Syllabus Winter 2025
Calculus for the Modern Engineer

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Likely Grading Policy (we are still in pilot mode):

- 10 % Final Exam. There is no midterm. The Final will take place on the date and time set by the UofM Registrar. You will be given a practice exam to help you prepare.
- 20% 8 HW sets consisting of a written part and a jupyter notebook in Julia. [No late HW accepted. In compensation, two lowest scores in each category among HWs 1 to 7 are dropped; HW 8 cannot be dropped]
- 20% for 5 Individually completed quizzes (open for one week) [lowest Quiz score is dropped]
- 8% Course Participation
  - 3 % total for midterm and end-of-term course evaluations
  - 5 % for lecture check-ins (random times on random days). To encourage synchronous (in-class or Zoom) participation versus just watching the lecture recordings, check-in forms are trivial when completed synchronously with the live lecture. When completed asynchronously, they require a 20- to 30-word summary of the lecture.
- 42% 3 individual Projects, unequally weighted. Projects can be turned in late with a penalty of 15% per day prorated hourly.
  - **17% Numerical Integration:** Uses linear acceleration collected from the IMU of a drone to its compute position and velocity.
  - **17% Gradient Descent with Equality Constraints:** Part 1 of the project begins with the ballistic equations, choosing the initial speed and angle of a basketball so that it makes a free throw. Part 2 applies optimization to floating base models of a gymnast and a 10-meter platform to create acrobatic maneuvers.
  - **8% (Shorter than the other two:) Modeling and Feedback Control of a Planar Version of ROB 311's BallBot:** The BallBot is very similar to a Segway. The complete control design for the Segway is given in the textbook, along with all steps for qualitative and numerical analysis of the controller gains. Students follow this method on the linear model of the BallBot, test the controller in simulation, first on the linearized model, and then on the full nonlinear model. Appropriate software tools to make these steps transparent and fun are provided.

1 Pre-calculus: Notation, Functions, and Various Algebraic Facts

- 1 hour lecture.
- Students are expected to review this material mostly on their own.
- 20 question Quiz over the material to ensure mastery.
- The Approximation Principle is highlighted in HW01,

Learning Objectives

By the end of this chapter, the student should be able to:

- Recognize calculus as the science of approximations.
- Interpret mathematical notation and appreciate its efficacy in conveying complex mathematical ideas succinctly.
- Revisit and refresh knowledge on key mathematical concepts that are crucial for understanding calculus.
- Develop an understanding of the importance of algorithms in mathematical problem-solving and analysis.
- Cultivate a taste for careful and precise mathematical reasoning.
- Explore Euler's number, e, and learn how it arose from a simple everyday question.

Outcomes

Upon successful completion of this chapter, students will be able to:

- Recognize the utility and importance of mathematical notation in the precise expression of mathematical concepts.
- Observe the Approximation Principle at work through the study of numbers like π, $\sqrt{2}$, and e.
- Understand and apply the Bisection Algorithm as an example of the Approximation Principle in numerical methods.
- Review and properly apply rules for manipulating inequalities.
- Reaffirm understanding of fundamental concepts such as functions, domains, ranges, and inverses.
- Conduct a thorough examination of roots and powers and their properties.
- Review and consolidate knowledge of the key characteristics of exponential and logarithmic functions.
- Utilize Euler's Formula to simplify complex trigonometric expressions effectively.
- Revisit (or learn) the Binomial Theorem and its applications in algebraic expansions.
- Learn how to effectively apply shifting and scaling operations to functions for various analytical purposes.

2 Calculus Foundations: Proofs, Finite Sums, Limits at Infinity, and Geometric Sums

- 4 hours lecture + recitation.
- Written and Julia HWs

Learning Objectives

By the end of this chapter, the student should be able to:

- Understand the significance of mathematical proofs and articulate their importance in Calculus and its applications.
- Master the technique of proof by induction and apply it to demonstrate the validity of mathematical statements.
- Comprehend the concept of limits and see some of their initial uses in Calculus.

