TUTORIAL 4: Boolean Algebra and Logic Simplification

1. Using any logic gates, draw the logic diagram of the given function. Do NOT simplify the function

$$X = \overline{A} + B(D + \overline{E}\,\overline{F})\overline{(A + C)}$$

2. Directly apply DeMorgan's law to the following expressions. You do NOT have to simplify the expressions.

i) $Y = \overline{(A + \overline{B} + C)} + \overline{\overline{C}\,D\,\overline{E}}$

ii) $Y = \overline{P + \overline{Q}\,\overline{(R + S\,P)}}$

3. Simplify the function using Boolean Algebra

$$Y = A\,B\,\overline{C} + A\,\overline{B} + A\,B\,C + \overline{A}\,\overline{B}$$

4. Develop a truth table for the following expression. From the truth table derive a standard product-of-sums (POS) expression.

$$f = (A + \overline{B})(A + C)(A + B + \overline{C})$$

5. Use a Karnaugh map to reduce the expression to a minimum sum-of-products (SOP) form

$$f = \overline{A}\,B(C\,\overline{D} + C\,D) + A\,C\,D$$

6. For the truth table given below,

A	B	C	*f*
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

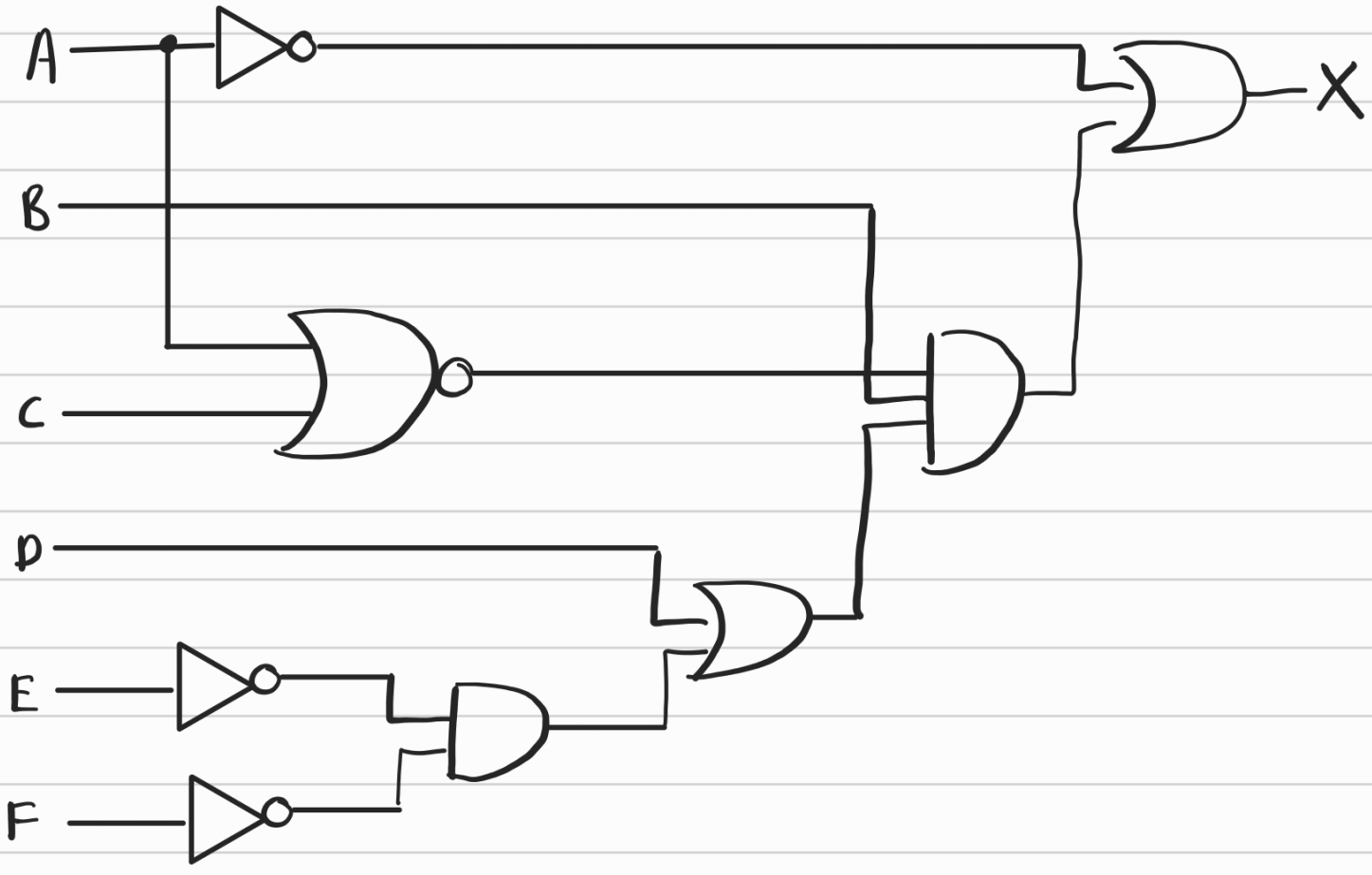
i) Express the output function, f in Standard Sum-of-Products (SOP) form.

