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A formula for Callable Floating Rate Notes

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Contents

Abstract

We derive a formula that allows pricing options on floating rate notes, and a second formula that is developed for the underlying bond. Both objects are the building blocks of a callable floater. We show how our formulas can be applied in real market situations by assuming that default intensity follows a CIR process. The formulas are then tested against monte carlo simulations under the assumption of CIR and Gamma OU dynamics.

Chapter 1

Introduction and theoretical framework

1.1 Introduction

Callable bonds are bonds that include a provision allowing the issuer to buy back the bond at a contractually predetermined time and price. This provision is a commonly shared feature among bonds issued in financial markets. For this reason, the literature dealing with the pricing of callable bonds is quite large. Further, a relatively large number of bonds pay floating coupons, which are popular in case of rising interest rate expectations, as this feature allows investors to reduce their interest rate exposure. Despite these facts, few papers address the pricing of callable floaters. In this regard, we note: Filipovic et al. (2009) value Bermudan callable snowball floaters using monte carlo simulation. Garcia et al. (2003) price options on floaters using a two factor HW-BK tree procedure as an approximation to value credit default swaptions. Brigo (2005) derives an equivalence between CDS and FRNs in order to derive a black-like formula for the option embedded in the callable floater. Those papers encompass existing methods to value callable floaters: trees, monte carlo simulation, and the equivalence with CDSs.

This thesis contributes to the literature by proposing two model-independent formulas: for the underlying floating rate bond, and for an option written on the floating rate bond. Those two objects constitute the building blocks of a callable, and equivalently puttable, floater. We then show how our formulas can be applied to real market situations by assuming that the default intensity follows the process developed by Cox et al. (1985). On the theoretical side, most closely related papers include: Jarrow et al. (2008) derive a reduced-form valuation model for callable bonds by assuming that solely the default intensity process drives the decision to call the bond. Our model builds upon Brigo, El-Bachir (2010) who use the Jamshidian (1989) decomposition to derive a semi-analytical formula for credit default swaptions. In regard to the pricing of non-callable FRNs, the standard metric used to quote FRNs is the discount margin. It is an ad hoc method defined as the spread over the index that equates the present value of future cash flows to the quoted FRN price. It is widely used as a quoting mechanism, but it is not a modelling formula because it involves no assumptions on the underlying stochastic process. On the modelling side, the valuation of non-callable floaters has been addressed among others by Longstaff and Schwartz (1995) who derived a closed-form expression for floating rate notes in the reduced-form framework.

This paper is structured as follows; First, we describe the theoretical framework in which we work. Then, we show how a formula for callable floaters can be derived within this framework. The remainder of this paper focuses on testing our results; Our formulas are tested against Monte Carlo simulation, then we test the validity of the deterministic interest rate assumption that supports our derivation.

1.2 Settings

We consider a probability space $(\Omega, \mathcal{G}, \mathrm{Q})$ with the standard brownian motion: $\mathrm{W} = \{W_t, t \geq 0\}$. Within this space, we define $B(t) = exp(\int_0^t r_s ds)$, as the bank account, with r the short rate, typically understood as a money market rate. The discount factor at time t for maturity T is therefore: $D(t,T) = B(t)/B(T) = exp(-\int_t^T r_s ds)$.

We work with the reduced-form modeling approach, in which the default time is defined as a stopping time τ modeled as the first jump of a Cox process $M(t;\lambda)$ as developed by Lando (1998):

$$\tau = inf\{t \in R_0^+ | M(t;\lambda) > 0\} \tag{1.1}$$

The Cox process is an inhomogeneous Poisson process with stochastic intensity parameter λ_t, which is the probability of default in the interval $[t, t+h]$ conditional on the absence of default up to time t:

$$\lambda_t = \lim_{h \to 0} \frac{1}{h} Q\{\tau \in [t, t+h] | \tau \geq t\} \tag{1.2}$$

For this reason, we work with an enlarged filtration $\mathcal{G} = \mathcal{F} \vee \mathcal{H}$, which is composed of $\mathcal{F}$:=F_t, the default-free filtration, and $\mathcal{H} := H_t$ where $H_t = \sigma(\{\tau < u, u \leq t\})$. This is the filtration containing default-related market information up to time t. From this definition, we infer that default-related information are generated solely by the Cox process. We obtain the probability of no default in the time interval [s;t] by starting from the equation linking the Poisson distribution and the Poisson process, and setting n=0:

$$Q(M(t;\lambda) - M(s;\lambda) = n) = \frac{(\int_s^t \lambda(u)du)^n}{n!} exp(-\int_s^t \lambda(u)du) \tag{1.3}$$

Chapter 2

A formula for callable floaters

2.1 Floating rate notes

A floating rate note (FRN, or floater) is a contract that allows the buyer to receive coupons at $L + X$, where L denotes an interest rate index, typically the LIBOR, plus X, the spread over the index, and at maturity T_b, a payment equal to the notional amount. Both coupons and notional payments are conditional on the absence of default of the issuer. In case of default, the bondholder receives a recovery amount proportional to the recovery rate REC, which is assumed deterministic.

