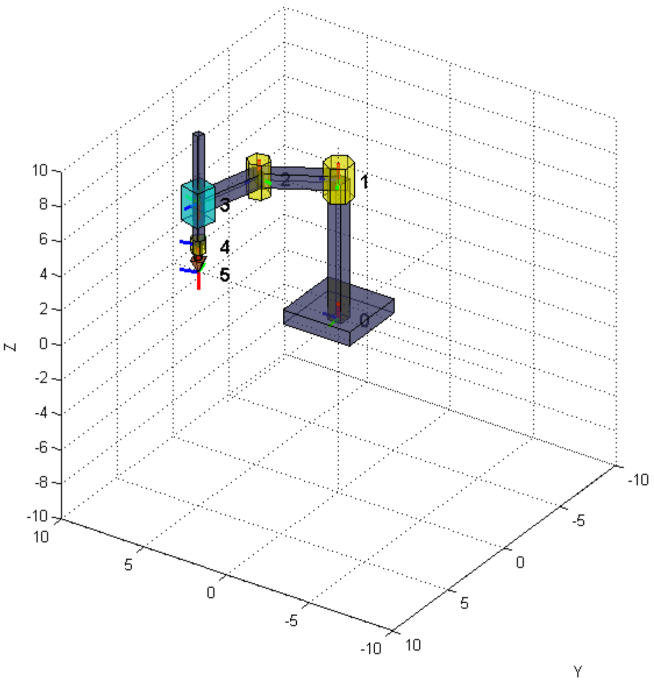
FORWARD AND INVERSE KINEMATICS

SCARA ROBOT



Michał Komorowski, January 2013, automatics and robotics

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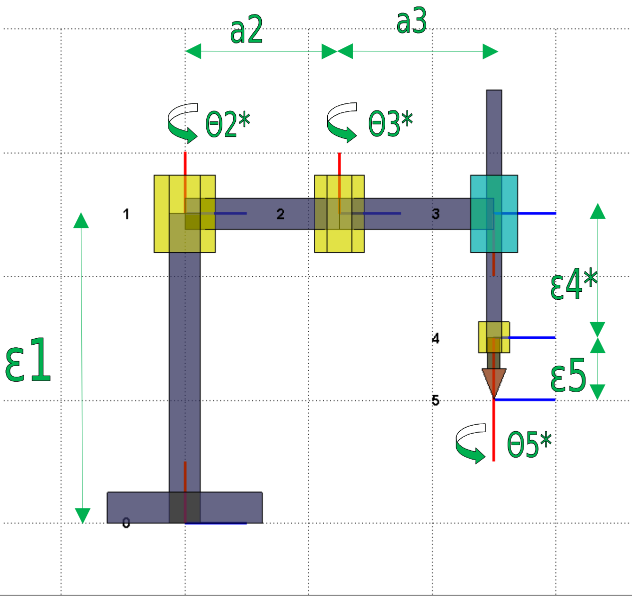
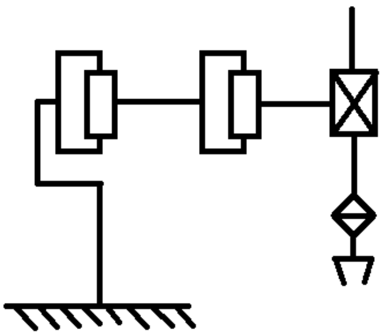
SCHEMATIC, D-H TABLE



SCARA schematic with coordinate systems
(red lines – Z axis, blue lines – X axis ; numbers near coordinate systems)

	$\boldsymbol{\Theta_i}$	$\boldsymbol{\varepsilon_i}$	$\mathbf{a_i}$	$\boldsymbol{\alpha_i}$
H01	0^o	ε_1	0	0^o
H12	Θ_2^*	0	a_2	0^o
H23	Θ_3^*	0	a_3	180^o
H34	0^o	ε_4^*	0	0^o
H45	Θ_5^*	ε_5	0	0^o



SCARA kinematic diagram

HOMOGENEOUS MATRICES

Homogeneous matrices were created with use of the D-H tables (previous page).
They describe positions and orientations of subsequent coordinate systems attached to the robot's joints and links – each relative to previous coordinate systems. Last matrix describes position and orientation of the end-effector, relative to the base coordinate system {0}.

D-H Table (joint variable marked with star *)

Θ_2^* - rotation angle - joint 1.

Θ_3^* - rotation angle – joint 2.

