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Accuracy improvement in nonlinear magnetostatic field computations with integral equation methods and indirect total scalar potential formulations

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Abstract

Purpose – The paper seeks to solve nonlinear magnetostatic field problems with the integral equation method and different indirect formulations.

Design/methodology/approach – To avoid large cancellation errors in cases where the demagnetizing field is high a difference field concept is used. This requires the computation of sources of the scalar potential of the excitation field.

Findings – A new formulation to compute these sources is presented. The improved computational accuracy is demonstrated with numerical examples.

Originality/value – The paper develops a novel formulation for the computation of sources of scalar excitation potential.

Keywords Magnetic fields, Numerical analysis

Paper type Research paper

Introduction

The integral equation method is an attractive way to solve magnetostatic field problems since the number of unknowns is always minimal. If linear problems are solved, neither the volume of the magnetizable material nor the surrounding air but only its surface has to be discretized. In the case of nonlinear problems, only the volume of the material has to be meshed additionally. Especially indirect formulations provide several advantages. For example, compared to direct formulation, there is only one unknown per node and there is no domain discrimination necessary to assemble the system matrix.

If the demagnetizing field (DF) is small compared to the excitation field, a reduced scalar potential formulation (RSP) can be used (Brebbia, 1978). In practice, this is the case when the iron core has a ring or frame shape (Bertotti, 1998). If the DF is large, huge cancellation errors occur. They can be avoided by using a total scalar potential formulation (TSP) (Mayergoyz *et al.*, 1987). In the indirect case, this is the so-called difference field concept (Kim *et al.*, 2000). It requires computation of sources of the exciting field on the surface of the magnetic material. We developed an alternative

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formulation for the computation of these sources since their accuracy mainly determines the one of the final solution.

Integral equation method

The magnetostatic field can be split into the sum of the known solenoidal excitation field H_C and the unknown induced irrotational field $\mathbf{H}_i$. The latter is caused by a magnetization $\mathbf{M}$ of the magnetic body $\mathbf{V}$:

$$\mathbf{H}_\mathrm{i} = -\nabla \int_\mathrm{V} \mathbf{M}(\mathbf{r}') \cdot \nabla' G(\mathbf{r}, \mathbf{r}')\mathrm{dV}', \tag{1}$$

which can be transformed with equivalent magnetic surface charges $\sigma = M \cdot \mathbf{n}$ and volume charges $\rho = \nabla \cdot \mathbf{M}$ to:

$$\mathbf{H}_\mathrm{i}(\mathbf{r}) = \oint_\mathrm{A} \sigma(\mathbf{r}')\nabla G(\mathbf{r}, \mathbf{r}')\mathrm{dA}' - \int_\mathrm{V} \rho(\mathbf{r}')\nabla G(\mathbf{r}, \mathbf{r}')\mathrm{dV}' \tag{2}$$

Together, this yields an equation for the unknown field strength:

$$\mathbf{H} = \mathbf{H}_\mathrm{C} + \mathbf{H}_\mathrm{i}. \tag{3}$$

Demanding $\operatorname{div} B = 0$ for points on the surface of the magnetic body yields an integral equation for the unknown equivalent surface and volume charges (McWhirter *et al.*, 1982; Lindholm, 1980):

$$\sigma(\mathbf{r}) - 2\lambda(\mathbf{r})\left[\int_\mathrm{A} \sigma(\mathbf{r}')\partial_\mathbf{n} G(\mathbf{r},\mathbf{r}')\mathrm{dA}' + \int_\mathrm{V} \rho(\mathbf{r}')\partial_\mathbf{n} G(\mathbf{r},\mathbf{r}')\mathrm{dV}'\right] = 2\lambda(\mathbf{r})\mathbf{n}(\mathbf{r})\cdot\mathbf{H}_\mathrm{C}(\mathbf{r}). \tag{4}$$

with the normal vector $\mathbf{n}$ pointing outwards from the body's surface $\mathbf{A}$ and:

$$\lambda(r) = \frac{\mu_r(\mathbf{r}) - 1}{\mu_r(\mathbf{r}) + 1}. \tag{5}$$

In general equation (4) gives inaccurate results, if the DF is small. This problem can be tackled by a TSP approach (Mayergoyz *et al.*, 1987) where in a first computation step sources of the excitation field are computed upon $\mathbf{A}$. They can be obtained by assuming infinite permeability so that with equation (3):

