Intrusive Probe Measuring Technique in Turbomachinery
Report

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1 Abstract

We examined with an invasive method the flow inside of a LTG (Lufttechnische Gesellschaft, Stuttgart) turbomachine which contains one stage. As a measuring tool a cobra-shaped 5-hole probe has been used which allows to analyse the total pressure, static pressure, Mach number and the two angles pitch and yaw. It has been inserted right after the stator. With an actuator the probe has been moved automatically to 20 points. This has been repeated for a range of 20°. All data were stored in LabView which has controlled the measuring tool too. In Matlab we anaysed the data and plotted them in 2D graphics.
The flow after the stator is highly unsteady. Even tough the measured steady data were expressive. By averaging over the massflow some unsteady influences were removed. The effect of the boundary layer around the blade was visible on most of the plots and also the radial pressure difference due to the rotating fluid.
The experiment showed that with this setup it is possible to improve the efficiency of blades by analysing the pressure and velocity distribution.

Contents

2 Introduction

To expand or compress a fluid in a turbomachine there is always a rotating row of blades combined with a stator, which is used to redirect the flow. The efficiency of the system is dependent on pressure losses over the blades. To improve the pressure distribution its important to know how it changes inside of the system.

2.1 Experimental setup

We analyze the axial fan LTG from figure 1 which sucks air from the environment and accelerates the axial fllow to a mean flow speed of $20.66\frac{m}{s}$ with a rotor. The engine has a power of 7kW and allows more than 4000 revolutions per minute.

To measure the pressure inside of the tube we use an invasive method.The probe is placed after the stator. It is connected to a traversing sledge with actuator which allows to move the probe radially and in addition to change the yaw angle φ.

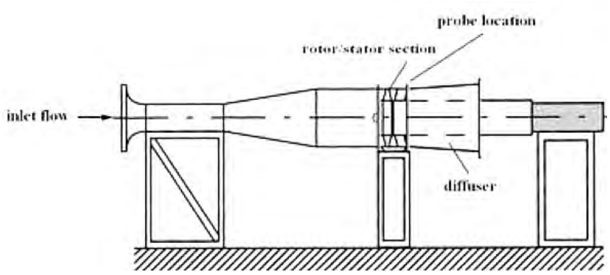


Figure 1: Sketch of the used LTG turbomachine with the measuring tool [1]

The probe is cobra-shaped like in figuer 3 and has a diameter of 2mm. At each position it moves till the front hole faces the flow. The hole facing the flow gives us the total pressure. With two oposing holes one can calculate the velocity and the direction of the flow shown in figure 4.

$$P_{tot} = P_{static} + \frac{1}{2}\rho v^2 \tag{1}$$

3 Method of attack

After turning on the power of the engine we measured the number of revolutions. This has been done with a stroboscope which enlighted the rotor. We changed its frequency until the system seemed to be inoperative. In other words until the rotor looked like if it was not moving. In this case the frequency of the light is the same as the one of the blades.

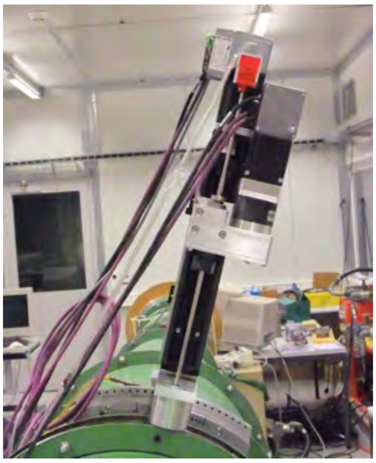


Figure 2: Traversing sledge with actuator to change the radial position and yaw angle φ of the probe

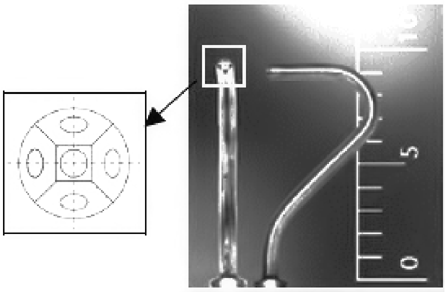


Figure 3: Cobra-shaped probe with 5 holes for measuring static, total pressure and the direction of the flow [1]

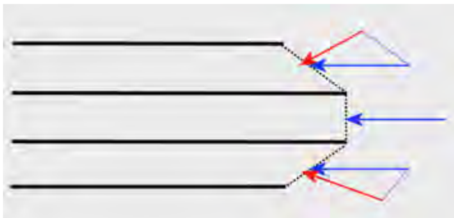


Figure 4: 2D model of the probe where the front hole is facing the flow [1]

The measurement of the pressure can be started by LabView. The actuator moves automatically in radial direction. The distance between two measuring points is $5.5mm$. After getting all data at one yaw angle the probe needs to be moved manually to the next angle where the measurement starts all over again. The evaluated range is $20°$ with a step size of $1°$. The data has been saved in a tex- file and is analyzed in MATLAB.

