AIP Journal of Applied Physics

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Citation: Journal of Applied Physics **79**, 5742 (1996); doi: 10.1063/1.362236
View online: http://dx.doi.org/10.1063/1.362236
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/79/8?ver=pdfcov
Published by the AIP Publishing

Experimental determination of an effective demagnetization factor for nonellipsoidal geometries

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For a nonellipsoidal magnetic sample, the internal magnetic field $\mathbf{H}_{\text{int}}(\mathbf{r})$ is inhomogeneous even when a uniform external magnetic field $\mathbf{H}_{\text{app}}$ is applied to it. In such a case the elements of the demagnetizing tensor become position-dependent, $\mathbb{N}=\mathbb{N}(\mathbf{r})$. The knowledge of the *local* demagnetization tensor is important for the analysis and design of devices using finite size and shape magnetic elements. The demagnetizing tensor elements can be calculated analytically and/or numerically, however, often a quick check of the extent of the inhomogeneity of the magnetization distribution of a sample, or a single approximate value of the demagnetizing factor for the given non-ellipsoidal geometry, would be satisfactory. Therefore a method to define and measure an effective demagnetizing tensor element N_{eff} for rectangular and circular shapes has been developed. Experiments, performed on 2×4 mm^2 yttrium iron garnet samples up to 220 μm thickness, show that the analytical approximation can be used to define an N_{eff}. But even for a length/thickness ratio of 200, the thin film approximation still is in an error of 3.7%. © *1996 American Institute of Physics*. [S0021-8979(96)75608-9]

I. INTRODUCTION

Ferrite elements are widely used in microwave devices, isolators, circulators, and phase shifters. The traditional elements have a spherical shape and uniform magnetization.

As the industry turns to monolithic integrated/hybrid nonreciprocal microwave devices, planar geometries have to be used. At microwave frequencies, the size of a distributed microstrip circulator is related to the wavelength in the ferrite ($\sim r/2$).

The magnetization distribution in modern planar ferrite elements, due to the nonellipsoidal shape of the ferrite, is no more uniform. This gives rise to nonuniform internal fields, affecting the operation of the device. In planar microwave devices, the thickness of the layers is in the range of 100 μm, and the lateral dimensions are comparable to the thickness so, the aspect ratio cannot be regarded as infinite. As a result, demagnetizing effects cannot be neglected. Due to inhomogeneous demagnetizing fields, the value of the external bias field in general has to be increased as compared to the case of the spherical geometry. Another problem, arising from the fact that the ferrite element is not a thin film, is that one has to solve the problem in three dimensions. The distribution of the internal magnetization of the ferrite and the electromagnetic field around the edges has a great important for device performance.[4] In designing magnetic devices, the range of N_{zz} is the measure of the inhomogeneity of the internal field in the sample.

The inhomogeneity of magnetization should be taken into account in device design, i.e., the knowledge of *local* distribution of demagnetization factors $N_{ij}(x,y,z)$ is necessary. If the *local* demagnetizing tensor is known, the equilibrium distribution of the magnetization can be calculated by micromagnetic methods.

The demagnetizing correction is nontrivial for samples in open magnetic circuits. An exact correction can be obtained only for ellipsoids, where both the magnetization $\mathbf{M}$ and the demagnetizing field $\mathbf{H}_d$ are uniform under a uniform applied field $\mathbf{H}_{\text{app}}$.[5–7] If the three principal ellipsoid axes coincide with the x, y, and z axes, then the internal field is:

$$\mathbf{H}_{in}=\mathbf{H}_{\text{app}}+\mathbf{H}_d=\mathbf{H}_{\text{app}}-\mathbb{N}\mathbf{M} \tag{1}$$

where $\mathbb{N}$ is a diagonal demagnetizing tensor, and $N_{xx}+N_{yy}+N_{zz}=1$, $0\leqslant N\leqslant 1$ in SI units, and $0\leqslant N\leqslant 4\pi$ in CGS units.

