ROB 101 - Computational Linear Algebra
Recitation #1

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1 Linear systems

A system of equations with no Non-linearity ($\cos, \sin, \log, x^2$ etc.)

1.1 Solution of Linear System of Equations

Lets work out some examples:
Find a solution by for the following set of equations by substitution, if it exists and also sketch a rough solution to corroborate:

1.1.1 Part A

Your Answer

$$x + 2y = 6 \quad X = \tag{1}$$
$$2x - y = 4 \quad y = \tag{2}$$

Answer

Step 1: Find y in terms of x from equation (2)

$y = 2x - 4$

Step 2: Replace y in eq 1

$x + 2(2x - 4) = 6$

$x + 4x - 8 = 6$

$5x = 14$

$x = \frac{14}{5}$

Step 3: Find y from the value of x.

$y = 2\left(\frac{14}{5}\right) - 4 = \frac{28}{5} - 4$

$y = \frac{8}{5}$

1.1.2 Part B

Your Answer

$$x + 2y = 6 \qquad X = \tag{3}$$

$$3x + 6y = 9 \qquad Y = \tag{4}$$

Answer Step 1:

$$y = \frac{9}{6} - \frac{3x}{6}$$

$$\Rightarrow y = \frac{3}{2} - \frac{x}{2}$$

Step 2:

$$x + 2\left(\frac{3}{2} - \frac{x}{2}\right) = 6$$

$$\cancel{x} + 3 - \cancel{x} = 6$$

$$3 = 6 \text{ [Contradiction]}$$

No Solution!

1.1.3 Part C

Your Answer

$$x + 2y = 6 \qquad X = \qquad (5)$$
$$3x + 6y = 18 \qquad Y = \qquad (6)$$

Answer: Step 1: $y = -\frac{x}{2} + 3$

Step 2: $x + 2\left(-\frac{x}{2} + 3\right) = 6$

$x - x + 6 = 6$

$6 = 6$ [Always True]

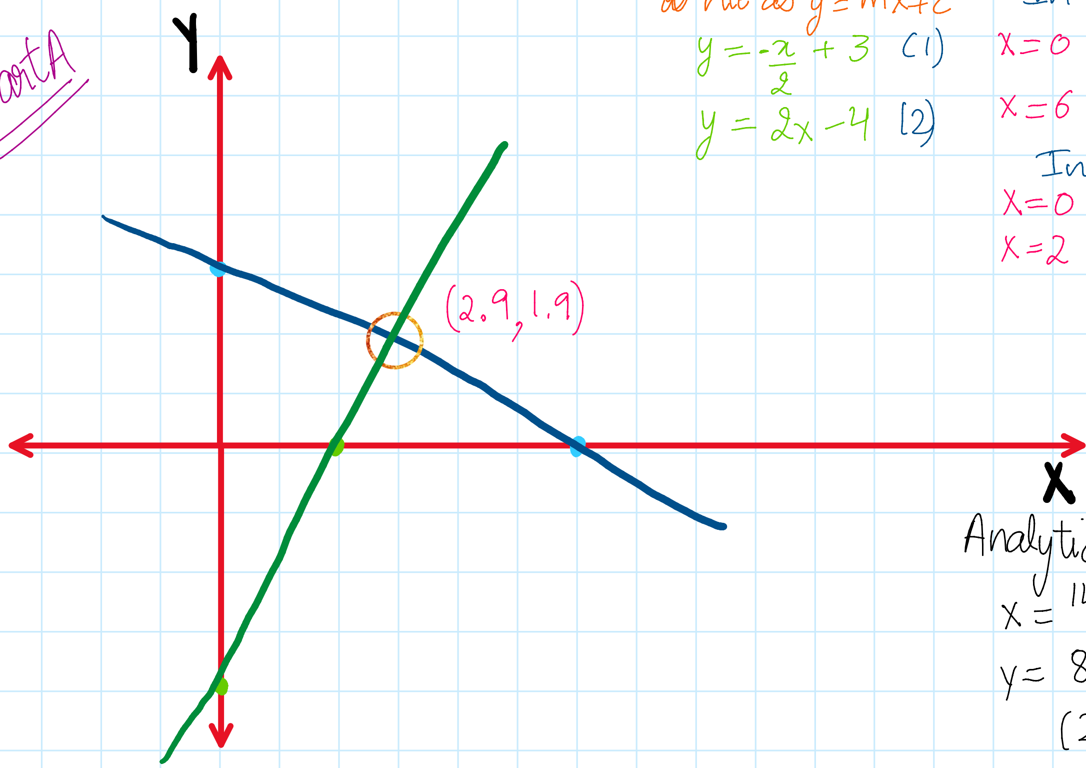
$x \in \mathbb{R}$ [x can be any number in the vast set of Real numbers, denoted by $\mathbb{R}$]

