Curve Construction

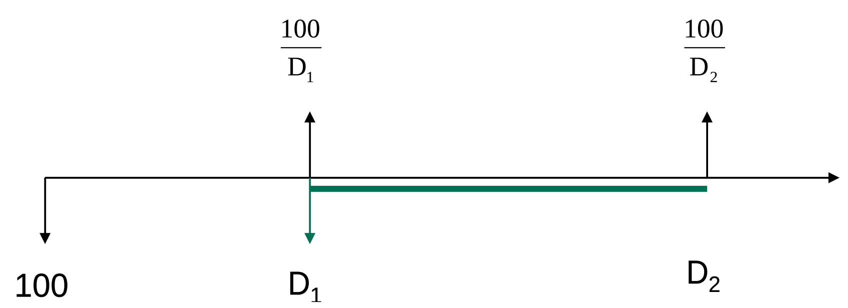
Yield Curve

- Market rate of interest for
  - Theoretical zero coupon instruments that
  - Matures at any future date
- Derived from
  - Prices of real financial instruments that
  - Trade in a liquid market

Input points

- Liquid market instruments do not exist for every possible date in the future
- Benchmark set or key points
  - Cash Rates
    - ON, TN, 3M, 6M, 1Y
  - Swaps Rates
    - 2Y, 3Y, 4Y, 5Y, 7Y, 10Y, 15Y, 20Y, 30Y
  - Liquid number of futures contract

Discount Factor & Forward Rate



$$\text{forward rate} = \left(\frac{\frac{100}{D_2}}{\frac{100}{D_1}} - 1 \right) \times \frac{\text{year}}{\text{period}} = \left(\frac{D_1}{D_2} - 1 \right) \times \frac{\text{year}}{\text{period}}$$

Cash

Cash rates on 8 March 2007 (Thursday, Base date)

ON	0.57%
TN	0.57%
3M	0.70625%

Basis is ACT/360, Number of days to spot is 2.

ON (Key Point 1)
Discount factor for Day 1 (Friday, 9 March). How?

TN (Key Point 2)
Discount factor for Day ?. How?

3M cash rate ended on ? (Day ?). Discount factor is ?

Future

First futures contract March 2007, started on Thursday, 22 March, 2007 or day 14 (Note: 21 March 2007 is a TOK holiday).

Infer a stub rate for the day 14 to such that the 3-month rate implied by the curve equals the 3-month cash rate entered. The reason is that 3m cash is the most liquid cash instrument at the short end.

Interpolate discount factor for 3M using discount factors for 1st and 2nd futures contracts (both discount factors still unknown).

What's another relationship relating discount factors of 1st and 2nd futures contracts?

Exponential Interpolation

Given 2 points x_1 and x_2 and a point x between them.

We assume that (x_1, y_1) and (x_2, y_2) lie on the curve

$$y = ke^{mx} \text{ where } k \text{ and } m \text{ are constants}$$

Solving for k and m,

$$m = \frac{\ln(y_2 / y_1)}{x_2 - x_1}$$

$$k = y_1 e^{-mx_1}$$

What's the implication for ON rates?

Futures

IMM futures price*:

22-Mar-07	99.307	20-Jun-07	99.3082
19-Sep-07	99.2235	19-Dec-07	99.1369
19-Mar-08	99.0504	18-Jun-08	98.968
17-Sep-08	98.8927	17-Dec-08	98.8175

*Assume convexity adjusted.

Mar-07 futures contract start on Thursday, 22-Mar-07 and runs to 20-Jun-07.

Discount factor for 20-Jun-07 ? (Key Point 4)

Futures

IMM futures price*:

22-Mar-07	99.307	20-Jun-07	99.3082
19-Sep-07	99.2235	19-Dec-07	99.1369
19-Mar-08	99.0504	18-Jun-08	98.968
17-Sep-08	98.8927	17-Dec-08	98.8175

*Assume convexity adjusted.

Jun-07 futures contract start on Wednesday, 20-Jun-07 and runs to 19-Sep-07.

Discount factor for 19-Sep-07 ?

Similar for other futures contracts

Swaps

3-year swap priced at 1.05625% against 6-month LIBOR. Spot, at 2 business days, Monday, 12 March 2007.

Date	**Day Count**		**Discount factor**
12 Mar 07 (Spot)	4	D_0	0.99993667
12 Sep 07	188	D_1	0.996373369
12 Mar 08	370	D_2	0.992289654
12 Sep 08	554	D_3	0.987313667
12 Mar 09	735	D_4	0.981676371
18 Mar 09 (futures)	741		0.981483207
14 Sep 09	921	D_5	?
12 Mar 10	1100	D_6	?

Swaps

The fixed leg cashflow must satisfy the following:

$$100D_0 = c_1D_1 + c_2D_2 + c_3D_3 + \ldots + c_5D_5 + (100 + c_6)D_6$$

D_5 and D_6 can be estimated by:

$$D_5 = 0.981483207 \times k^{180}$$
$$D_6 = 0.981483207 \times k^{(180+179)}$$
$$\text{where k is the constant overnight discount factor}$$

Solving iteratively, k = 0.999963597