ii) Express the output function, f in Standard Product-of-Sums (POS) form.

iii) If the following waveforms are applied to the inputs, A, B, and C of the logic circuit, draw the output waveform for the function, *f*.

1. Using any logic gates, draw the logic diagram of the given function. Do NOT simplify the function

$$X = \overline{A} + B(D + \overline{E}\,\overline{F})(\overline{A + C})$$



2. Directly apply DeMorgan's law to the following expressions. You do NOT have to simplify the expressions.

i) $Y = \overline{(A + \overline{B} + C)} + \overline{\overline{C}\,D\,\overline{E}}$

ii) $Y = \overline{P + \overline{Q}\,(\overline{R + S\,P})}$

i) $\overline{A}B\overline{C} + (C + \overline{D} + E)$

$= \overline{A}B\overline{C} + C + \overline{D} + E$

ii) $\overline{(P + \overline{Q})} + (R + SP)$

$= \overline{P}Q + R + SP$

3. Simplify the function using Boolean Algebra

$$Y = A\,B\,\overline{C} + A\,\overline{B} + A\,B\,C + \overline{A}\,\overline{B}$$

$Y = AB\overline{C} + A\overline{B} + ABC + \overline{A}\overline{B}$

$= AB(\overline{C} + C) + A\overline{B} + \overline{A}\overline{B}$

$= AB + A\overline{B} + \overline{A}\overline{B}$

$= A(B + \overline{B}) + \overline{A}\overline{B}$

$= A + \overline{A}\overline{B}$

$= A + \overline{B}$

4. Develop a truth table for the following expression. From the truth table derive a standard product-of-sums (POS) expression.

$$f = (A + \overline{B})(A + C)(A + B + \overline{C})$$

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$f = (A+B+C)(A+B+\overline{C})(A+\overline{B}+C)(A+\overline{B}+\overline{C})$

5. Use a Karnaugh map to reduce the expression to a minimum sum-of-products (SOP) form

$$f = \overline{A}B(C\overline{D} + CD) + ACD$$

$f = \overline{A}BC\overline{D} + \overline{A}BCD + ABCD + A\overline{B}CD$

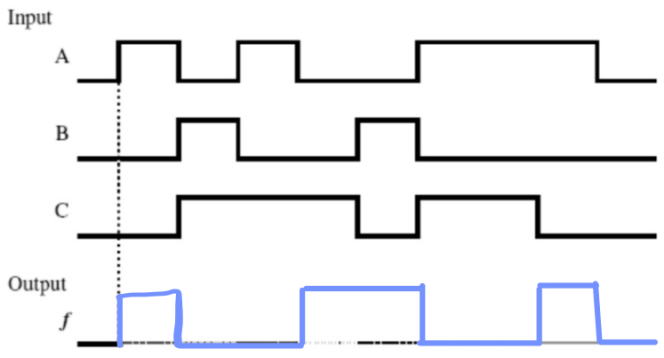
AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	0	1	0
10	0	0	1	0

A	B	C	D
0	1	1	1
0	1	1	0

$f = \overline{A}BC + ACD$

6. For the truth table given below,

A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



i) Express the output function, f in Standard Sum-of-Products (SOP) form.
ii) Express the output function, f in Standard Product-of-Sums (POS) form.
iii) If the following waveforms are applied to the inputs, A, B, and C of the logic circuit, draw the output waveform for the function, f.

