Technical report
PPPN Design and Hedging
FEHA2015

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1 Introduction

This part deals with the product's technical aspects. Its objective is to delimit the theoretical framework within which the product is priced and hedged. Then, to describe the data, the model's calibration, the resulting Monte Carlo pricing process, and the Greeks that will allow the bank to hedge the risk involved with selling the product, as well as a clear description of its profitability.

2 The Model

2.1 Settings

On a filtered probability space $(\Omega, \mathcal{F}, \mathcal{Q})$, we consider two standard brownian motions W $= \{W_t, t \geq 0\}$, and $\tilde{W} = \{\tilde{W}_t, t \geq 0\}$ correlated such that $Cov[dW_t d\tilde{W}_t] = \rho dt$.

S $= \{S_t, 0 \leq t \leq T\}$ is the underlying asset, the XLI Index, and $\Phi(u,t)$ the characteristic function of the random variable $log(S_t)$:

$$\Phi(u,t) = E[e^{iulog(S_t)}] \quad (1)$$

We assume the risk-free rate r to be continuously compounded throughout the sample period [0,T]. The dividend yield is denoted by q, and assumed to be constant and continously compounded.

2.2 The Heston Stochastic Volatility Model

Within the framework defined above, the Heston Stochastic Volatility model assumes that the underlying follows a classic geometric brownian motion defined such that:

$$dS_t = (r-q)S_t dt + \sigma_t S_t dW_t \tag{2}$$

With drift coefficient $(r-q)$, and volatility σ_t.

Notice that contrary to the Black-Scholes framework, the volatility is a function of time and follows a Cox-Ingersoll-Ross (CIR) stochastic process defined as:

$$d\sigma_t^2 = \kappa(\eta - \sigma_t^2)dt + \theta\sigma_t d\tilde{W}_t \tag{3}$$

With the vector of constant model parameters: $\{\kappa, \eta, \theta, \rho, \sigma_0\}$.

- $\kappa > 0$: Speed of mean reversion ;
- $\eta > 0$: Level of mean reversion ;
- $\theta > 0$: Volatility of the variance v_t;
- $-1 < \rho < 1$: Variance-underlying correlation
- $\sigma_0 > 0$: Initial vol;

Although we can now overcome the assumption of constant volality, which is perhaps the greatest B&S model shortcoming, we are now dealing with a much heavier parametrized model.

The Heston model's characteristic function is given by:

$$\begin{aligned} \Phi(u,t) = E[exp(iulog(S_t))] &= exp(iu(log(S_0) + (r-q)t)) \\ \times exp(\eta\kappa\theta^{-2}((\kappa - \rho\theta ui - d)t &- 2log((1-ge^{-dt})/(1-g)) \\ \times exp(\sigma_0^2\theta^{-2}((\kappa - \rho\theta ui - d)(1-e^{-dt})&/(1-ge^{-dt})) \end{aligned} \tag{4}$$

With:

$$d = ((\rho\theta ui - \kappa)^2 - \theta^2(-iu - u^2))^{.5} \tag{5}$$

$$g = (\kappa - \rho\theta ui - d)/(\kappa - \rho\theta ui + d) \tag{6}$$

3 The Product

3.1 Description

This is a structured product based on the XLI index performance. There is one single payment day, which is a contractually set maturity T. This implies no possibility of early redemption, even if the product is sold on the secondary market. For each N of notional investment at time t=0, the investor receives at time T, 0.95×N independently on the market performance, and an additional amount if the index never traded at or below a pre-determined barrier level H=75% of the initial index value S_0. The premium is computed as the notional value N times a participation rate p and the positive underlying return within the period [0,T].

The PPPN payoff function, $f(\tilde{S}_t)$, is written as:

$$f(\tilde{S}_t) = \begin{cases} 0.95N + pN(\frac{S_T - S_0}{S_0})^+ \quad m_T > HS_0 \\ 0.95N \quad m_T < HS_0 \end{cases}$$

Where:

$$m_T = min(S_t) \ t \in \,]0;T] \tag{7}$$

f is defined as a function of $\tilde{S}_t$, (and not S_T) to indicate that the note's payoff is a function of the underlying's historical prices due to the option's barrier component.

3.2 Pricing

To replicate this payoff, we first observe that in order to deliver $0.95N$ at maturity to the client, a given amount has to be invested at the risk-free rate. Concretely, this implies that the issuer must invest $0.95Ne^{-r(T-t_0)}$ on a risk-free asset (e.g zero-coupon German, or US bond). The difference can be invested in the option-component: $N - 0.95Ne^{-r(T-t_0)}$.

As the fraction $\frac{1}{S_0}$ can be placed outside of the option-component, this leaves us with the payoff of an ATM call option. Therefore, the PPPN payoff function $f(\tilde{S}_t)$, can be re-written as:

$$f(\tilde{S}_t) = \begin{cases} 0.95N + \frac{pN}{S_0} \times (S_T - S_0)^+ \quad m_T > HS_0 \\ 0.95N \quad m_T < HS_0 \end{cases}$$

We can conclude from the above payoff decomposition that this structured product can be decomposed into the sum of a riskless ZC bond, and an ATM DOBC option with barrier HS_0 and strike S_0. This implies that in order to value the product, we will have to price each sub-part of the PPPN separately. At time t=0, this gives:

$$V(\tilde{S}_t) = (0.95 \times e^{-r(T-t_0)}) + (B \times V_0^{DOBC}) \tag{8}$$

From this equation, we can infer two important conclusions: 1) the two main factors driving the PPPN value are volatility (which is paramount to determine the participation rate in the option-component), and interest rates, that determine the price of hedging the principal component. 2) To price the product, we have to further investigate the value of V_0^{DOBC}. Its correct valuation and hedging is the concern of the majority of this report.

The DOBC's payoff function can be found through the martingale relation:

$$V_0^{DOBC} = e^{-r(T-t_0)} E_Q[(S_T - K)^+ .1_{m_T > HS_0}] \tag{9}$$

In further computations, we assume no carrying costs, and no rebate associated with the DOBC option.

