Demo-PCA

February 7, 2020

[1]:
```
%pylab inline
```

```
Populating the interactive namespace from numpy and matplotlib
```

[2]:
```
from Python_code import examples as eg
import numpy as np
from numpy import *
import dionysus
```

The circular coordinates pipeline for examining different smoothness cost-functions:

```
Step 1. Getting the point cloud
Step 2. Computing the Vietoris-Rips filtration and its cohomology
Step 3. Selecting the Cocycle
Step 4. First smoothing using Least Squares (Optional)
Step 5. Second smoothing using a new cost function
```

0.1 Perform PCA (with different components) and compute the bottleneck distance between the PCA+CC persistent diagrams and the CC persistent diagrams above.

[3]:
```
import numpy as np
from Python_code import PCAtool as PCAtool
X=np.random.rand(300,2)
print(X)
X_new=PCAtool.pca(X,K=2)
import matplotlib.pyplot as plt
X_cen=X-np.mean(X,axis=0)
plt.scatter(X_cen[:,0], X_cen[:,1])
plt.scatter(X_new[:,0], X_new[:,1])
plt.show()
```

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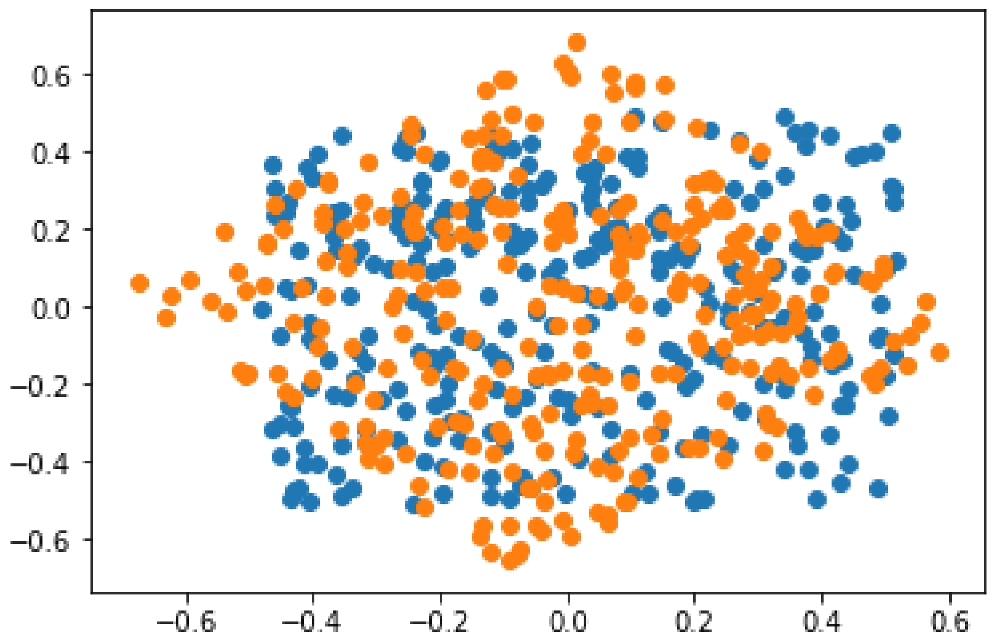

[4]:
```
#Let us compute the bottle-neck distance between full dataset and k-PCA dataset.
↪ k denotes the number of principal components.
prime=23
D=20
dat1=np.random.rand(50,D)
vr = dionysus.fill_rips(dat1, 2, 2.) #Vietoris-Rips complex
cp = dionysus.cohomology_persistence(vr, prime, True) #Create the persistent␣
↪cohomology
dgms = dionysus.init_diagrams(cp, vr) #Calculate the persistent diagram using␣
↪the designated coefficient field.
#record for plottings..
A=np.array([0,1,1])


for k in range(D):
    dat_k = PCAtool.pca(dat1,K=k+1)
    vr_k = dionysus.fill_rips(dat_k, 2, 4.) #Vietoris-Rips complex
```

```
    cp_k = dionysus.cohomology_persistence(vr_k, prime, True) #Create the␣
↪persistent cohomology
    dgms_k = dionysus.init_diagrams(cp_k, vr_k) #Calculate the persistent␣
↪diagram using the designated coefficient field.
    bdist_0 = dionysus.bottleneck_distance(dgms[0], dgms_k[0])
    bdist_1 = dionysus.bottleneck_distance(dgms[1], dgms_k[1])
    #dionysus.plot.plot_diagram(dgms_k[1], show=True)
    newrow=[k+1,bdist_0,bdist_1]
    #print(newrow)
    A = numpy.vstack([A, newrow])

print(A)
print(A[1,2])

fig, ax = plt.subplots()
ax.plot(A[:,0],A[:,1],c='b')#dimension 0 diagram distance
ax.plot(A[:,0],A[:,2],c='r')#dimension 1 diagram distance
ax.set(xlabel='# of principal components (k)', ylabel='Bottleneck distance',
       title='Bottleneck distance between full sample diagram and k-PCA sample␣
↪diagrams')
```