Step 3: $y = -\frac{x}{2} + 3$,

w.k.t. $x \in \mathbb{R}$, $y \in \mathbb{R}$

Infinite Number Of Solutions!

Part A



Write as $y = mx + c$

$y = -\frac{x}{2} + 3 \quad (1)$

$y = 2x - 4 \quad (2)$

In (1)

$x = 0 \quad y = 3 \quad (0,3)$

$x = 6 \quad y = 0 \quad (6,0)$

In (2)

$x = 0 \quad y = -4 \quad (0,-4)$

$x = 2 \quad y = 0 \quad (2,0)$

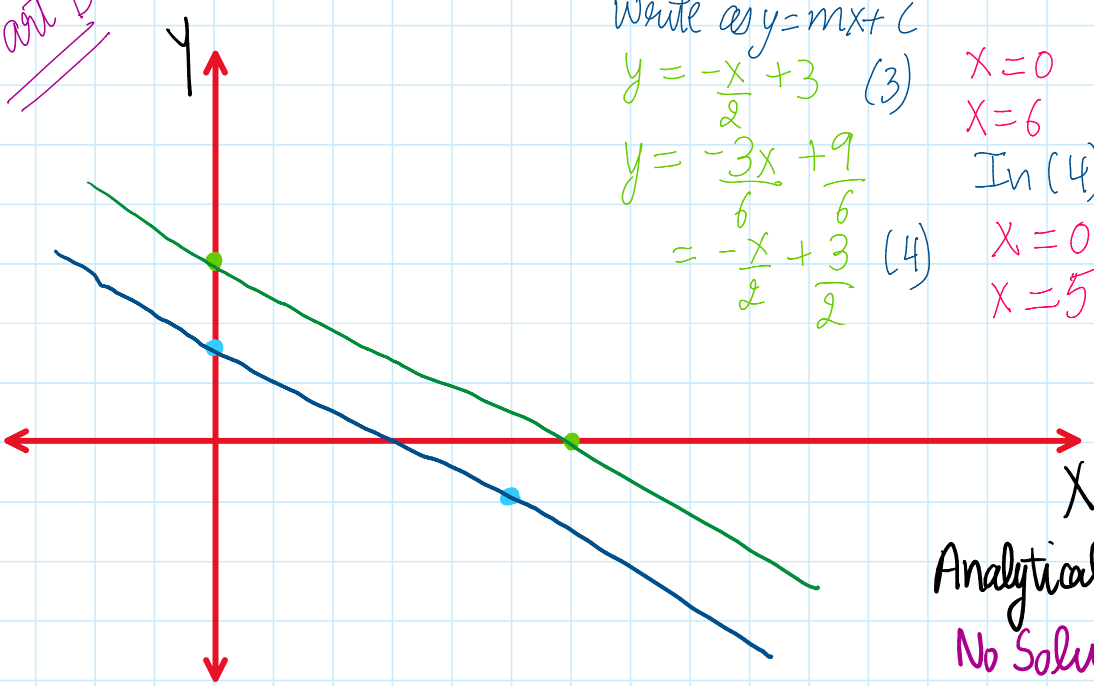
Analytical Solu^n was:-

$x = 14/5 = 2.8$

$y = 8/5 = 1.6$

$(2.8, 1.6)$

Part B



Write as $y = mx + c$

$y = -\frac{x}{2} + 3 \quad (3)$

$y = -\frac{3x}{6} + \frac{9}{6}$

$= -\frac{x}{2} + \frac{3}{2} \quad (4)$

In (3)

