Segmentation of Connected Handwritten Numerals by Graph Representation

Misako Suwa
Fujitsu Laboratories LTD.
suwa@jp.fujitsu.com

Abstract

This paper proposes a new algorithm for separating a touching pair of digits by using the graph-representation of the pattern. The segmentation can be regarded as grouping these edges and vertices into two disconnected sub-graphs. This process is executed by applying graph theory methods and certain heuristic rules. Since the boundaries of patterns are determined along the edges, the shapes of the segmented digits can be restored with high quality. The algorithm can segment not only simply connected cases but also multiply connected ones. The results of the performance evaluation using the NIST database are also presented.

1. Introduction

Although many studies have been reported on the segmentation of handwritten digits, it still remains a challenging problem. From the viewpoint of their target, the algorithms in these studies can be grouped into two types: those in which the number of touching digits is assumed to be two [1]-[11] and those in which it is not [12]-[14]. The proposed algorithm treats touching pairs of digits and belongs to the former group. It can be applied in cases such as the segmentation of digits written in individual dropout-color frames (see Fig. 1).

Most of these algorithms cannot be applied to single segment touching, multiply touching, and ligature touching, simultaneously. Additionally, all of them separate patterns along a straight line or line segments. Therefore, the segmented patterns sometimes have unnatural shapes.

Using the graph-representation strategy, the author proposes a new algorithm for segmenting both multiply connected and simply connected digits. It can also remove ligatures. A touching pattern is represented as a graph, and its separation is executed by using both graph theory techniques and heuristic rules. The boundaries of the digits are calculated such that the strokes have a uniform width. Therefore, they can be restored with high quality.

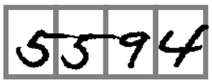


Fig. 1: Touching digits written in individual frames.

2. Types of Touching

Based on the ref. [9], we classified the touching types as follows:

a. **Simply connected**

Two patterns touch at single locations and no pseudo-loops are generated.

- **Single-point touching:** Two patterns touch at a single location, i.e., they share one point.
- **Single-segment touching:** Two patterns touch at a single location, i.e., they share one stroke.
- **Single-point touching by ligature:** Two patterns are connected by an unnecessary stroke.

b. **Multiply connected**

Two patterns touch at multiple locations. The author defines "N-tuply connected" as patterns that touch at N locations generating (N-1) pseudo-loops.

Fig. 2 shows the example of each type.

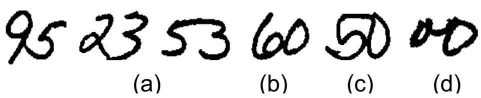


Fig. 2: Different types of touching digits: (a) single-point touching, (b) single-segment touching, (c) doubly connected, (d) ligature

3. Graph representation of the pattern

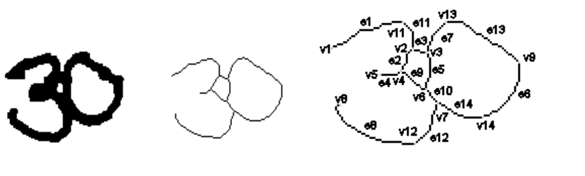
The Graph representation of a character image using thinning has been used in stroke-analysis studies of Kanji and Chinese characters. It is also applied to the segmentation of touching characters [9][15].

3.1. Preprocessing

First, smoothing and noise reduction is executed for a touching pattern to avoid the generation of spurious edges in the thinned pattern. Next the slant is normalized by the method given by Kimura *et al.* [16] and is thinned by using Hilditch's algorithm.

3.2. Graph representation

In order to represent a thinned pattern as a graph, adjacency and incidence relations between vertices and edges are calculated: The vertices of degrees 1, 3, and 4 (end point, T-joint point, and crossing point, respectively) are extracted by checking the eight neighbors of each foreground pixel of the thinned pattern. Then, every edge between two vertices is traced. If a significant turning point [6] exists on an edge, a degree-2 vertex is added to it. Spurious edges whose widths are less than that of the estimated stroke and are incident to a degree-1 vertex are eliminated. The degrees of the vertices incident with the edge are then recalculated. Coordinates of these vertices and each point of the edges are recorded in a file. An example of graph representation of a touching pattern is indicated in Fig. 3. After the adjacency and incidence relations are calculated, the circuit-base and edge cutset-base matrices are computed by using graph theory techniques.



