PHY 251, Fall 2009: Equations for Final Exam
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	kg	MeV/c^2	amu
m_e	9.11×10^{-31}	0.510999	0.000549
m_p	1.673×10^{-27}	938.272	1.007276
m_n	1.675×10^{-27}	939.566	1.008665
amu	1.660×10^{-27}	931.494	1

1 eV=1.602×10^{-19} Joule, $h=6.62\times10^{-34}$ J·sec=4.14×10^{-15} eV·sec. $hc=1239.8$ eV·nm. $k_B=1.38\times10^{-23}$ J/K=8.62×10^{-5} eV/K. $c=3.00\times10^8$ m/sec. $\epsilon_0=8.85\times10^{-12}$ in mks units. $\mu_B=e\hbar/(2m)=9.274\times10^{-24}$ J/T. 1 Gray=1 J/kg=100 rad. Sievert=Gray·RBE=100 rem. 1 Curie=3.7×10^{10} decay/sec. $N_A=6.02\times10^{23}$ atoms/mol.
Centripetal force to maintain circular motion: $\gamma mv^2/r$. Lorentz force: $q\vec{v}\times\vec{B}$. Volume element: $r^2dr\,\sin\theta d\theta\,d\varphi$. Constant a =: $x=x_0+v_0t+\frac{1}{2}at^2$, $v=v_0+at$, $v^2-v_0^2=2a(x-x_0)$.
$F=ma=dp/dt$, $E_k=\frac{1}{2}mv^2$, $p=mv$, $\gamma=\dfrac{1}{\sqrt{1-v^2/c^2}}$. $p=\gamma m_0v$, $F_\perp=\gamma m_0a$,
$F_\parallel=\gamma^3m_0a$. $\nu=\dfrac{\nu_0}{\gamma[1+(v/c)\cos\theta]}$ with $\theta=0$ for emitter moving directly away.

$$\begin{aligned} x_2 &= \gamma\,(x_1-vt_1) \\ y_2 &= y_1 \\ z_2 &= z_1 \\ t_2 &= \gamma\left(t_1-\frac{v}{c^2}x_1\right) \end{aligned} \qquad \begin{aligned} v_{2,x} &= \frac{v_{1,x}-v}{1-\dfrac{v\,v_{1,x}}{c^2}} \\ v_{2,y} &= \frac{v_{1,y}}{\gamma\left[1-\dfrac{v\,v_{1,x}}{c^2}\right]} \\ v_{2,z} &= \frac{v_{1,z}}{\gamma\left[1-\dfrac{v\,v_{1,x}}{c^2}\right]} \end{aligned} \qquad \begin{aligned} p_{x,2} &= \gamma\left(p_{x,1}-v(E/c^2)\right) \\ p_{y,2} &= p_{y,1} \\ p_{z,2} &= p_{z,1} \\ E_2 &= \gamma(E-vp_x). \end{aligned}$$

$E=E_0+E_k=m_0c^2+(\gamma-1)m_0c^2$, $E^2=E_0^2+p^2c^2$. $p=E/c$. $E=h\nu=hc/\lambda$.

$$u_\nu\,d\nu=\frac{8\pi h\nu^3}{c^3}\frac{1}{\exp[h\nu/kT]-1}\,d\nu \qquad\qquad \lambda_{\text{peak}}=hc/(4.965\,k_BT)$$

$E_k=h\nu-\phi$. $\lambda=h/p$, $\lambda_s-\lambda_0=\dfrac{h}{m_ec}(1-\cos\theta)$. $2d\sin\theta=n\lambda$.
$r_n=\dfrac{n^2}{Z}a_0$ with $a_0=\dfrac{\epsilon_0h^2}{m\pi e^2}=0.053$ nm, and $E_n=-\dfrac{Z^2}{n^2}E_0$ with $E_0=\dfrac{me^4}{8\epsilon_0^2h^2}=13.60$ eV.
$m_r=\dfrac{m_1m_2}{m_1+m_2}$. $\dfrac{1}{4\pi\epsilon_0}\dfrac{Ze^2}{r^2}=m\dfrac{v^2}{r}$. $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}\psi+U\psi=i\hbar\dfrac{\partial\psi}{\partial t}=E\psi$ if U is time independent.
$\Delta E\cdot\Delta t\gtrsim\hbar/2$, $\Delta x\cdot\Delta p\gtrsim\hbar/2$. For U constant:

$E>U$: $\psi=A\sin kx+B\cos kx,\ k=\sqrt{2m(E-U)}/\hbar$, and

$E<U$: $\psi=C\exp[-\alpha x],\ \alpha=\sqrt{2m(U-E)}/\hbar$.