ε_4^* - rotation angle - joint 3.

Θ_5^* - rotation angle – joint 4.
(end-effector orientation)

H01= Trans(0,0, ε_1)=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \epsilon 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

H12= Rot(Z,Θ_2^*)*Trans(a_2,0,0)=

$$\begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & \epsilon_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

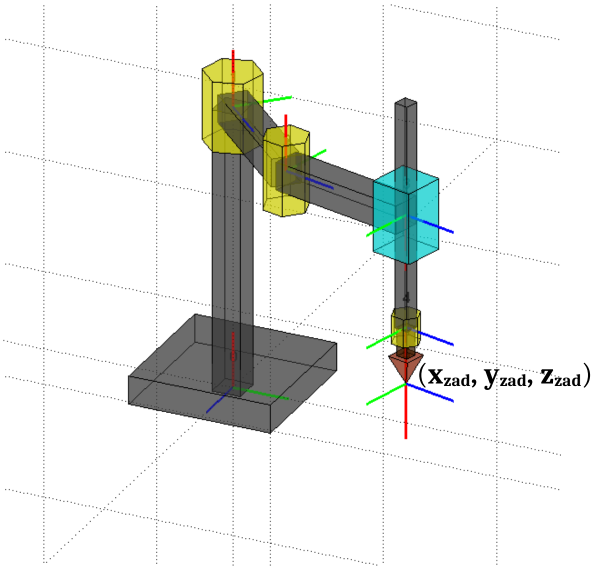
H23= Rot(Z,Θ_3^*)*Trans(a_3,0,0)* Rot(X,180°)=

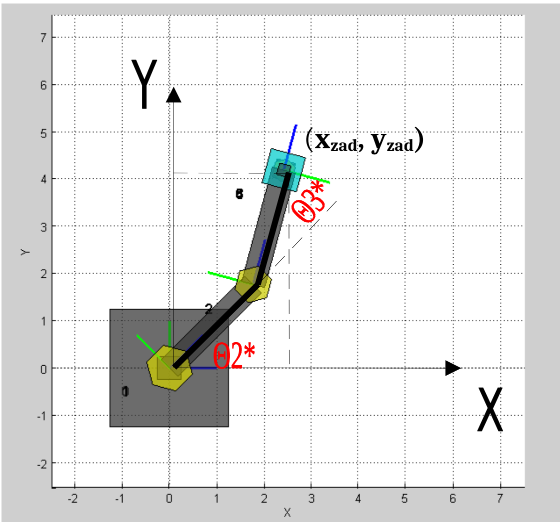
$$\begin{pmatrix} \cos(\theta_3) & \sin(\theta_3) & 0 & a_3\cos(\theta_3) \\ \sin(\theta_3) & -\cos(\theta_3) & 0 & a_3\sin(\theta_3) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

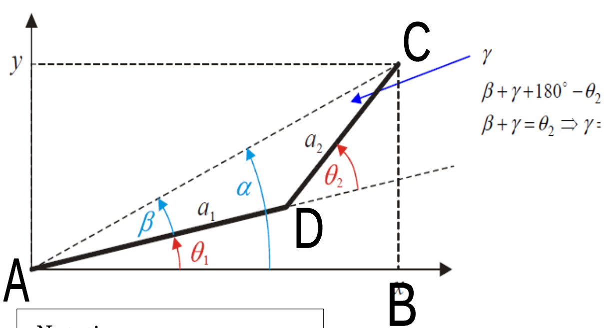
H05=H01*H12*H23*H34*H45=

$$\begin{pmatrix} \cos(\theta_2+\theta_3-\theta_5) & \sin(\theta_2+\theta_3-\theta_5) & 0 & a_3\cos(\theta_2+\theta_3)+a_2\cos(\theta_2) \\ \sin(\theta_2+\theta_3-\theta_5) & -\cos(\theta_2+\theta_3-\theta_5) & 0 & a_3\sin(\theta_2+\theta_3)+a_2\sin(\theta_2) \\ 0 & 0 & -1 & \epsilon_1-\epsilon_4-\epsilon_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

To solve the inverse kinematics we need to find relation between joint variables ($\boldsymbol{\Theta_2^*}$,$\boldsymbol{\Theta_3^*}$,$\boldsymbol{\varepsilon_4^*}$,$\boldsymbol{\Theta_5^*}$) and variables describing desired postition of the robot's end effector ($\mathbf{x_{zad}}$, $\mathbf{y_{zad}}$, $\mathbf{z_{zad}}$). Below a geometric approach is described to find target variables:







Notation:
$a_1:=l_{L2}$, $a_2:=l_{L3}$, $\Theta_2:=\Theta_3$,
$\Theta_1:=\Theta_2$

The Law of cosines in ΔADC gives:

$$\left(\sqrt{x_{zad}^{2}+y_{zad}^{2}}\right)^2=l_{L2}^{2}+l_{L3}^{2}-2\,l_{L2}l_{L3}\cos\left(\pi-\theta_3\right)$$

$$x_{zad}^{2}+y_{zad}^{2}=l_{L2}^{2}+l_{L3}^{2}+2l_{L2}l_{L3}\cos\left(\theta_3\right)$$

$$\cos\left(\theta_3\right)=\frac{x_{zad}^{2}+y_{zad}^{2}-l_{L2}^{2}-l_{L3}^{2}}{2\,l_{L2}l_{L3}}:=D$$

Utilizing the Pythagorean trigonometric identity:

$$\cos^2\left(\theta_3\right)+\sin^2\left(\theta_3\right)=1$$

leads to:

$$\sin\left(\theta_3\right)=\pm\sqrt{1-D^2}$$

Thus:

$$\theta_3=arctg\left(\pm\frac{\sqrt{1-D^2}}{D}\right)$$

There are two possible robot's elbow configurations, depending on a choice of a sign above. In the MATLAB visualisation code „+” sign was chosen. That implies „elbow down” configuration.

ΔADC angles analysis gives:

$$\alpha=\beta+\theta_2\rightarrow\theta_2=\alpha-\beta$$

$$\alpha=\arctan\left(\frac{y_{zad}}{x_{zad}}\right)$$

$$\gamma+\beta+\left(\pi-\theta_3\right)=\pi\rightarrow\gamma=\beta-\theta_3$$

From the Law of sines in ΔADC:

$$\frac{l_{L3}}{\sin\beta}=\frac{l_{L2}}{\sin\gamma}$$

$$\frac{l_{L3}}{\sin\beta}=\frac{l_{L2}}{\sin\left(\beta-\theta_3\right)}$$

$$\frac{l_{L3}}{\sin\beta}=\frac{l_{L2}}{\sin\theta_3\cos\beta-\sin\beta\cos\theta_3}$$

$$l_{L3}\sin\theta_3 cos\beta - l_{L3} sin\beta cos\theta_3 = l_{L2}\sin\beta$$

$$l_{L3}\sin\theta_3 cos\beta = l_{L3} sin\beta cos\theta_3 + l_{L2}\sin\beta$$

$$(l_{L3}\sin\theta_3) cos\beta = (l_{L3}\cos\theta_3 + l_{L2})\sin\beta$$

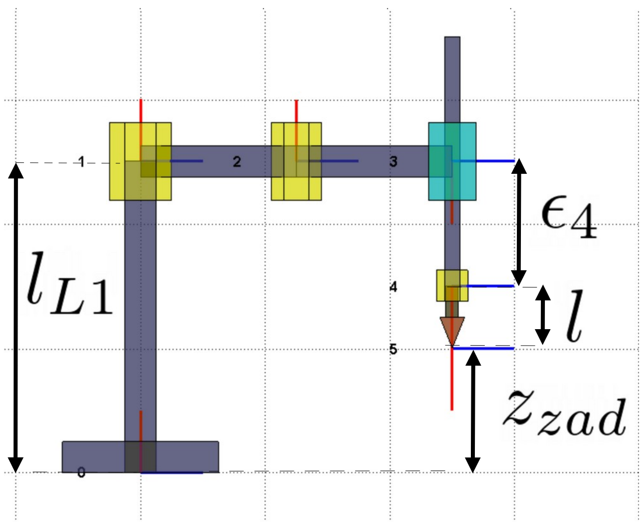
To get tangent of β *angle :*

$$\frac{sin\beta}{cos\beta} = \frac{l_{L3}\sin\theta_3}{l_{L3}\cos\theta_3 + l_{L2}}$$

$$tg\beta = \frac{l_{L3}\sin\theta_3}{l_{L3}\cos\theta_3 + l_{L2}}$$

$$\beta = arctg\left(\frac{l_{L3}\sin\theta_3}{l_{L3}\cos\theta_3 + l_{L2}}\right)$$

