# Apply Statistical Analysis

## Michael Dang

University of Missouri - Kansas City

April 28, 2025

## Assignment 4

Solutions are to be due on April 28th.

- (a) This is the two-factor experiment.
  - Treatment structure: 3 (height)  $\times$  2 (width) = 6 (treatment combinations)
  - Treatment combinations are:
    - (height 1, width 1)
    - (height 1, width 2)
    - (height 2, width 1)
    - (height 2, width 2)
    - (height 3, width 1)
    - (height 3, width 2)
- (b) All 20 supermarkets must be similar (under the similar environment), and the (Height, Width) treatment combinations were assigned completely randomly across them without any blocking.

(c) 
$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha \cdot \beta)_{ij} + \epsilon_{ijk}$$

### • Where:

 $y_{ijk}$  = the sales of the bakery's bread under the  $k^{\text{th}}$  supermarket of the  $i^{\text{th}}$  height and  $j^{\text{th}}$  width.

 $\mu = \text{Overall mean sales of the bakery's bread under all supermarkets.}$ 

 $\alpha_i$  = The effect of the  $i^{th}$  height on the response variable.

 $\beta_i$  = The effect of the  $j^{th}$  width on the response variable.

 $(\alpha \cdot \beta)_{ij}$  = The interaction effect of the  $i^{\text{th}}$  height and  $j^{\text{th}}$  width on the response variable.

 $\epsilon_{ijk}$  = Random experimental error of the  $k^{th}$  supermarket under the  $i^{th}$  height and the  $j^{th}$  width.

### • Where:

$$i = 1, 2, 3$$
  
 $j = 1, 2$   
 $k = 1, 2, ..., n_{ij}$ 

(d) • Model assumption:

$$\epsilon \stackrel{iid}{\sim} N(0, \sigma^2)$$

- Constant variance assumption.
- Normality assumption.
- (e) The estimated parameters of the model is below:

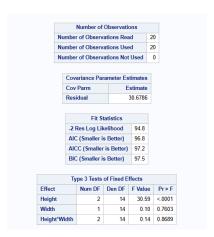


Figure 1: Estimated parameters of the model

• Constant variance assumption:

$$H_0: \sigma_1^2 = \sigma_2^2 = \dots = \sigma_6^2 = \sigma^2$$
 vs  $H_A: \text{Not all } \sigma_i^2$  are equal

- Levene's test:

$$0.3217 \text{ (p-value)} > 0.01 \text{ } (\alpha)$$

- Hence, fail to reject  $H_0$ .
- Constant variance assumption is satisfied.

		s Test for Homoge of Absolute Deviati			
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
HeightWidth	5	42.8333	8.5667	1.29	0.3217
Error	14	92.7778	6.6270		

Figure 2: Levene's test

• Normality assumption:

 $H_0$ : Residual are from a Normal population  $H_A$ : Residual are not from a Normal population

- K-S test:

$$> 0.15 \text{ (p-value)} > 0.05 \text{ } (\alpha)$$

- Hence, the normality assumption is satisfied.

Goodness-of-Fit Tests for Normal Distribution							
Test	Statistic		p Value				
Kolmogorov-Smirnov	D	0.10837667	Pr > D	>0.150			
Cramer-von Mises	W-Sq	0.02560585	Pr > W-Sq	>0.250			
Anderson-Darling	A-Sq	0.19881023	Pr > A-Sq	>0.250			

Figure 3: K-S test

(f) • Testing whether height affects sales:

$$H_0: \alpha_i = 0$$
 vs  $H_A: \alpha_i \neq 0$ 

- From figure 1 under the "Type 3 tests fixed effects" table:

$$< 0.0001 \text{ (p-value)} < 0.05 \text{ } (\alpha)$$

- Hence reject  $H_0$ .
- Therefore, with 95% confidence, we can conclude that the display height affects sales.
- Testing whether width affects sales:

$$H_0: \beta_j = 0$$
 vs  $H_A: \beta_j \neq 0$ 

- From figure 1 under the "Type 3 tests fixed effects" table:

0.7603 (p-value) > 0.05 (
$$\alpha$$
)

- Hence, fail to reject  $H_0$ .
- Therefore, with 95% confidence, we can conclude that the display width does not effects sales.
- Testing whether the interaction between height and width:

$$H_0: (\alpha \cdot \beta)_{ij} = 0$$
 vs  $H_A: (\alpha \cdot \beta)_{ij} \neq 0$ 

- From figure 1 under the "Type 3 tests fixed effects" table:

$$0.8689 \text{ (p-value)} > 0.05 \text{ } (\alpha)$$

- Hence, fail to reject  $H_0$ .
- Therefore, with 95% confidence, we can conclude that there is no significant interaction between display and width.
- Thus, the effect on height does not vary by width and the effect on width does not vary on height.