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A Sum Rule Concerning the Inhomogeneous Demagnetizing Field in Nonellipsoidal Samples*

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(Received March 19, 1962) Demagnetizing factor: N

It is shown that the inhomogeneous demagnetizing field occurring in uniformly magnetized, nonellipsoidal samples can be described by a generalized demagnetizing factor. The demagnetizing factor is a tensor and, in general, is different in different parts of the sample. The diagonal sum of this tensor field is unity at all points inside the sample and zero at all points outside the sample.

1. INTRODUCTION

THE concept of demagnetizing factors plays an important role in the theoretical interpretation of many ferromagnetic phenomena. It is well known that the presence of magnetic poles at the surface of a magnetized sample of a ferromagnetic material gives rise to a magnetic field which counteracts the applied field. In uniformly magnetized samples of ellipsoidal shape this "demagnetizing field" is uniform. The relation between the demagnetizing field $\mathbf{H}$ and the magnetization vector $\mathbf{M}$ may conveniently be expressed by the relation

$$H_i = -4\pi \sum_k N_{ik} M_k. \tag{1}$$

Here H_i is the ith component of the demagnetizing field, M_k the kth component of the magnetization vector, and the tensor N_{ik} is known as the demagnetizing factor. It can be shown that the diagonal sum of this tensor is equal to unity.

The purpose of this paper is to show how the concept of a demagnetizing factor can be applied to samples of nonellipsoidal shape and to prove a sum rule similar to the one applicable in ellipsoidal samples. Many experiments, including recent experiments on spin-wave propagation,[1] have been performed using samples with nonellipsoidal shape.

A theorem about the demagnetizing factors of such samples should be very helpful in the theoretical interpretation of these experiments. The theorem to be proved may be formulated as follows: Consider an arbitrarily shaped sample and assume that it is uniformly magnetized consecutively in three mutually perpendicular directions. The sum of the longitudinal demagnetizing fields (i.e., the components along the directions of magnetization) then equals $4\pi M$ for every point within the sample and equals zero for every point outside the sample.

2. DEFINITION OF DEMAGNETIZING FACTOR

Consider an arbitrarily shaped piece of a ferromagnetic material. We shall assume that the sample is uniformly magnetized in an arbitrary direction. Since the demagnetizing field is, in general, nonuniform, the state of uniform magnetization is rather difficult to realize experimentally. This state is approached, however, when the applied field is much larger than the saturation magnetization. In many experiments (such as ferromagnetic resonance experiments) the assumption of uniform magnetization is a very reasonable approximation.

In our discussion it will not be necessary to exclude the possibility that the sample has an arbitrary number of pores. The results are also valid for the compound demagnetizing field of several samples, provided that all are uniformly magnetized in the same direction and provided that the saturation magnetization is the same for all samples.

The demagnetizing field can be expressed as the gradient of a magnetostatic potential ψ,

is this possible because rot(H) = 0

$$\mathbf{H} = \boldsymbol{\nabla}\psi. \tag{2}$$

Since the divergence of the magnetic induction vanishes, the potential satisfies the relation

$$\nabla^2\psi = -4\pi\boldsymbol{\nabla}\cdot\mathbf{M}. \tag{3}$$

The solution of Eq. (3) can be obtained by noting that

$$\boldsymbol{\nabla}^2(1/|\mathbf{r}-\mathbf{r}'|) = -4\pi\delta(\mathbf{r}-\mathbf{r}'), \tag{4}$$

where $\delta(\mathbf{r}-\mathbf{r}')$ is the delta function. According to Eqs. (3) and (4), we have

$$\psi(\mathbf{r}) = \int \frac{(\boldsymbol{\nabla}\cdot\mathbf{M})_{r'}}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}' = -\int_{\text{surface}} \frac{\mathbf{M}\cdot\mathbf{n}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d^2\mathbf{r}'. \tag{5}$$

In the last step of (5) we have used the fact that the divergence of $\mathbf{M}$ vanishes everywhere except at the surfaces of the sample. The integration has to be carried out over all surfaces and $\mathbf{n}(\mathbf{r}')$ is the surface normal pointing towards the exterior (i.e., away from the ferromagnetic material).

