Aprendizado de Máquina e Reconhecimento de Padrões

Polynomial Logistic Regression

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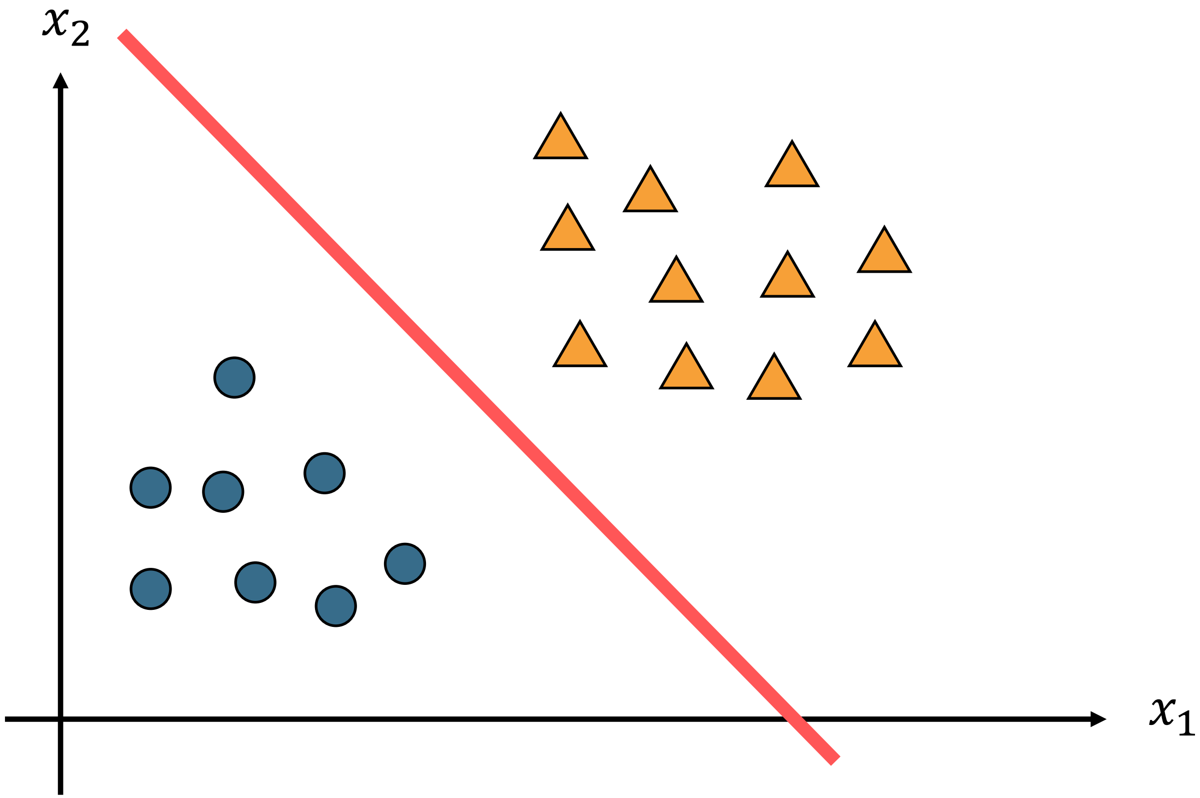
Linear and Nonlinear Separability of Classes

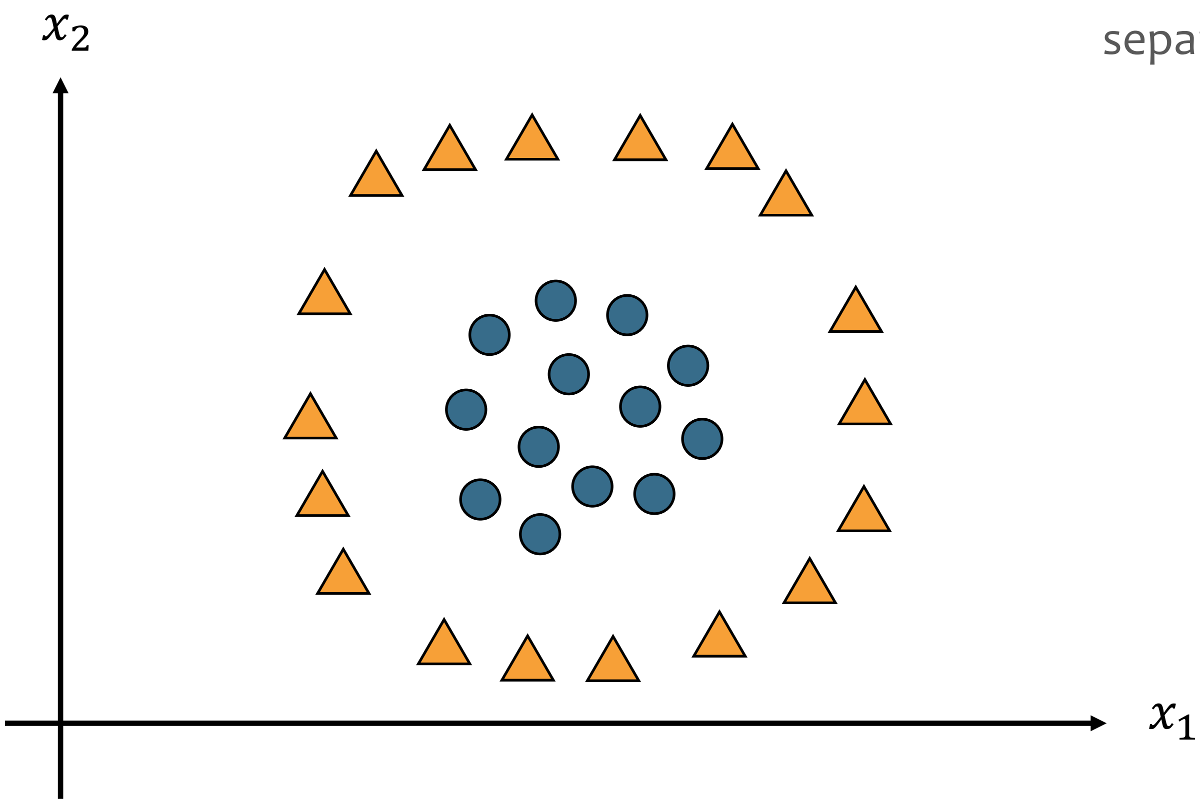
Several **classification problems** of different domains are **linearly separable**:

- **Their classes** can be separated in the feature space by **a linear model**.



Linear and Nonlinear Separability of Classes

However, there are **classification problems** where **there is no** linear separability **between classes**.



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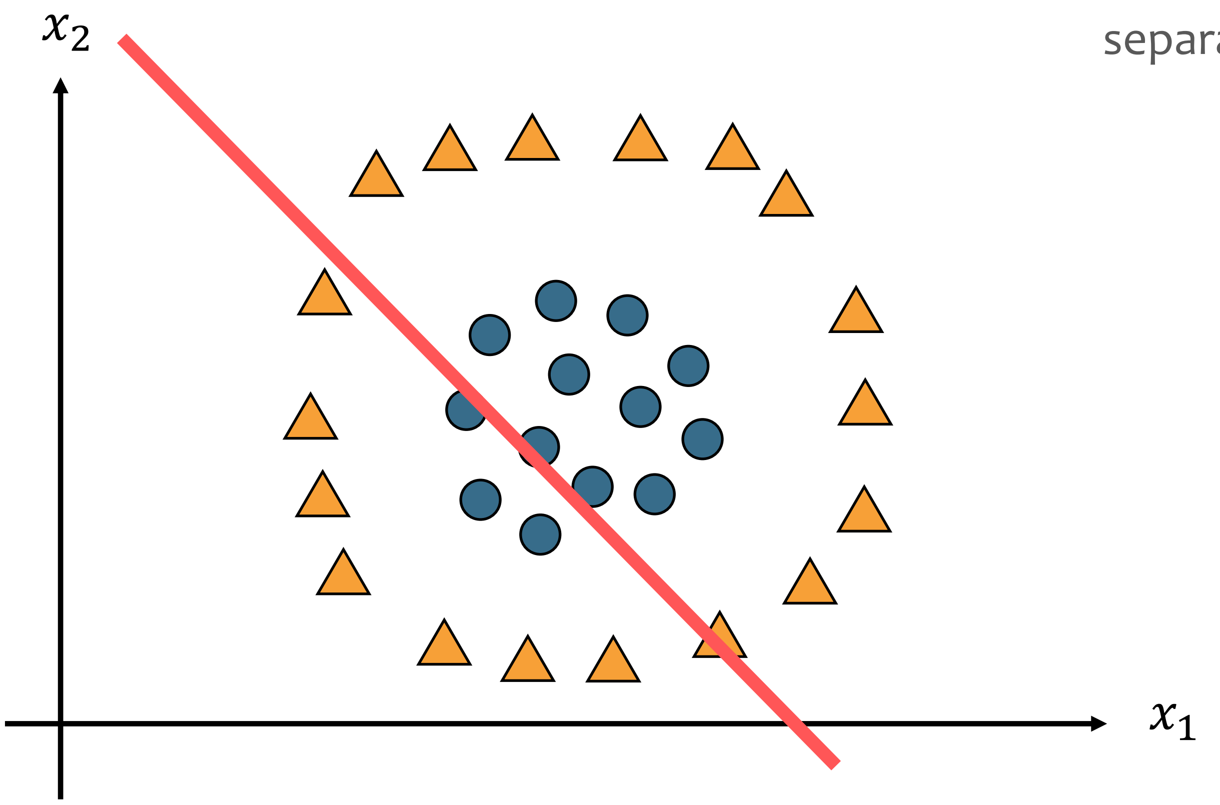

