3-D Magnetostatic Field Calculation by a Single Layer Boundary Integral Equation Method Using a Difference Field Concept

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***Abstract*—For an accurate analysis of three dimensional linear magnetostatic problems, a new boundary integral equation formulation is presented. This formulation adopts difference magnetic field concept and uses single layer magnetic surface charge as the unknown. The proposed method is capable of eliminating numerical cancellation errors inside ferromagnetic materials. In addition, computing time and storage memory are reduced by 75% in comparison with the reduced and total scalar potential formulations. Two examples are given to show its efficiency and accuracy.**

***Index Terms*—Difference magnetic field, Magnetic scalar potential, Magnetic surface charge, Single layer boundary integral equation.**

I. Introduction

TWO magnetic scalar potential formulations with two degrees of freedom have been widely used for the analysis of three dimensional (3-D) magnetostatic problems [1]–[3]. One introduces the reduced scalar potential for the induced magnetic fields due to the magnetization of iron. The other makes use of the total scalar potential inside iron materials and the reduced scalar potential outside the objects. However, both methods have deficiency in practical use, such as a considerable cancellation error in the reduced scalar potential and some cutting surface for the total scalar potential.

Recently, for computational efficiency, the single layer boundary integral equation (BIE) method has been developed [4], [5]. In this case, the unknown is an equivalent magnetic surface charge with one degree of freedom on the interface. But, this method produces the cancellation error inside the iron materials because it is based on the reduced scalar potential formulation mentioned above.

To get around these difficulties, a new single layer BIE formulation is presented in this paper. This formulation adopts the difference magnetic field concept in [6] and uses the equivalent magnetic surface charge as unknown. Therefore, the proposed method is capable of eliminating numerical cancellation errors inside the iron materials and can guarantee continuity and uniqueness of a scalar potential outside the materials. Further, computing time and storage memory are reduced by 75% in comparison with the reduced and total scalar potential formulation. Two examples are given to show its efficiency and accuracy.

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II. Single Layer BIE Formulations

A. The Existing Single Layer BIE Formulation

The existing single layer BIE is based on the reduced scalar potential formulation. The total magnetic field $\mathbf{H}$ in the whole space is expressed as the sum of the source field $\mathbf{H}^s$ and the induced magnetic field $\mathbf{H}^i$. The field $\mathbf{H}^i$ can be defined by a single layer magnetic surface charge σ_m distributed on the interface γ because it is an irrotational field.

$$\begin{aligned}\mathbf{H} &= \mathbf{H}^s + \mathbf{H}^i \\ &= \mathbf{H}^s - \int_\gamma \sigma_m \nabla G \, d\Gamma \end{aligned} \tag{1}$$

where G is the Green's function for 3-D magnetostatic problems. Applying (1) to the interface conditions with respect to magnetic flux density, the BIE is expressed in terms of a single layer magnetic surface source, σ_m, as in (2).

$$\frac{1}{2}\sigma_m + \lambda \int_\gamma \sigma_m \nabla G \cdot \mathbf{n} \, d\Gamma = \lambda \mathbf{H}^s \cdot \mathbf{n}$$
$$\lambda \equiv \frac{\mu_r - 1}{\mu_r + 1} \tag{2}$$

where μ_r denotes the relative permeability and means the normal vector outward to the interface γ.

By using the magnetic surface charge with one degree of freedom, this formulation can dramatically reduce computing time and storage memory compared with the reduced and total scalar potential formulations. However, the accurate solution inside ferromagnetic materials can not be obtained because of considerable cancellation errors.

B. The Proposed Single Layer BIE Formulation

To avoid the cancellation errors occurring in the existing BIE, this paper presents a new single layer BIE formulation. This adopts the difference magnetic field $\mathbf{h}$ that was applied to the finite element method (FEM) by Mayergoyz [6].

$$\mathbf{h} = \mathbf{H} - \mathbf{H}^o \tag{3}$$

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where $\mathbf{H}$ is the real field and $\mathbf{H}^o$ is the one obtained by assuming infinite permeability for the iron part.

According to this approach, the magnetic field $\mathbf{H}^o$ is first calculated by limiting the magnetic permeability μ_r in (2) to be infinite

$$\tfrac{1}{2}\sigma_m + \int_\gamma \sigma_m \nabla G \cdot \mathbf{n}\, d\Gamma = \mathbf{H}^s \cdot \mathbf{n}. \quad (4)$$

After solving (4) with respect to σ_m, the magnetic field $\mathbf{H}^o$ outside ferromagnetic materials is given from (1). At this time, it is clear that the magnetic field inside ferromagnetic materials is equal to zero.

