SECI1013: DISCRETE STRUCTURES
SESSION 2024/2025 – SEMESTER 1

ASSIGNMENT 4 (CHAPTER 4 AND 5)

INSTRUCTIONS:

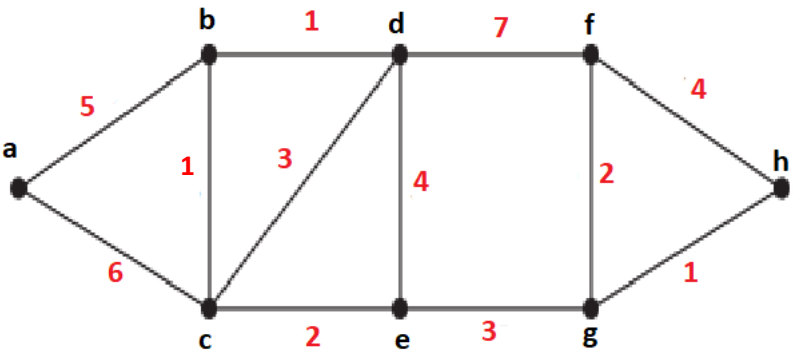
1. This assignment must be conducted in a group (3 or 4 students). Please clearly write the group members name & matric number in the front-page of the submission.
2. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in rejection of the submission of assignment.
3. For submission, scan and combine all answer/solution sheets as one PDF file. Then only **ONE** group member needs to submit on behalf of the group via e-learning **(Refer deadline in elearning)**
4. This assignment has 120 marks contribute 5% of overall course marks.

STRUCTURES:

1. Chapter 4 (4.7): Shortest Path Problem [25 Marks]
2. Chapter 4 (4.8): Tree [30 Marks]
3. Chapter 5 (5.1 and 5.2): Finite Automata [65 Marks]

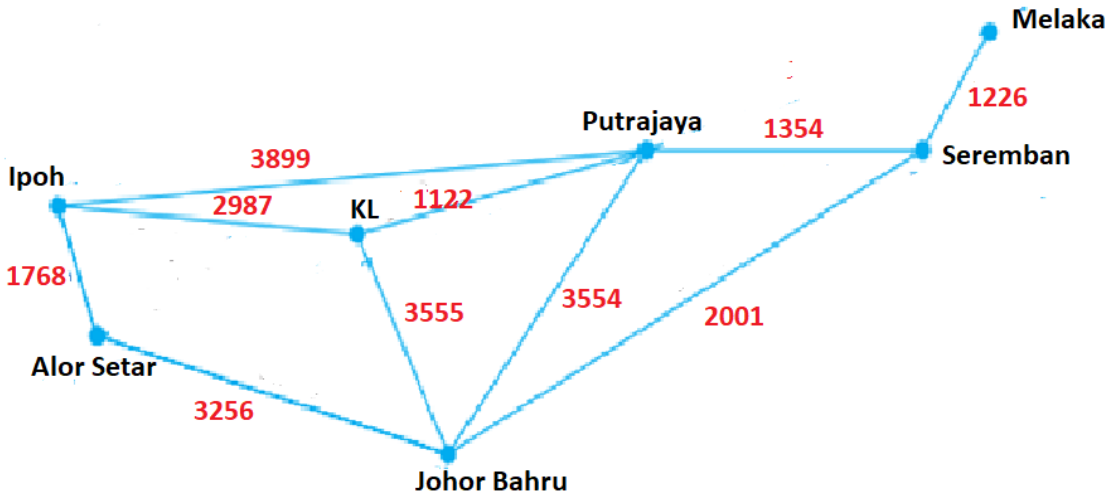
Chapter 4 (4.7): Shortest Path Problem [25 Marks]

1. Find the shortest path between a and h in the weighted graphs given. Show all the work procedures of Dijkstra's algorithm.



(10 Marks)

2. Given is the distance between an important city in Malaysia. The table shows the cost of transportation and time for travelling between cities.



Destination	Aeroplane		Car	
	Cost	Time spent	Cost	Time spent
Ipoh – Putrajaya	1500	1 H 30 MIN	700	4 H 30 MIN
Ipoh – Alor Setar	500	0 H 25 MIN	100	2 H 30 MIN
Ipoh – KL	1000	1 H 00 MIN	500	4 H 00 MIN
Alor Setar – Johor Bahru	1400	1 H 25 MIN	700	5 H 00 MIN
KL – Johor Bahru	2000	2 H 00 MIN	1000	6 H 00 MIN
KL – Putrajaya	500	0 H 30 MIN	100	2 H 30 MIN
Johor Bahru – Putrajaya	2000	2 H 00 MIN	1000	6 H 00 MIN

