SECI1013: DISCRETE STRUCTURES

SESSION 2024/2025 – SEMESTER 1

ASSIGNMENT 3 (CHAPTER 3 AND 4)

INSTRUCTIONS:

1. This assignment must be conducted in a group (3 or 4 students). Please clearly write the group members name & matric number in the front-page of the submission.
2. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in rejection of the submission of assignment.
3. For submission, scan and combine all answer/solution sheets as one PDF file. Then only **ONE** group member needs to submit on behalf of the group via e-learning **(Refer deadline in elearning)**
4. This assignment has 70 marks contribute 5% of overall course marks.

STRUCTURES:

1. Chapter 3 (3.5): Probability [35 Marks]
2. Chapter 4 (4.1 to 4.6): Graph Theory [35 Marks]

Chapter 3 (3.5): Probability [35 Marks]

1. A student wants to take a book from the boxes that are kept in the store. There are four boxes stored according to their subject category. Suppose a math book is three times more likely to be taken out than a chemistry book. Chemistry books, on the other hand, are twice as likely as biology, and biology and physics are equally likely to be chosen.

[10 Marks]

   i. What is the probability of being taken out for each subject? [5M]
   ii. Calculate the probabilities that Mathematics or Biology is taken out by the student. [5M]

2. If A and B are events of mutually exclusive and $P(A) = 0.4$ and $P(B) = 0.5$, find:

[8 Marks]

   i. $P(A \cup B)$
   ii. $P(A^c)$
   iii. $P(A^c \cap B)$

3. Assumed that there are 100 participants in a lucky draw competition. There are 3 prizes being offered which are the grand prize, second prize and third prize. The winners are randomly selected. What is the probability that Anis can win one of the prizes, if she participates in the competition?

[4 Marks]

4. The probability that a randomly chosen male has pneumonia problem is 0.40. Smoking has substantial adverse effects on the immune system, both locally and throughout the body. Evidence from several studies confirms that smoking is significantly associated with the development of bacterial and viral pneumonia. 80% of males who have pneumonia problem are smokers. Whilst 30% of males that do not have pneumonia problem are smokers.

[8 Marks]

   i. What is the probability that a male is chosen do not have pneumonia problem? [4M]

   ii. Determine the probability that a selected male has a pneumonia problem given that he is a smoker. [4M]

5. A store contains 1 pair of boots with each of the following colors are black, chocolate and yellow. Each pair is put together in a particular place. You enter into the dark store and pick randomly the boot without looking at it. Then, you replace it with another boots. What is the probability that you will choose the black pair of boots both times?

[5 Marks]

Chapter 4 (4.1 to 4.6): Graph Theory [35 Marks]

1. Explain the given keyword using your own word and represent your understanding by drawing the graph.
   a. Vertices
   b. Edges
   c. Adjacent Vertices
   d. Incident Edge
   e. Isolated Vertex
   f. Loop
   g. Parallel Edges

(7 Marks)

2. Let G = {V, E} be a graph. Draw a graph with the following specified properties
   a. An undirect graph having V ={v1, v2, v3, v4, v5, v6, v7, v8, v9} and E = {e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11, e12, e13, e14, e15}. Where *e*1 = (*v*1,*v*2), *e*2 = (*v*2 ,*v*3), *e*3 = (*v*2,*v*5), *e*4 = (*v*2,*v*4), *e*5 = (*v*4,*v*6) and *e*6 = (*v*5,*v*6), e7 = (v3,v7), e8 = (v6, v7), e9 = (v6, v8), e10 = (v7, v9), e11 = (v8, v9), e12 = (v5, v3), e13 = (v6,v6), e14 = (v5, v9) and e15 = (v9,v9).

      i. Find the degree of each vertex?
      ii. Find the adjacent matrix and incident matrix?

(3 Marks)

   b. A direct graph having V ={v1, v2, v3, v4, v5, v6, v7, v8, v9} and E = {e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11, e12, e13, e14, e15}. Where *e*1 = (*v*1,*v*2), *e*2 = (*v*2 ,*v*3), *e*3 = (*v*5,*v*2), *e*4 = (*v*2,*v*4), *e*5 = (*v*4,*v*6) and *e*6 = (*v*5,*v*6), e7 = (v3,v7), e8 = (v6, v7), e9 = (v8, v6), e10 = (v7, v9), e11 = (v9, v8), e12 = (v3, v5), e13 = (v6,v6), e14 = (v9, v5) and e15 = (v9,v9).
      i. Find the degree of each vertex?
      ii. Find the adjacent matrix and incident matrix?