4 Results

4.1 Pressures

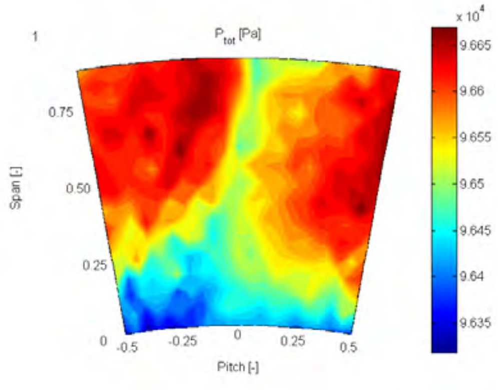


Figure 5: Total Pressure measured over the stator blade

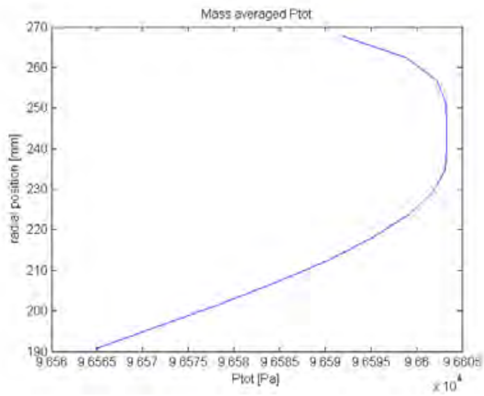


Figure 6: Massaveraged plot of the total pressure

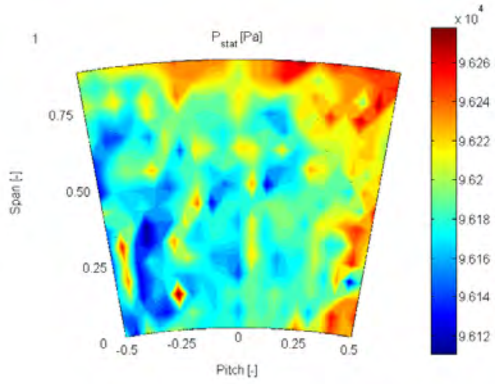
Figure 5 shows the total pressure over the evaluated area. In the middle of the picture at pitch angle 0 is a line of low pressure. At this place there is the stator blade. At the wall of the blade the velocity of the fluid is zero as a direct consequence of friction.

The lowest pressure is in the region between 0 and 25%. This can be explained again by friction at the inner tube. In addition another phenomenon takes influence. Due to the rotating fluid the pressure gets bigger closer to the wall and smaller around the axis.

Averaging over the massflow gives a more precise result, because it considers the different velocities and the varying cross-section surface. Figure 6 shows that the extremum is in the upper third of the radius. In this area the influence of the inner tube and the outer wall has its minimum.

The variations of static pressure are much smaller than the ones of total pressure. The maximum difference is roughly 18Pa. The influence of the blade is not obvious, but a small compression side on the left and a suction side on the right is visible. It can be observed that there is a small gradient from the wall directed to the axis, which means that the fluid gets radially compressed.

The massaveraged static pressure has its minimum at the inner tube and reaches its maximum at the wall. This has to do with the rotating fluid, which increases the pressure at the outer wall.



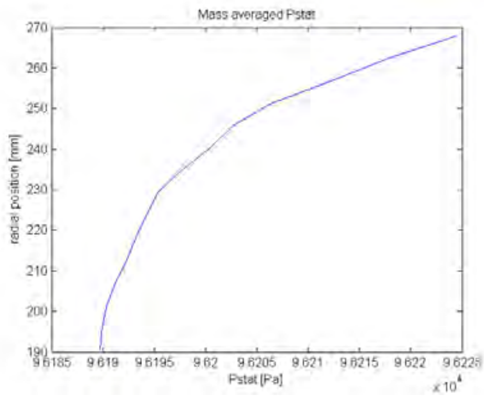
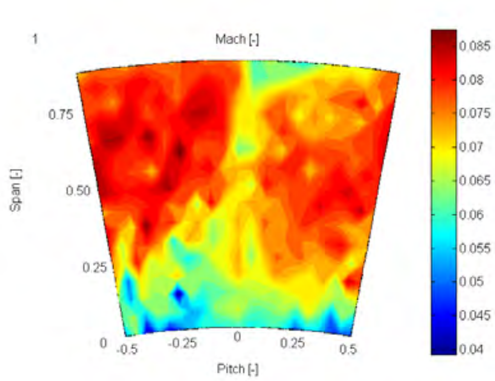


Figure 7: Static Pressure measured over the stator blade

Figure 8: Massaveraged plot of the static pressure

4.2 Mach number

In the visulisation of the Machnumber in figure 9 over the segment the influence of the blade is visible again through the turbulences which decreases the velocity. Due to the boundary layer the velocity close to the stator is much smaller than between two blades. The same influence has the inner tube which explains the radial gradient.



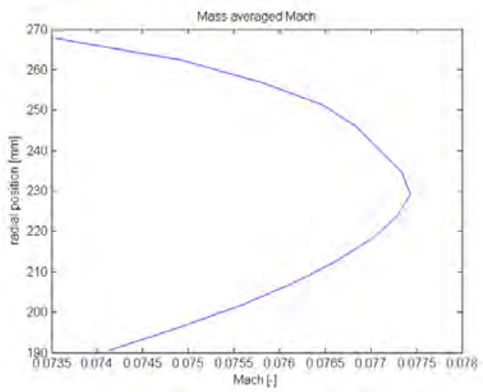


Figure 9: Calculated Machnumber in the evaluated range

Figure 10: Massaveraged plot of the Machnumber

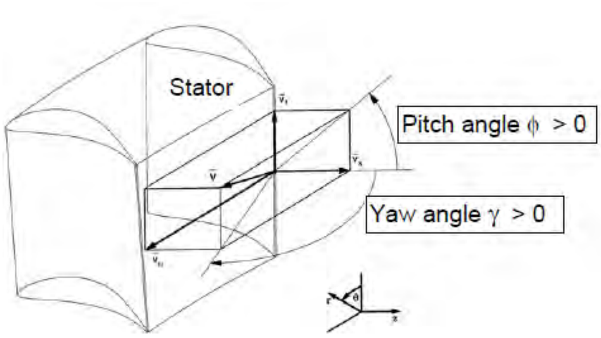


Figure 11: Sketch of the flow angles and velocities [1]

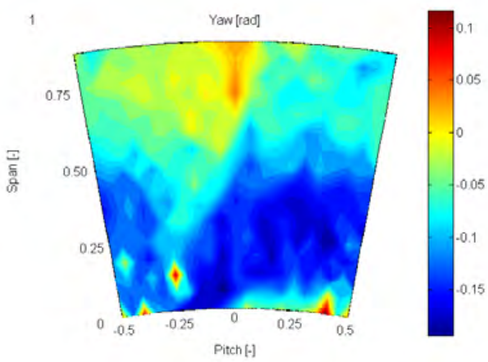


Figure 12: Yaw angle

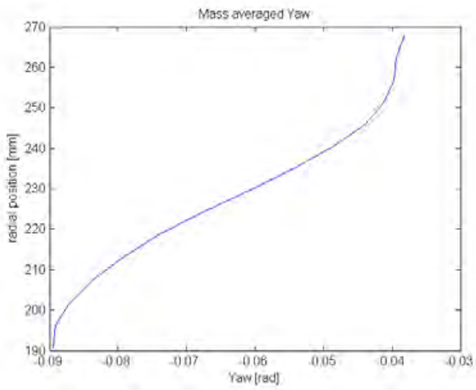


Figure 13: Massaveraged plot of the yaw angle

4.3 Angles

Figure 11 shows the two flow angles which have been measured and will be discussed in the following part.

