SECI1013-03 (DISCRETE STRUCTURE)

CHAPTER 3 IN SLIDE EXERCISE

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PART 1

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Chapter 3 - Part 1

Exercise 1

(i) Ways to choose a boy = 8 ways
Ways to choose a girl = 21 ways
Ways to choose a boy or a girl = 29 ways.

(ii) Way to select a Chinese = 7 ways
Ways to select a Malay = 29 - 7 = 22 ways
Ways to select a Chinese or a Malay = 29 ways

Exercise 2

Integers end with 1 → 11, 21, 31, 41 = 4 ways
Integers end with 7 → 7, 17, 27, 37, 47 = 5 ways
Integers end with 1 or 7 = 4 + 5 = 9 ways

Exercise 3

T_1: Choose first letter → 1 way
T_2: Choose second letter → 4 ways
T_3: Choose third letter → 3 ways
T_4: Choose fourth letter → 2 ways
There are $1 \times 4 \times 3 \times 2 = 24$ strings

Exercise 4

(i) Ways to select a boy = 8 ways
Ways to select a girl = 21 ways
Ways to select a boy and a girl = $8 \times 21 = 168$ ways

(ii) Ways to select a Chinese = 7 ways
Ways to select a Malay = 29 - 7 = 22 ways
Ways to select a Chinese and a Malay = $7 \times 22 = 154$ ways

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Exercise 5

(i) If Syed is chairperson → 1 x 5 x 4 = 20 ways
If Syed is secretary → 1 x 5 x 4 = 20 ways
If Syed is treasurer → 1 x 5 x 4 = 20 ways
In total, there are 20+20+20 = 60 ways.

(ii) If Tan is chairperson and Helmi is secretary → 2 x 4 = 8 ways
If Tan is secretary and Helmi is treasurer → 2 x 4 = 8 ways
If Tan is treasurer and Helmi is chairperson → 2 x 4 = 8 ways
In total, there are 8+8+8 = 24 ways

Exercise 6

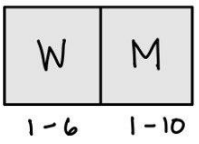
Distinct history books = 6
Distinct classics books = 9
Distinct law books = 7
Distinct education books = 4

History, classics and law → 6 x 9 x 7 = 378 ways
Classics, law and education → 9 x 7 x 4 = 252 ways
History, classics and education → 6 x 9 x 4 = 216 ways
History, law and education → 6 x 7 x 4 = 168 ways
In total, there are 378+252+216+168 = 1014 ways

PART 2

Exercise 1

In how many dance pairs (dance pairs means a pair (W,M), where W stands for a women and M for man), can be formed from a group of 6 women and 10 men?



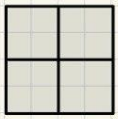
Total Dance Pairs $= P(6,1) \times P(10,1)$

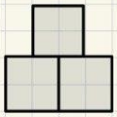
$= 6 \times 10$

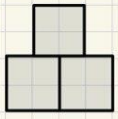
$= 60$

Exercise 2

- In how many ways can 10 distinct books be divided among 3 students if Khairin gets 4 books and Nurina and Sarah each get 3 books.

Khairin  ${}^{10}C_4 = 210$

Nurina  ${}^{10-4}C_3 = 35$

Sarah  ${}^{6-3}C_3 = 1$

$\therefore$ Total ways $= 210 \times 35 \times 1$

$= 7350$ ways

Exercise 3

- In how many ways can five people A, B, C, D, and E be seated around a circular table if

  a) A and B must sit next to each other

  b) A and B must not sit next to each other

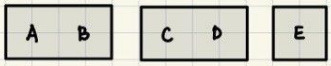
  c) A and B must be together and CD must be together

a) 

4 blocks in circular arrangement:

$(4-1)! \times 2! = 6 \times 2 = 12$ ways

A & B can switch

c) 

3 blocks in circular arrangement:

$(3-1)! = 2! = 2 \times 1 = 2$ ways

∴ Total ways = $2 \times 2! \times 2! = 8$ ways

b) A & B cannot sit next to each other

=

Total ways − A & B can sit next to each other

Total ways = $(5-1)! = 4! = 4 \times 3 \times 2 \times 1 = 24$ ways

∴ Total ways A & B cannot sit next to each other = $24 - 12 = 12$ ways

Exercise 4

A student is required to answer 7 out of 12 questions, which are divided into two groups, each containing 6 questions. The student is not permitted to answer more than 5 questions from either group. In how many different ways can the student choose the 7 questions?