Having α *and* β *and using* $\theta_2 = \alpha - \beta$ *leads to :*



$$\theta_2 = arctg\left(\frac{y_{zad}}{x_{zad}}\right) - arctg\left(\frac{l_{L3}\sin\theta_3}{l_{L3}\cos\theta_3 + l_{L2}}\right)$$

After analysis of the schematic one can write:

$$\varepsilon_4 = l_{L1} - l - z_{zad}$$

Last joint variable Θ_5 orients the end-effector in space. It is a rotation angle (Z axis) of the local coordinate system {5}. By fixing Θ_5 angle as 0, end-effector rotation angle with respect to coordinate system {0} equals $\Theta_2 + \Theta_3$.

If one want to control a global rotation angle of the end-effector, following formula shall be used:

$$\theta_5 = \theta_{5glob.} + \theta_2 + \theta_3$$

where : $\Theta_{5glob.}$ - desired angle of the end-effector relative to global coordinate system {1},
($\Theta_2 + \Theta_3$) – compensation term

The use of „+" sign in the compensation term should be clarified . Coordinate system {5} is oriented „upside-down" relative to a global coordinate system {0}, thus positive local angle of the end-effector implies negative global rotation angle of the end-effector. This concludes to:

$$\theta_5 \neq \theta_{5glob.} - (\theta_2 + \theta_3)$$

SCARA ROBOT SIMULATION PROGRAM DESCRIPTION

OUTLINE

Simulation program is divided into following main parts:

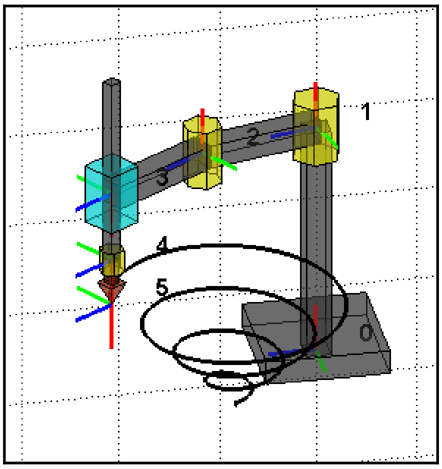
1. **Init the workspace (function:** ***InitSpace*****)**
2. **Load robot's dimensions, scale factor and robot's global orientation**
3. **Choice of one of the demo trajectories (variable:** ***mode*****):**
   3.1. Static configuration (mode=0)
   3.2. Trajectory 1: Circle (mode=1)
   3.3. Trajectory 2: Line (mode=2)
   3.4. Trajectory 3: „Fish" (mode=3)
   3.5. Trajectory 4: Quadrifolium (mode=4)
   3.6. Trajectory 5: Archimedean spiral (mode=5)
4. **Model animation, trajectory trace drawing, coordination system assignment (functions:** ***SCARA_3Dmodel*** **and** ***plot3*****)**

MODEL DRAWING FUNCTION

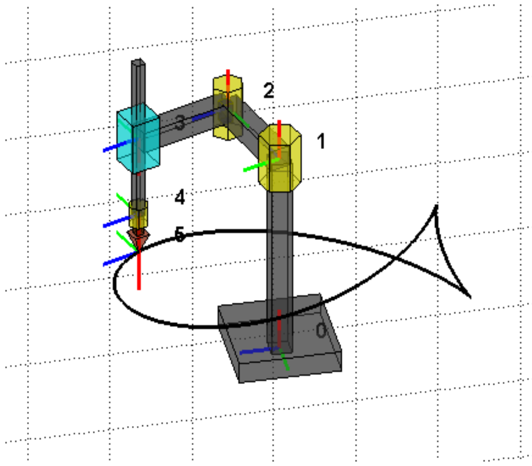
SCARA_3Dmodel (H_{loc}, Q, s) – inputs H_{loc} homogeneous matrix (which places the robot in desired position and with desired orientation), vector of joint variables Q and scale factor s. In the code user can define joints and links dimensions and their orientation in space defining forward kinematics (homogeneous matrices). SCARA_3Dmodel function calls four subfunctions: two for drawing linear and rotational joints (*J_ROT3D and J_TRANS3D),* one for drawing links (*L_BOX3D*) and one responsible for drawing the end-effector (*L_TOOL3D*).