The yaw angle γ is getting bigger with the distance from the inner tube. It gets its maximum at the outer wall close to the blade. It correlates with the tangential velocity. So the flow follows best the turbine blade as expected at the blade itself.

The influence of the blade on the pitch angle φ is slightly visible in figure 14. On the left of the stator the fluid is more redirected than on the right side. Tendentially the deviation is lower close to the outer wall. The pitch angle also correlates with the radial velocity.

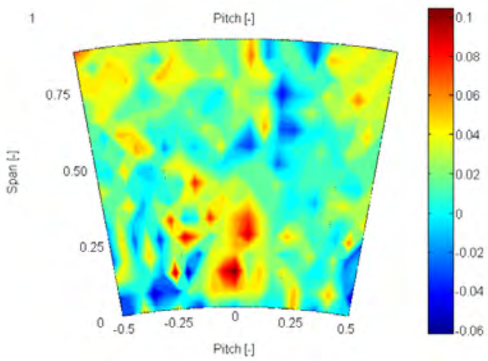


Figure 14: Pitch angle

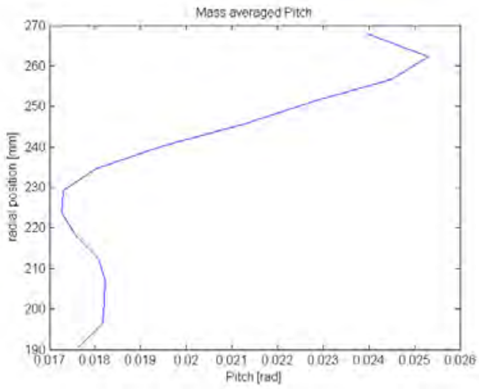


Figure 15: Massaveraged plot of the pitch angle

4.4 Velocities

The visualisation in figure 16 is quite similar to the one of the Machnumber in figure 9. Again the velocity in flow direction is smaller close to blade and getting bigger with the distance. The minimum is around the inner tube. The explanation ist the same as for the small Machnumer in this area. Due to friction the fluid is decelerated. This visualisation also correlates with the one of total pressure. In other words the total pressure gets its maximum where the velocity is at the maximum. This results of formula 1 which points out the relation between pressure and velocity.

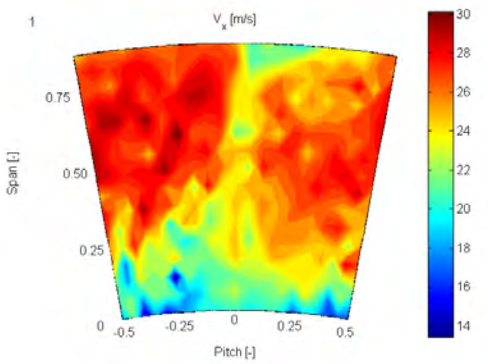


Figure 16: Velocity in x-direction

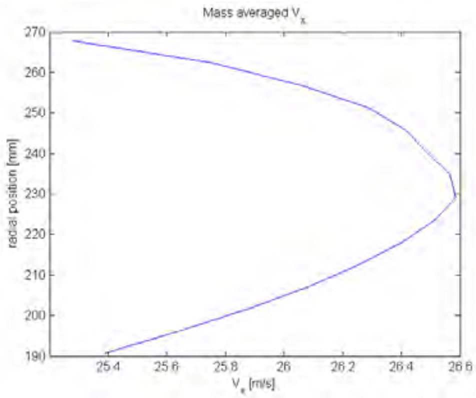


Figure 17: Massaveraged plot of the velocity in x direction

The tangential and the radial velocity are small compared to the axial speed. Since the goal is to have the same inlet and outlet velocity the result is fine.