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[4]:

```
[Text(0,0.5,'Bottleneck distance'),
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 Text(0.5,1,'Bottleneck distance between full sample diagram and k-PCA sample
```

```
diagrams')]
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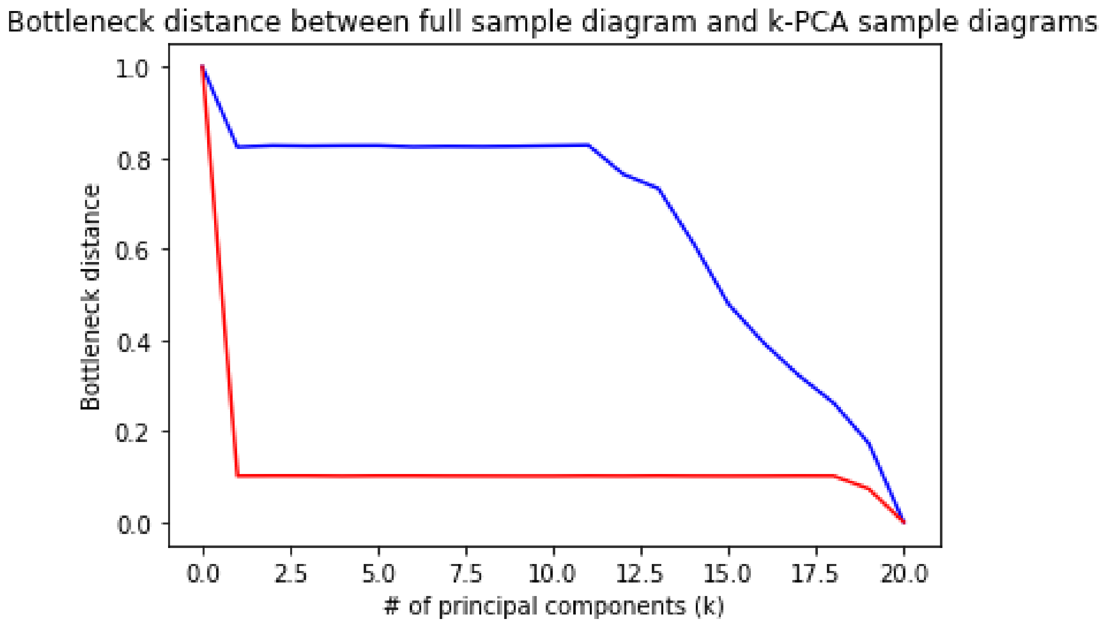

[5]:
```
type(dgms)
type(dgms[0])
#From diagrams to arrays
#np.array(dgms[0])
#From arrays to diagrams
#print( type([(0,1),(0.5,0.7)]) )
dionysus._dionysus.Diagram([(0,1),(0.5,0.7)])
```

[5]:
```
Diagram with 2 points
```

[6]:
```
def denoise_dgm(dgms,threshold=0.1):
    threshold = .1
    bars_0 = [np.array(bar).tolist() for bar in dgms[0] if bar.death-bar.birth␣
 ↪> threshold] #choosing cocycle that persist at least threshold=1.
    bars_1 = [np.array(bar).tolist() for bar in dgms[1] if bar.death-bar.birth␣
 ↪> threshold] #choosing cocycle that persist at least threshold=1.
    #cocycles = [cp.cocycle(bar.data) for bar in bars]
    #plt is the matplotlib incarnation.
    #print( type(bars_1) )
    #bars_1=np.array(bars_1)
    #print(type(bars_1[1]))
    #print(bars_1[1])
    #print(type(bars_1))
```

```
    dgm_denoise_0=bars_0
    dgm_denoise_1=bars_1
    #print(type(dgm_denoise_0))
    #dionysus.plot.plot_diagram(dgm_denoise_0, show=False)
    #dionysus.plot.plot_diagram(dgm_denoise_1, show=False)
    return([dgm_denoise_0,dgm_denoise_1])