4 Data

Our dataset consists of 236 observed vanilla options with maturities between 5 days and 1.67 year observed the 19/05/2015. Our data source is Yahoo! Finance. We assume the US treasuries to be perfectly default-free, and henceforth, the associated YTM as the risk-free rate r, which we interpolated in order to find the risk-free interest rate associated with each option maturity. The dividend yield q is included in Yahoo! Finance's index's description. It is equal to 1.95%, which we assume to be constant and continuously compounded during the analyzed period [0,T].

5 Calibration

The basic aim of calibration is to find a vector of parameters p that minimizes as much as possible the difference between the model option price with said vector of parameters, and the currently observed market prices.
The minimization problem can be expressed as:

$$min_p[Errors(p)] \tag{10}$$

We look for the global minimum over all p model parameters. Using the RMSE function is an industry practice. Nevertheless, we worked additionnaly with the aae, and arpe functions in order to test the robustness of our calibration procedure:

W-RMSE:

$$\sqrt{\sum_{options} \omega_i \frac{(V_{mkt} - V_{model})^2}{N_{options}}} \tag{11}$$

W-AAE:

$$\sum_{options} \omega_i \frac{|V_{mkt} - V_{model}|}{N_{options}} \tag{12}$$

W-ARPE

$$\frac{1}{N_{options}} \sum_{options} \omega_i \frac{|V_{mkt} - V_{model}|}{V_{mkt}} \tag{13}$$

At this point, it is important for us to underline that the set of parameters we want to find has to come from a set of options that are as liquid as possible. Indeed, the existence of a wide illiquidity premium embedded in the option price would bias our results. For this reason, we calibrate our model with vanilla options, (European calls and puts) that are priced through the Carr-Madan pricing formula:

$$C(K,T) = \frac{exp(-\alpha log(K))}{\pi} \int_0^{+\infty} exp(-iv log(K))\delta(v)dv \tag{14}$$

where

$$\delta(v) = \frac{exp(-rT)\phi(v(\alpha+1)i,T)}{\alpha^2 + \alpha v^2 + i(2\alpha+1)v} \tag{15}$$

As we know the characteristic function of the underlying process, we can work with this model, which allows for fast and accurate pricing of vanillas.

It is a well known fact that optimization results can be very dependent on the choice of the initial parameter set x_0. Since we cannot be sure whether our optimization routine has found a local, or a global minimum, we will verify whether our optimal parameter set is coherent by comparing our model prices with market prices, as well as our parameters values with typical values obtained in similar conditions. Further, we implemented a verification routine that counts the number of option priced outside of the market observed bid-ask spread. If 95% of options were priced within the bid-ask spread, we interpreted this result as suggesting a satisfying calibration, and a sound guess of initial parameters.

A further challenge we encoutered resides in the fact that liquidity varies significantly across our dataset; We observed wide irregular volume discrepencies between different strike and maturity ranges. If left un-checked, this factor would certainly bias our final estimation results. Therefore, we implemented the above-mentionned objective functions with a weighting according to open interest. All weights are positive, and sum to 1. Further, we conducted our calibration only on options with moneyness m = $K/S_0 \in [0.75, 1.35]$, and implemented the Feller condition, according to which:

$$2\kappa\eta \geq \theta^2 \tag{16}$$

If this condition is satisfied, the process nevers drops below zero. Note that altough this is ensured from a theoretical point of view, this might still happen in the simulation procedure due to discretization errors.

6 Simulation

Monte carlo simulations are built upon the law of large numbers; If X_n is a sequence of i.i.d. random variables, then:

$$\lim_{N\to\infty} \frac{1}{N}\sum_{j=1}^{N} X_j = \int xf(x)dx \tag{17}$$

In the context of exotic option pricing, Monte Carlo simulations are used to generate underlying values within a time interval t=[0,T]. Indeed, we want to evaluate $E_Q[g(S_T)] = E_Q[(S_T - K)^+.1_{m_T > HS_0}]$. To achieve this, we generate paths of $\{S_t, 0 \leq t \leq T\}$, and by taking their arithmetic average on a large enough number of draws, we intend to eventually converge to the value $E_Q[g(S_T)]$ that we want to evaluate. With this in mind, we recall from section (2) that the underlying asset S is an equity index that we assume to be driven by the stochastic differential equations:

$$dS_t = (r-q)S_t dt + \sigma_t S_t dW_t \tag{18}$$

and:

$$d\sigma_t^2 = \kappa(\eta - \sigma_t^2)dt + \theta\sigma_t d\tilde{W}_t \tag{19}$$

Simulation of these processes can only be achieved in discrete, but very small, time steps. Therefore, in order to simulate the index, as well as variance values, we first need to devise a discretization scheme in order to transform our - continuous-time SDE - model, into a discrete process. In this project, this will be achieved through the Euler discretization scheme, which is a standard practice in the industry. Altough we wish to remind that there exists other techniques meant to achieve the same result, such as the Milstein scheme, which adds second-order approximations to the discretization. Our discretized process now reads:

$$S_{t+1} = S_t + (r-q)S_t\Delta_t + \sigma_t S_t\sqrt{\Delta}_t l_n \tag{20}$$

and:

$$\sigma_{t+1}^2 = \sigma_t^2 + \kappa(\eta - \sigma_t^2)\Delta_t + \theta\sigma_t\sqrt{\Delta}_t v_n \tag{21}$$

Where v_n and l_n $\{n = 1, 2, ...T/\Delta_t\}$) are normally distributed random variables with dependence modeled through a cholesky decomposition, and Δ_t=1 day. One year is assumed to be comprised of 250 open market days. We perform our arithmetic average on 1.000.000 paths. While implementing the MC simulation, we were facing the risk that our variance becomes negative due to discretization errors. To avoid this, a reflection technique was implemented within the algorithm; If the variance output was negative, it was automatically converted into its absolute value, and then included into the simulation. Finally, the antithetic variable technique, which is efficient on monotonic payoffs such as the one studied, was implemented in order to reduce standard deviation of errors as well as computation time.