Johor Bahru – Seremban	2000	2 H 10 MIN	1000	6 H 20 MIN
Putrajaya – Seremban	500	0 H 20 MIN	100	2 H 30 MIN
Seremban – Melaka	500	0 H 20 MIN	100	2 H 00 MIN

a. Find the shortest route (in the distance) between
   i. Ipoh to Melaka
   ii. Alor Setar to Melaka

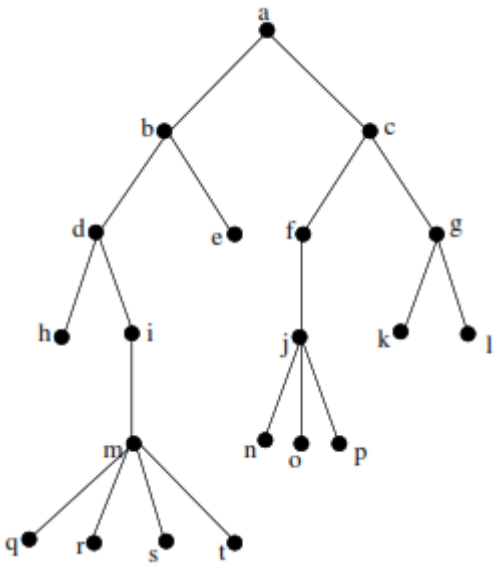
b. Find the cheapest way using an Aeroplane if we consider the shortest time traveling and distance between
   i. Ipoh to Melaka
   ii. Alor Setar to Melaka

c. Find the cheapest way using the car if we consider the shortest time travelling and distance between
   i. Ipoh to Melaka
   ii. Alor Setar to Melaka

(15 Marks)

Chapter 4 (4.8): Tree [30 Marks]

1. Answer the following questions based on the rooted tree shown below:



a. List the children of vertex j.
b. List the ancestors of vertex s.
c. List the siblings of vertex q.

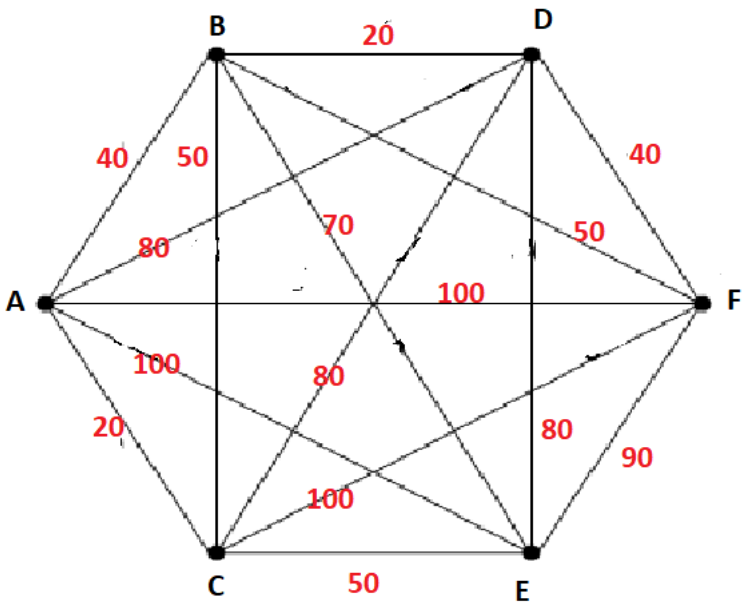
d. Find the number of leaves in this rooted tree.
e. List all level 3 vertices in this rooted tree.
f. Find the least m for which this tree is a rooted m-ary tree.
g. Find the height of this rooted tree.
h. Find the order that which you would visit the vertices of this tree if you use postorder traversal to visit the vertices.
i. Find the order that which you would visit the vertices of this tree if you use preorder traversal to visit the vertices.
j. Find the order that which you would visit the vertices of this tree if you use in-order traversal to visit the vertices.

(10 Marks)

2. A chain letter starts when a person sends a letter to 5 people. Each person sends the letter to 5 other people who have never received it or did not send it to anyone. Suppose that 20,000 people send out the letter before the chain ends and that no one receives more than one letter. How many people receive the letter? How many people do not send it out?

(10 Marks)

3. Find the minimum spanning tree using Kruskal's algorithm and provide the overall weight of the MST. Show all the procedures of the Kruskal algorithm and give the reason behind the path you choose.