'''
#Red highlight cocyles that persist more than threshold value on barcode, when␣
↪more than one cocyles have persisted over threshold values, this plots the␣
↪first one.
dionysus.plot.plot_bars(dgms[1], show=False)
plt.plot([[bar.birth,bar.death] for bar in dgms[1] if bar.death-bar.birth >␣
↪threshold][0],[[x,x] for x,bar in enumerate(dgms[1]) if bar.death-bar.birth␣
↪> threshold][0],'r')
plt.title('Showing the selected cycles on bar codes (red bars)')
plt.show()

#Red highlight ***ALL*** cocyles that persist more than threshold value on␣
↪diagram.
dionysus.plot.plot_diagram(dgms[1], show=False)
Lt1 = [[point.birth,point.death] for point in dgms[1] if point.death-point.
↪birth > threshold]
for Lt3 in Lt1:
    #print(Lt3)
    plt.plot(Lt3[0],Lt3[1],'ro')
plt.title('Showing the selected cycles on diagram (red points)')
plt.show()
'''
print(dgms_k)
test1=denoise_dgm(dgms_k)
test1[0]
```

```
[Diagram with 50 points, Diagram with 49 points, Diagram with 18424 points]
```

[6]:
```
[(0,inf),
 (0,1.40403),
 (0,1.3332),
 (0,1.4289),
 (0,1.40229),
 (0,1.403),
 (0,1.34595),
 (0,1.32887),
 (0,1.37156),
 (0,1.29584),
 (0,1.39248),
 (0,1.64584),
```

```
(0,1.23297),
(0,1.47286),
(0,1.39124),
(0,1.19594),
(0,1.12187),
(0,1.28163),
(0,1.4383),
(0,1.4698),
(0,1.20281),
(0,1.25048),
(0,1.31941),
(0,1.33678),
(0,1.17107),
(0,1.34541),
(0,1.42859),
(0,1.32976),
(0,1.54038),
(0,1.23253),
(0,1.32246),
(0,1.42239),
(0,1.2203),
(0,1.34523),
(0,1.22457),
(0,1.22807),
(0,1.33558),
(0,1.32693),
(0,1.08975),
(0,1.24706),
(0,1.20635),
(0,1.31939),
(0,1.53156),
(0,1.13285),
(0,1.01112),
(0,1.13553),
(0,1.30935),
(0,1.36507),
(0,1.27896),
(0,1.54502)]
```

[9]:

```
'''
#This is legacy code used to construct a dionysus.plot.plot_diagram object from␣
↪raw, it is necessary to compute bottleneck distance.
dgms_tmp=denoise_dgm(dgms_k,0.1)
dgms_base_0=dionysus._dionysus.Diagram()
dgms_base_1=dionysus._dionysus.Diagram()
#pybind issue, need to be correct instead of succint.
for itr in range(len(dgms_tmp[0])):
```

```
    dgms_base_0.append(dgms_tmp[0][itr])
for itr in range(len(dgms_tmp[1])):
    dgms_base_1.append(dgms_tmp[1][itr])
print(type(dgms_base[0]))
dionysus.bottleneck_distance(dgms_base_0, dgms_base_1)
'''
```

[9]:

```
'\ndgms_tmp=denoise_dgm(dgms_k,0.1)\ndgms_base_0=dionysus._dionysus.Diagram()\nd
gms_base_1=dionysus._dionysus.Diagram()\n#pybind issue, need to be correct
instead of succint.\nfor itr in range(len(dgms_tmp[0])):\n
dgms_base_0.append(dgms_tmp[0][itr])\nfor itr in range(len(dgms_tmp[1])):\n    d
gms_base_1.append(dgms_tmp[1][itr])\nprint(type(dgms_base[0]))\ndionysus.bottlen
eck_distance(dgms_base_0, dgms_base_1)\n'
```

[10]:

```
#Let us compute the bottle-neck distance between full dataset and k-PCA dataset.
 ↪ k denotes the number of principal components.
prime=23
D=10
#Here is the data insertion part
dat1=np.random.rand(50,D)
vr = dionysus.fill_rips(dat1, 2, 2.) #Vietoris-Rips complex
cp = dionysus.cohomology_persistence(vr, prime, True) #Create the persistent␣
 ↪cohomology
dgms = dionysus.init_diagrams(cp, vr) #Calculate the persistent diagram using␣
 ↪the designated coefficient field.
#record for plottings..
A=np.array([0,0,1,1])

