

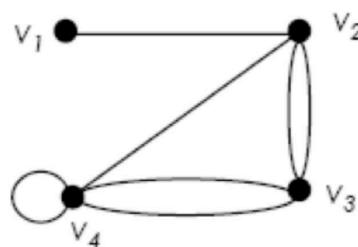
PART 1: GRAPH THEORY

EXERCISE 1



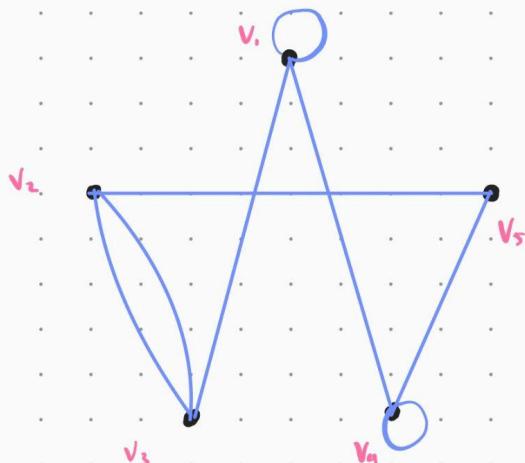
Exercise 1

- Find the degree of each vertex in the graph.



$$\begin{aligned}\deg(v_1) &= 1 \\ \deg(v_2) &= 4 \\ \deg(v_3) &= 4 \\ \deg(v_4) &= 5\end{aligned}$$

EXERCISE 2

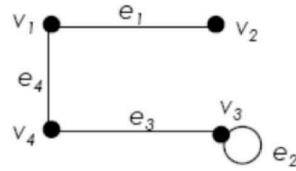


Draw the graph based on the following matrix:

$$A_G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

EXERCISE 3

- Find the adjacency matrix and the incidence matrix of the graph.



Adjacency

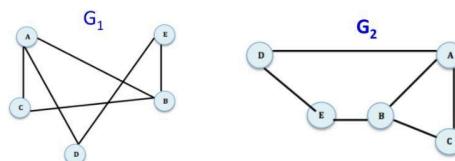
$$A_{G_1} = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 \\ v_1 & 0 & 1 & 0 & 1 \\ v_2 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 0 & 1 & 1 \\ v_4 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Incidence

$$I_{G_1} = \begin{bmatrix} e_1 & e_2 & e_3 & e_4 \\ v_1 & 1 & 0 & 0 & 1 \\ v_2 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 2 & 1 & 0 \\ v_4 & 0 & 0 & 1 & 1 \end{bmatrix}$$

EXERCISE 4

Q: Show that the following two graphs are isomorphic.



- both graphs have same number of vertices and same number of edges

- both graphs have same number of degree for corresponding vertices

• Vertices A and B from G_1 , & vertices A and B from G_2 have degree 3

• Vertices C, D, E from G_1 , & vertices C, D, B in G_2 have degree 2

- both graphs are simple graph and connected

• $f: G_1 \rightarrow G_2$

$$\begin{array}{l|l} G_1 = \{A, B, C, D, E\} & f(A)=a; f(B)=b; f(C)=c; f(D)=d; f(E)=e \\ G_2 = \{a, b, c, d, e\} & \end{array}$$

$$A_{G_1} = \begin{bmatrix} A & B & C & D & E \\ A & 0 & 1 & 1 & 1 & 0 \\ B & 1 & 0 & 1 & 0 & 1 \\ C & 1 & 1 & 0 & 0 & 0 \\ D & 1 & 0 & 0 & 0 & 1 \\ E & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$A_{G_2} = \begin{bmatrix} a & b & c & d & e \\ a & 0 & 1 & 1 & 1 & 0 \\ b & 1 & 0 & 1 & 0 & 1 \\ c & 1 & 1 & 0 & 0 & 0 \\ d & 1 & 0 & 0 & 0 & 1 \\ e & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

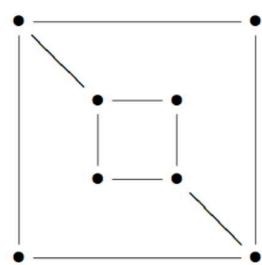
Thus, G_1 & G_2 are isomorphic

EXERCISE 5

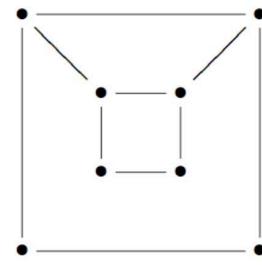
Exercise 5

Q: Is these two graphs are isomorphic?

