Local Demagnetizing Tensor Calculation for Arbitrary Non-ellipsoidal Bodies

Xiao Hua Huang and M. Pardavi-Horvath
Institute for Magnetic Research, The George Washington University, Washington, D.C. 20052, U.S.A.

***Abstract*—The distribution of the magnetization in planar ferrite devices is nonuniform due to the nonuniformity of the demagnetizing field. A method for calculating the local demagnetizing tensor for arbitrary 3-dimensional shapes, using the Finite Difference Method (FDM), is developed. The sample is discretized into rectangular elements and the magnetostatic interaction between each pair of elements is computed. The local demagnetizing tensor is obtained as the superposition of all interactions at the given element. With proper boundary conditions the method can be applied to the computation of the local demagnetizing tensor for arbitrarily shaped bodies. The local demagnetizing tensor can be used to calculate the non-uniform distribution in the ferrite, and to optimize the shape and size of the ferrite in the CAD of microwave devices.**

I. Introduction

Due to the recent rapid development of microwave technology, planar ferrites are used widely in non-reciprocal devices, especially in MMIC. Since the performance and losses of these devices (circulators, isolators, phase-shifters, filters, etc) are related to the uniformity of the magnetization of the ferrite elements, for reliable modeling and design of the planar devices in MIC and MMIC, the knowledge of the non-uniformity of the internal fields is essential.

In MMIC technology, the ferrite components are of non-ellipsoidal shapes. Rectangular slabs are used for filters, phase-shifters; circular or hexagonal/triangular disks for circulator. For these shapes, even in the case of a uniform external bias field, $\mathbf{H}_{dc}$, the internal field $\mathbf{H}_i(\mathbf{r})$ is non-uniform due to the deviation of the shape of the ferrite element from the ideal ellipsoid of the rotation [1] [2]. In this case the concept of the ***local*** tensor has to be introduced. For a non-ellipsoidal magnetic body with finite aspect ratio, the inhomogeneity of the internal field is described in the terms of elements of the local demagnetizing tensor, $\bar{\bar{\mathbf{N}}}(\mathbf{r})$:

$$\mathbf{H}_i(\mathbf{r}) = \mathbf{H}_{dc}(\mathbf{r}) - \bar{\bar{\mathbf{N}}}(\mathbf{r}) \cdot \mathbf{M}(\mathbf{r}) \tag{1}$$

In this case the demagnetizing tensor elements depend on the position in the sample, ie $N_{ij} = N_{ij}(\mathbf{r})$.

The goal of the present paper is to calculate numerically the 3-dimensional local demagnetizing tensor elements for the most widelly used shapes of microwave ferrites: rectangular slabs, circular and triangular/hexagonal disks. To calculate the demagnetizing tensor for arbitrary non-ellipsoidal samples, a three dimensional Finite Difference Method is applied. The code for the calculation of the local demagnetizing tensor elements can be used as a subroutine in the micromagnetic problem of computing the distribution of the magnetization direction in the ferrite, or calculating the losses in CAD programs.

II. General Theory

The surface divergence of the magnetization vector gives rise to the demagnetizing field. In order to calculate the demagnetizing field, we assume that the sample has been discretized into cubic elements, and assume that the magnetization vector is located at the center of each cubic cell. Therefore, the magnetic scalar potential due to the j^{th} element cube at the i^{th} element is given by:

$$\Psi_{i(mag)} = \frac{1}{4\pi} \int_{V_j} dV_j \mathbf{M}_j \cdot \nabla_j \left(\frac{1}{r_{ji}}\right) \tag{2}$$

where $\mathbf{M}_j$ is the magnetization vector in j^{th} element cube and $\nabla_j = \hat{x}\partial/\partial x_j + \hat{y}\partial/\partial y_j + \hat{z}\partial/\partial z_j$ in Cartesian coordinates; r_{ji} is the distance between i and j. Equation (2) may be written as:

$$\Psi_{i(mag)} = \frac{1}{4\pi} \left\{ \oint_{S_j} \frac{d\mathbf{S}_j \cdot \mathbf{M}_j}{r_{ji}} - \int_{V_j} dV_j \frac{\nabla_j \cdot \mathbf{M}_j}{r_{ji}} \right\} \tag{3}$$