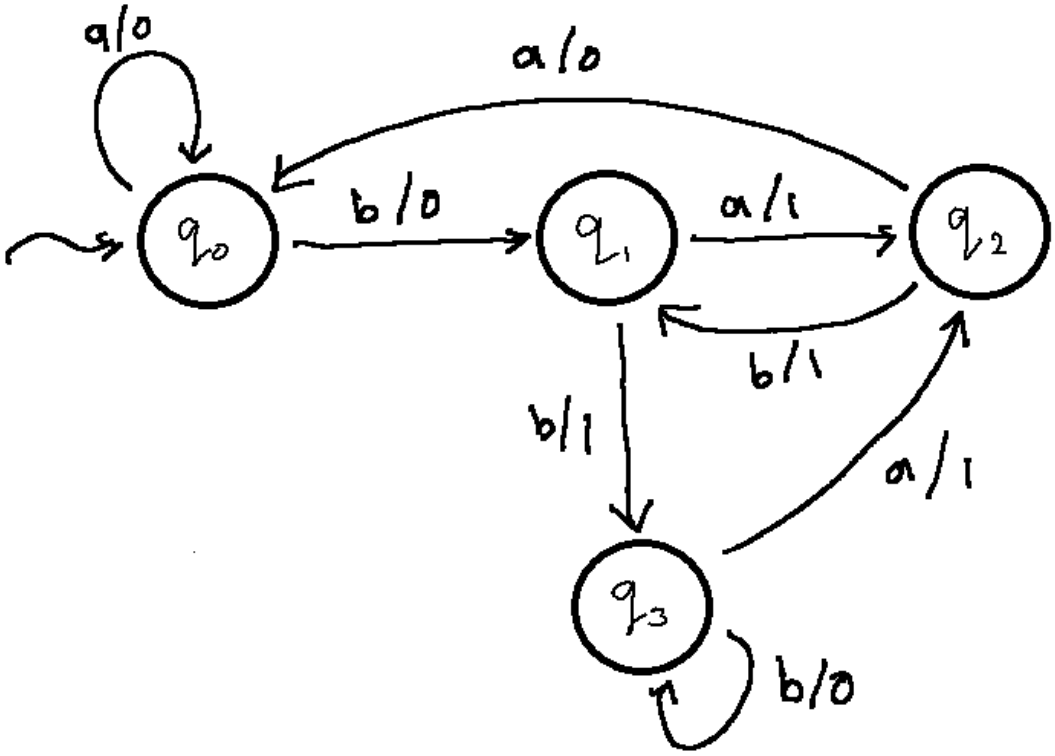
(10 Marks)

Chapter 5 (5.1 and 5.2): Finite Automata [65 Marks]

Question 1 [15 Marks]

A simple drink vending machine sells the can drinks that cost RM 2.00. The machine only accepts the coins of 50 and 100 cents. If the total coins exceed RM 2.00, the machine will accept the overpayment and will not give the change. Represent the vending machine with the input string is {50 and 100} and the initial state must be when no coin is put into the vending machine (where the cost is still RM 2.00). Represent the process for the drink vending machine using the DFA.

Question 2 [12 Marks]



State graph, G1

i. Draw the state transition table and the output function for machine M based on the state graph G1. [4 Marks]
ii. Write the output sequence of the input string and determine whether it will be accepted by the machine or not.
    a. abbaaab [4 Marks]
    b. bbbaababb [4 Marks]

Question 3 [8 Marks]

Let M = $\{\{q_0, q_1, q_2\}, \{a\}, q_0, f_s, \{q_1\}\}$, where f_s is defined as follows:

$$f_s\ (q_0, a) = q_1$$
$$f_s\ (q_1, a) = q_2$$
$$f_s\ (2, a) = q_2$$

i. Deterministic Finite Automata is a quintuple of M = {S, I, q_0, f_s, F}, where;
   - S → the finite states
   - I → the input alphabet
   - q_0 → the initial states
   - f_s → the state transition function
   - F → the final states

   State the elements that involve in M. [2M]

ii. Construct a transition table for the DFA given based on 1 (i). [3M]

iii. Shows the transition diagram that is extracted from 1 (ii). [3M]

Question 5 [20 Marks]

A machine M is written as M = $\{\{s_0, s_1\}, \{a, b\}, \{0, 1\}, s_0, f_s, f_0\}$, with the state transition function and output function as follows:

$$f_s\ (s_0, a) = s_1 \qquad f_0\ (s_0, a) = 1$$
$$f_s\ (s_0, b) = s_1 \qquad f_0\ (s_0, b) = 1$$
$$f_s\ (s_1, a) = s_0 \qquad f_0\ (s_1, a) = 0$$
$$f_s\ (s_1, b) = s_1 \qquad f_0\ (s_1, b) = 1$$

i. Draw a transition table of machine, M. [3M]

ii. Shows the transition diagram that for machine M. [5M]

iii. Shows the output sequence for the input and determine whether the output is accepted by machine M or not.

   a. abbab [4M]
   b. bbaa [4M]
   c. baaba [4M]

Question 6 [10 Marks]

ATM is a computerized machine that provides bank customers to gain access to their accounts using magnetic encoded plastic card and code number. It enables the customer to perform online transactions without involving cashier, clerk and bank teller. The customer make cash withdrawal, check account balances, transfer money as well as purchase prepaid mobile phone credit by using ATM card.