(a) (b) (c)

Fig. 3: Graph representation of a pattern: (a) original pattern, (b) thinned pattern, (c) graph representation.

3.3. Matrices relating to a graph

The following matrices relating to a graph are used for the algorithm. These notations and definitions follow ref. [17].

Let the connected graph $G(V, E)$ be composed of vertex set $V(G)$ and edge set $E(G)$. Let v_i and v_j be elements of V and e_j be an element of E.

- **Adjacency matrix *S*:** The rows and columns of S correspond to the vertices. Its element s_{ij} equals 1 or 0 depending on whether v_i and v_j are adjacent or not.
- **Incidence matrix *B*:** The rows and columns correspond to the vertices and edges respectively. Its element b_{ij} equals 1 or 0 depending on whether e_i and v_j are incident or not.
- **Edge cutset-base matrix *K*:** If E_1 is a subset of E, and graph $G(V, E\text{-}E_1)$ is disconnected, then E_1 is called an edge cutset of G. The order of a cutset equals the number of its edges. The rows and columns correspond to the cutsets and edges, respectively. Its element k_{ij} equals 1 or 0 depending on whether the i-th cutset-base $\boldsymbol{k}_i$ contains $\boldsymbol{e}_j$ or not.

4. Methodology

Separation of touching digits can be regarded as grouping the edges and vertices of the graph into disconnected sub-graphs. In this section, the method of grouping the edges using graph theory techniques and heuristic rules is described.

4.1. Segmentation by cutset-base

This method can be applied for the type of touching where the left and right parts of G are connected by a single horizontal edge e_H. We term this edge a "touching edge." If the edge is removed, then G is divided into two sub-graphs g_l and g_r. Most single-point touches and ligature touches belong to this type. Some single-segment touches also belong to this type depending on their thinning results (see Fig. 4). An example of K is shown in Fig. 7. After the calculation of the cutset-base matrix K, all its elements are checked and only order-1 cutsets are extracted. The initial candidates for touching edges are edges $\{e_i\}$ that belong to these cutsets. In order to reduce the number of candidates, the following pruning rules are applied:

- Initial and final points of the touching stroke are not endpoints. Therefore, e_i that is incident with a degree-1 vertex is excluded.

  A touching stroke e_H is not vertical. The aspect ratio a_i of e_i is greater than the threshold a_{th1}, then e_i is excluded from the candidates.
- A touching pattern is separated into left and right parts. The overlapping degree O_x along the x-axis of g_l and g_r is defined as:

  $$O_x = l_{lr} / (x_{\max} - x_{\min} + 1),$$

  where l_{lr} is the overlapping width of g_l and g_r and $(x_{\min}, x_{\max})$ are the minimum and maximum values of the x-coordinates of g_l and g_r, respectively (see Fig. 5). If O_x is greater than the threshold O_{xth}, then e_i is excluded from the candidates.

- Two digits are overlapping along the y-axis due to the lateral writing. The overlapping degree along y-axis O_y between g_l and g_r is defined in a similar manner as that for the x-axis. If O_y is less than O_{yth}, then e_i is excluded from the candidates.

After the determination of the candidates $\{e_{CH}\}$, all edges except e_{CH} are grouped into g_l and g_r. Then, e_{CH} is classified as left and/or right depending on overlapping degrees o_l and o_r along the x-axis between e_{CH} and each sub-graph. If o_l and o_r are nearly equal and their ratio r satisfies the condition,

$$r = \min(o_l, o_r) / \max(o_l, o_r) > r_{th},$$

then e_{CH} is divided at the centerline and grouped into both sides where r_{th} is its threshold. If their ratio does not satisfy the above condition, e_{CH} is grouped into the sub-graph with the larger overlap (see Fig. 6).