It is assumed that each of the elements is uniformly magnetized, then $\nabla_j \cdot \mathbf{M}_j = 0$ and

$$\Psi_{i(mag)} = \frac{1}{4\pi} \oint_{S_j} \frac{d\mathbf{S}_j \cdot \mathbf{M}_j}{r_{ji}}. \tag{4}$$

Manuscript received Feburary 18, 1996.
This work was supported in part by a grant from the ARPA/Microwave Ferrite Consortium.

and the demagnetizing field at the i^{th} element due to the j^{th} element is:

$$\begin{aligned}\mathbf{H}^{(j)}_{dem(i)} &= -\nabla_i \Psi_{i(mag)} \\ &= -\frac{1}{4\pi}\nabla_i \oint_{S_j} \frac{d\mathbf{S}_j \cdot \mathbf{M}_j}{r_{ji}} \\ &= \frac{1}{4\pi}\nabla_j \oint_{S_j} \frac{d\mathbf{S}_j \cdot \mathbf{M}_j}{r_{ji}},\end{aligned} \tag{5}$$

or, expressed in the form of tensor:

$$\mathbf{H}^{(j)}_{dem(i)} = \bar{\bar{\mathbf{D}}}_{ij} \cdot \mathbf{M}_j, \tag{6}$$

and $D_{xy} = D_{yx}$, $D_{zx} = D_{xz}$, $D_{yz} = D_{zy}$ due to the reciprocity of the interaction.

The calculation of $\bar{\bar{\mathbf{D}}}_{ij}$ can be performed by numerical integration. To simplify the integrations, the magnetization vector can be set in a certain direction. For example, in order to calculate D_{xx} and D_{xy}, one can let $M_y = M_z = 0$ and $M_x = 1$.

From equation (5) one has:

$$D_{xx} = \frac{H_x}{M_x} = \frac{1}{4\pi}\int \frac{x\ dy\ dz}{r^3}, \tag{7}$$

and

$$D_{xy} = \frac{H_y}{M_x} = \frac{1}{4\pi}\int \frac{y\ dy\ dz}{r^3}. \tag{8}$$

The tensor elements D_{xx} and D_{xy} can also be calculated using the dipole approximation. The interaction field $\mathbf{H}^{(j)}_{dem(i)}$ can be obtained by expanding $1/r_{ij}$ in a Taylor series. The first term in the series is the magnetic dipole field:

$$\mathbf{H}^{(j)}_{dem(i)} = \frac{3(\mathbf{M}\cdot\mathbf{r})\mathbf{r} - \mathbf{M}}{4\pi r^3} \times d^3, \tag{9}$$

where d is the unit length of the cubic element. Thus,

$$D_{xx}(x,y,z) = d^3 \times \frac{(3\ x^2/r^2 - 1)}{4\pi r^3}, \tag{10}$$

and

$$D_{xy}(x,y,z) = d^3 \times \frac{3xy}{4\pi r^5}. \tag{11}$$

There is an error in the dipole approximation method, which can be relatively high for the first neighbor interactions, however, computationally the dipole approximation is much more efficient [4].

The remaining tensor elements can be computed utilizing the symmetry properties of Eqs. (10) and (11), by permutation of variables x, y, and z. For symmetrical shapes there is no need to compute the tensor over the whole volume, but using symmetry transformations as in the case of a rectangular slab, the computation can be done for 1/8 of the volume and the elements transformed according to the symmetry of the system.

