Review for Final Exam

- Your cheat sheet should have math and code
- Upload it to Vocareum at beginning of the exam

1] $$I_z = \int_a^b \rho \cdot h \cdot \left[x^2 (f(x) - g(x)) + \frac{1}{3}\left(f^3(x) - g^3(x)\right)\right] dx$$

using QuadGK

2] TrapZ

```
Integral = 0.0
N = length(fSeq)
for i = 2:N

end
```

Negative area

```
for i = 2:N
  if fSeq[i] < 0
  Integral = Integral + fSeq[i] * dx
  end
end
```

$\overbrace{\text{fSeq[i] * dx}}^{\text{Right-sided Riemann Sum. Could do left.}}$ Could do as in Notebook

3]

3.1 $\lim_{x\to\infty} \frac{4x^4-5x+7}{-2x^3+3x^2-1} = -\infty$

3.2 $\lim_{x\to\pi^-} \sin(x) = 0$

$\lim_{x\to\pi^+} \frac{x-\pi}{x} = 0$

$\therefore \lim_{x\to\pi} g(x) = 0$

3.3 $\frac{d}{dx}(xe^x) = e^x + xe^x$

$\frac{d^2}{dx^2}(xe^x) = e^x + [e^x + xe^x] = 2e^x + xe^x$

3.4 $\left.\frac{d}{dx} x^{-2}\right|_{x=2} = \left.-2x^{-3}\right|_{x=2} = \frac{-2}{8} = -\frac{1}{4}$

4]

4.1 $\lim_{n\to\infty}\left(1-\frac{3}{n^2}\right)^{n^2} = \lim_{m\to\infty}\left(1-\frac{3}{m}\right)^m = e^{-3}$

4.2 $\lim_{x\to0^-}\frac{|x|}{x} = \lim_{x\to0^-}\frac{-x}{x} = -1$

4.3 $\frac{d}{dx}\left[(x^3+x)\sin(x)\right] = (3x^2+1)\sin(x) +$

$+ (x^3+x)\cos(x)$

4.4 $$\frac{d}{dx}\left[e^{x\sin(x)}\right]\Bigg|_{x=0} = e^{x\sin(x)}\cdot\left(x\cdot\sin(x)\right)'\Bigg|_{x=0}$$

$$= e^{x\sin(x)}\left(\sin(x)+x\cos(x)\right)\Bigg|_{x=0} = 0$$

5] Gradient descent for $x^* = \arg\min f(x)$

$g_1(x) = 0$

$g_2(x) = 0$

x_0 given

Let x_k = current value. Must solve for Δx s.t. $\nabla f(x_k)\cdot\Delta x < 0$

$\nabla g_i(x_k)\cdot\Delta x = 0$

$$\Delta x = -\nabla f(x_k) + \alpha_1\nabla g_1(x_k) + \alpha_2\nabla g_2(x_k)$$

$$= -\nabla f(x_k) + \underbrace{\begin{bmatrix}\nabla g_1(x_k) & \nabla g_2(x_k)\end{bmatrix}}_{G(x_k)}\underbrace{\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}}_{\alpha}$$

$$\therefore\ 0_{2\times1} = G(x_k)^T \cdot \Delta x = -G(x_k)^T \cdot \nabla f(x_k) + G(x_k)^T \cdot G(x_k) \cdot \alpha$$

$$\therefore\ G(x_k)^T \cdot G(x_k)\,\alpha = -G(x_k)^T \nabla f(x_k)$$

$x_k \leftarrow$ nlsolve(geq, x_0)

kmax = 1e4

$\Delta x \Leftarrow$ descent direction

```
while (||Δx|| ≥ Tol1) && (k ≤ kmax)

    x_k = x_k + s Δx

    if ||geq(x_k)|| ≥ Tol2

        x_k ← nlsolve(geq, x_k)

    end
    k = k+1

end

xStar = x_k

fStar = f(xStar)
```

6] Solve $\dot{x} = f(x)$, $x(0) = x_0$

A) Euler

```
tVec = 0:dt:tmax
N= length(tVec)

xMat = zeros(m, N)   where  m=length(x0)

xMat[:,1] = x0

for i = 2:N
  x= xMat[:, i-1]

  xMat[:,i]= x + f(x).dt

end
```

B) using DifferentialEquations

```
prob= ODEProblem( f, x0, tspan, params)

myODEsol = solve(prob, Tsit5())
```

We extract and plot the solution

7] Laplace Applied to ODEs

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 4u + \dot{u}$$

A) Compute G(s) by inspection

G_s = (s+4)/(s^3+6 s^2+11 s+6)

B) Use Sympy to solve for y(t) when $y(0)=\dot{y}(0)=\ddot{y}(0)=0$ and $u(t) = e^{-5t} u_s(t)$

$$U\_s = \frac{1}{s+5}$$

Y_s = G_s * U_s

y_resp_t = sympy.inverse_laplace_transform(Y_s , s, t)