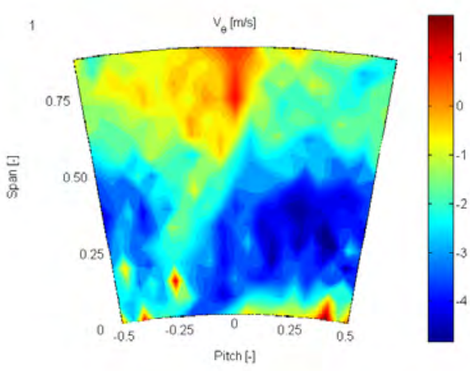


Figure 18: tangential velocity

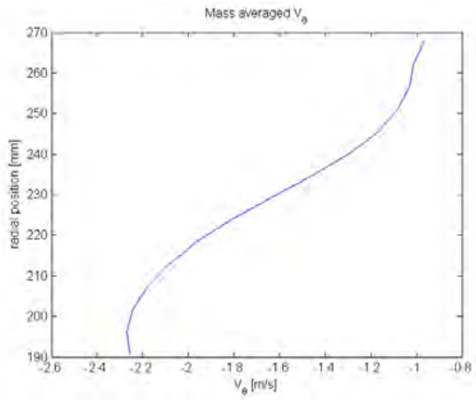


Figure 19: Massaveraged plot of tangential velocity

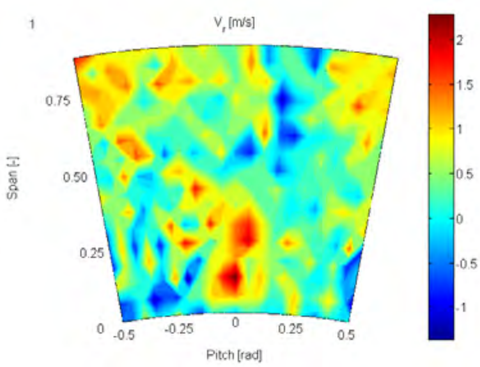


Figure 20: radial velocity

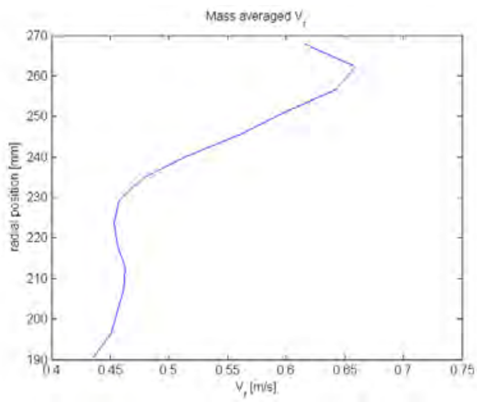


Figure 21: Massaveraged plot of the radial velocity

4.5 Velocity triangles

Table 1 shows that the deviation of the flow is higher than the angle of the blade. In the upper part of the stator the fluid gets less redirected.

Span	Yaw angle measured [°]	Yaw angle blade [°]
10%	-5.1392	-0.5
30%	-4.8716	4
50%	-3.8508	6
70%	-2.6410	8
90%	-2.2597	9

Table 1: Measured yaw angle and yaw angle of the blade

Figure 23 shows the velocity components of the fluid after passing the stator. In figure 22 the inlet and outlet velocity triangle are plottet together. The red vectors V_{2r} and V_{1r} are relative to the rotor. The vector C_x is both times at the same size because this was assumed in our calculation. The other assumptions was optimal flow with no deviation and incidence angle.

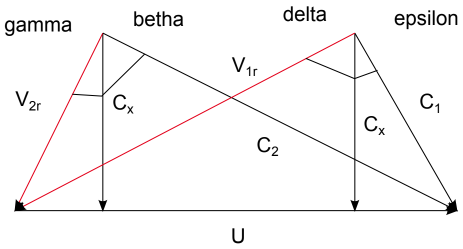


Figure 22: Velocity triangles Rotor and Stator inlet

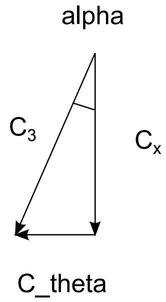


Figure 23: Stator Outlet velocity triangle

Span	α [°]	β [°]	γ [°]	δ [°]	ϵ [°]
10%	-0.5	51	-30.9134	32.4601	0
30%	4	45	-17.9594	34.0533	0
50%	6	41.5	-9.2143	35.8483	0
70%	8	39.5	-4.6183	36.6329	0
90%	9	37	1.2248	37.7733	0

Table 2: measured angles compared to the outlet angles

Span	C_1 [m/s]	C_2 [m/s]	C_3 [m/s]	V_{1r} [m/s]	V_{2r} [m/s]	U [m/s]	C_x
10%	25.2060	40.0527	25.2069	29.8732	29.3795	16.0333	25.2060
30%	26.0408	36.8272	26.1043	31.4306	27.3746	17.6000	26.0408
50%	26.5281	35.4201	26.6742	32.7277	26.8749	19.1667	26.5281
70%	27.8840	36.1367	28.1580	34.7475	27.9748	20.7333	27.8840
90%	28.7767	36.0323	29.1354	36.4059	28.7832	22.3000	28.7767

Table 3: Measured and calculated velocites after the rotor and stator

5 Conclusion

The experiment illustrated well the phenomenon inside of turbomachine and how important it is to know the pressures, velocities and directions of the flow inside to improve the blades. It also showed how complex the whole setup is to measure those values. Even the flow after the stator is highly unsteady this measurement method gave expressive results without having as much data as in an unsteady technique.
Due to the introduction into turbomachines the main differences between compressor and turbine blades became explicit.

References

[1] Intrusive Probe Measuring Technique in Turbomachinery. http://www.ifd.mavt.ethz.ch/education/Lectures/exp_methods/Handouts/Handouts2010/Lab_LTG_2010; visited on December 9th 2010.

