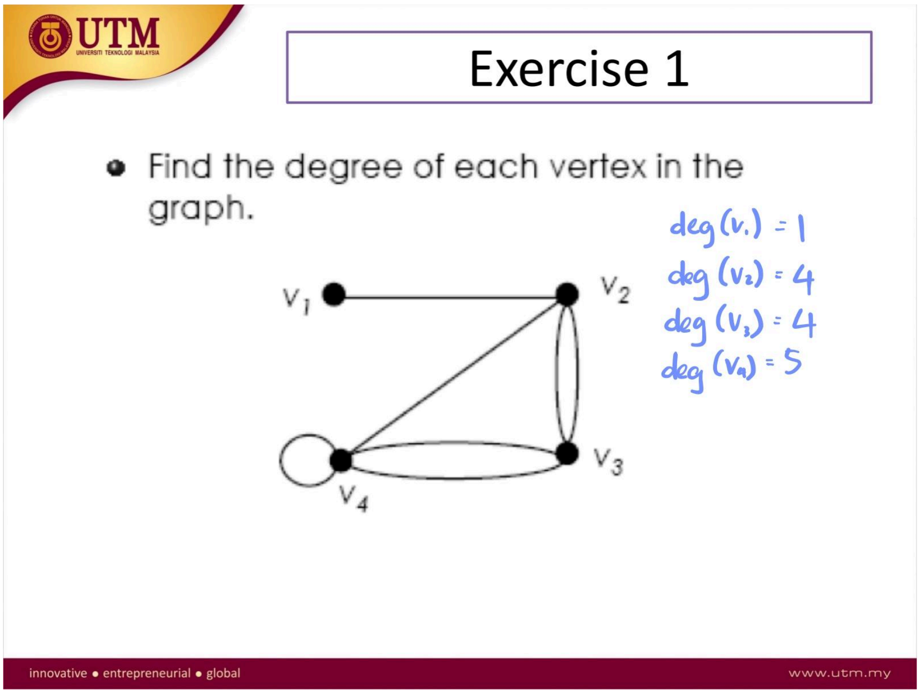
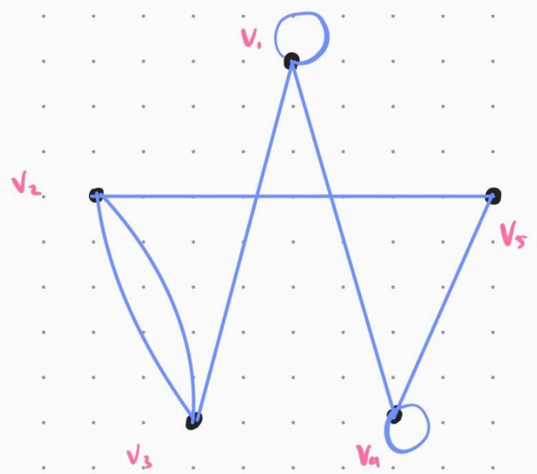
PART 1: GRAPH THEORY

EXERCISE 1



EXERCISE 2

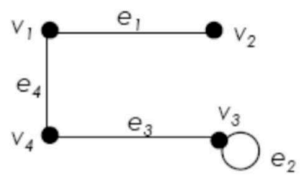


Draw the graph based on the following matrix:

$$A_G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

EXERCISE 3

- Find the adjacency matrix and the incidence matrix of the graph.



Adjacency

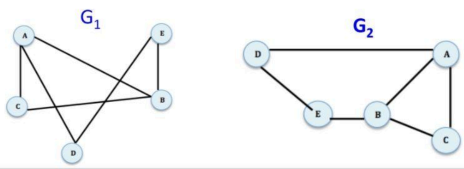
$$A_{G} = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 1 & 0 & 1 \\ v_2 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 0 & 1 & 1 \\ v_4 & 1 & 0 & 1 & 0 \end{array}$$

Incidence

$$I_{G} = \begin{array}{c|cccc} & e_1 & e_2 & e_3 & e_4 \\ \hline v_1 & 1 & 0 & 0 & 1 \\ v_2 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 2 & 1 & 0 \\ v_4 & 0 & 0 & 1 & 1 \end{array}$$

EXERCISE 4

Q: Show that the following two graphs are isomorphic.



