DIS 2B

GRAPHS

a graph is defined on a set of vertices and edges: $G = (V, E)$

	can repeat vertices	no repeated verts.
no restrictions on start/end	walk: a sequence of edges with possible repeats	path: a sequence of vertices with no repeats
start = end	tour: a walk that starts/ends at the same vertex	cycle: a path where the first/last vertex repeats

Eulerian means each edge is visited exactly once

→ Eulerian walk, Eulerian tour

INDUCTION ON GRAPHS

first decide: induction on vertices or edges?

BASE CASE

smallest graph where the claim applies

INDUCTIVE HYPOTHESIS

state the claim for n vertices / m edges

INDUCTIVE STEP

show the claim for n+1 vertices / m+1 edges by removing a vertex / edge and putting it back

ex. induction on vertices

→ take a graph with n+1 vertices

→ choose a vertex, remove it and its edges

→ use IH on the n vertex graph

→ add the vertex back and show the claim still holds

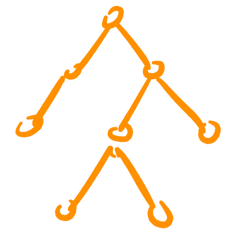
BUILD-UP ERROR

in the induction step, why must we remove and add back instead of just starting with a smaller graph and adding to it?

because of build-up error

we want to prove the claim for all graphs with $n+1$ vertices, but if we start from n vertices, we have to consider all the ways one could add an additional vertex.

TREES



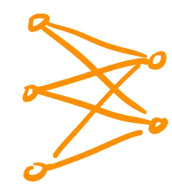
equivalent definitions:

- → connected acyclic
- → connected and $|E| = |V| - 1$
- → connected, and removing any edge disconnects it
- → connected, and adding an edge creates a cycle.

an exercise: prove that any of these implies the others

BIPARTITE GRAPHS

vertices can be split into 2 groups L and R where edges only go between the groups



COMPLETE GRAPHS

each vertex is connected to every other vertex

↳ $n(n-1)/2$ edges