CHAPTER 2 EXERCISE

GROUP 7

1. KAVIVARTHAN A/L MANNIVANAN
2. MICHELLE HO CHIA XIN
3. MUHAMMAD FIRDAUS BIN MD LANI
4. TAY XIN YING

Exercise

Define a relation R from $\mathbf{Z}$ to $\mathbf{Z}$ as follows: For all integer number m and n, $(m,n) \in \mathbf{Z}\times\mathbf{Z}$,

$$m\ R\ n \leftrightarrow m - n \text{ is even}$$

i) Is 4 R 0? yes

ii) Is 2 R 6? yes

iii) Is 3 R (-3)? yes

iv) Is 5 R 2? no, $5-2=3$ (odd)

v) List 5 integers that are related by R to 1.

$\mathbb{Z} = \{3, 5, 7, 9, 11\}$

To from	c_1	c_2	c_3	c_4	c_5
c_1		140	100	150	200
c_2	190		200	160	220
c_3	110	180		190	250
c_4	190	200	120		150
c_5	200	100	200	150	

If the relation R on the set of cities $A = \{c_1, c_2, c_3, c_4, c_5\}$: $c_i\ R\ c_j$ if and only if the cost of going from c_i to c_j is defined and less than or equal to RM180.

i) Find R.

ii) Matrices of relations for R

i) $R = \{(c_1, c_2), (c_1, c_3), (c_1, c_4), (c_2, c_4), (c_3, c_1), (c_3, c_2), (c_4, c_3), (c_4, c_5), (c_5, c_2), (c_5, c_4)\}$

ii)

$$M_R = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Let $A = \{1, 2, 3,4\}$ and R is a relation from A to A.

Suppose $R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

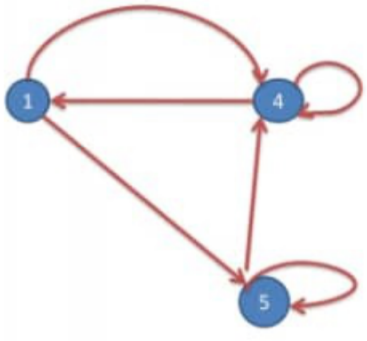
- What is R (represent)?
- What is matrix representation of R?

1. $(x,y) \in R$ if $x < y$, and $x, y \in A$

2.

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Let $A = \{1, 4, 5\}$ and let R be given by the digraph shown below. list in-degrees and out-degrees of all vertices.



	1	4	5
in degree	1	3	2
out degree	2	2	2

1. Let $A=\{1,2,3,4\}$ and let $R = \{(1,2), (2,2), (3,4), (4,1)\}$

Determine whether R symmetric, asymmetric or antisymmetric.

$(1,2) \in R$, $(2,1) \notin R$ (not symmetric)

$(1,2) \in R$, $(2,1) \notin R$
$(3,4) \in R$, $(4,3) \notin R$
$(4,1) \in R$, $(1,4) \notin R$
$(2,2)$, has loop (not asymmetric)

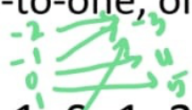
$(1,2) \in R$, $(2,1) \notin R$
$(3,4) \in R$, $(4,3) \notin R$
$(4,1) \in R$, $(1,4) \notin R$
$(2,2)$, $a=a$ (antisymmetric)

$\therefore R$ is antisymmetric

Exercise

Determine which of the relations f are functions from the set **X** to the set **Y**. In case any of these relations are functions, determine if they are one-to-one, onto **Y**, and/or bijection.

at most 1 at least 1 both



a) **X** = { -2, -1, 0, 1, 2 } , **Y** = { -3, 4, 5 } and f = { (-2,-3), (-1,-3), (0,4), (1,5), (2,-3) } onto Y

b) **X** = { -2, -1, 0, 1, 2 } , **Y** = { -3, 4, 5 } and f = { (-2,-3), (1,4), (2,5)} bijection

c) **X** = **Y**= { -3, -1, 0, 2 } and f = { (-3,-1), (-3,0), (-1,2), (0,2), (2,-1) } none

Exercise 1

A depositor deposits RM 10,000 in a savings account at a bank yielding 5% per year with interest compounded annually. How much money will be in the account after 30 years? Let P_n denote the amount in the account after n years.

$P_0 = 10000$

$P_n = P_{n-1} + 0.05P_{n-1}$

$P_1 = 10500$

$P_2 = 11025$

$a_n = ar^{n-1}$

$a_{30} = 10000(1.05)^{31-1}$ after 30 yrs = 31

$= 43219.42$

$P_n = P_0\left(1+\frac{r}{100}\right)^n$

$P_{30} = 10000\left(1+\frac{5}{100}\right)^{30}$

$= 43219.42$

Exercise 2

Consider the following sequence:

$a_1 = 1$ **1, 5, 9, 13, 17**

4 4 4 4

Find the recurrence relation that defines the above sequence.

$a_n = a_{n-1} + 4 \quad , n \geq 2$

A basketball is dropped onto the ground from a height of 15 feet. On each bounce, the ball reaches a maximum height 55% of its previous maximum height.

a)Write a recursive formula, a_n, that completely defines the height reached on the n_{th} bounce, where the first term in the sequence is the height reached on the ball's first bounce.

b)How high does the basketball reach after the 4_{th} bounce? Give your answer to two decimal places.

a) $a_n = 0.55a_{n-1} \quad , a_0 = 15 \quad , n \geq 1$

b) $a_1 = 0.55(a_{1-1}) = 0.55(15) = 8.25$

$a_2 = 0.55(a_{2-1}) = 0.55(8.25) = 4.5375$

$a_3 = 0.55(a_{3-1}) = 0.55(4.5375) = 2.4956$

$a_4 = 0.55(a_{4-1}) = 0.55(2.4956) = 1.3726$

≈ 1.37 ft