- both graphs have same number of vertices and same number of edges
- both graphs have same number of degree for corresponding vertices
  - Vertices A and B from G_1 & vertices A and B from G_2 have degree 3
  - Vertices C, D, E from G_1 & vartices C, D, E in G_2 have degree 2
- both graphs are simple graph and connected
- $f: G_1 \to G_2$

$G_1 = \{A, B, C, D, E\}$ | $f(A) = a$; $f(B) = b$; $f(C) = c$; $f(D) = d$; $f(E) = e$

$G_2 = \{a, b, c, d, e\}$

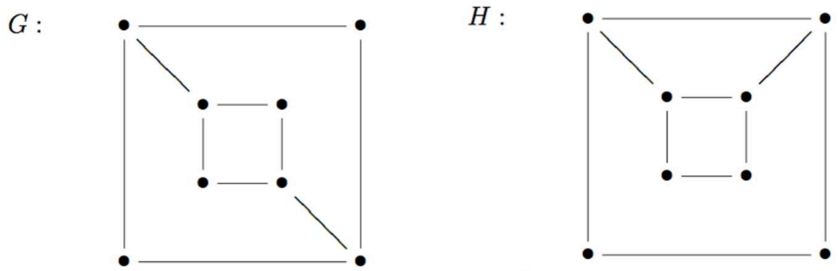
$$A_{G_1} = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 1 & 1 & 0 \\ B & 1 & 0 & 1 & 0 & 1 \\ C & 1 & 1 & 0 & 0 & 0 \\ D & 1 & 0 & 0 & 0 & 1 \\ E & 0 & 1 & 0 & 1 & 0 \end{array} \qquad A_{G_2} = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 1 & 1 & 1 & 0 \\ b & 1 & 0 & 1 & 0 & 1 \\ c & 1 & 1 & 0 & 0 & 0 \\ d & 1 & 0 & 0 & 0 & 1 \\ e & 0 & 1 & 0 & 1 & 0 \end{array}$$

Thus, G_1 & G_2 are isomorphic

EXERCISE 5

Exercise 5

Q: Is these two graphs are isomorphic?



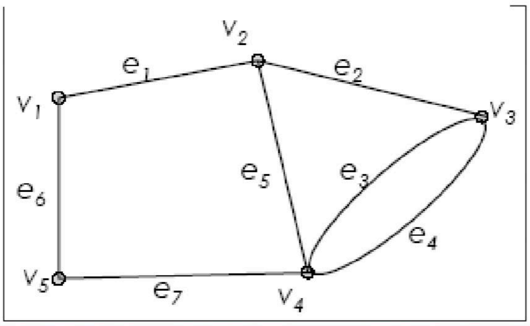
∴ No. The corresponding vertices in both graph don't have the same degree.

EXERCISE 6

Exercise 6

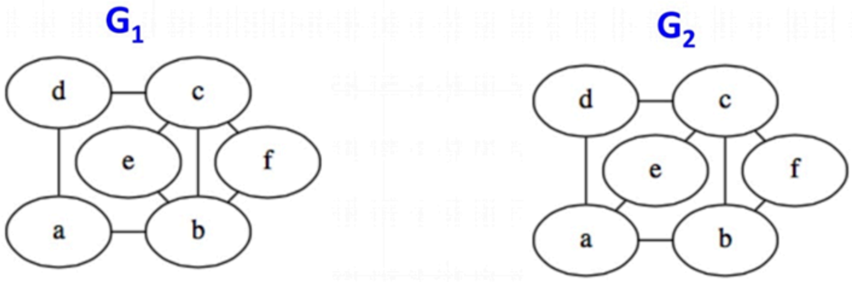
Tell whether the following is either a trail, path, circuit, simple circuit or none of these.

- $(v_2, e_2, v_3, e_3, v_4, e_4, v_3)$ trail
- $(v_4, e_7, v_5, e_6, v_1, e_1, v_2, e_2, v_3, e_3, v_4)$ simple circuit
- $(v_4, e_4, v_3, e_3, v_4, e_5, v_2, e_1, v_1, e_6, v_5, e_7, v_4)$ circuit



EXERCISE 7

Q: Which of the following graphs has Euler circuit?
Justify your answer.



G1

Vertex	a	b	c	d	e	f
Degree	2	4	4	2	2	2

∴ Every vertex has even degree.
Thus G_1 has Euler circuit.

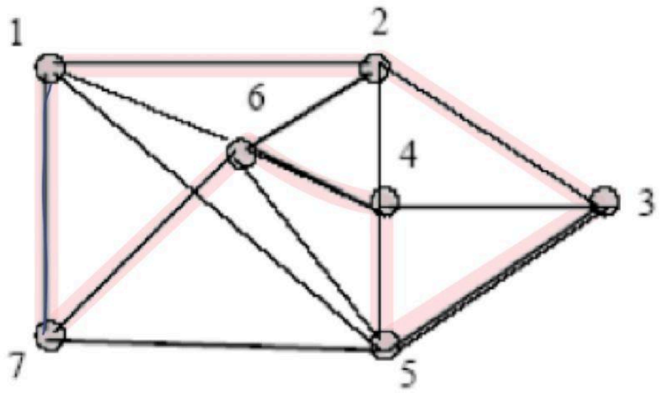
G2

Vertex	a	b	c	d	e	f
Degree	3	3	4	2	2	2

∴ Two vertices have odd degree.
Thus G_2 has Euler trail.

