Calculation Improvement of 3D Linear Magnetostatic Field Based on Fictitious Magnetic Surface Charge

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***Abstract*—This paper presents a mathematical model for calculating 3D linear magnetostatic fields with open boundaries. By using this improved method (based on the integral equations and magnetic surface charge), several field computation problems inside magnetic materials are being avoided. A comprehensive procedure is established for magnetic field computation within the free space and iron, as well as on their boundary—at a reduced computational cost. The proposed method was compared to the analytical solution for a test case.**

***Index Terms*—Biot–Savart law, fictitious magnetic charge, integral method, magnetostatic field calculation.**

I. Introduction

THE MATHEMATICAL model for solving 3D nonlinear magnetostatic fields based on integral equations introducing fictitious magnetic surface and volume charges is given in [1]. If a magnetic material is linear and homogeneous (which is often the case in practical problems), the mathematical model can be simplified since only fictitious magnetic surface charge remains unknown in the integral equations. In this case, volume numerical integration is not required [2], thus reducing the computational cost. Magnetic field calculation inside the magnetic material produces an enormous error [3]. To avoid this, a corrective surface charge, σ_c, is introduced. This fictitious charge is determined while respecting the physical image of the magnetic field inside the iron. In this way, a comprehensive mathematical procedure is developed that enables an accurate and speedy computation of the magnetic field in any point in space around the coil.

II. Mathematical Model

Consider the problem of Fig. 1, where a magnetic core ($\mu =$ constant) is in free space with a coil conductor of current density $\vec{J}$. A fictitious normalized magnetic surface charge density is introduced, given by

$$\sigma(P) = \frac{\sigma_m(P)}{\mu_0}. \tag{1}$$

The magnetostatic field can be found as the sum of two components,

$$\vec{H}(Q) = \vec{H}^{\vec{J}}(Q) + \vec{H}^{\sigma}(Q) \tag{2}$$

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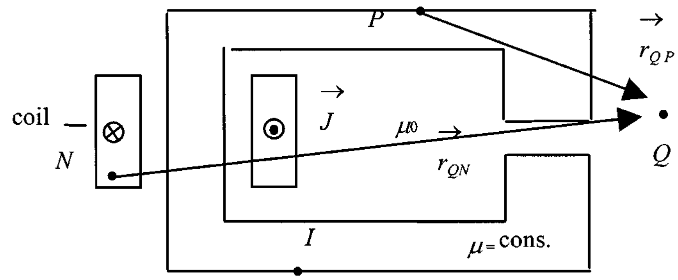

Fig. 1. Electromagnetic model.

where $\vec{H}^{\vec{J}}(Q)$ is the component due to the currant in free space (primary field), and $\vec{H}^{\sigma}(Q)$ is the component due to the fictitious magnetic surface charge to be determined (secondary field).

According to [1], σ is determined from the following integral equation,

$$\sigma(P) - \frac{\lambda}{2\pi}\int_{S_I}\sigma(P)\frac{(\vec{r}_{PI}, \vec{n}_P)}{r_{PI}^3}\,\mathrm{d}S_I = 2\lambda H_n^{\vec{J}}(P); \qquad \lambda = \frac{\mu_r - 1}{\mu_r + 1} \tag{3}$$

where $H_n^{\vec{J}}(P)$ is the normal component of the primary field at point P due to the coil current of density $\vec{J}$, and $\vec{n}_P$ is a unit vector normal to surface S of the magnetic core. The primary field at point Q can be determined using Biot–Savart's law for solid conductor [4],

$$\vec{H}^{\vec{J}}(Q) = \frac{1}{4\pi}\int_V \frac{\left[\vec{J}(N), \vec{r}_{QN}\right]}{r_{QN}^3}\,\mathrm{d}V_N \tag{4}$$

where

- V is the conductor volume,
- $\vec{J}$ is the coil current density and
- $\vec{r}_{QN}$ is the vector from conductor point N to field point Q.

