DIS 1B

INDUCTION: prove something for all natural numbers

0. choose a variable to induct on
   some are better options than others

1. prove the claim for the base case
   what's the smallest example?

2. state the induction hypothesis
   assume P is true for arbitrary k: $P(k)$

3. perform the inductive step
   then show that it is also true for $k+1$: $P(k+1)$

example

THEOREM 3.4 for $n \geq 1$, the sum of the first n odd numbers is n^2

⓪ we induct on n

① base case: the smallest case is $n=1$. $1^2 = 1$ ✓

② induction hypothesis: assume the sum of the first k odd numbers is k^2

③ induction step: show that the claim holds for $k+1$.

the $(k+1)$st odd number is $2k+1$.

by IH, the sum of the first k is k^2.

then the sum of the first $k+1$ is $k^2 + 2k + 1$

$= (k+1)^2$ ✓

STRONG INDUCTION

⓪ choose a variable to induct on

some are better options than others

① prove the claim for the base case

what's the smallest example?

② state the induction hypothesis

assume P is true for all i, $0 \le i \le k$:

$$\bigwedge_{i=0}^{k} P(i) = P(0) \wedge P(1) \wedge \ldots \wedge P(k)$$

③ perform the inductive step

then show that it is also true for $k+1$: $P(k+1)$

for examples: see theorems 3.6/3.7 in the notes

INDUCTION TIPS

→ write out a few cases to help you identify a pattern

→ try strengthening the induction hypothesis

a stronger hypothesis can better capture the true nature of the problem

$\boxed{\text{ex}}$ the sum of the first n odd #s is a square

↓

the sum of the first n odd #s is n^2

! this is different from strong induction

→ remember to use your hypothesis in your induction step

induction will not work without this

$\boxed{\text{think:}}$ can you break down the (k+1) case into the k case?