for thres in [0,0.1]:
    ######Thresholding the features on the persistent diagrams describing␣
 ↪persistent cohomology of VR complex(CC)
    dgms_tmp=denoise_dgm(dgms,thres)#thresholding CC diagrams
    dgms_base_0=dionysus._dionysus.Diagram()#take persistent points of dim 0
    dgms_base_1=dionysus._dionysus.Diagram()#take persistent points of dim 1
    #pybind issue, need to be correct instead of succint.
    for itr in range(len(dgms_tmp[0])):
        dgms_base_0.append(dgms_tmp[0][itr])
    for itr in range(len(dgms_tmp[1])):
        dgms_base_1.append(dgms_tmp[1][itr])
    #print(type(dgms_base[0]))
    #dionysus.bottleneck_distance(dgms_base_0, dgms_base_1)
    #######
    for k in range(D):
        dat_k = PCAtool.pca(dat1,K=k+1)
        vr_k = dionysus.fill_rips(dat_k, 2, 4.) #Vietoris-Rips complex
        cp_k = dionysus.cohomology_persistence(vr_k, prime, True) #Create the␣
 ↪persistent cohomology
```

```
        dgms_k = dionysus.init_diagrams(cp_k, vr_k) #Calculate the persistent␣
↪diagram using the designated coefficient field.
        #######Thresholding the features on the persistent diagrams describing␣
↪persistent cohomology of kPCA-VR complex(kPCA)
        #############Here we use the same thresholding for fairness. But this␣
↪configuration is easy to change.
        PCA_thres=thres
        #############
        dgms_k_tmp=denoise_dgm(dgms_k,PCA_thres)#thresholding CC diagrams
        dgms_k_0=dionysus._dionysus.Diagram()#take persistent points of dim 0
        dgms_k_1=dionysus._dionysus.Diagram()#take persistent points of dim 1
        #pybind issue, need to be correct instead of succint.
        for itr in range(len(dgms_k_tmp[0])):
            dgms_k_0.append(dgms_k_tmp[0][itr])
        for itr in range(len(dgms_k_tmp[1])):
            dgms_k_1.append(dgms_k_tmp[1][itr])
        #print(type(dgms_base[0]))
        #dionysus.bottleneck_distance(dgms_base_0, dgms_base_1)
        #######
        bdist_0 = dionysus.bottleneck_distance(dgms_base_0, dgms_k_0)
        bdist_1 = dionysus.bottleneck_distance(dgms_base_1, dgms_k_1)
        #dionysus.plot.plot_diagram(dgms_k[1], show=True)
        newrow=[k+1,thres, bdist_0,bdist_1]
        #print(newrow)
        A = numpy.vstack([A, newrow])
print(A)
#Column names of A
#1 number of principal components k;
#2 threshold used for filtering out topological features by persistence;
#3 Bottleneck distance between dimension 0 diagrams of CC and PCA
#3 Bottleneck distance between dimension 1 diagrams of CC and PCA
```

```
[[0.00000000e+00 0.00000000e+00 1.00000000e+00 1.00000000e+00]
 [1.00000000e+00 0.00000000e+00 5.32168150e-01 8.48593563e-02]
 [2.00000000e+00 0.00000000e+00 5.33484817e-01 8.48593563e-02]
 [3.00000000e+00 0.00000000e+00 5.33195853e-01 8.48593563e-02]
 [4.00000000e+00 0.00000000e+00 5.33411741e-01 8.46603960e-02]
 [5.00000000e+00 0.00000000e+00 5.30841947e-01 8.50518048e-02]
 [6.00000000e+00 0.00000000e+00 4.94617045e-01 8.49809349e-02]
 [7.00000000e+00 0.00000000e+00 3.85514319e-01 8.47640336e-02]
 [8.00000000e+00 0.00000000e+00 3.00773650e-01 8.47401023e-02]
 [9.00000000e+00 0.00000000e+00 1.68912739e-01 8.53575021e-02]
 [1.00000000e+01 0.00000000e+00 1.19829949e-07 5.98311871e-08]
 [1.00000000e+00 1.00000000e-01 5.32168150e-01 8.48593563e-02]
 [2.00000000e+00 1.00000000e-01 5.33484817e-01 8.48593563e-02]
 [3.00000000e+00 1.00000000e-01 5.33195853e-01 8.48593563e-02]
 [4.00000000e+00 1.00000000e-01 5.33411741e-01 8.46603960e-02]
```

```
[5.00000000e+00 1.00000000e-01 5.30841947e-01 8.50518048e-02]
[6.00000000e+00 1.00000000e-01 4.94617045e-01 8.49809349e-02]
[7.00000000e+00 1.00000000e-01 3.85514319e-01 8.47640336e-02]
[8.00000000e+00 1.00000000e-01 3.00773650e-01 8.47401023e-02]
[9.00000000e+00 1.00000000e-01 1.68912739e-01 8.53575021e-02]
[1.00000000e+01 1.00000000e-01 1.19829949e-07 5.98311871e-08]]
```

[ ]: