DIS 6A

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

"n choose k"

COUNTING

rules of counting

① to count the number of ways to make k choices with $n_1, n_2, \ldots n_k$ options for each choice: multiply $n_1 \times n_2 \times \ldots \times n_k$

ex. ordering 5 distinct letters

↳ $26 \times 25 \times 24 \times 23 \times 22 = \frac{26!}{21!}$

② to account for duplicate arrangements, divide by m, the number of duplicates (often used when order doesn't matter)

ex. selecting a set of 5 letters

ABCDE same as CBADE : 5! duplicates.

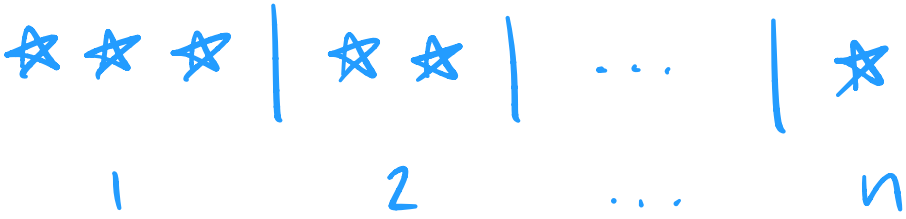
$$\frac{26!}{21!\,5!} = \binom{26}{5} = \binom{26}{21}$$

STARS AND BARS

goal: count the number of ways we can put
- k indistinguishable items into
- n distinct groups

arranging k stars into n groups:

use $n-1$ bars as dividers



$$\#\text{ ways} = \binom{n-1+k}{k} = \binom{\#\text{ stars} + \#\text{ bars}}{\#\text{ stars}}$$

choosing k spots out of $n-1+k$ to be the stars, and the others are bars