The D_{ij} tensor elements characterize the interaction of one cubic cell with the others. The local demagnetizing tensor elements can be calculated from the superposition of all interactions of the cell located at the given point. Assuming that the magnetization vector in each element is $\mathbf{m}_i(\mathbf{r}_i)$ and $|\mathbf{m}_i| = 1$, in order to compute $N^{(i)}_{zz}$, it is assumed that the whole sample is magnetized in z direction, i.e. $m_x = m_y = 0$ and $m_z = 1$, then

$$\begin{aligned}N^{(i)}_{zz} &= -\sum_j H^{(i)}_{dem(i)_z} \\ &= -\sum_j D^{(j)}_{zz}(\mathbf{r}_j),\end{aligned} \tag{12}$$

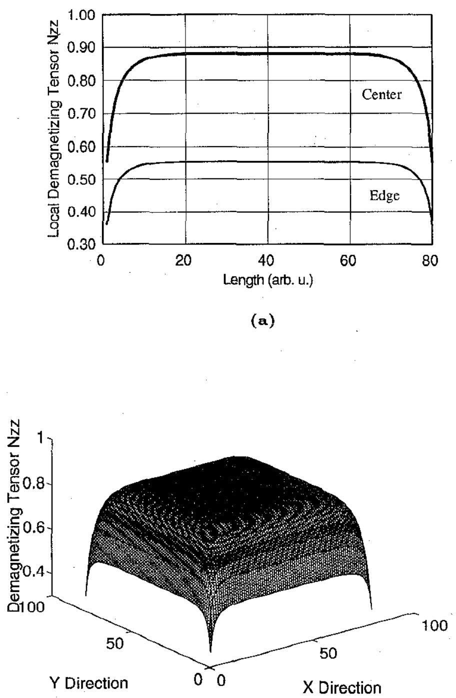
The remaining elements of demagnetizing tensor can be calculated in a similar way.

III. Results and Discussions

Since the ferrite is usually biased along the z direction perpendicular to its plane, thus, N_{zz} is the dominant term of the demagnetizing tensor. For this reason we illustrate the method on the example of N_{zz}.

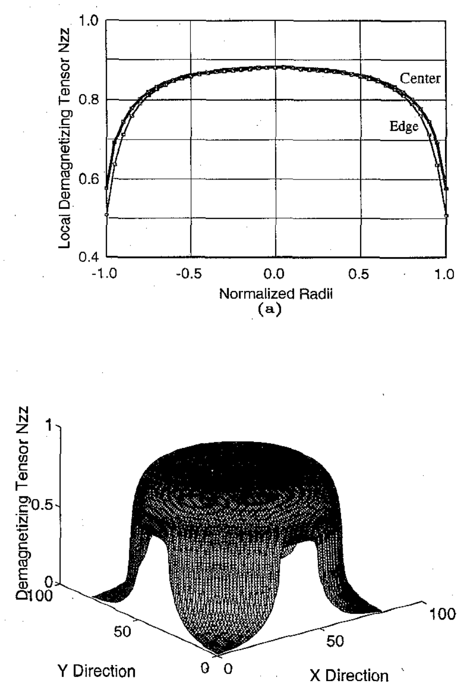
The 3-dimensional distribution of the N_{zz} component of the local demagnetizing tensor over a rectangular sample, discretized into $80 \times 80 \times 4$ elements, is given in Fig.1. The aspect ratio for this sample is quite large, $80/4 = 20$. Still, there is a substantial deviation from the thin film value, $N_{zz} = 1$. The effects of the edges and corners is clearly seen. While in the center $N_{zz} \simeq 0.9$; at the edges it reduced to about 0.55, and at the corners it is 0.35. As a consequence, the direction of the magnetization at the edges and corners will deviate substantially from that at the center of the sample, even in a field of $\mathbf{H}_{dc} = M_s\hat{z}$. For a lower aspect ratio the effect is even more pronounced, for a $10 \times 10 \times 4$ rectangular sample (aspect ratio 2.5) the non-uniform region extends practically all over the sample, and the maximum of N_{zz} at the center of the sample is only 0.65. These computed demagnetizing tensor elements are in excellent agreement with the value measured on single crystalline YIG samples of corresponding shapes [7].