2.2 Callable floating rate notes

A callable floating rate note (CFRN) is a bond with an embedded right, but not an obligation, to buy back the bond at time T_a and strike K. It can be decomposed into a non-callable bond, and a call option:

$$CFRN_t = FRN_t - V_t \tag{2.1}$$

Where V_t denotes the value of the embedded option, in other words, the option that the investor sells to the issuer that allows him to buy back the bond at a contractually predetermined strike K. This decomposition implies that we need a model to price both FRN_t, and V_t. The decomposition outlined above is a common market practice, and has been analyzed in Filipovic et al. (2009), who show on Bermudan callable snowball floaters that valuing both components without decomposition would lead to significant price underestimation.

2.3 Derivation

Proposition 1 • *The time t value of a maturity T_a call option written on a floating rate note with maturity T_b is:*

- $V_t = \mathbf{1}_{\{\tau \geq t\}}(exp(-\Gamma_{T_a}) \int_{T_a}^{T_b} Y(u)C(t, T_a, u, \lambda, \lambda^*)du)$
- *Where Y(u) is defined in (eq. 2.13), and C(.) represents the value of a call option as defined in (eq. 2.28).*

Proof 1

We start the proof with the payoff function of a discounted FRN, which is the definition in (2.1) expressed in mathematical terms:

$$\begin{aligned}\Pi_{FRN}(t) = -D(t, T_a)\mathbf{1}_{\{\tau \geq T_a\}} + \sum_{i=a+1}^{b} D(t, T_i)\delta_i(L(T_{i-1}, T_i) + X)\mathbf{1}_{\{\tau \geq T_i\}} \\ +D(t, T_b)\mathbf{1}_{\{\tau \geq T_b\}} + D(t, \tau)REC\mathbf{1}_{\{T_a \leq \tau \leq T_b\}}\end{aligned} \tag{2.2}$$

Since the option's payoff does not imply an initial payment other than K, we remove the initial payment term $-\mathbf{1}_{\{\tau \geq T_a\}}D(t, T_a)$, and apply the fundamental theorem of asset pricing adapted for the valuation of defaultable claims:

$$FRN_{T_a} = \mathbf{E_Q}[\Pi^*_{FRN}(T_a)|G_{T_a}] \tag{2.3}$$

And obtain, due to the linearity of expected values:

$$\begin{aligned}FRN_{T_a} = \mathbf{E_Q}[\sum_{i=a+1}^{b} D(T_a, T_i)\delta_i(L(T_{i-1}, T_i)]\mathbf{1}_{\{\tau \geq T_i\}}|G_{T_a})]+ \\ \mathbf{E_Q}[\sum_{i=a+1}^{b} D(T_a, T_i)\delta_i X)\mathbf{1}_{\{\tau \geq T_i\}}|G_{T_a})] + \mathbf{E_Q}[D(T_a, T_b)\mathbf{1}_{\{\tau \geq T_b\}}|G_{T_a})] \\ +\mathbf{E_Q}[D(T_a, \tau)REC\mathbf{1}_{\{T_a \leq \tau \leq T_b\}}|G_{T_a})]\end{aligned} \tag{2.4}$$

We will now assume that the interest rate r_t is deterministic. This assumption is justified with the following financial arguments; FRN's durations are typically low. This derives from the fact that both the floating and the final payment terms are inversely dependent on increasing interest rates, which will imply an increase in floating payments. On the other hand, this interest rate increase will push the present value of the final maturity payment downwards. The same reasoning applies if interest rates move in the opposite direction, which illustrates why FRNs carry low interest rate risk. This assumption can be further justified by the fact that the bond's value is mainly dependent on variations in the underlying index level between fixings, and is pushed to par periodically. The main risk factor that remains, should therefore be credit risk. For these reasons, we assume in further computations that the only factor driving the decision to call the bond is default intensity, in line with Jarrow et al. (2008). This conclusion can be confirmed from the point of view of the issuer; A better issuer's credit quality will imply that he can finance himself with a lower spread, for this reason, he will be incentivized to call the bond, and reissue a bond with a cheaper spread. To summarize, this assumption implies that intermediate cash flows are modelled as deterministic cash flows equivalently to dividends in equity derivatives valuation.