Fig. 4 Patterns where two parts are connected by e_H: (a) single-point touch, (b) ligature touch, (c) single-segment touch

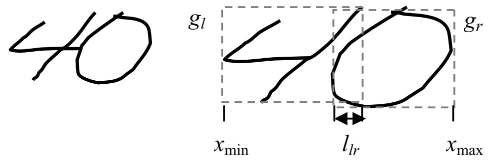


Fig. 5 Calculation of O_x: (a) G (b) sub-graphs g_l and g_r. Bounding boxes are added

Fig. 6 Grouping of edges: (a) e_{CH} belongs to g_l and g_r, (b) e_{CH} belongs to g_l

$$
\text{Cutset base number}\;
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7
\end{array}
\overset{\text{Edge number}}{\overset{\begin{array}{cccccccc}1&2&3&4&5&6&7&8\end{array}}{\begin{bmatrix}
1&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0\\
0&0&0&1&0&0&0&0\\
0&0&0&0&1&0&0&0\\
0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&1&1
\end{bmatrix}}}
$$

Fig. 7 *K* matrix of Fig. 6 (b) case: $e_{CH} = e_4$ and it is an element of the 4th cutset base

4.3. Ligature elimination

In the case of ligature elimination, we used the method proposed in ref. [9]: If e_{CH} protrudes from the bounding box of g_l or g_r, then the protruding part is regarded as a ligature and is eliminated (see Fig. 8).

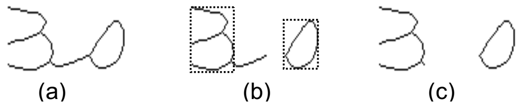


Fig. 8 Ligature elimination: (a) original graph, (b) after grouping, (c) ligature removal

4.4. Segmentation by edge duplication

This method can be applied for the type of touching where the left and right parts of G share a vertical edge e_V. If the edge is duplicated, then G can be divided into g_l and g_r. Its new adjacency matrix can be block-diagonalized after the appropriate permutations of its rows and columns. Each block corresponds to g_l and g_r, respectively. Most single-segment touches belong to this type. Fig. 9 and 10 depict the examples.

In order to determine a candidate touching edge, the following rules are applied for all edges $\{e_i\}$ of G:

- The vertices of e_V are T-joint or crossing points. Therefore, the e_i for which the degree of the vertex is less than or equal to two is excluded from the candidates.
- A touching stroke e_V is not horizontal. If the aspect ratio a_i of e_i is less than the threshold a_{th2}, then it is excluded from the candidates.
- An edge e_i is duplicated and the edges adjacent to it are appropriately grouped into the left and/or right parts. If G does not separate two disconnected sub-graphs after the process, then it is excluded.
- If O_y defined above is less than the threshold O_{yth}, then e_i is excluded from the candidates.

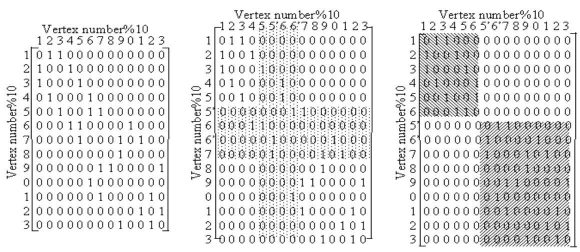


Fig. 9 Adjacency matrix of Fig.10 case: (a) original S, (b) after duplication of e_v, (c) after permutation of its rows and columns

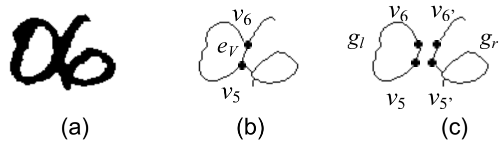


Fig. 10 Pattern where the left and right parts share e_V: (a) original image, (b) its graph, (c) duplication of e_V

4.4. Segmentation by the shortest path

If the above processes cannot find any candidates, then the pattern might be a multiply connected case. The v_t near the deepest point of the upper valley and the v_b near the highest point of the lower hill are decided by ref. [9] and the shortest path between them is calculated. It is regarded as a segmentation path. All edges that do not belong to the path are grouped into left or right parts depending on their locations and adjacency relations. Then, each edge e_i that belong to the path is checked and grouped to the left and/or right according to the following rules:

- A multiply connected case generates pseudo-loops near the centerline of G. If e_i is a part of a loop c and the x-coordinate x_c of the center of c satisfies

$$W/3 < x_c < 2W/3,$$

  then e_j is grouped into left or right parts; when e_i is located on the left side of c, it is grouped into the left and *vice versa*, where W is the width of G.
- If e_i does not satisfy the above condition, then it is duplicated and grouped into both sides.

