1. A mechanical system has a maximum response at 400.0 Hz when damping is turned off, and the frequency shifts by 0.3 Hz when damping is turned on. The mass of the oscillating object is 0.3 kg. Calculate the following: the spring constant of the system, its quality factor, the full-width at half maximum (FWHM) of its damped resonance curve, and the time it takes for oscillations to decrease to 1/10 of an original value.
   Answer: The spring constant is $k = m\omega_0^2 = (0.3)[2\pi \cdot (400)]^2 = 1.89 \times 10^6$ N/m. To find the quality factor $Q = \omega_0/\gamma$, we first find γ:

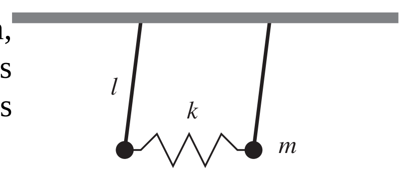
$$\begin{aligned} \omega^2 &= \omega_0^2 - \frac{\gamma^2}{4} \\ \gamma &= 2\sqrt{\omega_0^2 - \omega^2} \\ Q &= \frac{\omega_0}{\gamma} = \frac{\omega_0}{2\sqrt{\omega_0^2 - \omega^2}} = \frac{1}{2\sqrt{1 - \omega^2/\omega_0^2}} = \frac{1}{2\sqrt{1 - 399.7^2/400^2}} = 12.9 \end{aligned}$$

   The full-width at half-max or FWHM of the damped resonance curve is found from $2\Delta\omega = \omega_0/Q$ giving $2\Delta\omega = 2\pi(400)/(12.9) = 195$ radians/sec or 31 Hz. For damping, we have

$$\begin{aligned} \frac{E_0}{10} &= E_0 e^{-\gamma t_{10}} \\ \ln 10 &= \gamma t_{10} \\ t_{10} &= \frac{\ln 10}{\gamma} = \frac{\ln 10}{2\sqrt{(2\pi)^2(400^2 - 399.7^2)}} = 0.0118 \text{ seconds} \end{aligned}$$

   or 11.8 msec.

2. Describe the coupled oscillator system shown at right ($l = 0.2$ m, $m = 0.3$ kg, $k = 3$ N/m). What are its two resonant frequencies in Hertz, and (in just a few words) describe how the two masses move at each of the two frequencies?



   Answer: The resonance frequencies of the individual pendula are $\omega_0 = \sqrt{g/l} = \sqrt{9.8/0.2} = 7.00$ rad/sec or 1.11 Hz, and $\omega_c = \sqrt{k/m} = \sqrt{3/0.3} = 3.16$ rad/sec or 0.503 Hz. The two resonant frequencies are ω_0 where the two masses move left and right in synchrony (common mode motion), and differential mode motion where the two masses move towards and then away from each other at a frequency of

$$\omega' = \sqrt{\omega_0^2 + 2\omega_c^2} = \sqrt{7.00^2 + 2 \cdot 3.16^2} = 8.30 \text{ rad/sec}$$

   or 1.32 Hz.

3. As part of its ongoing attempt to demonstrate some utility of the International Space Station, NASA decides to send into orbit a piranha. In zero gravity, the fish is enclosed within a free-floating sphere of water ($n = 1.33$) that's 30 cm in diameter. An astronut peers in to look at the fish when it is at the opposite surface of the sphere. Where does the fish appear to be, and what is its magnification?

Answer: This is just like problem 5 on HW 5. We have $n_1 = 1$, $n_2 = 1.33$, $R = 15$ cm, and $s' = 2R = 30$ cm. Therefore

$$\begin{aligned}
\frac{n_1}{s} + \frac{n_2}{s'} &= \frac{n_2 - n_1}{R} \\
\frac{1}{s} + \frac{n_2}{2R} &= \frac{n_2 - 1}{R} \\
\frac{1}{s} &= \frac{1}{R}(n_2 - 1 - \frac{n_2}{2}) = \frac{1}{R}(1.33 - 1 - 1.33/2) = \frac{-.335}{R}
\end{aligned}$$

so $s = R/(-.335) = -15/.335 = -44.8$ cm. That is, the object appears to be 44.8 cm on the other side of the glass. Its magnification is

$$m = -\frac{n_1 s'}{n_2 s} = -\frac{1 \cdot (30 \text{ cm})}{1.33 \cdot (-44.8 \text{ cm})} = 0.503.$$

That is, the astronaut had better be careful, because the fish is closer (by 14.8 cm) and bigger (nearly double in size) than he thinks!

4. A double-slit diffraction pattern is formed using HeNe light at 632.8 nm. Each slit has a width of 50 μm. The pattern reveals that the third-order interference maxima are missing from the pattern, while the first and second order maxima are present. What is the largest possible value for the slit separation?
   Answer:
   For double-slit diffraction we have $I \propto (\sin^2 \beta/\beta^2)\cos^2\alpha$ with $\beta = \pi b f$ and $\alpha = \pi a f$ where $f = \theta/\lambda$ is a spatial frequency. Interference maxima occur when $\alpha = m\pi$ for the m^{th} order, leading to $f = m/a$ where in this case we know $m = 3$. Diffraction minima occur when $\beta = n\pi$, leading to $f = n/b$ where we know that $b = 50$ μm. These two conditions must coincide at the same spatial frequency, so $f = n/b = 3/a$ or $a = (3/n)b$. The largest value of a is when $n = 1$, so we have $a = (3/1)b = 150$ μm.

