10.7879 = \Delta X_{CD}$$

$$\underline{\Delta X_{CD} = 49.2121\ m}$$

Find the acceleration of the luge from point C to D (x dir).

$$v_f{}^2 = v_i{}^2 + 2a\Delta x$$

$$0 = 15.0646^2 + 2(a)(49.2121$$

$$\frac{-226.942}{2(49.2121)} = a$$

$$\underline{\vec{a}_x = -2.30575\frac{m}{s^2}} \leftarrow$$

Since the acceleration points opposite to the velocity, the luge is slowing down. Remember, the velocity at point C was transferred 100% into the horizontal x direction after hitting the ground.

Create a free body diagram (seen on the diagram, the box on the right hand part of the diagram) of the luge as it slides and list the sum of forces in the y direction to find F_N.

$$\sum F_y\colon F_{N_L} - F_g = m_L a_{L_y}$$

$$F_{N_L} - mg = 0$$

$$F_{N_L} = 25(9.8)$$

$$\underline{F_{N_L} = 245N} \uparrow$$

List the sum of the forces in the x direction, and substitute F_N in the equation of friction. Solve for μ_G.

$$\sum F_x\colon -F_f = m_L a_{L_x}$$

$$-(\mu F_N) = (25 * -2.30575)$$

$$-(\mu(245)) = -57.6439$$

$$\underline{\mu = 0.235281}$$

$$\boxed{\mu = 0.2353}$$