SECI1013: DISCRETE STRUCTURES
SESSION 2024/2025 – SEMESTER 1

ASSIGNMENT 1 (CHAPTER 1 AND 2)

INSTRUCTIONS:

1. This assignment must be conducted in a group (3 or 4 students). Please clearly write the group members name & matric number in the front-page of the submission.
2. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in rejection of the submission of assignment.
3. For submission, scan and combine all answer/solution sheets as one PDF file. Then only **ONE** group member needs to submit on behalf of the group via e-learning **(Refer elearning for Due Date)**
4. This assignment has 16 questions (80 marks), contribute 5% of overall course marks.

STRUCTURES:

1. Chapter 1 (All): Set Theory and Logic [40 Marks]
2. Chapter 2 (2.1 and 2.2): Relations and Functions [40 Marks]

Chapter 1 (All): Set Theory and Logic [40 Marks]

1. Let set U = {n | n ϵ whole numbers, 10 <= n <= 30};

   set G = {g | g ϵ even numbers};

   set F = {f | f ϵ natural numbers, f > 10 and f < 30};

   G ⊆ F;

   F ⊆ U.

   Answer all the questions for each of the following. [**10 Marks**]

   a. Write down set F and find |F|.
   b. Write down set G and find |G|.
   c. Construct a Venn diagram based on the given sets.
   d. Find cardinality of symmetric difference of set G and set F.

2. Let set A = {s, u, b};

   set B = {s, e, t};

   set C = {n, e, t}.

   Answer all the questions for each of the following in ascending order. [**5 Marks**]

   a. Find $|P(A)|$
   b. Find $A \cap B \cup C$
   c. Find $A - B$
   d. Find $B \times C$

3. Which of the following statements are propositions?

   State true or false. [**5 Mark**]

   a. The discrete structure implements set theory, relations, and functions to solve computer science problems.
   b. In computer science, the Boolean data type defines 0 = false and 1 = true.
   c. Let A and B be the subsets of U. $A \cup (A \cap B) = A$ can be proved by distributive, idempotent, and commutative laws.
   d. $a^2 - 2a + 1 = 0$; when $a \neq 1$
   e. $a^2 - b^2 = 0$; when $a=b$ or $a=-b$

4. Construct a truth table for each of the following conditional statements. [**4 Marks**]

   a. $(p \rightarrow q) \wedge (\neg p \leftrightarrow \neg q)$
   b. $(p \leftrightarrow q) \vee (\neg p \rightarrow \neg q)$

5. Given, $A = \neg p \wedge (\neg q \vee \neg r)$ and $B = p \vee (q \wedge r)$. State whether $A \equiv B$ or not. [**4.5 Marks**]

6. Given, $A = p \wedge (p \vee q)$ and $B = p \vee (p \wedge q)$. State whether $A \equiv B$ or not. [**2.5 Marks**]

7. Let P(x), Q(x), and R(x) be the statements.

   “x is a student,” “x is smart,” and “x is shy,” respectively.

   Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), and R(x), where the domain consists of all people. [**2 Marks**]

   a. Some students are shy.

b. All smart people are not shy.

8. Give direct proof to show a square of any negative numbers is positive. [**3.5 Marks**]

9. Give a proof by contradiction to show if C and D are sets, then C ∩ (D ∩ C') = {}. [**3.5 Mark**]

Chapter 2 (2.1 and 2.2): Relations and Functions [40 Marks]

10. Determine whether the relation *R* on set *Z* (set of integer number) is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. [**6 Marks**]

$$a \; R \; b \text{ if and only if } |a - b| = 2$$

11. Given a relation, *R* on *A* = {*a*, *b*, *c, d*} on as follows:

$$R = \{(a,a), (a,b), (a,d), (b,b), (b,c), (c,c), (c,d), (d,a), (d,d)\}$$

Show the matrix of relation, M_R and determine whether the relation, *R* is an equivalence relation. [**7 Marks**]

12. Let $f(x, y) = (2x - y, x - 2y)$; $(x, y) \in \mathbf{R} \times \mathbf{R}$, (**R** is set of real numbers.)

a) Show that *f* is one to one. [**4 Marks**]
b) Find f^{-1} [**7 Marks**]

13. Let a set *A* = {1, 2, 3}, *B* = {1, 2, 3, 4} and *C* = {1,2}. [**4 Marks**]

a) Draw the arrow diagram to define function *f: A*→*B* that is one-to-one but not onto.
b) List the three ordered pairs to define function *g: A*→*C* that is onto but not one-to-one.

