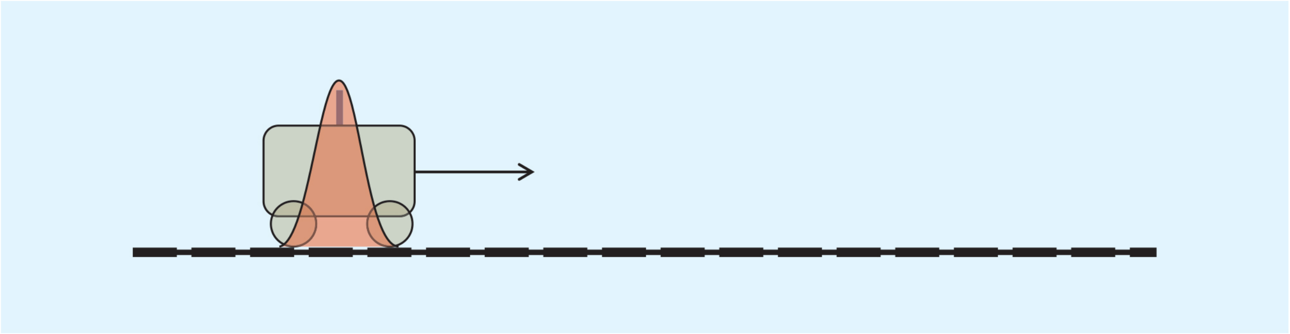
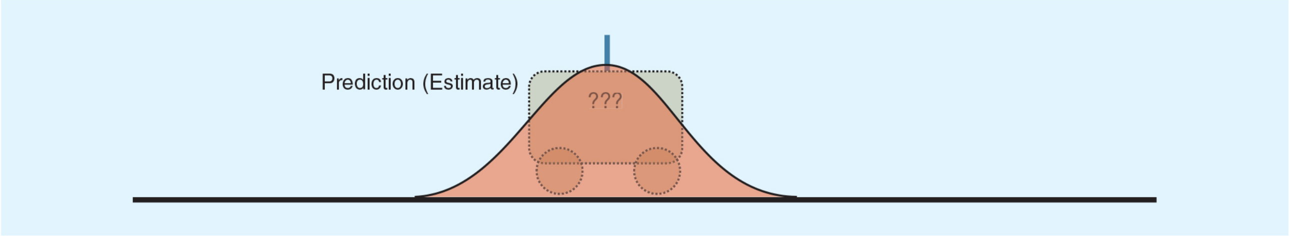
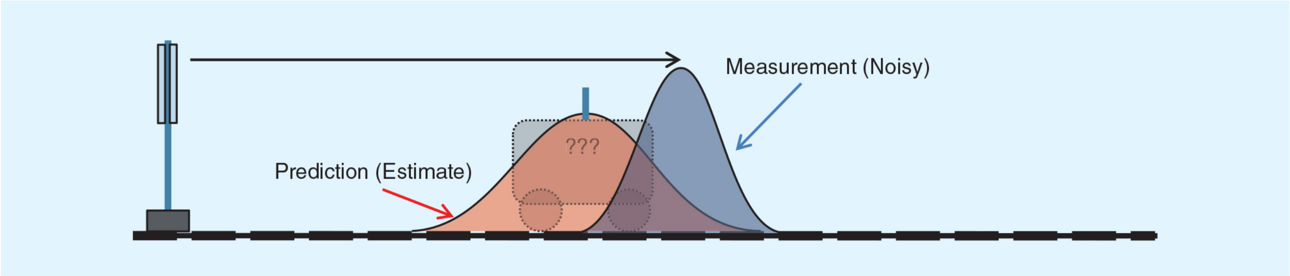


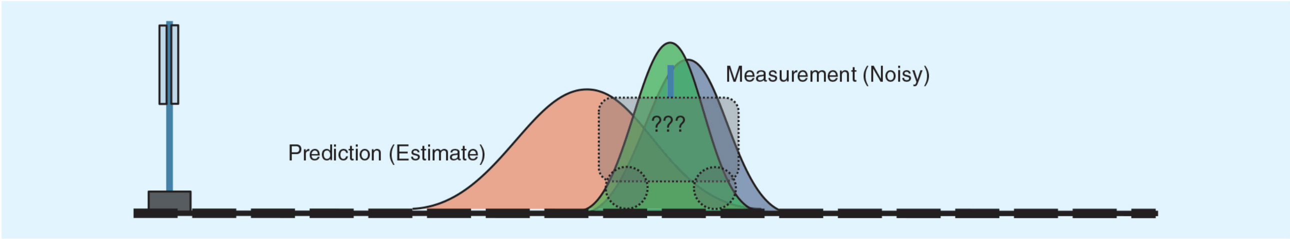
[FIG2] The initial knowledge of the system at time $t = 0$. The red Gaussian distribution represents the pdf providing the initial



[FIG3] **Here, the prediction of the location of the train at time $t = 1$ and the level of uncertainty in that prediction is shown. The confidence in the knowledge of the position of the train has decreased, as we are not certain if the train has undergone any accelerations or decelerations in the intervening period from $t = 0$ to $t = 1$.**



[FIG4] **Shows the measurement of the location of the train at time $t = 1$ and the level of uncertainty in that noisy measurement, represented by the blue Gaussian pdf. The combined knowledge of this system is provided by multiplying these two pdfs together.**



[FIG5] **Shows the new pdf (green) generated by multiplying the pdfs associated with the prediction and measurement of the train's location at time $t = 1$. This new pdf provides the best estimate of the location of the train, by fusing the data from the prediction and the measurement.**

The Kalman Filter

Definition of Terms:

$$
\begin{aligned}
\widehat{x}_{k|k} &:= \mathcal{E}\{x_k | y_0, \cdots, y_k\} \\
P_{k|k} &:= \mathcal{E}\{(x_k - \widehat{x}_{k|k})(x_k - \widehat{x}_{k|k})^\top | y_0, \cdots, y_k\}
\end{aligned}
$$

$$
\begin{aligned}
\widehat{x}_{k+1|k} &:= \mathcal{E}\{x_{k+1} | y_0, \cdots, y_k\} \\
P_{k+1|k} &:= \mathcal{E}\{(x_{k+1} - \widehat{x}_{k+1|k})(x_{k+1} - \widehat{x}_{k+1|k})^\top | y_0, \cdots, y_k\}
\end{aligned}
$$

Initial Conditions:

$$
\widehat{x}_{0|-1} := \bar{x}_0 = \mathcal{E}\{x_0\}, \quad \text{and} \quad P_{0|-1} := P_0 = \mathrm{cov}(x_0)
$$

For $k \geq 0$

Measurement Update Step:

$$
\begin{aligned}
K_k &= P_{k|k-1} C_k^\top \left(C_k P_{k|k-1} C_k^\top + Q_k \right)^{-1} \\
&\quad \text{(Kalman Gain)} \\
\widehat{x}_{k|k} &= \widehat{x}_{k|k-1} + K_k \left(y_k - C_k \widehat{x}_{k|k-1} \right) \\
P_{k|k} &= P_{k|k-1} - K_k C_k P_{k|k-1}
\end{aligned}
$$

Time Update or Prediction Step:

$$
\begin{aligned}
\widehat{x}_{k+1|k} &= A_k \widehat{x}_{k|k} \\
P_{k+1|k} &= A_k P_{k|k} A_k^\top + G_k R_k G_k^\top
\end{aligned}
$$

End of For Loop (Just stated this way to emphasize the recursive nature of the filter)