(3 Marks)

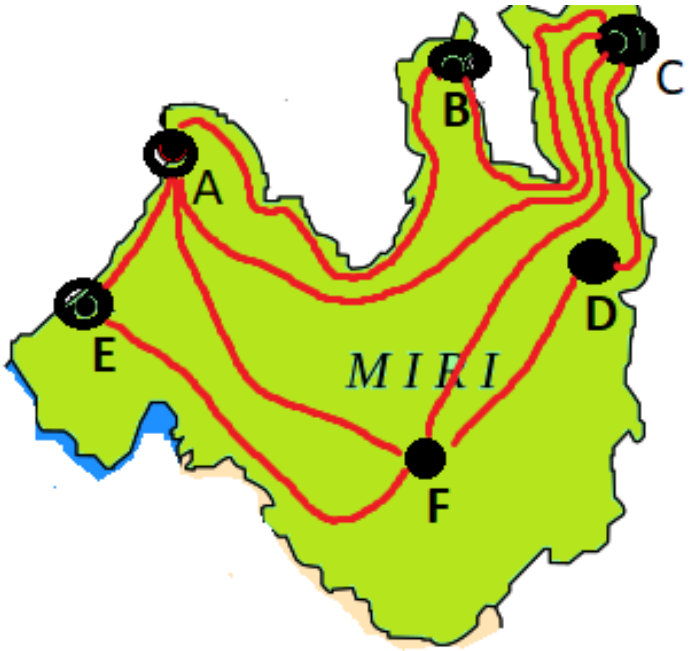
3. Draw an undirect graph for the following specified properties
   a. A graph with three vertices having the degrees of vertices 1, 3 and 4
   b. A simple graph with five vertices having the degrees of vertices 1, 3, 3, 3 and 4
   c. A simple graph in which each vertex has degrees 3 and 6 edges.

(7 Marks)

4. An undirect graph having V ={v1, v2, v3, v4, v5, v6, v7, v8, v9} and E = {e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11, e12}. Where *e*1 = (*v*1,*v*2), *e*2 = (*v*2 ,*v*8), *e*3 = (*v*8,*v*9), *e*4 = (*v*1,*v*3), *e*5 = (*v*3,*v*4) and *e*6 = (*v*3,*v*7), e7 = (v7,v6), e8 = (v3, v6), e9 = (v4, v5), e10 = (v6, v9), e11 = (v5, v8), e12 = (v5, v9). From the graph;
      i. Find all possible paths from v1 to v9?
      ii. Find all possible trails from v1 to v9?
      iii. Find the shortest and longest path from v1 to v9?
      iv. Find the shortest longest trail from v1 to v9?

(7 Marks)

5. Given is the map of Miri Town in Sarawak. There is 6 main district that is connected by the main road in Miri.

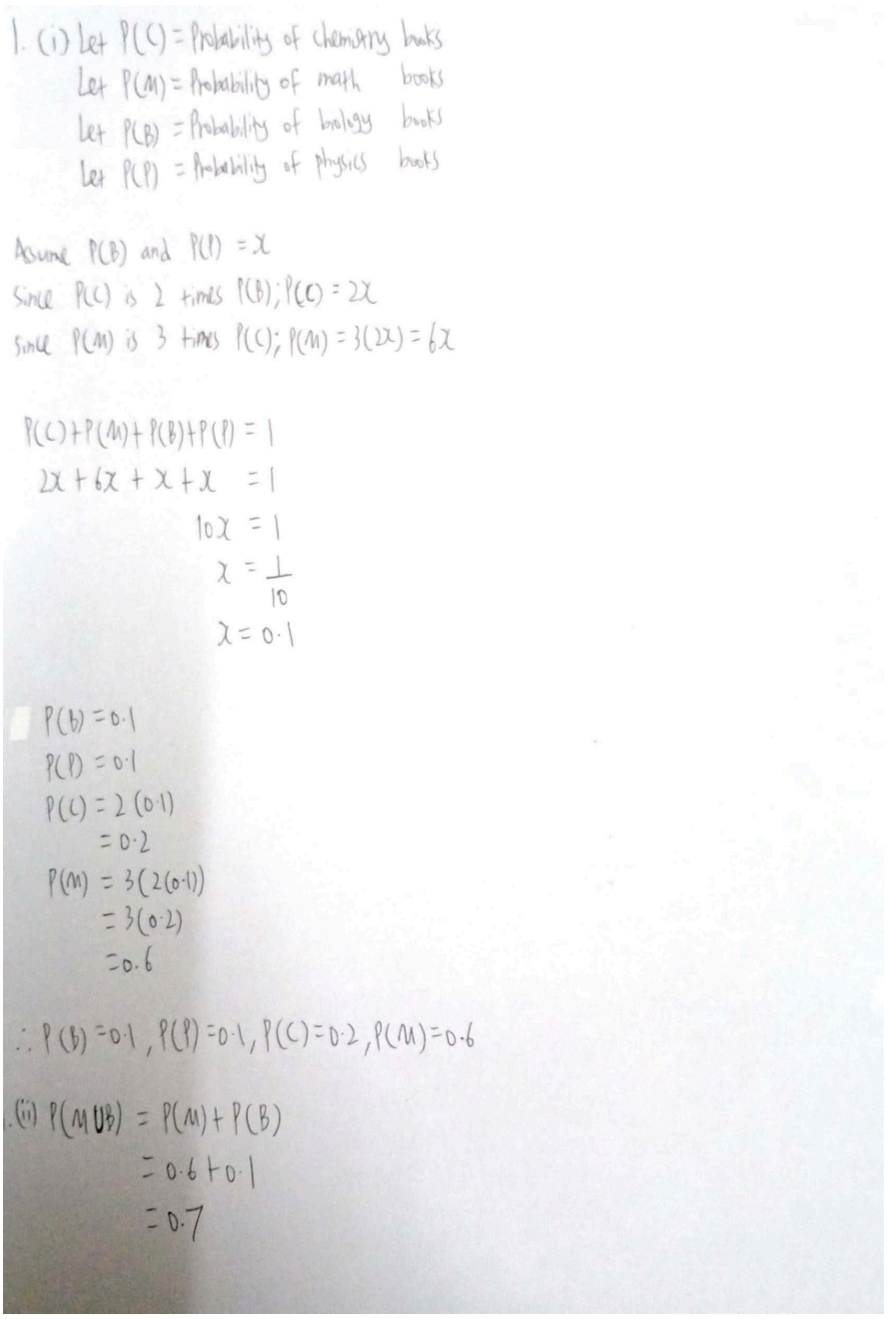


a. Find the possible Euler Path for this map?
b. Find the possible Euler Circuit for this map?
c. Find the possible Hamilton Circuit for this map?
d. Explain what the difference between Euler Circuit and Hamilton Circuit is?

(8 Marks)

Chapter 3 (3.5): Probability

QUESTION 1



1. (i) Let P(C) = Probability of chemistry books
Let P(M) = Probability of math books
Let P(B) = Probability of biology books
Let P(P) = Probability of physics books

Assume P(B) and P(P) = x
Since P(C) is 2 times P(B); P(C) = 2x
Since P(M) is 3 times P(C); P(M) = 3(2x) = 6x

$P(C)+P(M)+P(B)+P(P) = 1$
$2x + 6x + x + x = 1$
$10x = 1$
$x = \frac{1}{10}$
$x = 0.1$

$P(b) = 0.1$
$P(P) = 0.1$
$P(C) = 2(0.1)$
$= 0.2$
$P(M) = 3(2(0.1))$
$= 3(0.2)$
$= 0.6$

$\therefore P(B) = 0.1, P(P) = 0.1, P(C) = 0.2, P(M) = 0.6$

(ii) $P(M \cup B) = P(M) + P(B)$
$= 0.6 + 0.1$
$= 0.7$

QUESTION 2

2. (i) $P(A \cup B) = P(A) + P(B)$
$= 0.4 + 0.5$
$= 0.9$

(ii) $P(A^c) = 1 - P(A)$
$= 1 - 0.4$
$= 0.6$

(iii) $P(A^c \cap B)$ - Since A and B are mutually exclusive, $P(A^c \cap B) = P(B) = 0.5$

QUESTION 3

3. Total participants = 100
Total prize = 3

T_1: Anis win grand prize = $\frac{1}{100}$
$= 0.01$

T_2: Anis win second prize = $\frac{1}{100}$
$= 0.01$

T_3: Anis win third prize = $\frac{1}{100}$
$= 0.01$

Probability of Anis winning either one of the prize = $T_1 + T_2 + T_3$
$= 0.01 + 0.01 + 0.01$
$= 0.03$

QUESTION 4

Question 4

Let $P(P)$ = Probability a randomly chosen male has pneumonia = 0.40

$P(S)$ = Probability a male is smoker

$P(S|P)$ = Probability a male is smoker given he ~~have~~ has pneumonia = 0.80

$P(S|\neg P)$ = Probability a male is smoker given he doesn't have pneumonia = 0.30

i) $P(\neg P) = 1 - 0.40$

$= 0.60$

ii) $P(P|S) = \frac{P(P \cap S)}{P(S)}$

$P(S|P) = \frac{P(S \cap P)}{P(P)} = 0.80$

$\frac{P(S \cap P)}{0.40} = 0.80$

$P(S \cap P) = 0.80 \times 0.40$

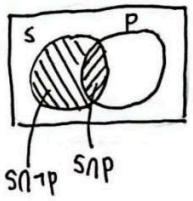
$= 0.32$

$P(S|\neg P) = \frac{P(S \cap \neg P)}{P(\neg P)} = 0.30$

$\frac{P(S \cap \neg P)}{0.60} = 0.30$

$P(S \cap \neg P) = 0.60 \times 0.30$

$= 0.18$



$P(S) = P(S \cap P) + P(S \cap \neg P)$

$= 0.18 + 0.32$

$= 0.50$

$P(P|S) = \frac{P(P \cap S)}{P(S)}$

$= \frac{0.32}{0.50}$

$= 0.64$

QUESTION 5

Question 5

$P(\text{Black}) = \frac{1}{3}$

B B

Probability choosing black boots both time:

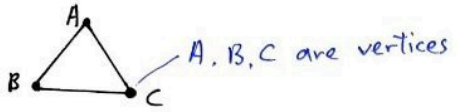
$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Chapter 4 (4.1 to 4.6): Graph Theory
QUESTION 1

Chapter 4

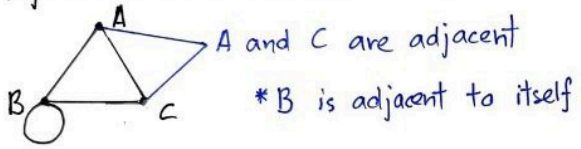
Question 1

1a) Vertices are the nodes or dots in a graph.



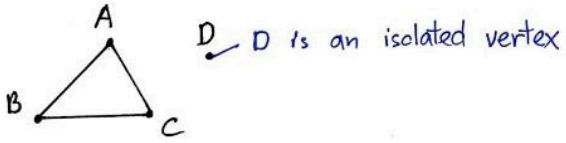
b) Edges are the line connecting two vertices in a graph.

c) Adjacent vertices are two vertices (dots) connected by an edge (line).

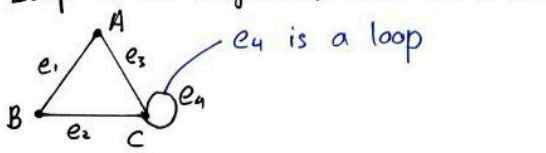


d) An edge is said to be incident to a vertex if it connects to that vertex

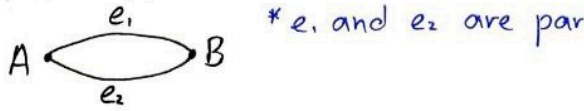
e) Isolated vertex is a vertex with no edges (lines) connected to it.



f) Loop is an edge (line) that starts and ends at the same vertex.