$$\mathbf{H} = \mathbf{H}_\mathrm{C} + \mathbf{H}_\mathrm{i}|_{\mu_r \to \infty} = 0 \tag{6}$$

which means that the sources of $\mathbf{H}_i$ are sources of $\mathbf{H}_C$ with opposite sign. Setting $\lambda = 1$ in equation (4) results in:

$$\frac{1}{2}\sigma_\infty(\mathbf{r}) + \int_\mathrm{A} \sigma_\infty(\mathbf{r}')\partial_\mathbf{n} G(\mathbf{r},\mathbf{r}')\mathrm{dA}' = \mathbf{n}(\mathbf{r})\cdot\mathbf{H}_\mathrm{C}(\mathbf{r}). \tag{7}$$

Once equation (7) has been solved, it can be used to substitute the right-hand side in equation (4) which gives:

$$\frac{1}{2}\delta\sigma(\mathbf{r})+\lambda(\mathbf{r})\left[\int_{\mathrm{A}}\delta\sigma(\mathbf{r}')\partial_{\mathbf{n}}G(\mathbf{r},\mathbf{r}')\mathrm{dA}'+\int_{\mathrm{V}}\rho(\mathbf{r}')\partial_{\mathbf{n}}G(\mathbf{r},\mathbf{r}')\mathrm{dV}'\right]=\frac{\lambda(r)-1}{2}\sigma_{\infty}(r) \quad (8)$$

where $\delta\sigma = \sigma - \sigma_{\infty}$ which are the sources of the difference field of the one with infinite permeability and the actual one. Whilst in linear problems, volume charges do not occur, for nonlinear problems, an additional equation for ρ is necessary. It is obtained by demanding $\mathrm{div}\mathbf{B}=0$ within $\mathbf{V}$. Two formulations have been investigated in this context (Krstajić *et al.*, 1992; Hafla *et al.*, 2005).

We experienced that equation (7) is inaccurate in cases where the DF is not very large compared to the excitation field. Therefore, we developed a novel formulation to compute σ_{∞}. The idea we hereby pursued was to exploit the analogy between a magnetizable body with infinite permeability to an electrical conductor. This can be seen by considering that tangential components of the magnetic field are continuous on **A**. For the magnetic induction follows:

$$\frac{1}{\mu_0\mu_r}B_{t,i} = \frac{1}{\mu_0}B_{t,o} \quad (9)$$

where the indices i and o denote values towards and opposite the normal vector direction, respectively. For infinite permeability $B_{t,o} = 0$ so:

$$H_{t,o} = 0 \quad (10)$$

which means that **A** is an equipotential surface. Also, Laplacian equation for the scalar magnetic potential holds. Together with equation (10), it can be followed that the potential is constant within V. The problem of computing σ_{∞} can, therefore, be solved analogously to finding the electrical surface charges upon a conductor of unknown potential.

With the magnetic scalar potential of the excitation field Ψ_{C} and the constant potential Ψ_{A} on the surface A:

$$\Psi_{\mathrm{A}} = \int_{\mathrm{A}} \sigma_{\infty}(r')G(r,r')\mathrm{dA}' + \Psi_{\mathrm{C}}(r) \quad (11)$$

which can be generalized to multiple bodies with surfaces $\mathbf{A}_i$:

$$-\sum_i \int_{\mathrm{A}_i} \sigma_{\infty}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\mathrm{dA}' + \sum_i \Psi_{\mathrm{A}_i} = \Psi_{\mathrm{C}}(\mathbf{r}) \quad (12)$$

where the known Ψ_{C} has been put on the right-hand side. Since, Ψ_{A_i} are unknown, additional equations are required. These are:

$$\oint_{\mathrm{A}_i} \sigma_{\infty}(r)\mathrm{dA} = 0 \quad (13)$$

because free equivalent charges do not exist. The coupled system equations (12) and (13) is a novel and alternative formulation to equation (7). After it has been solved,

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the sources of the TSP can be computed with equation (8). The potential Ψ_C can be determined as described in Hafla *et al.* (2005).