5. A halogen lamp with a filament area of $(2 \text{ mm}) \times (5 \text{ mm})$ puts out 25 Watts of light into 2π solid angle, evenly distributed over a wavelength range from 400 to 600 nm. What is the power that's emitted into a single spatially coherent mode, with 1000 waves of temporal coherence, at $\lambda = 500$ nm?
   Answer:
   Again, the coherent phase space is (full width)·(full angle)=λ in each direction. Therefore the 2D spatially coherent fraction of the output from this source is

   $$\frac{\lambda^2}{(\text{area}) \cdot (\text{solid angle})} = \frac{(500 \times 10^{-9} \text{ m})^2}{(2 \times 10^{-3})(5 \times 10^{-3})(2\pi)} = 4.0 \times 10^{-9}.$$

   The power is also distributed over a range from 400 to 600 nm or a bandwidth of 200 nm about 500 nm, whereas we want a bandwidth of (500 nm)/1000=0.5 nm to get our required temporal coherence. Therefore we have a temporally coherent fraction of

   $$\frac{0.5 \text{ nm}}{200 \text{ nm}} = 2.5 \times 10^{-3}$$

   so the power emitted into a single mode is

   $$(25 \text{ Watts}) \cdot (4.0 \times 10^{-9}) \cdot (2.5 \times 10^{-3}) = 2.5 \times 10^{-10} \text{ Watts}$$

6. What acoustic frequency is required of a plane acoustic wave, launched in an acousto-optic crystal, so that a HeNe laser beam is deflected by $1°$? The speed of sound in the crystal is 2500 m/s and its refractive index is $n = 1.6$.
   Answer:
   To get $\Phi_D = 1° = 2\theta$ on the exit, we need θ' inside the medium which is found by Snell's law in the small angle approximation, or $\theta' = \theta/n = \Phi_D/(2n)$. The Bragg grating period needed to get a deflection of θ' is found from $2d\sin(\theta') = \lambda$ or

$$d = \frac{\lambda}{2\sin(\theta')} = \frac{\lambda}{2\sin(\theta/n)} = \frac{\lambda}{2\sin\Big(\Phi_D/(2n)\Big)} = \frac{632.8 \times 10^{-9}\ \text{m}}{2\sin\Big(1°/(2\cdot 1.6)\cdot \pi/180°\Big)} = 5.8{\times}10^{-5}\ \text{m}$$

   or 58 μm. This gives an acoustical wavelength in the crystal. Now $v_{\text{sound}} = \lambda f$ so

$$f = \frac{v_{\text{sound}}}{\lambda} = \frac{2600\ \text{m/s}}{58 \times 10^{-6}\ \text{m}} = 4.5 \times 10^{7}\ \text{Hz}$$

   or 45 MHz.

7. How many lines must be ruled on a transmission grating so that it is just capable of resolving the sodium doublet (589.592 nm and 588.995 nm) in first diffraction order?
   Answer:
   We want to resolve a wavelength difference of

$$\frac{589.592 - 588.995}{0.5\cdot(589.592 + 588.995)} = 0.001013 = \frac{1}{987.1}$$

   To tell the difference between two wavelengths that differ by one part in 987, we have to count at least 987 waves of overlap which means we need 987λ of optical path length difference or 987 grating grooves. If we were to work in the m^{th} diffraction order, we'd have $m\lambda$ optical path length difference between grooves and we'd need only $987/m$ grooves.

8. Why is it that the refractive index for visible light increases with decreasing wavelength, and glass becomes opaque at shorter wavelengths? Limit your answer to two blue-book pages; use diagrams and equations if they are helpful to your discussion.
   Answer:
   Plasmon frequency centered on UV range. Damped-driven harmonic oscillator, and nature of the phase shift around an absorption resonance. Discussed in class many times...

9. Tell me about optical fiber communications, including the nature of the fibers, signal transmission schemes and bit rates, wavelengths, multiplexing, and signal switching. Limit your answer to two blue-book pages; use diagrams and equations if they are helpful to your discussion.
   Answer:
   Total internal reflection used to confine light into fiber core. Small diameter fiber with higher index core, lower index cladding. Single mode (few microns core) versus multimode fibers. Bit transitions at clock rate indicate 0; no transition indicates 1. Bit rates of 10 Gbps (gigabit per second) common at a single wavelength. Wavelength 1.54 μm but also multiplexing of multiple closely-spaced wavelengths with gratings used to separate them. Also time-multiplexing of low bit rate signals. Signal switching using microfabricated mirrors activated by electric fields (TI DLP, Lucent switches).