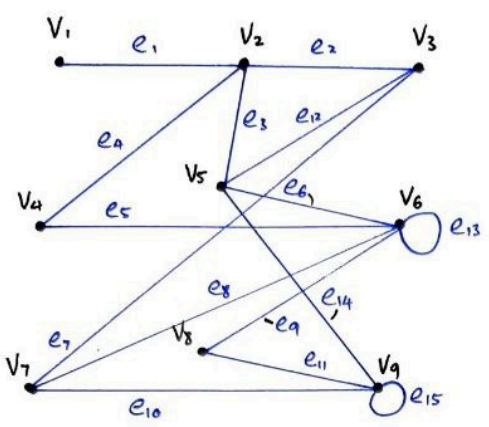
g) Parallel edges are two or more edges that connect the same pair of vertices.



QUESTION 2

Question 2

a)



i)

Vertex	Degree
V_1	1
V_2	4
V_3	3
V_4	2
V_5	4
V_6	6
V_7	3
V_8	2
V_9	5

ii) Adjacent

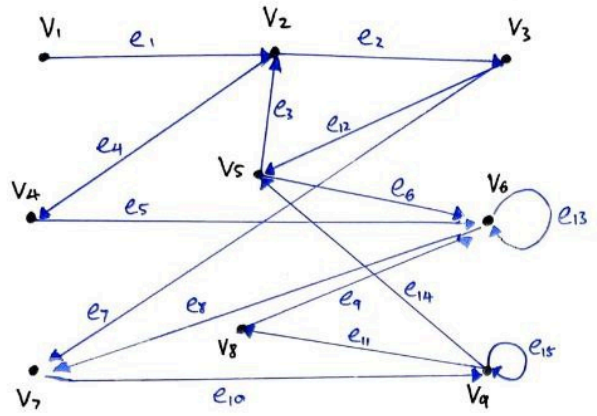
$$A_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_8 & V_9 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

ii) Incident

$$I_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{array} \begin{array}{c} \begin{array}{ccccccccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} & e_{14} & e_{15} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 2 \end{bmatrix} \end{array}$$

Question 2

b)



i)

Vertex	In degree	Out degree
V_1	0	1
V_2	2	2
V_3	1	2
V_4	1	1
V_5	2	2
V_6	4	2
V_7	2	1
V_8	1	1
V_9	2	3

ii) Adjacent

$A_{G_1} =$

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9
V_1	0	1	0	0	0	0	0	0	0
V_2	0	0	1	1	0	0	0	0	0
V_3	0	0	0	0	1	0	1	0	0
V_4	0	0	0	0	0	1	0	0	0
V_5	0	1	0	0	0	1	0	0	0
V_6	0	0	0	0	0	1	1	0	0
V_7	0	0	0	0	0	0	0	0	1
V_8	0	0	0	0	0	1	0	0	0
V_9	0	0	0	0	1	0	0	1	1

ii) Incident

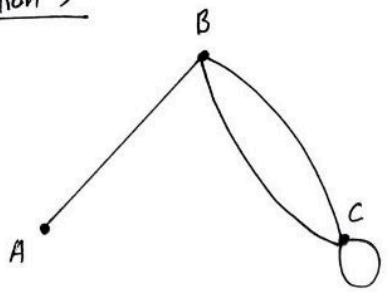
$I_{G_1} =$

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
V_1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
V_2	-1	1	-1	1	0	0	0	0	0	0	0	0	0	0	0
V_3	0	-1	0	0	0	0	1	0	0	0	0	1	0	0	0
V_4	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0
V_5	0	0	1	0	0	1	0	0	0	0	0	-1	0	-1	0
V_6	0	0	0	0	-1	-1	0	1	-1	0	0	0	1	0	0
V_7	0	0	0	0	0	0	-1	-1	0	1	0	0	0	0	0
V_8	0	0	0	0	0	0	0	0	1	0	-1	0	0	0	0
V_9	0	0	0	0	0	0	0	0	0	-1	1	0	0	1	1

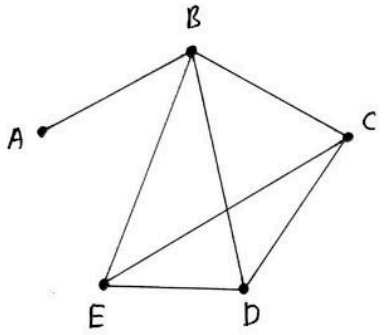
QUESTION 3

Question 3

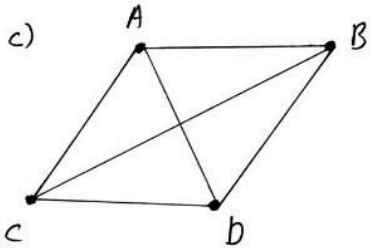
a)