$x = 0 \quad y = 3 \quad (0,3)$

$x = 6 \quad y = 0 \quad (6,0)$

In (4)

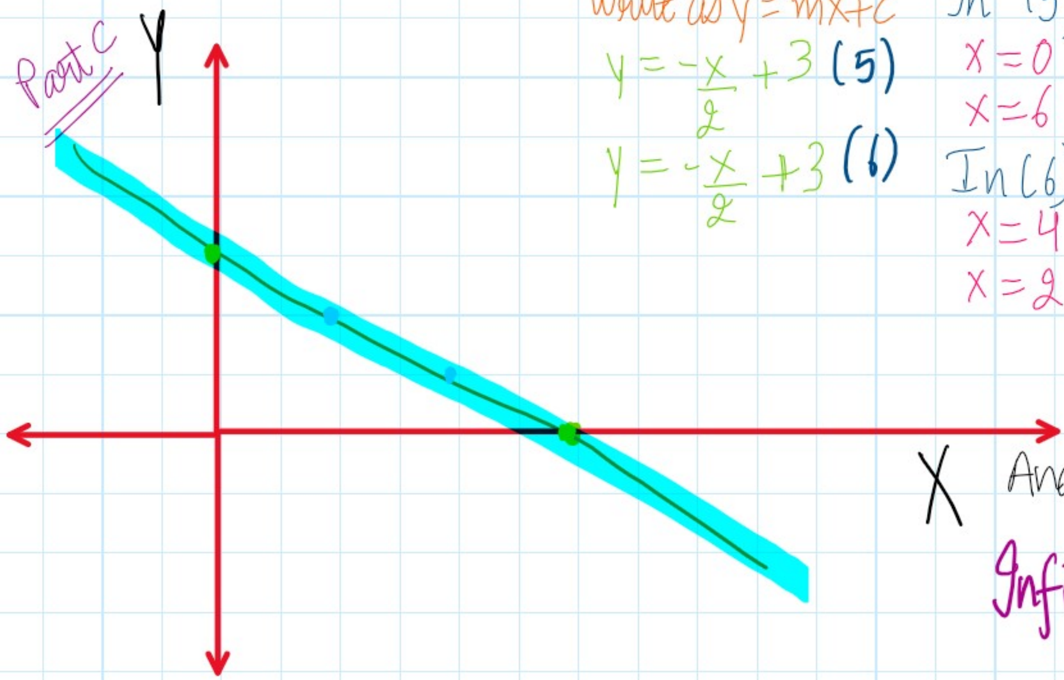
$x = 0 \quad y = 3/2 \quad (0, 3/2)$

$x = 5 \quad y = -1 \quad (5,-1)$

Analytical Solution

No Solution!

Part C



Write as $y = mx + c$

$y = -\frac{x}{2} + 3$ (5)

$y = -\frac{x}{2} + 3$ (6)

In (5)

$x = 0$ $y = 3$ $(0,3)$

$x = 6$ $y = 0$ $(6,0)$

In (6)

$x = 4$ $y = 1$ $(4,1)$

$x = 2$ $y = 2$ $(2,2)$

Analytical Solution:

Infinite No. of Solutions

Lets try and express these in the matrix format ($Ax = b$) and determine if the solution is unique $det(A) \neq 0$

Review: Determinant Facts:

- det(A) is a real number
- $Ax = b$, a system of equations with n equations and n unknowns has a unique solution for any b if an only if $det(A) \neq 0$
- When $det(A) = 0$, the system may have either infinite or no solution
- $det(A) = ad - bc$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Step 1: Order the equations and stack the vector of unknowns. (x)

Step 2: Move the constants to the RHS and stack them in order (b)

Step3: Find the coefficients of your unknowns to make the L.H.S. = Ax (A)

1. $x + 2y = 6$
   $2x - y = 4$

   $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \end{bmatrix}$ $b = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

