Socially Optimal Non-Discriminatory Restrictions for Continuous-Action Games

Michael Oesterle, Guni Sharon

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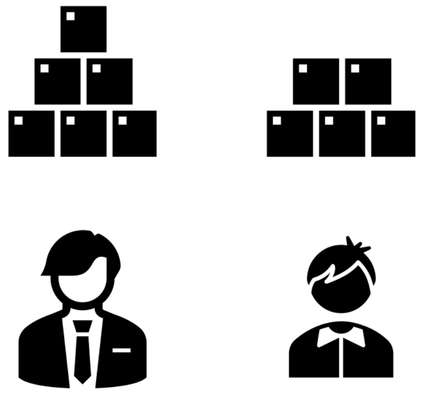
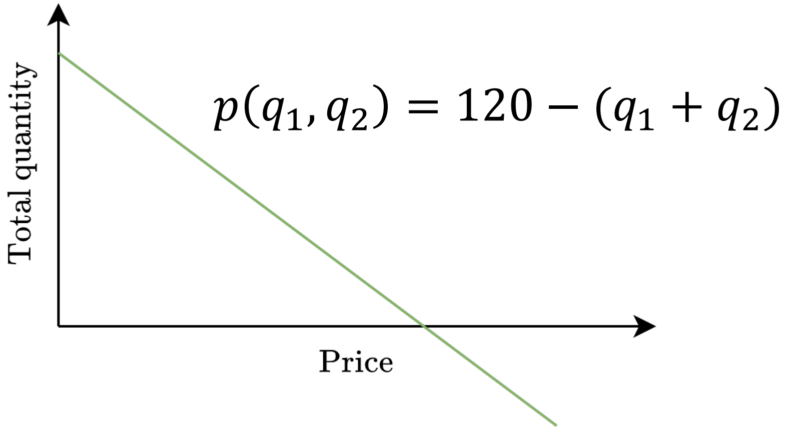
...and we can show how!

Multi-Player Games Need Governance For Optimal Social Welfare





Each player's profit is quantity times price:

$$u_i(q_1, q_2) = (p(q_1, q_2) - 12) \cdot q_i$$

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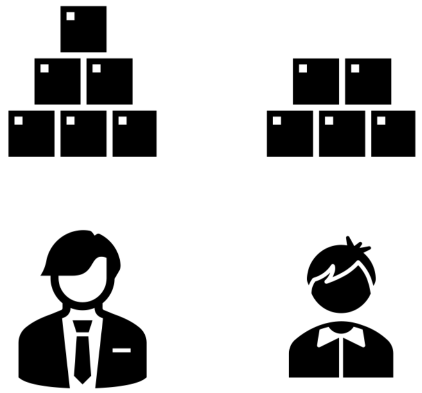

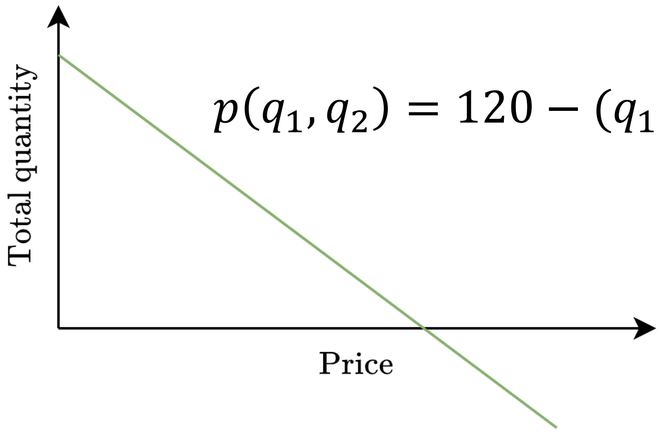

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- However, the social welfare (sum of profits) is maximized when both produce 27