Outcomes

Upon successful completion of this chapter, students will be able to:

- Apply logical rules to construct (simple) mathematical arguments.
- Learn there are at least two kinds of infinity.
- Analyze sums of powers of integers to build a foundational understanding of integration.
- Calculate limits at infinity for functions that are significant in Calculus.
- Acquire additional knowledge about Euler's number and its unique properties.

3 Definite Integration as the Signed Area Under a Curve

- 5 hours lecture + recitation.
- Written and Julia HWs

Learning Objectives

By the end of this chapter, the student should be able to:

- Define and explain the concept of a definite integral within the context of calculus.
- Execute accurate computations of definite integrals using appropriate mathematical techniques, such as the Trapezoidal Rule and Simpson's Rule.
- Describe and apply the basic properties of definite integrals to solve problems.
- Recognize and demonstrate the applications of definite integrals in various engineering scenarios.

Outcomes

Upon successful completion of this chapter, students will be able to:

- Construct and calculate Riemann lower and upper sums to approximate definite integrals.
- Acquire insight into what kinds of functions can be integrated.
- Employ numerical algorithms used in engineering practice to compute definite integrals.
- Utilize integration techniques to infer changes in position from a given velocity function, particularly in robotic applications.
- Identify the mathematical origins of parabolic trajectories in ballistic motion.
- Determine the area enclosed between two functions.
- Compute essential parameters for robotic models, such as total mass and center of mass.
- Learn how to trick a single-variable integral into computing the volume of a solid of revolution.

4 Properties of Functions: Left and Right Limits, Types of Continuity, Boundedness, and Generalizations of Max and Min

- 5 hours lecture + recitation.
- Written and Julia HWs

Remark: This Chapter reveals some of the mathematical backbone of Calculus. Traditionally, much of this material is placed earlier in a Calculus course. Because many of the topics are abstract and very technical, we delayed them until you've gotten the hang of combining programming and numerical calculations when learning mathematical concepts.

Learning Objectives

By the end of this chapter, the student should be able to:

- Appreciate the foundational role of calculus in mathematical modeling and problem-solving, integrating programming and numerical methods to solidify these concepts.
- Analyze the behavior of functions at specific points using the concept of one-sided limits.
- Thoroughly understand the nature of function continuity and discover that continuity comes in more than one flavor.
- Derive closed-form expressions for the integrals of exponential functions and apply these techniques to integrate trigonometric functions.
- Determine the boundedness of functions and understand the implications for mathematical analysis and applications.
- Recognize that while maximum and minimum values are important, they are not the full story.

Outcomes

Upon successful completion of this chapter, students will be able to:

- Acquire both intuitive and formal understandings of one-sided limits and their calculation.
- Apply numerical methods to estimate one-sided limits and assess the continuity of functions using the epsilon-delta definition.
- Recognize when it is permissible to take limits within functions and apply this knowledge to the integration of exponentials.
- Identify and analyze piecewise continuous functions and extend this understanding to a broader class of Riemann integrable functions.
- Review and apply the concepts of maximum and minimum function values, and explore alternative behaviors when these extrema do not exist.

5 Differentiation

- 5 hours lecture + recitation.
- Written and Julia HWs

Learning Objectives

By the end of this chapter, the student should be able to:

- Understand two conceptual views of a single-variable derivative
- Appreciate the effectiveness of various software tools for computing derivatives
- See real problems where single-variable derivatives are important in engineering.
- Make the leap to partial derivatives, which are single-variable derivatives applied to multivariable functions.

