Optimal Transport for Super Resolution Applied to Astronomy Imaging

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Super Resolution

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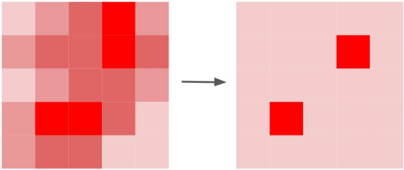

Optimal Transport

Definition

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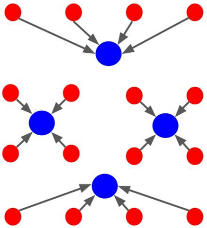
where $\Pi(\mu, \nu) = \{P \in \mathbb{R}^{n \times m} : \sum_{i=1}^{n} P_{ij} = \nu_j \ \forall j, \sum_{j=1}^{n} P_{ij} = \mu_i \ \forall i\}$.



Entropy

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For a probability mass function $p : \mathcal{J} \to \mathbb{R}$ (non-negative and sums to 1), we will use $H(p)$ to denote its entropy,

$$H(p) = -\sum_{\iota \in \mathcal{J}} p(\iota) \ln(p(\iota)). \tag{2}$$

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Key point: Sparse arrays have low entropy.

Entropic regularisation

Approximate optimal transport distances can be computed by solving the entropic regularization of optimal transport

$$\arg\min_{P\in\Pi(\mu,\nu)} \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}P_{ij} - \epsilon H(P).$$

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This is equivalent to the matrix scaling problem, more precisely finding positive multipliers $f \in \mathbb{R}^n, g \in \mathbb{R}^m$ such that the matrix

$$(f_i e^{-C_{ij}/\epsilon} g_j) \in \Pi(\mu, \nu).$$

Sinkhorn

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for f and then solving

$$\sum_{i=1}^{n} f_i e^{-C_{ij}/\epsilon} g_j = \nu_j$$

for g.

Wasserstein Distance Gradient

The multipliers f and g play the role of Lagrange multipliers and can be used to approximate the variation of $W(\mu, \nu)$ with respect to the first and second argument respectively.

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This can be used to implement gradient descent methods.

Wasserstein Inverse Problem for Super Resolution

For a measurement ν and positive regularization parameter λ, we define the *sparse approximation* of ν as a minimizer

$$\mu_* = \arg\min_{\mu \in \mathbb{P}(X)} d_W^{\epsilon}(\mu, \nu) + \lambda H(\mu). \tag{3}$$

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- We will let each entry in ν describe the intensity of a pixel in a 32×32 image and cost C_{ij} is chosen as the L^2-distance between the i^{th} pixel and j^{th} pixel.

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Algorithm 3: The Sinkhorn Algorithm for Optimal Transport Distances

$f = (1, \ldots, 1) \in \mathbb{R}^n$
$g = (1, \ldots, 1) \in \mathbb{R}^n$
while *f and g have not converged* **do**
  for $1 \leq i \leq n$ **do**
    $f_i = \mu_i / \left(\sum_j \exp(-C_{ij}/\epsilon) g_j \right)$
  end

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  end

end

$d_W^\epsilon = \sum_{i=1}^n \sum_{j=1}^n f_i g_j \exp(-C_{ij}/\epsilon) C_{ij}$

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Input: $X \in \mathbb{R}^{N \times m \times m}$: N images size $m \times m$, $\lambda \in \mathbb{R}$: positive noise level, $0 < \epsilon < 1$: optimal transportation regularization, $C \in \mathbb{R}^{m^2 \times m^2}$: cost matrix, $J_{\lambda,\epsilon}(x, v) := d_W^{\epsilon}(x, v) + \lambda H(v)$

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Algorithm 8: O.T. Super Resolution Clustering [Rawson and Hultgren, 2022]

$K = 0$
for $i = 1, 2, ..., N$ **do**
  $v = X_i$
  while v *has not converged* **do**
    $w = \nabla d^{\epsilon}_{W}(X_i, \cdot)|_v + \lambda \nabla H|_v$
    $w = w - \langle w, \frac{1}{m}\mathbb{1}\rangle \cdot \frac{1}{m}\mathbb{1}$