Typical PIN based ATM has following processes:

• Insert ATM card to establish interface. The card will be validated to ensure the correct ATM card is inserted. If the card is unsuccessfully validated, the card is ejected and ATM session is terminated.

• For valid ATM card, user is asked to enter the PIN and press the execution key for the system to match the PIN, if PIN does not matches, then user access is denied to the next stage and he or she is requested to repeat the operation with the correct PIN for a fixed two retries.

• If after the third time the PIN is unmatched, the card will be locked and ATM session is terminated.

• If the PIN matches then the transaction interface is displayed and user has to select the transaction that they wish to perform.

• After the machine performs the chosen transaction, it will prompt user if a new transaction is to be performed. If the response is 'Yes', the transaction interface is again displayed and if 'No' the transaction is terminated and card ejected and ATM session is terminated.

Based on the above description, the states, inputs and outputs are as follow:

States:

S_1: Welcome screen	S_6: Choosing transaction screen
S_2: Validating card	S_7: Performing transaction
S_3: 1st PIN entering screen	S_8: Asking for other transaction screen
S_4: 2nd PIN entering screen	S_9: Terminate ATM session
S_5: 3rd PIN entering screen	

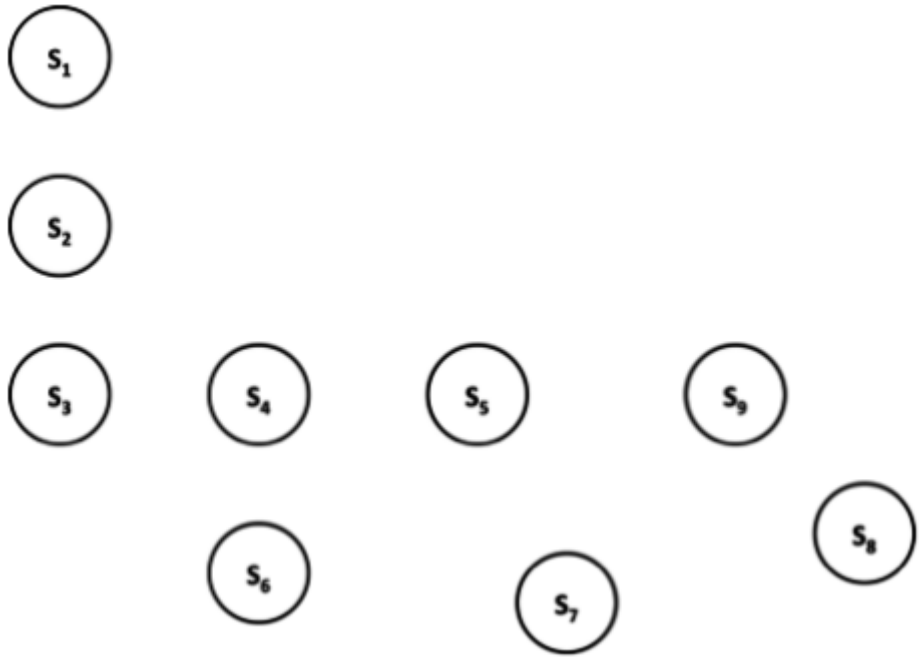
Inputs:

A: Insert ATM card	F: Transaction is selected
B: Valid ATM card	G: Complete transaction
C: Invalid ATM card	H: Yes for other transaction
D: Correct PIN	I: No for other transaction screen
E: Incorrect PIN	

States:

0: Nothing happened
1: Eject ATM card
2: Lock ATM card

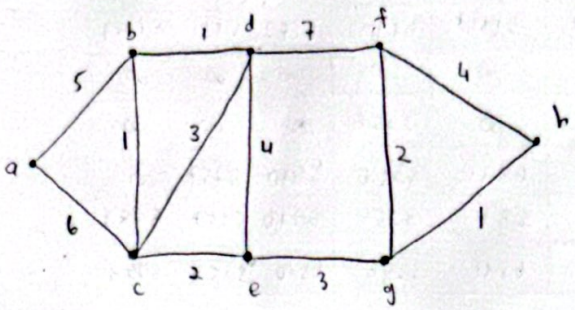
Redraw the transition diagram for the above system in your answer booklet and complete it.