7 Numerical results

By running our calibration routines on our dataset with different cost functions, we obtained three very similar, but distinct, sets of parameters:

	κ	η	θ	ρ	σ_0
w-rmse	4.9802	0.0359	0.5977	-0.8157	0.0919
w-arpe	4.9326	0.0347	0.5851	-0.6329	0.0857
w-aae	4.9047	0.0349	0.5740	-0.7499	0.0904

Table 1: Heston model - Risk neutral parameters

To select between them, we computed the error measures associated with each minimization algorithm:

$WRMSE$	$WARPE$	$WAAE$
0.0066	0.0306	0.0162

Table 2: Heston model - Global fit error measures

On this basis, we selected the parameters found through the weighted RMSE objective cost function, as they best minimize the distance between market observed, as well as model prices. The similarity between the parameters further suggests a robust calibration. With these parameters, we estimated 20 Monte carlo values for the DOBC option in order to test the robustness of the pricing algorithm:

	Mean	Median	Min.	Max.
Values	5.4468	5.4467	5.4389	5.4555

Table 3: DOBC - Monte carlo estimates

8 Profitability

Based on the product decomposition outlined in (3.2), we find the participation rate with which the product is priced at fair value equal to 64.90% . We set this participation rate at 45% in order to obtain a 2% profit from selling the product.

In order to determine this profit margin, we looked for studies on the pricing of similar products, and used those margins as a proxy for our competition; An average 4.5% overpricing across 214 issues has been found by Deng et al (2009) for ARBNs. Therefore, by setting a 2% profit margin, we intend to align ourselves clearly ahead of the competition. In practice, participation rates vary between approximately 30 percent up to several hundred percent. As those rates are directly dependent on the theoretical option value, one cannot infer anything on the product's fair pricing only from this number.

Note that leaving aside theoretical considerations (change of measure, martingale property..) one can see from the product decomposition that given the current low rate environment, it is particularly tempting to significantly enhance both the profit margin and the participation rate (and therefore the product's marketability) by securing the protected part with a very low risk, altough not "perfectly" risk-free bond: e.g. in the AA range.

9 Hedging the risk

Now that the product has been sold, we must hedge it; We need to build a portfolio Φ in order to reduce as much as possible the risk that the bank incurs from selling the PPPN.

To achieve this, we work within the B&S, constant volatility setting and compute sensitivity measures of the barrier option. Based on those metrics, which are derivatives, we will be able to hedge the exotic option with the underlying index, as well as a set of vanilla options.

We implemented the greeks for the DOBC option under the Black-Scholes model (details in appendix). Results were checked and confirmed by an exotic option greeks calculator.

Δ	Γ	υ	Θ
0.5026	0.0223	0.2738	-0.0048

Table 4: DOBC - Sensitivities

Delta hedge:

One can see from $e^{-r(T-t_0)}E_Q[(S_T-K)^+.1_{m_T>HS_0}]$ that the call option value is directly dependent on the underlying's S moves, which is therefore a significant source of risk that we have to hedge. Leaving aside any other, and specifically, second order effects, we want to obtain:

$$\Delta_\Phi = -[\Delta \times N \times \frac{p}{S_0}] + [\Delta \times N \times \frac{p}{S_0}] = 0. \tag{22}$$

The underlying has a delta equal to one. Therefore, we need to go long $\Delta \times N \times \frac{p}{S_0} = 3960$ units of the underlying.

Delta-vega hedge:

Besides first order sensitivity to underlying price moves, volatility, as a key driver of option value, is a major source of uncertainty. This part will implement a delta-vega hedging, which has to be achieved by using the underlying asset and vanillas. Indeed, the underlying allowed us to delta hedge because it has a delta of one. In regards to delta-vega hedging, one needs to underline that we are dealing with nonlinearities within our risk structure. Therefore, we need to use options to hedge our non-linear risk exposure.

Our initial portfolio vega is: $\upsilon_{DOBC} \times N \times \frac{p}{S_0}$ = -2157, which we have to hedge with $\upsilon_{mkt} = 0.1772$, or long 12172 options. This creates a residual delta of 2552 which has to be hedged with the underlying.

From available options, we chose the 70 strike call based on its characteristics, namely relatively cheap vega (see appendix).

Delta-vega-gamma-theta hedge:

For the same reason, we use K65 calls, K70 calls, and K30 puts (options 1,2,3 respectively), in order to solve the following system:

$$\begin{cases} \Gamma_{initial} + \omega_1 \Gamma_{option1} + \omega_2 \Gamma_{option2} + \omega_3 \Gamma_{option3} = 0 \\ \upsilon_{initial} + \omega_1 \upsilon_{option1} + \omega_2 \upsilon_{option2} + \omega_3 \upsilon_{option3} = 0 \\ \Theta_{initial} + \omega_1 \Theta_{option1} + \omega_2 \Theta_{option2} + \omega_3 \Theta_{option3} = 0 \end{cases}$$

We solve the system, obtain $w_i = [175, 2157, -37]$ i=[1,2,3] and hedge the residual delta with the underlying by following the same procedure as above. However in this case, working in a constant vol world can create problems; The matrix is ill-conditionned, which implies that our system's solutions are very sensitive to small variations in the inputs.

Comments on the hedging strategy:

In perfect markets, it is optimal to set up a hedge, and then rebalance it dynamically. In practice, one cannot avoid market frictions. For this reason, hedging a barrier option dynamically would immediately result in high hedging costs. In other words, it is in our interest to modify the hedge as less as possible in order to save on transaction costs, by adopting a quasi-static (as static as possible) hedging strategy. This has an important practical implication: we will prefer hedging by keeping as close as possible to the product's maturity. On top of this, in this market, it is not likely that we can be sure of vanilla options liquidity when closing the position. Therefore, it is better to let them expire.

