DIS 12A

PROBABILITY DENSITY FUNCTION

the PDF of a continuous RV X is a function f

1. $f(x) \geq 0$ for all $x \in \mathbb{R}$
2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

and we can define $P(a \leq X \leq b) = \int_a^b f(x)\,dx$

! $f(x) \neq P(X=x)$ (PDF $\neq$ probability)

CUMULATIVE DISTRIBUTION FUNCTION

the CDF of a continuous RV X is a function F

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(z)\,dz$$

note that: $f(x) = \frac{d}{dx}F(x)$ and $F(x) \to 1$ as $x \to \infty$

CONTINUOUS ANALOGS OF PAST CONCEPTS

expectation: $E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$

variance: $\text{var}(X) = E[X^2] - E[X]^2$

$$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - \left(\int_{-\infty}^{\infty} x f(x)\, dx\right)^2$$

joint distribution: $P(a \le X \le b,\ c \le Y \le d)$

$$= \int_c^d \int_a^b \underbrace{f(x,y)}_{\text{joint density}}\, dx\, dy$$

$f(x,y) \ge 0$, $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1$

independence: X, Y independent iff

$$f(x,y) = f_X(x) f_Y(y) \quad \forall x, y$$

UNIFORM DISTRIBUTION

$X \sim \text{uniform}(a, b)$

$f(x) = \frac{1}{b-a}$ $(a \leq x \leq b)$

$E(X) = \frac{a+b}{2}$

$F(x) = \frac{x}{b-a}$

$\text{var}(X) = \frac{b^2 - a^2}{12}$

EXPONENTIAL DISTRIBUTION

$X \sim \text{Exponential}(\lambda)$

$f(x) = \lambda e^{-\lambda x}$

$E[X] = \frac{1}{\lambda}$

$F(x) = 1 - e^{-\lambda x}$

$\text{var}(X) = \frac{1}{\lambda^2}$

exponential is the continuous version of geometric

↳ λ is the rate at which an event occurs

↳ X describes how long we wait until the event