Numerical results

We discretized the proposed formulation with Galerkin's method and isoparametric second order elements. As linear solver GMRES in combination with a Jacobi preconditioner was used. Nonlinear problems can be tackled by solving stepwise linear problems (Krstajić *et al.*, 1992; Hafla *et al.*, 2005), therefore, only linear examples are presented. Volume meshes are used for field evaluation only. Our calculations were run on a 2 GHz Intel Xeon computer. Details to the investigated problems are listed in Table I:

Steel plate

The problem of the steel plate shown in Figure 1 has been solved. Geometrical dimensions are identical to those from TEAM Workshop Problem 13 (Nakata *et al.*, 1990). The magnetic field $\mathbf{H}_{\sigma_\infty}$ due to σ_∞ has been computed along the *z*-axis which lies within the steel plate and on the symmetry axis of the coil which has 1,000AT. According to equation (6), the accurate value $\mathbf{H}_{\sigma_\infty}$ is identical to $\mathbf{H}_\mathrm{i}$. Therefore, the deviation between the calculated field $\mathbf{H}_{\sigma_\infty}$ and $\mathbf{H}_\mathrm{i}$ gives the numerical error which is shown in Figure 2. The proposed formulation to compute σ_∞ is superior to the conventional formulation equation (7).

C-shaped magnet

The magnetic field of the C-shaped magnet with circular cross-section shown in Figure 3 has been investigated. The relative permeability of the iron core is 100 and the ampereturns are 1,000. For comparison, the solution obtained by BEM-FEM (Kurz *et al.*, 1997) computation was used. The results shown in Figure 4 show that the proposed TSP formulation gives better accuracy then the conventional TSP and the RSP formulations.

Table I.
DOF and computation time for σ_∞ of investigated problems

	DOF of σ_∞	Computation time for σ_∞ in seconds: Conventional formulation	Proposed formulation (time accounting for computation of Ψ_C)
Team 13 plate	2,906	406	641 (407)
C-shaped magnet	1,826	183	357 (250)
Contactor	12,530	1,119	1, 676 (300)

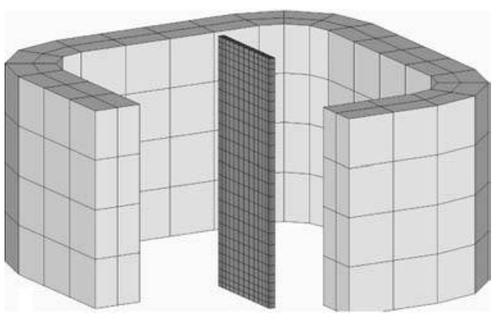

Figure 1.
Plate from TEAM Workshop Problem 13

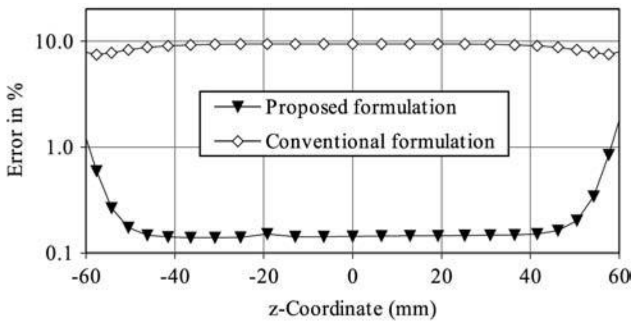

Figure 2.
Error of magnetic field due to σ_∞

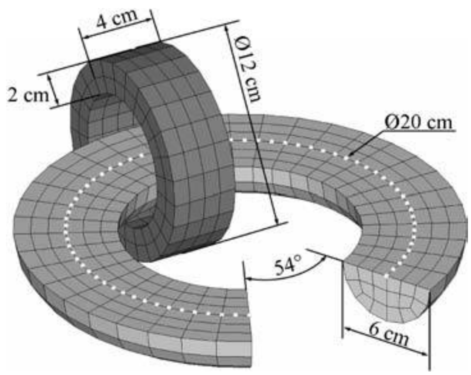

Figure 3.
C-shaped magnet. For graphical representation, the upper half of the magnetizable material is omitted to show the dotted evaluation path