The typical geometry for a microwave circulator is the disk shape, with an aspect ratio on the order of ten. In this case the field is directed along the surface normal of the disk. The demagnetizing field of a hollow cylinder (a "buble domain") has been calculated previously in [5], and it can be applied directly to the case of a circular disk, letting the internal radius r to approach zero. A singularity occurs at $r = 0$, although the $N_{zz}(\mathbf{r})$ element of the tensor should be continuous across the center. The analytical continuation of $N_{zz}(\mathbf{r})$ is used to calculate it at $r = 0$. However, the previous method of computing



(b)

Fig. 1. (a) $N_{zz}(x,y,z)$-Demagnetizing tensor element for a $80 \times 80 \times 4$ rectangular slab;(b) global-view of $N_{zz}(x,y,z)$.



(b)

Fig. 2. (a)$N_{zz}(x,y,z)$-demagnetizing tensor element for a circular disk, aspect ratio $R/T = 10 : 1$, (b) global-view of $N_{zz}(x,y,z)$.

in Cartesian coordinates can be directly applied to any geometry.

The discretization error around the edges can be controlled by choosing an appropriate fine grid. Fig.2 is illustrating the method for the case of a circular disk, having a radius/thickness ratio of 10. The inhomogeneity of $N_{zz}(\mathbf{r})$ is pronounced around the edges. The volume ratio of the sample affected by this inhomogeneity increases with incresing thickness for the same radius; it is decreasing with increasing radius, as expected.

IV. Conclutions

With the introduction of planar geometry and diminishing sizes of the ferrite elements in microwave devices, the problem of the non-uniformity of the internal fields becomes very important. This inhomogeneity is caused by the non-uniform demagnetizing field inside of a non-ellipsoidal body. The local demagnetizing tensor is computed, using the numerical method of Finite Differences. This method is shown to be generally applicable for the calculation of the local demagnetizing tensor for arbitrary shaped non-ellipsoidal bodies.

One of the consequences of the non-uniform magnetization is the occurrence of magnetostatic modes, contributing to the losses by increasing the linewidth and by the substantial in-plane component of the magnetization. Another effect is that bias fields higher than for a uniformly magnetized body are needed for the operation of the device. The results can be applied to the CAD of microwave ferrite devices to optimize the geometry and minimize the losses.

Acknowledgment

The authors express their appreciation to the members of the Institute for Magnetics Research, especially to E. Della Torre for many useful discussions.

References

[1] W.F.Brtown, Jr., *Magnetostatic principles in ferromagnetism*, North-Holland, Amsterdam, 1962.

[2] E. Schlömann, "A sum rule concerning the inhomogeneous demagnetizing field in non-ellipsoidal samples", *J. Appl. Phys.*, **33**, 2825, 1962.

[3] R. I. Joseph and E. Schlömann., "Demagnetizing Field in Nonellipsoidal Bodies.",*J. Appl. Phys*; **36**, 1579, 1965

[4] D. Chen, J. A. Brug and R. B. Goldfarb, "Demagnetizing Factors for Cylinders",*IEEE Trans. Mag.*; **27**, 3601, 1991.

[5] G. Kadar, C.J. Hegedus and E. Della Torre, "Cylindrical Demagnetizing Matrix", *IEEE Trans. Mag.*; **14**, 276, 1978.

[6] Yingdong Yang, "Magnetization Process of Fine Particles", Ph. D dissertation, Carnegie Mellon University, 1989.

[7] G.Zheng, M. Pardavi-Horvath and X. Huang, "Experimental Determination of An Effective Demagnetization Factor for Non-ellipsoidal Geometries", *40thMMM*, Philadelphia, 1995, **EP-21**.