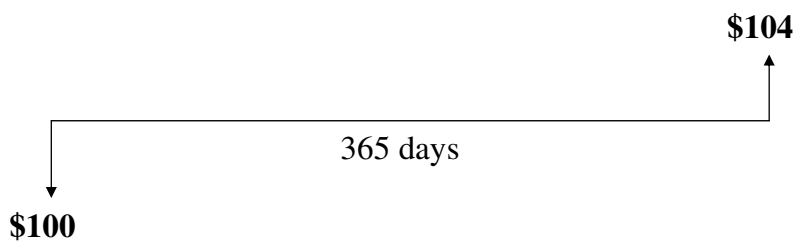
Financial Arithmetic

Simple Interest

$$\text{Total Proceed} = \text{Principal} \times \left(1 + \text{interest rate} \times \frac{\text{days}}{\text{year}}\right)$$

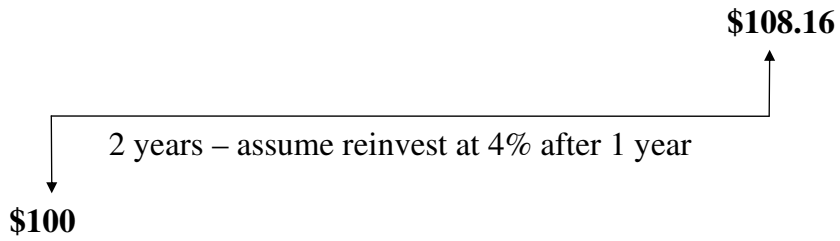
$$\$104 \quad = \quad \$100 \times \left(1 + \quad 4\% \quad \times \frac{365}{365}\right)$$



Compound Interest

$$\text{Total Proceed} = \text{Principal} \times \left(1 + \text{interest rate} \times \frac{\text{days}}{\text{year}}\right)^{N}$$

$$\$108.16 = \$100 \times \left(1 + 4\% \times \frac{365}{365}\right)^{2}$$



Nominal and Effective Rates

- Consider 4% per annum and *quarterly* interest payments
- Assume reinvest at 4%

$$\text{Total Return} = \text{Principal} \times \left(1 + \frac{\text{interest rate}}{n}\right)^{n}$$

$$\$104.06 = \$100 \times \left(1 + \frac{4\%}{4}\right)^{4}$$

Nominal and Effective Rates

- 4% is the nominal rates
- 4.06% is the effective rates

$$\text{effective rate} = \left[\left(1+\frac{\text{nominal rate}}{\text{n}}\right)^{\text{n}} - 1\right]$$

$$\text{nominal rate} = \left[(1+\text{effective rate})^{\frac{1}{\text{n}}} - 1\right] \times \text{n}$$

Example

5% is the nominal interest rate quoted for a 1-year deposit when the interest is paid all at maturity. What is the quarterly equivalent?

$$\left[(1.05)^{\frac{1}{4}} - 1\right] \times 4 = 4.91\%$$

Effective Rates

The interest rate for a 5-month (153-day) investment is 10.2%. What is the effective rates?

$$\text{Effective rate} = \left(1 + 0.102 \times \frac{153}{365}\right)^{\frac{365}{153}} - 1 = 10.50\%$$

$$\text{Effective Rate} = \left(1 + \text{nominal rate} \times \frac{\text{days}}{\text{year}}\right)^{\frac{365}{\text{days}}} - 1$$

Interest Rate

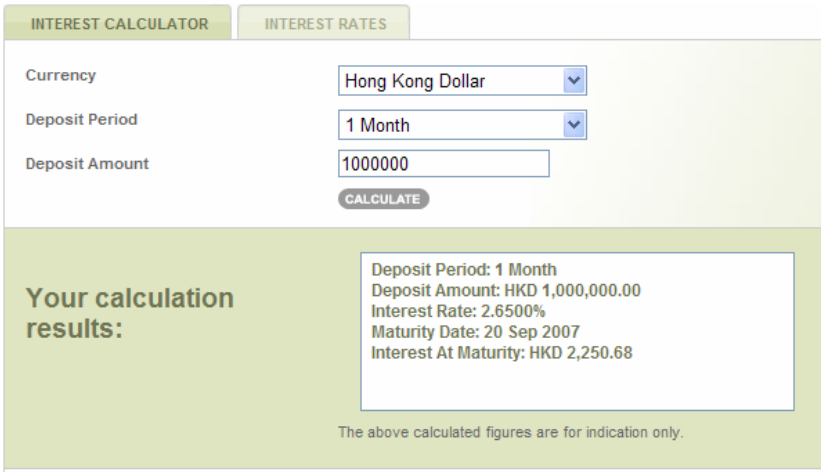
- The period for which the investment/loan will last
- The absolute period to which the quoted interest rate applies
  - 10% for 6-month?
- The frequency with which interest is paid