14. Function *f* and *g* are defined by formulas as shown below.

$$f(x) = x^3 \quad \text{and} \quad g(x) = x - 1, \text{ for all real number } x.$$

i) Find $g \circ f$ and $f \circ g$. [**4 Marks**]
ii) Determine whether $g \circ f$ equals $f \circ g$. [**2 Marks**]

15. Let B = {0,1}. Give a recurrence relation for the strings of length *n* in *B* * that do not contain 01.
    (*B* * is the set of all string over *B*) [**3 Marks**]

16. A game is played by moving a marker ahead either 2 or 3 steps on a linear path. Let c_n be the number of different ways a path of length *n* can be covered. Given,

    $$c_n = c_{n-2} + c_{n-3},\ c_1=0,\ c_2=1,\ c_3=1$$

    Write a recursive algorithm to compute c_n. [**3 Marks**]

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SECI1013-03 (DISCRETE STRUCTURE)

ASSIGNMENT 1 (CHAPTER 1 & 2)

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QUESTIONS 1, 2, 3

Assignment 1

Chapter 1

Question 1

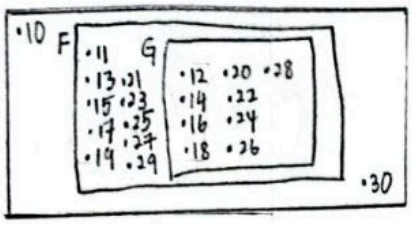
a. $F = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

$|F| = 19$

b. $G = \{12, 14, 16, 18, 20, 22, 24, 26, 28\}$

$|G| = 9$

c.



d. $F - G = \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$

$|F-G| = 10$

Question 2

a) $|P(A)| = 2^3$

$= 8$

b) $A \cap B = \{s\}$

$A \cap B \cap C = \{s, n, e, t\}$

c) $A - B = \{u, b\}$

d) $B = \{s, e, t\}$, $C = \{n, e, t\}$

$B \times C = \{(s,n), (s,e), (s,t), (e,n), (e,e), (e,t), (t,n), (t,e), (t,t)\}$

Question 3

a. Proposition, true

b. Proposition, true

c. Proposition, true

d. Proposition, false

e. Proposition, true

QUESTION 4

Question 4

a.

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg p \leftrightarrow \neg q$	$(p \to q) \wedge (\neg p \leftrightarrow \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

b.

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$\neg p \to \neg q$	$(p \leftrightarrow q) \vee (\neg p \to \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	F
F	F	T	T	T	T	T

QUESTIONS 5, 6, 7

Question 5

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg q \vee \neg r$	$q \wedge r$	A	B
T	T	T	F	F	F	F	T	F	T
T	T	F	F	F	T	T	F	F	T
T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	T	T	F	F	T
F	T	T	T	F	F	F	T	F	T
F	T	F	T	F	T	T	F	T	F
F	F	T	T	T	F	T	F	T	F
F	F	F	T	T	T	T	F	T	F

$\therefore A \not\equiv B.$

Question 6

p	q	$p \vee q$	$p \wedge q$	A	B
T	T	T	T	T	T
T	F	T	F	T	T
F	T	T	F	F	F
F	F	F	F	F	F

$\therefore A \equiv B.$

Question 7

(a) $\exists(x)(P(x) \wedge R(x))$

(b) $\forall(x)\ (Q(x) \rightarrow \neg R(x))$

QUESTION 8

Question 8

For all negative integers x, x^2 is positive.

let $-a$ is a negative integer:

$$(-a)^2 = (-a).(-a)$$
$$= a^2$$

The multiplication of two same signs, either positive or negative, brings positive sign as its outcome.

Therefore, $x = -a$.

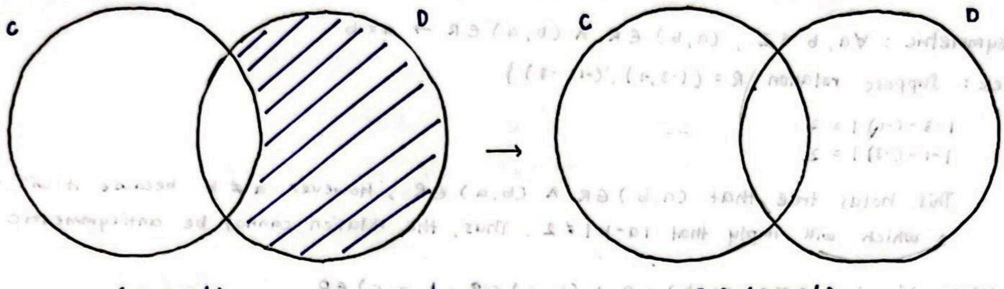
Since $x<0$ and $x^2>0$, we can conclude that $\forall(x)\ (x<0 \rightarrow x^2>0)$.