EXERCISE 8

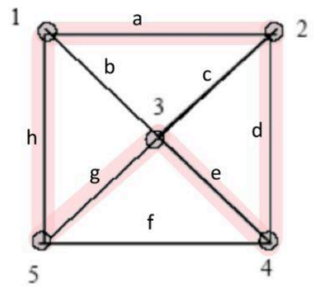
Question: Is this graph has Hamiltonian cycle?



This graph has Hamiltonian cycle.
(1, 2, 3, 5, 4, 6, 7)

EXERCISE 9

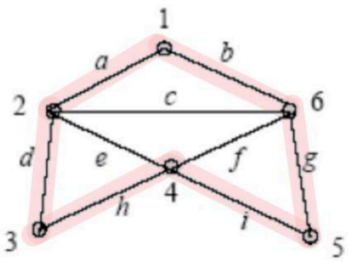
Question: Prove that this graph has Hamiltonian circuit.



The Hamiltonian circuit is (1, a, 2, d, 4, e, 3, g, 5, h, 1)

EXERCISE 10

Find a Hamiltonian circuit in this graph.

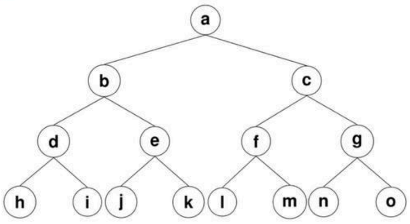


(1, b, 6, g, 5, i, 4, h, 3, d, 2, a, 1)

PART 2 : TREES

EXERCISE 1

Exercise 1



Find:

- Ancestors of m
- Descendents of g
- Parent of *e*
- Children of *e*
- Sibling of *h*

- Ancestors of m = {m, f, c, a}
- Descendants of g = {g, n, o}
- Parent of e = {b}
- Children of e = {j, k}
- Sibling of h = {i}

EXERCISE 2

Exercise 2

- How many matches are played in a tennis tournament of 27 players?

Total edges = total matches played

Total nodes = total players

∴ Total matches played = Total players − 1

= 27 − 1

= 26 matches

EXERCISE 3

Exercise 3

Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.)

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leaves = 1000 $\quad$ $l = n - i$

$m = 2$ $\quad$ $= (mi+1) - i$

$i = ?$

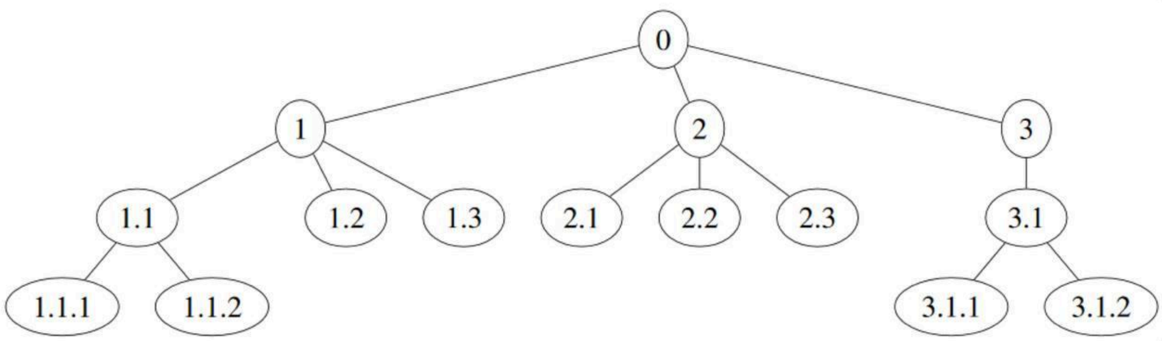
$1000 = (2i+1) - i$

$1000 = i+1$

$i = 999$

EXERCISE 4

Exercise 4



Find the lexicographic ordering of the above tree.