TRAJECTORY GENERATION FUNCTION (MODE_XXX FILES)

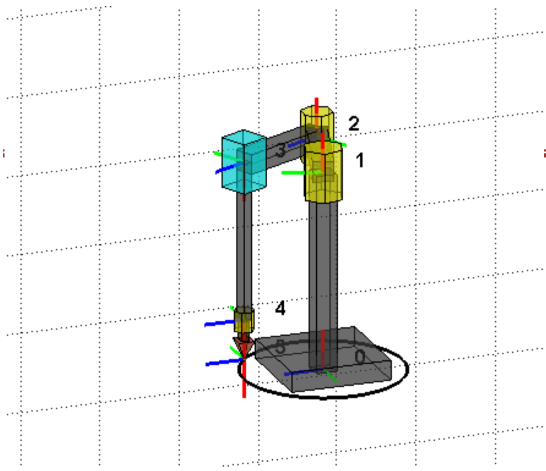
In each for loop iteration current trajectory point in space is computed and joint variable recalculated via inverse kinematics.



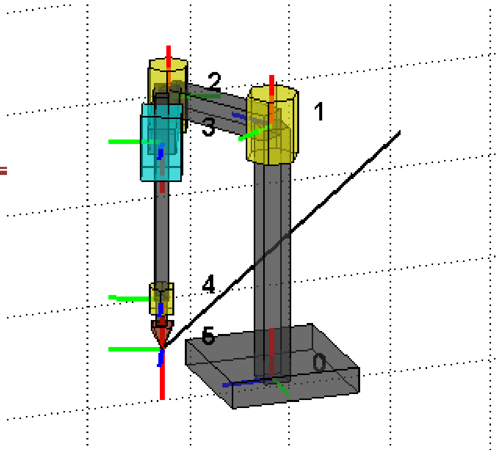
ARCHIMEDEAN SPIRAL (MODE=5)



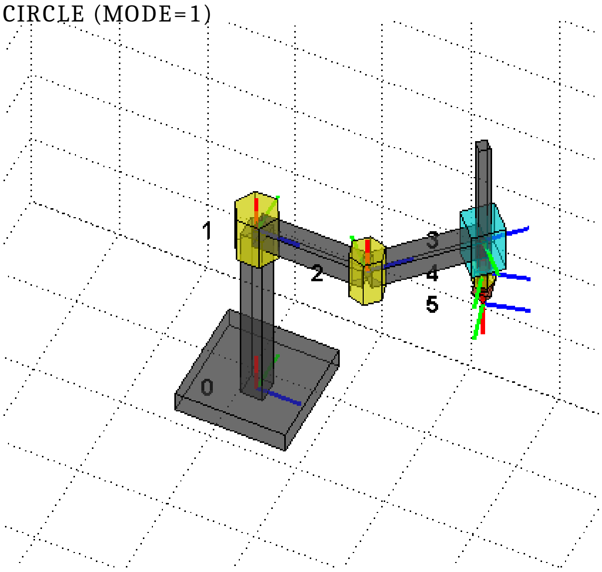
FISH (MODE=3)



CIRCLE (MODE=1)

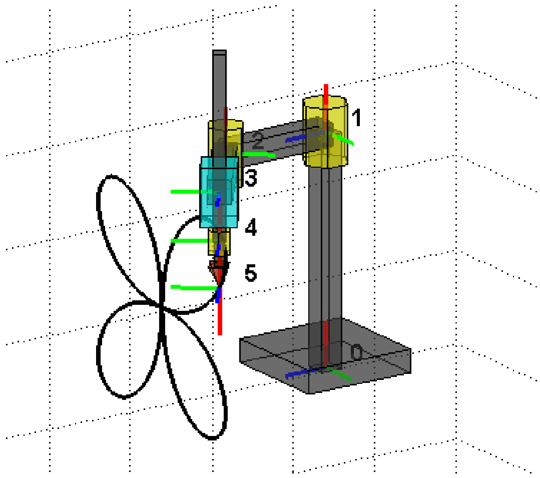


LINE (MODE=2)



ARBITRARY STATIC CONFIGURATION:

$\boldsymbol{\Theta}_2=0$, $\boldsymbol{\Theta}_3=45$ $\mathbf{E}_4=1$ $\boldsymbol{\Theta}_5=30$



QUADRIFOLIUM (MODE=4)

References

- Jezierski E.: *Dynamika robotów*, Wydawnictwa Naukowo-Techniczne, 2006
- http://www.mathworks.com/help/rptgenext/ug/documentation.html
- www.wolframalpha.com (parametric curves)