Hence, proven ⋇

QUESTION 9

9. Give a proof by contradiction to show if C and D are sets, then $C \cap (D \cap C') = \{\}$

Supposition : Suppose C & D are sets , $C \cap (D \cap C') \neq \{\}$



$(D \cap C')$ $\qquad$ $C \cap (D \cap C')$

Thus, $C \cap (D \cap C')$ is an empty set , a contradiction . Which proves that the original supposition is false.

$\therefore$ Therefore if C&D are sets , then $C \cap (D \cap C') = \{\}$

Contradiction : if C & D are sets , then $C \cap (D \cap C') \neq \{\}$

if $C \cap (D \cap C') \neq \{\}$, then there exists an element x such that

$$x \in C \cap (D \cap C')$$

This implies :

1. $x \in C$
2. $x \in (D \cap C')$

From 2 : x must exist and x must exist C' for 2 to be true

However since $x \in C$, x cannot exist in C' at the same time .

$\therefore$ Thus $C \cap (D \cap C') = \{\}$

QUESTION 10

Chapter 2 : (2.1 & 2.2) Relations and Functions

10. Determine whether the relation R on set Z (set of integer) is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive.

$$\boxed{a \ R \ b \text{ if and only if } |a-b| = 2}$$

$Z = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$

Reflexive : $\forall a \in Z$, $(a,a) \in R$

Check : $|-1-(-1)| = 0$

$|0-0| = 0$

$|1-1| = 0$

$\rightarrow$ R is not reflexive.

Irreflexive : $\forall a \in Z$, $(a,a) \notin R$

Check : since the relation doesn't contain any pair of the form (a,a), thus the relation will be irreflexive.

Symmetric : $\forall a, b \in Z$, $(a,b) \in R \rightarrow (b,a) \in R$

Check : Suppose R is symmetric , $R = \{(-3,-1), (-1,-3), (0,2), (2,0), (2,4), (4,2)\}$

$|-3-(-1)| = 2$

$|-1-(-3)| = 2$

$|0-2| = 2$

$|2-0| = 2$

$|2-4| = 2$

$|4-2| = 2$

$$M_R = \begin{array}{c|c} & \begin{matrix} -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

The relation is symmetric

Asymmetric : $\forall a, b \in \mathbb{Z}, (a, b) \in R \rightarrow (b, a) \notin R$

Check : Since the relation is symmetric, thus the relation cannot be asymmetric because $\forall a, b \in \mathbb{Z}, (a, b) \in R \rightarrow (b, a) \notin R$ does not hold true.

Antisymmetric : $\forall a, b \in \mathbb{Z}, (a, b) \in R \wedge (b, a) \in R \rightarrow a = b$

Check : Suppose relation $R = \{(-3, -1), (-1, -3)\}$

$|-3-(-1)| = 2$

$|-1-(-3)| = 2$

This holds true that $(a, b) \in R \wedge (b, a) \in R$, however $a \neq b$ because it will yield 0 which will imply that $|a-b| \neq 2$. Thus, the relation cannot be antisymmetric.

Transitive : $\forall a, b, c \in \mathbb{Z}, (a, b) \in R \wedge (b, c) \in R \rightarrow (a, c) \in R$

Check : Suppose $a = 1$, $b = 3$, $c = 5$

Method #1

$|1-3| = 2$ and $|3-5| = 2$ However, $|1-5| \neq 2$

Thus, $(1, 5) \notin R$ and the relation is not transitive

Method #2

Suppose : $R = \{(-1, 3), (3, -1), (0, 2), (2, 0)\}$

$$M_R = \begin{array}{c c} & \begin{array}{ccccc} -1 & 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_R \otimes M_R = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore M_R \otimes M_R \neq M_R$. Thus, the relation is not transitive.

QUESTION 11

11. Given a relation, R on $A = \{a, b, c, d\}$ on as follows:

$$\boxed{R = \{(a,a), (a,b), (a,d), (b,b), (b,c), (c,c), (c,d), (d,a), (d,d)\}}$$

Show that the matrix of relation, M_R and determine whether the relation, R is an equivalence relation.

$$M_R = \begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

* Relation is reflexive

Symmetric: if $M_R = M_R^T$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$M_R \neq M_R^T$ * Relation is not symmetric

Transitive: $M_R \otimes M_R = M_R$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$M_R \otimes M_R \neq M_R$ * Relation is not transitive

∴ Relation, R is reflexive, but not symmetric nor transitive. Thus relation R is not an equivalent relation.

