CHAPTER 1 EXERCISE
GROUP 7

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Example

A={a,b,c,d,e,f,g,h}
B={b,d,e}
C={a,b,c,d,e}
D={r,s,d,e}

Proper subset of A ??

Ans: $B \subseteq A$
$C \subseteq A$
$D \nsubseteq A$

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Exercise

Determine whether each pair of sets is equal

{1, 2, 2, 3}, {1, 3, 2}

Ans: Yes, they are equal

Exercise

let $X = \{1, 2, 2, \{1\}, 9\}$
$|X| = 4$
$|P(X)| = 2^4 = 16$

If M is finite, determine the $|M|$

a) $M = \{1, 2, 3, 4\}$
b) $M = \{4, 4, 4\}$
c) $M = \{\}$
d) $M = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

a) $|M| = 4$
b) $|M| = 1$
c) $|M| = 0$
d) $|M| = 3$

Exercise

- Let,

$U = \{ \cancel{a}, b, \cancel{c}, d, e, \cancel{f}, g, h, i, j, k, l, \cancel{m} \}$

$A = \{ a, \cancel{c}, f, \cancel{m} \}$

$B = \{ b, \cancel{c}, g, h, \cancel{m} \}$

$(A \cap B) = \{c, m\}$

$|A \cup B| = |A| + |B| - |A \cap B|$
$= 4 + 5 - 2$
$= 7$

- Find:

$|A \cup B|$, $A - B$ dan A'.

$A - B = \{a, f\}$
$A' = \{b, d, e, g, h, i, j, k, l\}$

Exercise

- Let A, B and C be sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$

- Prove that $B = C$

$$B = B \cap (A \cup B)$$
$$= B \cap (A \cup C)$$
$$= (B \cap A) \cup (B \cap C)$$
$$= (A \cap C) \cup (B \cap C)$$
$$= (C \cap A) \cup (C \cap B)$$
$$= C \cap (A \cup B)$$
$$= C \cap (A \cup C)$$
$$= C \text{ [proven]}$$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Exercise

- $A = \{a, b\}$, $B = \{1, 2\}$, $C = \{x, y\}$

- Determine the following set nad their cardinality,
  a) $B \times C$
  b) $A \times B \times C$,

a) $B \times C = \{(1, x), (1, y), (2, x), (2, y)\}$

$|B \times C| = 2 \times 2$
$= 4$

b) $A \times B \times C = \{(a, 1, x), (a, 1, y), (a, 2, x), (a, 2, y), (b, 1, x), (b, 1, y), (b, 2, x), (b, 2, y)\}$

$|A \times B \times C| = 2^3 = 8$

Exercise

Suppose x is a particular real number. Let ***p***, ***q*** and ***r*** symbolize "$0 < x$", "$x < 3$" and "$x = 3$", respectively. Write the following inequalities symbolically:

a) $x \leq 3$ $q \vee r$: $x<3$ or $x=3$

b) $0 < x < 3$ $p \wedge q$

c) $0 < x \leq 3$ $p \wedge (q \vee r)$

Example

Construct the truth table for,

A = $\neg(p \vee q) \rightarrow (q \wedge p)$

Solution:

p	*q*	$(p \vee q)$	$\neg(p \vee q)$	$(q \wedge p)$	A
T	T	T	F	T	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	F	F

Exercise

Propositional functions ***p***, ***q*** and ***r*** are defined as follows:

p is "$n = 7$"

q is "$a > 5$"

r is "$x = 0$"

Write the following expressions in terms of ***p***, ***q*** and ***r***, and show that each pair of expressions is **logically equivalent**. State carefully which of the above laws are used at each stage.

(a) $((n = 7)$ or $(a > 5))$ *and* $(x = 0)$ $(p \vee q) \wedge r$

$((n = 7)$ *and* $(x = 0))$ or $((a > 5)$ *and* $(x = 0))$ $(p \wedge r) \vee (q \wedge r)$

(b) $\neg((n = 7)$ *and* $(a \leq 5))$ $\neg(p \wedge \neg q)$

$(n \neq 7)$ or $(a > 5)$ $\neg p \vee q$

(c) $(n = 7)$ or $(\neg((a \leq 5)$ *and* $(x = 0)))$ $p \vee (\neg(\neg q \wedge r))$

