Force – Magnetism

Review from last time

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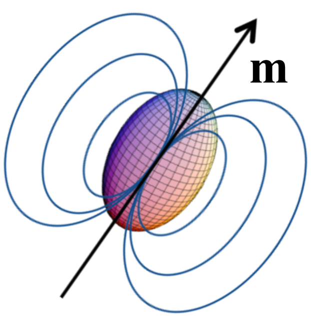
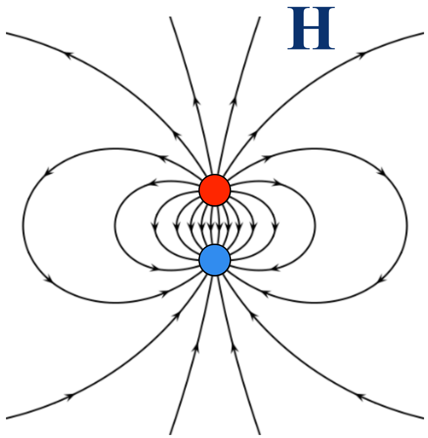
“Few subjects in science are more difficult to understand than magnetism.”

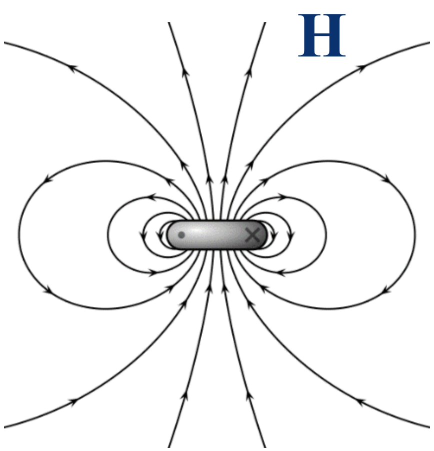
- Encyclopaedia Britannica

The Magnetic Dipole

- The dipole is the most elementary entity in magnetism
- It has a magnetic pole (similar to electric charge) on one end and a second, equal but opposite, pole on the other.
- Generates a magnetic field, with field lines going from the north to the south pole
- An electrical current moving in a loop creates a dipole. Other examples: compass needle, bar magnet.
- Described by a vector $\mathbf{m}$ called magnetic moment
  - $\mathbf{m}$ points from south to north pole
  - The norm of $\mathbf{m}$ is called the dipole strength (in Am^2)







Maxwell's Equations (1884)

- We will only consider magnetics:
  - Ampère's circuit law:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Can be neglected for low frequencies (<10^{14}Hz)

= 0 if there are no field sources in the region of interest:
- No flowing currents
- No permanent magnets
- Magnetostatics

  - Gauss's law for magnetism:

$$\nabla \cdot \mathbf{B} = 0$$

- What is the difference between $\mathbf{H}$ and $\mathbf{B}$?
  - A magnetic field $\mathbf{H}$ (in A/m) gives rise to a magnetic induction or flux density $\mathbf{B}$ (in T) in a medium/material with permeability μ (in N/A^2 or henry/m or Tm/A)

$$\mathbf{B} = \mu \mathbf{H}$$

  - In general, as $\mathbf{B}$ and $\mathbf{H}$ are vectors, μ is a 3x3 tensor and the relationship between $\mathbf{B}$ and $\mathbf{H}$ is nonlinear and anisotropic, even temperature dependent
  - $\mathbf{H}$ does not depend on the medium, whereas $\mathbf{B}$ does.
  - $\mathbf{B}$ is the number of magnetic flux lines cutting through a perpendicular plane of given area.

$$\mathbf{B} = \mu \mathbf{H}$$

- $\mathbf{B}$ is a measure of how a material reacts to a magnetic field $\mathbf{H}$, and we can use the permeability μ to classify magnetic materials
  - Linear and isotropic materials (μ is a positive scalar)

$$\begin{aligned} \mu &= \mu_0 \mu_r \\ \mu_0 &= 4\pi \times 10^{-7}\ \mathrm{Tm/A} \end{aligned}$$

"Permeability of free space"

- μ_r is the relative permeability
  - diamagnetic $\mu_r < 1$ — decrease in flux density — repelled by magnet
  - paramagnetic $\mu_r = 1 \ldots 10$
  - ferromagnetic $\mu_r \gg 10$
  - vacuum/air $\mu_r = 1$

