Michael Dang - 16257750
MATH434
HW3 Section 7.3

(1)

b) $10x_1 - x_2 = 9 \Rightarrow x_1 = (9 + x_2)/10$

$-x_1 + 10x_2 - 2x_3 = 7 \Rightarrow x_2 = (7 + 2x_3 + x_1)/10$

$-2x_2 + 10x_3 = 6 \Rightarrow x_3 = (6 + 2x_2)/10$

$x^{(0)} = 0$

Iteration

	$x^{(0)}$	1	2
x_1	0	$x_1 = (9+0)/10 = 9/10 = \boxed{.9}$	$x_1 = (9+.7)/10 = .97$
x_2	0	$x_2 = (7+2(0)+0)/10 = 7/10 = \boxed{.7}$	$x_2 = (7+2(.6)+.9)/10 = .91$
x_3	0	$x_3 = (6+2(0))/10 = 6/10 = \boxed{.6}$	$x_3 = (6+2(.7))/10 = .74$

(2) Problem 9

$2x_1 - x_2 + x_3 = -1$

$2x_1 + 2x_2 + 2x_3 = 4$

$-x_1 - x_2 + 2x_3 = -5$

has the sol: $(1, 2, -1)^t$

Section 7.4

③ Problem 2

a) $4x_1 - x_2 - x_3 = 5 \Rightarrow x_1 = (5 + x_2 + x_3)/4$

$-x_1 + 3x_2 + x_3 = -4 \Rightarrow x_2 = (-4 + x_1 - x_3)/3$

$2x_1 + 2x_2 + 5x_3 = 1 \Rightarrow x_3 = (1 - 2x_1 - 2x_2)/5$

$\omega = 1.1$, $x^{(0)} = 0$

$$x_i^{(k+1)} = (1-\omega)x_i^k + \frac{\omega}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij}x_j^k\right)$$

$$\Rightarrow x_1^{(k+1)} = 1.1\left[\frac{(5 + x_2 + x_3)}{4}\right] - (.1)x_1^{(k)}$$

$$\Rightarrow x_2^{(k+1)} = 1.1\left[\frac{(-4 + x_1 - x_3)}{3}\right] - (.1)x_2^{(k)}$$

$$\Rightarrow x_3^{(k+1)} = 1.1\left[\frac{(1 - 2x_1 - 2x_2)}{5}\right] - (.1)x_3^{(k)}$$

Iteration

k	x_1	x_2	x_3
0	0	0	0
1	$(1.1)(5/4) = 1.375$	$(1.1)(-4/3) = -1.467$	$(1.1)(1/5) = .22$
2	$(1.1)\left[\frac{(5 + 1.467 + .22)}{4}\right] - (.1)(1.375) = .8946$	$(1.1)\left[\frac{(-4 + 1.375 - .22)}{3}\right] - (.1)(-1.467) = -.8965$	$(1.1)\left[\frac{(1 - 2(1.375) - 2(-1.467))}{5}\right] - (.1)(.22) = .2385$

Section 7.5

④ Problem 2

b) $\begin{bmatrix} 58.9 & .03 \\ -6.1 & 5.31 \end{bmatrix} = A$

$K_\infty[A] = \|A\|_\infty \cdot \|A^{-1}\|_\infty$

$$A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{(5.31)(58.9) - (.03)(-6.1)}\begin{bmatrix} 5.31 & -.03 \\ 6.1 & 58.9 \end{bmatrix}$$

$$= \frac{1}{312.942}\begin{bmatrix} 5.31 & -.03 \\ 6.1 & 58.9 \end{bmatrix}$$

$$= \begin{bmatrix} .017 & -9.586 \times 10^{-5} \\ .0195 & .1882 \end{bmatrix}$$

$\|A\|_\infty = |58.91| + |1.031| = 58.93$

$\|A^{-1}\|_\infty = |1.0195| + |1.1882| = .2077$

$\Rightarrow cond_\infty(A) = \|A\|_\infty \cdot \|A^{-1}\|_\infty = 58.93 \times (.2077)$

$= \boxed{12.24} \Rightarrow$ ill-condition

(5) Problem 4

b) $58.9x_1 + .03x_2 = 59.2$

$-6.1x_1 + 5.31x_2 = 47$

$x = (1, 10)^t$

$\tilde{x} = (1.02, 9.98)^t$

Compute $\|x - \tilde{x}\|_\infty$ (i.e. error)

$$\frac{K_\infty(A)\ \|b - A\tilde{x}\|_\infty}{\|A\|_\infty}$$

$\|x - \tilde{x}\|_\infty = \|(1, 10) - (1.02, 9.98)\|_\infty$

$= \|(-.02, .02)\|_\infty$

$= \boxed{.02}$

$$\frac{K_\infty(A)\ \|b - A\tilde{x}\|_\infty}{\|A\|_\infty} = 12.24 \times \frac{528.65}{58.93} = \boxed{109.803}$$

← from previous problem

$$A\tilde{x} = \begin{bmatrix} 58.9 & .03 \\ -6.1 & 5.31 \end{bmatrix} \begin{bmatrix} 1.02 \\ 9.98 \end{bmatrix} = \begin{bmatrix} (58.9)(9.98) + (.03)(1.02) \\ (-6.1)(1.02) + (5.31)(9.98) \end{bmatrix}$$

$$= \begin{bmatrix} 587.85 \\ 46.77 \end{bmatrix}$$

$$\Rightarrow b - A\tilde{x} = \begin{bmatrix} 59.2 \\ 47 \end{bmatrix} - \begin{bmatrix} 587.85 \\ 46.77 \end{bmatrix} = \begin{bmatrix} -528.65 \\ .23 \end{bmatrix}$$

$\Rightarrow \|b - A\tilde{x}\|_\infty = 528.65$

$\Rightarrow \|A\|_\infty = 58.93$