$G:$



$H:$



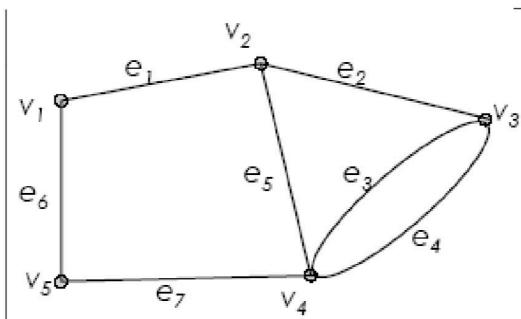
∴ No. The corresponding vertices in both graph don't have the same degree.

EXERCISE 6

Exercise 6

Tell whether the following is either a trail, path, circuit, simple circuit or none of these.

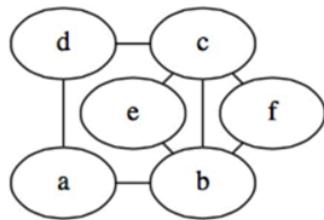
- $(v_2, e_2, v_3, e_3, v_4, e_4, v_3)$ **trail**
- $(v_4, e_7, v_5, e_6, v_1, e_1, v_2, e_2, v_3, e_3, v_4)$ **simple circuit**
- $(v_4, e_4, v_3, e_3, v_4, e_5, v_2, e_1, v_1, e_6, v_5, e_7, v_4)$ **circuit**



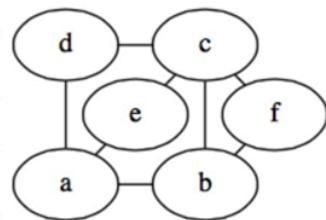
EXERCISE 7

Q: Which of the following graphs has Euler circuit?
Justify your answer.

G_1



G_2



G_1

Vertex	a	b	c	d	e	f
Degree	2	4	4	2	2	2

\therefore Every vertex has even degree.
Thus G_1 has Euler circuit.

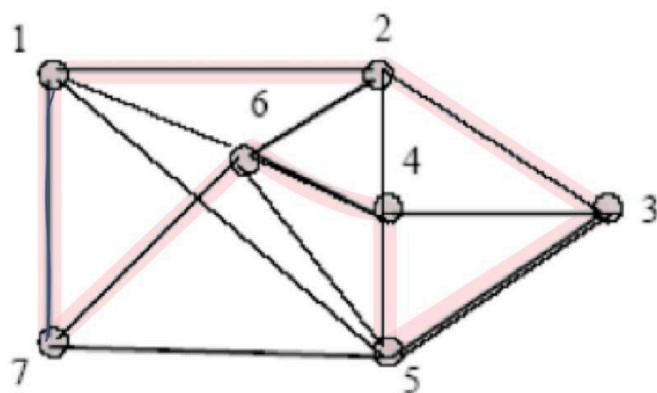
G_2

Vertex	a	b	c	d	e	f
Degree	3	3	4	2	2	2

\therefore Two vertices have odd degree.
Thus G_2 has Euler trail.

EXERCISE 8

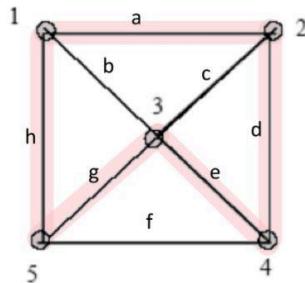
Question: Is this graph has Hamiltonian cycle?



