Integral Equation Method Using Total Scalar Potential for the Simulation of Linear or Nonlinear 3D Magnetostatic Field with Open Boundary

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***Abstract*–A method of simulating both linear and nonlinear 3D magnetostatic field with open boundary is described. It is based on the integration with total scalar potential on the surface (linear problems) or in the volume (nonlinear problems) of magnetic materials. For nonlinear problems, this procedure can avoid the cancellation errors and is convenient because of the use of total scalar potential and the volume integration only. It was tested by a given example and was applied to a deflection field calculation of multipole yoke.**

I. Introduction

There have been many methods to solve linear and nonlinear magnetostatic fields. FEM is the most popular, but it is not efficient in solving open boundary problems. In recent years, the methods based on the integration have demonstrated many advantages. These methods include BEM and IEM (Integral Equation Method). BEM is based on the reduced scalar potential [1], the reduced and total scalar potential [2] or the total scalar potential and its normal derivative [3]. IEM has many forms according to the unknown variables in the integral equation such as the vector potential **A**, the magnetization **M**, the reduced scalar potential ϕ or the total scalar potential ψ combined with reduced scalar potential ϕ [4] and the magnetic charges [5].

It is clear that the use of scalar potential is simpler and requires less memory than the use of vector potential **A** and magnetization **M** because of less unknown variables.

For nonlinear problems, the **H** in nonlinear regions may have large errors [4]. The total scalar potential and its normal derivative [3] or the double scalar potential ϕ-ψ [2], [4] was used to overcome this difficulty. But, in the case of complicated boundaries of nonlinear regions, the advantages of these methods are diminished due to the complicated interface boundary conditions. In addition, the volume and surface numerical integration are required in these methods [2]-[5], so the calculation cost is very large.

The IEM presented in this article uses only the total scalar potential instead of the double unknown variables to model magnetostatic fields and uses a modified Gauss integration method [7] to handle the numerical integration with singularity. For nonlinear problems, this method only needs to calculate the volume numerical integration instead of both the volume and surface numerical integration. It is more convenient than BEM and ϕ-ψ methods especially for 3D nonlinear problems with complicated magnetic material surfaces and open boundaries and offers satisfactory accuracy.

Manuscript received Nov. 1, 1993. This work was supported in part by the National Eight-Five Important Science & Technology Plan under the Grant No. 85-803-08

II. Theory and Formulas

A. Integral Equation with Total Scalar Potential

The total space magnetostatic field contributed by current excitations and magnetized materials is

$$\mathbf{H} = \mathbf{H}_c + \mathbf{H}_m \tag{1}$$

In the regions without current, **H** can be represented by the gradient of the total scalar potential [2], [4]

$$\mathbf{H} = -\nabla\Psi \tag{2}$$

$$\Psi = \Phi_m + \Phi_c \tag{3}$$

According to the electric-magnetic theory [6], we have

$$\Phi_m = \frac{1}{4\pi}\iiint_{\Omega_m} \frac{\mathbf{M}\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dv' \tag{4}$$

M denotes the magnetization.

$$\mathbf{M} = \chi\mathbf{H} = -(\mu_r - 1)\nabla\Psi \tag{5}$$

χ is the coefficient of magnetization, μ_r is the relative magnetic permeability of material.

Outside the current regions, Φ_c which is produced by current distribution can be calculated as follows

$$\Phi_c = -\int_{p_0}^{p(\mathbf{r})} \mathbf{H}_c(\mathbf{r})\cdot d\mathbf{l} \tag{6}$$

p_0 is the space point which is assumed at zero potential. The integral routine should not pass through current regions.

Then we can obtain the integral equation with the total scalar potential

$$\Psi = -\frac{1}{4\pi}\iiint_{\Omega_m} \chi\nabla\Psi\cdot\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} dv' + \Phi_c \tag{7}$$

When (7) is used in magnetic materials, we can change it to (8) through Green's Law

$$\Psi = -\frac{1}{4\pi\mu_r}\iint_{S_m}\chi\Psi\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\cdot d\mathbf{s}' + \frac{1}{4\pi\mu_r}\iiint_{\Omega_m}\Psi\nabla\chi\cdot\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}dv' + \frac{1}{\mu_r}\Phi_c \tag{8}$$

In the linear magnetic fields, $\nabla\chi=0$, so:

$$\Psi = -\frac{1}{4\pi\mu_r}\iint_{S_m}\chi\Psi\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\cdot d\mathbf{s}' + \frac{1}{\mu_r}\Phi_c \tag{9}$$

Equation (7) is the volume integral equation for nonlinear problems and (9) is the surface integral equation for linear problems.