ASSIGNMENT 4

(HAPUA (4-7)

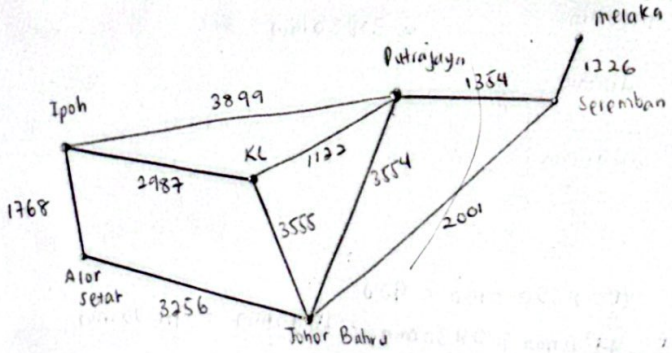
1.



Heration	S	N	L(a)	L(b)	L(c)	L(d)	L(e)	L(f)	L(g)	L(h)
0	{}	{a,b,c,d,e,f,g,h}	0	∞	∞	∞	∞	∞	∞	∞
1	{a}	{b,c,d,e,f,g,h}	0	5	6	∞	∞	∞	∞	∞
2	{a,b}	{c,d,e,f,g,h}	0	5	6	6	∞	∞	∞	∞
3	{a,b,c}	{d,e,f,g,h}	0	5	6	6	8	∞	∞	∞
4	{a,b,c,d}	{e,f,g,h}	0	5	6	6	8	13	∞	∞
5	{a,b,c,d,e}	{f,g,h}	0	5	6	6	8	13	11	∞
6	{a,b,c,d,e,g}	{f,h}	0	5	6	6	8	13	11	12
7	{a,b,c,d,e,g,h}	{f}	0	5	6	6	8	13	11	12

Shortest path : a → c → e → g → h = 12

2.



a) i. Ipoh to melaka

Iteration	S	N	L(I)	L(AS)	L(KL)	L(JB)	L(P)	L(S)	L(M)
0	{}	{I,AS,KL,JB,P,S,M}	0	∞	∞	∞	∞	∞	∞
1	{I}	{AS,KL,JB,P,S,M}	0	1768	2987	∞	3899	∞	∞
2	{I,AS,}	{KL,JB,P,S,M}	0	1768	2987	3256	3899	∞	∞
3	{I,AS,KL}	{JB,P,S,M}	0	1768	2987	3256	3899	∞	∞
4	{I,AS,KL,JB}	{P,S,M}	0	1768	2987	3256	3899	5257	∞
5	{I,AS,KL,JB,P}	{S,M}	0	1768	2987	3256	3899	5253	∞
6	{I,AS,KL,JB,P,S}	{M}	0	1768	2987	3256	3899	5253	6479
7	{I,AS,KL,JB,P,S,M}	{}	0	1768	2987	3256	3899	5253	6479

shortest distance : I → P → S → M = 6479

2. a) ii) Alor Setar to melaka

Iteration	S	N	L(AS)	L(KL)	L(JB)	L(P)	L(S)	L(M)
0	{ }	{AS, KL, JB, P, S, M}	0	∞	∞	∞	∞	∞
1	{AS}	{KL, JB, P, S, M}	0	∞	3256	∞	∞	∞
2	{AS, JB}	{KL, P, S, M}	0	6811	3256	6810	5257	∞
3	{AS, JB, S}	{KL, P, M}	0	6811	3256	6810	5257	6483
4	{AS, JB, S, M}	{KL, P}	0	6811	3256	6810	5257	6483

shortest distance : AS → JB → S → M : 6483

b) i. Ipoh to melaka

shortest distance : I → P → S → M

destination	Aeroplane	
	cost	time
Ipoh → Putrajaya	1500	1H30 min
Putrajaya → Seremban	500	0H20 min
Seremban → melaka	500	0H20 min

cost : 1500 + 500 + 500 = 2500

time : 90 + 20 + 20 = 130 min
= 2H10 min

ii. Alor Setar to melaka

shortest distance : AS → JB → S → M :

destination	aeroplane	
	cost	time
Alor Setar → Johor Bahru	1400	1H00min
Johor Bahru → Seremban	2000	2H10min
Seremban → melaka	500	0H20min

cost : 1400 + 2000 + 500 = 3900

time : 1H + 2H10min + 0H20min
= 3H30min

c) i) Ipoh to melaka

destination	car	
	cost	time
Ipoh → Putrajaya	700	4H30min
Putrajaya → Seremban	100	2H30 min
Seremban → melaka	100	2H00min

cost : 700 + 100 + 100 = 900

time : 4H30 min + 2H30min + 2H00min = 9H 00 min

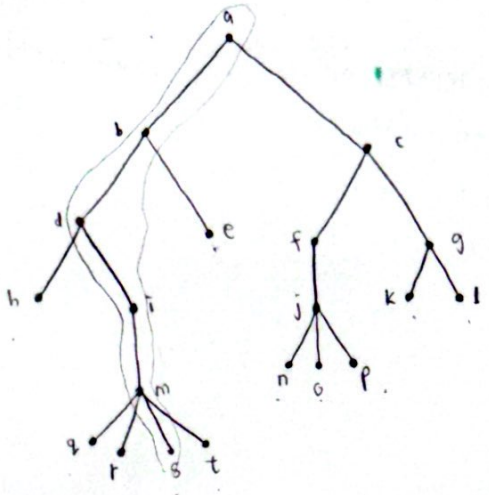
ii) Alor Setar to melaka

destination	car	
	cost	time
Alor Setar → Johor Bahru	700	5H00min
Johor Bahru → Seremban	1000	6H20min
Seremban → melaka	100	2H00min