This graph has Hamiltonian cycle.
(1, 2, 3, 5, 4, 6, 7)

EXERCISE 9

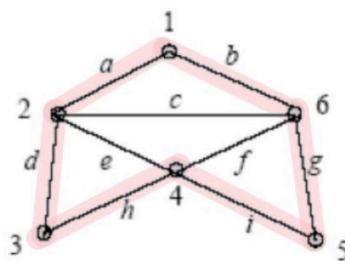
Question: Prove that this graph has Hamiltonian circuit.



The Hamiltonian circuit is $(1, a, 2, d, 4, e, 3, g, 5, h, 1)$

EXERCISE 10

Find a Hamiltonian circuit in this graph.

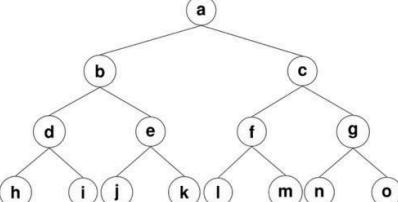


$(1, b, 6, g, 5, i, 4, h, 3, d, 2, a, 1)$

PART 2 : TREES

EXERCISE 1

 Exercise 1



Find:

- Ancestors of m
- Descendants of g
- Parent of e
- Children of e
- Sibling of h

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• Ancestors of m = {m, f, c, a}
• Descendants of g = {g, n, o}
• Parent of e = {b}
• Children of e = {j, k}
• Sibling of h = {i}

EXERCISE 2

 Exercise 2

- How many matches are played in a tennis tournament of 27 players?
 $\text{Total edges} = \text{total matches played}$
 $\text{Total nodes} = \text{total players}$

$$\begin{aligned}\therefore \text{Total matches played} &= \text{Total players} - 1 \\ &= 27 - 1 \\ &= 26 \text{ matches}\end{aligned}$$

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EXERCISE 3

Exercise 3

Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.)

leaves = 1000 $\ell = n - i$
 $m = 2$ $= (m+1) - i$
 $i = ?$

$$1000 = (2^i + 1) - i$$

$$1000 = 2^i + 1$$

$$i = 999$$

EXERCISE 4

Exercise 4

```

graph TD
    0((0)) --- 1((1))
    0 --- 2((2))
    0 --- 3((3))
    1 --- 1_1((1.1))
    1 --- 1_2((1.2))
    1 --- 1_3((1.3))
    2 --- 2_1((2.1))
    2 --- 2_2((2.2))
    2 --- 2_3((2.3))
    3 --- 3_1((3.1))
    3 --- 3_1_2((3.1.2))
    1_1 --- 1_1_1((1.1.1))
    1_1 --- 1_1_2((1.1.2))
  
```

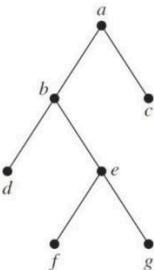
Find the lexicographic ordering of the above tree.

$0 < 1 < 1.1 < 1.1.1 < 1.1.2 < 1.2 < 1.3 < 2 < 2.1 < 2.2 < 2.3 < 3 < 3.1$
 $< 3.1.1 < 3.1.2$

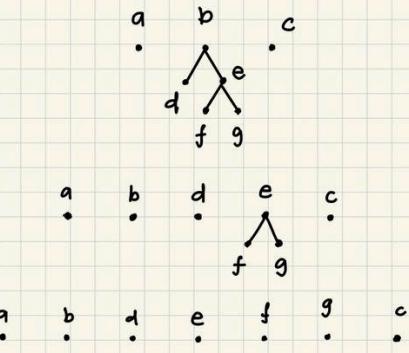
EXERCISE 5

 **Exercise 5**

Determine the order in which a preorder traversal visits the vertices of the given ordered rooted tree.



Preorder Traversal: Root, Left subtree, Right subtree



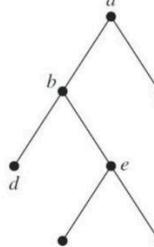
EXERCISE 6

 **Exercise 6**

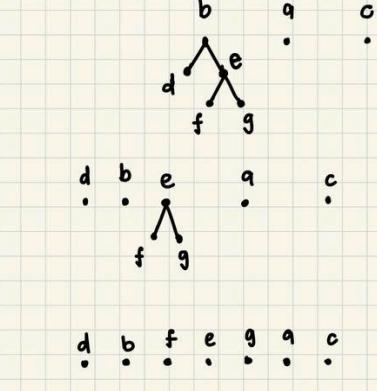
Inorder Traversal :

Left subtree
↳ Root
↳ Right subtree

Determine the order in which an inorder traversal visits the vertices of the given ordered rooted tree.