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    $\alpha = \sup\{\alpha \in \mathbb{R} : J_{\lambda,\epsilon}(v) > J_{\lambda,\epsilon}(v - \alpha w)\}$
    $\alpha = \min\{0.01, \alpha\}$

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        $v = v - \alpha w$; $v = diag(\mathbb{1}_{v>0})\, v$; $v = v/\|v\|_1$
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    $\alpha = \min\{0.01, \alpha\}$
    $v = v - \alpha w;\ v = diag(\mathbb{1}_{v>0})\ v;\ v = v/\|v\|_1$
  end
  $V_i = v;\ \delta = \max V_i$
  if $rank(H_0(V_i^{-1}([0.75\delta,\ \delta]))) == 1$ **then**
    $K_i = 1$
  end
end

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Optimal Transport Bound on Gaussian Noise

Theorem ([Rawson and Hultgren, 2022])

Let $\nu = \frac{1}{k}\sum_{i=1}^{k} \delta_{p_i}$ where $p_i \in \mathbb{R}^d$ and $\tilde{\nu}$ be the noisy signal acquired by sampling, for each i, n points according to a normal distribution centered at p_i with independent components of variance σ^2.

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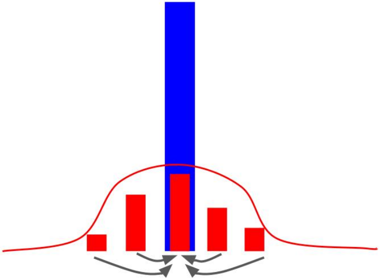

Reconstructing the Support of a Sparse Signal

Theorem ([Rawson and Hultgren, 2022])

Assume ν is a sparse signal and $\bar{\nu}$ is a noisy signal such that $d_w(\nu, \bar{\nu}) < \delta$.

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Assume ν is a sparse signal and $\bar{\nu}$ is a noisy signal such that $d_w(\nu, \bar{\nu}) < \delta$. Then the solution of

$$\mu = \arg \min_{\mu : d_W(\bar{\nu}, \mu) \leq \delta} H(\mu)$$

will identify the structure of μ, i.e. have the same support as μ, if $||\nu||_0 \leq ||\mu||_0$ for all μ such that $d_W(\mu, \bar{\nu}) < 2\delta$, with equality only if μ and ν has the same support.

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Reconstructing the Support of a Sparse Signal [Rawson and Hultgren, 2022]

Remark

The conditions in Theorem 4 can be summarized as a low enough noise level δ and enough sparsity of the true signal ν (making it a local minimizer of the L^0-norm).

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Remark

The conditions in Theorem 4 can be summarized as a low enough noise level δ and enough sparsity of the true signal ν (making it a local minimizer of the L^0-norm). It is interesting to note that these conditions are essentially necessary: if the inequality in Theorem 4 is violated by some μ closer than δ to $\bar{\nu}$, then the solution of (4) *does not identify the structure of ν.*

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Noise is high entropy, hence it is expected that the noise can be removed by minimizing the entropy.

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Remark

Noise is high entropy, hence it is expected that the noise can be removed by minimizing the entropy. However, if the signal-to-noise ratio is too low, this reconstruction is underdetermined.

Reconstructing the Support of a Sparse Signal

Theorem ([Rawson and Hultgren, 2022])

Fix a positive probability vector $\nu \in \mathbb{R}^d_{>0}$ *such that all elements of* ν *are distinct.*

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Fix a positive probability vector $\nu \in \mathbb{R}^d_{>0}$ such that all elements of ν are distinct. Then the sparse recovery is continuous to perturbations around ν for small λ, i.e. for every $\epsilon' > 0$ there exists $\delta > 0$, such that if $d_W(\nu, \nu') < \delta$, $\mu_* = \arg\min_{\mu \in \mathbb{P}(X) : d_W(\mu,\nu) < \lambda} H(\mu)$, and $\mu'_* = \arg\min_{\mu \in \mathbb{P}(X) : d_W(\mu,\nu') < \lambda} H(\mu)$