$0 < 1 < 1.1 < 1.1.1 < 1.1.2 < 1.2 < 1.3 < 2 < 2.1 < 2.2 < 2.3 < 3 < 3.1$

$< 3.1.1 < 3.1.2$

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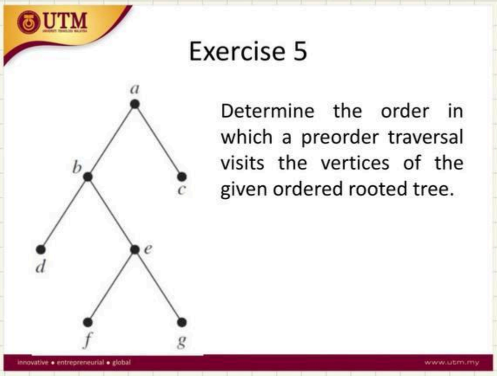
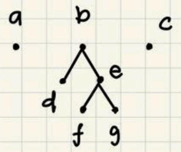
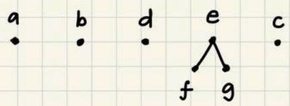
EXERCISE 5



Preorder Traversal : Root , Left subroot, Right subroot





a b d e f g c

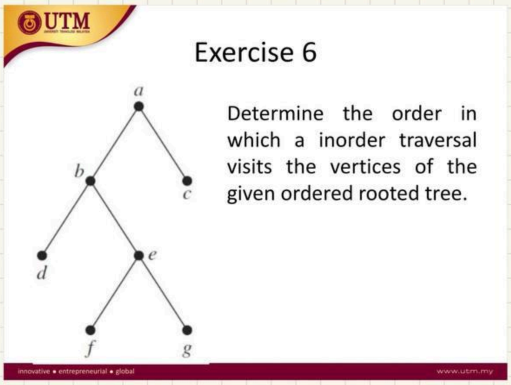
EXERCISE 6

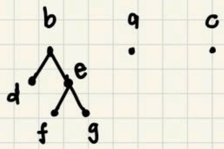
Inorder Traversal :