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EXERCISE 7

Postorder Traversal :

Left Subroot
Right Subroot
Root

Exercise 7

Determine the order in which a postorder traversal visits the vertices of the given ordered rooted tree.

The postorder traversal sequence is: d, f, g, e, b, c, a.

EXERCISE 8

Exercise 8, 9, 10

Determine the order of preorder (8), inorder (9) and postorder (10) of the given rooted tree.

Preorder Traversal: Root, Left Subroot, Right Subtree

Preorder traversal sequence: a, b, d, e, i, m, n, o, f, j, k, h, c, g, l

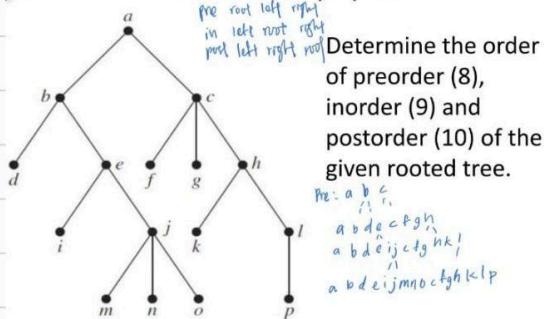
Inorder traversal sequence: m, n, o, i, d, e, f, j, k, l, a, b, c, g, h

Postorder traversal sequence: m, n, o, i, d, e, f, j, k, l, g, h, c, b, a

EXERCISE 9



Exercise 8, 9, 10



Determine the order of preorder (8), inorder (9) and postorder (10) of the given rooted tree.

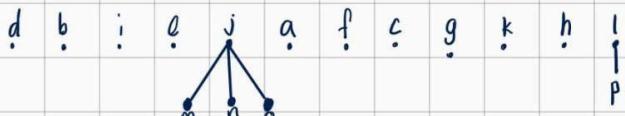
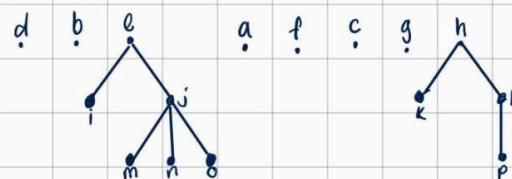
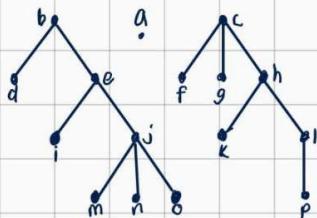
Pre: a b c
In: d e f j g h l
Post: a b d e i j c g h k l p

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Exercise 9

inorder : left root right



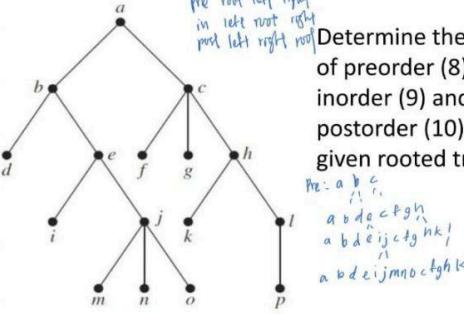
∴ d b i e m n o j a f c g k h l !

EXERCISE 10



Exercise 8, 9, 10

pre root left right
in left root right
post left right root



Determine the order of preorder (8), inorder (9) and postorder (10) of the given rooted tree.