B. Discretization of Integral Equation

Taking the nonlinear problem as an example, we can discretize (7) using the isoparametric element of first order

$$\Psi = -\frac{1}{4\pi}\sum_{n=1}^{N_v}\iiint_{\Omega_n}(\sum_{k=1}^{8}C_k\chi_k)\{(\sum_{k=1}^{8}\mathbf{D}_k\Psi_k)\cdot\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\}dv' + \Phi_c \tag{10}$$

$$C_k = \frac{1}{8}(1+\xi_l\xi)(1+\eta_l\eta)(1+\zeta_n\zeta) \qquad l,m,n=1,2,3\ldots,8 \tag{11}$$

k is the number of the node of a volume element. C_k are the shape functions of unit cube which link each element in global coordinates to the local coordinates. D_k is the gradient of C_k. ξ,η,ζ are the local coordinates.

Then, we can get a set of equations in the unknown total scalar potential of volume mesh node

$$[\mathbf{A}_{ij}][\Psi_j] = [\mathbf{B}_j] \tag{12}$$

$$\mathrm{A}_{ij} = \begin{cases} 1+\dfrac{1}{4\pi}\sum_{l=1}^{N_{ej}}\{\iiint_{\Omega_l}(\sum_{k=1}^{8}C_k\chi_k)(\mathbf{D}_{j,l}\cdot\dfrac{\mathbf{r}_i-\mathbf{r}'}{|\mathbf{r}_i-\mathbf{r}'|^3})dv'\} & (i=j) \\ \dfrac{1}{4\pi}\sum_{l=1}^{N_{ej}}\{\iiint_{\Omega_l}(\sum_{k=1}^{8}C_k\chi_k)(\mathbf{D}_{j,l}\cdot\dfrac{\mathbf{r}_i-\mathbf{r}'}{|\mathbf{r}_i-\mathbf{r}'|^3})dv'\} & (i\neq j) \end{cases} \tag{13}$$

$$[\mathbf{B}_j] = [\Phi_{cj}] \tag{14}$$

N_{ej} is the total number of volume elements including the j node, $D_{j,l}$ is the gradient of shape function of the first order corresponding to the j node in the l volume element. k is the number of the node of the l volume element.

$$\mathbf{D}_{j,l} = \nabla C_{j,l} = [\mathbf{J}]^{-1}\left[\frac{\partial C_{j,l}}{\partial\xi} \quad \frac{\partial C_{j,l}}{\partial\eta} \quad \frac{\partial C_{j,l}}{\partial\zeta}\right]^{\mathrm{T}} \tag{15}$$

$[\mathrm{J}]$ is the Jacobian matrix.

The coefficients A_{ij} are calculated numerically by Gauss numerical integration formula.

C. Solving the Nonlinear Equation

Since χ is unknown in nonlinear problems, (7) must be solved iteratively starting with a certain value of χ. Generally, χ of the first step was given the value of linear problem solution. For the further steps, $\chi^{(n)}$ was calculated from the $\mathbf{H}^{(n)}$ through the **B-H** characteristics. The $\mathbf{H}^{(n)}$ can be calculated from the gradient of the $\Psi^{(n)}$

$$\mathbf{H}_i^{(n)} = \frac{1}{N_i}\sum_{j=1}^{N_i}(\sum_{k=1}^{8}\mathbf{D}_{i,j,k}\Psi_k^{(n)}) \tag{16}$$

N_i is the total number of volume elements including the i node, $\mathbf{D}_{i,j,k}$ is the gradient of shape function corresponding to the i node in the j element.

The criteria of convergence is that the maximum of relative error of χ of two steps is less than a given error. In order to get fast convergence, the $\chi^{(n)}$ was calculated as

$$\chi^{(n)} = \omega(\frac{\mathbf{B}^{(n)}}{\mu_0\mathbf{H}^{(n)}}-1)+(1-\omega)\chi^{(n-1)} \tag{17}$$

ω is the super relaxation coefficient: $1<\omega<2$.

D. Calculation of Magnetic Fields

After calculating the distribution of the total scalar potential in magnetic materials, we can calculate the magnetic field of space points (nonlinear problems):

$$\mathbf{H} = \frac{1}{4\pi}\iiint_{\Omega_m}\{\nabla\chi\nabla\Psi\cdot\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}+\chi(\nabla(\nabla\Psi\cdot\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}))\}dv' + \mathbf{H}_c \tag{18}$$

When the space point is in the magnetic material, we can use interpolation to calculate the field, because the field of mesh node has been calculated.

III. Integration with Singularity

When calculating the coefficient A_{ii} (nonlinear problems), we should handle the integration with singularity as follows

$$\iiint_{\Omega}\mathbf{f}\cdot\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}dv' \qquad \mathbf{r}\in\Omega \tag{19}$$

Ω is the integration domain of a unit cube, for the use of isoparametric element of first order, $\mathbf{r}$ denotes one of the vectors of the eight nodes of a unit cube.