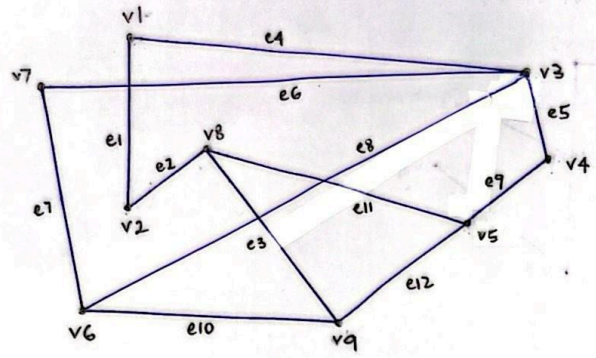
b)



c)



QUESTION 4



i. Find all possible path from v1 to v9?

Path #1 : (v1, e1, v2, e2, v8, e3, v9)
Path #2 : (v1, e1, v2, e2, v8, e11, v5, e12, v9)
Path #3 : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e6, v7, e7, v6, e10, v9)
Path #4 : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e8, v6, e10, v9)
Path #5 : (v1, e4, v3, e6, v7, e7, v6, e10, v9)
Path #6 : (v1, e4, v3, e8, v6, e10, v9)
Path #7 : (v1, e4, v3, e5, v4, e9, v5, e12, v9)
Path #8 : (v1, e4, v3, e5, v4, e9, v5, e11, v8, e3, v9)

ii Find all possible trails from v1 to v9?

Trail #1 : (v1, e1, v2, e2, v8, e3, v9)
Trail #2 : (v1, e1, v2, e2, v8, e11, v5, e12, v9)
Trail #3 : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e6, v7, e7, v6, e10, v9)
Trail #4 : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e8, v6, e10, v9)
Trail #5 : (v1, e4, v3, e6, v7, e7, v6, e10, v9)
Trail #6 : (v1, e4, v3, e8, v6, e10, v9)
Trail #7 : (v1, e4, v3, e5, v4, e9, v5, e12, v9)
Trail #8 : (v1, e4, v3, e5, v4, e9, v5, e11, v8, e3, v9)
Trail #9 : (v1, e4, v3, e8, v6, e7, v7, e6, v3, e5, v4, e9, v5, e12, v9)
Trail #10 : (v1, e4, v3, e6, v7, e7, v6, e8, v3, e5, v4, e9, v5, e12, v9)

iii. Find the longest path and the shortest path from v1 to v9?

Shortest path : (v1, e1, v2, e2, v8, e3, v9) / (v1, e4, v3, e8, v6, e10, v9)

Longest path : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e6, v7, e7, v6, e10, v9)

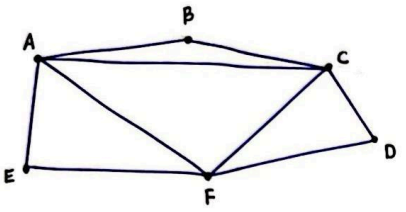
iv. Find the shortest and logest trail from v1 to v9?

Shortest : (v1, e1, v2, e2, v8, e3, v9) / (v1, e4, v3, e8, v6, e10, v9)

Longest : (v1, e1, v2, e2, v8, e11, v5, e9, v4, e5, v3, e6, v7, e7, v6, e10, v9)

QUESTION 5

b.



i. Find the possible Euler path for the path?

↳ (E, A, B, C, A, F, C, D, F, E)

ii. Find the possible Euler Circuit for the map?

↳ (F, E, A, B, C, D, F, A, C, F)

iii. Find the Hamiltonian Circuit for this map?

↳ (D, C, B, A, E, F, D)

iv. Differences between Euler & Hamiltonian Circuit.

Euler Circuit	Hamiltonian Circuit
- Visits every edge exactly once	- Visit every vertex exactly once.
- Every vertex must be visited at least once.	- Every edge doesn't need to be visited at least once.