Group #1 : 6 questions , answers 2 questions

$$C(6,2) = \frac{6!}{2!(6-2)!} = 15$$

Group #2 : 6 questions , answers 5 questions

$$C(6,5) = \frac{6!}{5!(6-5)!} = 6$$

case #1

Group #1 : 6 questions , answers 3 questions

$$C(6,3) = \frac{6!}{3!(6-3)!} = 20$$

Group #2 : 6 questions , answers 4 questions

$$C(6,4) = \frac{6!}{4!(6-4)!} = 15$$

case #2

∴ case #3 , answer 5 then 2
case #4 , answer 4 then 3

Total ways = $(15 \times 6)(2) + (15 \times 20)(2)$
= 780 ways

Exercise 5

There is a box containing identical blue, green, pink, yellow, red and dark blue balls. In how many ways we can select 4 balls?

$n = 6, \quad r = 4$

$$C(6+4-1, 4) = \frac{(6+4-1)!}{4!(6-1)!}$$

$$= \frac{9!}{4!5!}$$

$$= 126 \text{ ways}$$

PART 3

Exercise 1

- How many students must be in a course to guarantee at least two students receive same score in the test, if the test is graded on a scale from 0 to 100. Explain your answer.

n

k

m

n - the number of students

m - all possible scores (0-100)

$M = \{x \mid x \in \mathbb{Z}, 0 \le x \le 100\}$, $|M| = 101$

$k = 2$

The cardinality of the set of all possible scores is 101, if each student was to obtain a unique score, then there would be 101 students. However, to guarantee that at least two students obtain the same score, then we need to add one more student.

$\therefore n = m(k-1) + 1$

$n = 101(2-1) + 1$

$n = 102$

$\therefore$ 102 students

Exercise 2

- Show that every set of 15 socks chosen among 14 pairs of socks contains at least one matched pair.

(n) pigeons $\Rightarrow$ 14 pairs - 28 total socks

(k) pigeonholes $\Rightarrow$ 15 socks

$n > k$, thus there must exist at least one matched pair

$\therefore$ proven by the 1st form of the pigeonhole principle

PART 4

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Chapter 3 - Part 4

Exercise 1

$P(E) = \frac{C(12,4)}{C(15,4)}$

$= \frac{33}{91}$

Exercise 2

(i) $P(S) = 2^6$

$= 64$

Let denote event A = "less than 2 heads"

$P(A) = \frac{7}{64}$

(ii) $P(A') = 1 - P(A)$

$= 1 - \frac{7}{64}$

$= \frac{57}{64}$

Exercise 3

P(car accident) = 0.09

P(intoxicated) = 0.32

P(car accident while intoxicated) = 0.15

P(car accident or intoxicated) = P(car accident) + P(intoxicated) - P(car accident while intoxicated)

= 0.09 + 0.32 - 0.15

= 0.26

Exercise 4

Let denote event A = "rain"

Let denote event B = "high barometric pressure"

$P(B) = 0.85$

$P(A \cap B) = 0.15$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.85} = \frac{3}{17}$$

Exercise 5

$$P(R|L) = \frac{P(R \cap L)}{P(L)}$$

$$P(R \cap L) = P(RTL) + P(RT^cL) = \frac{1}{12} + \frac{1}{24} = \frac{1}{8}$$

$$P(L) = P(RTL) + P(RT^cL) + P(R^cTL) + P(R^cT^cL) = \frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16} = \frac{11}{48}$$

$$P(R|L) = \frac{\frac{1}{8}}{\frac{11}{48}} = \frac{6}{11}$$

Exercise 6

Let denote event H = "code written by Hana"
Let denote event A = "code written by Amir"
Let denote event D = "code written by Dani"
Let denote event B = "Buggy codes"

$P(H) = 0.30$
$P(A) = 0.45$
$P(D) = 0.25$
$P(H|B) = 0.03$
$P(A|B) = 0.02$
$P(D|B) = 0.05$

$$P(B|D) = \frac{P(D|B)\,P(D)}{P(H|B)\,P(H) + P(A|B)\,P(A) + P(D|B)\,P(D)}$$
$$= \frac{(0.05)(0.25)}{(0.03)(0.30) + (0.02)(0.45) + (0.05)(0.25)}$$
$$= 0.4098$$

Exercise 7

Let denote event H = "Halim passes the final exam"
Let denote event A = "Aina passes the final exam"

$P(H) = 0.85$
$P(A) = 0.70$

(i) $P(H') = 1 - P(H)$
$= 1 - 0.85$
$= 0.15$

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(ii) $P(H \cap A) = P(H) \cdot P(A)$

$= (0.85)(0.70)$

$= 0.595$

(iii) $P(H' \cap A') = P(H') \cdot P(A')$

$= (0.15)(1 - 0.70)$

$= (0.15)(0.30)$

$= 0.045$

(iv) $P(H \cap A') + P(H' \cap A)$

$= P(H) \cdot P(A') + P(H') \cdot P(A)$

$= (0.85)(0.30) + (0.15)(0.70)$

$= 0.255 + 0.105$

$= 0.36$