   $det(A) = 1(-1) - 2(2) = -1 - 4 = -5$

   $det(A) \neq 0$ $\Rightarrow$ Unique Solution

2. $x + 2y = 6$
   $3x + 6y = 10$

   $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \end{bmatrix}$ $b = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$

   $det(A) = 1(6) - 2(3) = 0$

   $det(A) = 0$ $\Rightarrow$ Either infinite or no solution

3. $x + 2y = 6$
   $3x + 6y = 18$

   $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \end{bmatrix}$ $b = \begin{bmatrix} 6 \\ 18 \end{bmatrix}$

   $det(A) = 1(6) - 2(3) = 0$

   $det(A) = 0$ $\Rightarrow$ Either infinite or no solution

4. $4x = 10$ "Missing Coefficients"
   $x + 6y = 2$

   $A = \begin{bmatrix} 4 & 0 \\ 1 & 6 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \end{bmatrix}$ $b = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$

   $det(A) = 4(6) - 0(1) = 24$

   $det(A) \neq 0$ $\Rightarrow$ Unique Solution

5. $y - 2x = 4$
   $y = 2$

   $A = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \end{bmatrix}$ $b = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

   $det(A) = -2(1) - 1(0) = -2$

   $det(A) \neq 0$ $\Rightarrow$ Unique Solution

6. $2x - y = 3$
   $6x - 3y = 1$

   $A = \begin{bmatrix} -3 & 6 \\ -1 & 2 \end{bmatrix}$ $x = \begin{bmatrix} y \\ x \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

   $det(A) = -3(2) - 6(-1) = 0$

   $det(A) = 0$ $\Rightarrow$ Either infinite or no solution

2 Quadratic equation

$$ax^2 + bx + c = 0$$

So on the x-y axis we want to plot:

$$y = ax^2 + bx + c$$

Finding the roots at $y = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac \geq 0$, Roots are real
If $b^2 - 4ac < 0$, Roots are complex

Lets see some examples and how to plot these standard quadratic forms:

$$x^2 - 10x + 21 = 0 \tag{7}$$

Answer Find the roots of the quadratic Equation, Your Answer $x =$

$a = 1$, $b = -10$, $c = 21$

$$x = \frac{10 \pm \sqrt{100 - 4(1)(21)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{100 - 84}}{2}$$

$$= \frac{10 \pm \sqrt{16}}{2}$$

Discriminant $= 16 > 0$
$\Rightarrow$ Roots are real and distinct

$$= \frac{10 \pm 4}{2}$$

$$= \frac{14}{2}, \frac{6}{2}$$

$$= 7, 3$$

Answer Finding the roots, $x^2 - 10x + 25 = 0$ Your Answer (8)

$X =$

$a = 1,\ b = -10,\ c = 25$

$$X = \frac{10 \pm \sqrt{100 - 4(1)(25)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{100 - 100}}{2}$$

Discriminant $= 0 \geq 0$,
Roots are real and repeated

$$= \frac{10}{2}, \frac{10}{2}$$

$$= 5, 5$$

$$x^2 - 10x + 26 = 0 \tag{9}$$

Your Answer

$x =$

Answer

$a=1,\ b=-10,\ c=26$

$$x = \frac{10 \pm \sqrt{100-4(1)(26)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{100-104}}{2}$$

$$= \frac{10 \pm \sqrt{-4}}{2}$$

Discriminant $= -4 < 0$.

$\Rightarrow$ Roots are complex or imaginary

$$= \frac{10 \pm 2\sqrt{-1}}{2}$$

Note: $\sqrt{-1}$ is donoted a i, and is a complex number

$$= 5+i,\ 5-i$$

We need Atleast 3 Points to define a Parabola

Part A

$y = x^2 - 10x + 21$

Put the roots in eq.