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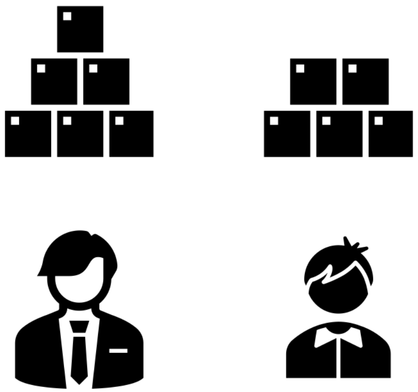

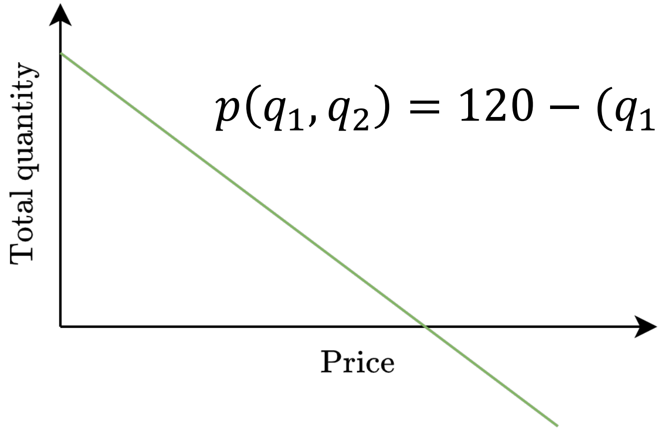

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What is the best strategy for a player to make maximum profit?

- If the players can choose their quantities freely, they each produce 36
- However, the social welfare (sum of profits) is maximized when both produce 27
- **Restricting the quantities by introducing an upper bound increases social welfare**

What Can A Governance Do?

- Reward shaping: Change which actions players prefer
  Example: Marginal Cost Tolling (Sharon et al. 2017) for traffic networks

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- Reward shaping: Change which actions players prefer
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- **Action space shaping: Change which actions players can take**
  Example: Road closures for traffic networks

Specifically: Compute socially optimal action space restrictions

What Is Restriction-Based Governance?

- Making the action space smaller can be an effective means to increase social welfare

Discrete action space

Continuous action space

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- By allowing the same actions for all players, we create a “fair”, non-discriminatory governance (a reward-shaping mechanism like Vickrey-Clarke-Groves (VCG) would not usually do this)

What Is The Goal Of Our Paper?

In a **Normal-Form Game** with a **social utility** function,

find a **subset of the action space**

which results in the **maximum social welfare**,

when all **players act selfishly**

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  - ➢ Always remove half-open intervals, based on a resolution parameter
- Output the restriction with the highest social welfare
  - ➢ Apply a depth-first search over possible restrictions

What Are The Technical Details?

2.1 Model

Let $G = (N, A, \boldsymbol{u})$ be a Normal-Form Game with player set $N = \{1, ..., n\}$ and action space A which applies to all players ("uniform" NFG). Writing product sets and vectors of variables in bold face, a joint action is given by $\boldsymbol{a} \in \boldsymbol{A} := A^N$. The players' utility functions are $\boldsymbol{u} = (u_i)_{i \in N}$, where $u_i : \boldsymbol{A} \to \mathbb{R}$. Moreover, let $\mathfrak{u} : \boldsymbol{A} \to \mathbb{R}$ be a *social utility function*. We call $G = (N, A, \boldsymbol{u}, \mathfrak{u})$ a *social game*.