We translate in mathematical terms the fact that the risk-free interest rate process does not drive the decision to call by assuming that the risk-free interest rate r_t is constant, and therefore deterministic, which implies r_t=r $\forall$t $\in R_0^+$. This assumption leaves only λ_t being stochastic:

$$\begin{aligned} FRN_{T_a} &= \mathbf{E_Q}[\sum_{i=a+1}^{b} exp(-r\int_{T_a}^{T_i} dt)\delta_i(L(T_{i-1},T_i)+X)]\mathbf{1}_{\{\tau\geq T_i\}}|G_{T_a})] \\ +exp(-r\int_{T_a}^{T_b} dt)&\mathbf{E_Q}[\mathbf{1}_{\{\tau\geq T_b\}}|G_{T_a})] + REC\ \mathbf{E_Q}[exp(-r\int_{T_a}^{u} du)\mathbf{1}_{\{T_a\leq\tau\leq T_b\}}|G_{T_a})] \end{aligned} \tag{2.5}$$

Or:

$$\begin{aligned} FRN_{T_a} &= \mathbf{1}_{\{\tau\geq T_a\}}(\sum_{i=a+1}^{b} \delta_i(L(T_{i-1},T_i)+X)D(T_a,T_i)H(T_a,T_i)] \\ +D(T_a,T_b)H(T_a,T_b) &+ REC\int_{T_a}^{T_b} D(T_a,u)\mathbf{E_Q}[\lambda_u exp(-\int_{T_a}^{s}\lambda_s\ ds)|G_{T_a})]du) \end{aligned} \tag{2.6}$$

With the survival probability $H(t,T)$ defined as:

$$H(t,T) = \mathbf{E_Q}[exp(-\int_t^T \lambda_s\ ds)|F_t)] \tag{2.7}$$

Using the fact that:

$$\mathbf{E_Q}[\lambda_T exp(-\int_t^T \lambda_s\ ds)|F_t)] = -\frac{\partial H(t,T)}{\partial T} \tag{2.8}$$

We obtain:

$$\begin{aligned} FRN_{T_a} &= \mathbf{1}_{\{\tau\geq T_a\}}(\sum_{i=a+1}^{b} \delta_i(L(T_{i-1},T_i)+X)D(T_a,T_i)H(T_a,T_i) \\ &+D(T_a,T_b)H(T_a,T_b) - REC\int_{T_a}^{T_b} D(T_a,u)\frac{\partial H(T_a,u)}{\partial u}du) \end{aligned} \tag{2.9}$$

By integrating by parts the third term (details in appendix), we obtain:

$$\int_{T_a}^{T_b} D(T_a,u)\frac{\partial H(T_a,u)}{\partial u}du = -1 + \int_{T_a}^{T_b}(\Delta_{T_b} D(T_a,u) + q(u))H(T_a,u)du \tag{2.10}$$

Where $\Delta_{T_b}(u)$ is the dirac measure centered at T_b, and we defined $q(u) = -\frac{\partial D(T_a,u)}{\partial u} = D(T_a,u).r$. We group together all terms:

$$\begin{aligned} FRN_{T_a} = \mathbf{1}_{\{\tau \geq T_a\}}(\sum_{i=a+1}^{b} [\delta_u(L(T_{u-1},u) + X)]D(T_a,u)H(T_a,u)]du \\ +D(T_a,T_b)H(T_a,T_b) + REC - REC\int_{T_a}^{T_b}(\Delta_{T_b}(u)D(T_a,u) + q(u))H(T_a,u)du)]) \end{aligned} \tag{2.11}$$

We write this expression in term of integrals by using dirac's measure:

$$\begin{aligned} FRN_{T_a} = \mathbf{1}_{\{\tau \geq T_a\}}(\int_{T_a}^{T_b} [\delta_u(L(u-1,u) + X)]D(T_a,u)H(T_a,u)]du \\ + \int_{T_a}^{T_b} D(T_a,u)H(T_a,u)\Delta_{T_b}(u)du + \int_{T_a}^{T_b} REC\Delta_{T_b}(u)du \\ -REC\int_{T_a}^{T_b}(\Delta_{T_b}(u)D(T_a,u) + q(u))H(T_a,u)du)) \end{aligned} \tag{2.12}$$

By defining:

$$\begin{aligned} c(u) = \delta_u(L(u-1,u) + X)D(T_a,u) + D(T_a,u)\Delta_{T_b}(u) \\ -REC(\Delta_{T_b}(u)D(T_a,u) + q(u)) + \frac{REC\Delta_{T_b}(u)}{H(T_a,u,\lambda)} \\ c(u) = Y(u) + \frac{REC\Delta_{T_b}(u)}{H(T_a,u,\lambda)} \end{aligned} \tag{2.13}$$

We defined $c(u)$ using these two different expressions, for clarity of notation, and since we will use both expressions in the derivation. We then may rewrite this expression as:

$$FRN_{T_a} = \mathbf{1}_{\{\tau \geq T_a\}}\int_{T_a}^{T_b} c(u)H(T_a,u)du \tag{2.14}$$

From a financial point of view, this equation represents a portfolio of coupons discounted by survival probabilities. $c(u)$ is expressed as follows in order to obtain the FRN value expressed as a sum of monotonic functions.

We turn to the pricing of the option *per se*; The payoff function of a European call option on this FRN is:

$$\Pi_{T_a} = \mathbf{1}_{\{\tau \geq T_a\}}(FRN(T_a, T_b) - K)^+ \tag{2.15}$$

We compute a λ^* constrained to be strictly positive and to simultaneously satisfy the following condition:

$$K = \int_{T_a}^{T_b} c(u, \lambda^*)H(T_a, u, \lambda^*)du = \int_{T_a}^{T_b} (Y(u) + \frac{REC\Delta_{T_b}(u)}{H(T_a, u, \lambda^*)})H(T_a, u, \lambda^*)du \tag{2.16}$$

We recombine this term with the FRN option payoff, and observe that the terms in $\frac{REC\Delta_{T_b}(u)}{H(T_a,u,.)}$ are vanishing:

$$\Pi_{T_a} = \mathbf{1}_{\{\tau \geq T_a\}}[\int_{T_a}^{T_b} Y(u)(H(T_a, u, \lambda) - H(T_a, u, \lambda^*)du\ ]^+ \tag{2.17}$$

Using the Jamshidian (1989) decomposition, we re-express the sum above as:

$$\Pi_{T_a} = \mathbf{1}_{\{\tau \geq T_a\}}(\int_{T_a}^{T_b} Y(u)[H(T_a, u, \lambda) - H(T_a, u, \lambda^*)]^+ du) = \mathbf{1}_{\{\tau \geq T_a\}}\pi^*_{T_a} \tag{2.18}$$

From a financial point of view, this means that we converted an option on a portfolio into a portfolio of options.

According to the key lemma proved in Bielecki et al. (2004), the value of a call option on the FRN is:

$$V_t = D(t, T_a)\mathbf{E_Q}[\mathbf{1}_{\{\tau \geq T_a\}}\pi^*_{T_a}|G_t] = \mathbf{1}_{\{\tau \geq t\}}D(t, T_a)\mathbf{E_Q}[exp(-\int_t^{T_a} \lambda_s ds)\pi^*_{T_a}|F_t] \tag{2.19}$$

We rewrite the full payoff function:

$$V_t = \mathbf{1}_{\{\tau \geq t\}}D(t, T_a)\mathbf{E_Q}[exp(-\int_t^{T_a} \lambda_s ds) \int_{T_a}^{T_b} Y(u)[H(T_a, u, \lambda) - H(T_a, u, \lambda^*)]^+ du|F_t] \tag{2.20}$$

By interchanging the integrals:

$$V_t = \mathbf{1}_{\{\tau \geq t\}} \int_{T_a}^{T_b} Y(u)\mathbf{E_Q}[exp(-\int_t^{T_a} \lambda_s ds)D(t, T_a)(H(T_a, u, \lambda) - H(T_a, u, \lambda^*)\ )^+|F_t]du \tag{2.21}$$

In order to apply the fundamental theorem of risk neutral valuation to the previous expression, we define:

$$N(T_a, t, T_a) = exp(r(T_a - t))N(t, t, T_a) = \mathbf{E_Q}[exp(\int_t^{T_a} \lambda_s ds)|F_t] \tag{2.22}$$

$N(T_a, t, T_a)$ can be interpreted as the expected value in T_a of a bank account with an instantaneous return equal to the instantaneous credit spread given by the market. Since it is a tradable asset, its return under the measure Q is equal to the risk-free rate, which would otherwise result in an arbitrage.

