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Demagnetizing factors for rectangular ferromagnetic prisms

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An analytic expression is given for the magnetometric demagnetizing factors of the general rectangular prism, with special emphasis on the particular case of a square cross section. It is argued that this demagnetizing factor should be used in numerical computations that assume such prisms, when comparing the results with the theoretical study of ellipsoids. © *1998 American Institute of Physics*. [S0021-8979(98)04506-X]

Demagnetizing factors for both ellipsoidal and nonellipsoidal bodies have been studied for more than 100 years. The results for ellipsoids are fully presented in Ref. 1 with all the necessary features for any practical application. The case of a circular cylinder is most thoroughly covered in Ref. 2. For prisms, however, tables can only be found in the literature for two particular cases. One is the formula and a table given[3] for the case of an infinite prism, magnetized transversely. The other is a short table[4] for the case of a square cross section. A partial derivation of the formula for prisms can be found in Ref. 5, as a part of a more general study, but this article is more concerned with the demagnetizing *field* than with the demagnetizing factors, and it uses a rather complicated notation. It should be noted that the demagnetizing field in non-ellipsoidal bodies is not uniform. It is a function of space in the body. Therefore, in order to define a constant demagnetizing factor, which depends only on the geometry, it is necessary to define some sort of averaging of this field. In the literature there is also an often quoted, but less often used, study[6] of the demagnetizing energies in an obscure journal which is not easy to find nowadays.

The knowledge of the theoretical demagnetizing factors of prisms has become necessary because of the large number of recent numerical computations[7] of the magnetization processes in such prisms. Any comparison of these computations with the theory of the processes in other bodies, such as ellipsoids, should involve the elimination of the appropriate demagnetization. Of course, it is possible to include a computation of the demagnetization in these numerical studies, especially when some workers[8] claim anyway that it is simpler to compute all the magnetostatic coefficients numerically, ''although complicated analytic forms for the interaction energy can be obtained.'' But when the demagnetization is *not* computed, or at least not reported, it should be helpful to have an expression which may be used for comparing one published result with another. Such an expression is given here.

We consider a uniform and homogeneous ferromagnetic particle in the shape of a rectangular prism, and we define the origin of a Cartesian coordinate system at the center of this prism. More specifically, we assume (as in Ref. 5) that the prism extends over the volume $-a\leqslant x\leqslant a$, $-b\leqslant y\leqslant b$ and $-c\leqslant z\leqslant c$, see Fig. 1. If this prism is saturated along z, a surface charge is created on its faces $z=\pm c$. The potential due to this charge can be calculated by well-known integrals on these surfaces, and the magnetic field is the gradient of that potential. It takes another integration of this field over the prism volume to obtain the magnetostatic self-energy, but all these integrations are nearly the same as in Ref. 9. On the whole, the algebra is non-trivial but straightforward in principle. The *magnetometric* demagnetizing factor in the z-direction, D_z, is defined as the factor that makes the magnetostatic self-energy per unit volume equal to $2\pi D_z M_s^2$. Therefore, all these calculations lead to

$$\begin{aligned}\pi D_z=&\frac{b^2-c^2}{2bc}\ln\left(\frac{\sqrt{a^2+b^2+c^2}-a}{\sqrt{a^2+b^2+c^2}+a}\right)+\frac{a^2-c^2}{2ac}\ln\left(\frac{\sqrt{a^2+b^2+c^2}-b}{\sqrt{a^2+b^2+c^2}+b}\right)+\frac{b}{2c}\ln\left(\frac{\sqrt{a^2+b^2}+a}{\sqrt{a^2+b^2}-a}\right)+\frac{a}{2c}\ln\left(\frac{\sqrt{a^2+b^2}+b}{\sqrt{a^2+b^2}-b}\right)\\&+\frac{c}{2a}\ln\left(\frac{\sqrt{b^2+c^2}-b}{\sqrt{b^2+c^2}+b}\right)+\frac{c}{2b}\ln\left(\frac{\sqrt{a^2+c^2}-a}{\sqrt{a^2+c^2}+a}\right)+2\arctan\left(\frac{ab}{c\sqrt{a^2+b^2+c^2}}\right)+\frac{a^3+b^3-2c^3}{3abc}\\&+\frac{a^2+b^2-2c^2}{3abc}\sqrt{a^2+b^2+c^2}+\frac{c}{ab}\left(\sqrt{a^2+c^2}+\sqrt{b^2+c^2}\right)-\frac{(a^2+b^2)^{3/2}+(b^2+c^2)^{3/2}+(c^2+a^2)^{3/2}}{3abc}.\end{aligned}\tag{1}$$

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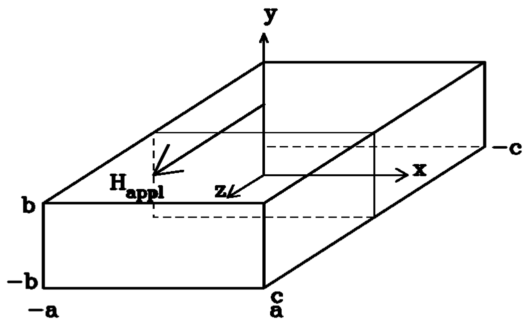

FIG. 1. The coordinate system used in the calculations. Its origin is at the center of the rectangular prism. The field H_{appl} is applied along the z axis.