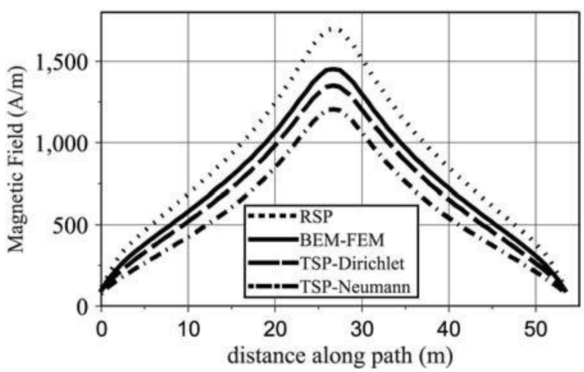

Figure 4.
Norm of magnetic field along evaluation path

Contactor

As an example with multiple magnetizable parts, the method has been applied to the contactor shown in Figure 5(a). The total height of the set-up is 61 mm and a relative permeability 100 was assumed. With the proposed formulation, the agreement of magnetic field computed along the evaluation path shown in Figure 5(b) with results obtained BEM-FEM computation is better than with the conventional formulation (Figure 6).

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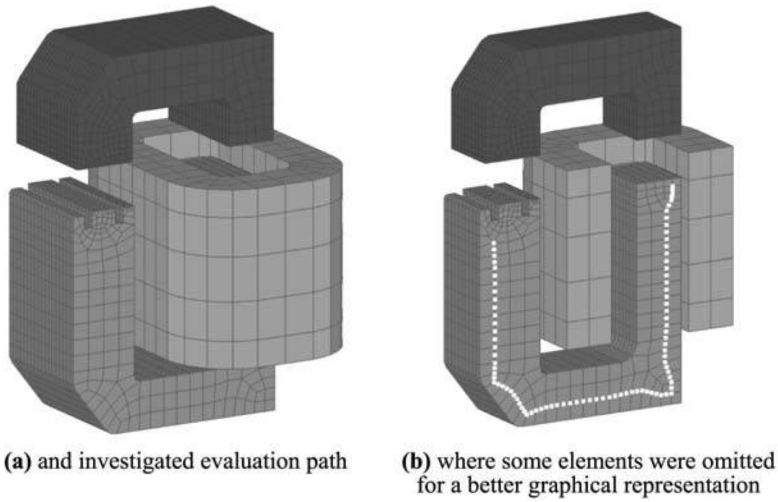

Figure 5.
Used mesh of investigated contactor

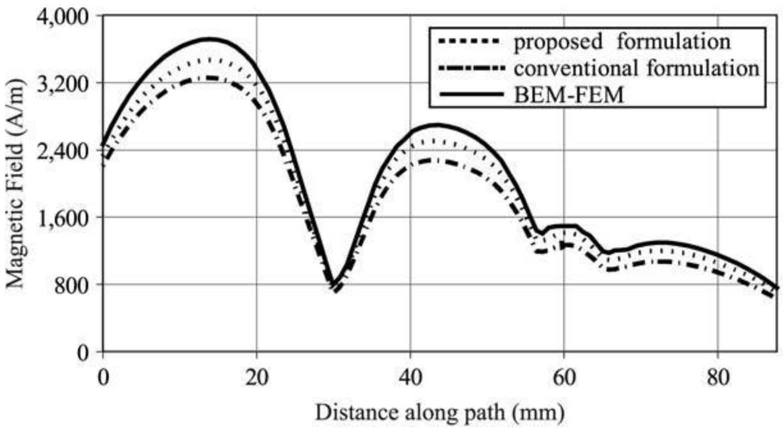

Figure 6.
Magnetic field computed along evaluation path in Figure 5 (b)

The potentials Ψ_{A_1} and Ψ_{A_2} of the upper and the lower part appear as unknowns in equation (12). They have been computed to $\Psi_{A_1} = 29.5\,\mathrm{A}$ and $\Psi_{A_2} = 73.0\,\mathrm{A}$,, respectively. Owing to numerical errors, the total magnetic charge does not vanish as demanded in equation (13). The residual charges are $\oint_{A_1} \sigma_\infty(r)\mathrm{dA} = -2.6 \times 10^{-4}\,\mathrm{Vs}$ and $\oint_{A_2} \sigma_\infty(r)\mathrm{dA} = 1.9 \times 10^{-4}\,\mathrm{Vs}$.

Conclusions

A novel formulation for the computation of sources of scalar excitation potential has been developed that exploits an analogy to electrostatics. It allows application of the difference field concept for a wider range of problems with increased accuracy. It can be applied to both linear and nonlinear problems.

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