Pre: a b c
d e f g h i j k l p

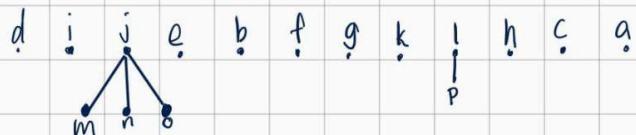
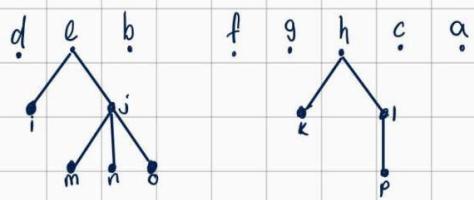
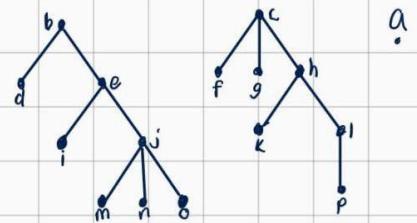
In: a b d e c f g h i j k l p

Post: a b d e i j f g h k l p

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Exercise 10

Postorder: left right root



∴ d i m n o j e b f g k l ! h c a

EXERCISE 11

Exercise 11, 12, 13

root left right

Determine the order of preorder (11),

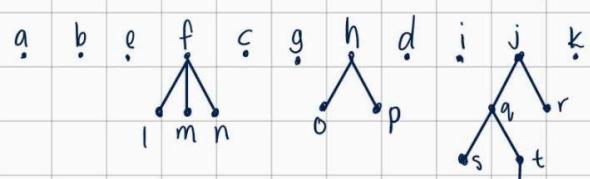
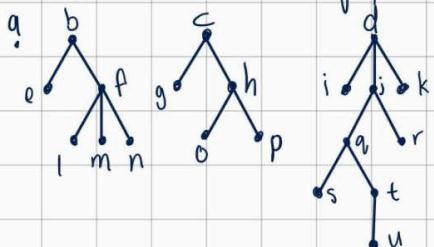
a b c d
e f g h i j k
l m o p q r
s t u

a b e f l m n c g h o p d i j k
a b e f l m n c g h o p d i j g s t r k
a b e f l m n c g h o p d i j g s t r k

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Exercise 11

Preorder: root left right



a b e f ! m n c g h o p d i j k



a b e f ! m n c g h o p d i j ? t r k

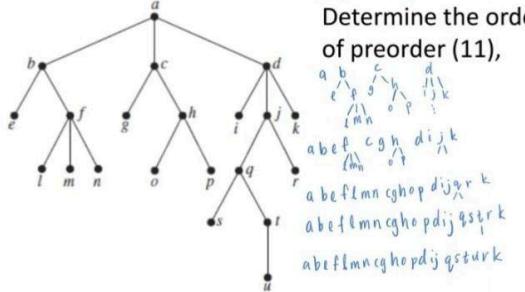
a b e f ! m n c g h o p d i j ? s t u r k