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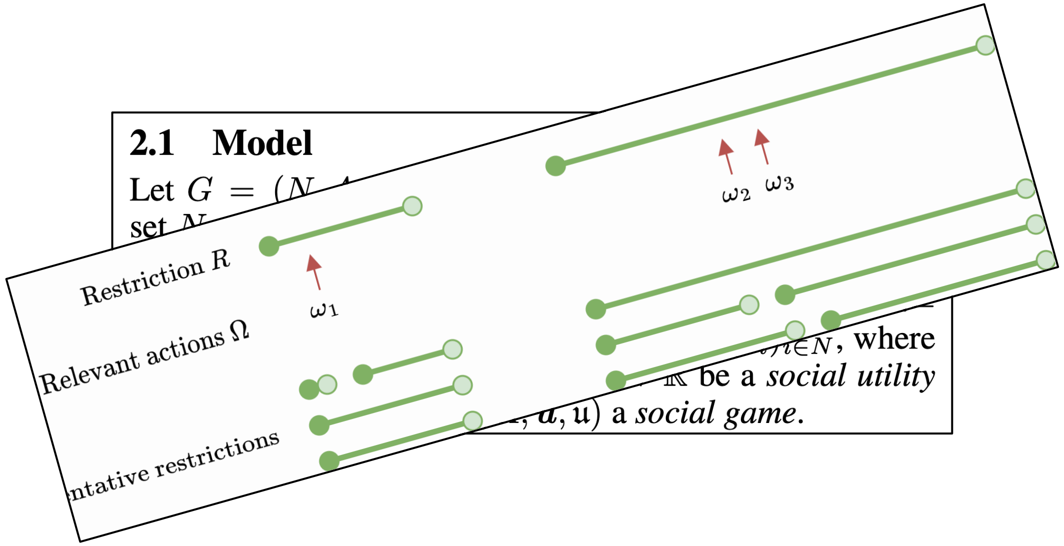

What Are The Technical Details?

Algorithm 1: Socially Optimal Action-Space Restrictor (SOAR)

Data: Social Game $G = (N, A, \boldsymbol{u}, \mathfrak{u})$, equilibrium oracle μ, resolution ϵ

Result: Socially optimal restriction $\hat{R}^* \subseteq A$

$(\hat{R}^*, \hat{u}^*) \leftarrow (A, \mathfrak{u}(\mu(A)))$
$Q \leftarrow$ Queue with content A
while Q *is not empty* **do**
    $R \leftarrow Q.dequeue()$
    `// Loop through relevant actions`
    for $\omega \in \Omega(\mu(R))$ **do**
        $R' \leftarrow R.remove(\mathcal{U}_\epsilon(\omega))$ `// Tentative restriction`
        if R' *is not empty and has not been explored before* **then**
            $Q.enqueue(R')$
            if $\mathfrak{u}(\mu(R')) > \hat{u}^*$ **then**
                $(\hat{R}^*, \hat{u}^*) \leftarrow (R', \mathfrak{u}(\mu(R')))$
            end
        end
    end
end
return $\hat{R}^*$

What Are The Technical Details?

Proposition 1. *Given some $x \in \mathbb{R}$, let $\mathcal{U}_\epsilon(x) := [x-\epsilon, x+\epsilon)$ denote the half-open ϵ-neighborhood of x, and for a vector $\boldsymbol{x} \in \mathbb{R}^N$, let $\mathcal{U}_\epsilon(\boldsymbol{x}) := \cup_{i \in N} \mathcal{U}_\epsilon(x_i) \subseteq \mathbb{R}$ (note that this neighborhood is still one-dimensional!). Assume that $\boldsymbol{a} \in \boldsymbol{A}$ is a joint action such that $\mathcal{N} \subseteq \boldsymbol{R}$ with $R := A \setminus \mathcal{U}_\epsilon(\boldsymbol{a})$. Then*

$$\mathcal{N} \subseteq \mathcal{N}|_R \, ,$$

which means that invalidating actions within the ϵ-neighborhood of $\boldsymbol{a}$ removes none of the Nash Equilibria from G.