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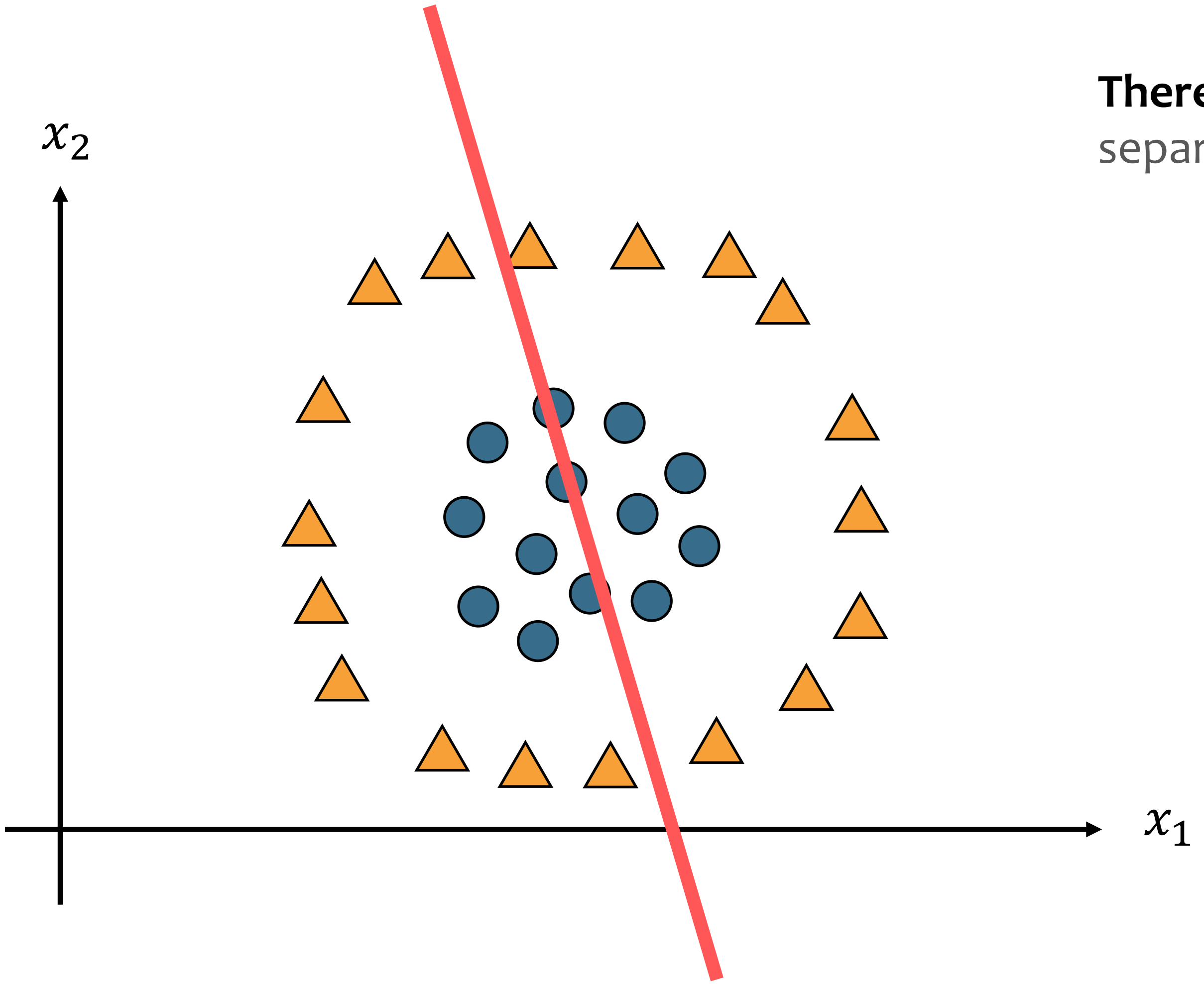

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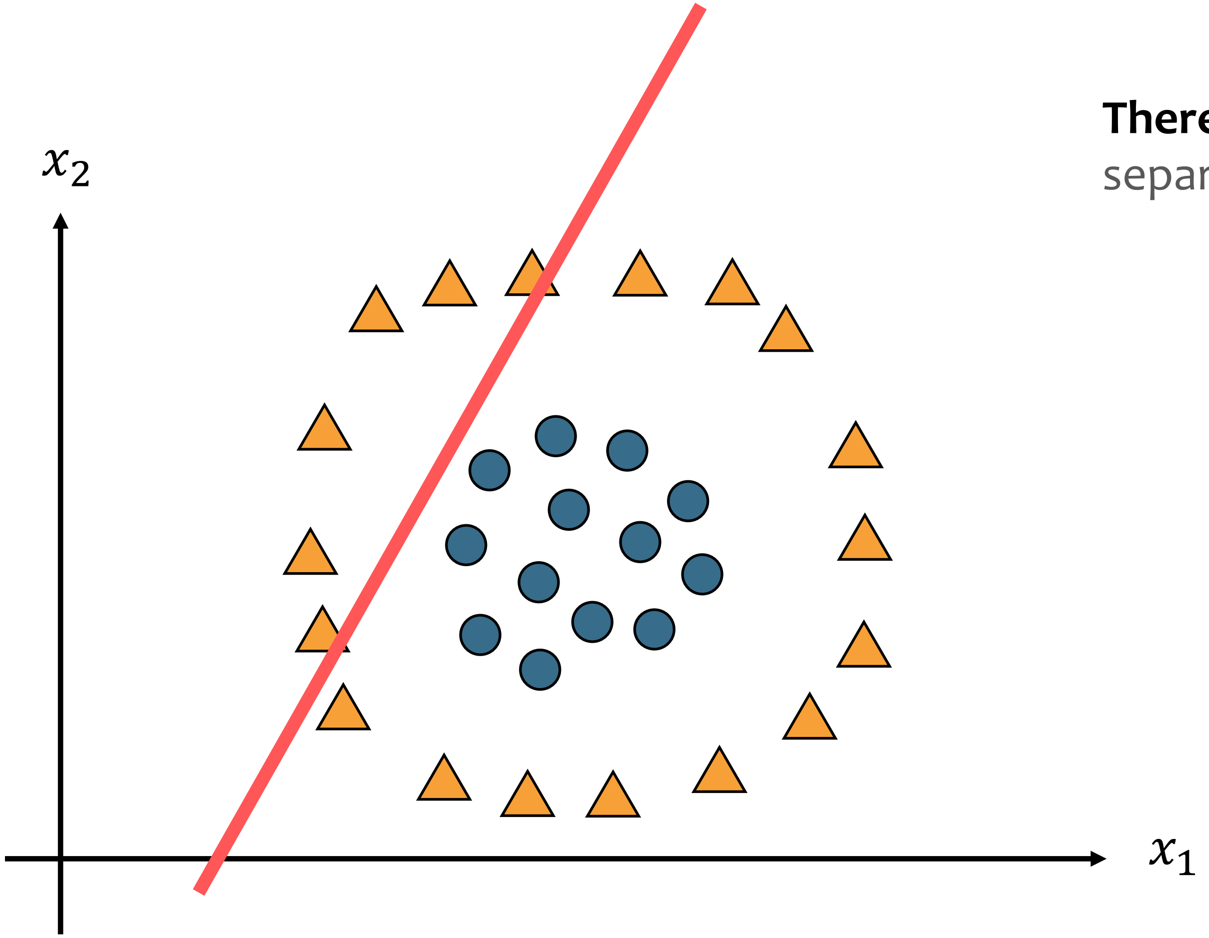

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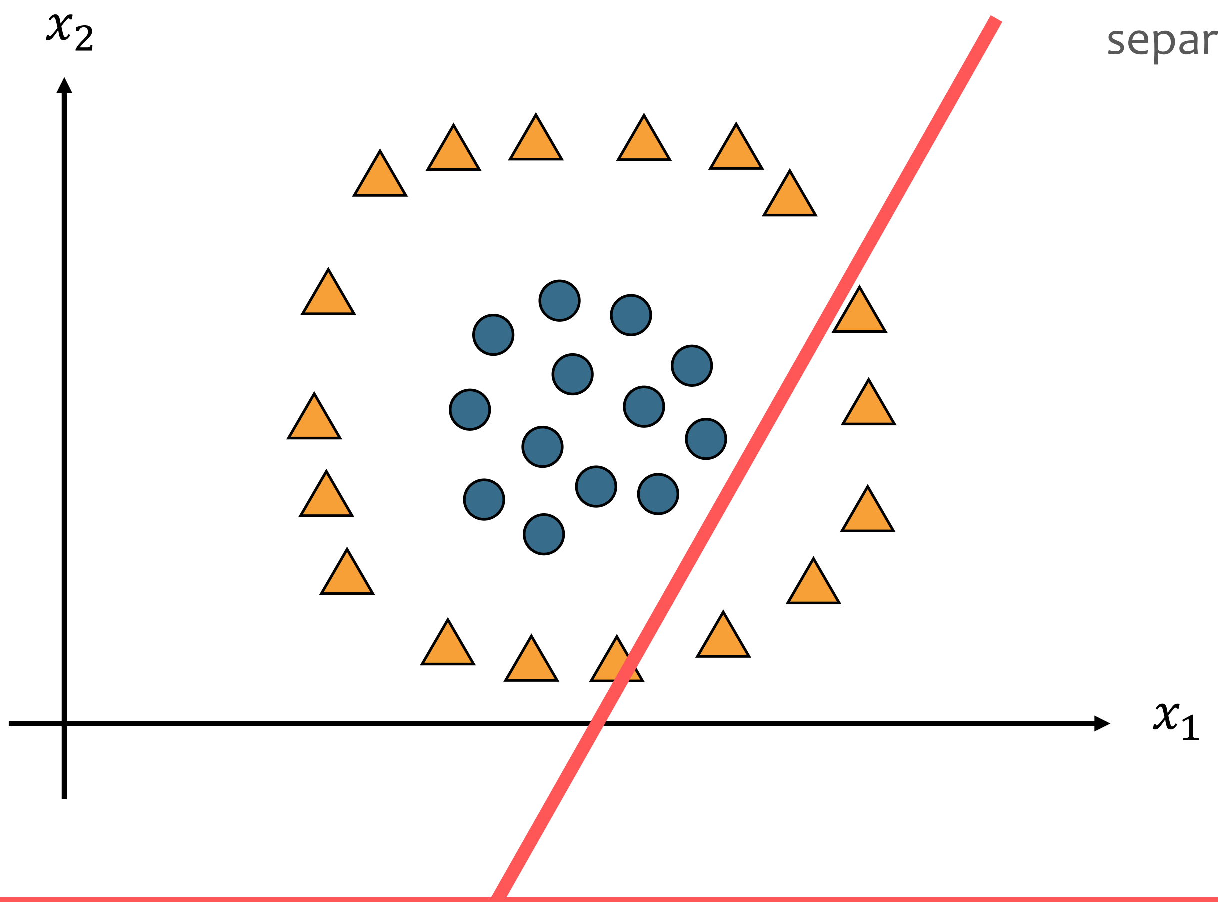

