Mathematical Induction

Mathematical Induction is a special way of proving things. It has only 2 steps:

- Step 1. Show it is true for the **first one**
- Step 2. Show that if **any one** is true then the **next one** is true

Then **all** will be true

Have you heard of the "Domino Effect"?

- Step 1. The first domino falls
- Step 2. If any domino falls, the next domino falls

So ... **all dominos will fall!**

That is how it works.

In the world of numbers we say:

- Step 1. Show it is true for **n=1**
- Step 2. Show that if **n=k** is true then **n=k+1** is also true

How to Do it

Step 1 is usually easy, you just have to prove it is true for **n=1**

Step 2 is best done this way:

- **Assume** it is true for **n=k**
- **Prove** it is true for **n=k+1** (you can use the **n=k** case as a **fact**.)

Step 2 can often be ***tricky*** ... because you may need to use imaginative tricks to make it work!

Like in this example:

Example: 3^n-1 is a multiple of 2

Is that true? Let us find out.

1. Show it is true for **n=1**

$$3^1-1 = 3-1 = 2$$

Yes 2 is a multiple of 2. That was easy.

3^1-1 is true

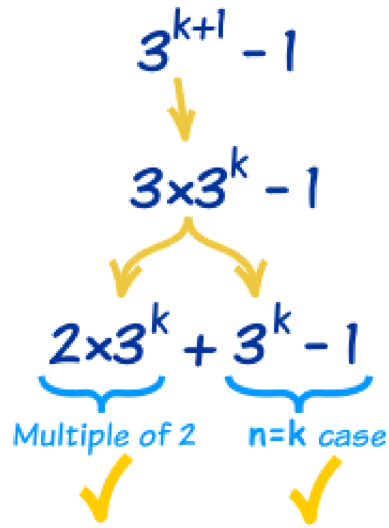
2. Assume it is true for **n=k**

3^k-1 is true

(Hang on! How do we know that? We don't!
It is an **assumption** ... that we treat
as a fact for the rest of this example)

Now, prove that $\mathbf{3^{k+1}-1}$ is a multiple of 2

$\mathbf{3^{k+1}}$ is also $\mathbf{3 \times 3^k}$



And then split **3×** into **2×** and **1×**

And each of these are multiples of 2

Because:

- $\mathbf{2 \cdot 3^k}$ is a multiple of 2 (you are multiplying by 2)
- $\mathbf{3^k-1}$ **is true** (we said that in the assumption above)

So:

$3^{k+1}-1$ is true

DONE!

Did you see how we used the $\mathbf{3^k-1}$ case as being **true**, even though we had not proved it? That is OK, because we are relying on the **Domino Effect** ...

... we are asking **if** any domino falls will the **next one** fall?

So we take it as a fact (temporarily) that the "**n=k**" domino falls (i.e. $\mathbf{3^k-1}$ is true), and see if that means the "**n=k+1**" domino will also fall.

Tricks

I said before that you often need to use imaginative tricks.

A common trick is to rewrite the **n=k+1** case into 2 parts:

- one part being the **n=k** case (which is assumed to be true)
- the other part can then be checked to see if it is also true

We did that in the example above, and here is another one:

Example: Adding up Odd Numbers

$$1 + 3 + 5 + ... + (2n-1) = n^2$$

1. Show it is true for **n=1**

$$1 = 1^2 \text{ is True}$$

2. Assume it is true for **n=k**

$$1 + 3 + 5 + ... + (2k-1) = k^2 \text{ is True}$$

Now, prove it is true for "k+1"

$$1 + 3 + 5 + ... + (2k-1) + (2(k+1)-1) = (k+1)^2 \text{ ... ?}$$

We know that $\mathbf{1 + 3 + 5 + ... + (2k-1) = k^2}$ (the assumption above), so we can do a replacement for all but the last term:

$$\mathbf{k^2} + (2(k+1)-1) = (k+1)^2$$

Now expand all terms:

$$k^2 + 2k + 2 - 1 = k^2 + 2k+1$$

And simplify:

$$k^2 + 2k + 1 = k^2 + 2k + 1$$

They are the same! So it is true.

So:

$$1 + 3 + 5 + ... + (2(k+1)-1) = (k+1)^2 \text{ is True}$$

DONE!

So there you have it!

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