A Matlab code

```
%---------------------------------------------------------------%
% Pressure Sensing
%---------------------------------------------------------------%

clc
clear all
close all

%-------------------%
% Reading input file
filename = 'C:\Users\workstation\Documents\My Dropbox\Studium\experimentelle↙
Methoden\Pressure Sensing\Bericht\MATLAB\5HP_data_points.txt';

d = dir([filename]);
if length(d)>0    % the file exists
 disp([ ' reading: ' filename]);

 fid = fopen(filename,'r');
 [line1,count] = fscanf(fid,'%s',[1,32])

 col = 18; % number of columns

 [A,count] = fscanf(fid,'%g',[col,inf]); % reading parameters into Matrix A
 fclose(fid);
end

A=A';

%---------------------------------------------------------------%
% constants
%---------------------------------------------------------------%
T=22+273.15; % Table 2 of experimental explanation
R=287;
k=1.4;
m_dot=3.3;
%density=3.3/(pi*59.39*0.25^2/4);
%v_mean=20.66;
dRi=5.5;

%---------------------------------------------------------------%
% Post Processing
%---------------------------------------------------------------%

% polar coordinates -> cartesian coordiantes
Theta=10-reshape(A(:,3),16,[]);
Theta=pi*Theta./180; % Grad -> Radiant
radius=300.5-reshape(A(:,4),16,[]);

[x,y]=pol2cart(Theta,radius);

% 2D plot Ptot
figure(1)

Ptot=reshape(A(:,8),16,[]);
contourf(y,x,Ptot,30)
```

```
hold on;
contour(y,x,Ptot,30)
axis off

text(-60,277,'1')
text(-55,183+23.5*3,'0.75')
text(-50,183+23.5*2,'0.50')
text(-45,183+23.5,'0.25') % (277-183)*25%/100=23.5
text(-40,183,'0')

text(-32,181,'-0.5','HorizontalAlignment','center')
text(-32/2,183,'-0.25','HorizontalAlignment','center')
text(0,185,'0','HorizontalAlignment','center')
text(32/2,183,'0.25','HorizontalAlignment','center')
text(32,181,'0.5','HorizontalAlignment','center')

text(0,175,'Pitch [-]','HorizontalAlignment','center')
text(0,277,'P_{tot} [Pa]','HorizontalAlignment','center')
text(-65,183+23.5*2,'Span [-]','Rotation',90,'HorizontalAlignment','center')

axis([-50 50 180 280])
colorbar


% 2D plot Pstat
figure(2)
Pstat=reshape(A(:,9),16,[]);
contourf(y,x,Pstat,30)
hold on;
contour(y,x,Pstat,30)
axis off

text(-60,277,'1')
text(-55,183+23.5*3,'0.75')
text(-50,183+23.5*2,'0.50')
text(-45,183+23.5,'0.25') % (277-183)*25%/100=23.5
text(-40,183,'0')

text(-32,181,'-0.5','HorizontalAlignment','center')
text(-32/2,183,'-0.25','HorizontalAlignment','center')
text(0,185,'0','HorizontalAlignment','center')
text(32/2,183,'0.25','HorizontalAlignment','center')
text(32,181,'0.5','HorizontalAlignment','center')

text(0,175,'Pitch [-]','HorizontalAlignment','center')
text(0,277,'P_{stat} [Pa]','HorizontalAlignment','center')
text(-65,183+23.5*2,'Span [-]','Rotation',90,'HorizontalAlignment','center')

axis([-50 50 180 280])
colorbar

% 2D plot Mach number
figure(3)
hold on
Mach=reshape(A(:,10),16,[]);
```

```
contourf(y,x,Mach,30)
hold on;
contour(y,x,Mach,30)
axis off

text(-60,277,'1')
text(-55,183+23.5*3,'0.75')
text(-50,183+23.5*2,'0.50')
text(-45,183+23.5,'0.25') % (277-183)*25%/100=23.5
text(-40,183,'0')

text(-32,181,'-0.5','HorizontalAlignment','center')
text(-32/2,183,'-0.25','HorizontalAlignment','center')
text(0,185,'0','HorizontalAlignment','center')
text(32/2,183,'0.25','HorizontalAlignment','center')
text(32,181,'0.5','HorizontalAlignment','center')

text(0,175,'Pitch [-]','HorizontalAlignment','center')
text(0,277,'Mach [-]','HorizontalAlignment','center')
text(-65,183+23.5*2,'Span [-]','Rotation',90,'HorizontalAlignment','center')

axis([-50 50 180 280])
colorbar

% 2D plot Yaw angle
figure(4)
hold on
Yaw=reshape(A(:,6),16,[]);
Yaw=pi*Yaw./180;
contourf(y,x,Yaw,30)
hold on;
contour(y,x,Yaw,30)
axis off

text(-60,277,'1')
text(-55,183+23.5*3,'0.75')
text(-50,183+23.5*2,'0.50')
text(-45,183+23.5,'0.25') % (277-183)*25%/100=23.5
text(-40,183,'0')

text(-32,181,'-0.5','HorizontalAlignment','center')
text(-32/2,183,'-0.25','HorizontalAlignment','center')
text(0,185,'0','HorizontalAlignment','center')
text(32/2,183,'0.25','HorizontalAlignment','center')
text(32,181,'0.5','HorizontalAlignment','center')

text(0,175,'Pitch [-]','HorizontalAlignment','center')
text(0,277,'Yaw [rad]','HorizontalAlignment','center')
text(-65,183+23.5*2,'Span [-]','Rotation',90,'HorizontalAlignment','center')



axis([-50 50 180 280])
colorbar
```

```
% 2D plot Pitch angle