For nonellipsoidal samples, if the sample is placed in a uniform applied field $\mathbf{H}_{\text{app}}$ along its axis, a demagnetization factor N can be defined as the ratio of the average demagnetizing field to the average magnetization of the entire sample:[8]

$$\int_S \mathbf{H}_d(\mathbf{r})d\mathbf{S}=-N\int_S \mathbf{M}(\mathbf{r})d\mathbf{S}. \tag{2}$$

But now N is the function of the aspect ratio, $m=L/D$ (length to diameter). So, the ellipsoidal approximation is no longer suitable and a relationship between the shape and size of the ferrite element and the demagnetizing factors should be developed. The demagnetizing factors, and/or the local demagnetizing tensor elements have been calculated analytically in Refs. 1–3 using various approximations. Nonuniform demagnetizing factors have been calculated in Ref. 1, based on a first order approximation, that the direction of the magnetization coincides with the direction of the local magnetic field at any point within the sample. This approximation may cause a 20% error. It is desirable to know the difference between the real demagnetizing factors and that calculated by Ref. 1.

In the present work an experimental method was developed to determine an *effective* demagnetizing factor N_{eff} for thick, small rectangular and circular ferrite samples. The results are compared to N_{eff}, derived from Ref. 1, based on a technique of statistical averaging over the volume.

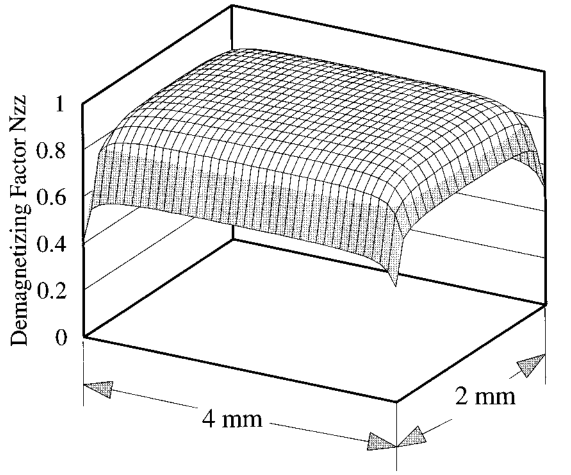

FIG. 1. Calculated distribution of local demagnetization factors $N_{zz}(x,y)$ on the center plane, $z=0$, of a $2\times4\times0.135$ mm^3 YIG sample.

II. EXPERIMENTS

The method to determine an *effective* demagnetizing factor for a nonellipsoidal sample is based on the measurement of the in-plane and out-of-plane major hysteresis loops using a vibrating sample magnetometer (VSM). Experiments were performed on 4×2 mm^2 rectangular yttrium iron garnet (YIG) films ($4\pi M_s = 1780$ G), in the thickness range from 4.7 to 220 μm; on 3–5 mm square YIG films of 140 μm thickness; and on round polycrystalline MnMg ferrites ($4\pi M_s = 2350$ G) of 0.15 in. diameter, thickness from 0.005 in. to 0.025 in. The measured YIG samples were grown by liquid phase epitaxy on (111) GGG substrates. All samples were cut from the same wafer, and polished to the final thickness. The measured magnetization curves have been corrected for the paramagnetic contribution of the GGG substrate.

The procedure is illustrated for the case of the rectangular geometry. The dimensions of the rectangular sample are: $2a\times2b\times2h=2\times4\times0.135$ mm^3. For this configuration, the 3D distribution of demagnetizing factors, $N_{ij}(x,y,z)$ has been calculated. Figure 1 illustrates $N_{zz}(x,y)$ in the central plane ($z=0$) of the YIG sample. For this geometry the distribution of the magnetization is mostly uniform; inhomogeneous fields occur only around the edges of the sample.

The maximum N_{zz} is at the center on the top surface. The smallest value is at the corners of the sample. If $N_{zz}(x,y)$ is calculated over planes, corresponding to different z, then after averaging $N_{zz}(x,y)$ over the plane, the maximum average Max N_{zz} will appear on the top and bottom planes of sample, while the minimum N_{zz} will be on the center plane of the sample. We expect that the experimentally determined $N_{zz,\mathrm{eff}}$ will correspond to the average of $N_{zz}(x,y,z)$ all over the sample.

