

Reply With Quote

Opalg
MHB Oldtimer

MHB Moderator

MHB Math Helper



Status: Offline

Join Date: Feb 2012

Location: Leeds, UK

Posts: 1,480

Thanks: 481 times

Thanked: 4510 times

Awards: 

DECEMBER 23RD, 2012, 12:21

#3

Originally Posted by **matqkks**

I have been trying to prove the following result:
If A is real symmetric matrix with an eigenvalue λ of multiplicity m then λ has m linearly independent e.vectors.
Is there a simple proof of this result?

This is a slight variation of Deveno's argument. I will assume you already know that the eigenvalues of a real symmetric matrix are all real.

Let A be an $n \times n$ real symmetric matrix, and assume as an inductive hypothesis that all $(n-1) \times (n-1)$ real symmetric matrices are diagonalisable. Let λ be an eigenvalue of A , with a normalised eigenvector x_1 . Using the Gram-Schmidt process, form an orthonormal basis $\{x_1, x_2, \dots, x_n\}$ with that eigenvector as its first element.

Let P be the $n \times n$ matrix whose columns are x_1, x_2, \dots, x_n , and denote by $T: \mathbb{R}^n \rightarrow \mathbb{R}^n$ the linear transformation whose matrix with respect to the standard basis is A . Then P is an orthogonal matrix ($P^T = P^{-1}$), and the matrix of T with respect to the basis $\{x_1, x_2, \dots, x_n\}$ is $P^T A P$. The (i, j) -element of that matrix is $(P^T A P)_{ij} = \langle A x_j, x_i \rangle$. In particular, the elements in the first column are

$$(P^T A P)_{i1} = \langle A x_1, x_i \rangle = \langle \lambda x_1, x_i \rangle = \begin{cases} \lambda & (i = 1) \\ 0 & (i > 1) \end{cases}$$

(because the vectors x_i are orthonormal). Thus the first column of $P^T A P$ has λ as its top element, and 0 for each of the other elements. Since $P^T A P$ is symmetric, the top row also consists of a λ followed by all zeros. Hence the matrix $P^T A P$ looks like this:

$$\begin{bmatrix} \lambda & 0 & \dots & 0 \\ 0 & & & \\ \vdots & & B & \\ 0 & & & \end{bmatrix},$$

where B is an $(n-1) \times (n-1)$ real symmetric matrix. By the inductive hypothesis, B is diagonalisable, so there is an orthogonal $(n-1) \times (n-1)$ matrix Q such that $Q^T B Q$ is

diagonal. Let

$$R = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & & & \\ \vdots & & Q & \\ 0 & & & \end{bmatrix}.$$

Then $R^T P^T A P R$ is diagonal, as required.

Was sich überhaupt sagen lässt, lässt sich klar sagen; und wovon man nicht reden kann, darüber muss man schweigen.

(Anything that can be said at all, can be said clearly; and whereof one cannot speak, thereon one must be silent.)

– Ludwig Wittgenstein's good advice for forum contributors, in *Tractatus Logico-Philosophicus*.

[Reply With Quote](#)

[▲ Go to First Post](#)

« Singular Value decomposition | An inverse of the adjoint »

Similar Threads

[SOLVED] Incorporate axis symmetric case

By dwsmith in forum Mathematics Software and Calculator Discussion

Replies: 0

Last Post: December 6th, 2012, 14:56

Inverse of a Symmetric Matrix

By OhMyMarkov in forum Linear and Abstract Algebra

Replies: 2

Last Post: October 6th, 2012, 11:28

[SOLVED] Eigenvalues

By Sudharaka in forum Pre-Algebra and Algebra

Replies: 2

Last Post: May 27th, 2012, 09:02

Matrix Theory...showing that matrix is Unitary

By cylv89 in forum Linear and Abstract Algebra

Replies: 3

Last Post: March 27th, 2012, 17:00

A and B are two symmetric matrices

By Yankel in forum Linear and Abstract Algebra

Replies: 4

Last Post: January 27th, 2012, 09:17

-- Math Help Boards

Have a small screen? Select Math Help Boards for Small Screens!

Powered by vBulletin Copyright © 2000 - 2012, Jelsoft Enterprises Ltd.

Search Engine Optimisation provided by DragonByte SEO v1.0.15 (Pro) - vBulletin Mods & Addons Copyright © 2014 DragonByte Technologies Ltd.

Feedback Buttons provided by Advanced Post Thanks / Like (Pro) - vBulletin Mods & Addons Copyright © 2014 DragonByte Technologies Ltd.

© 2012-2014 Math Help Boards

[FORUMS](#)

[RULES](#)

[POTW](#)

[CONTACT](#)

[TOP](#)

