Final Lab

Last name, First name: _____

Student ID Number: _____

Instructions

For the Final Exam, follow the following guidelines:

(1) **Solutions**:
  i.   To answer the problems, you may use the code you wrote throughout the semester, or build a new code.
  ii. All work must be done only by you. No outside help should be taken. If you don't work independently, it is always evident at first glance.

(2) Submission:
  i.   This exam has 5 problems, one bonus problem (last question), 9 pages, and 100 points.
  ii. Bonus problem is optional. If you get full credit from this exam, and you choose to solve the bonus problem, the credit for the bonus problem will be added to your grades for Lab assignments/project.
  iii. Upload your solutions as a single PDF file on Canvas.
  iv. The due date for the final exam is Thursday, May 12, 2022, at 11:00 PM Central Time.

Problem 1. For the following first-order ODE:

$$y't + 2y + e^{3t} = 0$$

1.1. Find the explicit solution using MATLAB built-in *dsolve()*.

Matlab Code *(7 points)*:

Copy and paste the solution from Command Window *(3 points)*:

1.2. Find the solution of the given ODE with the initial condition $y(0) = 0$.

Matlab Code *(3 points)*:

Copy and paste the solution from Command Window *(2 points)*:

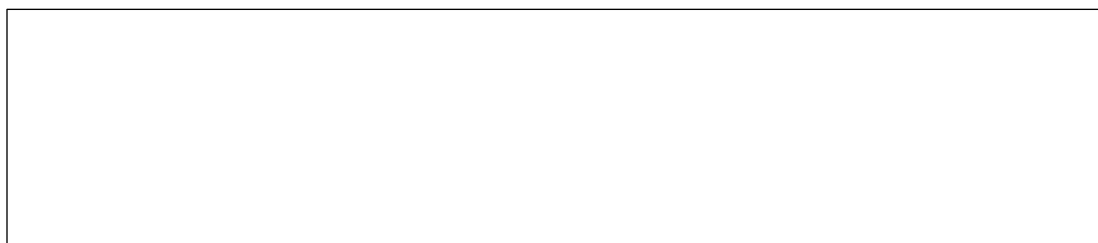1.3. Plot the solution of the IVP obtained from problem 1.2 with the following properties:
- Set x-axis limits to range from 5 to 10 ;
- Set y-axis limits to range from -3×10^{11} to 1×10^{11};
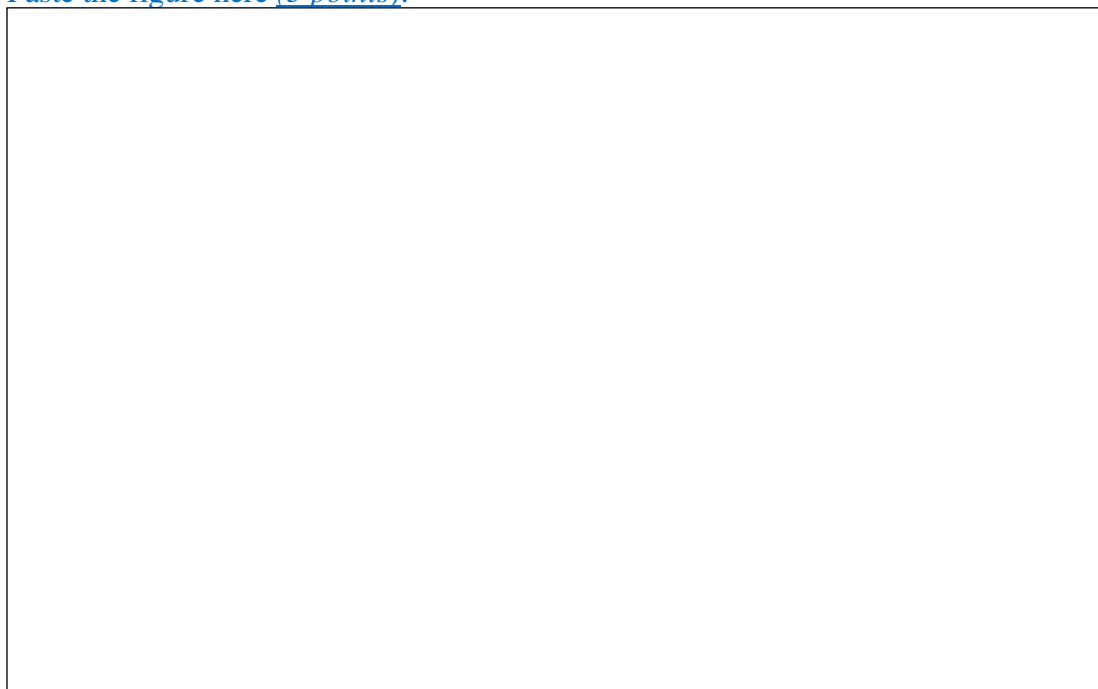
Copy and paste the figure here *(5 points)*:

Problem 2. Consider the IVP $\frac{dy}{dx} = \frac{-xy}{\sqrt{6-y^2}}$ with multiple initial conditions $y(0) = 0.5, 1.0, 1.5,$ and 2.0. Find the numerical solutions using MATLAB *ode45* solver with 0.0001 and 0.01 absolute and relative error tolerances, respectively. Plot the solutions in the same figure with the following properties: (i) set x-axis and y-axis limits to ranges from 0 to 5 and 0 to 1.0, respectively; and (ii) add legends to display the initial condition corresponding to each curve. (Hint: you may use the command *hold on* to retain the current plot, and use for loop to pass initial conditions at each iteration.)

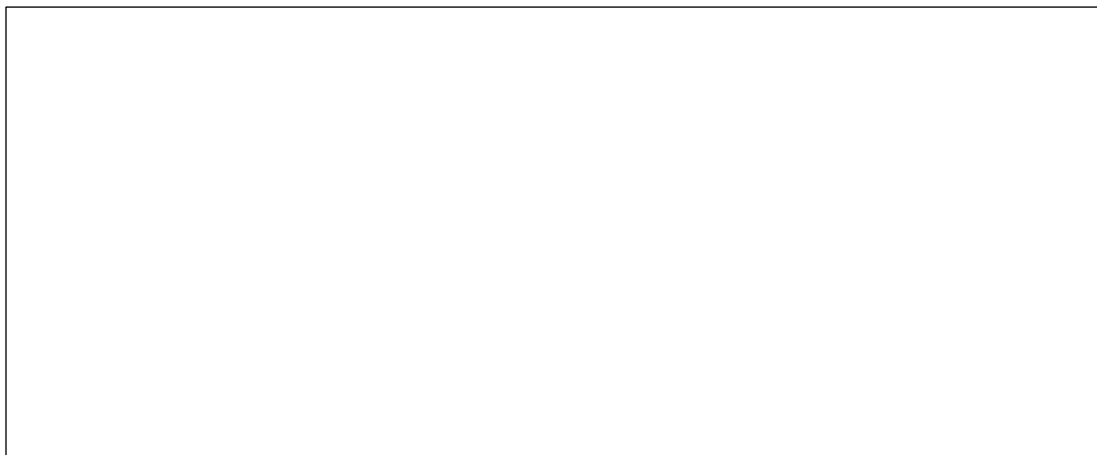Matlab Code *(10 points)*:

Paste the figure here *(5 points)*:

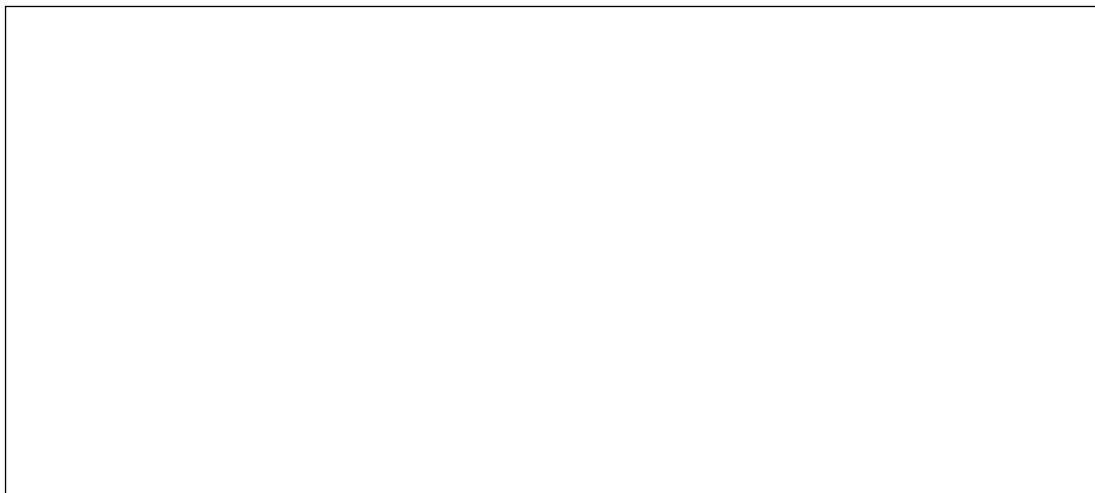Problem 3. Consider the differential equation given by

$$y' = \sin(y)\cos(t)$$

(i). Plot direction field over the interval $(-5,5) \times (0,5)$.