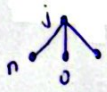
cost : 700 + 1000 + 100 = 1800

time : JB 5H 00 min + 6H 20 min + 2H 00 min = 13H 20 min

(4.8)



a) children of vertex j

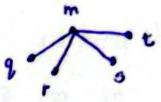


n, o, p are children of j

b) ancestors of vertex s.

a, b, d, i, m are ancestors of s

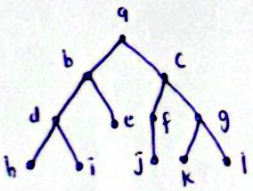
c) siblings of vertex q.



r, s, t are siblings of q

d) number of leaves : 11

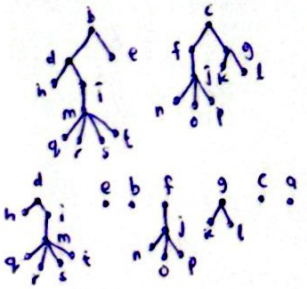
e) list all level 3 vertices



f) m = 4

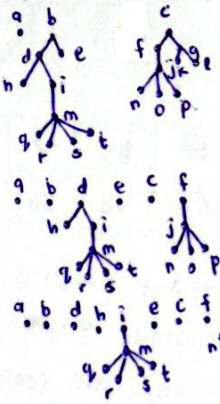
g) height = 5

h) post-order (left, right, root)



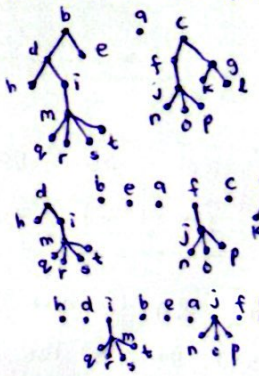
h q r s t m i d e b n o p j f k e g c a

i) preorder (root, left, right)



a b d h i m q r s t e c f j n o p g k l

j) inorder (left, root, right)



h d i b e a j f c k g l

h d m i b e q a n o j p f c k g l

h d q r m s t i b e a n o j p f c k g l

m=5

(2.) A chain letter starts when a person sends a letter to 5 people. Each person sends the letter to 5 other people who have never received it or did not send it to anyone. Suppose that 20 000 people send out the letter i=20 000 before the chain ends and that no one receives more than one letter.

How many people receive the letter? n=?

How many people do not send it out? l=?

using 5-ary tree. m=5

$n = mi + 1$

$= 5(20\,000) + 1$

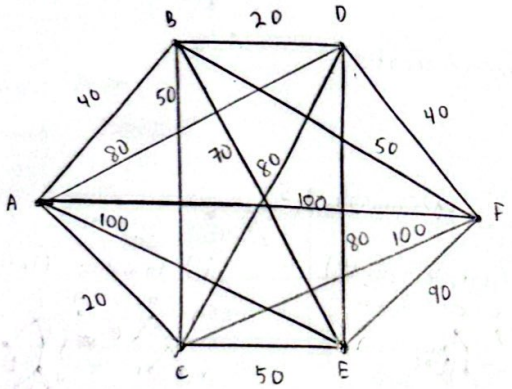
$= 100\,001$ people receive the letter

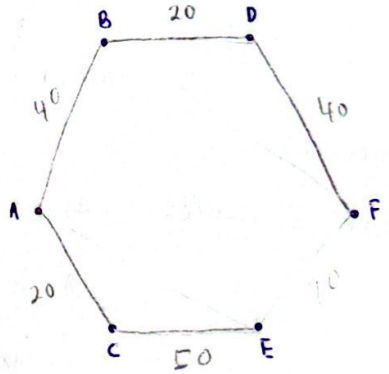
$l = (m-1)i + 1$

$= (5-1)(20\,000) + 1$

$= 80\,000$ do not send it out

3.