Next, consider the difference field $\mathbf{h}$ which can be constructed as a perturbation of $\mathbf{H}^o$ due to saturation effect. Because the difference field defined by (3) is irrotational in the whole space, it can be written in terms of a single layer magnetic surface charge $\tilde{\sigma}_m$ as

$$\mathbf{h} = -\int_\gamma \tilde{\sigma}_m \nabla G\, d\Gamma \quad (5)$$

where $\tilde{\sigma}_m$ is associated with the magnetic field penetrated through the ferromagnetic materials when the permeability μ_r becomes finite. From (5), it is known that this formulation always satisfies the continuity and uniqueness of a scalar potential. The interface condition for the difference field system is derived from the relations of the difference field $\mathbf{h}$ and the real field $\mathbf{H}$.

$$\mu^+ \mathbf{h}^+ \cdot \mathbf{n} = \mu^-(\mathbf{h}^- + \mathbf{H}^o) \cdot \mathbf{n} \quad (6)$$

where superscripts, + and −, denote the ferromagnetic and the nonferromagnetic region, respectively. The single layer BIE with respect to $\mathbf{h}$ is given from (5) and (6).

$$\frac{1}{2}\tilde{\sigma}_m + \lambda \int_\gamma \tilde{\sigma}_m \nabla G \cdot \mathbf{n}\, d\Gamma = -\frac{1}{\mu_r + 1}\mathbf{H}^o \cdot \mathbf{n} \quad (7)$$

After (4) and (7) are solved successively, the field $\mathbf{H}^-$ outside the ferromagnetic materials and $\mathbf{H}^+$ in these objects are obtained in terms of the surface magnetic charges distributed over the interface.

$$\mathbf{H}^- = \mathbf{H}^s - \int_\gamma [\sigma_m + \tilde{\sigma}_m] \nabla G\, d\Gamma,$$
$$\mathbf{H}^+ = -\int_\gamma \tilde{\sigma}_m \nabla G\, d\Gamma. \quad (8)$$

The proposed method enables us to evaluate the magnetic field inside ferromagnetic materials without any cancellation error and guarantees the continuity and uniqueness of the scalar potential. The system matrix assembly process can be performed only once because the left sides of (4) and (7) are almost identical. Thus, this method keeps the computational efficiency similar to the existing single layer BIE.

III. Numerical Results

In order to verify the proposed method, two examples of linear magnetostatic problems are presented. With the zero order elements, the single layer BIE's, (2), (4) and (7), are discretized to yield the system matrix equations. The numerical results are compared with other calculation methods.

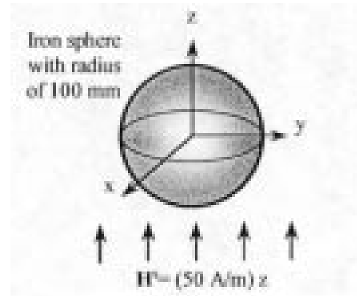

Fig. 1. Iron sphere in a uniform field.

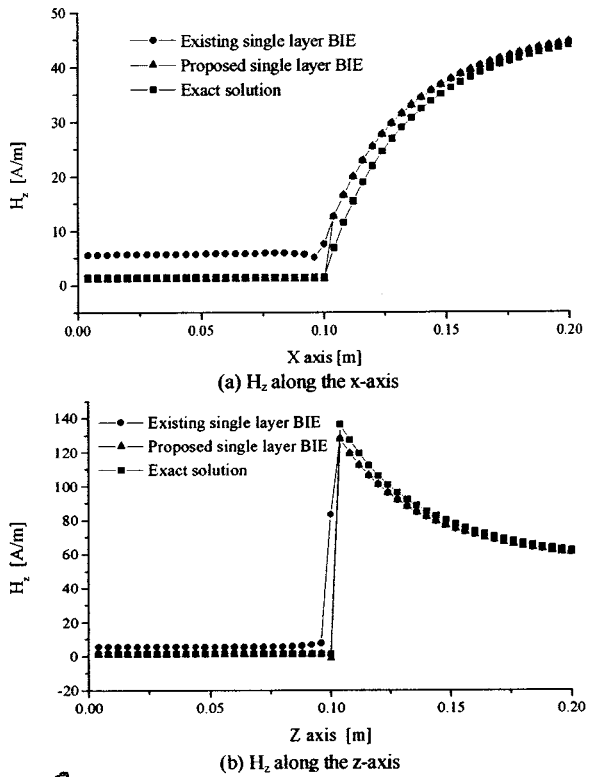

Fig. 2. Comparison of the magnetic field intensities in iron sphere. (a) H_z along the x-axis. (b) H_z along the z-axis.

TABLE I
CALCULATION ERROR WITH RESPECT TO THE EXACT SOLUTION

	μ_r=100	μ_r=1000
Existing single layer BIE	300 %	3070 %
Proposed single layer BIE	13 %	16 %

A. An Iron Sphere in a Uniform Field

The iron sphere, which has an exact solution of the magnetic fields, is depicted in Fig. 1. The boundary mesh consists of 608 triangular surface elements.

Fig. 2 shows that the analytical and numerical calculation results are compared on the reference axes in Fig. 1 when the relative permeability is 100. It is evident that the proposed method yields much better solution than the existing single layer BIE.