In order to cancel the singularity's influence on the standard Gauss integration formula, we use a nonlinear

polynomial transformation of odd order in the local parameter plane [7] (see Fig. 1.)

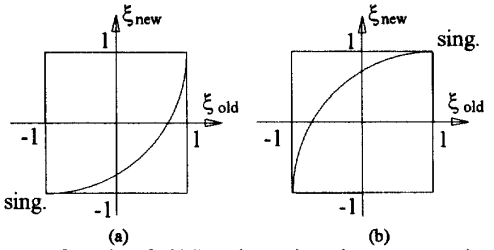

Fig. 1. transformation of old Gauss integration point to new one using n-th order transformation
(a) singularity point at $\xi = -1$ (b) singularity point at $\xi = 1$

The polynomial transformation is:

$$\xi_{new} = 2\left(\frac{\xi_{old}+1}{2}\right)^{2n+1} - 1 \qquad \xi_{sing.} = -1$$
$$\xi_{new} = 2\left(\frac{\xi_{old}-1}{2}\right)^{2n+1} + 1 \qquad \xi_{sing.} = +1 \tag{20}$$

ξ_{old} is the integration point of standard Gauss method, n is a integration constant, in general, it is a integer greater than one. The choice of n should consider the influence of the word length of a computer.

The standard Gauss integration formula can be modified to

$$\int_{-1}^{1} f(x)dx = \int_{-1}^{1} f(x(\xi))\frac{\partial \xi_{new}}{\partial \xi_{old}} d\xi_{old} \approx \sum_{i=1}^{n} w_i(\xi_i) f(x(\xi))\frac{\partial \xi_{new}}{\partial \xi_{old}}\Big|_{\xi=\xi_i} \tag{21}$$

The modified Gauss weights are

$$w_{new} = w_{old}\frac{\partial \xi_{new}}{\partial \xi_{old}} \tag{22}$$

It is clear that the new weight coefficient will be approaching zero when ξ towards singular point and the singular influence on the standard Gauss integration can be cancelled.

By repeating the use of each local coordinate ξ, η, ζ, we can solve the three dimension singular integration of (19).

IV. Example and Application

A. Nonlinear Problem

In order to examine the method above, we apply it to an example presented by [3].

The structure of the core was shown in page 1101 of [3], the iron was discretized into 512 cubes with 729 total volume mesh nodes. Fig. 2 is the equipotential lines of total scalar potential on the iron surface. Fig 4. is the distribution of **B** along X axis compared to the result of FEM and [3]. The χ of first step is 500, super relaxation coefficient ω=1.5, after 7 steps, the maximum relative error of χ of two steps is less than 0.001.

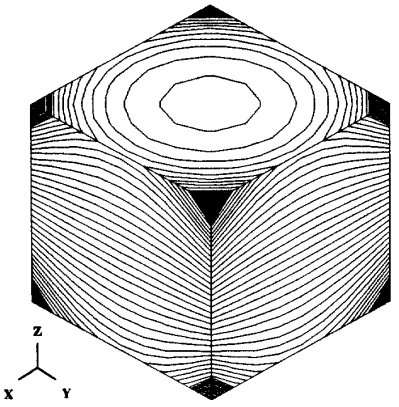

Fig. 2. Equipotential lines on the surface of iron when $J=1\times10^7$ A/m^2

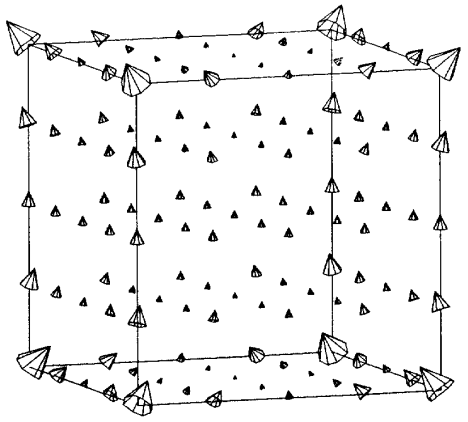

Fig. 3. Magnetic flux density distribution in the iron

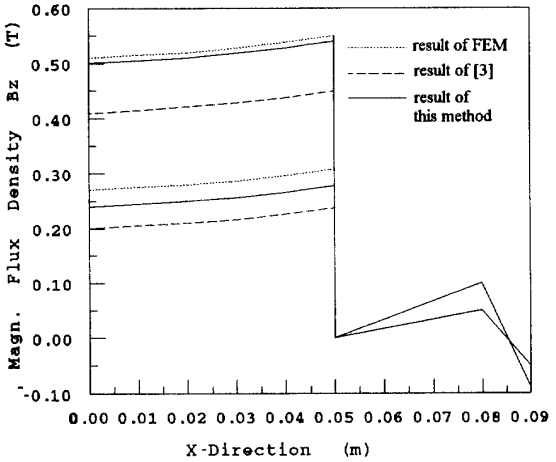

Fig. 4. B_z along X axis compared to the result of FEM and [3] when $J=5\times10^6$A/m^2 and $J=1\times10^7$A/m^2