Proof. Let $\boldsymbol{x} \in \mathcal{N}$ be an NE over the action space A, and let R be defined as in the statement of the proposition. Then

$$\begin{aligned} & x_i \in \mathcal{B}_i(x_{-i}) \; \forall i \in N \\ \implies \; & u_i(\boldsymbol{x}) \geq u_i(a', \boldsymbol{x}_{-i}) \; \forall a' \in A \; \forall i \in N \\ \overset{R \subseteq A}{\implies} \; & u_i(\boldsymbol{x}) \geq u_i(a', \boldsymbol{x}_{-i}) \; \forall a' \in R \; \forall i \in N \\ \overset{\boldsymbol{x} \in \boldsymbol{R}}{\implies} \; & x_i \in \mathcal{B}_i|_R(x_{-i}) \forall i \in N \end{aligned}$$

ion-Space Re-

), equilibrium

$\hat{R}^* \subseteq A$

nt actions

Tentative

re

ntative

8 Q

9 **if**

10

11 **en**

12 **end**

13 **end**

14 **end**

15 **return** $\hat{R}^*$

Theorem 1. *Let $G = (N, A, \boldsymbol{u}, \mathfrak{u})$ be a social game. If Assumption 2 holds, and for a sufficiently small $\epsilon > 0$, Algorithm 1 finds an optimal restriction R^*.*

Proof. SOAR terminates after finitely many steps: Any tentative restriction R' produced by a reduction of some $R \in Q$ continues a chain of increasingly constrained restrictions, as in Proposition 2, and the length of such a chain is bounded by $\lceil \frac{b-a}{\epsilon} \rceil$.

At the point of termination, Q is empty. Condition (i) in Proposition 4 does not hold anymore, which means that $\hat{R}^*$ is indeed an optimal restriction. $\square$

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Experiments

Parameterized Cournot Game (CG)

In [ ]:

```
results = []
epsilon, decimals = 0.1, 3
solver = IntervalUnionRestrictionSolver(epsilon=epsilon)
progress_bar = display(progress(0, 100), display_id=True)
lambda_min, lambda_max = 10.0, 200.0
lambdas = list(np.round(np.arange(lambda_min, lambda_max, 1.0), decimals=decimals))

print(f'Solving {len(lambdas)} Cournot games...')
for i, lambda_ in enumerate(lambdas):
  progress_bar.update(progress(i, len(lambdas)))

  u_1 = QuadraticTwoPlayerUtility(0, [-1.0, 0.0, -1.0, lambda_, 0.0, 0.0])
  u_2 = QuadraticTwoPlayerUtility(1, [0.0, -1.0, -1.0, 0.0, lambda_, 0.0])

  a = IntervalUnion([(0.0, lambda_)])
  g = GovernedNormalFormGame(a, [u_1, u_2], u_1 + u_2)

  results.append(solver.solve(g, nash_equilibrium_oracle=worst_hill_climbing_nash_equilibrium))

progress_bar.update(progress(len(lambdas), len(lambdas)))

print('Done!')
```

return $\hat{R}^*$

Proposition 4 does not hold anymore, which means that $\hat{R}^*$ is indeed an optimal restriction. $\square$

How Do We Measure If This Works?

By how much does it get better?

- (Absolute and relative) improvement of social welfare

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- (Absolute and relative) improvement of social welfare

How restrictive do we have to be?