Example

Deposit period	HK $10,000 to HK $99,999	HK $100,000 to HK $499,999	HK $500,000 to HK $999,999	HK $1,000,000 or above
1 day				2.5000%
1 week	2.5000%	2.5000%	2.5000%	2.5000%
2 weeks	2.5000%	2.5000%	2.5000%	2.5000%
1 month	2.5000%	2.5500%	2.6000%	2.6500%
2 months	2.5000%	2.5500%	2.6000%	2.6500%
3 months	2.5500%	2.6000%	2.6500%	2.7000%
6 months	2.6000%	2.6500%	2.7000%	2.7500%
9 months	2.6500%	2.7000%	2.7500%	2.8500%
12 months	2.7500%	2.8500%	2.9500%	3.0500%

Interest is calculated on the following year basis:
Hong Kong Dollar - 365 days or 366 days (in leap years), Pound Sterling, Singapore Dollar, Thai Baht - 365 days and other currencies - 360 days.

Example – Cont'd



$$\text{Interest At Maturity} = \$1000000 \times \frac{2.65}{100} \times \frac{31}{365}$$

Continuous Compounding

Daily equivalent rate

Equivalent rate with daily compounding for an annual rate of 9.3%

$$\left[(1+0.093)^{\frac{1}{365}}-1\right]\times 365 = 8.894\%$$

Continuously compounded rate

$$= \mathrm{LN}(1.093) = 8.893\%$$

Continuous Compounding

Continuously compounded rate

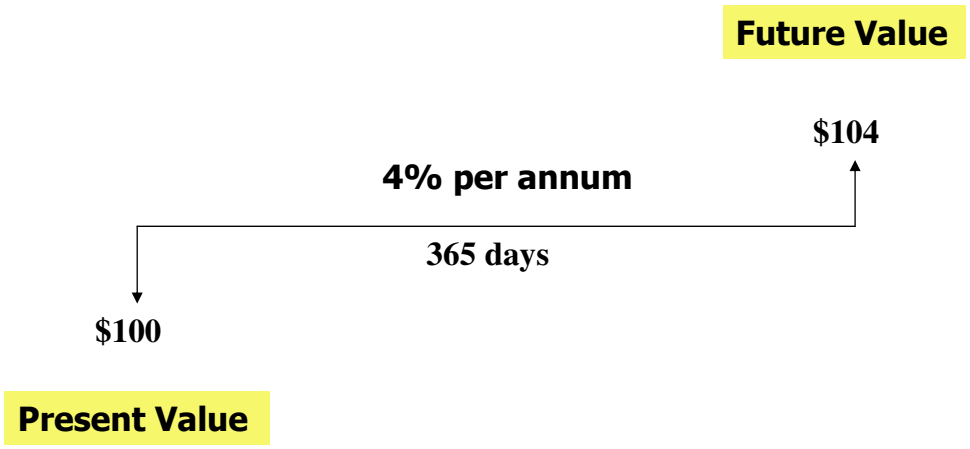
$$= \frac{365}{\text{days}} \times \mathrm{LN}\left(1+\mathrm{i}\times\frac{\text{days}}{\text{year}}\right)$$

where i is the nominal rate for days

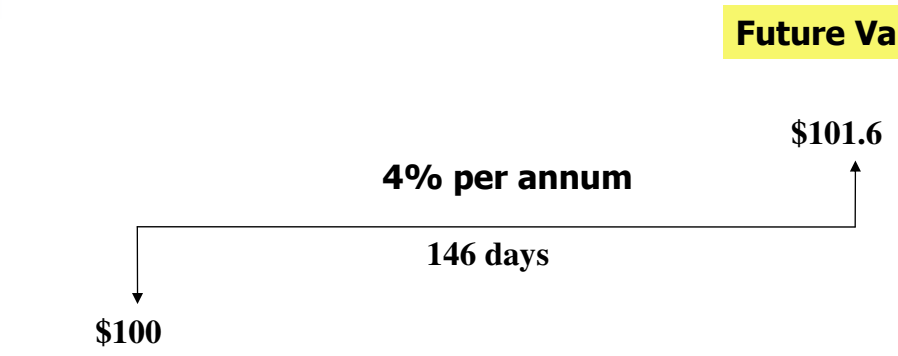
Or,

$$\mathrm{i} = \frac{\text{year}}{\text{days}} \times \left(\mathrm{e}^{\mathrm{r}\times\frac{\text{days}}{365}}-1\right)$$

Future Value/Present Value



Future Value/Present Value



$$100 = \frac{101.6}{\left(1 + 0.04 \times \frac{146}{365}\right)}$$

Future Value/Present Value

For short-term investments

$$FV = PV \times \left(1 + i \times \frac{\text{days}}{\text{year}}\right)$$

$$PV = \frac{FV}{\left(1 + i \times \frac{\text{days}}{\text{year}}\right)}$$

Yield/Rate of Return

For short-term investments

$$\text{yield} = \left(\frac{FV}{PV} - 1\right) \times \frac{\text{year}}{\text{days}}$$

$$\text{effective yield} = \left(1 + \text{yield} \times \frac{\text{days}}{\text{year}}\right)^{\frac{365}{\text{days}}} - 1$$

$$\text{effective yield} = \left(\frac{FV}{PV}\right)^{\frac{365}{\text{days}}} - 1$$