Outcomes

Upon successful completion of this chapter, students will be able to:

- Apply the definition of the derivative as rise over run in the limit.
- Understand the centrality of the derivative for linear approximation of a function.
- Compute a few derivatives via the rise over run definition.
- Obtain a sense of when common functions are differentiable and when they are not.
- Learn how to compute derivatives with various software packages.
- Master the Rules of Differentiation and Understand their Origin.
- Apply single-variable derivatives to determine speed from position.
- Apply the fact that a strictly positive derivative implies the function is strictly monotonically increasing.
- Use L'Hôpital's Rule for limits of indeterminate form.
- Compute Jacobians, gradients, and Hessians, with examples using software.
- Understand and use the total derivative.

6 Engineering Applications of the Derivative

- 5 hours lecture + recitation.
- Written and Julia HWs

Learning Objectives

By the end of this chapter, the student should be able to:

- Analyze and model engineering problems using the principles of calculus, specifically through the application of derivatives to understand system behaviors.
- Develop strategies for solving optimization problems, both with and without constraints.
- Critically assess the role of derivatives in determining system dynamics and understand the significance of dynamic equations in engineering contexts.
- Apply the concept of energy to the analysis of mechanical systems using the framework of Lagrangian mechanics.

Outcomes

Upon successful completion of this chapter, students will be able to:

- Calculate path length and arc length for given paths.
- Solve engineering problems involving root finding and minimization.
- Use gradient descent to find local minima of functions and understand its limitations.
- Apply second derivative tests to determine the nature of critical points in functions.
- Solve optimization problems involving equality and inequality constraints using Lagrange multipliers.
- Derive and apply Lagrange's equations to solve problems in dynamics.
- Compute kinetic and potential energy for mechanical systems.
- Model and analyze the motion of complex systems like multi-link manipulators using Lagrange's formalism.
- Determine moments of inertia for various bodies and understand their effects on rotational motion.

7 Antiderivatives and the Fundamental Theorems of Calculus

- 5 hours lecture + recitation.
- Written and Julia HWs

Learning Objectives

By the end of this chapter, students should be able to:

- Understand that antidifferentiation is essentially the inverse operation of differentiation.
- Comprehend the two Fundamental Theorems of Calculus and explain the precise sense in which differentiation and integration are inverse operations on functions.
- Understand that while antidifferentiation and definite integration are technically distinct operations, they are often perceived as identical in the context of Calculus education. This perception is partially justified by their closely related concepts. However, it's crucial to acknowledge the potential drawbacks of this viewpoint, particularly the confusion and discouragement that can arise from the complex rules associated with manually computing antiderivatives.
- Appreciate the historical and practical context of computing antiderivatives by hand, and recognize the modern approaches that render hand computations almost obsolete for all but the simplest of cases.

Outcomes

Upon successful completion of this chapter, students will:

- Be introduced to the concept of antiderivatives and familiarize themselves with elementary techniques for finding them.
- Gain a thorough understanding of the Fundamental Theorems of Calculus, including their implications in both geometric and analytic contexts.
- Develop an insight into the interplay between differentiation, antiderivatives, and definite integration, enhancing their comprehension of calculus as a whole.
- Cultivate an appreciation for the traditional methods of finding antiderivatives while also understanding the value and efficiency of using computational tools for real-world applications.

8 Improper Integrals

- 2 hours lecture + recitation.
- Written and Julia HWs

Learning Objectives

By the end of this chapter, students should be able to:

- Define improper integrals and understand their significance within the context of calculus.
- Identify integrals that may pose difficulties due to their unbounded nature or the behavior of the function being integrated.
- Answer the pressing question: can an integral be doubly improper?
- Explore the applications of improper integrals, emphasizing their practical importance in Statistics and Probability.

Outcomes

Upon successful completion of this chapter, students will:

- Master the technique of integrating over unbounded domains using limits.
- Learn to manage and integrate functions with singularities or discontinuities by applying limits to circumvent infinite behavior at finite points, thus handling vertical asymptotes effectively.
- Gain a comprehensive understanding of the analytical methods for solving improper integrals, employing antiderivatives to facilitate calculation.
- Acquire skills in applying numerical methods, specifically Julia's `QuadGK`, for evaluating improper integrals, particularly when analytical solutions are challenging to obtain.
- Delve into two probability theory examples that utilize improper integrals, reinforcing the concept's application in real-world scenarios and theoretical studies.