As there are no traded options within the product's maturity range, 1.67Y maturity options have been chosen, which is the highest, and closest, available maturity. This means that at expiration the hedge will have to be re-adjusted, which will come at a certain, but unknown, cost. One should also note that in practice, we would not construct an hedge for every single option individually, but on a portfolio of options. The second main consideration after option maturity, is the choice of specific options within the maturity range. Here again, we focus on the cost of setting up a hedging strategy, as we observe that the greeks display clear differences between traded options, and their associated mids (see appendix).

Barrier option are seldom challenging to hedge due to barrier risk. However, in this case, barrier risk is low. It can be explained by studying the product's payoff: if the option knocks-out, we simply lose a liability, and have to liquidate the hedge. Therefore, in normal market conditions, the risk management of a down and out barrier call option is not likely to be difficult in opposition to options with potentially large gammas, such as options that can be knocked out in the money, as this risk structure implies two close, but distinct events. Further, the short position in the DOBC implies that we are long skew. In such a setting, vega and gammas are increasing if the market rallies, and decreasing if there is a sell-off.

Finally, we are left with the most direct part of the hedging strategy: hedging the principal part by investing $0.95 \times N \times e^{-r(T-t_0)}$ at the risk free rate in order to deliver the protected principal at maturity.

3 Concluding remarks

By running estimations on a quite "exotic" dataset characterized by wide bid/ask spreads and few liquid options, judging by our goodness of fit measures, we can affirm that we have obtained a satisfying calibration.

Yet it is important to underline that a good calibration is not a protection against model risk. This phenomenon has been studied by Schoutens et al. (2004) and Hardle et al. (2007), who showed that although calibration risk for equity vanilla options is low, wide discrepancies in the pricing of associated exotics can be obtained from near-perfectly calibrated models.

This conclusion has significant implications for this project:
1) By setting the participation rate, a safety margin has to be taken into account to compensate for model risk.
2) Even with a safety margin, one has to be cautious about the pricing of the exotic component, as calibration on vanilla options only determine marginal distributions, but not the underlying stochastic process.

References

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Appendix

DOBC Greeks:

$$\Delta_B = -\frac{H^2}{S_0^2} e^{-qT} N[-d_{bs1}(H^2/S_0, K)] \tag{23}$$

$$\Lambda_B = \frac{H^2}{S_0^3} e^{-qT} [2N[-d_{bs1}(H^2/S_0, K)] + \frac{n(d_{bs1}(H^2/S_0, K))}{\sigma\sqrt{T}} \tag{24}$$

$$\upsilon_B = \sqrt{T} K e^{-rT} n(d_{bs}(H^2/S_0, K)) > 0 \tag{25}$$

Where

$$d_{bs}(H^2/S_0, K) = \frac{ln(H^2/S_0 K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} \tag{26}$$

and

$$d_{1bs}(H^2/S_0, K) = d_{bs}(H^2/S_0, K) + \sigma\sqrt{T} \tag{27}$$

$$\Delta_{DI} = \frac{H}{S_0}^{\frac{2\omega}{\sigma}} [\Delta_B - \frac{2\omega}{S_0 \sigma^2} C_{bs}(\frac{H^2}{S_0}, K)] \tag{28}$$

$$\Lambda_{DI} = \frac{H}{S_0}^{\frac{2\omega}{\sigma}} [\frac{2\omega(2\omega + \sigma^2)}{S^2 \sigma^4} C_{bs}(\frac{H^2}{S_0}, K) + \Lambda_B - \frac{4\omega}{S_0 \sigma^2} \Delta_B] \tag{29}$$

$$\upsilon_{DI} = \frac{H}{S_0}^{\frac{2\omega}{\sigma}} [\upsilon_B - \frac{4(r - g)}{\sigma^3} C_{bs}(\frac{H^2}{S_0}, K) ln(\frac{H}{S_0})] \tag{30}$$

$$\Delta_{DO} = \Delta_{vanilla} - \Delta_{DI} \tag{31}$$

$$\Lambda_{DO} = \Lambda_{vanilla} - \Lambda_{DI} \tag{32}$$

$$\upsilon_{DO} = \upsilon_{vanilla} - \upsilon_{DI} \tag{33}$$

$$\Theta_{DO} = B_1 - B_2 \tag{34}$$

Where

$$B_1 = -.5\sigma S_0 f(n(X)/\sqrt{T}) + S_0 f N(X) q - K d N((X - \sigma\sqrt{T}) r \tag{35}$$

$$B_2 = \frac{H}{S_0}^{2\lambda} S_0 f n(y) .5(\sigma/\sqrt{T})) + \frac{H}{S_0}^{2\lambda-2} [q S_0 f (\frac{H}{S_0}^{2}) N(y) - r K d N((y - \sigma\sqrt{T}] \tag{36}$$

With:

$f = e^{-qT}$

$d = e^{-rT}$

$\theta_{-} = ((r-q)/\sigma) + (\sigma/2)$

$\mu = \sigma\theta_{-}$

$\lambda = 1 + (\mu/\sigma^2)$

$p = (\mu + \sigma^2) T$

$y = (log((H^2/S_0)K)) + p)/(\sigma\sqrt{T})$

$X = (log(S_0/K) + p)/(\sigma\sqrt{T})$

Figure 1: All maturities - Calibrated option values

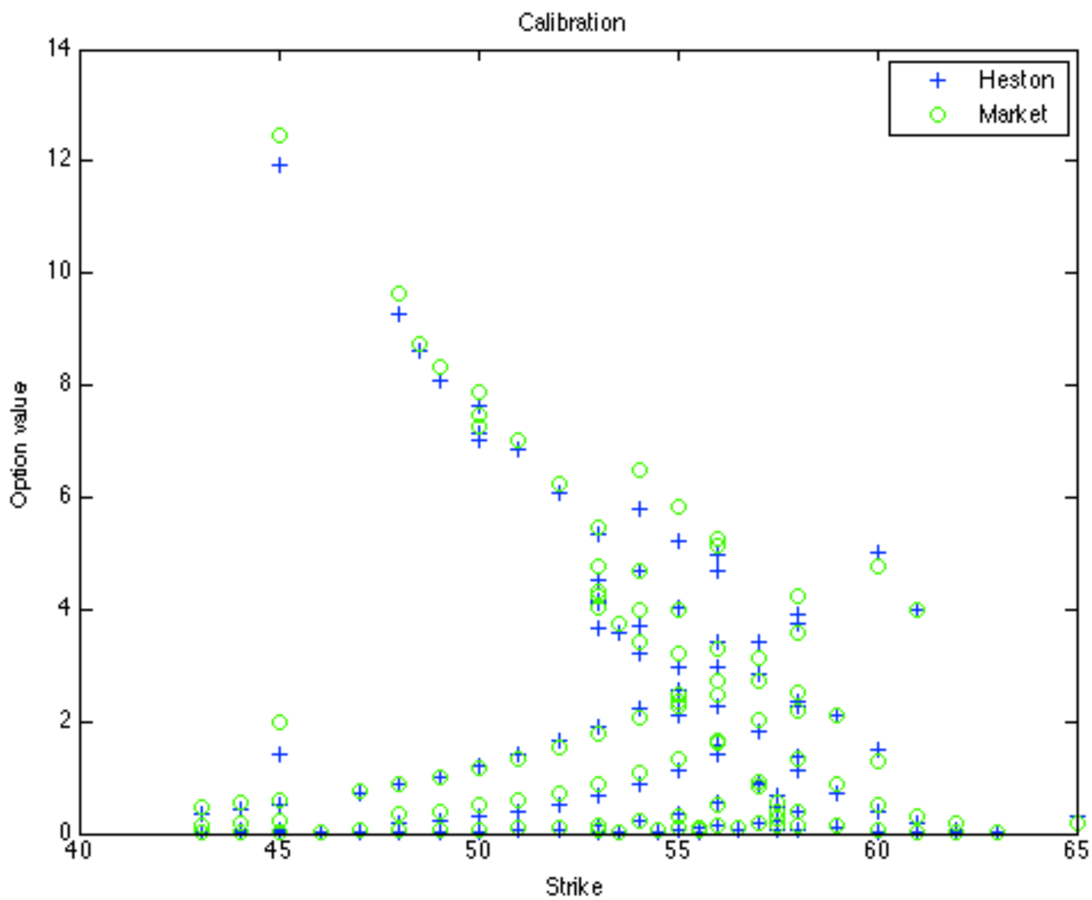


Figure 2: DOBC value sensitivity - 1st order - Delta

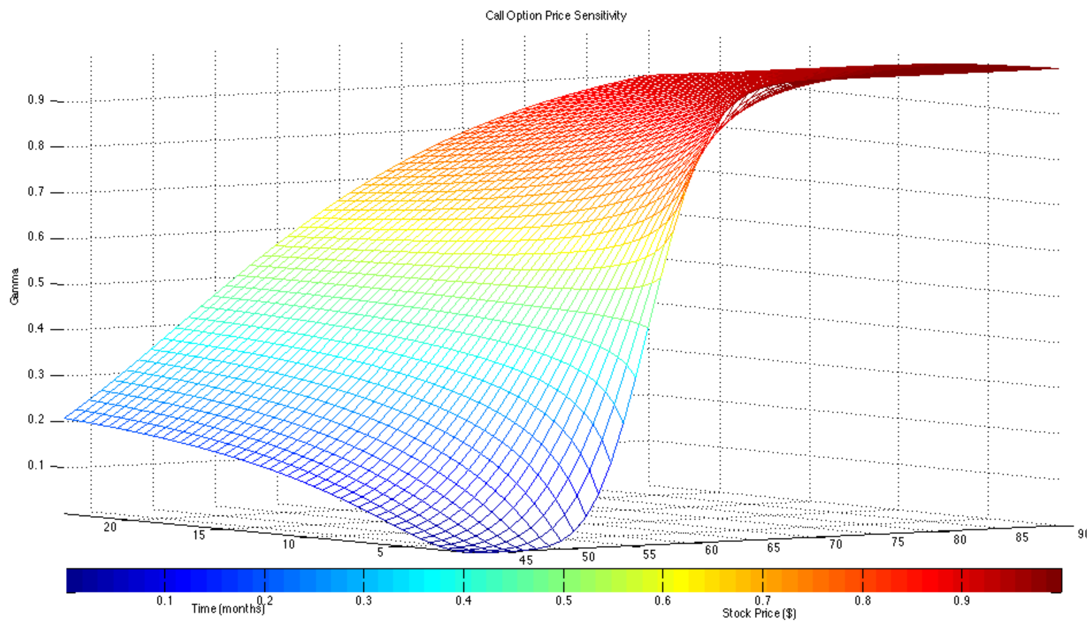


Figure 3: DOBC value sensitivity - 2nd order - Gamma

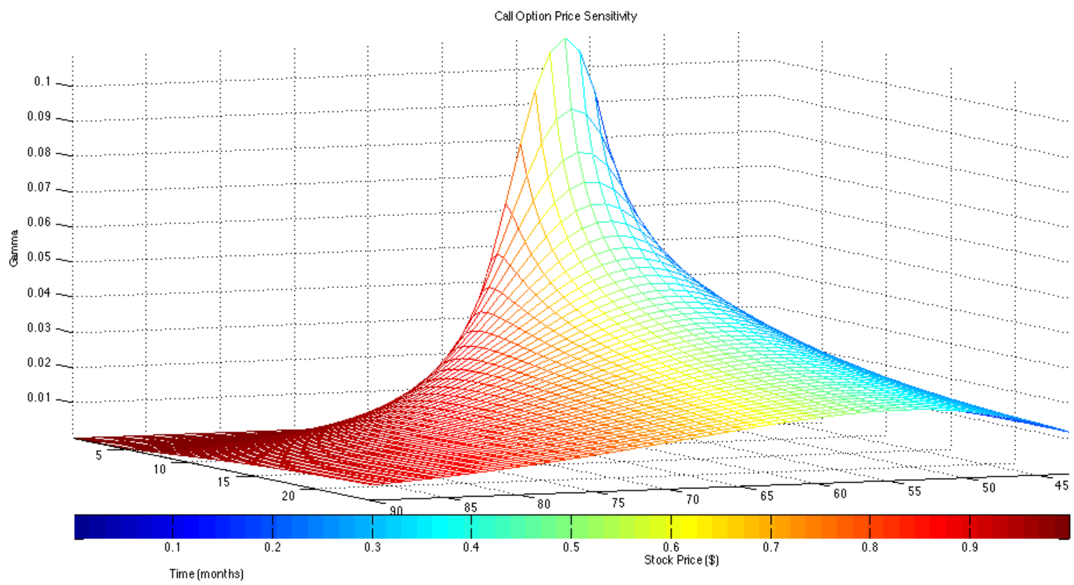


Figure 4: DOBC value sensitivity - 1st order - Theta

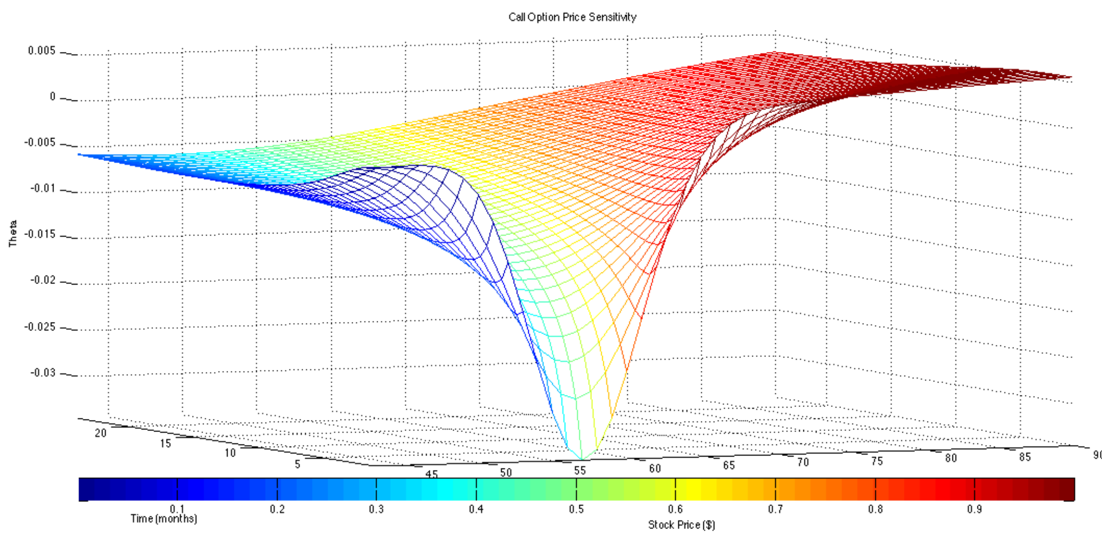


Figure 5: Vanilla sensitivity - 1.67Y maturity - Delta