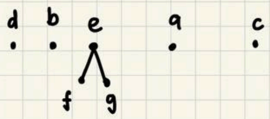
Left subroot

Root

Right subroot

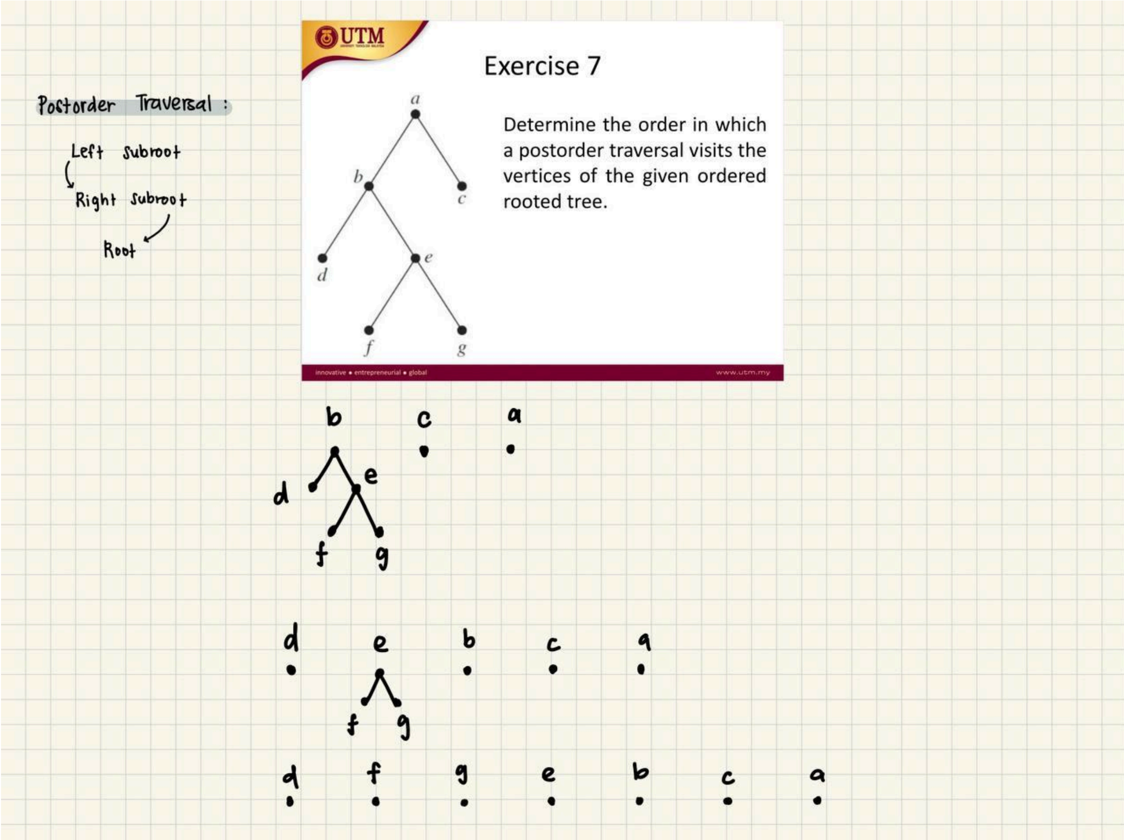




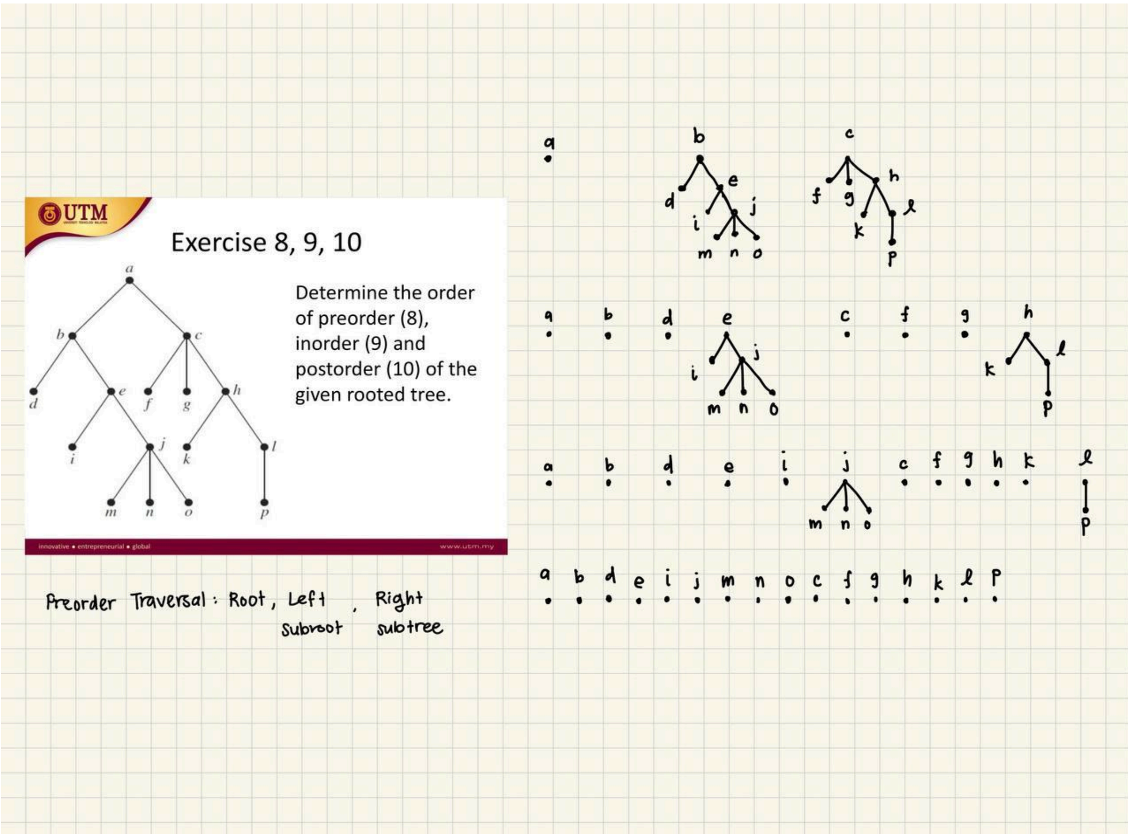


d b f e g a c

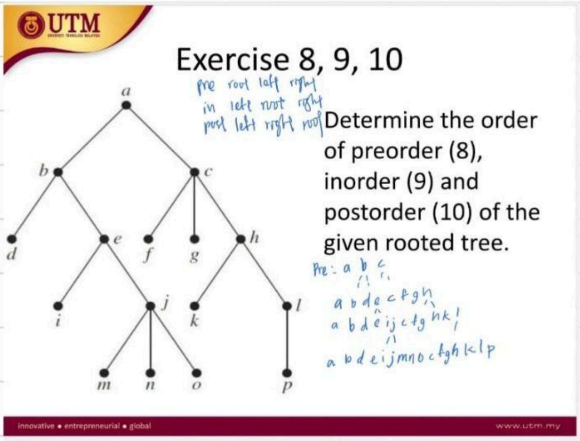
EXERCISE 7



EXERCISE 8

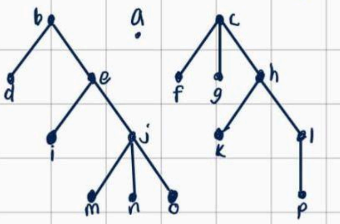


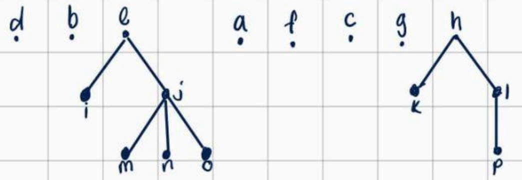
EXERCISE 9

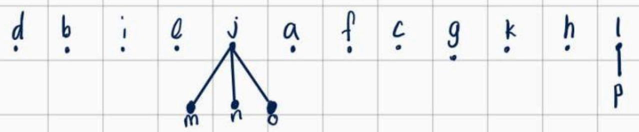


Exercise 9

inorder : left root right

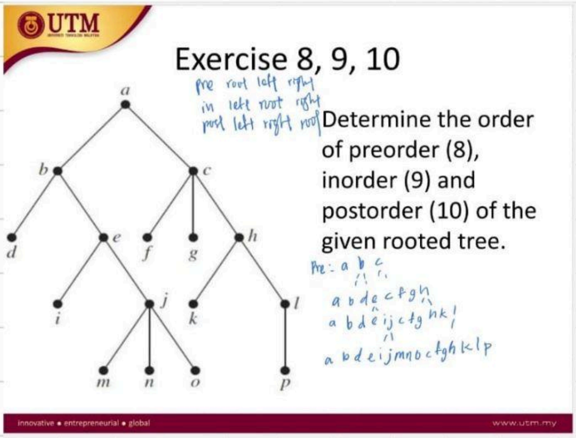






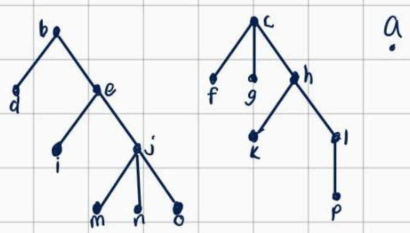
∴ d b i e m j n o a f c g k h p l

EXERCISE 10

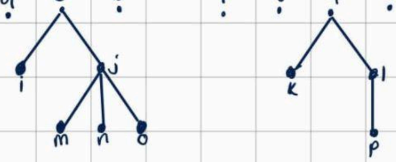


Exercise 10