(paramagnetic and ferromagnetic) concentrate flux — attracted to magnet

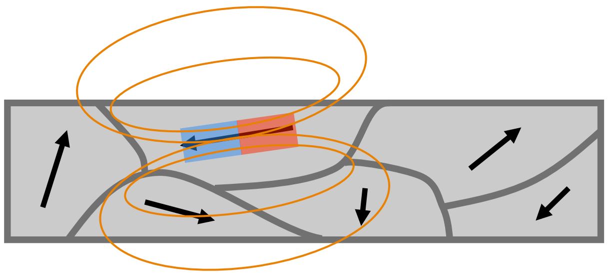
In air, $\mathbf{B}$ and $\mathbf{H}$ are equivalent and often no distinction is made between them (that's why $\mathbf{B}$ is also sometimes called a "magnetic field")

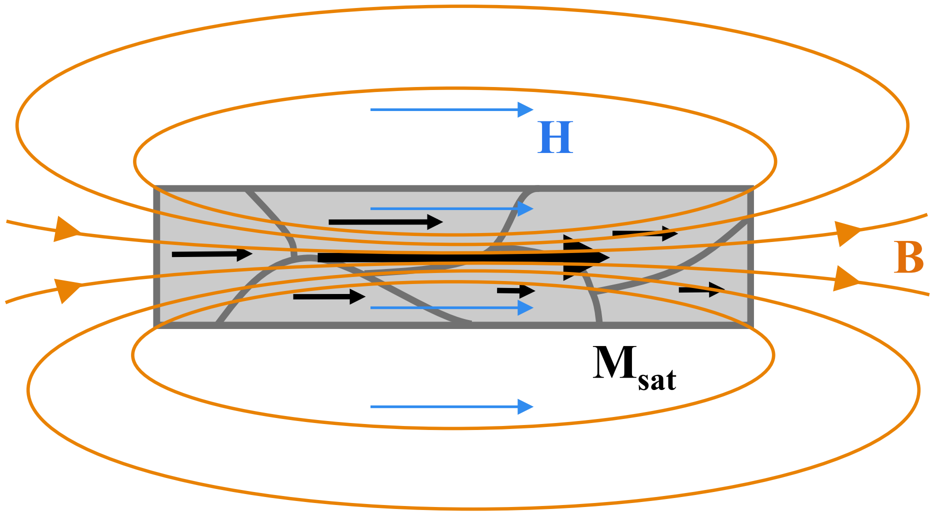
Commonly known as *magnetic materials*. Examples are iron, nickel, cobalt and their alloys