QUESTION 12

12. Let $f(x,y) = (2x-y, x-2y)$; $(x,y) \in R \times R$, (R is the set of real numbers)

a) Show that f is one-to-one

If f is one-to-one, $f(x_1, y_1) = f(x_2, y_2)$ then $x_1 = x_2$ and $y_1 = y_2$

$$f(x_1, y_1) = f(x_2, y_2)$$

$f(x_1) = f(x_2)$

$2x_1 - y_1 = 2x_2 - y_2 \quad \cdots ①$

$f(y_1) = f(y_2)$

$x_1 - 2y_1 = x_2 - 2y_2 \quad \cdots ②$

From ① : $y_1 = 2x_1 - 2x_2 + y_2 \quad \cdots ③$

③ → ② : $x_1 - 2(2x_1 - 2x_2 + y_2) = x_2 - 2y_2$

$x_1 - 4x_1 + 4x_2 - \cancel{2y_2} = x_2 - \cancel{2y_2}$

$-3x_1 = -3x_2$

$x_1 = x_2$

From ② : $x_1 = x_2 - 2y_2 + 2y_1 \quad \cdots ④$

④ → ① : $2(x_2 - 2y_2 + 2y_1) - y_1 = 2x_2 - y_2$

$\cancel{2x_2} - 4y_2 + 4y_1 - y_1 = \cancel{2x_2} - y_2$

$3y_1 = 3y_2$

$y_1 = y_2$

∴ This shows that f is a one-to-one function.

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b) Find f^{-1}

$f(x,y) = (2x-y, x-2y)$

$(x,y) = f^{-1}(2x-y, x-2y) \rightarrow (x,y) = f^{-1}(a,b)$

let $a = 2x - y$, $b = x - 2y$

$a = 2x - y \quad \cdots ①$

$b = x - 2y \quad \cdots ②$

From ① : $x = \dfrac{a+y}{2} \quad \cdots ③$

③ → ② : $b = \dfrac{a+y}{2} - 2y$

$2b = a + y - 4y$

$y = \dfrac{a-2b}{3}$

From ② : $y = \dfrac{x-b}{2} \quad \cdots ④$

④ → ① : $a = 2x - \dfrac{(x-b)}{2}$

$2a = 4x - x + b$

$x = \dfrac{2a-b}{3}$

$\therefore f^{-1}(a,b) = (x,y)$

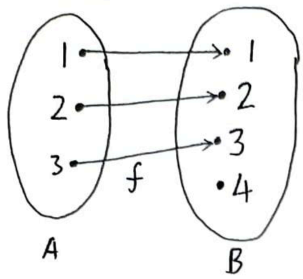
$f^{-1}(a,b) = \left(\dfrac{2a-b}{3}, \dfrac{a-2b}{3}\right)$

$f^{-1}(x,y) = \left(\dfrac{2x-y}{3}, \dfrac{x-2y}{3}\right)$

QUESTIONS 13, 14, 15

Question 13

a)



b) $g = \{(1,1), (2,2), (3,2)\}$

Question 14

i) $f(x) = x^3$

$g(x) = x - 1$

$g \circ f = g[f(x)]$

$= x^3 - 1$

$f \circ g = f[g(x)]$

$= (x-1)^3$

ii) $g[f(x)] = x^3 - 1$

$f[g(x)] = (x-1)^3$

$= (x-1)^2(x-1)$

$= (x^2 - 2x + 1)(x-1)$

$= x^3 - x^2 - 2x^2 + 2x + x - 1$

$= x^3 - 3x^2 + 3x - 1$

Thus, $g[f(x)] \neq f[g(x)]$

Question 15

only "000..." "111..." "...1100...." allowed

$n=1$	$= 0, 1$	$a_1 = 2$
$n=2$	$= 00, 11, 10$	$a_2 = 3$
$n=3$	$= 000, 111, 100, 110$	$a_3 = 4$
$n=4$	$= 0000, 1111, 1000, 1100, 1110$	$a_4 = 5$
$n=5$	$= \underbrace{00000, 11111}_{2}, \underbrace{10000, 11000, 11100, 11110}_{a_{n-2}}$	$a_5 = 6$

$\therefore a_n = a_{n-2} + 2, \; n \geq 2, \; a_1 = 2, \; a_2 = 3$

QUESTION 16

Question 16

Input = n

Output = C_n

```
Cn {
  if (n=1)
     return 0
  else if (n=2 or n=3)
     return 1
  else
     return Cn-2 + Cn-3
}
```