AC	BD	AB	DF	BC	CE	BF	BE	CD	DE	AD	AF	AE	CF
20	20	40	40	50	50	50	70	80	80	80	100	100	100

AC and BD → has the least weightage which is 20

AB and DF → the next least weightage which is 30

CE → BC and BF create a cycle when its connected

overall weight of the MST = 50 + 20 + 40 + 20 + 40 = 170

(5.1 and 5.2)

I = {50, 100}

q_0 = no coin is put into the vending machine (2.00)

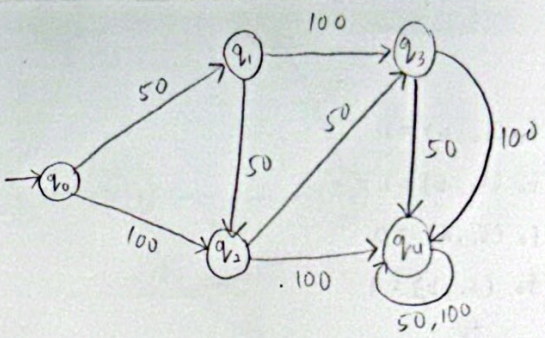
q_1 = accepts 50 cents (1.50)

q_2 = accepts 100 cents (1.00)

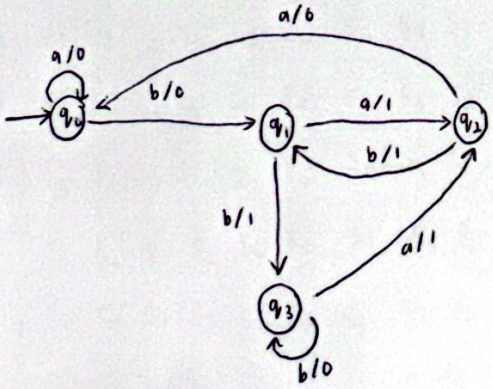
q_3 = accepts 150 cents (0.50)

q_4 = accepts 200 cents (0.00)

f_s	50	100
q_0	q_1	q_2
q_1	q_2	q_3
q_2	q_3	q_4
q_3	q_4	q_4
q_4	q_4	q_4



2)



i)

	f_s		f_o	
	a	b	a	b
q_0	q_0	q_1	0	0
q_1	q_2	q_3	1	1
q_2	q_0	q_1	0	1
q_3	q_2	q_3	1	0

$G1 = \{S, I, O, q_0, f_s, f_o\}$

$S = \{q_0, q_1, q_2, q_3\}$

$I = \{a, b\}$

$O = \{0, 1\}$

q_0 = initial state

ii) a) abbaaab

$q_0 \xrightarrow[0]{a} q_0 \xrightarrow[0]{b} q_1 \xrightarrow[1]{b} q_3 \xrightarrow[1]{a} q_2 \xrightarrow[0]{a} q_0 \xrightarrow[0]{a} q_0 \xrightarrow[0]{b} q_1$: not accepted by machine

Output = 0

b) bbbaabab

$q_0 \xrightarrow[0]{b} q_1 \xrightarrow[1]{b} q_3 \xrightarrow[0]{b} q_3 \xrightarrow[1]{a} q_2 \xrightarrow[0]{a} q_0 \xrightarrow[0]{b} q_1 \xrightarrow[1]{a} q_2 \xrightarrow[1]{b} q_1 \xrightarrow[1]{b} q_3$

: accepted by machine

Output = 1

3) $M = \{\{q_0, q_1, q_2\}, \{a\}, q_0, f_s, \{q_1\}\}$,

$f_s(q_0, a) = q_1$

$f_s(q_1, a) = q_2$

$f_s(2, a) = q_2$

i. $S = \{q_0, q_1, q_2\}$

$I = \{a\}$

$q_0 = q_0$

$f_s = f_s$

$F = q_1$

ii.

f_s	a
q_0	q_1
q_1	q_2
q_2	q_2

iii.

4) $f_s(S_0, a) = S_1$ $\quad f_o(S_0, a) = 1$

$f_s(S_0, b) = S_1$ $\quad f_o(S_0, b) = 1$

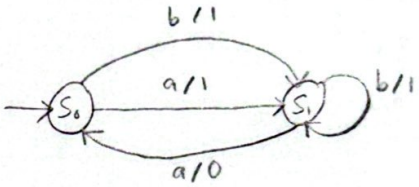
$f_s(S_1, a) = S_0$ $\quad f_o(S_1, a) = 0$

$f_s(S_1, b) = S_1$ $\quad f_o(S_1, b) = 1$

i.

	f_s		f_o	
	a	b	a	b
S_0	S_1	S_1	1	1
S_1	S_0	S_1	0	1

ii.



iii. a) abbab

$S_0 \xrightarrow[1]{a} S_1 \xrightarrow[1]{b} S_1 \xrightarrow[1]{b} S_1 \xrightarrow[0]{a} S_0 \xrightarrow[1]{b} S_1$

output : 1

∴ accepted by machine M

b) bbaa

$S_0 \xrightarrow[1]{b} S_1 \xrightarrow[1]{b} S_1 \xrightarrow[0]{a} S_0 \xrightarrow[1]{a} S_1$