Infinite well: $E_n=n^2\pi^2\hbar^2/(2mL^2)$ with $\psi=\sqrt{2/L}\sin(n\pi x/L)$
Harmonic oscillator: $E_n=(n+\frac{1}{2})\hbar\omega$, $\psi_0=A\exp[-\omega mx^2/(2\hbar)]$.
Coulomb potential: $\psi(r,\theta,\varphi)=R_{n,l}(r)\Theta_{l,m_l}(\theta)\Phi_{m_l}(\varphi)=R_{n,\ell}(r)Y_\ell^{m_\ell}(\theta,\varphi)$.
$\langle f\rangle=\int f\,P(x)\,dx$. $\Delta x=\sqrt{\langle x^2\rangle-\langle x\rangle^2}$
Energy order of shells: $1s<2s<2p<3s<3p<4s\lesssim3d<4p<5s<4d<5p$

$< 6s < 4f \lesssim 5d < 6p < 7s < 6d \lesssim 5f \ldots$

$|L| = \sqrt{\ell(\ell+1)}\hbar$, $L_z = m_\ell \hbar$, $\vec{\mu}_L = -(e/2m)\vec{L}$. $|S| = \sqrt{s(s+1)}\hbar$, $S_z = m_s\hbar$, $\vec{\mu}_s = -(e/m)\vec{S}$.
$U = m_\ell \mu_B B = 2 m_s \mu_B B$
$n(E)\,dE = f(E)\,g(E)\,dE$. Gibbs factor: $\exp[(N\mu - E)/(k_B T)]$.
$f_{\mathrm{MB}}(E) = \dfrac{1}{\exp[E/k_B T]}$, $f_{\mathrm{FD}}(E) = \dfrac{1}{\exp[(E-E_F)/(k_B T)]+1}$, $f_{\mathrm{BE}}(E) = \dfrac{1}{\exp[E/(k_B T)]-1}$.
$E_{\mathrm{F}} = \dfrac{h^2}{8m}\left(\dfrac{3N}{\pi V}\right)^{2/3}$, $C_V = \dfrac{9Nk_B^2}{2E_{F0}}T$. $\dfrac{A_{21}}{B_{21}} = 8\pi h \nu^3/c^3$. $B_{12} = B_{21}$.
$R = r_0 A^{1/3}$ with $r_0 = 1.2 \times 10^{-15}$ m. BE $= -a_1 A + a_2 A^{2/3} + a_3 \dfrac{Z^2}{A^{1/3}} + a_4 \dfrac{(N-Z)^2}{A}$ with $a_1 = 15.5$ MeV, $a_2 = 16.8$ MeV, $a_3 = 0.72$ MeV, and $a_4 = 19$ MeV. $N = N_0 \exp[-\lambda t]$, activity $A = \lambda N$.

$$\int x^m\, e^{ax}\,dx = e^{ax}\sum_0^m (-1)^r \frac{m!\,x^{m-r}}{(m-r)!\,a^{r+1}} \qquad \int_0^\infty x^n\, e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$
$$\int_0^\infty x^{2n}\, e^{-ax^2}\,dx = \frac{1\cdot 3\cdot 5\ldots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}} \qquad \int_0^\infty x^{2n+1}\, e^{-ax^2}\,dx = \frac{n!}{2a^{n+1}}$$
$$\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{\sin(2ax)}{4a} \qquad \int \cos^2(ax)\,dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$(1+x)^n \simeq 1 + nx$ for $x \ll 1$.