Mostly of interest for us

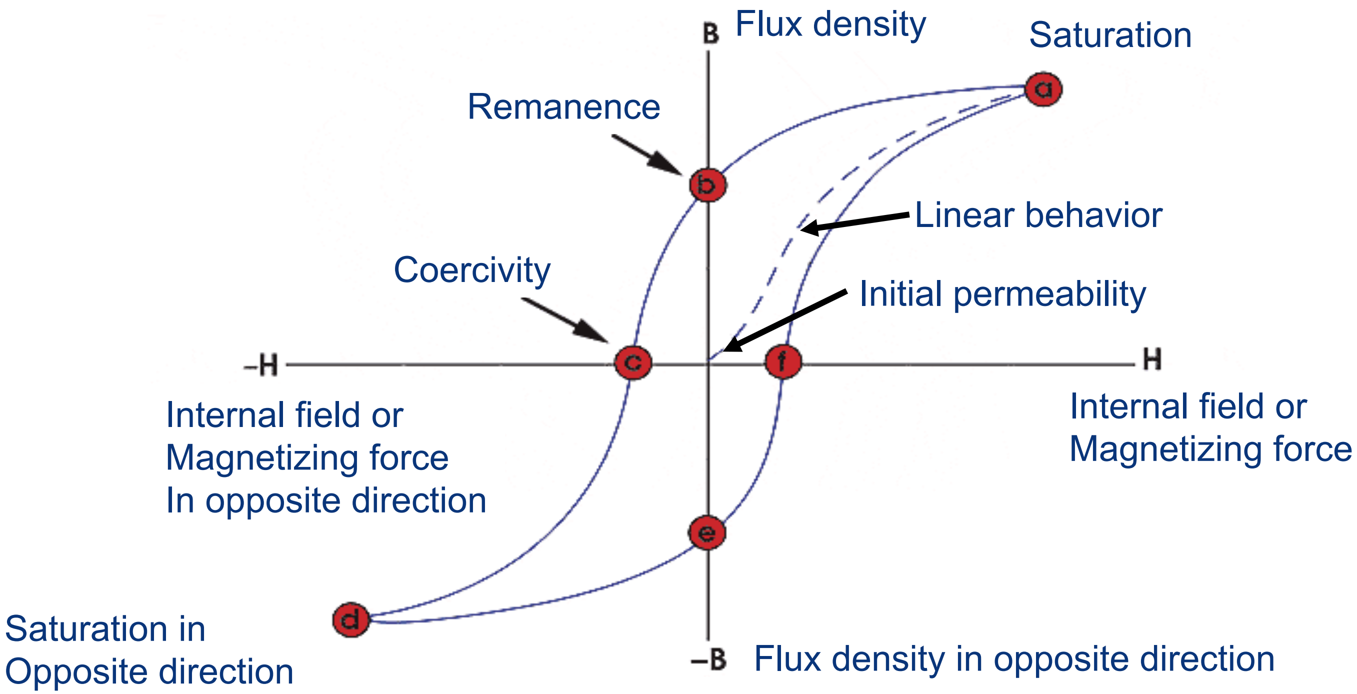
A Closer Look at Ferromagnetism

- What happens when a ferromagnetic material is placed in a sufficiently strong magnetic field?
  - The domain walls move
  - The domains reorient parallel to the applied field and a net field is generated. If all domains are oriented, **saturation** occurs. Increasing the applied field further has no more effect.
  - The phenomenon that the material undergoes when it is placed in the magnetic field is called **magnetization** (and denoted by the vector $\mathbf{M}$).
  - The behavior when the external field is turned off leads to further classification:
    - In *soft magnetic* materials, the domain walls will again reorient at random orientations and no or a weak net field is observed
    - In *hard magnetic* materials, the domain walls will remain reoriented, creating a permanent magnet.



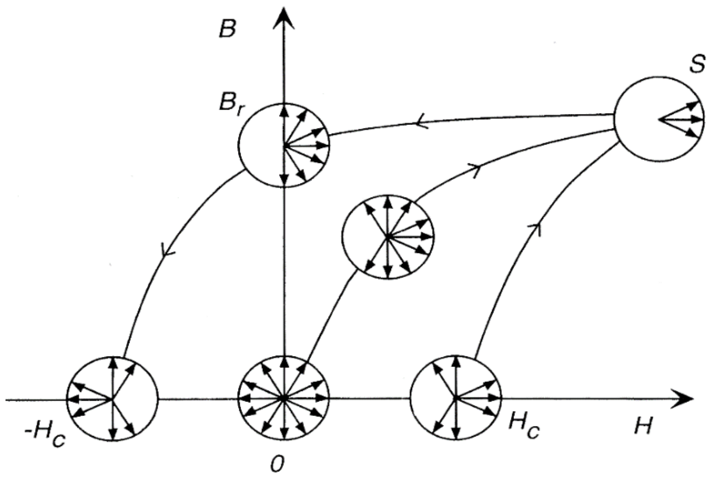


- The magnetization process is conveniently presented in (equivalent) B-H or M-H diagrams.

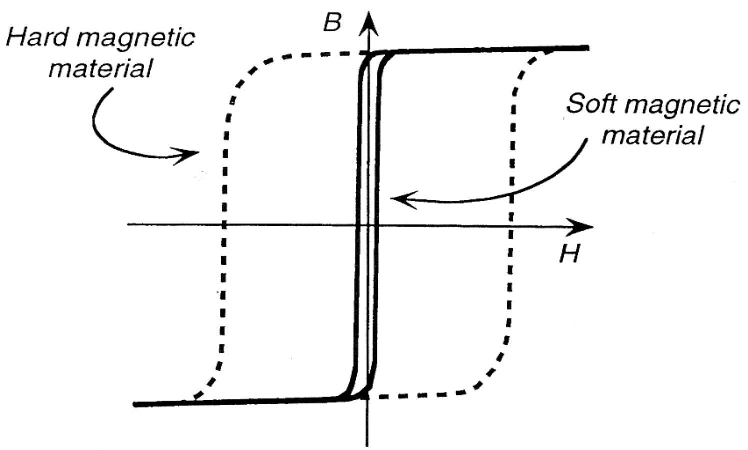


The Hysteresis Loop

- The demagnetizing behavior (when lowering the field and/or applying an opposite field) determines hardness or softness.
  - Quantification by Coercivity (H_c): the (negative) field required for B=0
  - Hard magnets: $|H_c|$ > 10000 A/m
  - Soft magnets: $|H_c|$ < 1000 A/m