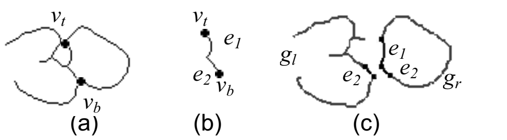


Fig. 11 Segmentation path: (a) graph G, (d) the shortest path, (c) grouping edges

4.5. Restoration of the boundaries of digits

The boundaries of digits are calculated away from the rightmost part of g_l and leftmost part of g_r by half of the estimated stroke width (see Fig. 12).

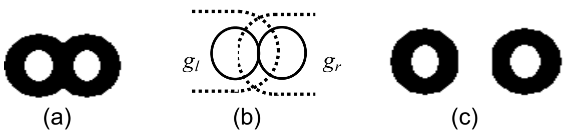


Fig. 12 Restoration of boundaries: (a) original image, (b) boundaries of g_l and g_r, (c) separation result

4.5. Scoring candidates

If there still exists plural candidates, they are scored and orders are given to them. We used the following simple function for the score Sc here:

$$Sc = (1.0 - \frac{\min(w_l, w_r)}{\max(w_l, w_r)}) \times (1.0 - \frac{\min(h_l, h_r)}{\max(h_l, h_r)}),$$

where (w_l, w_r) and (h_l, h_r) are the widths and heights of the left and right segmented patterns, respectively.

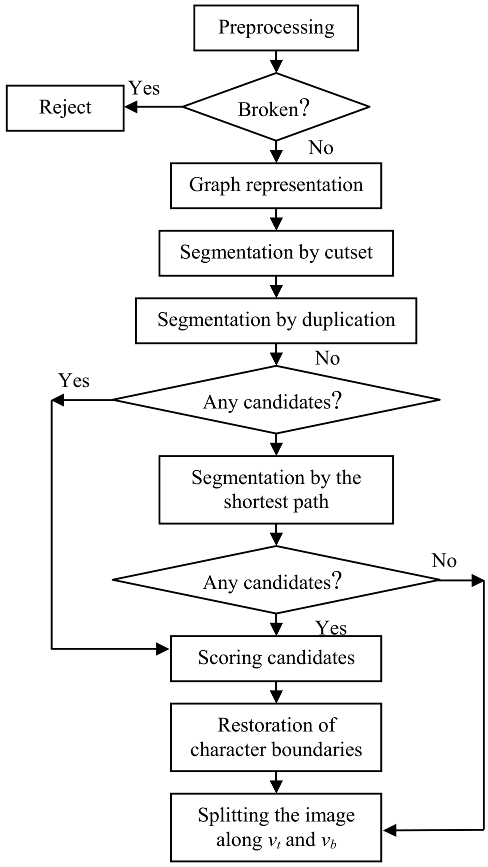


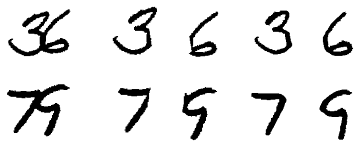
Fig. 13 Algorithm flow

5. Experimental results and discussion

We evaluated the performance of our method using touching images extracted from the NIST-19 database. The test data used 2000 pairs of touching digits and another 1000 pairs were used to construct rules and determine the thresholds. Using the method in Fig. 13, 88.4% of the patterns were correctly segmented. Table 1 shows the comparison with certain algorithms. The segmentation rates were recalculated to total 100% of the sum of the correct rate, error rate, and rejection rate. The quality of the segmented images appears to be improved in the case of touching digits that share a stroke. Some examples are shown in Fig. 14, and correctly segmented patterns and errors are shown in Fig. 15 (a) and (b).

