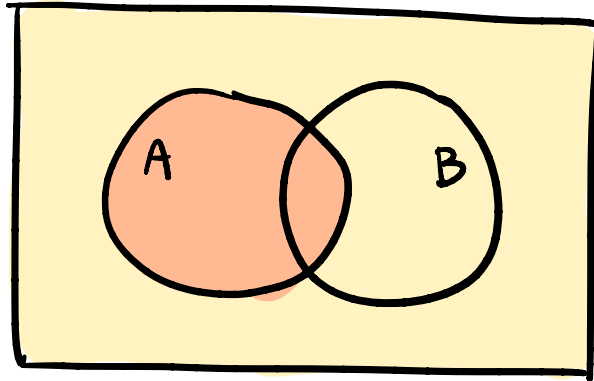
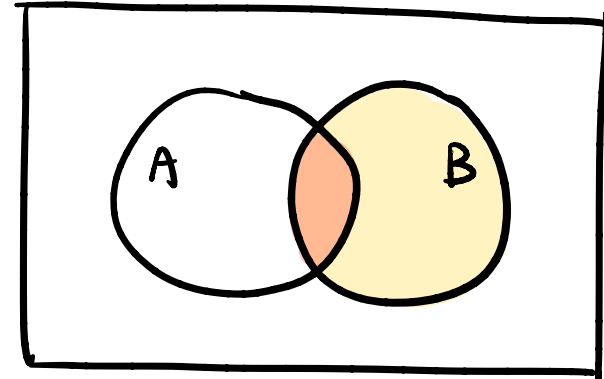


# CONDITIONAL PROBABILITY



$P(A)$



$P(A|B)$

$P(A|B)$  = probability of A given B  
(aka conditioning on event B)

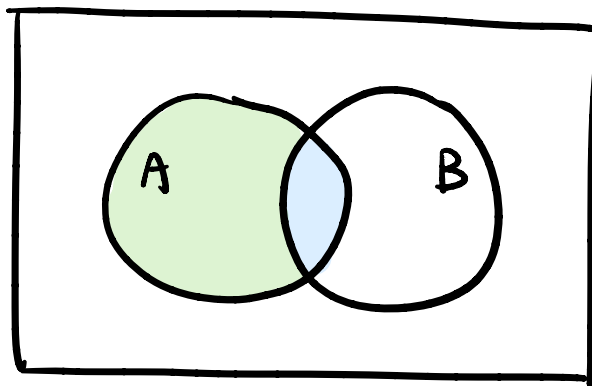
## Baye's Rule

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A) P(A)}{P(B)}$$

↑  
posterior

↖  
prior

## TOTAL PROBABILITY



$$\begin{aligned} P(A) &= P(A \cap \bar{B}) + P(A \cap B) \\ &= P(A|\bar{B})P(\bar{B}) + P(A|B)P(B) \end{aligned}$$

generalized total probability:

$$P(A) = \sum_{i=1}^n P(A|B_i)P(B_i)$$

$$\text{where } \sum_{i=1}^n P(B_i) = 1$$

## INDEPENDENCE

events  $A$  and  $B$   
are independent

$$\Leftrightarrow P(A \cap B) = P(A)P(B)$$

$$\Leftrightarrow P(A|B) = P(A)$$

$$\Leftrightarrow P(B|A) = P(B)$$

intuitively,  $A$  happening doesn't change the probability that  $B$  happens, and vice versa

events  $A_1, A_2, \dots, A_n$  are mutually independent if for all subsets of events

$$P(A_1 \cap \dots \cap A_j) = P(A_1) \dots P(A_j)$$

events are pairwise independent if they are mutually independent for subsets of size 2