We defined $N(T_a, t, T_a)$ in this manner in order to re-express the survival probability $H(t, T_a, \lambda)$ as a function of tradable assets: $N(t, t, T_a)$ and a ZC bond:

$$H(t, T_a, \lambda) = \mathbf{E_Q}[exp(-\int_t^{T_a} \lambda_s ds)|F_t] = \frac{exp(-r(T_a - t))}{N(t, t, T_a)} \tag{2.23}$$

Since H is a function of λ_t, we define the following Ito process under the real probability measure:

$$d\lambda_t = \alpha(t, \lambda_t)dt + \beta(t, \lambda_t)dW_t \tag{2.24}$$

We apply Ito's Lemma on $H(t, T, \lambda)$, and obtain the general diffusion equation of H, valid for any assumption on $\alpha(t, \lambda_t)$, and $\beta(t, \lambda_t)$:

$$dH(t, T, \lambda) = \mu_t H(t, T, \lambda)dt - v_t H(t, T, \lambda)dW_t \tag{2.25}$$

Where μ_t, and v_t are functions of $\alpha(t, \lambda_t)$, and $\beta(t, \lambda_t)$ obtained by straightforward computations. By applying Girsanov's theorem, we switch to the risk neutral measure by using:

$$MP_{CR}(t) = \frac{\mu_t - r}{v_t} \tag{2.26}$$

Which can be interpreted as a market price of credit risk. We then obtain:

$$dH(t, T, \lambda) = rH(t, T, \lambda)dt - v_t H(t, T, \lambda)dW_t^* \tag{2.27}$$

Since H, and N are tradable assets, their return under the risk neutral measure are equal to the risk-free rate, and H is a martingale when discounted by the risk-free bank account. The objective of the previous reasoning was to outline a theoretical grounding that would allow to apply the fundamental theorem of risk neutral valuation to the following expression:

$$C(t, T_a, u, \lambda, \lambda^*) = \mathbf{E_Q}[D(t, T_a)(H(T_a, u, \lambda) - H(T_a, u, \lambda^*)\ )^+|F_t] \tag{2.28}$$

Where $C(t, T_a, u, \lambda, \lambda^*)$ represents a call option on a zero coupon bond with the hazard rate λ_t as underlying process.

An interesting feature of this formula consists in the fact that it permits different assumptions on the intensity process as long as the process satisfies the positivity condition, such as lognormal or CIR, since by definition, probabilities must be positive. A suitable process for λ_t should further be solvable for a zero coupon bond option expression. As the CIR process satisfies both conditions, we will further assume that λ_t is driven by a CIR process.

Proposition 2 *The time t value of a callable floating rate note with maturity T_b, with one embedded call option of maturity T_a is:*

$$CFRN_t = \mathbf{1}_{\{\tau \geq t\}}\left(\int_t^{T_b} c(u)H(t,u)du - exp(-\Gamma_{T_a}) \int_{T_a}^{T_b} Y(u)C(t,T_a,u,\lambda,\lambda^*)du\right)$$

Proof 2 *Proof follows directly from the decomposition outlined in (2.2), and from the bond and option expressions obtained in (eq. 2.14), and (proposition 1.), respectively.*

We conclude this section by noticing that (proposition 1.) can be modified in order to obtain the formula of a put option, which can then be recombined using (proposition 2.) in order to derive the formula of a puttable floater.

Chapter 3

Numerical results and conclusions

3.1 Testing the model against Monte Carlo simulations

This section is devoted to testing the formula that we derived in section 2 by means of Monte Carlo simulation. Although Monte Carlo techniques are best suited for the pricing of path-dependent payoffs, instead of trees, we test our model against Monte Carlo simulations because this technique has the significant advantage over trees that the intensity process cannot reach negative values, which is a shortcoming of trees in the CIR model, even if usual stability conditions are satisfied, such as the Feller condition and: $\theta \approx \frac{2\kappa\theta}{k+h}$. In other words, monte carlo simulation is better suited than trees to model the CIR process in a precise manner, which is necessary for testing purposes.

3.1.1 CIR process - calibration

The default intensity λ_t is modelled as a CIR process:

$$d\lambda_t = \kappa(\theta - \lambda_t)dt + \sigma\sqrt{\lambda_t}dW_t \tag{3.1}$$

The formula derived in Cox et al. (1985) can be applied in the credit space due to the structural equivalence between zero coupon bond prices and survival probabilities:

$$H(t,T) = A(t,T)e^{-B(t,T)\lambda_t} \tag{3.2}$$

With:

$$A(t,T) = [\frac{2h\ exp((k+h)(T-t)/2)}{2h + (k+h)(exp((T-t)h) - 1)}]^{2k\theta/\sigma^2} \tag{3.3}$$

$$B(t,T) = \frac{2(exp(h(T-t)) - 1)}{2h + (k+h)(exp((T-t)h) - 1)} \tag{3.4}$$

$$h = \sqrt{\kappa^2 + 2\sigma^2} \tag{3.5}$$

In other words, a survival probability can be treated as a zero coupon bond price with r_t replaced by λ_t as pointed out in Brigo, Mercurio (2006). We calibrate a CIR process on AAPL CDSs, and obtain the following parameter set:

