TUTORIAL 2: Number Systems and Codes

1. Convert the binary numbers to its decimal equivalent.

   a. $10110_2 = 2^4 + 2^2 + 2^1 = 22_{10}$

   b. $\overset{7}{1}00\overset{4}{1}0\overset{2}{1}0\overset{0}{1}_2 = 2^7 + 2^4 + 2^2 + 2^0 = 149_{10}$

   c. $\overset{6}{1}\overset{5}{1}0\overset{3}{1}0\overset{1}{1}\overset{0}{1}_2 = 2^6 + 2^5 + 2^3 + 2^1 + 2^0 = 107_{10}$

   d. $\overset{11}{1}00\overset{8}{1}0000\overset{3}{1}00\overset{0}{1}_2 = 2^{11} + 2^8 + 2^3 + 2^0 = 2313_{10}$

   e. $\overset{8}{1}00\overset{5}{1}0\overset{3}{1}\overset{2}{1}00_2 = 2^8 + 2^5 + 2^3 + 2^2 = 300_{10}$

2. Convert the decimal numbers to its binary equivalent using **repetitive division method**.

   a. 77_{10}

   b. 96_{10}

   c. 205_{10}

   d. 1040_{10}

   e. 3216_{10}

3. Convert the decimal numbers in Question (2) to its binary equivalent using **weighted summation** method.

4. Convert the decimal numbers to its binary equivalent (to four radix point)

   a. 1305.375_{10}

   b. 111.33_{10}

   c. 301.12_{10}

   d. 164.875_{10}

   e. 1000.01_{10}

5. Convert the hexadecimal numbers to its decimal equivalent.

   a. 743_{16}

   b. 2000_{16}

   c. $7FF_{16}$

   d. $ABCD_{16}$

   e. 165_{16}

6. Convert the decimal numbers to its hexadecimal equivalent.

   a. 59_{10}

   b. 1024_{10}

   c. 2313_{10}

   d. $65{,}536_{10}$

   e. 919_{10}

2. Convert the decimal numbers to its binary equivalent using **repetitive division method**.

a. $77_{10} = 1001101_2$

b. $96_{10} = 1100000_2$

c. $205_{10} = 11001101_2$

d. $1040_{10} = 10000010000_2$

e. $3216_{10} = 110010010000_2$

a) 2 | 77
2 | 38 1
2 | 19 0
2 | 9 1
2 | 4 1
2 | 2 0
2 | 1 0
0 1

b) 2 | 96
2 | 48 0
2 | 24 ... 0
2 | 12 0
2 | 6 0
2 | 3 0
2 | 1 1
0 1

c) 2 | 205
2 | 102 1
2 | 51 0
2 | 25 1
2 | 12 1
2 | 6 0
2 | 3 0
2 | 1 1
0 1

d) 2 | 1040
2 | 520 0
2 | 260 0
2 | 130 0
2 | 65 0
2 | 32 1
2 | 16 0
2 | 8 0
2 | 4 0
2 | 2 0
2 | 1 0
0 1

e) 2 | 3216
2 | 1608 .. 0
2 | 804 ... 0
2 | 402 ... 0
2 | 201 ... 0
2 | 100 1
2 | 50 0
2 | 25 0
2 | 12 1
2 | 6 0
2 | 3 0
2 | 1 1
0 1

3. Convert the decimal numbers in Question (2) to its binary equivalent using **weighted summation** method.

	1	2	4	8	16	32	64	128	256	512	1024	2048
a)	1	0	1	1	0	0	1					
b)	0	0	0	0	0	1	1					
c)	1	0	1	1	0	0	1	1				
d)	0	0	0	0	1	0	0	0	0	0	1	
e)	0	0	0	0	1	0	0	1	0	0	1	1