B. Linear Problem

Using (9), we have solved a complex problem of deflection field simulation in the multipole yoke used in super-high-resolution CRT [8]. The structure of the core is shown in Fig. 5 in which the surface of the core is discretized into isoparametric rectangle, The coil is winded in the slots and the number of horizontal and vertical deflection coil in every slot or the excitation is distributed according to the sine and cosine function with the angle θ between a slot and X-Y plane:

$$(NI)_H = k_H \cos(\theta), \quad (NI)_V = k_V \sin(\theta) \qquad (23)$$

Fig. 6 is the equipotential lines of total scalar potential on the plane at Z=5mm. Fig. 7 is the distribution of $\mathbf{B}_y$ along Z axis with different μ_r. Fig. 8 is the result compared to measured .

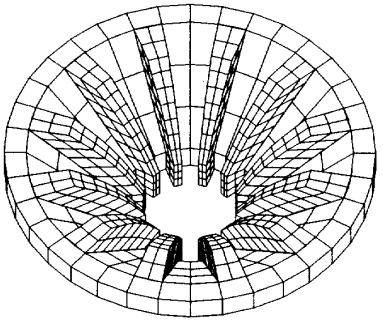

Fig. 5. The structure and surface mesh of the core of deflection yoke

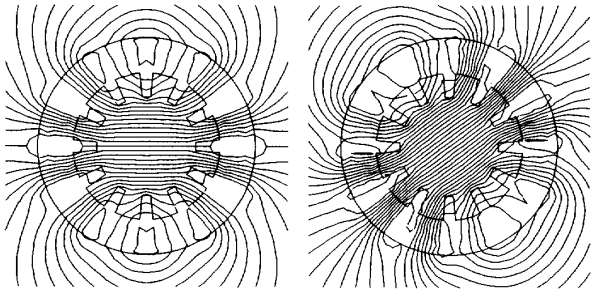

Fig. 6. Equipotential lines on the plane at Z=5.0mm
(a) $(NI)_H$ =20.0cos(θ) (A), $(NI)_V$=0.0 (A), μ_r=3000
(b) $(NI)_H$ =40.0cos(θ) (A), $(NI)_V$=30.0sin(θ) (A), μ_r=3000

V. Conclusion

The IEM for magnetostatic field problems with high efficiency has been presented. For nonlinear materials and open boundary condition, it has smaller calculation requirements and fast convergence, and is more convenient and simpler than FEM and other double variable methods. The use of the total scalar potential and a modified Gauss integration formula can increase the accuracy greatly.

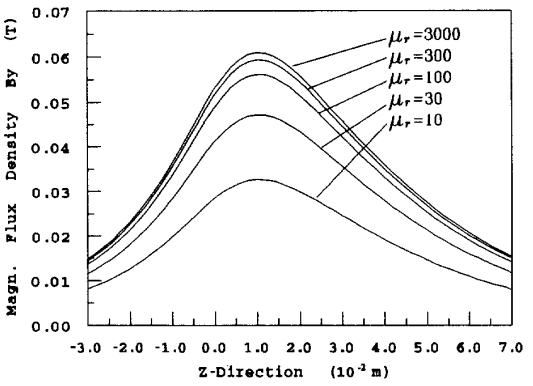

Fig. 7. $\mathbf{B}_y$ along Z axis with different μ_r
$(NI)_H$ =1000.0cos(θ) (A), $(NI)_V$=0.0.

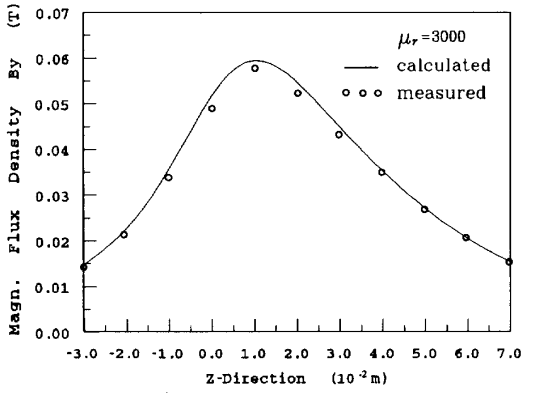

Fig. 8. Calculated and measured $\mathbf{B}_y$ along Z axis
$(NI)_H$ =1000.0cos(θ) (A), $(NI)_V$=0.0.

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