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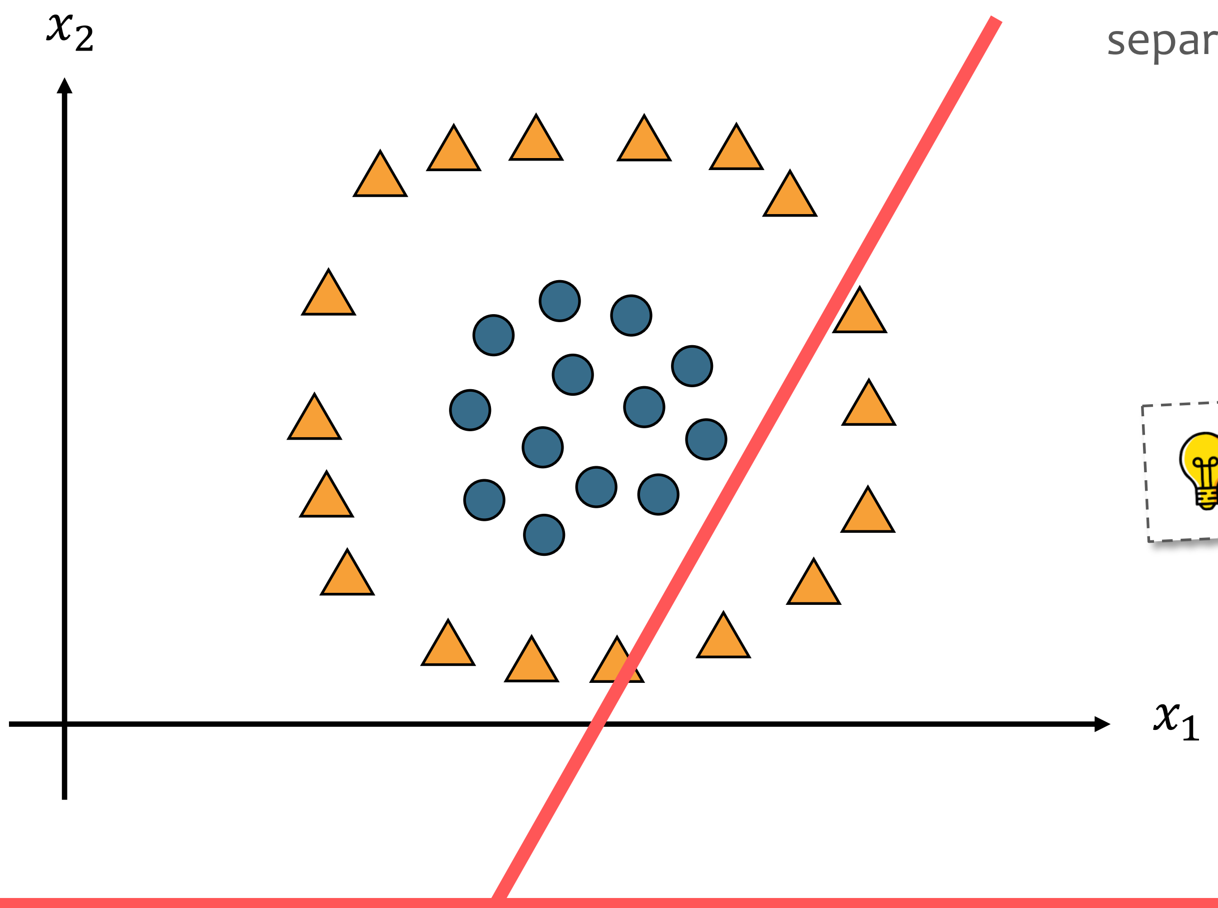

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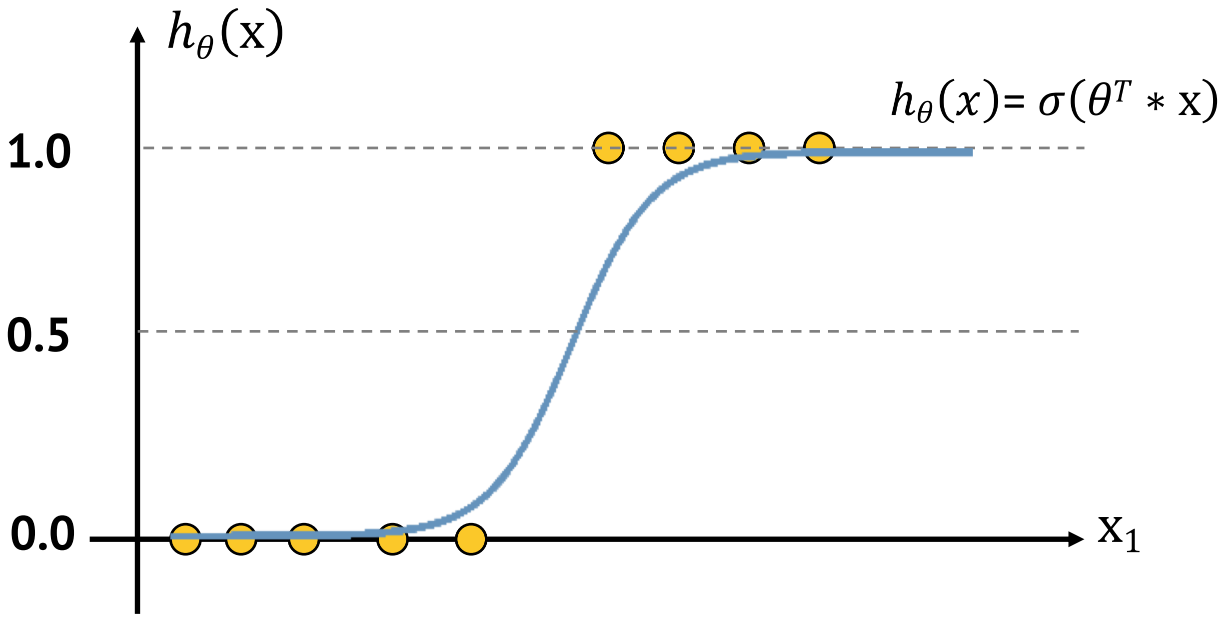
These problems require **nonlinear models.**

Logistic Regression



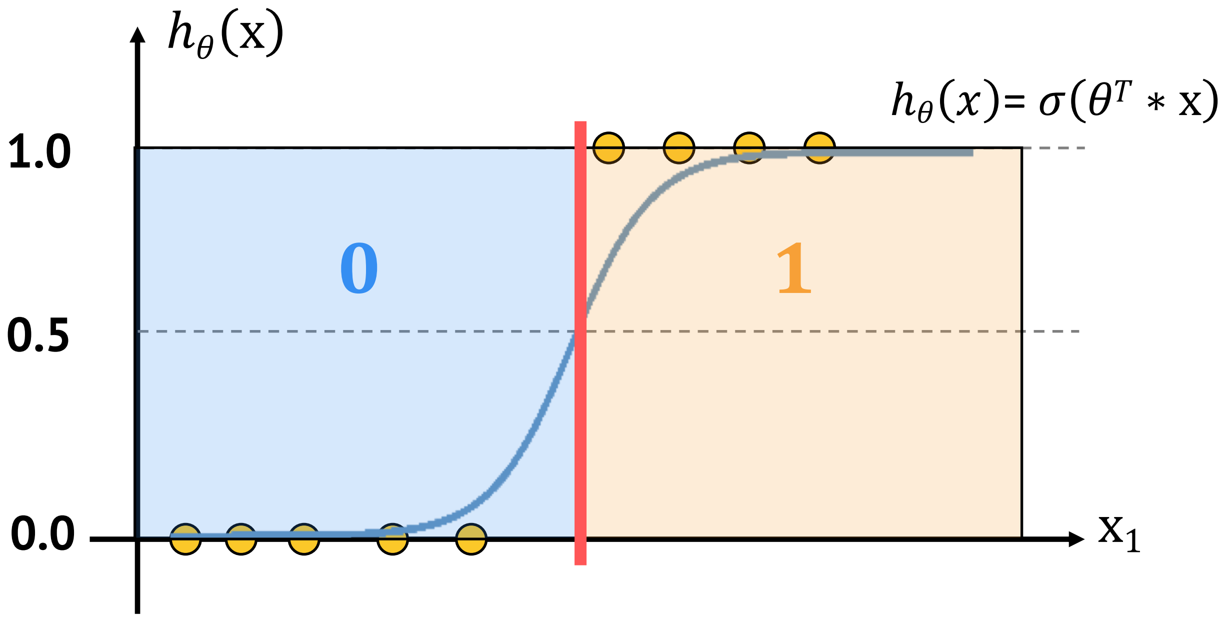
$$\boxed{h_\theta(\mathrm{x}) = \sigma(\theta^T * \mathrm{x}) = \frac{1}{1+e^{-(\theta^T * \mathrm{x})}}}$$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix} \qquad \mathrm{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$h_\theta(\mathrm{x}) = \mathrm{P}(y = 1 | \mathrm{x}; \theta)$$

estimated probability of the observation x is **the positive class** $(y = 1)$ given a model with parameter set θ.

Logistic Regression



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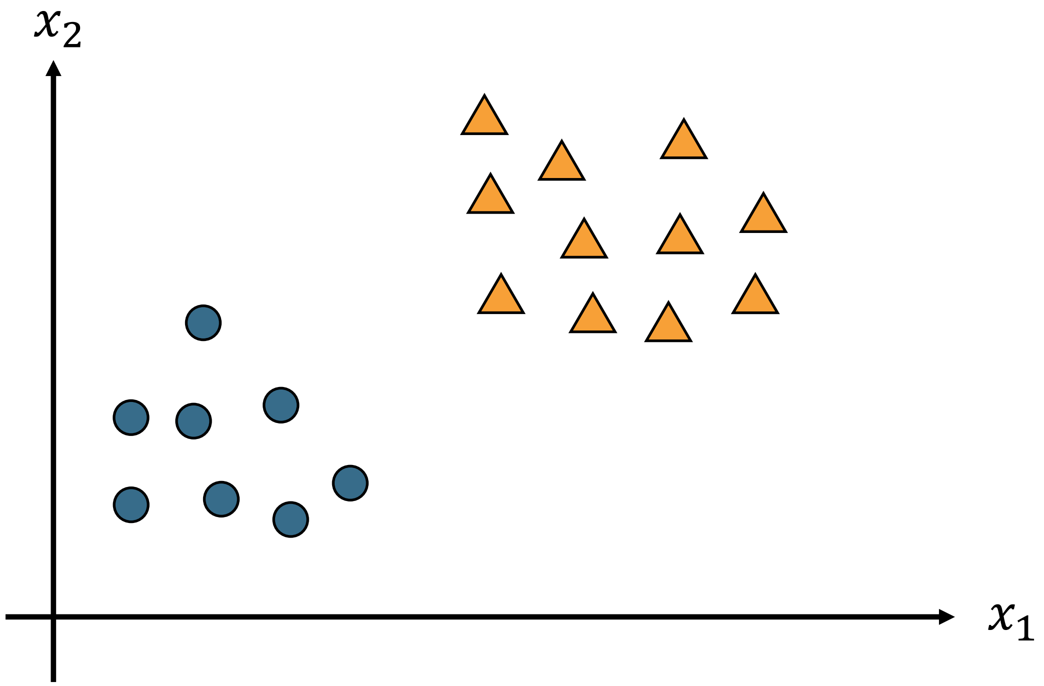