	κ	θ	σ	λ_0
λ_t	0.1052	0.0106	0.0472	0.0003

Table 3.1: AAPL - Intensity process calibration

3.1.2 Gamma Ornstein-Uhlenbeck process - calibration

In this section we calibrate default intensity under the assumption that it follows a Gamma Ornstein-Uhlenbeck process. This is achieved to compare the classic CIR process with a stochastic process that is entirely driven by jumps. The Ornstein-Uhlenbeck process is expressed by the following stochastic differential equation:

$$d\lambda_t = -\eta\lambda_t dt + dz_t \tag{3.6}$$

Where $\eta \in R_0^+$ and z_t is the background driving Lévy process defined as:

$$z_t = \sum_{n=1}^{N_t} x_n \tag{3.7}$$

N_t is a Poisson process with intensity parameter a, and x_n are i.i.d. exponentially distributed variables. In this setting, the characteristic function of $\Lambda_t = \int_0^t \lambda_s ds$ is:

$$\begin{gathered}\phi_{Gamma-OU}(u,t,\eta,a,b,\lambda_0) = \\ exp(\frac{iu\lambda_0}{\eta}(1-e^{-\eta t}) + \frac{\eta a}{iu-\eta b}(b\ log(\frac{b}{b-iu\eta^{-1}(1-e^{-\eta t})}) - iut))\end{gathered} \tag{3.8}$$

Recall that the characteristic function of Λ_t is defined as:

$$\phi_{Gamma-OU}(u,t,\eta,a,b,\lambda_0) = E[exp(iu\Lambda_t)] \tag{3.9}$$

Therefore, by setting iu=-1, we obtain a closed-form expression for the survival probability under Gamma OU dynamics which is calibrated to a CDS term structure by using standard optimization procedures.

More precisely, a parameter set that matches as closely as possible market implied survival probabilities to model probabilities is derived by root mean square error (RMSE) minimization. The parameter set is exhibited in table 3.2:

	a	b	η	λ_0
λ_t	0.2990	6.2695	0.1383	0.0001

Table 3.2: AAPL - Intensity process calibration

The Ornstein-Uhlenbeck process is then simulated using the following scheme:

$$\lambda_{n\Delta t} = e^{-\eta\Delta t}\lambda_{n-1\ \Delta t} + \sum_{n=N_{(n-1)\Delta t+1}}^{N_{n\Delta t}} x_n e^{-u_n\eta\Delta t} \tag{3.10}$$

Where u_n are uniform random variables, and x_n are i.i.d. exponentially distributed random variables with parameter b.

3.1.3 Numerical results

The testing procedure is applied on a callable bond with the following features:

	T_a	T_b	r_0	X	REC	δ
FRN_t	4	5	0.01	.01734	0	.5

Table 3.3: Callable bond - characteristics

We then approximate the discounted expected value of the embedded option with Monte Carlo simulation where λ_t follows CIR and Gamma OU processes:

$$V_t = \mathbf{1}_{\{\tau\geq t\}} D(t,T_a)\mathbf{E_Q}[exp(-\int_t^{T_a}\lambda_s ds)(FRN_{T_a} - K)^+|F_t] \tag{3.11}$$

FRN_t is defined in (eq. 2.14) and the square root diffusion is discretized using the scheme outlined in Andersen (2006). By decomposing the underlying process, the option value in the CIR model can be shown to be:

$$\begin{aligned} C_t(.) = D(t,T_a)[H(T_a,T_b,\lambda)\chi^2(2\bar{r}[\rho+\psi+B(T_a,T_b)]; \frac{4k\theta}{\sigma^2}, \frac{2\rho^2\lambda_t exp(h(T_a-t))}{\rho+\psi+B(T_a,T_b)}) \\ -H(T_a,T_b,\lambda^*)\chi^2(2\bar{r}[\rho+\psi]; \frac{4k\theta}{\sigma^2}, \frac{2\rho^2\lambda_t exp(h(T_a-t))}{\rho+\psi})] \end{aligned} \tag{3.12}$$

With $\rho, \psi, \bar{r}$ described in appendix.