Long-Term Investment

$$FV = PV \times (1+i)^N$$

$$PV = \frac{FV}{(1+i)^N}$$

$$yield = \left(\frac{FV}{PV}\right)^{\frac{1}{N}} - 1$$

Example

I invest $138 now. After 64 days I receive back a total (principal + interest) of $139.58. What is my yield on this investment?

$$yield = \left(\frac{139.58}{138.00} - 1\right) \times \frac{365}{64} = 0.0653 = 6.53\%$$

Discount Factors

$$PV = FV \times \text{Discount Factor}$$

For simple interest

$$\text{Discount Factor} = \frac{1}{1 + i \times \frac{\text{days}}{\text{year}}}$$

For compound interest

$$\text{Discount Factor} = \left(\frac{1}{1+i}\right)^{N}$$

Examples

What is the 3-year discount factor based on a 3-year interest rate of 8.5% compounded annually?

$$\text{discount factor} = \frac{1}{(1+0.085)^3} = 0.7829$$

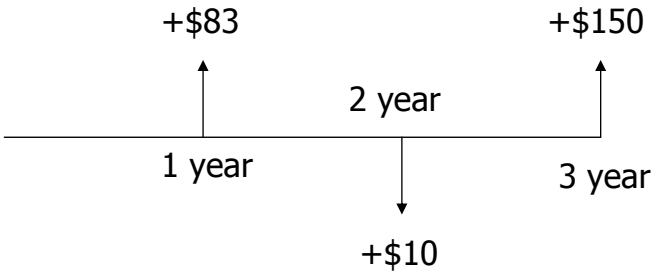
What is the present value of $100 in 3 years time?

$$\$100 \times 0.7829 = \$78.29$$

Net Present Value

NPV = sum of all the present values

Cashflow

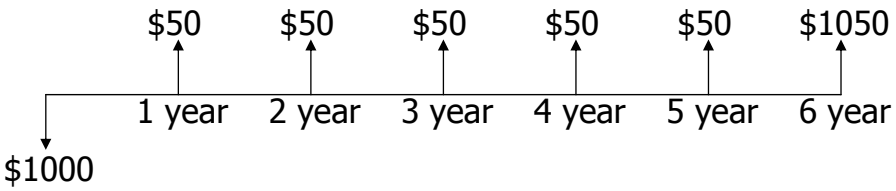


Discounting at rate of 7.5%

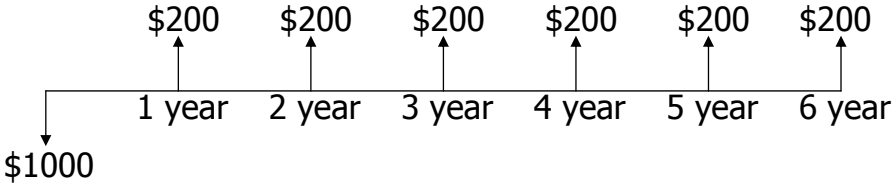
$$NPV = \frac{83}{(1.075)} - \frac{10}{(1.075)^2} + \frac{150}{(1.075)^3}$$

Which is better?

Investment 1:



Investment 2:



Internal Rate of Return

Investment 1:

$$IRR = 5.0000\%$$

$$1000 = \frac{50}{(1+IRR)} + \frac{50}{(1+IRR)^2} + \frac{50}{(1+IRR)^3} + \frac{50}{(1+IRR)^4} + \frac{50}{(1+IRR)^5} + \frac{1050}{(1+IRR)^6}$$

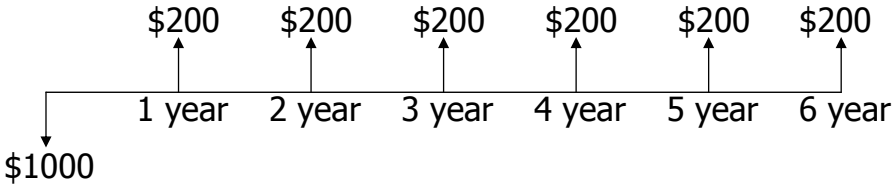
Investment 2:

$$IRR = 5.4718\%$$

$$1000 = \frac{200}{(1+IRR)} + \frac{200}{(1+IRR)^2} + \frac{200}{(1+IRR)^3} + \frac{200}{(1+IRR)^4} + \frac{200}{(1+IRR)^5} + \frac{200}{(1+IRR)^6}$$

Annuity

A regular stream of future cash receipts which can be purchased by an initial cash investment.



$$yield = Internal\ Rate\ of\ Return$$