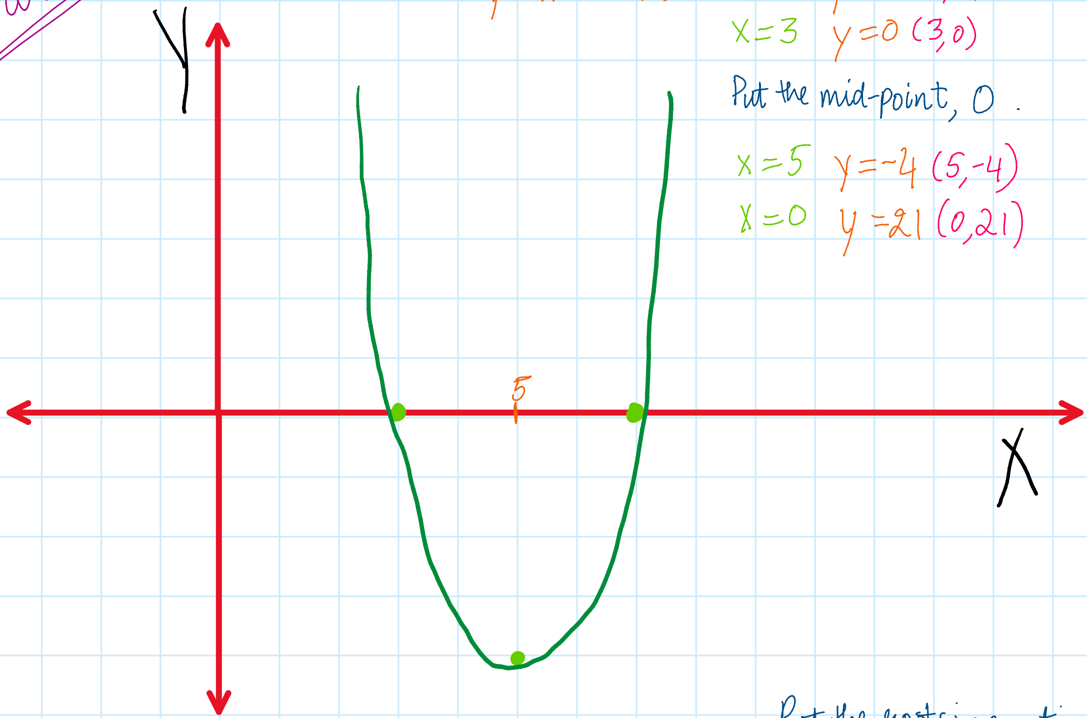
$x = 7$ $y = 0$ $(7,0)$

$x = 3$ $y = 0$ $(3,0)$

Put the mid-point, 0 .