Distribution of domain orientations at various points along the BH loop



BH loops for soft and hard magnetic materials

- A hysteresis loop behaviour can be observed
  - **B** depends not only on **H**, but also on the state of the material

Magnetization

- Magnetization is a vector field $\mathbf{M} = \mathbf{M}(x,y,z)$ (as are $\mathbf{B}$ and $\mathbf{H}$)
- The constitutive relationship is:
$$\begin{aligned} \mathbf{B} &= \mu_0(\mathbf{H} + \mathbf{M}) \\ &= \mu_0\mathbf{H} + \mu_0\mathbf{M} \end{aligned}$$
- In hard magnetic materials (permanent magnets)
  - Once magnetized: $\mathbf{M}$ is independent of $\mathbf{H}$
  - The remanence is $\mathbf{B}_r = \mu_0\mathbf{M}$
- In soft magnetic materials, $\mathbf{M}$ and $\mathbf{H}$ are related by the susceptibility tensor:
$$\mathbf{M} = \chi\mathbf{H}$$
  - This looks similar to the permeability
$$\mathbf{B} = \mu\mathbf{H}$$

$$\mathbf{M} = \chi\mathbf{H}$$

- The relationship between the tensors μ_r and χ can be derived by noting that

$$\begin{aligned}\mathbf{B} &= \mu_0\mathbf{H} + \mu_0\mathbf{M} \\ &= \mu_0\mathbf{H} + \mu_0\chi\mathbf{H} \\ &= \mu_0(\mathbb{1} + \chi)\mathbf{H} \\ &= \mu\mathbf{H}\end{aligned}$$

- Thus we have

$$\chi = \frac{\mu}{\mu_0} - \mathbb{1}$$

- And for the linear region (χ is a scalar)

$$\chi = \mu_r - 1$$

- Since μ_r and χ are equivalent we can also use the susceptibility to distinguish between diamagnetic (χ < 0), para- and ferromagnetic (χ > 0) and nonmagnetic (χ = 0) materials.

- We now know that a ferromagnetic material will magnetize if it is placed in a magnetic field.
- From experience, we also know that a piece of iron will rotate and move if a magnet is brought close to it.
- Thus, the magnetization of a material is related to the forces and torques exerted on it in a magnetic field.
- More precisely, the forces and torques depend on the field around the body.

A Closer Look at the Torque

- We have $$\mathcal{T} = \mu_0 v \mathbf{M} \times \mathbf{H}$$

- Observe that the torque
  - Depends on the magnitude of the applied field $\mathbf{H}$ (as opposed to the force).
  - Vanishes when $\mathbf{M}$ and $\mathbf{H}$ are aligned.
  - Is maximum at a specific angle
    - For hard magnetic materials, $\mathbf{M}$ is independent of the applied field and the maximal torque occurs, when $\mathbf{M}$ and $\mathbf{H}$ are perpendicular.
    - For the soft magnetic materials $\mathbf{M}$ depends on its internal field and the torque relationship is nonlinear.

A Closer Look at the Force

- We have $\mathbf{F} = \mu_0 v(\mathbf{M}\cdot\nabla)\mathbf{H}$
- Assuming there is no current flowing in the body (J=0), and using the resulting constraint from Maxwell's equations $\nabla \times \mathbf{H} = 0$ we can rewrite the force into a more intuitive expression:

$$\mathbf{F} = \mu_0 v \left[\mathbf{M}\cdot\frac{\partial \mathbf{H}}{\partial x}, \quad \mathbf{M}\cdot\frac{\partial \mathbf{H}}{\partial y}, \quad \mathbf{M}\cdot\frac{\partial \mathbf{H}}{\partial z}\right]^T$$

- Observe that the force
  - Depends on the dot product between $\mathbf{M}$ and the spatial derivative of $\mathbf{H}$.
    - Maximum is achieved when they are aligned
  - The magnitude of $\mathbf{H}$ does not play a role
  - For a nonzero magnetization, a force component will vanish if
    - the field is constant along that direction
    - $\mathbf{M}$ is perpendicular to the spatial derivative of $\mathbf{H}$ in that direction
  - Remember that $\mathbf{M}$ is constant for hard magnetic materials and depends on the internal field for soft magnetic materials