Please identify anyone, whether or not they are in the class, with whom you discussed your homework. This problem is worth 1 point, but on a multiplicative scale (multiplied by the rest). Collaboration is encouraged.

1. (10 points) Consider the following family of Quasi-Newton methods in Algorithm 1

Algorithm 1 The family of Quasi-Newton methods.

Input: Choose $\boldsymbol{x}^{(0)} \in \mathbb{R}^n$
Input: A symmetric positive definite matrix $\boldsymbol{H}_0 \in \mathbb{R}^{n\times n}$.
Output: An approximation to $\boldsymbol{x}^* := \arg\min_{\boldsymbol{x}\in\mathbb{R}^n} f(\boldsymbol{x})$.
for $k = 0, 1, 2, \ldots$ until converging **do**
  Compute $\boldsymbol{g}^{(k)} = \nabla f(\boldsymbol{x}^{(k)})$.
  Compute $\boldsymbol{d}^{(k)} = -\boldsymbol{H}_k \boldsymbol{g}^{(k)}$.
  Find $\alpha_k = \arg\min_{\alpha\ge 0} f\left(\boldsymbol{x}^{(k)} + \alpha \boldsymbol{d}^{(k)}\right)$.
  Update $\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \alpha_k \boldsymbol{d}^{(k)}$.
  Update $\boldsymbol{H}_{k+1}$.
end for

Suppose that the function we wish to minimize is a standard quadratic function:

$$f(\boldsymbol{x}) := \frac{1}{2}\boldsymbol{x}^\top \boldsymbol{Q}\boldsymbol{x} - \boldsymbol{b}^\top \boldsymbol{x} + c, \quad \text{where } \boldsymbol{Q} \in \mathbb{R}^{n\times n}, \boldsymbol{Q} = \boldsymbol{Q}^\top, \boldsymbol{Q} > 0.$$

(a) Suppose $\{\boldsymbol{H}_k\}$ is given, find an expression for α_k in terms of $\boldsymbol{Q}$, $\boldsymbol{H}_k$, $\boldsymbol{g}^{(k)}$, and $\boldsymbol{d}^{(k)}$. You can exploit the existing formula for the steepest descent.

(b) Given a sufficient condition on $\boldsymbol{H}_k$ for α_k to be positive at each k.

2. (10 points) Consider $\boldsymbol{x} = (x_1, x_2)^\top$ and the following quadratic function $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^\top \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \boldsymbol{x} - \boldsymbol{x}^\top \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 7$$

Let the initial guess $\boldsymbol{x}^{(0)} = \boldsymbol{0}$.

(a) Applying the rank-one update method to find the minimizer.

(b) Use the rank-one update method to generate two $\boldsymbol{Q}$-orthogonal directions (Ref. Homework 3).