i) $f = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

ii) $f = (A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

7. Below is the truth table of a three-input XOR gate.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

i) Express Y in standard sum-of-product (SOP) variable form.
ii) Express Y in standard product-of-sum (POS) variable form

i) $Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

ii) $Y = (A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

8. Construct the truth table for

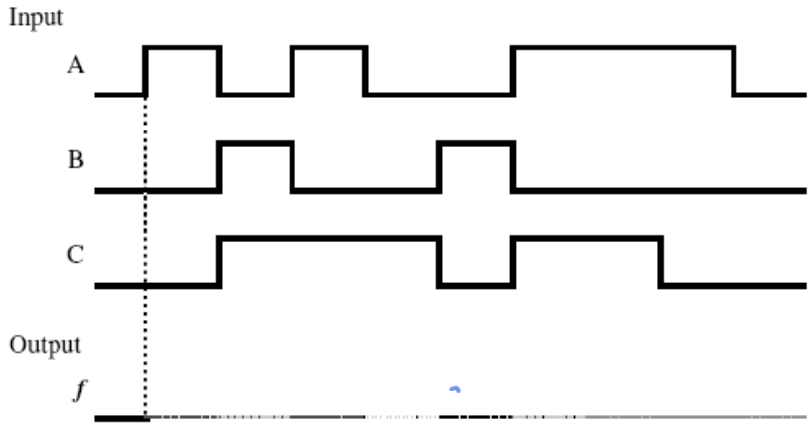
SOP

$$Y = ABC + A\bar{B}C + A\bar{C}B$$

$Y = A\bar{B}C + (\bar{A}+\bar{B})C + (\bar{A}+\bar{C})B$

$= A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}BC + AB\bar{C}$

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



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A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

i) Express Y in standard sum-of-product (SOP) variable form.
ii) Express Y in standard product-of-sum (POS) variable form

8. Construct the truth table for

$$Y = A\overline{BC} + \overline{AB}\,C + \overline{AC}\,B$$

9. Using any logic gates, draw the logic diagram of

$$P = \overline{\overline{(X + Z)} + \overline{Y}(W + \overline{XZ})}$$

10. Using Karnaugh Map,find the minimum SOP expression for the following function

i)

$$Y = \overline{P}\,\overline{Q}\,\overline{R}\,\overline{S} + \overline{P}\,Q\,\overline{R}\,S + \overline{P}\,Q\,R\,\overline{S} + PQ\,\overline{R}\,\overline{S} + P\,\overline{Q}\,\overline{R}\,\overline{S} + \overline{P}\,\overline{Q}\,R\,\overline{S} + P\,Q\,\overline{R}\,S$$

ii)

$$G(w, x, y, z) = \sum m(1, 3, 14, 15) + d(0, 2, 6, 8, 13)$$

11. Implement $Y = \overline{a}(b + \overline{d}(\overline{\overline{ac} + e}))$ using any type of logic devices

12. Using Karnaugh Map, find the minimum SOP of the given m-notation

$$F(p, q, r) = \sum m(0,1,3,4,6)$$

13. Prove that

$$x\,y + x\,\overline{y} + \overline{x}\,\overline{y} = x + \overline{y}$$

10. Using Karnaugh Map,find the minimum SOP expression for the following function

i)

$$Y = \overline{P}\,\overline{Q}\,\overline{R}\,\overline{S} + \overline{P}\,\overline{Q}\,\overline{R}S + \overline{P}\,\overline{Q}R\,\overline{S} + \overline{P}\,\overline{Q}RS + P\overline{Q}\,\overline{R}\,\overline{S} + \overline{P}Q\,\overline{R}\,\overline{S} + P\overline{Q}R\,\overline{S}$$

ii)

$$G(w, x, y, z) = \sum m(1, 3, 14, 15) + d(0, 2, 6, 8, 13)$$

i)

PQ \ RS	00	01	11	10
00	1	1	0	1
01	1	0	0	0
11	1	0	0	0
10	1	0	0	1

$Y = \overline{R}\,\overline{S} + \overline{P}\overline{Q}\overline{R} + R\overline{S}$

ii)

wx \ yz	00	01	11	10
00	x	1	1	x
01	0	0	0	x
11	0	x	1	1
10	x	0	0	0

$G = \overline{w}\,\overline{x} + wxy$

13. Prove that

$$xy + x\overline{y} + \overline{x}\,\overline{y} = x + \overline{y}$$

$$xy + x\bar{y} + \bar{x}\bar{y} = x(y + \bar{y}) + \bar{x}\bar{y}$$
$$= x + \bar{x}\bar{y}$$
$$= x + \bar{y} \quad \text{[proven]}$$