In the following figure, we compare option values obtained using (proposition 1.) with values obtained using Monte Carlo:

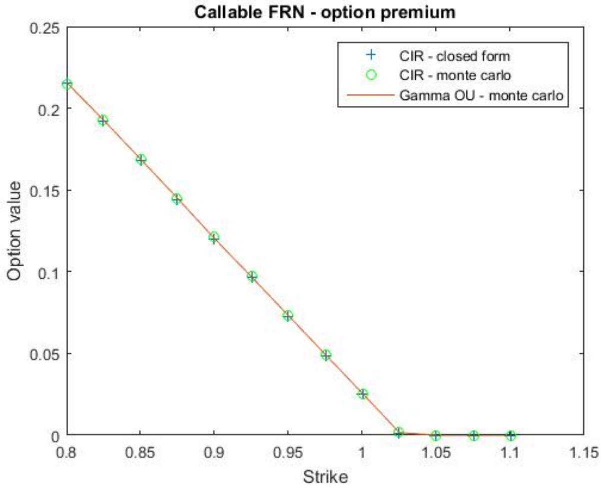

Figure 3.1: Model comparison against Monte Carlo simulation

We repeat the same experiment, and compare the formula obtained in (eq. 2.14) with the following expected value approximated under the assumption of Gamma OU and CIR dynamics:

$$FRN_{t,T_b} = \mathbf{E}_{\mathbf{Q}}[\Pi^{*}_{FRN}(t,T_b)|G_t] \tag{3.13}$$

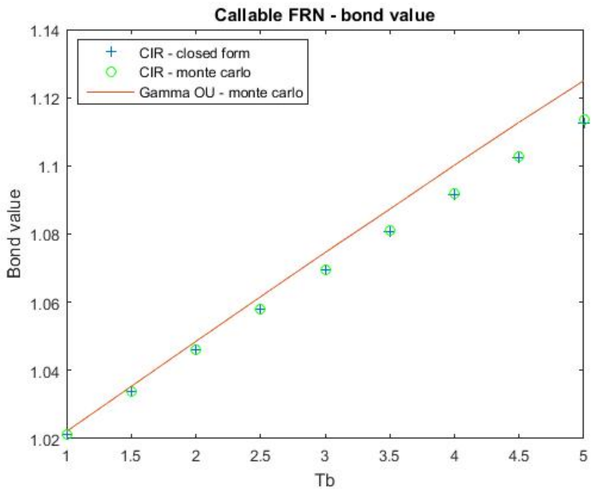

Figure 3.2: Model comparison against Monte Carlo simulation

Both processes yield similar results in shorter maturities, and the difference between computed results becomes progressively more pronounced in higher maturities, and therefore longer simulated paths. Notice that at issuance, the spread X is adjusted to the bond's market value. Since we are testing the model, we do not adjust the spread for each maturity. This, and the assumption of deterministic interest rates, explains why computed prices are increasing with each subsequent maturity.

3.1.4 Sensitivity analysis

In this section, we conduct a sensitivity analysis in the CIR model of the formulas we previously derived. This is achieved by modifying one parameter at a time while keeping the other parameters fixed:

	κ	θ	σ	λ_0	REC	r_0	X
V_t	-	-	+	-	+	-	+
FRN_t	-	-	+	-	+	-	+

Table 3.4: Sensitivity to parameter variations

3.2 Testing the assumption of deterministic interest rates

Until now, the assumption of deterministic interest rates has been supported only by financial arguments. In this part of the paper, we will resort to numerical tests in order to verify the validity of this assumption. More precisely, we will compare FRN values computed by assuming r_t to be stochastic and deterministic, where r_0 is the starting value of the stochastic process in case of stochastic interest rates, and the flat short rate r_0 under the assumption of deterministic interest rates. Both values will be compared along different maturities. In order to achieve this, we do not resolve to Monte Carlo simulation, but instead, we work with closed-form expressions. We follow this approach because an expected value approximated by Monte Carlo simulation always remains an approximation, even for a high number of simulated paths. In other words, by resorting to closed-form expressions, we want to be certain that minimal differences between computed values are not caused by imperfect convergence of the Monte Carlo algorithm.

Interest rate process parameters used in the testing procedure, and results are included in tables (3.5), and (3.6), respectively:

	κ	θ	σ	r_0
r_t	0.3023	0.0159	0.0713	0.0100

Table 3.5: CIR interest rate process - parameters

Table 3.6: Comparison of stochastic and deterministic interest rates.