EXERCISE 12

 **Exercise 11, 12, 13**

Determine the order of preorder (11),

root left right



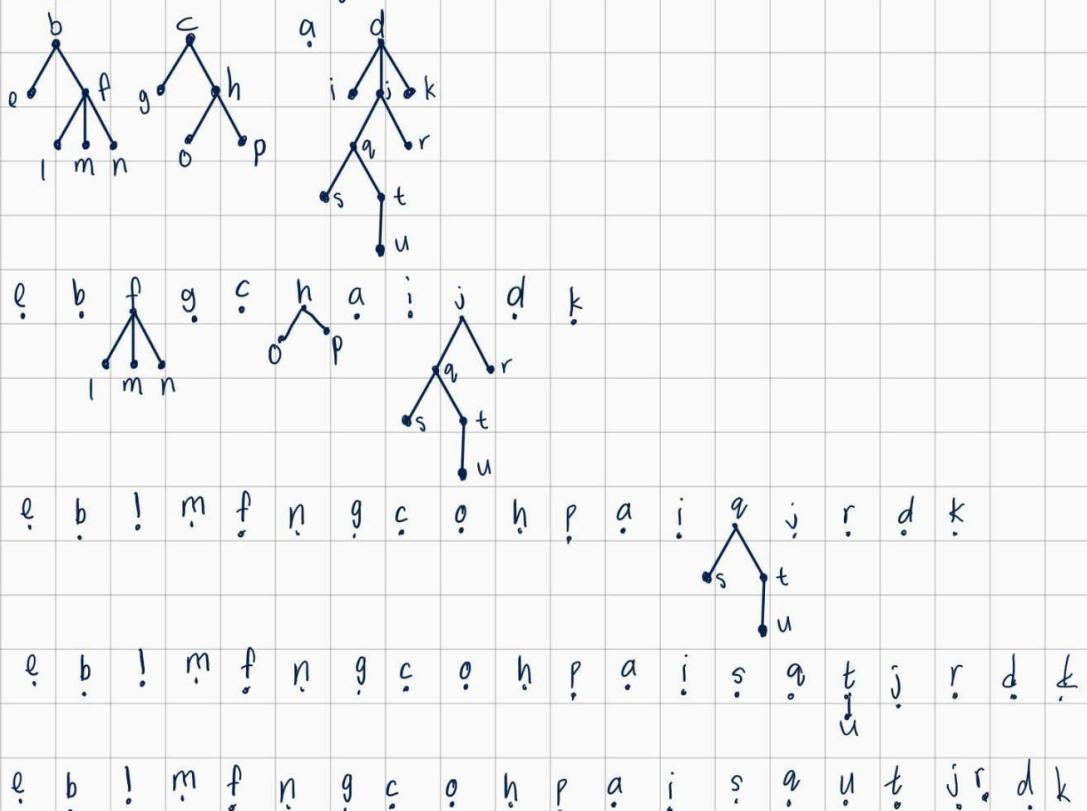
abeflmncghopdijqrstu

abeflmncghopdijqrsturk

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Exercise 12

inorder : left root right



EXERCISE 13

Exercise 11, 12, 13

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Determine the order of preorder (11),

root left right

a b e f l m n c g h o p i s t j r d k u

a b e f l m n c g h o p i s t j r d k u

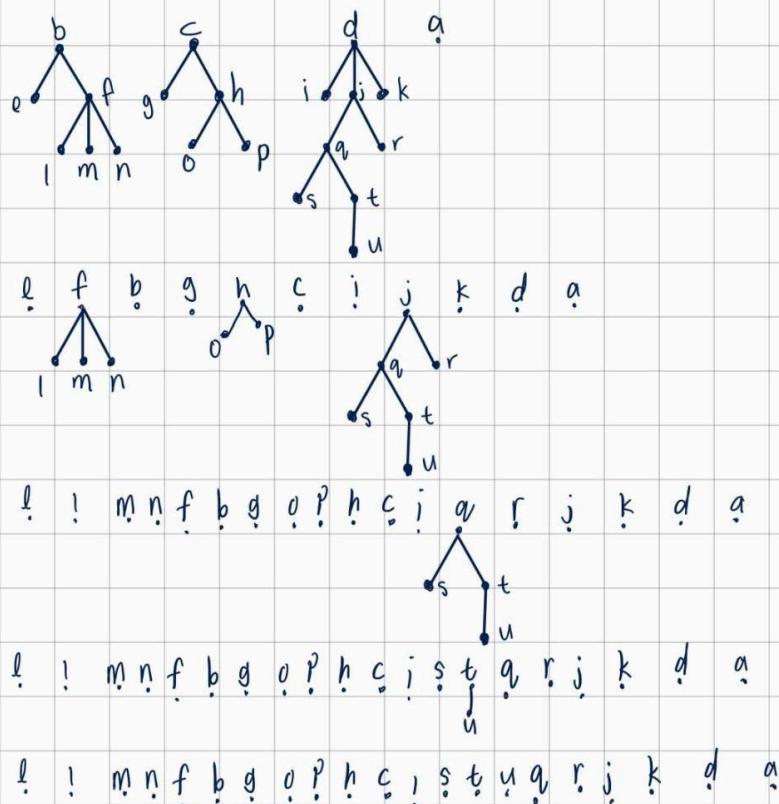
a b e f l m n c g h o p i s t j r d k u

a b e f l m n c g h o p i s t j r d k u

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Exercise 13

Postorder: left right root



EXERCISE 14



Exercise 14

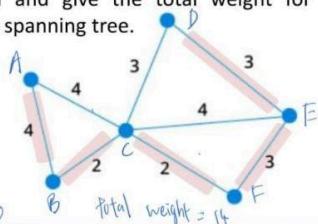
Find the minimum spanning tree using Kruskal's Algorithm and give the total weight for the minimum spanning tree.