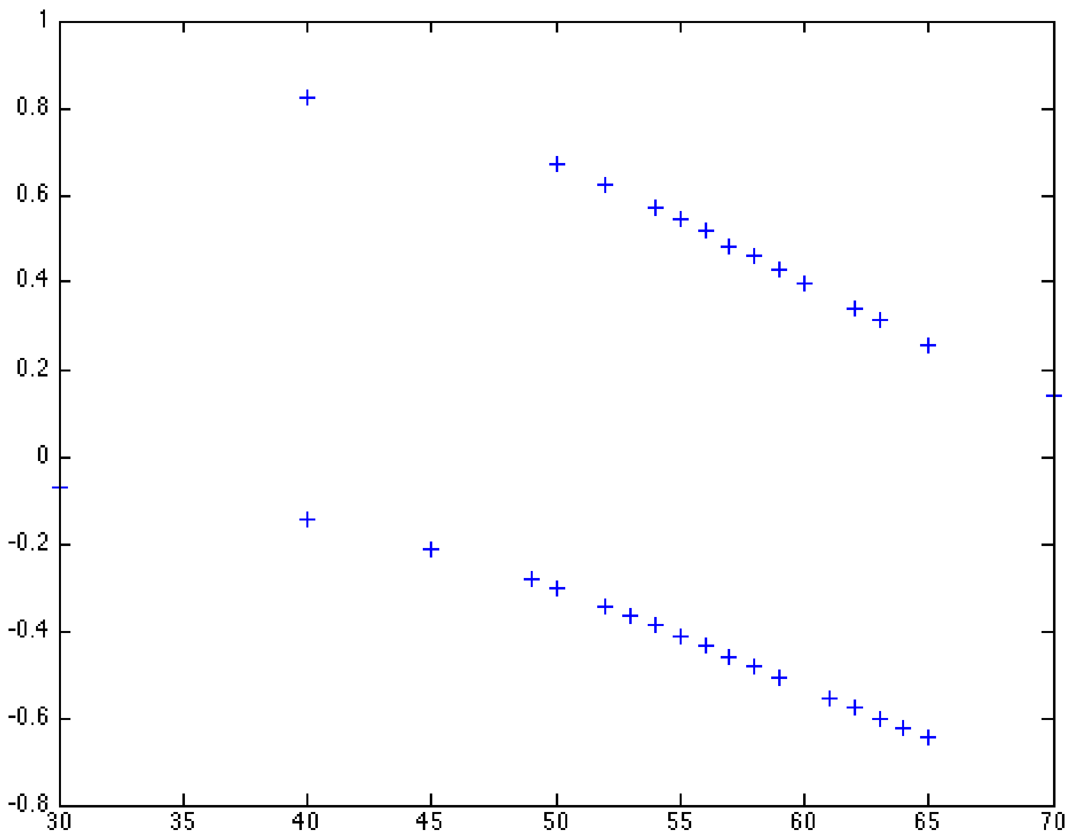


Figure 6: Vanilla sensitivity - 1.67Y maturity - Gamma

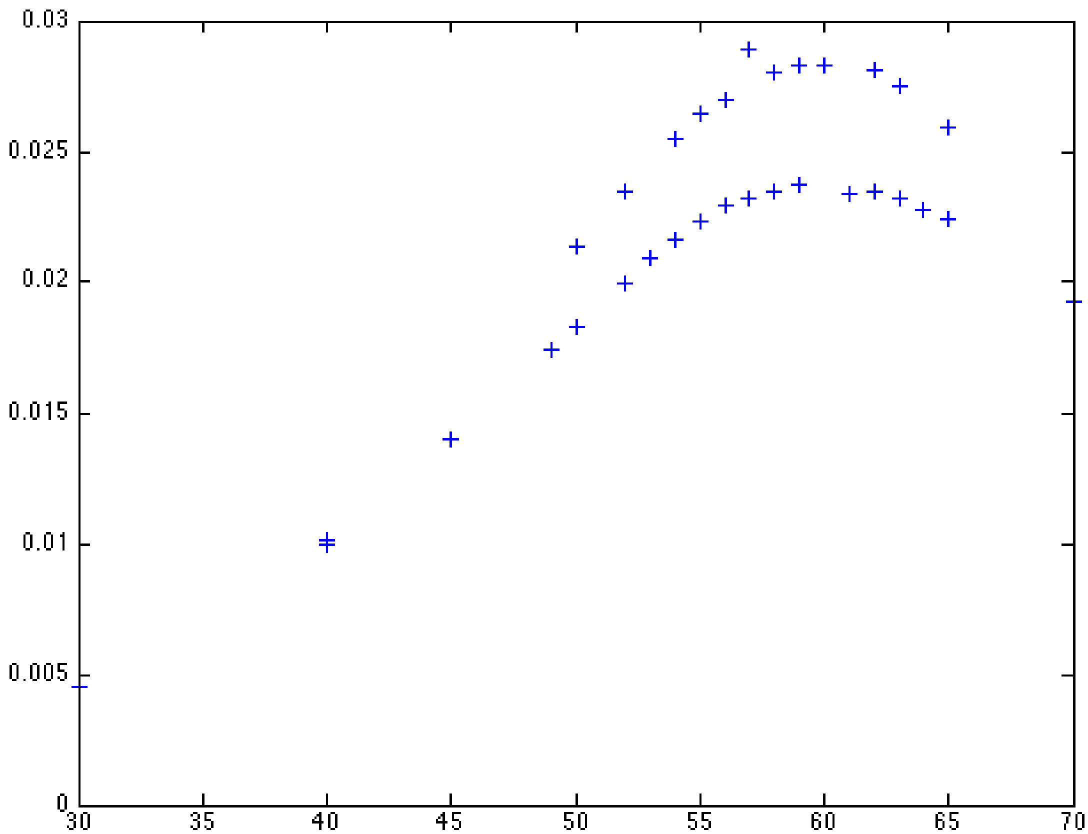


Figure 7: Vanilla sensitivity - 1.67Y maturity - Vega

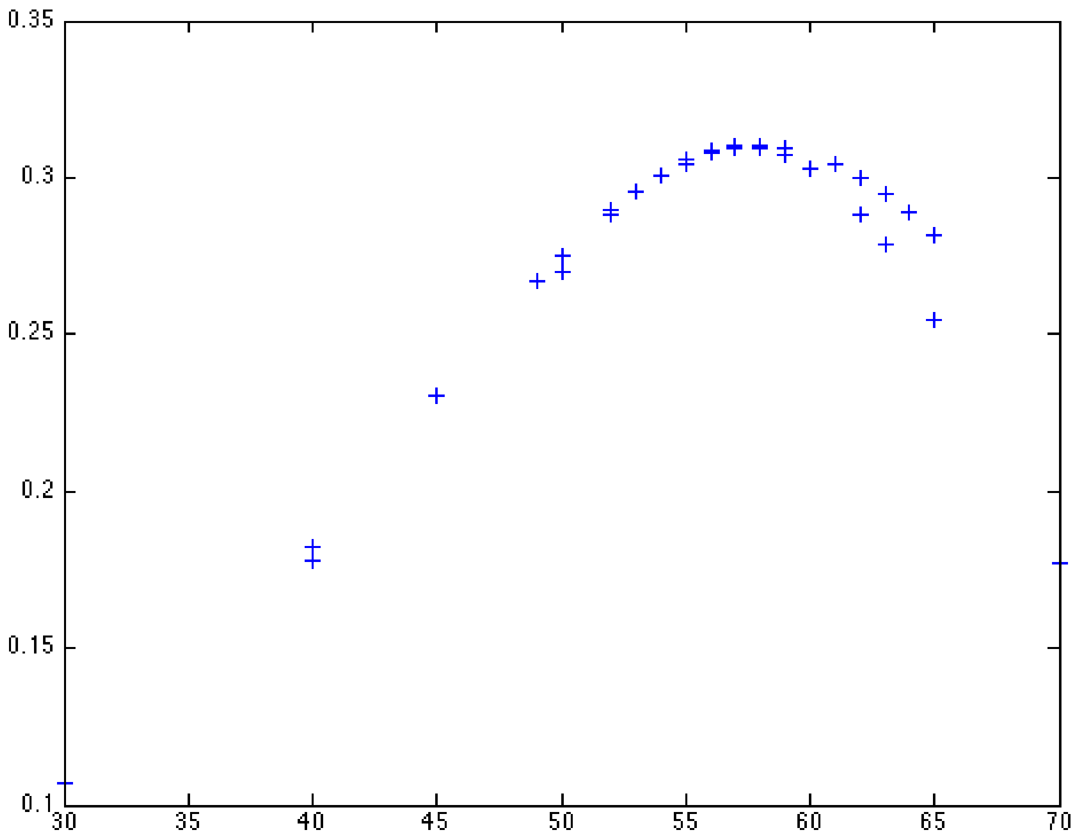
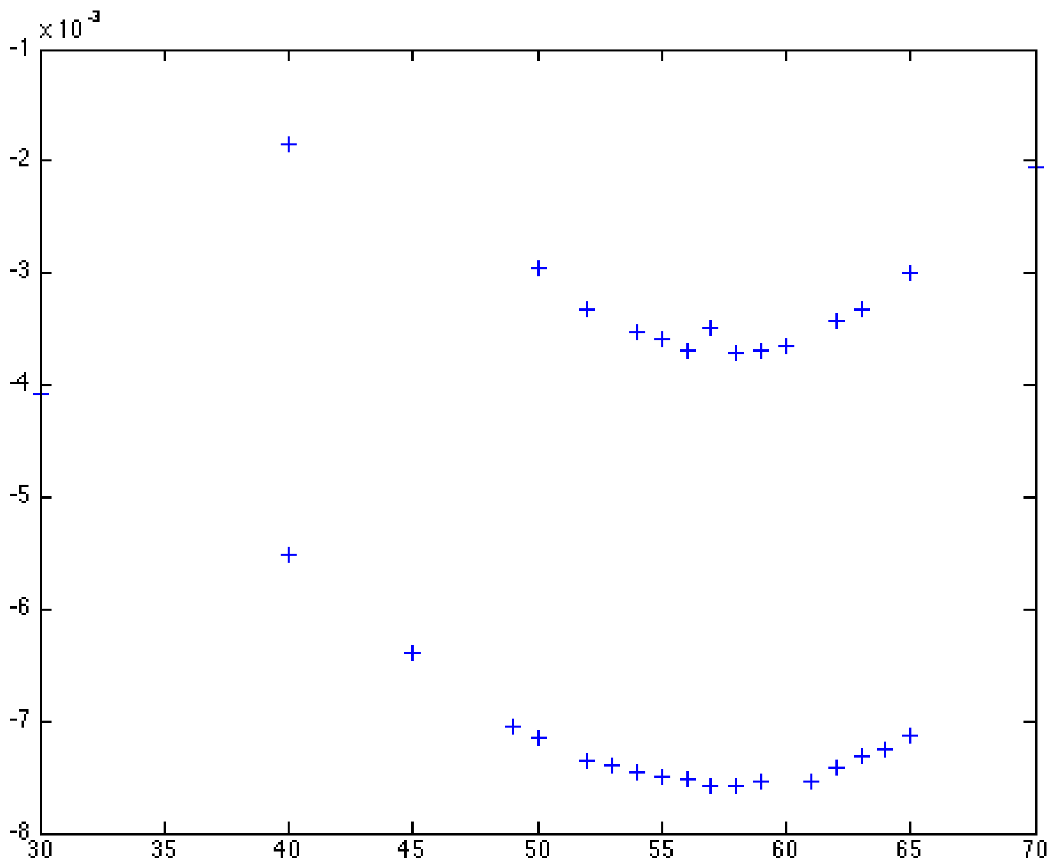


Figure 8: Vanilla sensitivity - 1.67Y maturity - Theta



Strike	Mid	Delta	Gamma	Vega	Theta
40	17.75	0.821457	0.0101153	0.177759	-0.00184063
50	9.225	0.672926	0.0213627	0.270202	-0.00295117
52	7.825	0.624558	0.0234259	0.287893	-0.00331587
54	6.475	0.572553	0.0254934	0.300996	-0.00353029
55	5.85	0.544897	0.0264798	0.30556	-0.00358731