$((n = 7)$ or $(a > 5))$ or $(x \neq 0)$ $(p \vee q) \vee \neg r$

a) $(p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$ prove it

p	q	r	$p \vee q$	$p \wedge r$	$q \wedge r$	$(p \vee q) \wedge r$	$(p \wedge r) \vee (q \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

b) $\neg(p \wedge \neg q) \equiv \neg p \vee q$

p	q	$\neg q$	$\neg p$	$(p \wedge \neg q)$	$\neg(p \wedge \neg q)$	$\neg p \vee q$
T	T	F	F	F	T	T
T	F	T	F	T	F	F
F	T	F	T	F	T	T
F	F	T	T	F	T	T

c) $p \vee (\neg(\neg q \wedge r)) \equiv (p \vee q) \vee \neg r$

p	q	r	$\neg q$	$(\neg q \wedge r)$		$\neg r$	$p \vee q$	$(p \vee q) \vee \neg r$	$p \vee (\neg(\neg q \wedge r))$
T	T	T	F	F	T	F	T	T	T
T	T	F	F	F	T	T	T	T	T
T	F	T	T	T	F	F	T	T	T
T	F	F	T	F	T	T	T	T	T
F	T	T	F	F	T	F	T	T	T
F	T	F	F	F	T	T	T	T	T
F	F	T	T	T	F	F	F	F	F
F	F	F	T	F	T	T	F	T	T

Propositions ***p***, ***q***, ***r*** and ***s*** are defined as follows:
p is "I shall finish my Coursework Assignment"
q is "I shall work for forty hours this week"
r is "I shall pass Maths"
s is "I like Maths"

Write each sentence in symbols:
(a) I shall not finish my Coursework Assignment.
(b) I don't like Maths, but I shall finish my Coursework Assignment.
(c) If I finish my Coursework Assignment, I shall pass Maths.
(d) I shall pass Maths only if I work for forty hours this week and finish my Coursework Assignment.

a) $\neg p$ ✓
b) $(\neg s) \wedge p$ ✓
c) $p \longrightarrow r$ ✓
d) $r \leftrightarrow (q \wedge p)$ ✓

Write each expression as a sensible (if untrue!) English sentence:
(e) $q \vee p$
(f) $\neg p \rightarrow \neg r$

e) I shall work for forty hours this week or finish my Coursework Assignment. ✓

f) If I don't finish my Coursework Assignment, I shall not pass Maths. ✓

$p \vee (q \wedge \neg p)$

$p \vee q$

p	q	$\neg p$	$(q \wedge \neg p)$	$p \vee (q \wedge \neg p)$	$p \vee q$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	F	F	F

$\therefore$ logical equivalent

$(\neg p \wedge q) \vee (p \wedge \neg q)$ – a

$(\neg p \wedge \neg q) \vee (p \wedge q)$ – b

p	q	$\neg p$	$\neg q$	$\neg p \wedge q$	$p \wedge \neg p$	$(\neg p \wedge q) \vee (p \wedge \neg q)$	$\neg p \wedge \neg q$	$p \wedge q$	$(\neg p \wedge \neg q) \vee (p \wedge q)$
T	T	F	F	F	F	F	F	T	T
T	F	F	T	F	F	F	F	F	F
F	T	T	F	T	F	T	F	F	F
F	F	T	T	F	F	F	T	F	T

$\therefore$ not logical equivalent

1. Prove that if x is an even integer, then $x^2 - 6x + 5$ is odd
(Direct Proof)

$x = 2a$

$x^2 - 6x + 5 = (2a)^2 - 6(2a) + 5$

$= 4a^2 - 12a + 5$

$= 4a^2 - 12a + 4 + 1$

$= 2(2a^2 - 6a + 2) + 1$

odd number $= \underline{2m + 1}$; where $m = 2a^2 - 6a + 2$ is an integer

proven #

2. Prove that if n is an integer and n^3+5 is odd, then n is even
(Indirect Proof)

$p \to q \equiv \neg q \to \neg p$

If n^3+5 is odd, then n is even. $\equiv$ If n is odd, then n^3+5 is even.

*assume n is odd is true.

$n = 2a + 1$

$n^3 + 5 = (2a+1)^3 + 5$

$= (2a+1)^2(2a+1) + 5$

$= (4a^2 + 4a + 1)(2a+1) + 5$

$= 8a^3 + 8a^2 + 2a + 4a^2 + 4a + 1 + 5$

$= 8a^3 + 12a^2 + 6a + 6$

$= 2(4a^3 + 6a^2 + 3a + 3)$

$= 2t$; where $t = 4a^3 + 6a^2 + 3a + 3$ is an integer

Hence, the statement if n is odd, then n^3+5 is even is true

3. Prove that if x is odd (p), then x^2 is odd (q) (Contradiction)

Proof :

1. contradiction : suppose x is odd, x^2 is even

2. $x = 2a + 1$

$x^2 = (2a+1)^2$

$= 4a^2 + 4a + 1$

$= 2(2a^2 + 2a) + 1$

$= 2m + 1$; $m = 2a^2 + 2a$ and is an integer (odd).

in supposition, x^2 is even ; after prooving, x^2 is odd

$\therefore$ contradiction happens. Supposition could not be true.