$x = 5$ $y = -4$ $(5,-4)$

$x = 0$ $y = 21$ $(0,21)$



Part B

$y = x^2 - 10x + 25$

Put the roots in equation

$x = 5$ $y = 0$ $(5,0)$

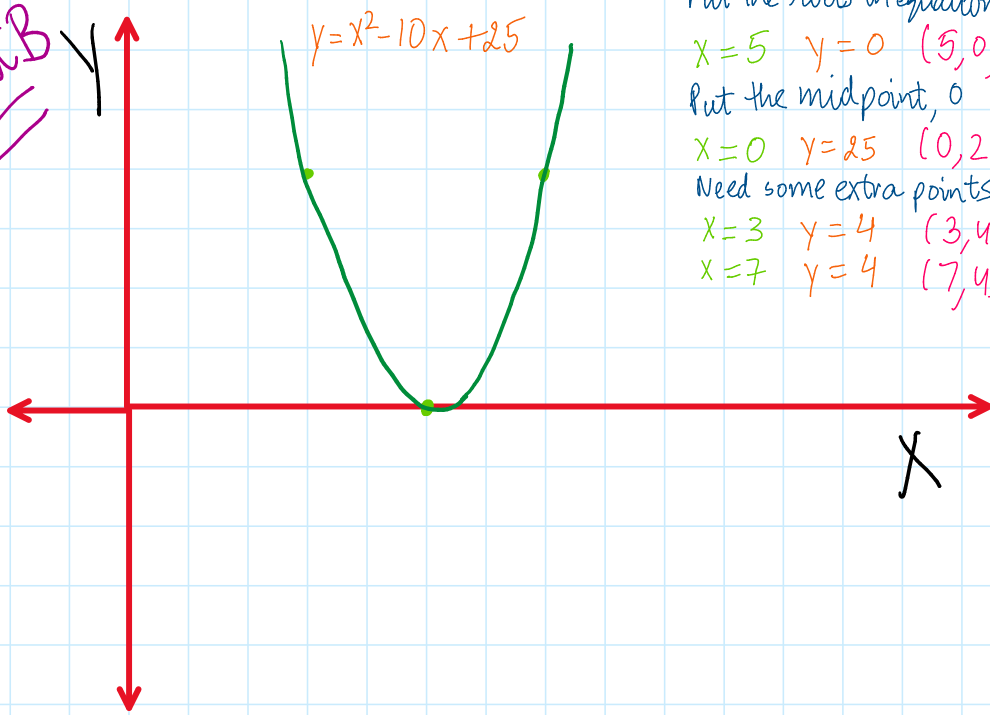
Put the midpoint, 0

$x = 0$ $y = 25$ $(0,25)$

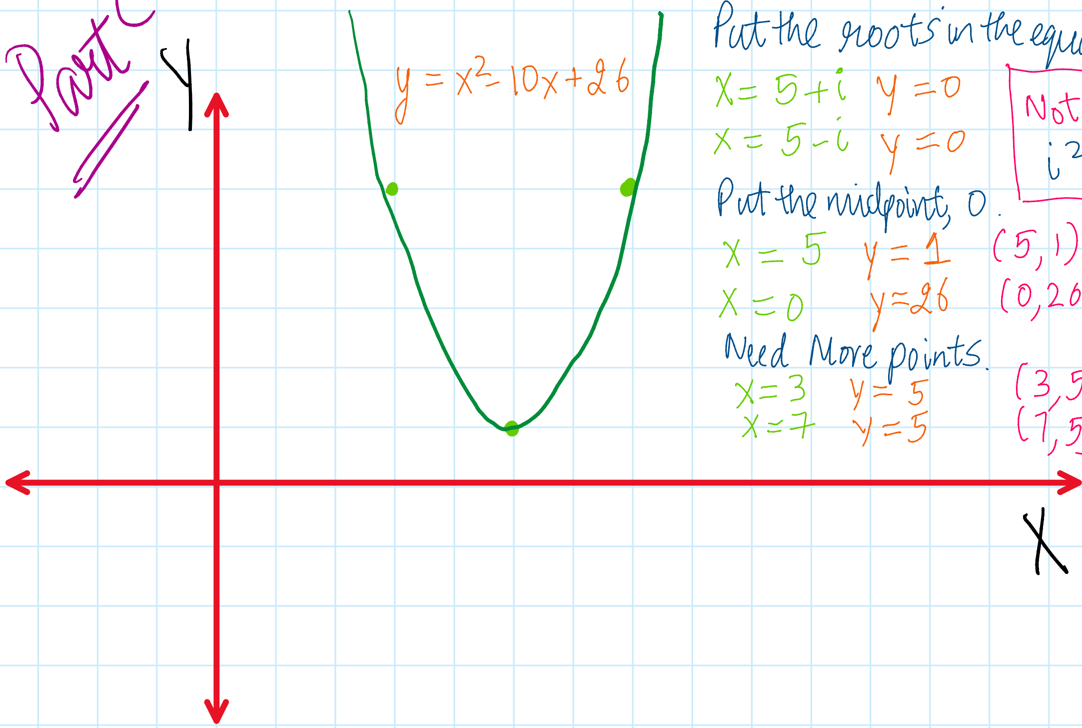
Need some extra points

$x = 3$ $y = 4$ $(3,4)$

$x = 7$ $y = 4$ $(7,4)$



Part C



Put the roots in the equation

$x = 5 + i \quad y = 0$

$x = 5 - i \quad y = 0$

Note: $i^2 = -1$

Put the midpoint, 0.

$x = 5 \quad y = 1 \quad (5, 1)$

$x = 0 \quad y = 26 \quad (0, 26)$

Need More points.

$x = 3 \quad y = 5 \quad (3, 5)$

$x = 7 \quad y = 5 \quad (7, 5)$