figure(5)
hold on
Pitch=reshape(A(:,7),16,[]);
Pitch=pi*Pitch./180;
contourf(y,x,Pitch,30)
hold on;
contour(y,x,Pitch,30)
axis off

text(-60,277,'1')
text(-55,183+23.5*3,'0.75')
text(-50,183+23.5*2,'0.50')
text(-45,183+23.5,'0.25') % (277-183)*25%/100=23.5
text(-40,183,'0')

text(-32,181,'-0.5','HorizontalAlignment','center')
text(-32/2,183,'-0.25','HorizontalAlignment','center')
text(0,185,'0','HorizontalAlignment','center')
text(32/2,183,'0.25','HorizontalAlignment','center')
text(32,181,'0.5','HorizontalAlignment','center')

text(0,175,'Pitch [-]','HorizontalAlignment','center')
text(0,277,'Pitch [-]','HorizontalAlignment','center')
text(-65,183+23.5*2,'Span [-]','Rotation',90,'HorizontalAlignment','center')

axis([-50 50 180 280])
colorbar

% 2D Velocities v_x
figure(6)
hold on
V_x=Mach.*sqrt(k*R*T)./(1.+tan(Pitch).^2.+tan(Yaw).^2).^0.5;
contourf(y,x,V_x,30)
hold on;
contour(y,x,V_x,30)
axis off

text(-60,277,'1')
text(-55,183+23.5*3,'0.75')
text(-50,183+23.5*2,'0.50')
text(-45,183+23.5,'0.25') % (277-183)*25%/100=23.5
text(-40,183,'0')

text(-32,181,'-0.5','HorizontalAlignment','center')
text(-32/2,183,'-0.25','HorizontalAlignment','center')
text(0,185,'0','HorizontalAlignment','center')
text(32/2,183,'0.25','HorizontalAlignment','center')
text(32,181,'0.5','HorizontalAlignment','center')

text(0,175,'Pitch [-]','HorizontalAlignment','center')
text(0,277,'V_x [m/s]','HorizontalAlignment','center')
text(-65,183+23.5*2,'Span [-]','Rotation',90,'HorizontalAlignment','center')

axis([-50 50 180 280])
```

```
colorbar

% 2D Velocities v_r
figure(7)
hold on
V_r=V_x.*tan(Pitch);
contourf(y,x,V_r,30)
hold on;
contour(y,x,V_r,30)
axis off

text(-60,277,'1')
text(-55,183+23.5*3,'0.75')
text(-50,183+23.5*2,'0.50')
text(-45,183+23.5,'0.25') % (277-183)*25%/100=23.5
text(-40,183,'0')

text(-32,181,'-0.5','HorizontalAlignment','center')
text(-32/2,183,'-0.25','HorizontalAlignment','center')
text(0,185,'0','HorizontalAlignment','center')
text(32/2,183,'0.25','HorizontalAlignment','center')
text(32,181,'0.5','HorizontalAlignment','center')

text(0,175,'Pitch [rad]','HorizontalAlignment','center')
text(0,277,'V_r [m/s]','HorizontalAlignment','center')
text(-65,183+23.5*2,'Span [-]','Rotation',90,'HorizontalAlignment','center')

axis([-50 50 180 280])
colorbar

% 2D Velocities v_theta
figure(8)
hold on
V_theta=V_x.*tan(Yaw);
contourf(y,x,V_theta,30)
hold on;
contour(y,x,V_theta,30)
axis off

text(-60,277,'1')
text(-55,183+23.5*3,'0.75')
text(-50,183+23.5*2,'0.50')
text(-45,183+23.5,'0.25') % (277-183)*25%/100=23.5
text(-40,183,'0')

text(-32,181,'-0.5','HorizontalAlignment','center')
text(-32/2,183,'-0.25','HorizontalAlignment','center')
text(0,185,'0','HorizontalAlignment','center')
text(32/2,183,'0.25','HorizontalAlignment','center')
text(32,181,'0.5','HorizontalAlignment','center')

text(0,175,'Pitch [-]','HorizontalAlignment','center')
text(0,277,'V_{\theta} [m/s]','HorizontalAlignment','center')
text(-65,183+23.5*2,'Span [-]','Rotation',90,'HorizontalAlignment','center')
```

```
axis([-50 50 180 280])
colorbar


%---------------------------------------------------------------%
% mass averaged plot
%---------------------------------------------------------------%

radial_pos=radius(:,1)+dRi/2;

% preparation for mass flow
% mean velocity
for i=1:20
    for j=1:15
        V_x_mean(j,i)=(V_x(j,i)+V_x(j,i+1)+V_x(j+1,i)+V_x(j+1,i+1))/4;
    end
end
% mean Ptot
for i=1:20
    for j=1:15
        Ptot_mean(j,i)=(Ptot(j,i)+Ptot(j,i+1)+Ptot(j+1,i)+Ptot(j+1,i+1))/4;
    end
end
% density
density_mean=Ptot./(R*T);


% Ptot
for i=1:20
    for j=1:15
        Ptot_mean(j,i)=(Ptot(j,i)+Ptot(j,i+1)+Ptot(j+1,i)+Ptot(j+1,i+1))/4;

    end
end

Ptot_sum=0;
m_dot_cell_tot=0;
for j=1:15
    for i=1:20
        dA=dRi*pi*1*(2*radius(j,i)+dRi)/360;
        m_dot_cell=dA*density_mean(j,i)*V_x_mean(j,i);
        Ptot_sum=Ptot_sum+Ptot_mean(j,i)*m_dot_cell;
        m_dot_cell_tot=m_dot_cell_tot+m_dot_cell;
    end
    Ptot_mmean(j)=Ptot_sum;
    Ptot_mmean(j)=Ptot_mmean(j)/m_dot_cell_tot;
end

figure(9)
plot(Ptot_mmean,radial_pos(2:16))
title('Mass averaged Ptot')
xlabel('Ptot [Pa]')
ylabel('radial position [mm]')
```

```
% Pstat
for i=1:20
    for j=1:15
        Pstat_mean(j,i)=(Pstat(j,i)+Pstat(j,i+1)+Pstat(j+1,i)+Pstat(j+1,i+1))/4;

    end
end

Pstat_sum=0;
m_dot_cell_tot=0;
for j=1:15
