ROB 201 - Calculus for the Modern Engineer
HW #5

Prof. Grizzle

Check Canvas for due date and time

Remark: There are six (6) HW problems plus a *Jupyter notebook* to complete and turn in.

1. Read Chapter 5 of our ROB 201 Textbook, CALCULUS FOR THE MODERN ENGINEER. Based on your reading of the Chapter, summarize in your own words:

   (a) the purpose of Chapter 05;

   (b) two things you found the most DIFFICULT.

   There are no "right" or "wrong" answers, but no answer means no points. The goal is to reflect a bit on what you are learning and why.

2. Explain your steps while computing the derivatives of the following functions and state which of the Product, Ratio (Quotient), or Chain (Composition) Rule or Rules is/are being used. There is no need to call out the rule for differentiating a linear combination.

   (a) Differentiate the function $f(x) = (2x^3 + x)(x^2 - 3)$ with respect to x, **without multiplying it out**. Leave the answer "unsimplified".

   (b) Find the derivative of $g(x) = \frac{e^x}{x^2+1}$ with respect to x. You can simplify the final answer or not. We'll accept almost any correct answer, and if we mark a correct answer wrongly, we'll fix it.

   (c) Differentiate the function $\gamma(x) = e^{3x^2+2x-5}$ with respect to x.

   (d) Differentiate $\varphi(x) = (x^3 - 2x^2 + 4)(\cos(x^2))$ with respect to x.

3. Where applicable, use L'Hôpital's Rule to evaluate the following limits. If it is not applicable, explain why. In all cases, show your work.

   (a) $\lim_{x\to 1} \frac{\sin(\pi x)}{(x-1)}$

   (b) $\lim_{x\to 1} x^2 e^{-x}$

   (c) $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x$ using L'Hôpital's Rule.

   Because $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$, you know the answer should be e. We want you to show it using L'Hôpital's Rule.

4. Compute the following partial derivatives using the rules of single-variable differentiation. You are allowed to just give the answers, but, if you go that route, it will be hard to earn partial credit in case of an error. It's your call.

   (a) $\frac{\partial}{\partial x} \left(\frac{e^x}{y^2+1}\right)$

   (b) $\frac{\partial}{\partial y} \left((2x^3 + x)(y^2 - 3)\right)$

   (c) $\frac{\partial}{\partial z} \left(\frac{e^x}{y^2+1}\right)$

5. Compute the following "vector derivatives".

   (a) $\mathrm{Jac}_f(x) := \frac{\partial f(x)}{\partial x}$ for $f : \mathbb{R}^3 \to \mathbb{R}$ by $f(x_1, x_2, x_3) = \sin(x_1) \cdot \cos((x_3)^2)$. For this part, explain your reasoning.
   (b) $\nabla f(x)$ for $f : \mathbb{R}^3 \to \mathbb{R}$ by $f(x_1, x_2, x_3) = \sin(x_1) \cdot \cos(e^{x_2 x_3})$. For this part you can just give the answer, but of course, showing work is always a good idea.

6. You are given a function $z = f(x, y)$, where $x = x(t)$ and $y = y(t)$, and it's known that $f(x, y) = x^2 y + e^{xy}$, $x(t) = \sin(t)$, and $y(t) = e^t$. **Find the total derivative $\boldsymbol{\frac{dz}{dt}}$ at** $t = 0$.

Hints

Prob. 1 Write approximately 15 or more words for each part of the question.

Prob. 2 Explain a strategy, and then document it. For example, "Apply the Composition Rule" with $f(x) = \cos(x)$ and $g(x) = \sqrt{x}$. $f'(x) = ...$, $g'(x) = ...$ and hence, for $x > 0$, $\left(\cos(\sqrt{x})\right) = ...$

Prob. 3 For part (c), use

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e^{\lim_{x\to\infty}\ln\left(\left(1+\frac{1}{x}\right)^x\right)}$$

and then first evaluate

$$\lim_{x\to\infty}\ln\left(\left(1+\frac{1}{x}\right)^x\right)$$

after using one of your log rules.

Even after the hint, you may want to attend Recitation. The problem will not be discussed during office hours.

Prob. 4 For partial derivatives with respect to y, all other variables are treated as constants. For example, because $\frac{d}{dy}\left(y^2+1\right) = 2y$, we conclude that: $\frac{\partial}{\partial y}\left(e^x\left(y^2+1\right)\right) = e^x\frac{\partial}{\partial y}\left(y^2+1\right) = e^x\cdot 2\cdot y = 2y\,e^x$. You can express the answer in either form, or $2e^x\,y$, etc.

Prob. 5 Be careful to apply the Chain Rule correctly. It would be silly not to check your answers with "software".

Prob. 6 You need to find $\left.\frac{dz}{dt}\right|_{t=0}$. You are not asked to compute the general form of $\frac{dz}{dt}(t)$. Your final answer is a number.