(ii). Find and plot the numerical solutions of given ODE with initial conditions $y(0) = -2, -1,$ $0, 1,$ and 2 with domain $[0\ 5]$. Note: Five solution curves and the direction field must be rendered in the same graph.

Matlab Code *(10 points)*:

Paste the figure here *(10 points)*:

Problem 4. As we had in the lab project, The Euler and Runge-Kutta (RK) methods are used to approximate the solutions for the ODEs. The IVP $y'(3 + t^2) - y^2 - 2ty = 0$, $y(0) = 0.5$ is given in interval [0 , 1.0]:

4.1. Using your MATLAB codes, complete the following table by finding the exact and approximate values of the solution of the given IVP at t = 0.1 and 0.5 using the Euler and RK methods with the step size h= 0.05. Round numbers to **six** decimal places. *(9 points)*

Table. Approximated and exact solutions with h=0.05

t	Approximated solution obtained from Euler method	Approximated solution obtained from RK method	Exact solution	Absolute error
0.1				
0.5				

4.2. Find the approximated solution of the given IVP at t=0.1 using **hand calculation**, as we did in class, from both Euler and RK methods with a step size of 0.05 and verify the approximated values obtained in Problem 4.1. **(Show your work)**

4.2.1. Euler Method *(4 points)*

4.2.2. RK method. *(8 points)*

Problem 5. Consider the following second-order ODE with coefficient parameters α and β.
$$-3u'' - \alpha u' + \beta u = 0, \quad u'(0) = u(0) = 1$$
Find and plot the general solutions for problems 5.1, 5.2, and 5.3 using MATLAB.

5.1. $\alpha = 3$ and $\beta = -1$; *(8 points)*

General Solution:

Figure:

5.2. $\alpha = 6$ and $\beta = -3$; *(8 points)*
General Solution:

Figure:

5.3. $\alpha = 3$ and $\beta = 1$; *(8 points)*
General Solution:

Figure:

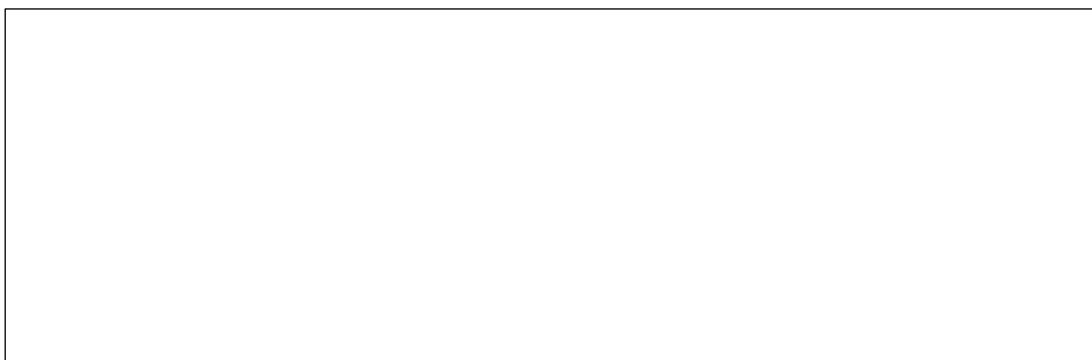Bonus Problem (optional): Solve the system of ODE

$$\begin{pmatrix} \dfrac{dx}{dt} \\ \dfrac{dy}{dt} \end{pmatrix} = \begin{bmatrix} -4 & -3 \\ 2 & 3 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{bmatrix} -5 \\ -2 \end{bmatrix}$$

with initial condition $x(0) = y(0) = 1$.

(Hint: First, transform the given matrix-form system to a system of first-order ODEs. Also, you may find the following document helpful *"Ordinary Differential Equations in MATLAB"* by P. Howard. This document has been posted by Dr. Mc Coy on Canvas>Files on January 19, 2022.)

(i)   Find the solutions $x(t)$ and $y(t)$. *(10 points)*

Matlab Code

(ii)   Plot the solutions in the same figure. Set x-axis limits to range from 0 to 0.5 ; Set y-axis limits to range from -2 to 1.5. *(5 points)*

Paste the figure here

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