Solving (3) for the surface charge density $\sigma(S)$, the secondary field at point Q is calculated by the integral

$$\vec{H}^{\sigma}(Q) = \int_{S_I}\sigma(I)\frac{\vec{r}_{QI}}{r_{QI}^3}\,\mathrm{d}S_I. \tag{5}$$

As stated above, expression (2) renders the exact magnetic field in free space, but is erroneous in the magnetic core [3]. Regarding the fact that the vectors of the primary and the secondary field inside the magnetic core are almost parallel but opposite in direction (demagnetization effect), one can introduce a

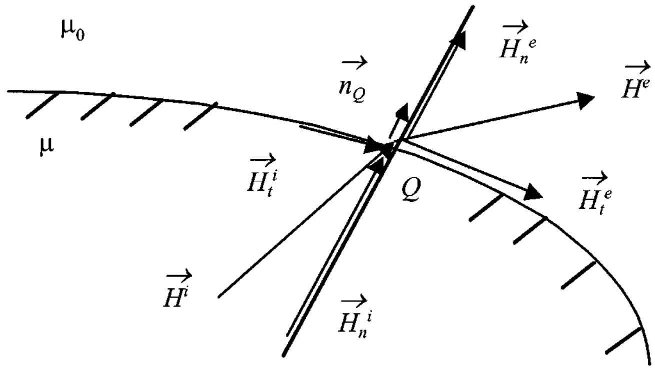

Fig. 2. The conditions on the boundary.

corrective surface charge density σ_c, in order to enable computation of the field inside the core as well. Since $\mu_r \gg 1$ ($\lambda \to 1$) is assumed, σ_c is obtained from the following equation:

$$\sigma_c(P) - \frac{1}{2\pi}\int_{S_I}\sigma_c(I)\frac{(r_{PI}, \vec{n}_P)}{r_{PI}^3}\,\mathrm{d}S_I = 2H_n^{\vec{J}}(P). \qquad (6)$$

The magnetic field due to the corrective magnetic surface charge is

$$\vec{H}^{\sigma_c}(Q) = \int_{S_I}\sigma_c(I)\frac{\vec{r}_{QI}}{r_{QI}^3}\,\mathrm{d}S_I. \qquad (7)$$

According to the above expressions, one can assemble a simple procedure for calculating magnetic field either inside or outside the magnetic core, or on their boundary. Normal and tangential components of the magnetic field on the boundary between the free space and the core are shown in Fig. 2.

I) For point Q inside the magnetic core,

$$\vec{H}^i(Q) = \vec{H}^\sigma(Q) - \vec{H}^{\sigma_c}(Q) \qquad (8)$$

II) For point Q on the boundary the core and free space (facing the core),

$$\begin{aligned}\vec{H}^{i-e}(Q) = {}&\vec{H}^\sigma(Q) - \vec{H}^{\sigma_c}(Q) + \frac{\sigma(Q)+\sigma_c(Q)}{2\mu_r}\vec{n}_Q\\ &+ \frac{1-\mu_r}{\mu_r}\left[\vec{n}_Q(\vec{H}^\sigma(Q) - \vec{H}^{\sigma_c}(Q))\right]\vec{n}_Q \qquad (9)\end{aligned}$$

III) For point Q on the boundary the core and free space (facing free space),

$$\vec{H}^{e-i}(Q) = \vec{H}^\sigma(Q) - \vec{H}^{\sigma_c}(Q) + \frac{\sigma(Q)+\sigma_c(Q)}{2}\vec{n}_Q \qquad (10)$$

IV) For point Q in free space

$$\vec{H}^e(Q) = \vec{H}^{\vec{J}}(Q) + \vec{H}^\sigma(Q) \qquad (11)$$

In all the four cases one has to calculate $\sigma(Q), \sigma_c(Q), \vec{H}^\sigma(Q)$ and $\vec{H}^{\sigma_c}(Q)$ from (3), (5)–(7) respectively. In case IV only primary field calculation is required, but not the secondary field due to the corrective magnetic charge. In this way, one can considerably reduce the computational cost.