The demagnetizing field is now according to Eqs. (2) and (5) in component form:

$$H_i(\mathbf{r}) = \partial\psi/\partial r_i = -4\pi \sum N_{ik}(\mathbf{r}) M_k, \tag{6}$$

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[1] J. R. Eshbach, Phys. Rev. Letters 8, 357 (1962).

where the demagnetizing factor $N_{ik}(\mathbf{r})$ is given by

$$N_{ik}(\mathbf{r})=\frac{1}{4\pi}\int_{\text{surface}}\frac{(r_i'-r_i)}{|r'-r|^3}n_k(\mathbf{r}')d^2\mathbf{r}'. \qquad (7)$$

3. SUM RULE

According to Eq. (7) the diagonal sum of the demagnetizing factor at any point $\mathbf{r}$ is given by a surface integral over the normal component of $-(4\pi)^{-1}\nabla_{r'}|\mathbf{r}'-\mathbf{r}|^{-1}$, where the gradient must be taken with respect to $\mathbf{r}'$.

Using Gauss' theorem we convert this surface integral into a volume integral:

$$\sum_i N_{ii}(\mathbf{r})=-\frac{1}{4\pi}\int\nabla_{r'}{}^2\frac{1}{|r-r'|}d^3r'. \qquad (8)$$

Finally with the help of Eq. (4), we obtain

$$\sum_i N_{ii}(\mathbf{r})=\begin{matrix}1 & \text{if } \mathbf{r} \text{ is inside sample,}\\ 0 & \text{if } \mathbf{r} \text{ is outside sample.}\end{matrix} \qquad (9)$$

This is the mathematical expression of the theorem described verbally at the end of Sec. 1.

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Variation of Dielectric Constant with Voltage in Ferroelectrics and Its Application to Parametric Devices

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A functional relationship between dielectric constant and voltage for a ferroelectric in the paraelectric state is derived, and Fourier capacitance coefficients for different applied voltage functions are computed. It is shown that because the loss tangent of a ferroelectric in the paraelectric state is proportional to frequency, it can be represented in the same way as a varactor; i.e., as a series resistance and capacitance. It is concluded that since the power-handling capability of single-crystalline ferroelectric is virtually unlimited, and since in many cases it has lower loss than a varactor diode, it will soon find wide application in parametric device design.

I. INTRODUCTION

NONLINEAR capacitance voltage elements are playing an increasingly more important role in microwave device design. Varactor diodes, for example, are being extensively used in parametric amplifiers, harmonic generators, and switches. Recently new ferroelectric materials have been found that have excellent nonlinear capacitance voltage variation with low loss and show promise of being useful in frequency multipliers, parametric amplifiers, phase shifters, and modulators. Because of their greater power handling capability and lower loss, ferroelectrics might eventually replace varactor diodes in many applications and be useful in the millimeter wave region where varactor diodes are too lossy to use.

However, this is still in the future, for unlike varactor diodes whose microwave properties have been extensively investigated and characterized, few functional relationships for ferroelectrics have as yet been worked out. In addition, there seems to be quite some disagreement as to the magnitude of the properties of ferroelectrics that have been measured. Many measurements made on bulk pieces do not agree with those made on very small samples. These problems have made it very difficult to determine the nonlinear characteristics of the various ferroelectric materials. And this, in turn, makes it difficult to tell which are the best applications of ferroelectrics.

Thus far, few investigators except Rupprecht *et al.*[1] and Diamond[2] have attempted to give an explicit relationship between dielectric constant and the externally applied field. Rupprecht's theory (based on Slater's theory) will be seen to be valid for small fields only. Diamond developed his theory to apply to polycrystalline material which, while important, is unhandy since it requires a computer. In addition, it only applies to polycrystalline materials which are inherently more lossy.[3] Stern and Lurio[4] applied the phenomenological theory of Devonshire[5,6] to determine the change in dielectric constant with applied field with quite good agreement. Here again, however, small biasing fields were assumed.

We shall follow a treatment based on Slater's and Devonshire's work to obtain a relationship which is

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