why? because (U*AU)* = U*A*(U*)* = U*AU (since A* = A, if A is hermetian).

but...if T is upper-triangular, AND hermetian, it must be LOWER triangular, as well. which means when A is hermetian, T is DIAGONAL. furthermore, since the diagonal elements of T equal their own complex-conjugates, T is a REAL matrix.

observation #2: any real symmetric matrix is hermetian. thus any real symmetric matrix is unitarily similar to a real diagonal matrix, D.

observation #3: the eigenvalues of A are the eigenvalues of D, and vice-versa.

proof: det(xI - A) = det(U^{-1}(xI - A)U) = det(U^{-1}(xI)U - U^{-1}AU) = det(xI - D)

observation #4: since the eigenvalues of A (a real symmetric matrix) are real, the eigenvectors are likewise real. thus we may take U to be a real unitary matrix, that is, an orthogonal one.

now suppose that a real, symmetric matrix A has an eigenvalue of (algebraic) multiplicity m. since A is orthogonally similar to a diagonal matrix D with the same eigenvalues (via an orthogonal matrix Q), D has m diagonal entries that are the same.

note that Q^{-1}AQ = D, means AQ = QD. since Q is invertible, its columns are linearly independent (it's square of full rank).

suppose $\{u_{j_1},\dots,u_{j_m}\}$ are the columns of Q corresponding to the diagonal entry λ of D repeated m times (the eigenvalue of algebraic multiplicity m). then:

$$A(u_{j_k}) = (u_{j_k})^T D = \lambda u_{j_k}\ k = 1,\dots,m$$

giving m linearly independent eigenvectors of the eigenspace E_λ.

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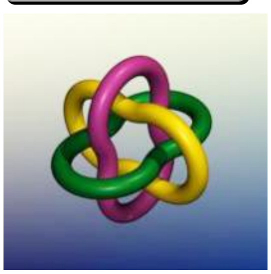

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DECEMBER 23RD, 2012, 12:21 **#3**

> Originally Posted by **matqkks**
>
> I have been trying to prove the following result:
> If A is real symmetric matrix with an eigenvalue lambda of multiplicity m then lambda has m linearly independent e.vectors.
> Is there a simple proof of this result?

This is a slight variation of Deveno's argument. I will assume you already know that the eigenvalues of a real symmetric matrix are all real.

Let A be an $n\times n$ real symmetric matrix, and assume as an inductive hypothesis that all $(n-1)\times(n-1)$ real symmetric matrices are diagonalisable. Let λ be an eigenvalue of A, with a normalised eigenvector x_1. Using the Gram–Schmidt process, form an orthonormal basis $\{x_1,x_2,\ldots,x_n\}$ with that eigenvector as its first element.

Let P be the $n\times n$ matrix whose columns are $x_1,x_2,\ldots,x_n$, and denote by $T:\mathbb{R}^n\to\mathbb{R}^n$ the linear transformation whose matrix with respect to the standard basis is A. Then P is an orthogonal matrix ($P^{\mathrm{T}} = P^{-1}$), and the matrix of T with respect to the basis $\{x_1,x_2,\ldots,x_n\}$ is $P^{\mathrm{T}}AP$. The (i,j)-element of that matrix is $(P^{\mathrm{T}}AP)_{ij} = \langle Ax_j,x_i\rangle$. In particular, the elements in the first column are

$$(P^{\mathrm{T}}AP)_{i1} = \langle Ax_1,x_i\rangle = \langle \lambda x_1,x_i\rangle = \begin{cases}\lambda & (i=1)\\ 0 & (i>1)\end{cases}$$

(because the vectors x_i are orthonormal). Thus the first column of $P^{\mathrm{T}}AP$ has λ as its top element , and 0 for each of the other elements. Since $P^{\mathrm{T}}AP$ is symmetric, the top row also consists of a λ followed by all zeros. Hence the matrix $P^{\mathrm{T}}AP$ looks like this:

$$\begin{bmatrix}\lambda & 0 & \ldots & 0\\ 0 & & & \\ \vdots & & B & \\ 0 & & & \end{bmatrix},$$

where B is an $(n-1)\times(n-1)$ real symmetric matrix. By the inductive hypothesis, B is diagonalisable, so there is an orthogonal $(n-1)\times(n-1)$ matrix Q such that $Q^{\mathrm{T}}BQ$ is diagonal. Let

$$R=\begin{bmatrix}1 & 0 & \dots & 0\\ 0 & & & \\ \vdots & & Q & \\ 0 & & & \end{bmatrix}.$$

Then $R^{\mathrm{T}}P^{\mathrm{T}}APR$ is diagonal, as required.

Was sich überhaupt sagen lässt, lässt sich klar sagen; und wovon man nicht reden kann, darüber muss man schweigen.

(Anything that can be said at all, can be said clearly; and whereof one cannot speak, thereon one must be silent.)

– Ludwig Wittgenstein's good advice for forum contributors, in *Tractatus Logico-Philosophicus*.

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