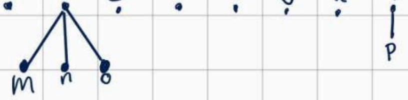
Postorder: left right root



d e b f g h c a

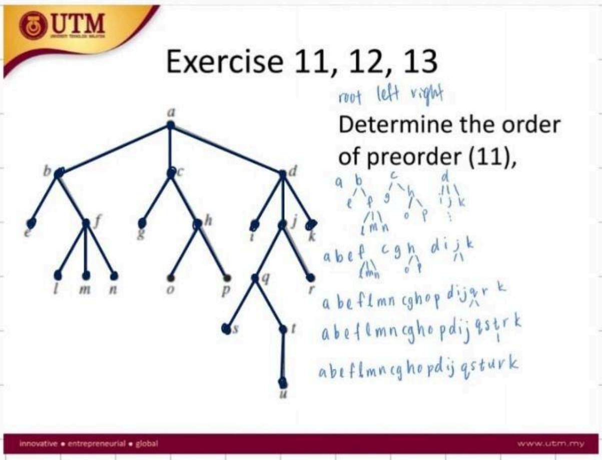


d i j e b f g k l h c a



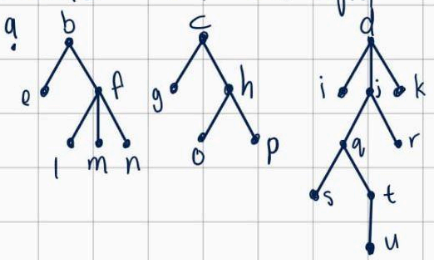
∴ d i m n o j e b f g k p l h c a

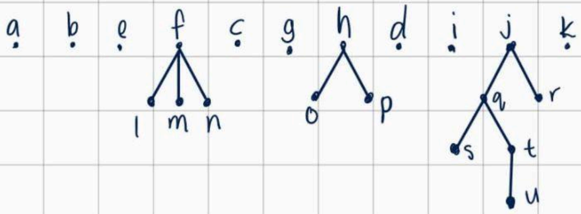
EXERCISE 11

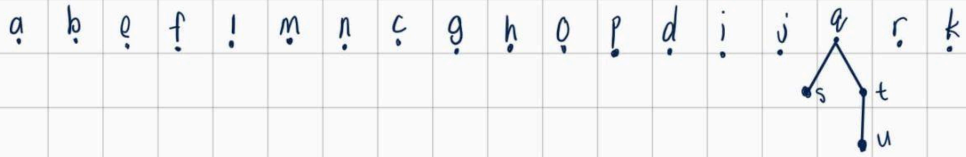


Exercise 11

Preorder: root left right



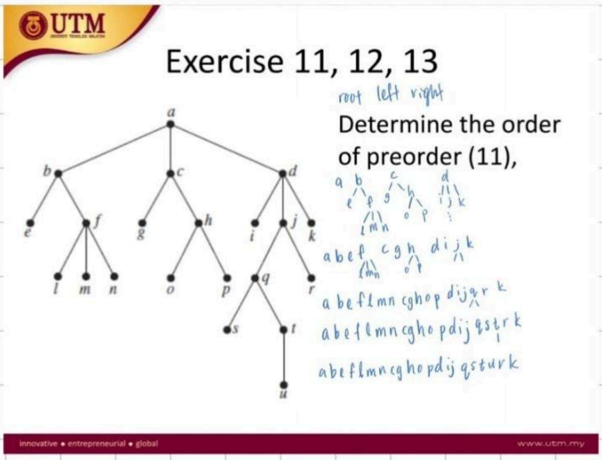




a b e f l m n c g h o p d i j q s t r k
u

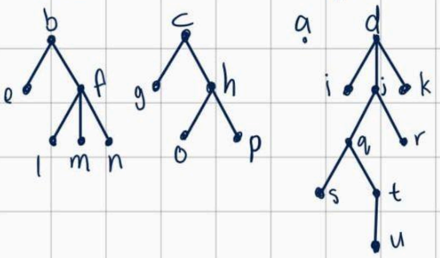
a b e f l m n c g h o p d i j q s t u r k

EXERCISE 12

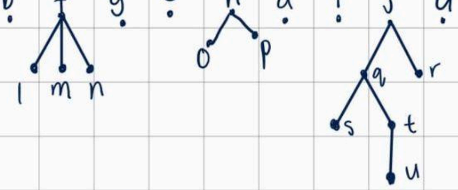


Exercise 12

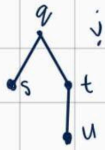
inorder : left root right



e b f g c h a i j d k



e b l m f n g c o h p a i q j r d k

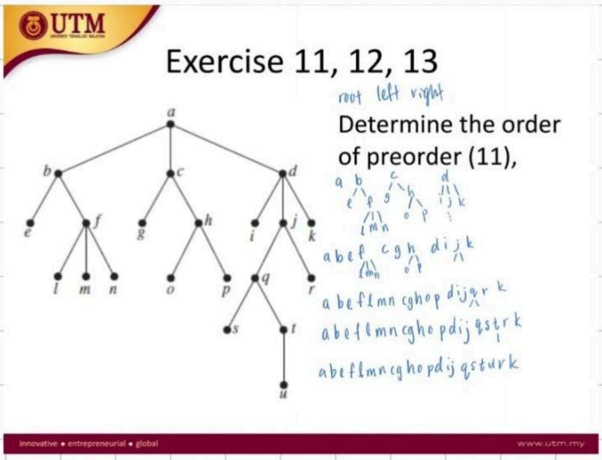


e b l m f n g c o h p a i s q t j r d k

u

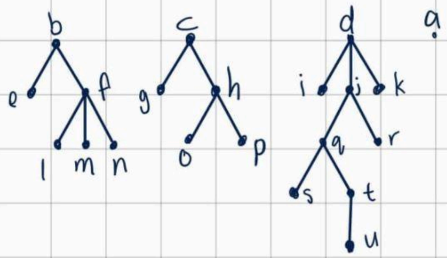
e b l m f n g c o h p a i s q u t j r d k

EXERCISE 13



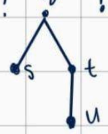
Exercise 13

Postorder : left right root



e f b g h c i j k d a

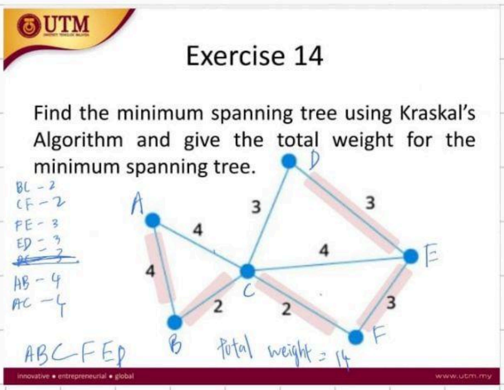
e l m n f b g o p h c i q r j k d a



e l m n f b g o p h c i s t q r j k d a

e l m n f b g o p h c i s t u q r j k d a

EXERCISE 14



Edges:

BC – 2

CF – 2

FE – 3

ED – 3

DC – 3

AB – 4

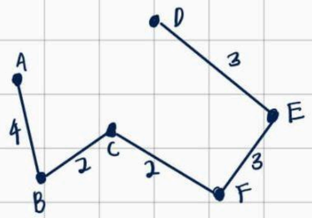
AC – 4

CE – 4

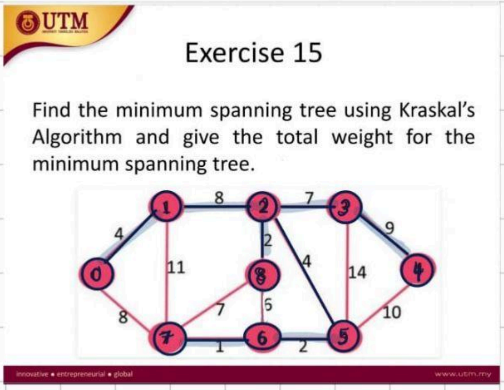
indicates selected edges

Total weight = 2+2+3+3 + 4 = 14

Minimum spanning tree:



EXERCISE 15



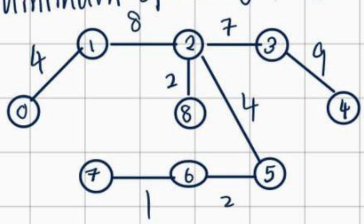
Edges

6,7 — 1	2,3 — 7
2,8 — 2	0,7 — 8
6,5 — 2	1,2 — 8
0,1 — 4	3,4 — 9
2,5 — 4	4,5 — 10
6,8 — 6	1,7 — 11
7,8 — 7	3,5 — 14

indicates selected edges

Total weight = 1+2+2+4+4+7+8+9 = 37

Minimum spanning tree :



PART 3: SHORTEST PATH
EXERCISE 1

Chapter 4 - Part 3

EXERCISE 1

Iteration	S	N	L(S)	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(T
0	{ }	{S,A,B,C,D,E,F,T}	0	∞	∞	∞	∞	∞	∞	∞
1	{S}	{A,B,C,D,E,F,T}	$\boxed{0}$	9	14	15	∞	∞	∞	∞
2	{S,A}	{B,C,D,E,F,T}	$\boxed{0}$	$\boxed{9}$	14	15	33	∞	∞	∞
3	{S,A,B}	{C,D,E,F,T}	$\boxed{0}$	$\boxed{9}$	$\boxed{14}$	15	32	44		
4	{S,A,B,C}	{D,E,F,T}	$\boxed{0}$	$\boxed{9}$	$\boxed{14}$	$\boxed{15}$	32	35		59
5	{S,A,B,C,D}	{E,F,T}	$\boxed{0}$	$\boxed{9}$	$\boxed{14}$	$\boxed{15}$	$\boxed{32}$	34		57
6	{S,A,B,C,D,E}	{F,T}	$\boxed{0}$	$\boxed{9}$	$\boxed{14}$	$\boxed{15}$	$\boxed{32}$	$\boxed{34}$	45	40
7	{S,A,B,C,D,E,T}	{F}	$\boxed{0}$	$\boxed{9}$	$\boxed{14}$	$\boxed{15}$	$\boxed{32}$	$\boxed{34}$	45	$\boxed{40}$

Shortest path: $S \to B \to D \to E \to T$

$S \to B : 14$

$B \to D : 18$

$D \to E : 2$

$E \to T : 6$

$S \to B \to D \to E \to T : 14 + 18 + 2 + 6 = 40 = L(T)$

EXERCISE 2

Exercise 2

Iteration	S	N	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(G)	L(H)
0	{ }	{A,B,C,D,E,F,G,H}	0	∞						
1	{A}	{B,C,D,E,F,G,H}	$\boxed{0}$	1	2					
2	{A,B}	{C,D,E,F,G,H}	$\boxed{0}$	$\boxed{1}$	2	6	4			
3	{A,B,C}	{D,E,F,G,H}	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	5	4	6		
4	{A,B,C,E}	{D,F,G,H}	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	5	$\boxed{4}$	6	11	
5	{A,B,C,D,E}	{F,G,H}	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	$\boxed{5}$	$\boxed{4}$	6	11	
6	{A,B,C,D,E,F}	{G,H}	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	$\boxed{5}$	$\boxed{4}$	$\boxed{6}$	11	8
7	{A,B,C,D,E,F,H}	{G}	$\boxed{0}$	$\boxed{1}$	$\boxed{2}$	$\boxed{5}$	$\boxed{4}$	$\boxed{6}$	11	$\boxed{8}$

Shortest path: A → C → F → H

A → C: 2

C → F: 4

F → H: 2

A → C → F → H : 2+4+2 = 8 = L(H)