Special Cases

If $\mu_r \to \infty$ $\lambda \to 1$, $\sigma(Q) \to \sigma_c(Q)$ we have the following results.

I) For point Q inside the magnetic core,

$$\vec{H}^i(Q) = 0 \qquad (12)$$

II) For point Q on the boundary the core and free space (facing the core),

$$\vec{H}^{i-e}(Q) = 0 \qquad (13)$$

III) For point Q on the boundary the core and free space (facing free space),

$$\vec{H}^{e-i}(Q) = \sigma(Q)\vec{n}_Q \qquad (14)$$

IV) For point Q in free space,

$$\vec{H}^e(Q) = \vec{H}^{\vec{J}}(Q) + \vec{H}^\sigma(Q) \qquad (15)$$

These results are well known in literature.

III. Numerical Procedure

Equations (3), (5)–(7) are solved, in general, with the Extended Boundary Element (EBE) method by employing surface elements (SE). These elements are the approximated parts of the boundary surfaces or the parts of the surface that can be described analytically [1]. In the EBE method, functions approximating the elements are unknown. However, these functions are described using a set of basis functions for simplicity [5]. If the function values in the m appropriately selected points of the element are known, the function value in the arbitrary point of the element can be represented as

$$f = \sum_{l=1}^{m} N_l f_l = [N^{(s)}][f^{(s)}]. \qquad (16)$$

For the geometry, elements use a quadratic approximation or are represented by partial elements described analytically. For the unknown function $\sigma(Q)$ and $\sigma_c(Q)$ on SE, we use constant or linear functions. Finally, fictitious magnetic surface charges in the 3D linear magnetic field computation are determined by solving the following set of linear algebraic equations

$$[\sigma] - \lambda[A][\sigma] = [B]. \qquad (17)$$

IV. Examples

As an illustration, let us solve the well-known example of a magnetic sphere in a primary homogeneous magnetic field $\vec{H}_0$, (see Fig. 3), [6].

a) For the point of calculation inside the sphere, we have

$$H_x^i(x, y, z) = 0, \qquad H_y^i(x, y, z) = 0,$$
$$H_z^i(x, y, z) = \frac{3H_0}{\mu_r + 2}. \qquad (18)$$

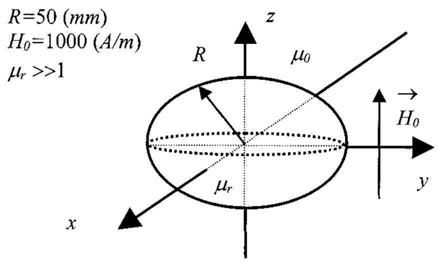

Fig. 3. The sphere in homogeneous magnetic field H_0.

TABLE I
COMPARISON OF TWO APPROACHES

(x,y,z) (mm)	Analytical H(A/m)	Numerical H(A/m)	Error (%)
(0,0,0)	0	0	0
(20,30,10)	0	0	0
(-50,20,10)	315.347	316.785	-0.457
(50,20,0)	199.589	200.398	-0.405
(0,0,51)	2884.645	2886.186	-0.053
(100,0,100)	1024.242	1024,793	-0.054

b) For the point of calculation outside the sphere, we have

$$\begin{aligned}H_x^e(x, y, z) &= 3\frac{\mu_r - 1}{\mu_r + 2}\frac{R^3 H_0 xz}{(x^2+y^2+z^2)^{5/2}}\\ H_y^e(x, y, z) &= 3\frac{\mu_r - 1}{\mu_r + 2}\frac{R^3 H_0 yz}{(x^2+y^2+z^2)^{5/2}}\\ H_z^e(x, y, z) &= H_0 + 3\frac{\mu_r - 1}{\mu_r + 2}\frac{R^3 H_0 z^2}{(x^2+y^2+z^2)^{5/2}}\\ &\quad - \frac{\mu_r - 1}{\mu_r + 2}\frac{R^3 H_0}{(x^2+y^2+z^2)^{3/2}}. \end{aligned} \tag{19}$$

Due to symmetry, the magnetic field can be calculated in 1/8 part of the sphere only. The chosen mesh consists of 71 triangle SE and 51 nodes. The field computations by the analytical and EBE method are given in Table I. There is very good agreement between the results obtained by these two very different calculation methods.