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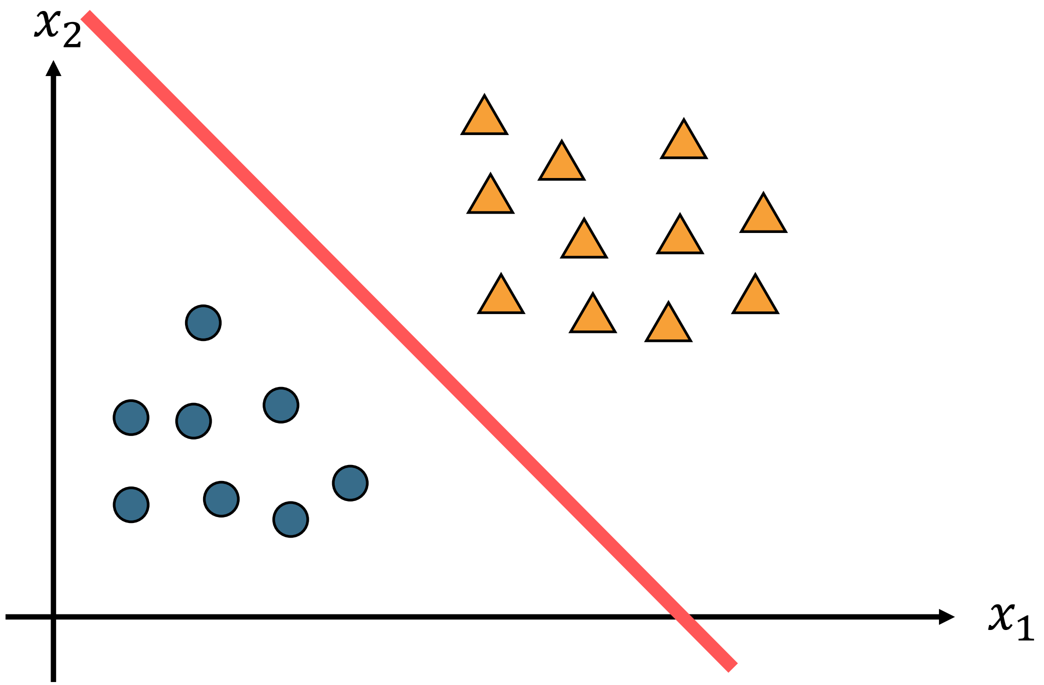

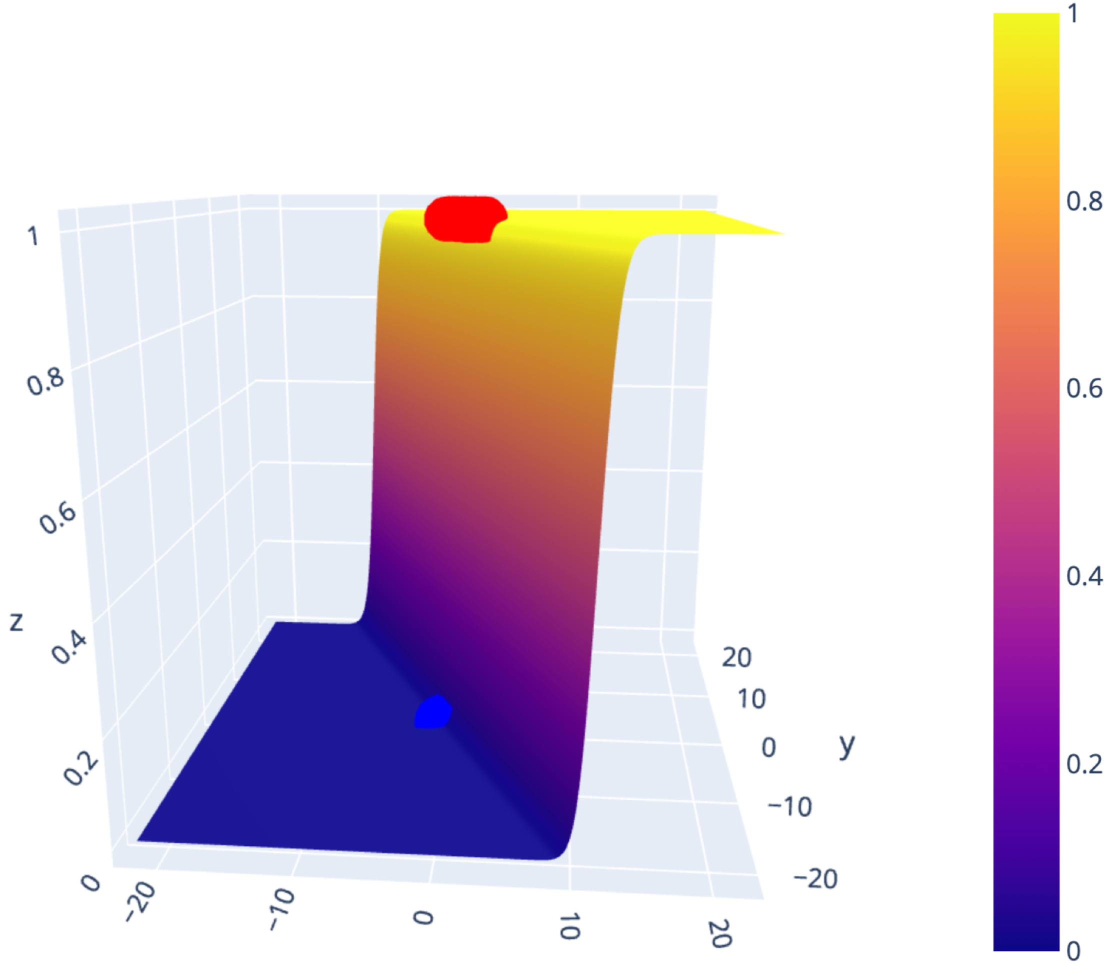

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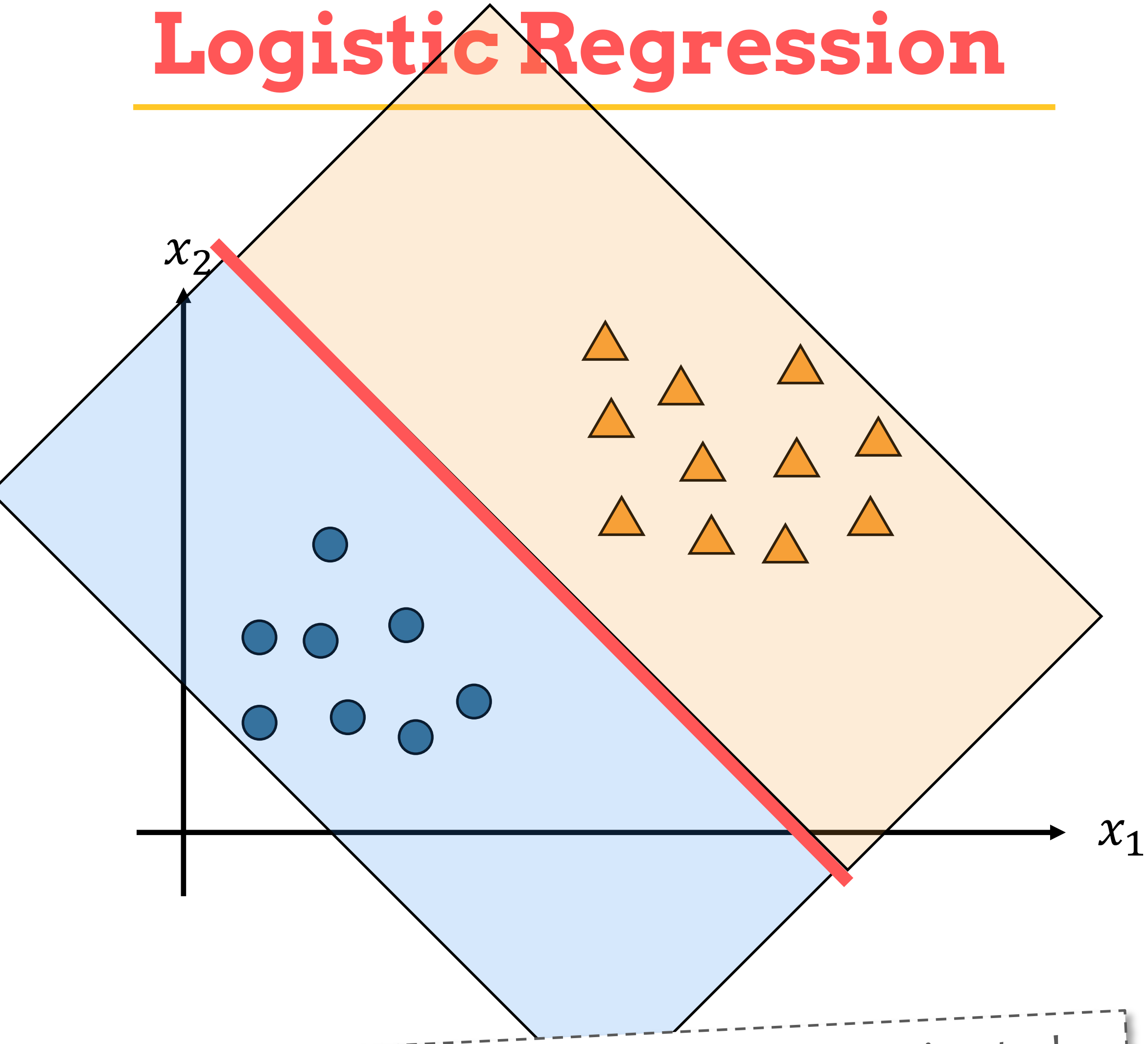