Output : 1

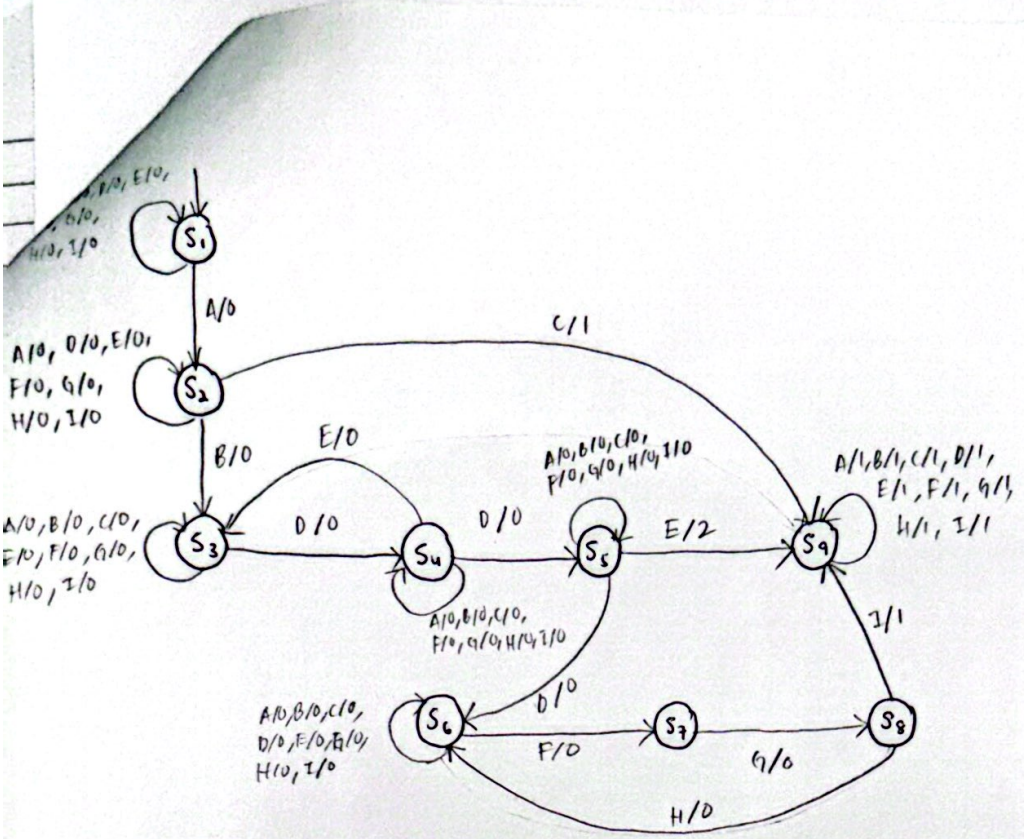
∴ accepted by machine M

c) baaba

$S_0 \xrightarrow[1]{b} S_1 \xrightarrow[0]{a} S_0 \xrightarrow[1]{a} S_1 \xrightarrow[1]{b} S_1 \xrightarrow[0]{a} S_0$

output : 0

∴ not accepted by machine M



	fs									fo								
	A	B	C	D	E	F	G	H	I	A	B	C	D	E	F	G	H	I
S_1	S_2	S_1	S_1	S_1	S_1	S_1	S_1	S_1	S_1	0	0	0	0	0	0	0	0	0
S_2	S_2	S_3	S_9	S_2	S_2	S_2	S_2	S_2	S_2	0	0	1	0	0	0	0	0	0
S_3	S_3	S_3	S_3	S_4	S_3	S_3	S_3	S_3	S_3	0	0	0	0	0	0	0	0	0
S_4	S_4	S_4	S_4	S_5	S_3	S_4	S_4	S_4	S_4	0	0	0	0	0	0	0	0	0
S_5	S_5	S_5	S_5	S_6	S_9	S_5	S_5	S_5	S_5	0	0	0	0	2	0	0	0	0
S_6	S_6	S_6	S_6	S_6	S_6	S_7	S_6	S_6	S_6	0	0	0	0	0	0	0	0	0
S_7	S_7	S_7	S_7	S_7	S_7	S_7	S_8	S_7	S_7	0	0	0	0	0	0	0	0	0
S_8	S_8	S_8	S_8	S_8	S_8	S_8	S_8	S_6	S_9	0	0	0	0	0	0	0	0	1
S_9	S_9	S_9	S_9	S_9	S_9	S_9	S_9	S_9	S_9	1	1	1	1	1	1	1	1	1