The next example is the sphere of the following characteristic parameters: $R = 50$ (mm), $H_0 = 100$ (A/m), $\mu_r = 10$. The relevant magnetic field distributions obtained in two different ways are given in Figs. 4 and 5 for Case #1 ($y = z = 0$; $x \in [0, 0.16]$ m) and Case #2 ($x = y = 0$; $z \in [0, 0.16]$ m), respectively. We also appreciate an agreement for the points on the boundary, inside and outside the sphere.

V. CONCLUSION

The linear 3D magnetostatic field problem has been solved using an improved extended boundary element method where consistent expressions are obtained in order to calculate efficiently magnetic field either inside or outside magnetic material including its boundary.

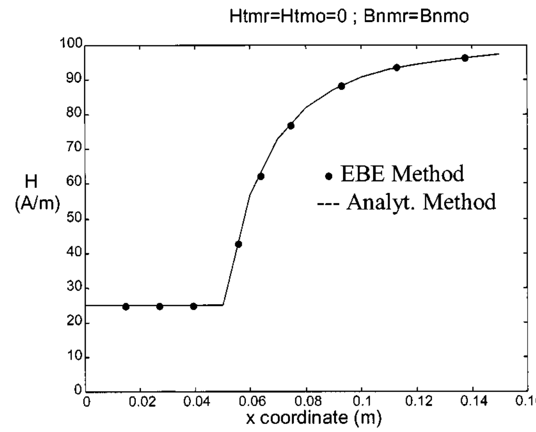

Fig. 4. Magnetic field distribution and boundary conditions (Case #1).

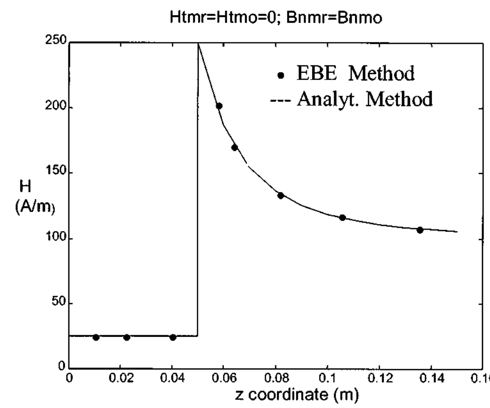

Fig. 5. Magnetic field distribution and boundary conditions (Case #2).

REFERENCES

[1] B. Krstajic, Z. Andjelic, S. Milojkovic, S. Babic, and S. Salon, "Nonlinear 3D magnetostatic field calculation by the integral equation method with surface and volume magnetic charges," *IEEE Trans. Mag.*, vol. 28, pp. 1088–1091, Mar. 1992.
[2] L. Han, L.-S. Tong, and J. Yang, "Integral equation method using total scalar potential for the simulation of linear or nonlinear 3D magnetostatic field with open boundary," *IEEE Trans. Mag.*, vol. 30, pp. 2897–1900, Sept. 1994.
[3] I. D. Mayergoyz, *Mathematical Models of Hystereses*. New York: Springer-Verlag, 1991.
[4] S. Babic and M. M. Gavrilovic, "New expression for calculating magnetic fields due to current-carrying solid conductors," *IEEE Trans. Magnetics*, vol. 33, pp. 4134–4136, Sept. 1997.
[5] S. J. Salon, *Finite Method Analysis of Electrical Machines*. Boston, MA: Kluwer Academic, July 1995.
[6] J. D. Jackson, *Classical Electrodynamics*. New York: Wiley, 1962.