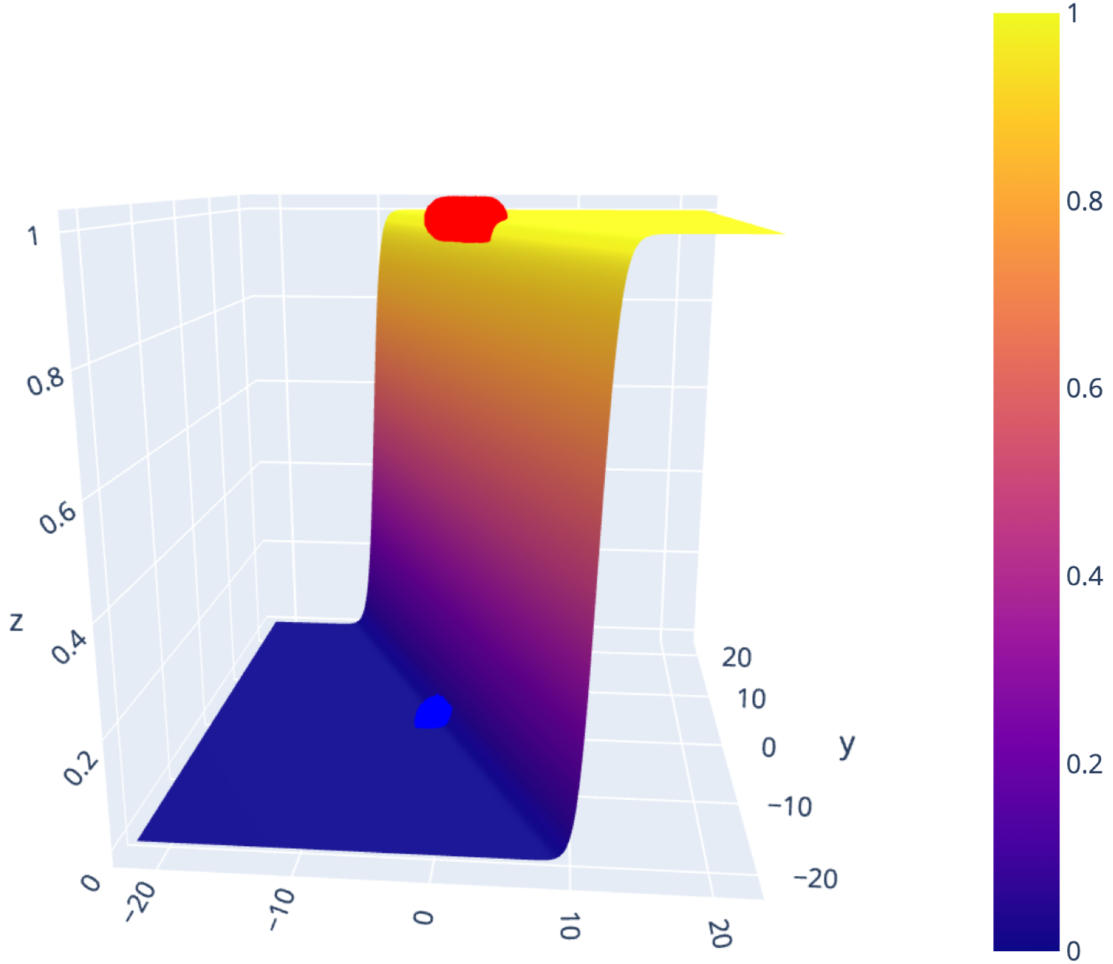

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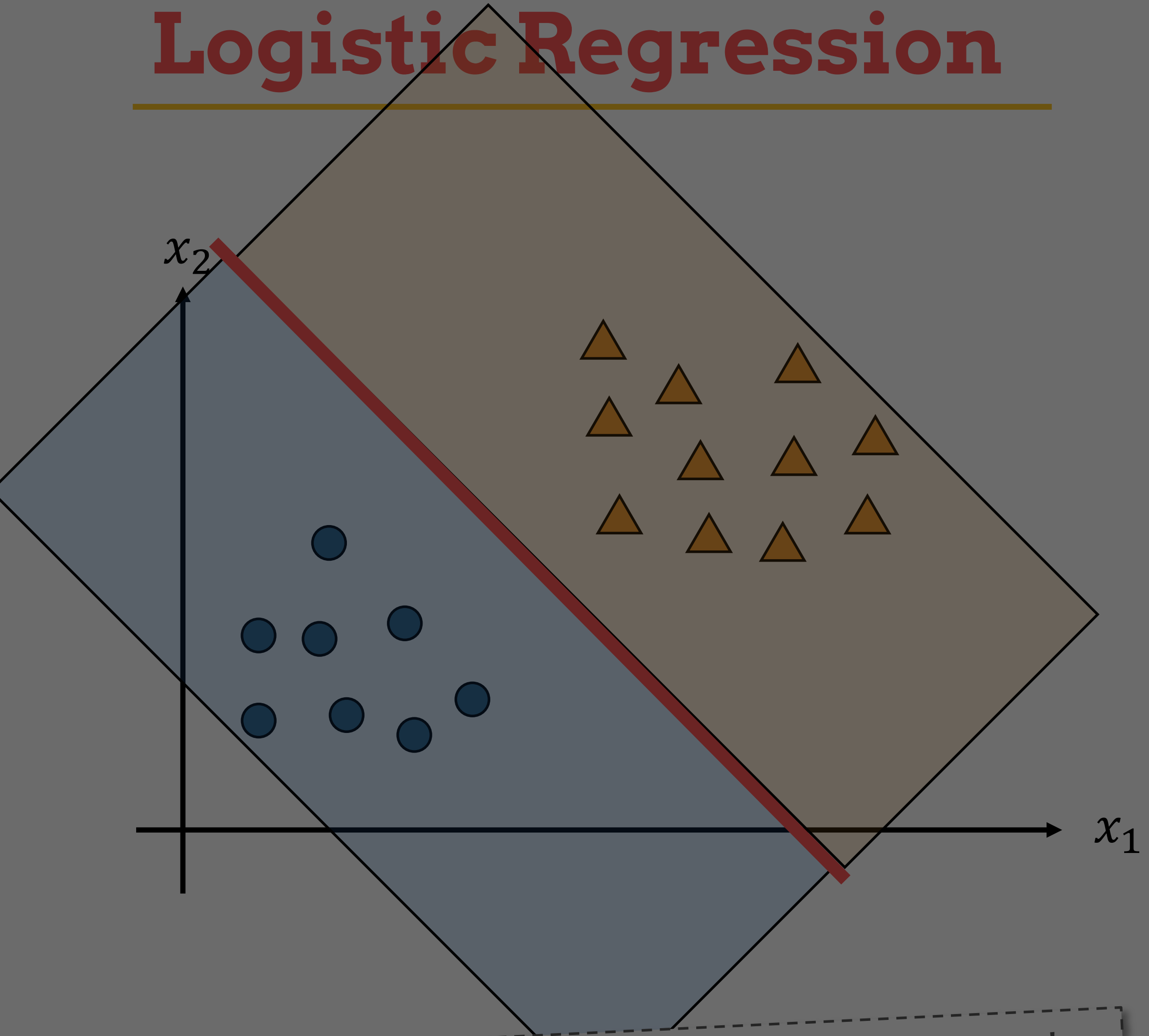

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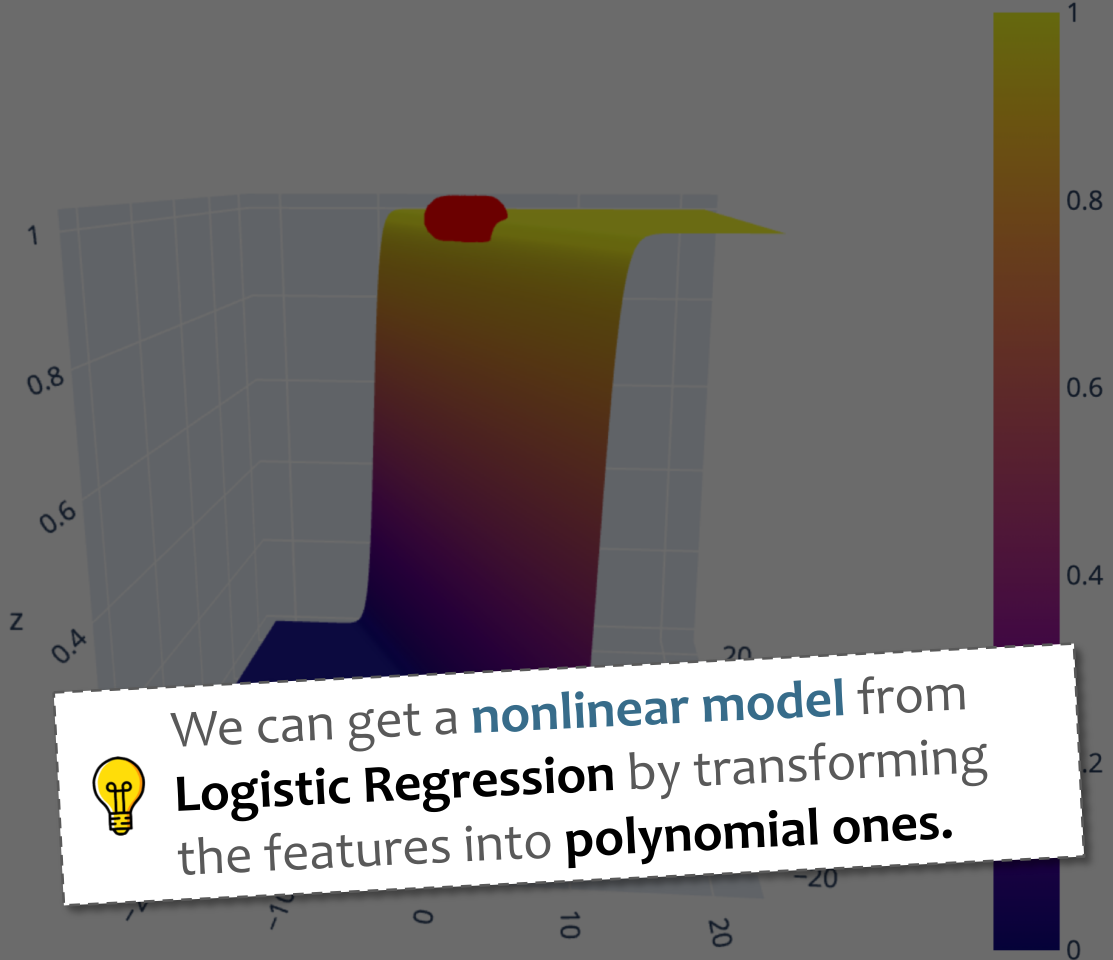

💡 We can get a **nonlinear model** from **Logistic Regression** by transforming the features into **polynomial ones.**

Polynomial Logistic Regression

$n = 2$

two features

$$h_\theta(\mathrm{x}) = \sigma(\theta^T * \mathrm{x}) = \sigma(\theta_0 + \theta_1 \mathrm{x}_1 + \theta_2 \mathrm{x}_2) = \frac{1}{1+e^{-(\theta_0+\theta_1\mathrm{x}_1+\theta_2\mathrm{x}_2)}}$$

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`sklearn.preprocessing.PolynomialFeatures`

https://scikit-learn.org/stable/modules/generated/sklearn.preprocessing.PolynomialFeatures.html

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Degrees (d)**: 2**

$n = 5$

$$h_\theta(\mathrm{x}) = \sigma(\theta^T * \mathrm{x}) = \sigma(\theta_0 + \theta_1 \mathrm{x}_1 + \theta_2 \mathrm{x}_2 + \theta_3 \mathrm{x}_1^2 + \theta_4 \mathrm{x}_1 \mathrm{x}_2 + \theta_5 \mathrm{x}_2^2) = \frac{1}{1+e^{-(\theta_0+\theta_1 \mathrm{x}_1+\theta_2 \mathrm{x}_2+\theta_3 \mathrm{x}_1^2+\theta_4 \mathrm{x}_1 \mathrm{x}_2+\theta_5 \mathrm{x}_2^2)}}$$

