Apply Statistical Analysis

Michael Dang

University of Missouri - Kansas City

April 28, 2025

Assignment 4

Solutions are to be due on April 28th.

(a)
- This is the two-factor experiment.
- Treatment structure: 3 (height) $\times 2$ (width) $= 6$ (treatment combinations)
- Treatment combinations are:
  - (height 1, width 1)
  - (height 1, width 2)
  - (height 2, width 1)
  - (height 2, width 2)
  - (height 3, width 1)
  - (height 3, width 2)

(b) All 20 supermarkets must be similar (under the similar environment), and the (Height, Width) treatment combinations were assigned completely randomly across them without any blocking.

(c)

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha \cdot \beta)_{ij} + \epsilon_{ijk}$$

- Where:

$$\begin{aligned}
y_{ijk} &= \text{the sales of the bakery's bread under the } k^{\text{th}} \text{ supermarket} \\
&\quad \text{of the } i^{\text{th}} \text{ height and } j^{\text{th}} \text{ width.} \\
\mu &= \text{Overall mean sales of the bakery's bread under all supermarkets.} \\
\alpha_i &= \text{The effect of the } i^{th} \text{ height on the response variable.} \\
\beta_j &= \text{The effect of the } j^{th} \text{ width on the response variable.} \\
(\alpha \cdot \beta)_{ij} &= \text{The interaction effect of the } i^{\text{th}} \text{ height and} j^{\text{th}} \text{ width on the} \\
&\quad \text{response variable.} \\
\epsilon_{ijk} &= \text{Random experimental error of the } k^{th} \text{ supermarket under} \\
&\quad \text{the } i^{th} \text{ height and the } j^{th} \text{ width.}
\end{aligned}$$

- Where:

$$\begin{aligned}
i &= 1,\ 2,\ 3 \\
j &= 1,\ 2 \\
k &= 1,\ 2,\ ...,\ n_{ij}
\end{aligned}$$

(d)
- Model assumption:

$$\epsilon \overset{iid}{\sim} N(0, \sigma^2)$$

- Constant variance assumption.
- Normality assumption.

(e)
- The estimated parameters of the model is below:

Number of Observations	
Number of Observations Read	20
Number of Observations Used	20
Number of Observations Not Used	0

Covariance Parameter Estimates	
Cov Parm	Estimate
Residual	30.6786

Fit Statistics	
-2 Res Log Likelihood	94.8
AIC (Smaller is Better)	96.8
AICC (Smaller is Better)	97.2
BIC (Smaller is Better)	97.5

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
Height	2	14	30.59	<.0001
Width	1	14	0.10	0.7603
Height*Width	2	14	0.14	0.8689

Figure 1: Estimated parameters of the model

- Constant variance assumption:

$$H_0 : \sigma_1^2 = \sigma_2^2 = ... = \sigma_6^2 = \sigma^2 \quad \text{vs} \quad H_A : \text{Not all } \sigma_i^2 \text{ are equal}$$

  - Levene's test:

$$0.3217 \;\; (\text{p-value}) > 0.01 \;\; (\alpha)$$

  - Hence, fail to reject H_0.
  - Constant variance assumption is satisfied.

The GLM Procedure

Levene's Test for Homogeneity of Sales Variance ANOVA of Absolute Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
HeightWidth	5	42.8333	8.5667	1.29	0.3217
Error	14	92.7778	6.6270		

Figure 2: Levene's test

- Normality assumption:

$$H_0\text{: Residual are from a Normal population}$$
$$H_A\text{: Residual are not from a Normal population}$$

  - K-S test:

$$> 0.15 \;\; (\text{p-value}) > 0.05 \;\; (\alpha)$$

  - Hence, the normality assumption is satisfied.

Goodness-of-Fit Tests for Normal Distribution				
Test	Statistic		p Value	
Kolmogorov-Smirnov	D	0.10837667	Pr > D	>0.150
Cramer-von Mises	W-Sq	0.02560585	Pr > W-Sq	>0.250
Anderson-Darling	A-Sq	0.19881023	Pr > A-Sq	>0.250

Figure 3: K-S test

(f)
- Testing whether height affects sales:

$$H_0 : \alpha_i = 0 \quad \text{vs} \quad H_A : \alpha_i \neq 0$$

  - From figure 1 under the "Type 3 tests fixed effects" table:

$$< 0.0001 \;\; (\text{p-value}) < 0.05 \;\; (\alpha)$$

- Hence reject H_0.
- Therefore, with 95% confidence, we can conclude that the display height affects sales.

- Testing whether width affects sales:

$$H_0 : \beta_j = 0 \quad \text{vs} \quad H_A : \beta_j \neq 0$$

  - From figure 1 under the "Type 3 tests fixed effects" table:

$$0.7603 \ \text{(p-value)} > 0.05 \ (\alpha)$$

  - Hence, fail to reject H_0.
  - Therefore, with 95% confidence, we can conclude that the display width does not effects sales.

- Testing whether the interaction between height and width:

$$H_0 : (\alpha \cdot \beta)_{ij} = 0 \quad \text{vs} \quad H_A : (\alpha \cdot \beta)_{ij} \neq 0$$

  - From figure 1 under the "Type 3 tests fixed effects" table:

$$0.8689 \ \text{(p-value)} > 0.05 \ (\alpha)$$

  - Hence, fail to reject H_0.
  - Therefore, with 95% confidence, we can conclude that there is no significant interaction between display and width.
  - Thus, the effect on height does not vary by width and the effect on width does not vary on height.

□