DIS 12B

GAUSSIAN (NORMAL) DISTRIBUTION

$X \sim \mathcal{N}(\mu, \sigma^2)$

$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}$

$E[X] = \mu$

$var(X) = \sigma^2$

the standard normal is denoted by $\mathcal{N}(0,1)$

↳ any Gaussian RV can be expressed as

$$X = \sigma Z + \mu \ , \quad Z \sim \mathcal{N}(0,1)$$

a linear combination of independent Gaussians:

$$X \sim \mathcal{N}(\mu_X, \sigma_X^2) \ , \quad Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2) \ , \quad Z = aX + bY$$

$$Z \sim \mathcal{N}(a\mu_X + b\mu_Y, \ a^2\sigma_X^2 + b^2\sigma_Y^2)$$

CENTRAL LIMIT THEOREM

for i.i.d. RVs $X_1, X_2, \dots X_n$ with $E(X_i) = \mu$, $\mathrm{var}(X_i) = \sigma^2$,
define $S_n = \sum_{i=1}^{n} X_i$. then,

$$\frac{S_n - n\mu}{\sigma\sqrt{n}} \to \mathcal{N}(0,1) \quad \text{as} \quad n \to \infty$$

equivalently, $\frac{S_n}{n} \to \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$ as $n \to \infty$