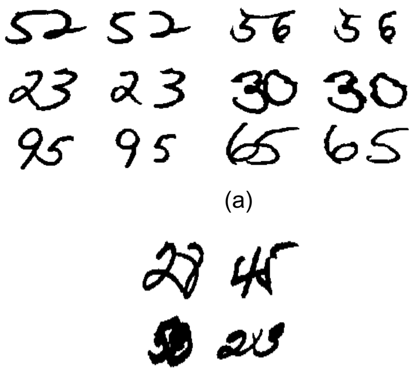
Table 1: Performance comparison

Algorithm	Correct	Error rate	Rejection	DB
[10]	80.83%	19.17%	-	Original
[7] H.R.	69.25%	2.15%	28.60%	NIST
[7] L.R.	88.15%	7.15%	4.70%	NIST
[8]	88.06%	3.94%	8.00%	NIST+org.
Proposed	88.40%	8.45%	3.15%	NIST



(a) (b) (c)

Fig. 14 Comparison of segmented patterns: (a) original image, (b) Method [8], (c) proposed



(a)

(b)

Fig. 15 Segmentation results: (a) correctly segmented patterns, (b) errors

6. Conclusions and remarks

A new algorithm was proposed for segmenting simply and multiply connected digits. After thinning the binary pattern, the edges and vertices are extracted and the pattern is represented as a connected graph. Candidate segmentation paths are calculated using both graph theory techniques and heuristic rules. Ligatures are eliminated using the rules proposed by Elnagar *et al*. The boundaries of the digits are calculated to make the width of the touching strokes uniform. As a result, the separated digits have a more natural shape than those produced by algorithms that split patterns using a straight line or line segments.

7. References

[1] M. Shridhar and A. Badreldin, "Recognition of Isolated and Simply Connected Handwritten Numerals", *Pattern Recognition*, **Vol. 19**, No. 1, 1986, pp. 1-12

[2] F. Kimura and M. Shridhar, "Recognition of Connected Numerals", *ICDAR '91*, 1991, pp. 731-739

[3] N.W. Strathy *et al.*, "Segmentation of Handwritten Digits Using Contour Features", *ICDAR '93*, 1993, pp. 577-580

[5] J.M. Westall and M.S. Narasimha, "Vertex Directed Segmentation of Handwritten Numerals", *Pattern Recognition*, **Vol. 26**, No. 10, 1993, pp. 1473-1486

[6] Z. Shi and C. Govindaraju, "Segmentation and Recognition of Connected Handwritten Numeral Strings", *Pattern Recognition*, **Vol. 30**, No. 9, 1997, pp. 1501-1504

[7] Z. Lu *et al.*, "A Background-thinning-based Approach for Separating and Recognizing Connected Handwritten Digit Strings", *Pattern Recognition*, **Vol. 32**, No. 6, 1999, pp. 921-933

[8] Y-K. Chen and J-F. Wang, "Segmentation of Single- or Multiple-Touching Handwritten Numeral String Using Background and Foreground Analysis", *IEEE Trans. on PAMI*, **Vol. 22**, No. 10, 2000, pp. 1304-1317.

[9] A. Elnagar and R. Alhajj, "Segmentation of Connected Handwritten Numeral Strings", *Pattern Recognition*, **Vol. 36**, No. 3, 2003, pp. 625-634

[10] M. Cheriet *et al.*, "Background Region-based Algorithm for the Segmentation of Connected Digits", *ICPR '92,* 1992,

[11] U. Pal *et al.*, "Touching Numeral Segmentation Using Water Reservoir Concept", *Pattern Recognition Letters*, **Vol. 24**, 2003, pp. 261-272
pp. 619-622

[12] B. Zhao *et al.*, "A New Method for Segmenting Unconstrained Handwritten Numeral String", *ICDAR '96*, 1997, pp. 524-527

[13] Z. Shi *et al.*, "A System for Segmentation and Recognition of Totally Unconstrained Handwritten Numeral Strings", *ICDAR '97*, 1997, pp. 455-458

[14] H. Fujisawa *et al.*, "Segmentation Method for Character Recognition: From Segmentation to Document Structure Analysis", *Proc. of IEEE*, **Vol. 80**, No. 7, 1992, pp. 1079-1092

[15] M. Suwa, "Segmentation of Touching Handwritten Japanese Characters Using Graph Theory Method", *Proc. of SPIE*, **Vol. 4307**, 2001, pp. 280-289

[16] F. Kimura *et al.*, "Improvements of A Lexicon Directed Algorithm for Recognition of Unconstrained Handwritten Words", *ICDAR '93*, 1993, pp. 18-22

[17] V. Chachra, P.M. Chare and J.M. Moore, "Applications of Graph Theory Algorithms", *Elsevier North Holland*, 1979