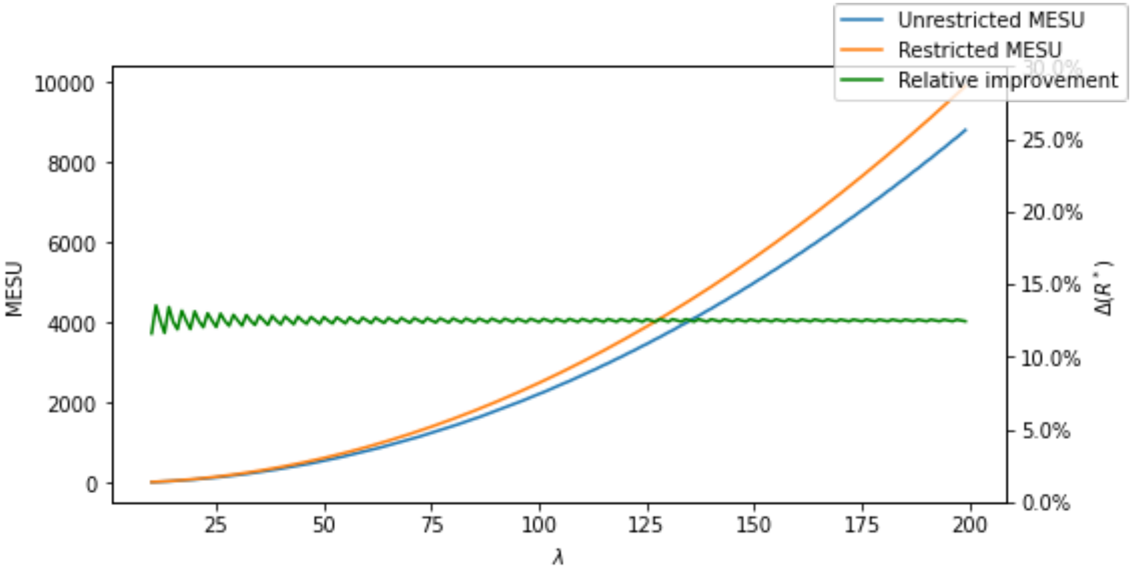
- Degree of restriction (proportion of allowed and forbidden actions)

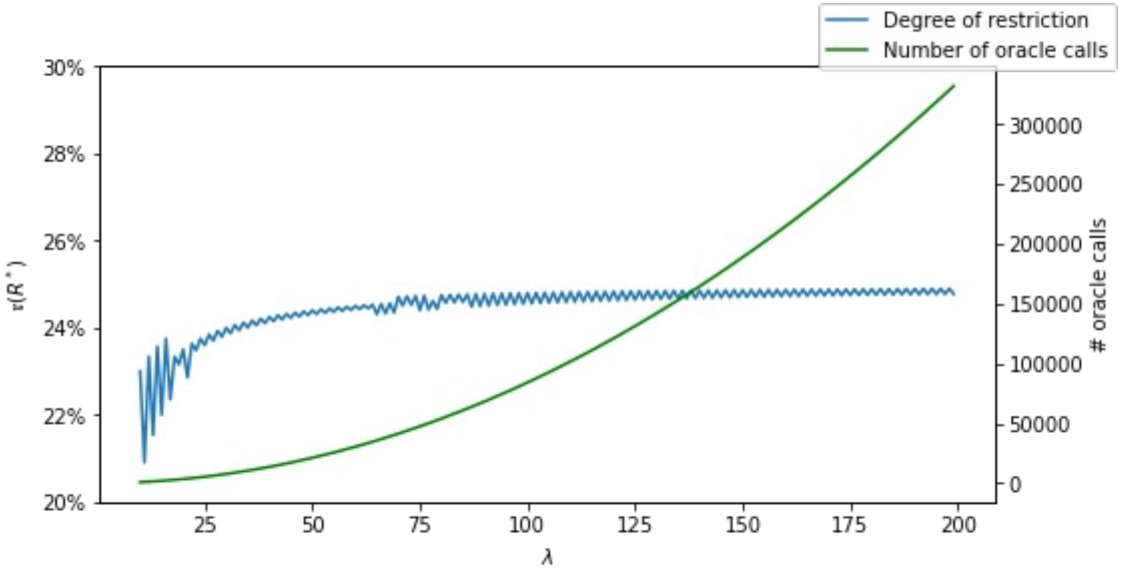
How long does it take to find the optimal restriction?

- Number of tentative restrictions / oracle calls

Does It Work?

Yes!





MESU = Minimum Equilibrium Social Utility
Degree of restriction = Proportion of forbidden actions

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… we can (currently) only handle one-dimensional action spaces

… non-discriminatory restrictions only make sense in so-called coordination games

Key Message and Contact

What: Non-discriminatory action space restrictions in multi-player games

Why: Optimizing and standardizing regulatory restrictions

How: Informed depth-first search on the action space

https://github.com/michoest/aaai-2023
(Repository with paper and code)

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Right now!

Tonight!

Anytime!