    for i=1:20
        dA=dRi*pi*1*(2*radius(j,i)+dRi)/360;
        m_dot_cell=dA*density_mean(j,i)*V_x_mean(j,i);
        Pstat_sum=Pstat_sum+Pstat_mean(j,i)*m_dot_cell;
        m_dot_cell_tot=m_dot_cell_tot+m_dot_cell;
    end
    Pstat_mmean(j)=Pstat_sum;
    Pstat_mmean(j)=Pstat_mmean(j)/m_dot_cell_tot;
end

figure(10)
plot(Pstat_mmean,radial_pos(2:16))
title('Mass averaged Pstat')
xlabel('Pstat [Pa]')
ylabel('radial position [mm]')


% Mach number
for i=1:20
    for j=1:15
        Mach_mean(j,i)=(Mach(j,i)+Mach(j,i+1)+Mach(j+1,i)+Mach(j+1,i+1))/4;

    end
end

Mach_sum=0;
m_dot_cell_tot=0;
for j=1:15
    for i=1:20
        dA=dRi*pi*1*(2*radius(j,i)+dRi)/360;
        m_dot_cell=dA*density_mean(j,i)*V_x_mean(j,i);
        Mach_sum=Mach_sum+Mach_mean(j,i)*m_dot_cell;
        m_dot_cell_tot=m_dot_cell_tot+m_dot_cell;
    end
    Mach_mmean(j)=Mach_sum;
    Mach_mmean(j)=Mach_mmean(j)/m_dot_cell_tot;
end

figure(11)
plot(Mach_mmean,radial_pos(2:16))
title('Mass averaged Mach')
xlabel('Mach [-]')
ylabel('radial position [mm]')
```

```
% Yaw angle
for i=1:20
    for j=1:15
        Yaw_mean(j,i)=(Yaw(j,i)+Yaw(j,i+1)+Yaw(j+1,i)+Yaw(j+1,i+1))/4;

    end
end

Yaw_sum=0;
m_dot_cell_tot=0;
for j=1:15
    for i=1:20
        dA=dRi*pi*1*(2*radius(j,i)+dRi)/360;
        m_dot_cell=dA*density_mean(j,i)*V_x_mean(j,i);
        Yaw_sum=Yaw_sum+Yaw_mean(j,i)*m_dot_cell;
        m_dot_cell_tot=m_dot_cell_tot+m_dot_cell;
    end
    Yaw_mmean(j)=Yaw_sum;
    Yaw_mmean(j)=Yaw_mmean(j)/m_dot_cell_tot;
end

figure(12)
plot(Yaw_mmean,radial_pos(2:16))
title('Mass averaged Yaw')
xlabel('Yaw [rad]')
ylabel('radial position [mm]')


% Pitch angle
for i=1:20
    for j=1:15
        Pitch_mean(j,i)=(Pitch(j,i)+Pitch(j,i+1)+Pitch(j+1,i)+Pitch(j+1,i+1))/4;

    end
end

Pitch_sum=0;
m_dot_cell_tot=0;
for j=1:15
    for i=1:20
        dA=dRi*pi*1*(2*radius(j,i)+dRi)/360;
        m_dot_cell=dA*density_mean(j,i)*V_x_mean(j,i);
        Pitch_sum=Pitch_sum+Pitch_mean(j,i)*m_dot_cell;
        m_dot_cell_tot=m_dot_cell_tot+m_dot_cell;
    end
    Pitch_mmean(j)=Pitch_sum;
    Pitch_mmean(j)=Pitch_mmean(j)/m_dot_cell_tot;
end

figure(13)
plot(Pitch_mmean,radial_pos(2:16))
title('Mass averaged Pitch')
xlabel('Pitch [rad]')
ylabel('radial position [mm]')
```

```
% V_x
for i=1:20
    for j=1:15
        V_x_mean(j,i)=(V_x(j,i)+V_x(j,i+1)+V_x(j+1,i)+V_x(j+1,i+1))/4;

    end
end

V_x_sum=0;
m_dot_cell_tot=0;
for j=1:15
    for i=1:20
        dA=dRi*pi*1*(2*radius(j,i)+dRi)/360;
        m_dot_cell=dA*density_mean(j,i)*V_x_mean(j,i);
        V_x_sum=V_x_sum+V_x_mean(j,i)*m_dot_cell;
        m_dot_cell_tot=m_dot_cell_tot+m_dot_cell;
    end
    V_x_mmean(j)=V_x_sum;
    V_x_mmean(j)=V_x_mmean(j)/m_dot_cell_tot;
end

figure(14)
plot(V_x_mmean,radial_pos(2:16))
title('Mass averaged V_x')
xlabel('V_x [m/s]')
ylabel('radial position [mm]')


% V_r
for i=1:20
    for j=1:15
        V_r_mean(j,i)=(V_r(j,i)+V_r(j,i+1)+V_r(j+1,i)+V_r(j+1,i+1))/4;

    end
end

V_r_sum=0;
m_dot_cell_tot=0;
for j=1:15
    for i=1:20
        dA=dRi*pi*1*(2*radius(j,i)+dRi)/360;
        m_dot_cell=dA*density_mean(j,i)*V_x_mean(j,i);
        V_r_sum=V_r_sum+V_r_mean(j,i)*m_dot_cell;
        m_dot_cell_tot=m_dot_cell_tot+m_dot_cell;
    end
    V_r_mmean(j)=V_r_sum;
    V_r_mmean(j)=V_r_mmean(j)/m_dot_cell_tot;
end

figure(15)
plot(V_r_mmean,radial_pos(2:16))
title('Mass averaged V_r')
xlabel('V_r [m/s]')
```

```
ylabel('radial position [mm]')


% V_theta
for i=1:20
    for j=1:15
        V_theta_mean(j,i)=(V_theta(j,i)+V_theta(j,i+1)+V_theta(j+1,i)+V_theta(j+1,↙
i+1))/4;

    end
end

V_theta_sum=0;
m_dot_cell_tot=0;
for j=1:15
    for i=1:20
        dA=dRi*pi*1*(2*radius(j,i)+dRi)/360;
        m_dot_cell=dA*density_mean(j,i)*V_x_mean(j,i);
        V_theta_sum=V_theta_sum+V_theta_mean(j,i)*m_dot_cell;
        m_dot_cell_tot=m_dot_cell_tot+m_dot_cell;
    end
    V_theta_mmean(j)=V_theta_sum;
    V_theta_mmean(j)=V_theta_mmean(j)/m_dot_cell_tot;
end

figure(16)
plot(V_theta_mmean,radial_pos(2:16))
title('Mass averaged V_\theta')
xlabel('V_\theta [m/s]')
ylabel('radial position [mm]')


%---------------------------------------------------------------%
% velocity triangles (difference between measured an blade angle)
%---------------------------------------------------------------%

% blade parameters
alpha_10=-0.5*pi/180;
alpha_30=4*pi/180;