56	5.275	0.516972	0.0269796	0.308512	-0.0036882
57	4.6	0.483997	0.0289306	0.309874	-0.00348814
58	4.225	0.458636	0.028046	0.309364	-0.00371533
59	3.7	0.428903	0.0283208	0.307037	-0.00368955
60	3.26	0.399553	0.028331	0.302897	-0.00365138
62	2.44	0.338978	0.0280753	0.288443	-0.00343549
63	2.09	0.310701	0.0274937	0.27888	-0.00332386
65	1.49	0.254902	0.0259007	0.254443	-0.00300504
70	0.535	0.139653	0.019281	0.177212	-0.00204825
30	0.435	-0.0696315	0.00452007	0.10703	-0.00407274
40	1.255	-0.145915	0.00999129	0.182488	-0.00550987
45	2	-0.212165	0.0139802	0.230357	-0.00640128
49	2.875	-0.281447	0.0174113	0.267017	-0.0070484
50	3.105	-0.300681	0.0183326	0.275046	-0.00714945
52	3.7	-0.342242	0.0199954	0.289429	-0.00735783
53	4.025	-0.364237	0.0209014	0.295453	-0.00739776
54	4.35	-0.387137	0.0216647	0.30059	-0.00745022
55	4.75	-0.410627	0.022284	0.304676	-0.00750474
56	5.15	-0.434871	0.0229173	0.307654	-0.00751463
57	5.6	-0.458668	0.0231912	0.309365	-0.00757023
58	6.1	-0.482883	0.0234709	0.30988	-0.00757762
59	6.6	-0.50779	0.0237657	0.309122	-0.00753373
61	7.75	-0.552745	0.0234051	0.304432	-0.00754757
62	8.325	-0.578145	0.0235024	0.299872	-0.00740737
63	8.95	-0.60078	0.0232339	0.294626	-0.00732316
64	9.65	-0.621715	0.0227951	0.288765	-0.00725076
65	10.35	-0.64308	0.022369	0.281762	-0.00713259

Table 5: Vanilla sensitivities - 1.67Y maturity