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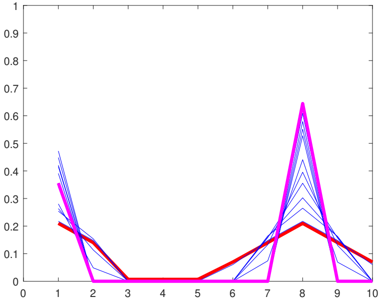

(a) Sparsity level $\lambda = 10$. Solution $\nu = (0.35, 0, 0, 0, 0, 0, 0, 0.65, 0, 0)$.

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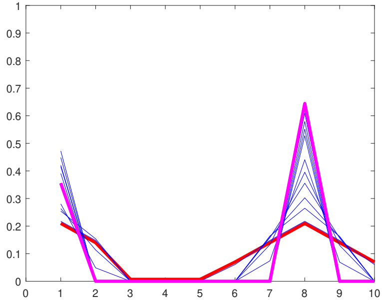

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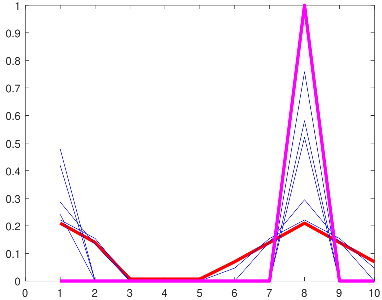

(b) Sparsity level $\lambda = 100$. Solution $\nu = (0, 0, 0, 0, 0, 0, 0, 1, 0, 0)$.

Figure: Plot of super resolution O.T. method. Red line is initial distribution. Blue lines are steps along gradient. Pink line is final, converged distribution.

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Star Clustering Application

- The formation and evolution of star clusters [Pérez et al., 2021]

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- Algorithmically detect star clusters in images of sky patches
- State of the art method trains a convolutional neural network (CNN) to classify each region in an image as containing a star cluster or not [Pérez et al., 2021]
- Neural networks are notoriously computationally expensive, sensitive to noise, and inflexible to appending or removing data variables

Star Clustering Application [Rawson and Hultgren, 2022]

Table: Confusion matrix of O.T. method on LEGUS data compared to StarcNet [Pérez et al., 2021]. Column gives StarcNet classification and row gives O.T. classification [Rawson and Hultgren, 2022].

	StarcNet Cluster	StarcNet Not Cluster
O.T. Cluster	25% (32)	13.3% (17)
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- The accuracy rate of clustering without sparsifying is 46% w.r.t. the CNN.
- The O.T. method increases in accuracy to 74% with respect to the CNN.

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Star Clustering Application [Rawson and Hultgren, 2022]

Table: Confusion matrix of O.T. method on LEGUS data compared to StarcNet [Pérez et al., 2021]. Column gives StarcNet classification and row gives O.T. classification [Rawson and Hultgren, 2022].

	StarcNet Cluster	StarcNet Not Cluster
O.T. Cluster	25% (32)	13.3% (17)
O.T. Not Cluster	12.5% (16)	49.2% (63)

- The accuracy rate of clustering without sparsifying is 46% w.r.t. the CNN.
- The O.T. method increases in accuracy to 74% with respect to the CNN.
- The CNN accuracy rate is 86% with respect to experts, but even experts agree with each other only around 70%-75% [Wei et al., 2020].

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- The CNN accuracy rate is 86% with respect to experts, but even experts agree with each other only around 70%-75% [Wei et al., 2020].
- Our method provides a very high performance given that no neural network training, which often takes weeks of compute time, is required.

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Conclusion

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- We proved that optimal transportation will reconstruct sparse sources and is robust to noise
- Relevant for correcting distortions and noise in imaging, ex: star cluster detection
- A predictive model for star clusters can produce a *policy* that informs where future surveys should look for star clusters [Rawson and Freeman, 2021, Freeman and Rawson, 2021].

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End

Thank You!

Questions?

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