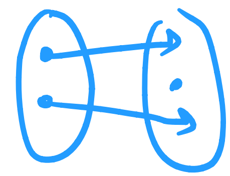
DIS 6B

BIJECTIONS

let $f: X \to Y$ be a function that maps from a set X to a set Y

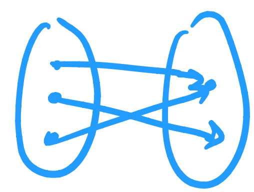
→ f is an injection if each x is mapped to a unique y

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

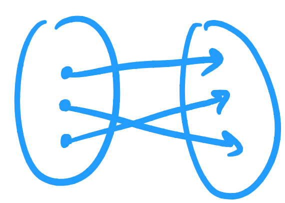


→ f is a surjection if every y has at least one x mapped to it

$$\forall y \exists x \, (f(x) = y)$$



→ f is a bijection iff it is both injective and surjective



COUNTABILITY

to prove two sets have the same cardinality, we show there is a bijection between them

a set S is countable

$$\Updownarrow$$

there is a bijection between S and $\mathbb{N}$ or a subset of $\mathbb{N}$

some infinite sets:

COUNTABLE	UNCOUNTABLE
$\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{N} \times \mathbb{N}$	$\underbrace{P(\mathbb{N})}_{\text{power set: the set of all subsets.}}$, $\mathbb{R}$

COUNTABILITY PROOFS

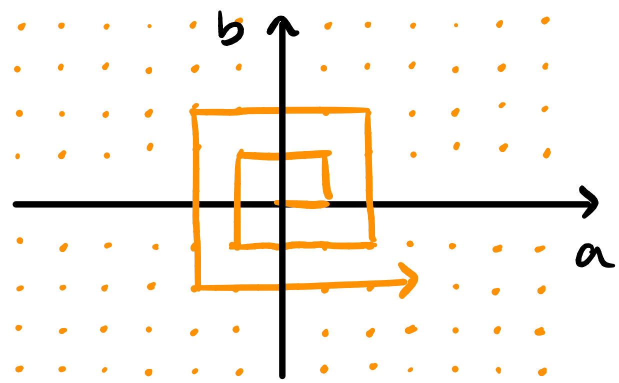
proving COUNTABILITY

↳ bijection to a known countable set.

ex. $\mathbb{Q}$ is countable

$q \in \mathbb{Q}, \quad q = a/b$

$(a, b) \leftrightarrow n$



proving UNCOUNTABILITY

↳ bijection to a known uncountable set

↳ diagonalization

ex. $\mathbb{R}$ is uncountable

↓

or $[0, 1]$

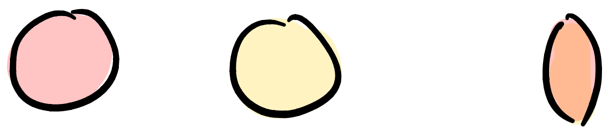
0	0.51249356...
1	0.14162985...
2	0.94782712...
3	0.53098175...
n	0.7691...

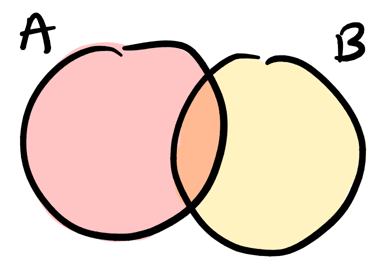
<u>PIE</u> (PRINCIPLE OF INCLUSION - EXCLUSION)

goal: count the objects in a union of subsets without overcounting

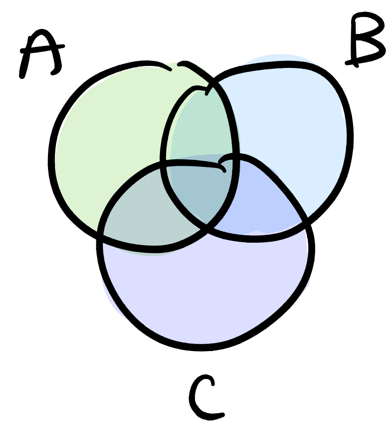
ex. two subsets A and B

$$|A \cup B| = |A| + |B| - |A \cap B|$$





$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$



there is a general formula for n sets...
but don't just memorize it

COMBINATORIAL PROOFS

goal: use a "story" to prove a combinatorial identity

! this is not a proof through algebraic manipulation of the expressions

think about different ways you might choose things from a group.

→ can you break the task into smaller choices?

→ the values in the expressions are hints about how the choices are made

→ sums = different cases

→ products = series of choices