N_{eff} can be determined from in-plane and out-of-plane hysteresis loops. For the case of negligible anisotropy (as compared to $4\pi M_s$), the loops are expected to be congruent after correcting for the demagnetizing factors. Based on this assumption, the expression $H_{\mathrm{in}} = H_{\mathrm{app}} - 4\pi M_s N_{\mathrm{eff}}$ has been used to find N_{eff}, i.e., to correct the in-plane and out-of-plane loops until they coincide. First, it is assumed that the samples are thin enough, therefore, N_{xx} approaches zero; i.e., the in-plane hysteresis loop is not corrected. Then, the out-of-plane loop should be corrected with a proper N_{zz} until it overlaps with the in-plane loop. This procedure gives the minimum of N_{zz}. Next, it is considered that the inhomogeneous field exists not only in the out-of-plane direction, but also in-plane. So, both hysteresis loops should be corrected by N_{zz} and N_{xx} to arrive to the same loop. The final hysteresis loops, after corrected by demagnetizing factors, should not "overshoot," i.e., have a negative slope. This limiting N_{zz} is the maximum N_{zz}, and Max N_{zz} = Min $N_{zz} + N_{xx}$. The values of Max N_{zz} and Min N_{zz}, determined experimentally, are used to verify the numerical results of Refs. 1–3.

TABLE I. Thickness dependence of demagnetizing factors for MgMn-ferrite discs, diameter: D=0.15 in., height: L=0.010, 0.015, and 0.020 in.

	N	D/L=15	D/L=10	D/L=7.5
N_{xx}	Theory (2)	0.0482	0.0696	0.0894
	Measured	0.0562	0.0785	0.0955
N_{zz}	Theory (2)	0.904	0.861	0.821
	Theory (3)	0.8478	0.7967	0.7533
	Measured	0.758	0.725	0.698

III. RESULTS AND DISCUSSION

The correction procedure has been applied to a series of YIG samples of same size, but having different thickness, and same thickness but different size; and circular MgMn ferrites of different thickness.

A. Circular ferrite samples

Figures 2(a) and 2(b) illustrate the in-plane hysteresis loops before and after the correction was performed for circular ferrite samples of different aspect ratios (D=0.15 in.) and are given in Table I. Theoretical values for N_{xx} in Table 1 are calculated by Eq. (2), and the corresponding $N_{zz} = 1-2N_{xx}$ are given in Table I. The difference between the experimental data and the theoretical results of the ellipsoidal approximation (2) is much larger than expected. Therefore, the solenoid approximation of Eq. (3) has been used to calculate the effective demagnetizing factors of circular samples:

$$N_{zz}=1-4LL_s/(\mu_0\pi D^2), \tag{3}$$

where L_s contains the complete elliptic integrals, and μ_0 is the permeability of vacuum. Table I shows that both Eqs. (2) and (3) underestimate the effect of the shape, i.e., the inhomogeneity of the internal field for disk shaped samples.

B. Rectangular YIG samples

Measurements were done on 4×2 mm^2 YIG films up to 220 μm thickness. The theoretical results for the local demagnetizing tensor element $N_{zz}(x,y,z)$, are based on the first order approximation of Ref. 1. Calculated and measured demagnetizing factors are given in Table II and Fig. 3. In the theoretical estimation column, Max N_{zz} is the average of