The other two demagnetizing factors, D_x and D_y, can be derived from this equation by applying twice the cyclic permutation $c \rightarrow a \rightarrow b \rightarrow c$. It is readily seen that these factors obey the relation

$$D_x + D_y + D_z = 1, \tag{2}$$

as they should.

In the particular case $b \rightarrow \infty$, by using the notation

$$p = \frac{c}{a}, \tag{3}$$

Eq. (1) reduces to

$$\pi D_z = \frac{1-p^2}{2p} \ln(1+p^2) + p \ \ln p + 2 \arctan\left(\frac{1}{p}\right), \tag{4}$$

which is the equation published[3] by Brown.

In the particular case of a square cross section, $a=b$, the use of the same notation of Eq. (3) in Eq. (1) leads to

$$\begin{aligned}\pi D_z = & \left(p - \frac{1}{p}\right) \ln\left(\frac{\sqrt{p^2+2}+1}{\sqrt{p^2+2}-1}\right) + \frac{2}{p} \ \ln(\sqrt{2}+1) \\ & + p \ \ln\left(\frac{\sqrt{p^2+1}-1}{\sqrt{p^2+1}+1}\right) + 2 \arctan\left(\frac{1}{p\sqrt{p^2+2}}\right) \\ & + \frac{2(1-p^2)}{3p}\sqrt{p^2+2} + \frac{2(1-p^3)}{3p} - \frac{2^{3/2}}{3p} \\ & + \frac{2}{3}\sqrt{p^2+1}\left(2p - \frac{1}{p}\right).\end{aligned} \tag{5}$$

The special value for a cube,

$$D_z = \frac{1}{3} \quad \text{for } p=1, \tag{6}$$

can be obtained by substituting $p=1$ in Eq. (5). It may also be deduced from Eq. (2) and the symmetry argument that all three factors are the same for a cube. Other demagnetizing factors have been computed from Eq. (5) for some ratios, p, of typical recording particles, or of some of the particles in recent experiments[10] on individual Ni particles. These values are listed in Table I, together with the values for prolate spheroids[1] for comparison.

TABLE I. The demagnetizing factor, D_z^s, of a prolate spheroid and the magnetometric demagnetizing factor, D_z^p, of a square prism, for an aspect ratio, p.

p	D_z^s	D_z^p
2.0	0.17356	0.19832
3.0	0.10871	0.14036
4.0	0.075407	0.10845
5.0	0.055821	0.088316
6.0	0.043230	0.074466
7.0	0.034609	0.064363
8.0	0.028421	0.056670
9.0	0.023816	0.050617
10.0	0.020286	0.045731
11.0	0.017515	0.041705

Much more elongated nickel particles than the entries in Table I, such as those in Refs. 11 and 12, have a negligible demagnetizing field, so that it is not necessary to know it for analyzing experimental data. Besides, computer resources are inadequate anyway for accurate computations of such particles. For the range of p tabulated in Table I, however, demagnetization affects the theoretical nucleation field (or switching field) very considerably. Therefore, the large difference between D_z^s and D_z^p cannot be ignored, and in as much as square prisms are assumed[13] in the computations, the proper demagnetizing factors must be used.

Generally speaking, one should avoid the *assumption* of a prism (which is highly unlikely to be the shape of the experimental particles) in the analysis, unless there is experimental evidence that the demagnetizing factors are close to the values in Table I. For this purpose, the demagnetization can be estimated from the approach to saturation parallel and perpendicular to the long axis, or from other experimental data, such as the apparent anisotropy in the ferromagnetic resonance, or similar data. Assuming that the shape is that of a prism, in order to compare with computational results for such a shape, as is only too often done, can only be misleading.

Real small particles are neither ellipsoids nor prisms, both geometries being only approximations. In comparing theoretical results it should therefore be borne in mind that the demagnetization of a prism is larger than that of an ellipsoid with the same p, thus making its internal field smaller than that of the ellipsoid, for the same value of the applied filed. Since theories actually rely on the *internal* field, comparison makes sense between two cases that have the same demagnetizing factors, and not between those with the same p. Thus, according to Table I, the theory of a prism with $p=11$ should be compared with that of a prolate spheroid with an aspect ratio of about 6, and not 11.

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