Very similar to **Polynomial Regression.**

Degrees (d): 2

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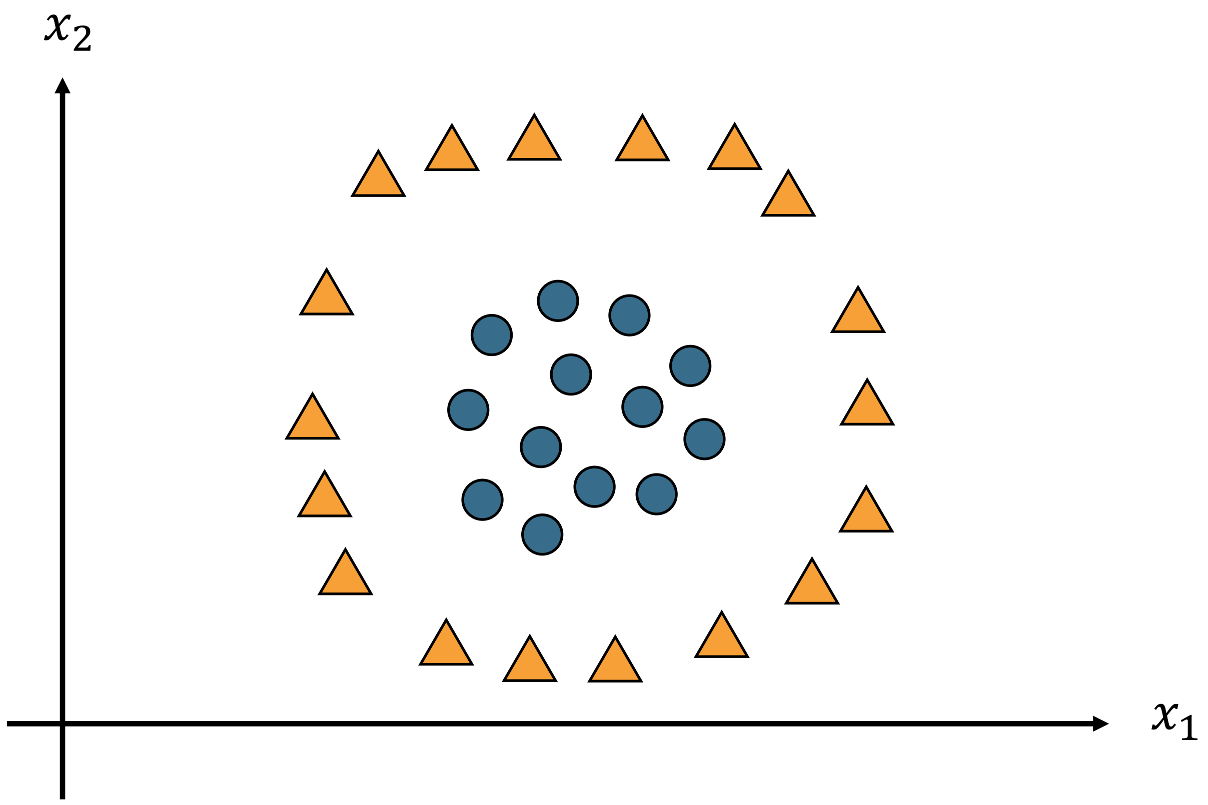
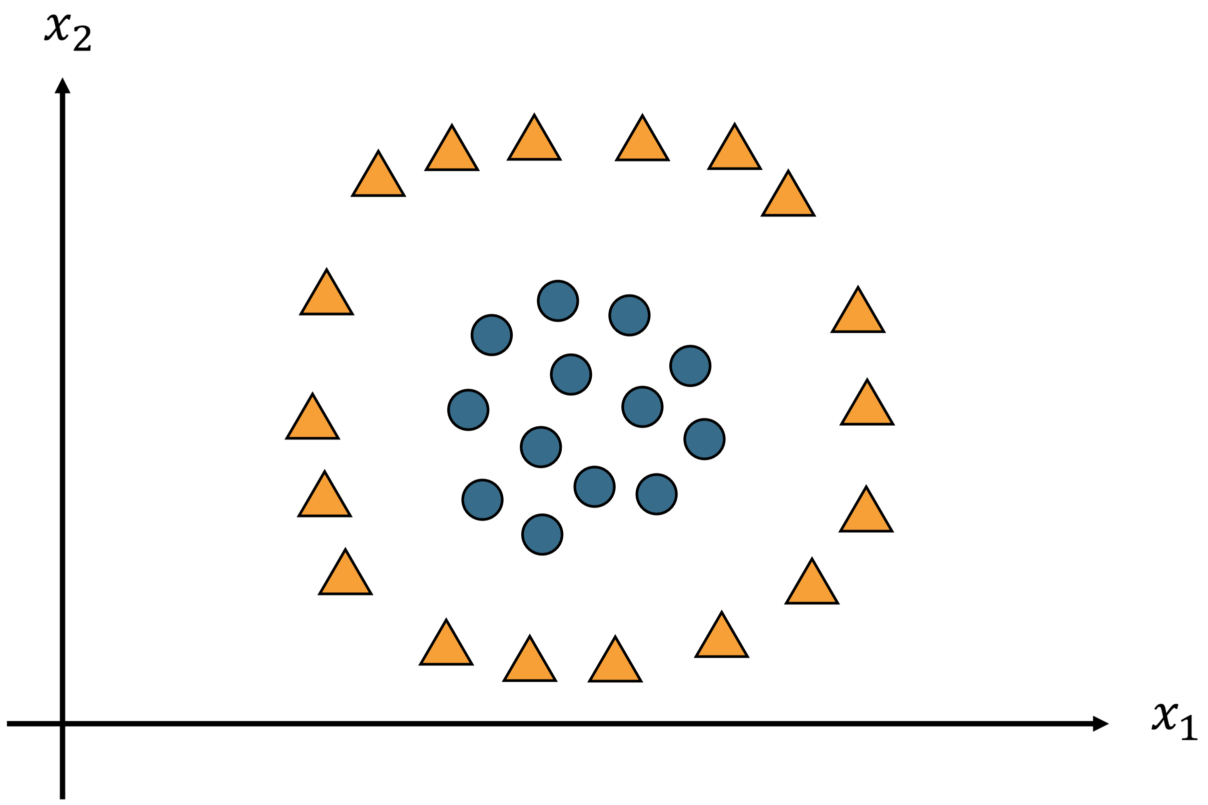
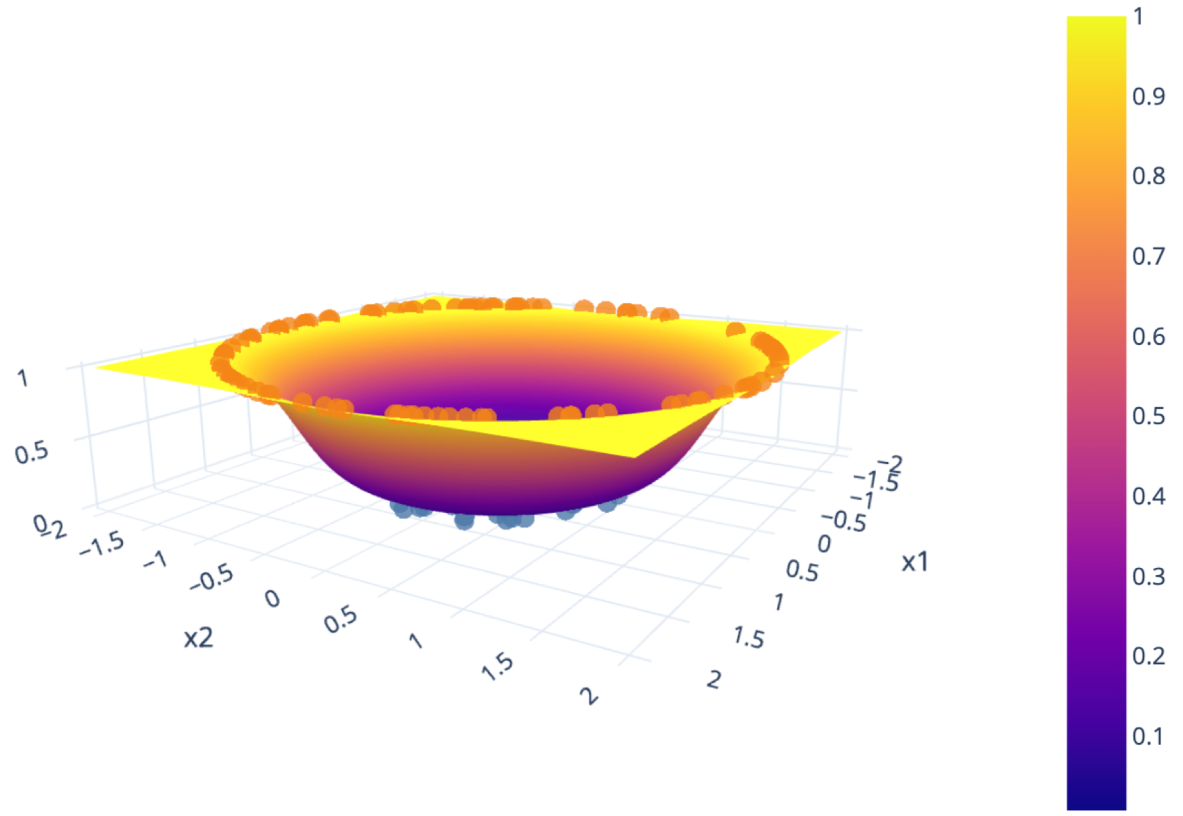


Degrees (d)**: 2**

$$h_\theta(x) = \sigma(\theta^T * x) = \sigma(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_1 x_2 + \theta_5 x_2^2) = \frac{1}{1+e^{-(\theta_0+\theta_1 x_1+\theta_2 x_2+\theta_3 x_1^2+\theta_4 x_1 x_2+\theta_5 x_2^2)}}$$