9 Ordinary Differential Equations

- 8 hours lecture + recitation.
- Written and Julia HWs

Learning Objectives

By completing this chapter, students will:

- Gain an introduction to differential equations, focusing on their significance and the various classifications.
- Learn the foundational theories and techniques for solving one-dimensional first-order ordinary differential equations (ODEs).
- Acquire knowledge on the fundamentals and solution methods for vector first-order ODEs.
- Explore the application of numerical methods in solving vector first-order ODEs.
- Delve into the characteristics of linear systems of first-order ODEs and their implications for mathematical modeling.

Outcomes

Upon mastering the content of this chapter, students will be equipped to:

- Understand the definition and classification of ODEs based on their order, linearity, and whether they are homogeneous or non-homogeneous.
- Engage with various strategies for tackling first-order ODEs, covering separable, linear, and select nonlinear equations.
- Dive into the theory and application of solving systems of linear differential equations, highlighting their importance across diverse applications.
- Comprehend the utilization of ODEs in modeling and solving real-world problems across various domains.
- Grasp the fundamentals of numerical methods designed for solving ODEs in scenarios where analytic solutions are challenging or unattainable.
- Deeply investigate linear systems of ODEs, focusing on:
  - The structure and solution of linear vector ODEs, symbolized as $\dot{x} = Ax, x(t_0) = x_0$.
  - The concept and significance of the matrix exponential, denoted as e^{At}.
  - Essential attributes of the matrix exponential, such as $\frac{d}{dt}e^{At} = Ae^{At} = e^{At}A$.
  - Methods for solving linear vector ODEs, exemplified by $x(t) = e^{A(t-t_0)}x_0$.
  - The role of complex scalar exponentials, $e^{(a+\mathbf{i}\omega)t}$, in the context of ODE solutions.
  - The interplay between eigenvectors, eigenvalues, and the matrix exponential, such as $Av = \lambda v \implies e^{A(t-t_0)}v = e^{\lambda(t-t_0)}v$.
  - The criteria for stability in linear systems, $\text{real}(\lambda) < 0 \implies \lim_{t\to\infty} e^{A(t-t_0)}v = 0_{n\times 1}$, underlining how the real parts of eigenvalues influence the system's long-term behavior.
  - The application of these principles to general initial conditions when A has a complete set of eigenvectors.

10 Laplace Transforms through the Lens of Feedback Control

- 10 hours lecture + recitation.
- Written and Julia HWs

Learning Objectives

- To introduce the concept and mathematical theory of Laplace Transforms.
- To become familiar with common Laplace transforms and their properties.
- To understand the process and methods of finding inverse Laplace transforms.
- To apply Laplace transforms for solving linear ordinary differential equations (ODEs).
- To define and derive transfer functions for linear time-invariant systems.
- To understand the importance and application of transfer functions in the design of Singe-Input Single-Output (SISO) feedback loops.
- To use appropriate software tools for each of the above topics.

Outcomes

- Gaining a foundational understanding of Laplace transforms and their application in engineering problems.
- Mastery of standard Laplace transforms of common functions and their key properties.
- Ability to understand inverse Laplace transforms using techniques such as partial fraction decomposition and tools in Julia.
- Proficiency in applying Laplace transforms to simplify and solve linear ODEs with initial conditions.
- Comprehensive understanding of transfer functions, their derivation, and significance in system analysis.
- Applying knowledge of transfer functions to design and analyze SISO feedback loops in engineering systems, particularly in robotics.

Balance of Time

- Reinforcing key topics in integration.
- Reinforcing key topics in differentiation.
- Reinforcing key topics in ODEs