alpha_50=6*pi/180;
alpha_70=8*pi/180;
alpha_90=9*pi/180;

betha_10=51*pi/180;
betha_30=45*pi/180;
betha_50=41.5*pi/180;
betha_70=39.5*pi/180;
betha_90=37*pi/180;

r_10=192.4; % between 14, 15
r_30=211.2; % between 11, 12
r_50=230; % between 8, 9
r_70=248.8; % between 5, 6
r_90=267.6; % between 2, 3
```

```
% difference to measured Yaw angle
degree_10=180/pi*((r_10-radial_pos(15))/(radial_pos(14)-radial_pos(15))*(Yaw_mmean↙
(14)-Yaw_mmean(15))+Yaw_mmean(15))
degree_30=180/pi*((r_30-radial_pos(12))/(radial_pos(11)-radial_pos(12))*(Yaw_mmean↙
(11)-Yaw_mmean(12))+Yaw_mmean(12))
degree_50=180/pi*((r_50-radial_pos(9))/(radial_pos(8)-radial_pos(9))*(Yaw_mmean(8)-↙
Yaw_mmean(9))+Yaw_mmean(9))
degree_70=180/pi*((r_70-radial_pos(6))/(radial_pos(5)-radial_pos(6))*(Yaw_mmean(5)-↙
Yaw_mmean(6))+Yaw_mmean(6))
degree_90=180/pi*((r_90-radial_pos(3))/(radial_pos(2)-radial_pos(3))*(Yaw_mmean(2)-↙
Yaw_mmean(3))+Yaw_mmean(3))

difference_10=(r_10-radial_pos(15))/(radial_pos(14)-radial_pos(15))*(Yaw_mmean(14)-↙
Yaw_mmean(15))+Yaw_mmean(15)-alpha_10
difference_30=(r_30-radial_pos(12))/(radial_pos(11)-radial_pos(12))*(Yaw_mmean(11)-↙
Yaw_mmean(12))+Yaw_mmean(12)-alpha_30
difference_50=(r_50-radial_pos(9))/(radial_pos(8)-radial_pos(9))*(Yaw_mmean(8)-↙
Yaw_mmean(9))+Yaw_mmean(9)-alpha_50
difference_70=(r_70-radial_pos(6))/(radial_pos(5)-radial_pos(6))*(Yaw_mmean(5)-↙
Yaw_mmean(6))+Yaw_mmean(6)-alpha_70
difference_90=(r_90-radial_pos(3))/(radial_pos(2)-radial_pos(3))*(Yaw_mmean(2)-↙
Yaw_mmean(3))+Yaw_mmean(3)-alpha_90


%---------------------------------------------------------------%
% calculating metal rotor angles
%---------------------------------------------------------------%

% Stages:
% 1 before Rotor -> 2 behind Rotor -> 3 behind Stator
% Assumption: We took the nearest radius to the percent

rpm=5000;
omega=rpm/60;

% 10%
speed=r_10*10^-3*omega
% stage 3
v_xmean=(r_10-radial_pos(15))/(radial_pos(14)-radial_pos(15))*(V_x_mmean(14)-↙
V_x_mmean(15))+V_x_mmean(15)
v_thetamean=tan(alpha_10)*v_xmean
c_3_10=sqrt(v_xmean^2+v_thetamean^2)
% stage 2
c_2_10=v_xmean/cos(betha_10)
v_2r_10=sqrt(v_xmean^2+(speed-v_xmean*tan(betha_10))^2)
gamma_10=atan((speed-v_xmean*tan(betha_10))/v_xmean)
%stage 3
c_1_10=v_xmean
delta_10=atan(speed/c_1_10)
v_r1_10=speed/sin(delta_10)

% 30%
speed=r_30*10^-3*omega
% stage 3
v_xmean=(r_30-radial_pos(12))/(radial_pos(11)-radial_pos(12))*(V_x_mmean(11)-↙
```

```
V_x_mmean(12))+V_x_mmean(12)
v_thetamean=tan(alpha_30)*v_xmean
c_3_30=sqrt(v_xmean^2+v_thetamean^2)
% stage 2
c_2_30=v_xmean/cos(betha_30)
v_2r_30=sqrt(v_xmean^2+(speed-v_xmean*tan(betha_30))^2)
gamma_30=atan((speed-v_xmean*tan(betha_30))/v_xmean)
%stage 3
c_1_30=v_xmean
delta_30=atan(speed/c_1_30)
v_r1_30=speed/sin(delta_30)

% 50%
speed=r_50*10^-3*omega
% stage 3
v_xmean=(r_50-radial_pos(9))/(radial_pos(8)-radial_pos(9))*(V_x_mmean(8)-V_x_mmean↙
(9))+V_x_mmean(9)
v_thetamean=tan(alpha_50)*v_xmean
c_3_50=sqrt(v_xmean^2+v_thetamean^2)
% stage 2
c_2_50=v_xmean/cos(betha_50)
v_2r_50=sqrt(v_xmean^2+(speed-v_xmean*tan(betha_50))^2)
gamma_50=atan((speed-v_xmean*tan(betha_50))/v_xmean)
%stage 3
c_1_50=v_xmean
delta_50=atan(speed/c_1_50)
v_r1_50=speed/sin(delta_50)

% 70%
speed=r_70*10^-3*omega
% stage 3
v_xmean=(r_70-radial_pos(15))/(radial_pos(14)-radial_pos(15))*(V_x_mmean(14)-↙
V_x_mmean(15))+V_x_mmean(15)
v_thetamean=tan(alpha_70)*v_xmean
c_3_70=sqrt(v_xmean^2+v_thetamean^2)
% stage 2
c_2_70=v_xmean/cos(betha_70)
v_2r_70=sqrt(v_xmean^2+(speed-v_xmean*tan(betha_70))^2)
gamma_70=atan((speed-v_xmean*tan(betha_70))/v_xmean)
%stage 3
c_1_70=v_xmean
delta_70=atan(speed/c_1_70)
v_r1_70=speed/sin(delta_70)

% 90%
speed=r_90*10^-3*omega
% stage 3
v_xmean=(r_90-radial_pos(15))/(radial_pos(14)-radial_pos(15))*(V_x_mmean(14)-↙
V_x_mmean(15))+V_x_mmean(15)
v_thetamean=tan(alpha_90)*v_xmean
c_3_90=sqrt(v_xmean^2+v_thetamean^2)
% stage 2
c_2_90=v_xmean/cos(betha_90)
v_2r_90=sqrt(v_xmean^2+(speed-v_xmean*tan(betha_90))^2)
gamma_90=atan((speed-v_xmean*tan(betha_90))/v_xmean)
```

```
%stage 3
c_1_90=v_xmean
delta_90=atan(speed/c_1_90)
v_r1_90=speed/sin(delta_90)


% output
delta_10=delta_10*180/pi
delta_30=delta_30*180/pi
delta_50=delta_50*180/pi
delta_70=delta_70*180/pi
delta_90=delta_90*180/pi

gamma_10=gamma_10*180/pi
gamma_30=gamma_30*180/pi
gamma_50=gamma_50*180/pi
gamma_70=gamma_70*180/pi
gamma_90=gamma_90*180/pi
```