Maturity	Δ	$FRN_t(r.stoch)$	$FRN_t(r.det)$
1	-0.0006	1.0207	1.0213
2	0.0007	1.0468	1.0461
3	0.0029	1.0724	1.0695
4	0.0055	1.0971	1.0917
5	0.0083	1.1210	1.1127
6	0.0111	1.1438	1.1326
7	0.0138	1.1655	1.1516
8	0.0164	1.1861	1.1697
9	0.0188	1.2057	1.1869
10	0.0210	1.2242	1.2033
11	0.0228	1.2418	1.2190
12	0.0245	1.2585	1.2340
13	0.0259	1.2742	1.2484
14	0.0271	1.2892	1.2621
15	0.0280	1.3033	1.2753
16	0.0287	1.3167	1.2880
17	0.0292	1.3294	1.3002
18	0.0295	1.3414	1.3119
19	0.0296	1.3528	1.3232
20	0.0295	1.3636	1.3341

We observe minor differences, denoted with the symbol Δ, among short maturities that become progressively larger for higher maturities. This is a satisfying result because most often, FRNs are issued with maturities varying between one and three years. We interpret the progressively higher differences observed in longer maturity FRNs as a result of the higher number of intermediate cash flows which are affected by the assumption on the short interest rate.

Although this extension will not be implemented in this paper, our framework could be extended into a two factor model with stochastic r_t and λ_t, which would be suitable to model FRNs with higher maturities or with a higher interest rate sensitivity due to a particular choice of reference interest rate index. This would be similar to the modelling of long-dated equity derivatives where the interest rate sensitivity, although negligible for short maturities, is taken into account with a stochastic r_t.

3.3 Conclusions

We derived a formula for callable and puttable floaters based on a decomposition into a bond component, and an option component under the assumption that the default event is the first jump of a Cox process. Our main contribution to the existing literature resides in a formula for the embedded option that supports any positive stochastic process solvable for a zero coupon bond option expression. The formulas are then tested against Monte Carlo simulation under the assumption that default intensity follows a CIR process.

We then calibrated a Gamma Ornstein-Uhlenbeck process to the same CDS term structure and obtained bond and option values matching very closely the results obtained under the assumption of CIR dynamics, which is in line with the findings of Cariboni and Schoutens (2008).

Further, we tested the assumption of deterministic interest rates that supports our derivation. Among shorter maturities, bond values obtained under the assumption of deterministic interest rates prove to be very close to the values obtained under the assumption of stochastic interest rates. The progressively higher differences observed in longer maturity FRNs are interpreted as a direct consequence of the higher number of intermediate cash flows which are affected by the assumption on the short rate.

Further research could investigate the impact of the liquidity premium in the valuation of the CFRN's bond component. The closed-form expression could be further adapted to take simultaneously into account CIR and jump dynamics by means of a J-CIR++ process.

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Appendices

.1 Integration by parts

$$
\begin{aligned}
&\int_{T_a}^{T_b} D(T_a,u)\frac{\partial H(T_a,u)}{\partial u}du \\
&= D(T_a,T_b)H(T_a,T_b) - 1 - \int_{T_a}^{T_b} \frac{\partial D(T_a,u)}{\partial u} H(T_a,u)du \\
&= \int_{T_a}^{T_b} \Delta_{T_b} D(T_a,u)H(T_a,u)du - 1 + \int_{T_a}^{T_b} q(u)H(T_a,u)du \\
&= -1 + \int_{T_a}^{T_b} (\Delta_{T_b} D(T_a,u) + q(u))H(T_a,u)du
\end{aligned} \tag{14}
$$

Where Δ_{T_b} is the dirac measure centered at T_b and $q(u) = -\frac{\partial D(T_a,u)}{\partial u} = D(T_a,u).r$

.2 CIR closed form option expression

$$
\begin{aligned}
C_t(.) = D(t,T_a)[H(T_a,T_b,\lambda)\chi^2(2\bar{r}[\rho+\psi+B(T_a,T_b)]; \frac{4k\theta}{\sigma^2}, \frac{2\rho^2\lambda_t exp(h(T_a-t))}{\rho+\psi+B(T_a,T_b)}) \\
-H(T_a,T_b,\lambda^*)\chi^2(2\bar{r}[\rho+\psi]; \frac{4k\theta}{\sigma^2}, \frac{2\rho^2\lambda_t exp(h(T_a-t))}{\rho+\psi})]
\end{aligned} \tag{15}
$$

$$
\rho = \frac{2h}{\sigma^2(exp(h(T_a-t))-1)} \tag{16}
$$

$$
\psi = \frac{k+h}{\sigma^2} \tag{17}
$$

$$
\bar{r} = \frac{ln(A(T_a,T_b)/H(T_a,T_b,\lambda^*))}{B(T_a,T_b)} \tag{18}
$$

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