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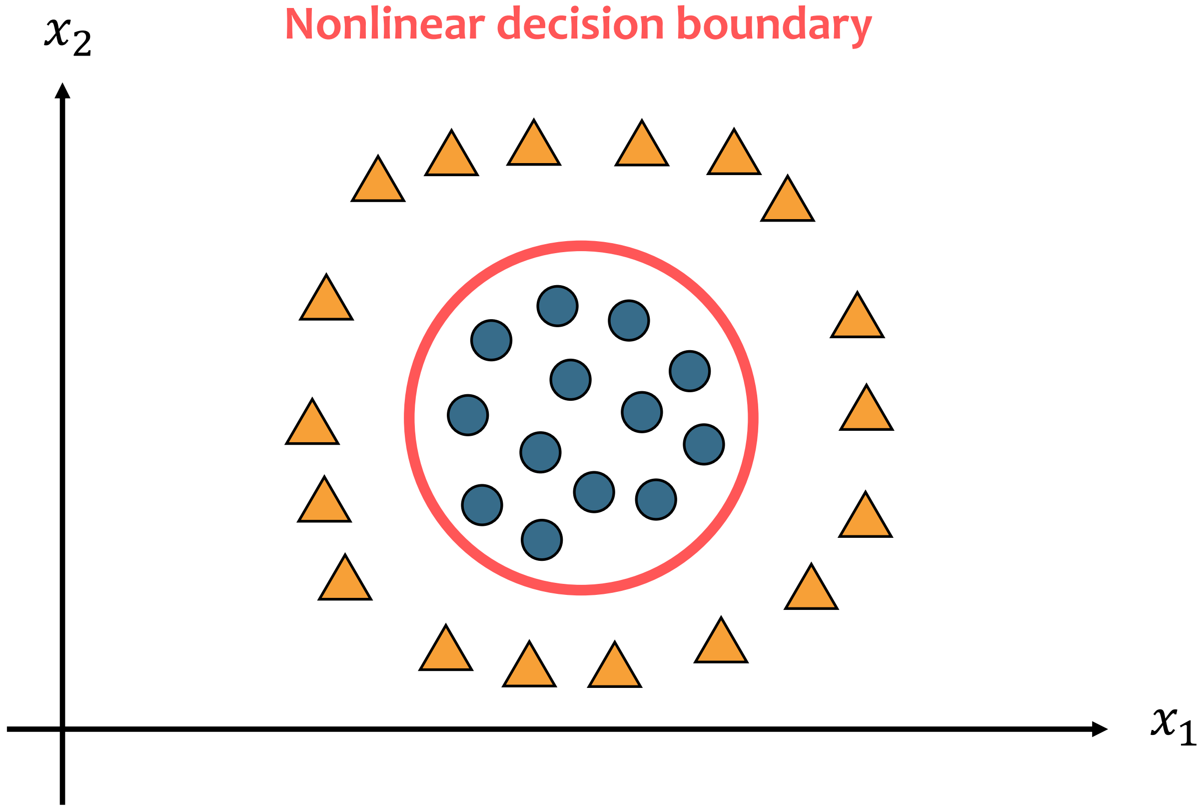

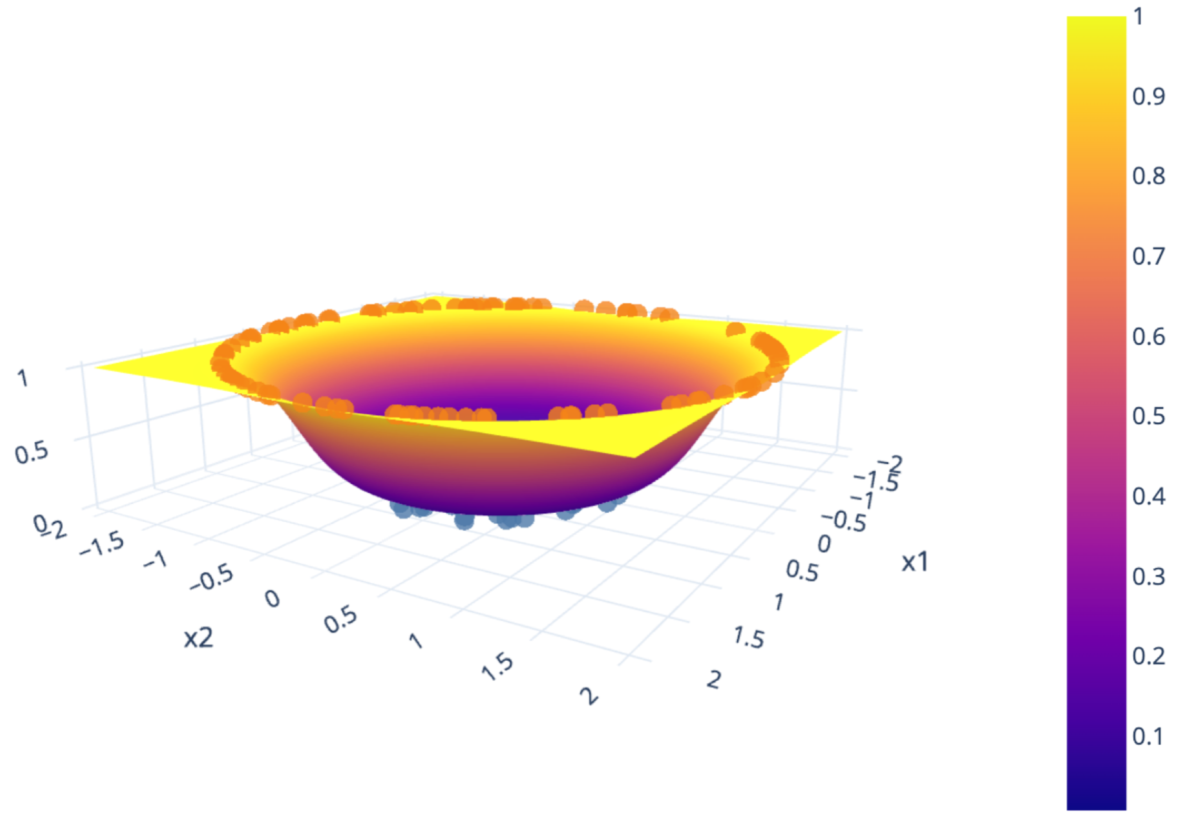

Degrees (d)**: 50**

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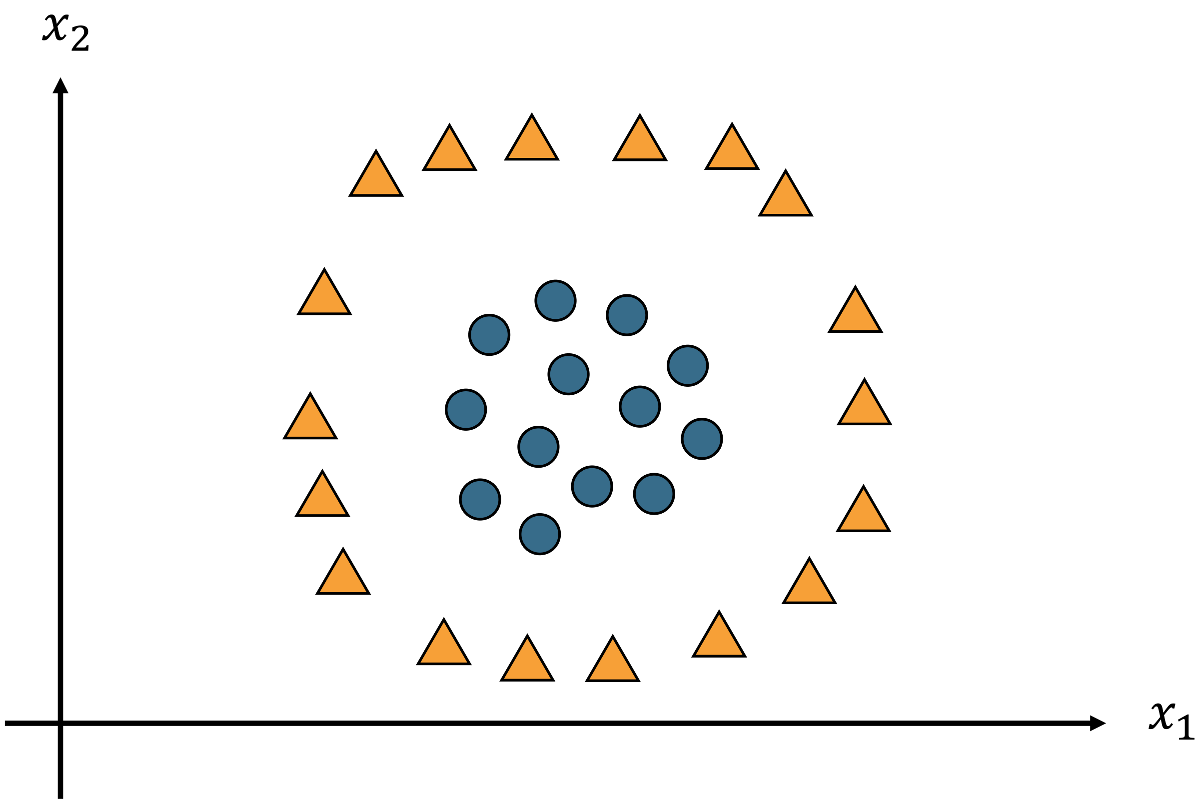

Degrees (d)**: 30**

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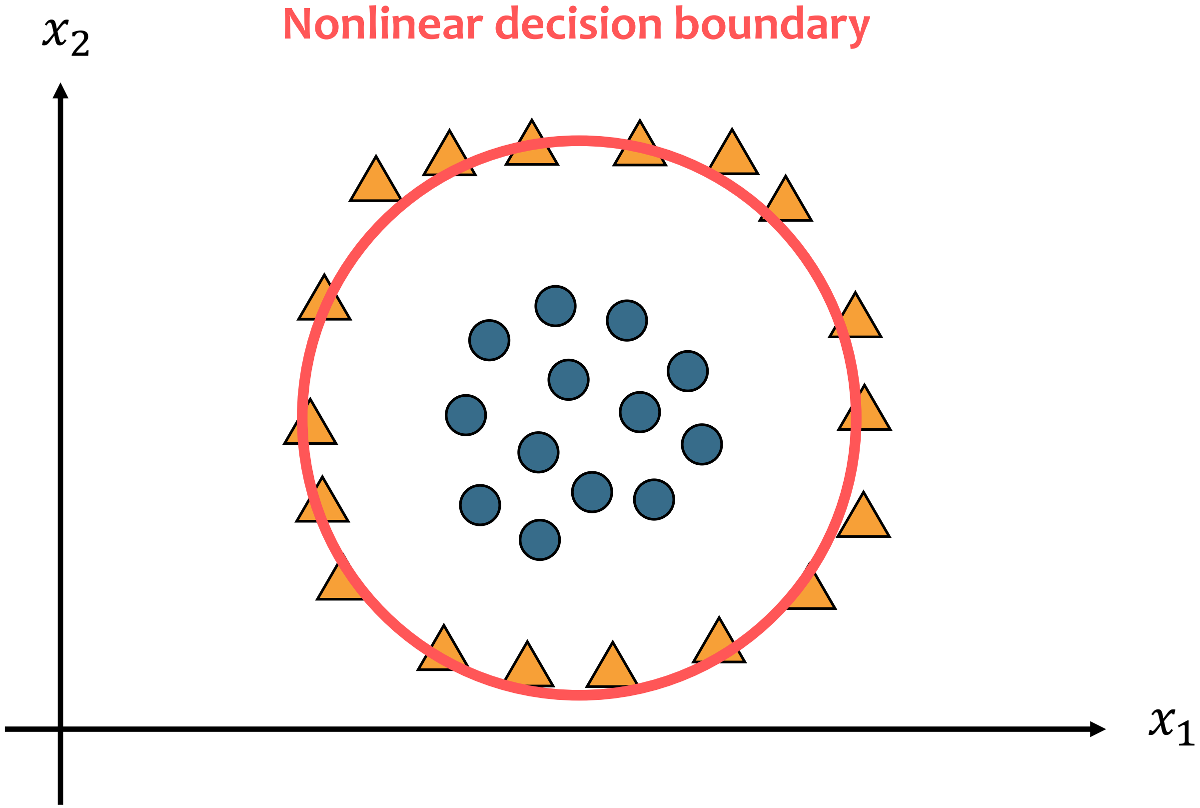

