# ROB 201 - Calculus for the Modern Engineer
# HW #2

Prof. Grizzle

**Check Canvas for due date and time**

**Remark:** There are six (6) HW problems plus a *Jupyter notebook* to complete and turn in.

1. (a) Create a "Cheat Sheet" for the first two chapters of the textbook. You'll receive the same score for a handwritten solution as a typeset solution. Here is an example from ROB 101.

   (b) Note any material where you found the explanation confusing or difficult to master.

2. For the strictly increasing function, $f(x) = \frac{x}{2} + 2\,x^2$, determine lower and upper bounds for the area under the function for the interval $[0, 2]$, using $n := 4$, $\Delta x := \frac{2-0}{4} = 0.5$, and $x_i := (i-1)\cdot\Delta x$, $1 \leq i \leq 5$.

   (a) Give a numerical value for $\text{Area}_4^{\text{Low}}$.
   (b) Show your steps for computing $\text{Area}_4^{\text{Low}}$.

3. A 3D printer is building a 3D object by depositing layers of material. The thickness of each layer is designed to be half that of the previous layer. The first layer is 4 mm thick. The 3D printer is programmed to stop after depositing 10 layers.

   (a) What is the thickness of the 10th layer?
   (b) Calculate the total thickness of the object after the 10th layer has been deposited.

4. Compute the following limits using the "Easy Way" as explained in the textbook. You can just give an answer; you do not need to show your work.

   (a) $\displaystyle\lim_{x\to\infty} \frac{7x^5 - 2x^3 + x - 4}{3x^5 + x^2 + 1}$

   (b) $\displaystyle\lim_{x\to\infty} \frac{3x^2 - x + 1}{4x^4 + 2x^3 - x + 3}$

   (c) $\displaystyle\lim_{x\to\infty} \frac{2 + x}{x^{\frac{1}{3}} - 3}$

   (d) $\displaystyle\lim_{x\to-\infty} \frac{4x^4 - x^2 + 5}{3x^3 + 2x + 1}$

   (e) $\displaystyle\lim_{x\to-\infty} 2^{-x}\frac{4x^4 - x^2 + 5}{3x^3 + 2x + 1}$

   (f) $\displaystyle\lim_{x\to\infty} \pi^{0.01\,x}\frac{3x^2 - x + 1}{4x^4 + 2x^3 - x + 3}$

5. This problem focuses on the Binomial Theorem from Chapter 1.

(a) For the first six binomial coefficients, where $k$ ranges from $0$ to $5$, give their formulas and their values.

(b) Compute the expansion of $(1 + \frac{1}{x})^5$.

6. **The Cantorian Clan's Eternal Craftsmanship:** The Cantorian Clan is renowned across the galaxy for their unparalleled craftsmanship. Each day, they manufacture **Harmonic Orbs**, fantastical objects with weights measured precisely in grams. The Cantorians have a unique tradition: every day, they craft one orb for each of their beloved *harmonic fractions*, that is, for every number of the form:

$$\frac{1}{i}, \quad \text{where } i = 2, 3, 4, \ldots$$

For example:

- On the first day, they craft orbs with weights $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ grams.
- On the second day, they craft orbs with the same weights, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ grams, and so on.

The Cantorians' lineage is eternal, and their production continues forever. At the end of time, their collection of all Harmonic Orbs is complete.

**The Questions:**

(a) Is the set of Harmonic Orbs produced by the Cantorian Clan on Day 1 **countable** or **uncountable**? Explain why.

(b) How does the total number of Harmonic Orbs produced after two days (i.e., adding up both days' production) compare to the number of Harmonic Orbs produced after one day? Give an answer and then very briefly explain it.

(c) How does the total number of Harmonic Orbs produced at the end of time (i.e., adding up the total production over days $1, 2, 3, \ldots$) compare to the number of Harmonic Orbs produced after one day? Give an answer and then very briefly explain it.

**Remark:** In HW #8, we will compute the total weight in grams of the Cantorian Clan's daily orb manufacturing. For **zero bonus points:** Can you guess the total weight of each day's orbs to within one gram?

# Hints

**Prob. 1** Nothing to add.

**Prob. 2** Below is the upper bound worked out. You should follow a similar pattern for the lower bound. Don't be shocked when your lower bound is far from the upper bound of $8\frac{3}{4}$. For $n = 500$, the two answers are (lowerSum = 6.315344000000007, upperSum = 6.351344000000007, estIntegral = 6.333344000000007, pmError = 0.018000000000000238), but of course, you would never compute that by hand.

$$
\begin{aligned}
\text{Area}_4^{\text{Up}} &= \sum_{i=1}^{4} f(x_{i+1}) \cdot \Delta x \\
&= \sum_{i=1}^{4} \left(f(i \cdot \Delta x)\right) \cdot \Delta x \\
&= \sum_{i=1}^{4} \left(\frac{i \cdot 0.5}{2} + 2 \cdot (i \cdot 0.5)^2\right) \cdot 0.5 \\
&= 0.5 \cdot \left[\left(\frac{0.5}{2} + 2 \cdot (0.5)^2\right) + \left(\frac{2 \cdot 0.5}{2} + 2 \cdot (2 \cdot 0.5)^2\right) + \left(\frac{3 \cdot 0.5}{2} + 2 \cdot (3 \cdot 0.5)^2\right) + \left(\frac{4 \cdot 0.5}{2} + 2 \cdot (4 \cdot 0.5)^2\right)\right] \\
&= 0.5 \cdot \left[(0.25 + 0.5) + (0.5 + 2) + (0.75 + 4.5) + (1 + 8)\right] \\
&= 0.5 \cdot \left[0.75 + 2.5 + 5.25 + 9\right] \\
&= 0.5 \cdot \left[17.5\right] \\
&= 8.75
\end{aligned}
$$

**Prob. 3** Check out geometric sums.

**Prob. 4** None given.

**Prob. 5** $\binom{5}{0} = \frac{5!}{0!5!} = 1$

**Prob. 6** You have numerous resources at your disposal. In fact, you have so many that we will not discuss this problem in Office Hours so that you learn to take advantage of your resources!

- Check out the textbook's Table of Contents and click on it to go to the section on Countable Sets (see Chapter 2).
- If the textbook does not help you to have a general idea of what it means for a set to be countable, provide the following **prompt to Maizey or UM GPT:**

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  Give me a basic introduction to countable sets w/o using the word injective.
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  Once you understand the most basic definition, you can ask Maizey for a "little more depth". If it jumps too far ahead, ask it to back up. Ask for examples you can work and answer!

- Post on Piazza and seek help from your classmates (recall, we are deliberately not helping you on this problem so that you are motivated to explore various avenues of assistance). Posting and answering questions on Piazza is not considered cheating in CALCULUS FOR THE MODERN ENGINEER.