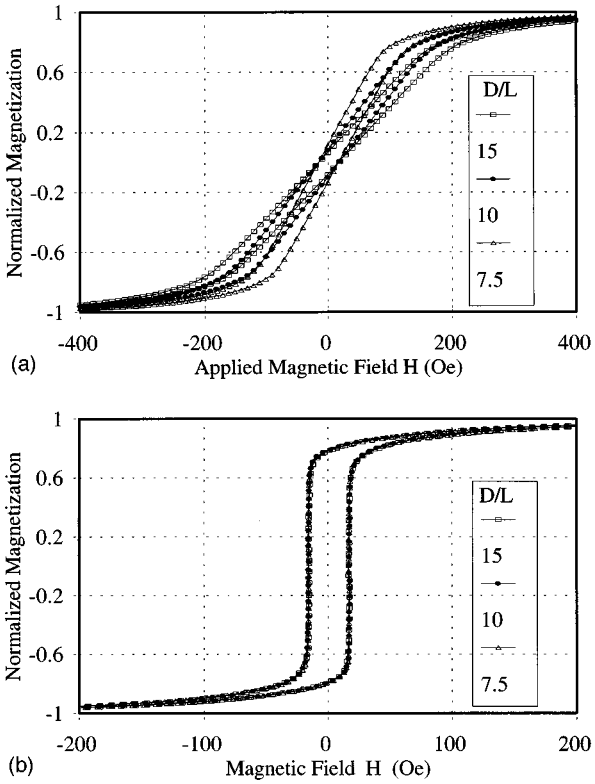

FIG. 2. In plane hysteresis loops for MnMg ferrite disks, $D=0.15$ in., $L=0.010$, 0.015, and 0.020 in.; (a) as measured, (b) after correcting by the demagnetizing factors N_{xx} and N_{zz}.

demagnetization factors on the top (bottom) plane, min N_{zz} is on the center plane; Avg N_{zz} is the average all over the sample. The agreement between the averaged values of the first order approximation of Ref. 1 and the measured demagnetization factors is in good agreement with Ref. 1, however the experimental data suggest a different slope of the curve.

C. Shape dependence of the demagnetizing factor

Measurements of the shape dependence of the magnetization curves are performed on $h=140$-μm-thick square ($L=3, 4$, and 5 mm) YIG samples with aspect ratio of $20 \leqslant L/h \leqslant 200$. The calculated and measured values of N_{zz} are given in Table III. The difference between the measured Max N_{zz} and the calculated value is 3% for the smallest

TABLE II. Shape dependence of demagnetizing factors for square YIG samples (side L, thickness $h=140$ μm).

Demagnetization factor N			$L/h=21.4$	$L/h=28.6$	$L/h=35.7$	$L/h=214.2$
N_{xx}	Measured		0.042	0.0368	0.030	(theor.)
N_{zz}	Theory	Max	0.864	0.867	0.868	0.964
		Min	0.814	0.817	0.818	0.956
		Ave	0.828	0.830	0.831	0.960
	Measured	Max	0.840	0.856	0.873	
		Min	0.798	0.820	0.843	
	Error		3.0%	1.2%	0.6%	⋯

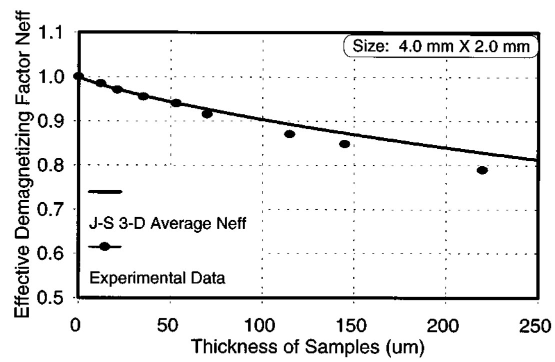

FIG. 3. Thickness dependence of the measured effective demagnetizing factor N_{zz} of 2×4 mm^2 epitaxial YIG films (solid circles), compared to the averaged values calculated from Ref. 1 (solid line).

sample, and it is less than 1% for the 5 mm sample. However, for $L/h=214$ the calculated Max N_{zz} is only 0.964, i.e., the validity of the thin film approximation is still worse than 3%.

V. CONCLUSIONS

The internal field of a finite size, finite thickness magnetic material is nonuniform, which affects the operation of microwave ferrite devices. The knowledge of the *local* demagnetizing tensor elements $N_{ij}(x,y,z)$ is necessary to analyze and design such devices. Analytical calculations of the demagnetizing tensor elements are based on approximations. To investigate the applicability of these theories, a method, based on in-plane and out-of-plane hysteresis loop measurements, has been developed to define and measure an effective demagnetizing factor for nonellipsoidal samples. The measurements dependence of the effective N_{zz}, N_{xx} on thickness, sample size, and shape have been performed on rectangular YIG and circular MgMn ferrite samples. The measured data have been compared to the properly averaged, calculated results, and it is concluded, that the agreement between the calculated and measured N_{eff} values for rectangular YIG samples is very good, however, the available analytical calculations underestimate the inhomogeneity for the case of disc-shaped samples.

ACKNOWLEDGMENTS

Stimulating discussions with colleagues from the Ferrite Development Consortium and the Magnetics Research Institute are appreciated. This work was partially supported by ARPA Ferrite Development Consortium.

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