Table I presents the calculation errors for the same number of surface elements when the permeability is 100 and 1000, respectively. It is shown that the proposed method provides a stable solution regardless of the permeability. It is thought that the use of

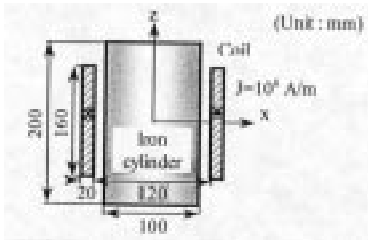

Fig. 3. Iron cylinder in a cylindrical coil.

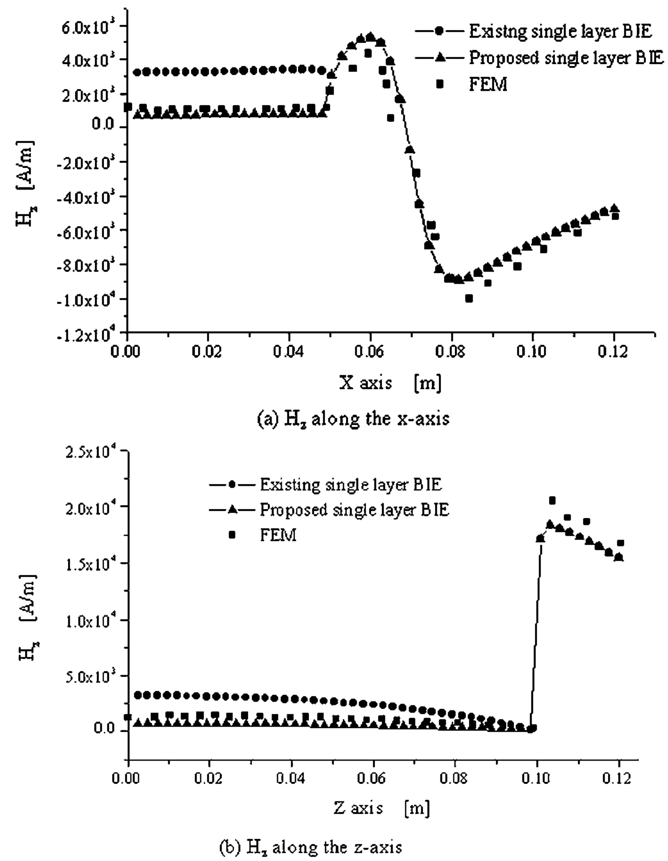

Fig. 4. Comparison of the magnetic field intensities in iron cylinder. (a) H_z along the x-axis. (b) H_z along the z-axis.

planar and zero order element in the proposed methods causes the numerical results to be a little off the exact solutions.

B. An Iron Cylinder in the Field of a Cylindrical Coil

This example validates the presented method in a case where a current source is included in magnetostatic systems. The dimensions and conditions of the iron cylinder are presented in Fig. 3. Due to the symmetry, the half part of the iron cylinder is considered. The boundary mesh consists of 686 rectangular surface elements.

For comparison, this test problem was solved by two single layer BIE methods and 3-D axisymmetric FEM. The FEM uses a linear shape function. When the relative permeability is 100, the calculation results are compared in Fig. 4.

IV. Conclusion

The proposed method is very efficient for 3-D linear magnetostatic field calculation since it does not need to compute a magnetic scalar potential due to the current source field and the size of system matrix is reduced to half of the size of the reduced and total scalar potential formulations. In addition, two numerical examples show that it gives relatively accurate and stable solutions inside magnetic materials.

It is thought that the proposed method can be extended to the analysis of the 3-D nonlinear magnetostatic problems.

References

[1] D. A. Lindholm, "Notes on boundary integral equations for three-dimensional magnetostatics," *IEEE Trans. Magn.*, vol. 16, pp. 1409–1413, 1980.
[2] J. P. Peng, S. Salon, and M. V. K. Chari, "A comparison of finite element and boundary element formulations for three-dimensional magnetostatic problems," *IEEE Trans. Magn.*, vol. 20, pp. 1950–1952, 1984.
[3] W. M. Rucker and K. R. Richter, "Three-dimensional magnetostatic field calculation using boundary element method," *IEEE Trans. Magn.*, vol. 24, pp. 23–26, 1988.
[4] J. Shen, Z. Andjelic, and B. Schaub, "A hybrid single and dual layer boundary integral equation formulation for 3-D eddy currents," *IEEE Trans. Magn.*, vol. 34, pp. 2636–2639, 1998.
[5] B. Krstajic *et al.*, "Nonlinear 3D magnetostatic field calculation by the integral equation method with surface and volume magnetic charges," *IEEE Trans. Magn.*, vol. 28, pp. 1088–1091, 1992.
[6] I. D. Mayergoyz *et al.*, "A new scalar potential formulation for three-dimensional magnetostatic problems," *IEEE Trans. Magn.*, vol. 23, pp. 3889–3894, 1987.