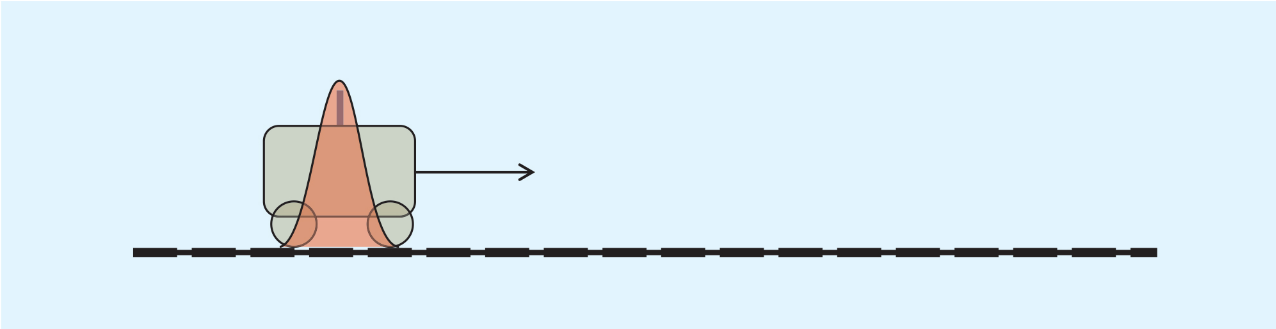
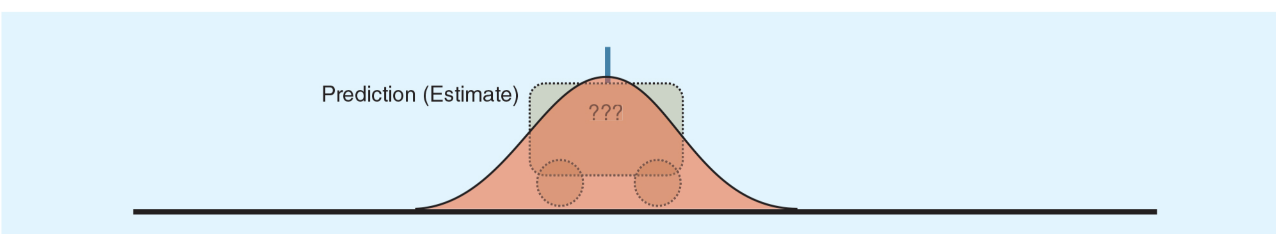
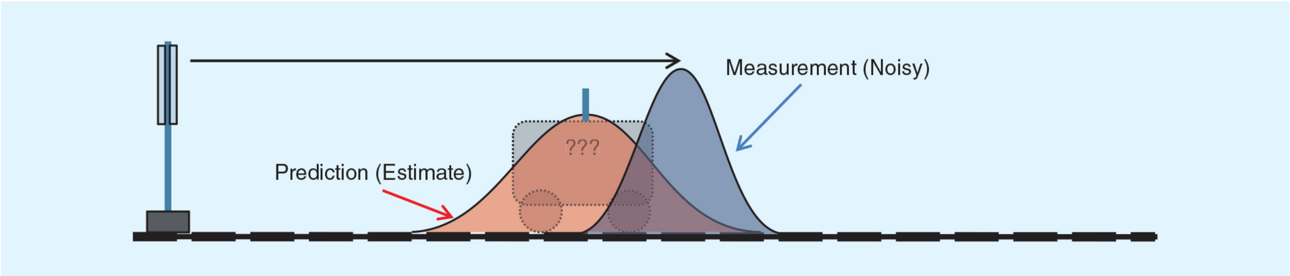


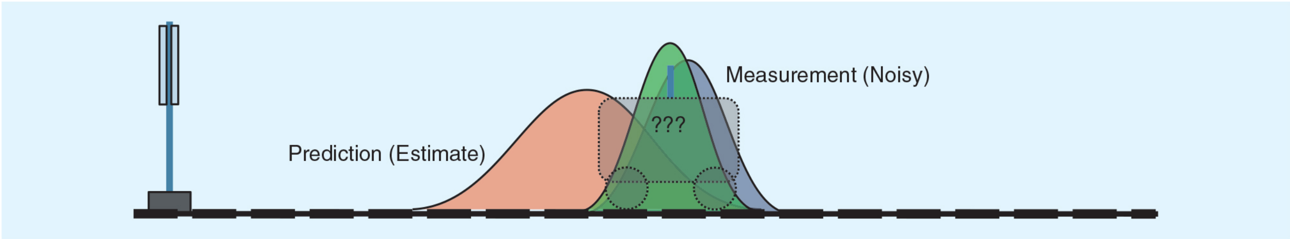
[FIG2] The initial knowledge of the system at time $t = 0$. The red Gaussian distribution represents the pdf providing the initial



[FIG3] Here, the prediction of the location of the train at time $t = 1$ and the level of uncertainty in that prediction is shown. The confidence in the knowledge of the position of the train has decreased, as we are not certain if the train has undergone any accelerations or decelerations in the intervening period from $t = 0$ to $t = 1$.



[FIG4] Shows the measurement of the location of the train at time $t = 1$ and the level of uncertainty in that noisy measurement, represented by the blue Gaussian pdf. The combined knowledge of this system is provided by multiplying these two pdfs together.



[FIG5] Shows the new pdf (green) generated by multiplying the pdfs associated with the prediction and measurement of the train's location at time $t = 1$. This new pdf provides the best estimate of the location of the train, by fusing the data from the prediction and the measurement.

Predict-Update Equations

First, recall the time discrete Kalman filter equations (see MI37) and the predict-update equations as below:

Predict:

$$\hat{\mathbf{x}}_{t|t-1} = \mathbf{F}_t \hat{\mathbf{x}}_{t-1|t-1} + \mathbf{B}_t \mathbf{u}_t$$
$$\mathbf{P}_{t|t-1} = \mathbf{F}_t \mathbf{P}_{t-1|t-1} \mathbf{F}_t^T + \mathbf{Q}_t$$

Update:

$$\hat{\mathbf{x}}_{t|t} = \hat{\mathbf{x}}_{t|t-1} + \mathbf{K}_t \left(\mathbf{y}_t - \mathbf{H}_t \hat{\mathbf{x}}_{t|t-1}\right)$$
$$\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{H}_t^T \left(\mathbf{H}_t \mathbf{P}_{t|t-1} \mathbf{H}_t^T + \mathbf{R}_t\right)^{-1}$$
$$\mathbf{P}_{t|t} = (\mathbf{I} - \mathbf{K}_t \mathbf{H}_t) \mathbf{P}_{t|t-1}$$

where

$\hat{\mathbf{x}}$: Estimated state.

$\mathbf{F}$: State transition matrix (i.e., transition between states).

$\mathbf{u}$: Control variables.

$\mathbf{B}$: Control matrix (i.e., mapping control to state variables).

$\mathbf{P}$: State variance matrix (i.e., error of estimation).

$\mathbf{Q}$: Process variance matrix (i.e., error due to process).

$\mathbf{y}$: Measurement variables.

$\mathbf{H}$: Measurement matrix (i.e., mapping measurements onto state).

$\mathbf{K}$: Kalman gain.

$\mathbf{R}$: Measurement variance matrix (i.e., error from measurements).

Subscripts are as follows: $t|t$ current time period, $t-1|t-1$ previous time period, and $t|t-1$ are intermediate steps.