4. Convert the decimal numbers to its binary equivalent (to four radix point)

a. 1305.375_{10} = 10100011001.0110_2

b. 111.33_{10} = 1101111.0101_2

c. 301.12_{10} = 100101101.0001_2

d. 164.875_{10} = 10100100.1110_2

e. 1000.01_{10} = 1111101000.0000_2

a) 1305.375_{10}

2 | 1305
2 | 652 1
2 | 326 0
2 | 163 0
2 | 81 1
2 | 40 1
2 | 20 0
2 | 10 0
2 | 5 0
2 | 2 1
2 | 1 0
0 1

$0.375 \times 2 = 0.75$ ... 0
$0.75 \times 2 = 1.5$ 1
$0.5 \times 2 = 1.0$ 1
$0 \times 2 = 0$ ... 0

b) 2 | 111
2 | 55 ... 1
2 | 27 ... 1
2 | 13 ... 1
2 | 6 ... 1
2 | 3 ... 0
2 | 1 ... 1
0 ... 1

$0.33 \times 2 = 0.66$ ... 0
$0.66 \times 2 = 1.32$ ... 1
$0.32 \times 2 = 0.64$ ... 0
$0.64 \times 2 = 1.28$ ... 1

c) 2 | 301
2 | 150 ... 1
2 | 75 ... 0
2 | 37 ... 1
2 | 18 ... 1
2 | 9 ... 0
2 | 4 ... 1
2 ... 0

2 | 2
2 | 1 ... 0
0 ... 1

$0.12 \times 2 = 0.24$ ... 0
$0.24 \times 2 = 0.48$ ... 0
$0.48 \times 2 = 0.96$ ... 0
$0.96 \times 2 = 1.92$ ... 1

d) 2 | 164
2 | 82 ... 0
2 | 41 ... 0
2 | 20 ... 1
2 | 10 ... 0
2 | 5 ... 0
2 | 2 ... 1
2 | 1 ... 0
0 ... 1

$0.875 \times 2 = 1.75$ ... 1
$0.75 \times 2 = 1.5$ ... 1
$0.5 \times 2 = 1.0$ ... 1

e) 1000.01_{10}

2 | 1000
2 | 500 ... 0
2 | 250 ... 0
2 | 125 ... 0
2 | 62 ... 1
2 | 31 ... 0
2 | 15 ... 1
2 | 7 ... 1
2 | 3 ... 1
2 | 1 ... 1
0 ... 1

$0.01 \times 2 = 0.02$ ... 0
$0.02 \times 2 = 0.04$ ... 0
$0.04 \times 2 = 0.08$ ... 0
$0.08 \times 2 = 0.16$ ... 0

5. Convert the hexadecimal numbers to its decimal equivalent.

a. 743_{16}

b. 2000_{16}

c. $7FF_{16}$

d. $ABCD_{16}$

e. 165_{16}

a) $743_{16} = 7(16^2) + 4(16^1) + 3(16^0)$

$= 1859_{10}$ ✓

b) $2000_{16} = 2(16^3)$

$= 8192_{10}$ ✓

c) $7FF_{16} = 7(16^2) + 15(16^1) + 15(16^0)$

$= 2047_{10}$ ✓

d) $ABCD_{16} = 10(16^3) + 11(16^2) + 12(16^1) + 13(16^0)$

$= 43981_{10}$ ✓

e) $165_{16} = 1(16^2) + 6(16^1) + 5(16^0)$

$= 357_{10}$ ✓

6. Convert the decimal numbers to its hexadecimal equivalent.

a. $59_{10} = 3B_{16}$ ✓

b. $1024_{10} = 400_{16}$ ✓

c. $2313_{10} = 909_{16}$ ✓

d. $65{,}536_{10} = 10000_{16}$ ✓

e. $919_{10} = 397_{16}$ ✓

a)
16 | 59
16 | 3 .... 11 = B
0 ... 3

b)
16 | 1024
16 | 64 0
16 | 4 ... 0
0 4

c)
16 | 2313
16 | 144 9
16 | 9 ... 0
0 9

d)
16 | 65536
16 | 4096 .. 0
16 | 256 ... 0
16 | 16 ... 0
16 | 1 ... 0
0 ... 1

e)
16 | 919
16 | 57 ... 7
16 | 3 9
0 3

7. Convert the octal numbers to its decimal equivalent.

   a. 56_8
   b. 467_8
   c. 1000_8
   d. 2341_8
   e. 31456_8

8. Convert the decimal numbers to its octal equivalent.

   a. 59_{10}
   b. 1024_{10}
   c. 2313_{10}
   d. 65536_{10}
   e. 919_{10}

9. When a large decimal number is to be converted to binary, it is **sometimes** easier to convert it to hex and then from hex to binary. Try this procedure for 3216_{10} and compare it with the procedure used in Question (2).

10. Convert the binary numbers in Question (1) to

    a. hexadecimal
    b. octal

11. Convert the binary numbers to its equivalent hexadecimal and octal values.

    a. 10111010100.101
    b. 1000001101111.01

12. Encode the decimal numbers in BCD.

    a. 77_{10}
    b. 96_{10}
    c. 205_{10}
    d. 1040_{10}
    e. 3216_{10}

13. The following numbers are in BCD. Convert them to decimal.

    a. 100101110101010010
    b. 010101010101
    c. 0111011101110101

14. Which of the following numbers are valid BCD values?

    a. 0110101100110001
    b. 1001110000001000
    c. 00000000000

7. Convert the octal numbers to its decimal equivalent.

a. 56_8

b. 467_8

c. 1000_8

d. 2341_8

e. 31456_8

a) $56_8 = 5(8^1) + 6(8^0)$
$= 46_{10}$ ✓

b) $467_8 = 4(8^2) + 6(8^1) + 7(8^0)$
$= 311_{10}$ ✓

c) $1000_8 = 1(8^3)$
$= 512_{10}$ ✓

d) $2341_8 = 2(8^3) + 3(8^2) + 4(8^1) + 1(8^0)$
$= 1249_{10}$ ✓

e) $31456_8 = 3(8^4) + 1(8^3) + 4(8^2) + 5(8^1)$
$+ 6(8^0)$
$= 13102_{10}$ ✓

8. Convert the decimal numbers to its octal equivalent.

a. $59_{10} = 73_8$ ✓

b. $1024_{10} = 2000_8$ ✓

c. $2313_{10} = 4411_8$ ✓

d. $65536_{10} = 200000_8$ ✓

e. $919_{10} = 1627_8$ ✓

a)
8 | 59
8 | 7 3
0 7

b)
8 | 1024
8 | 128 0
8 | 16 0
8 | 2 0
0 2

c)
8 | 2313
8 | 289 1
8 | 36 ... 1
8 | 4 ... 4
0 ... 4

d)
8 | 65536
8 | 8192 0
8 | 1024 0
8 | 128 0
8 | 16 ... 0
8 | 2 ... 0
0 2

e)
8 | 919
8 | 114 ... 7
8 | 14 ... 2
8 | 1 ... 6
0 1

9. When a large decimal number is to be converted to binary, it is **sometimes** easier to convert it to hex and then from hex to binary. Try this procedure for 3216_{10} and compare it with the procedure used in Question (2).

$10 \rightarrow 16 \rightarrow 2$

16 | 3216
16 | 201 ...0
16 | 12 .. 9
0 ... 12 = C

$3216_{10} = C90_{16}$

$C90_{16} = 1100\ 1001\ 0000_2$

10. Convert the binary numbers in Question (1) to

a. hexadecimal 16

b. octal

a. 10110_2
b. 10010101_2
c. 1101011_2
d. 100100001001_2
e. 100101100_2

a) $10110_2 = 0001\ 0110$

$= 15_{16}$ 16_{16}

$10010101_2 = 1001\ 0101$

$= 95_{16}$

$1101011_2 = 0110\ 1011$

$= 6B_{16}$

$100100001001_2 = 1001\ 0000\ 1001$

$= 909_{16}$

$100101100_2 = 0001\ 0010\ 1100$

$= 12C_{16}$

b) $10110_2 = 010\ 110$

$= 26_8$

$010010101_2 = 010\ 010\ 101$

$= 225_8$

$1101011_2 = 001\ 101\ 011$

$= 153_8$

$100100001001_2 = 100\ 100\ 001\ 001$

$= 4411_8$

$100101100_2 = 100\ 101\ 100$

$= 454_8$

11. Convert the binary numbers to its equivalent hexadecimal and octal values.

a. 10111010100.101

b. 10000011011111.01

a) 10111010100.101_2

$= 0101\ 1101\ 0100\ .\ 1010$

$= 5D4.A_{16}$

010111010100.101_2

$= 010\ 111\ 010\ 100\ .\ 101$

$= 2\textcircled{6}24.5_8$

2724.5_8

b) 1000 0011 01111.01$_2$

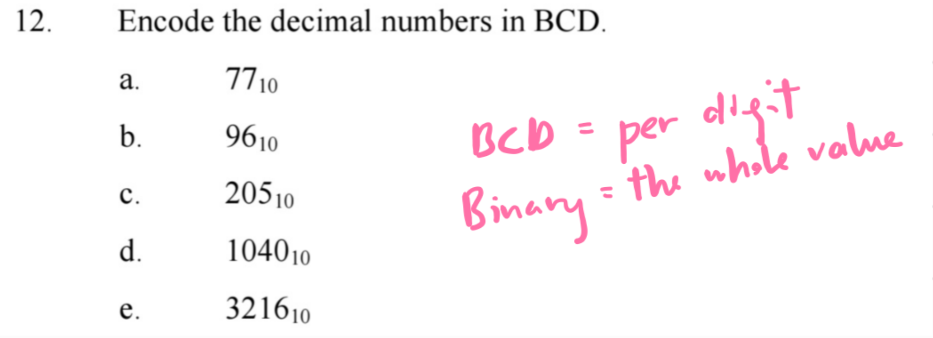
= 0001 0000 0110 1111.0100

= 106F.4$_{16}$ ✓

1000 0011 01111.01$_2$

= 001 000 001 101 111 . 010

= 10157.2$_8$ ✓



12. Encode the decimal numbers in BCD.

a. 77_{10}

b. 96_{10}

c. 205_{10}

d. 1040_{10}

e. 3216_{10}

BCD = per digit

Binary = the whole value

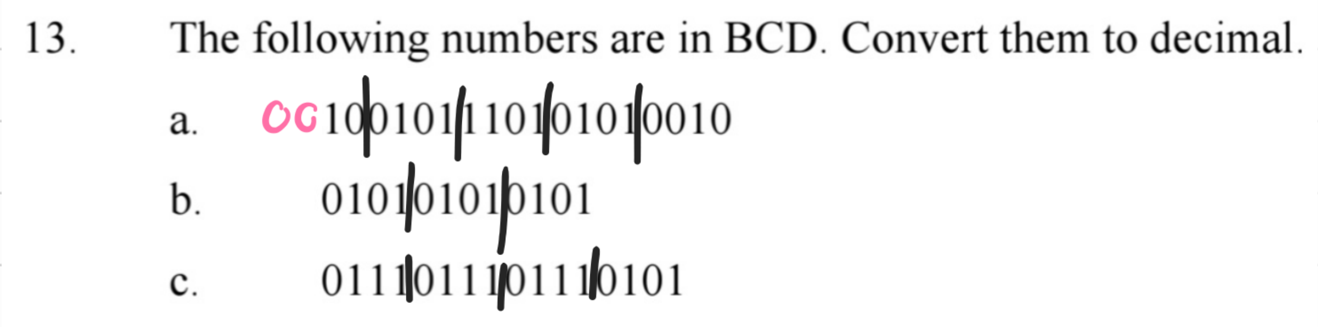
a) 0111 0111

b) 1001 0110

c) 0010 0000 0101

d) 0001 0000 0100 0000

e) 0011 0010 0001 0110



13. The following numbers are in BCD. Convert them to decimal.

a. 00 1001011101010010

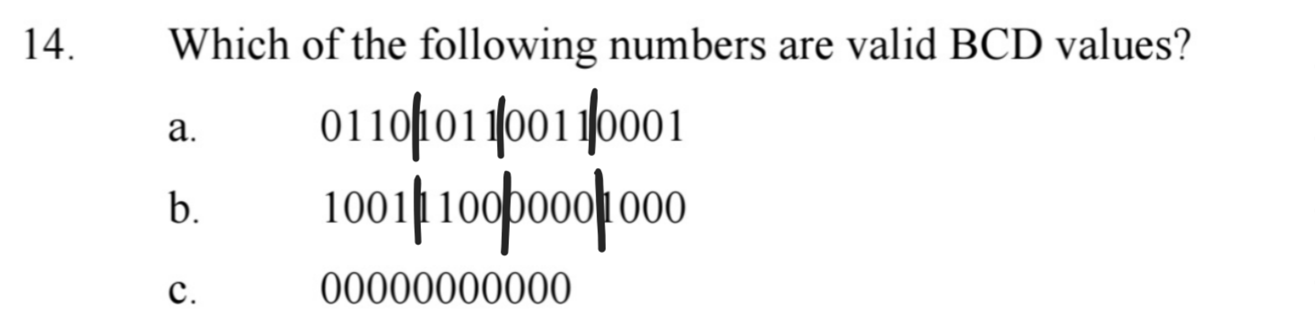
b. 010101010101

c. 0111011101110101

a) Invalid BCD code

b) 555_{10}

c) 7775_{10}



14. Which of the following numbers are valid BCD values?

a. 0110101100110001

b. 1001110000001000

c. 000000000000

a) Invalid ($1011 = 11_{10}$)

b) Invalid ($1100 = 12_{10}$)

c) Valid

15. What is the largest BCD-encoded decimal value that can be represented in three bytes?

16. For the question below please refer to the ASCII table.
    a. What is the most significant nibble of the ASCII code for letter E?
    b. Represent the statement “X = 3 × Y” in ASCII code. Attach each character values with even parity bit.
    c. The following bytes (shown in hex) represent a person’s name as it would be stored in the computer’s memory. Determine the name of each person.
        i. 42 72 61 64 20 50 69 74 74
        ii. 41 6E 67 65 6C 69 6E 61

17. Calculate the lower and upper bound of signed number for 7-bit number system using the representation of
    a. sign and magnitude
    b. 1’s complement
    c. 2’s complement

18. Calculate the binary signed values in the representation format of (i) sign and magnitude, (ii) 1’s complement and 2’s complement using 8-bit number system.
    a. $+ 55_{10}$
    b. $+ 127_{10}$
    b. $- 87_{10}$
    c. $- 128_{10}$

19. Given a number system specification: **size of a number is 6 bit, including the sign bit AND signed numbers using 2’s complement**

    Calculate and show your working for the arithmetic operations below.
    a. 18 + 3
    b. -18 + 3
    c. 18 – 3
    d. -18 - 3

15. What is the largest BCD-encoded decimal value that can be represented in three bytes?

1001 1001 1001 $= 999_{10}$

16. For the question below please refer to the ASCII table.

a. What is the most significant nibble of the ASCII code for letter E?

b. Represent the statement “X = 3 × Y” in ASCII code. Attach each character values with even parity bit.

c. The following bytes (shown in hex) represent a person’s name as it would be stored in the computer’s memory. Determine the name of each person.

i. 42 72 61 64 20 50 69 74 74

ii. 41 6E 67 65 6C 69 6E 61

a) 45_{16} = 0100 0101

∴ 0100

b) 'X' = 58(hex) = 1011000 (binary) = 1101 1000 (even parity)

' ' = 20(hex) = 0100000 (binary) = 1010 0000 (even parity)

'=' = 3D(hex) = 011 1101 (binary) = 1011 1101 (even parity)

'3' = 33 (hex) = 0110011 (binary) = 0011 0011 (even parity)

'*' = 2A = 0101010 = 1010 1010

'Y' = 59 = 1011001 = 0101 1001

ASCII = 11011000 10111101 00110011 10101010 01011001 (no spacing)

ci) Brad Pitt

ii) Angelina

17. Calculate the lower and upper bound of signed number for 7-bit number system using the representation of

a. sign and magnitude

b. 1’s complement

c. 2’s complement

a) lower $= -(2^{n-1}-1)$

$= -(2^{7-1}-1)$

$= -63$

upper $= (2^{n-1}-1)$

$= (2^{7-1}-1)$

$= 63$

b) lower $= -(2^{n-1}-1)$

$= -(2^{7-1}-1)$

$= -63$

upper $= (2^{n-1}-1)$

$= 63$

c) lower $= -(2^{n-1})$

$= -(2^{7-1})$

$= -64$

upper $= (2^{n-1}-1)$

$= (2^{7-1}-1)$

$= 63$

18. Calculate the binary signed values in the representation format of (i) sign and magnitude, (ii) 1's complement and 2's complement using 8-bit number system.

a. $+55_{10}$

b. $+127_{10}$

b. -87_{10}

c. -128_{10}

ai) 2 | 55
2 | 27 ... 1
2 | 13 ... 1
2 | 6 ... 1
2 | 3 ... 0
2 | 1 ... 1
0 ... 1

$+55_{10} = +(0110111)$
$= 00110111$

ii) 1's = 11001000

iii) 2's = 11001000
\+ 1
11001001

bi) 2 | 127
2 | 63 ... 1
2 | 31 ... 1
2 | 15 ... 1
2 | 7 ... 1
2 | 3 ... 1
2 | 1 ... 1
0 ... 1

$+127_{10} = +(1111111)$
$= 01111111$

ii) 1's = 10000000

ii) 2's = 1000 0000
\+ 1
1000 0001

ci) 2 | 87
2 | 43 ... 1
2 | 21 ... 1
2 | 10 ... 1
2 | 5 ... 1
2 | 2 ... 1
2 | 1 ... 0
0 ... 1

$-87_{10} = -(101\ 1111)$
$= 1101\ 1111$

ii) 1's = 0010 0000

ii) 2's = 0010 0000
\+ 1
0010 0001

di) 2 | 128
2 | 64 0
2 | 32 0
2 | 16 0
2 | 8 0
2 | 4 0
2 | 2 0
2 | 1 0
0 ... 1

$-128_{10} = X$

lower bound 1's = -127

upper 1's = 128

Out of range

19. Given a number system specification: **size of a number is 6 bit, including the sign bit AND signed numbers using 2's complement**

Calculate and show your working for the arithmetic operations below.

a. 18 + 3

b. -18 + 3

c. 18 – 3

d. -18 - 3

a) +18 = 010010

+3 = 000011

$$\begin{array}{r} 010010 \\ +\ 000011 \\ \hline 010101 \end{array}$$

$+18 + 3 = 010101_2$

b) -18 = -(010010)

1's = 101101

$$\begin{array}{r} 2's = 101101 \\ +\qquad\quad 1 \\ \hline 101110 \end{array}$$

$$\begin{array}{r} -18 + 3 = 101110 \\ +\ 000011 \\ \hline 110001 \end{array}$$

$-18 + 3 = 110001_2$

c) -3 = -(+3)

= -(000011)

1's = 111100

$$\begin{array}{r} 2's = 111100 \\ +\qquad\quad 1 \\ \hline 111101 \end{array}$$

$$\begin{array}{r} 18 + (-3) = 010010 \\ +\ 111101 \\ \hline 1001111 \end{array}$$

$18 - 3 = 001111_2$

d) -18 = 101110 (2's)

-3 = 111101 (2's)

$$\begin{array}{r} -18 + (-3) = 101110 \\ +\ 111101 \\ \hline 1101011 \end{array}$$

$-18 - 3 = 101011_2$