~~BC = 2
CF = 2
FE = 3
ED = 3
AB = 4
AC = 4~~

~~ABC-FEP~~

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Edges :

BC - 2

CF - 2

FE - 3

ED - 3

DC - 3

AB - 4

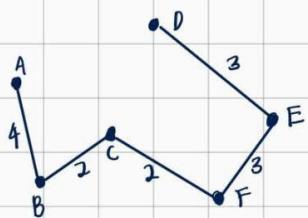
AC - 4

CE - 4

indicates selected edges

$$\text{Total weight} = 2 + 2 + 3 + 3 + 4 = 14$$

Minimum spanning tree :

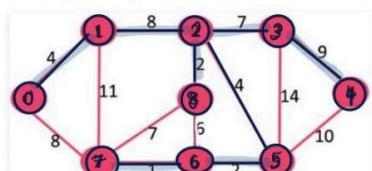


EXERCISE 15



Exercise 15

Find the minimum spanning tree using Kruskal's Algorithm and give the total weight for the minimum spanning tree.



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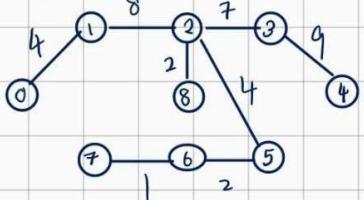
Edges

- 6,7 - 1
- 2,3 - 7
- 2,8 - 2
- 0,7 - 8
- 6,5 - 2
- 1,2 - 8
- 0,1 - 4
- 3,4 - 9
- 2,5 - 4
- 4,5 - 10
- 6,8 - 6
- 1,7 - 11
- 7,8 - 7
- 3,5 - 14

indicates selected edges

$$\text{Total weight} = 1 + 2 + 2 + 4 + 4 + 7 + 8 + 9 = 37$$

Minimum spanning tree :



PART 3: SHORTEST PATH

EXERCISE 1

Chapter 4 - Part 3

EXERCISE 1

Iteration	S	N	L(S)	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(T)
0	{S}	{S, A, B, C, D, E, F, T}	0	∞						
1	{S}	{A, B, C, D, E, F, T}	0	9	14	15	∞	∞	∞	∞
2	{S, A}	{B, C, D, E, F, T}	0	9	14	15	33	∞	∞	∞
3	{S, A, B}	{C, D, E, F, T}	0	9	14	15	32	44		
4	{S, A, B, C}	{D, E, F, T}	0	9	14	15	32	35	59	
5	{S, A, B, C, D}	{E, F, T}	0	9	14	15	32	34	51	
6	{S, A, B, C, D, E}	{F, T}	0	9	14	15	32	34	45	40
7	{S, A, B, C, D, E, T}	{F}	0	9	14	15	32	34	45	40

Shortest path: S → B → D → E → T

S → B : 14

B → D : 18

D → E : 2

E → T : 6

$$S \rightarrow B \rightarrow D \rightarrow E \rightarrow T : 14 + 18 + 2 + 6 = 40 = L(T)$$

EXERCISE 2

Exercise 2

Iteration	S	N	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(G)	L(H)
0	{ }	{A, B, C, D, E, F, G, H}	0	∞						
1	{A}	{B, C, D, E, F, G, H}	0	1	2					
2	{A, B}	{C, D, E, F, G, H}	0	1	2	6	4			
3	{A, B, C}	{D, E, F, G, H}	0	1	2	5	4	6		
4	{A, B, C, E}	{D, F, G, H}	0	1	2	5	4	6	11	
5	{A, B, C, D, E}	{F, G, H}	0	1	2	5	4	6	11	
6	{A, B, C, D, E, F}	{G, H}	0	1	2	5	4	6	11	8
7	{A, B, C, D, E, F, G}	{H}	0	1	2	5	4	6	11	7

Shortest path: A → C → F → H

A → C: 2

C → F: 4

F → H: 2

$$A \rightarrow C \rightarrow F \rightarrow H : 2 + 4 + 2 = 8 = L(H)$$