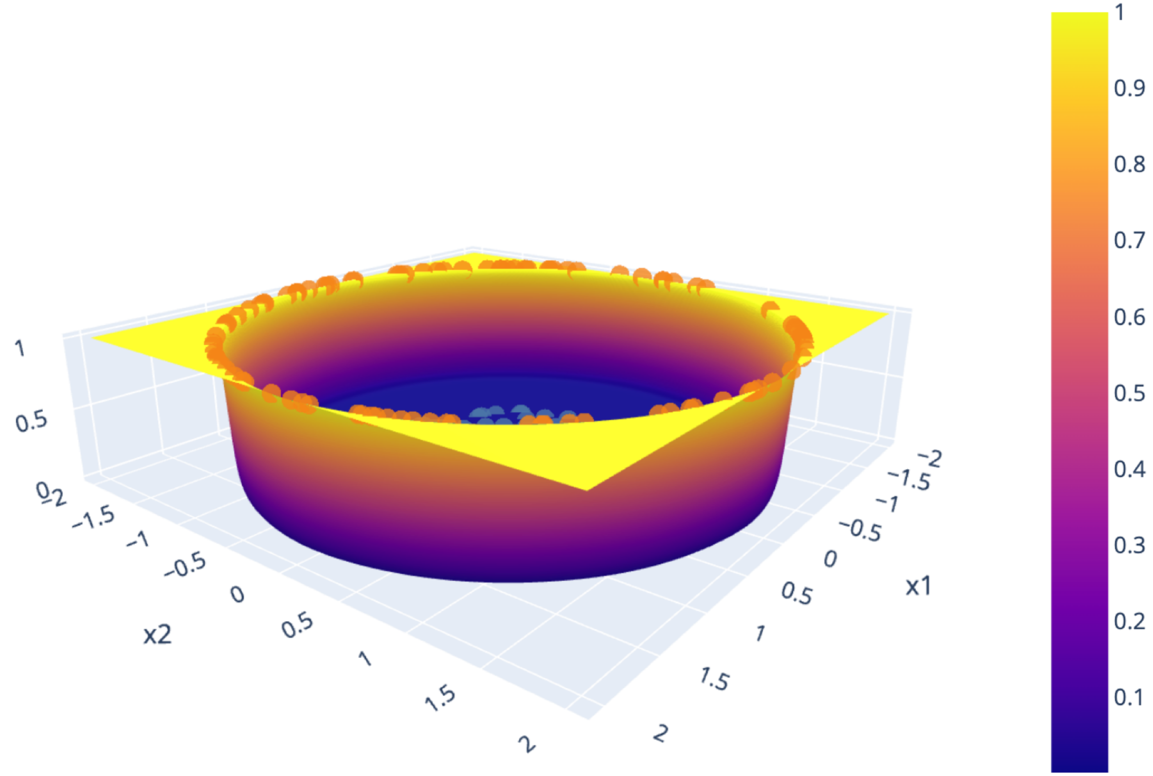

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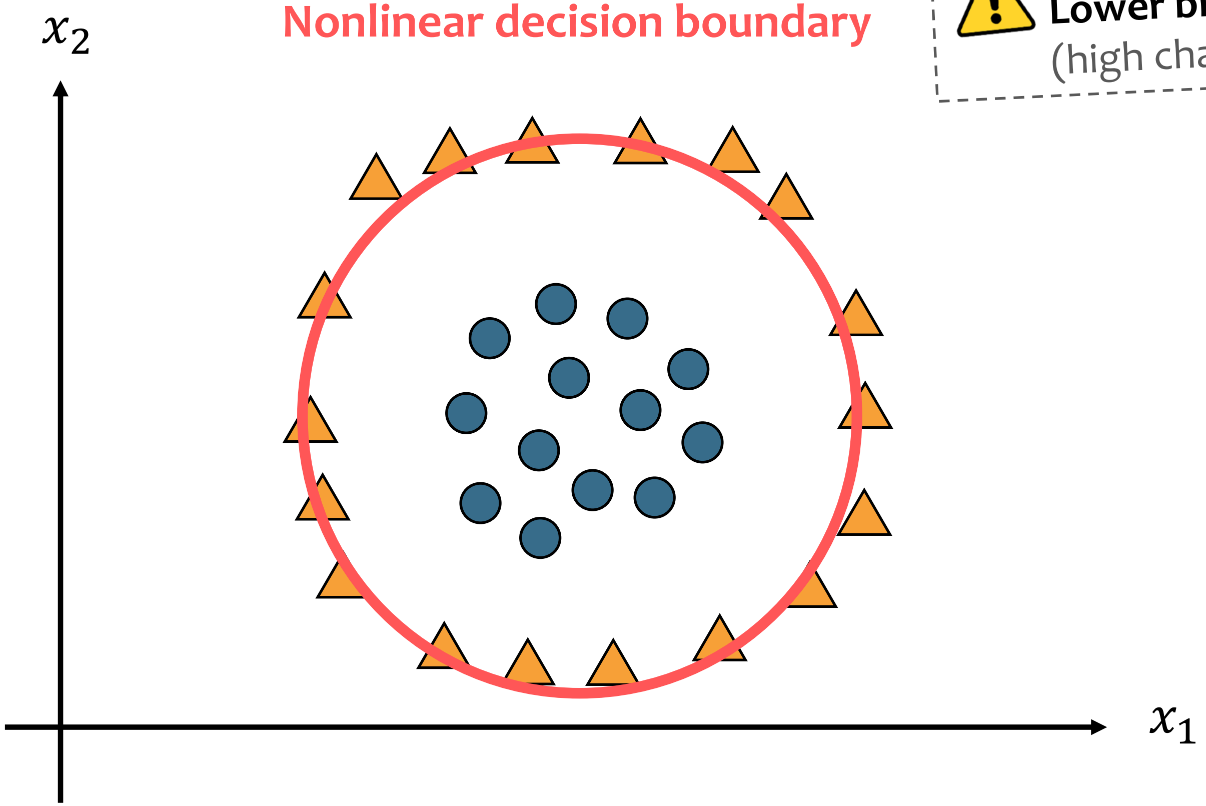
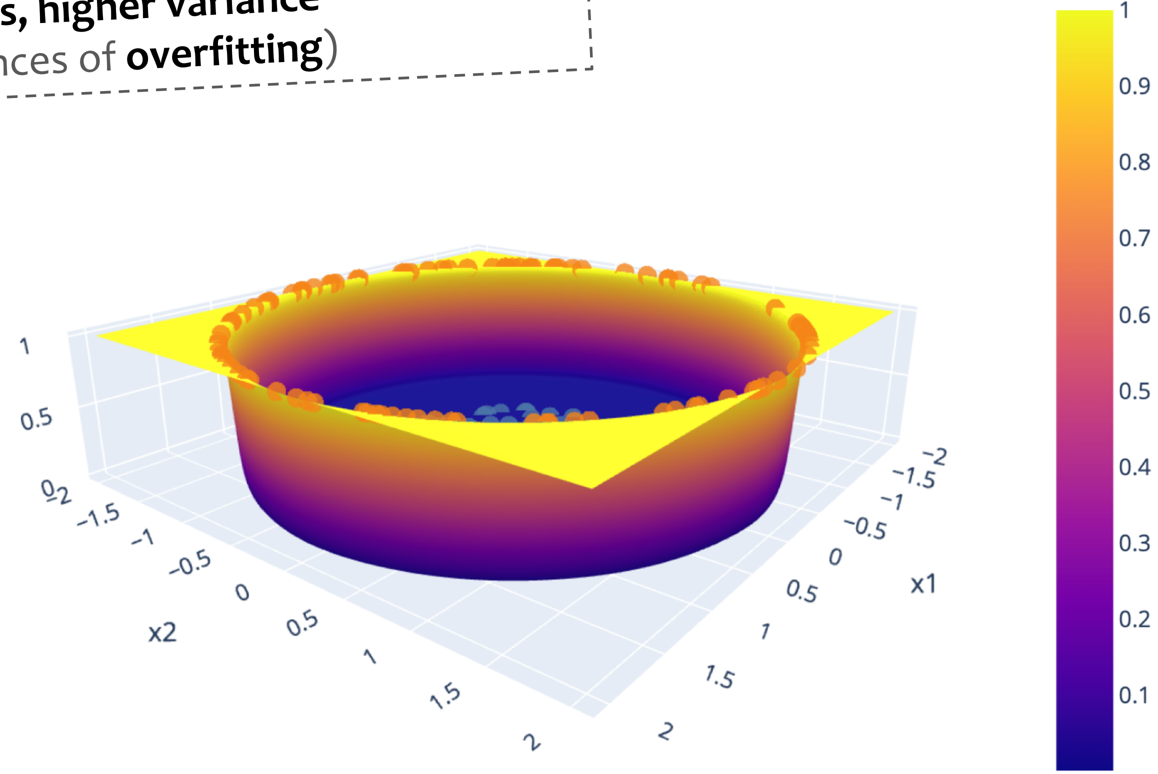
A really **more complex nonlinear model**:
Lower bias, higher variance
(high chances of **overfitting**)





Let's code it!

Polynomial Logistic Regression

Degree = 2

```
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import PolynomialFeatures
from sklearn.preprocessing import StandardScaler
from sklearn.linear_model import LogisticRegression

pol_log_reg_clf = Pipeline([
    ('pol_feats', PolynomialFeatures(degree=2, include_bias=False)),
    ("std_scaler", StandardScaler()),
    ("log_reg", LogisticRegression(random_state=42)),
])

pol_log_reg_clf.fit(X_train, y_train)
```

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