ROB 201 - Calculus for the Modern Engineer
HW #7

Prof. Grizzle

Check Canvas for due date and time

Remark: There are six (6) HW problems plus a *Jupyter notebook* to complete and turn in.

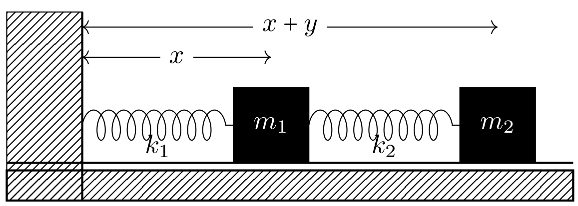
1. Read Chapter 7 of our ROB 201 Textbook, `Calculus for the Modern Engineer`. Based on your reading of the Chapter, summarize in your own words:

   (a) the purpose of Chapter 07;

   (b) two things you found the most DIFFICULT.

   There are no "right" or "wrong" answers, but no answer means no points. The goal is to reflect a bit on what you are learning and why.

2. A kind of Double Slinky: Consider a mechanical system consisting of two springs and two masses sliding on a level frictionless table as depicted below. Note that y is the distance from the center of the first mass to the second mass.



   The system has

   - potential energy, $PE = \frac{1}{2}k_1(x - x_0)^2 + \frac{1}{2}k_2(y - y_0)^2$, where $k_1 > 0$ and $k_2 > 0$ are spring constants having units of N/m (Newtons-per-meter) and where $x_0 > 0$ and $y_0 > 0$ are the rest positions of the springs in units of meters;
   - kinetic energy, $KE = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(\dot{x} + \dot{y})^2$, where $m_1 > 0$ and $m_2 > 0$ are the masses in units of kilograms.

   Derive BY HAND the Equations of Motion (EoM) from Lagrange's equations, assuming there are no external forces. **Provide adequate intermediate steps so that your reasoning can be evaluated**. Optional: You can use the code from JuliaHW06 to check your answer.

3. These are short-answer questions about the Fundamental Theorems of Calculus. For each part, you can just give the final answer followed by a very short comment about why your answer is correct or a one-step solution and a brief comment. There is not really any computational work to show.

   (a) Let $f(x) = 3x^2+2x-1$ and define the function $g(x) := \int_1^x f(t)\,dt$ as an indefinite integral of f. Use the First Fundamental Theorem of Calculus to find the derivative of $g(x)$.

   (b) Suppose $\alpha(x) = \sin(x^3)\cdot\exp\left(\frac{1}{1+x^2}\right)$ and $\alpha'(x) = \frac{x\left(3x(1+x^2)^2\cos(x^3)-2\sin(x^3)\right)}{(1+x^2)^2}\cdot\exp\left(\frac{1}{1+x^2}\right)$. Evaluate $\displaystyle\int_0^{\sqrt[3]{\pi}} \alpha'(x)\,dx$.

   (c) Suppose $h(x)$ is a continuous function satisfying $\int_2^x h(t)\,dt = x^2 - 4x + 4$. Find $h(x)$.

4. By hand, find antiderivatives for the following functions. Show your work and/or explain your reasoning as the problem may require. If you are using a method highlighted in the section of the textbook, **The Art of the Antiderivative: Inverting Differentiation Rules to Find Antiderivatives**, call it out.

   (a) $f(x) = \frac{x^k}{k!}$

   (b) $g(x) = x^2 \exp(x^3)$

5. By hand, determine the following integrals. Show your work and/or explain your reasoning as the problem may require. If you are using a method highlighted in the section of the textbook, **The Art of the Antiderivative: Inverting Differentiation Rules to Find Antiderivatives**, call it out.

   (a) $\int x \cos(x)\, dx$

   (b) $\int \frac{3}{x^2+5x+6}\, dx$

6. By hand, determine the following integrals. Show your work and/or explain your reasoning as the problem may require. If you are using a method highlighted in the section of the textbook, **The Art of the Antiderivative: Inverting Differentiation Rules to Find Antiderivatives**, call it out.

   (a) $\int \frac{1}{\sqrt{9-x^2}}\, dx$, for all $x \in [0, 3)$.

   (b) $\int x^2 \sin(x)\, dx$

Hints

Prob. 1 Write approximately 15 or more words for each part of the question.

Prob. 2 EoM are also called the Robot Equations.

Prob. 3 For part (c), while you may be tempted to answer as was done here, please be a bit more specific! All kidding aside, the problem is a bit different than examples worked in the textbook and may require some thinking. Once you have an answer, it is easy to check it. **We will NOT say anymore in office hours or Piazza.**

Prob. 4 What does it mean to call out a method from **The Art of the Antiderivative: Inverting Differentiation Rules to Find Antiderivatives**? It means to mention "Integrating the Differential", "u-Substitution", etc., if one of them is applicable. Here is part (a) worked out for you. Note the inclusion of the constant of integration.

For $f(x) = \frac{x^k}{k!}$, the antiderivative can be found by recognizing that the derivative of x^{k+1} is $(k+1)x^k$. Therefore, we can reverse this process to find the antiderivative:

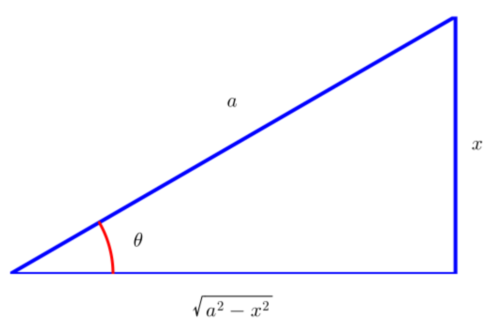
$$F(x) = \int \frac{x^k}{k!}\,dx = \frac{1}{k!}\int x^k\,dx = \frac{x^{k+1}}{(k+1)k!} + C = \frac{x^{k+1}}{(k+1)!} + C$$

To call out the method, you can cite either **the power rule for integration** or **the fundamental rule of recognizing a total differential**.

You still need to turn in a solution to part (a).

Prob. 5 Nothing more to add.

Prob. 6 Part-(a) requires a trig substitution. Recall this diagram from the textbook. The diagram is valid whether the radical is in the numerator or the denominator of the integrand.



Part-(b) requires integration by parts, twice.