ESE 271 Final Exam Name:

Fall, 2008 ID Number:

Do not place your answers on this front page.

Each problem is worth 25 points.

Prob. 1:

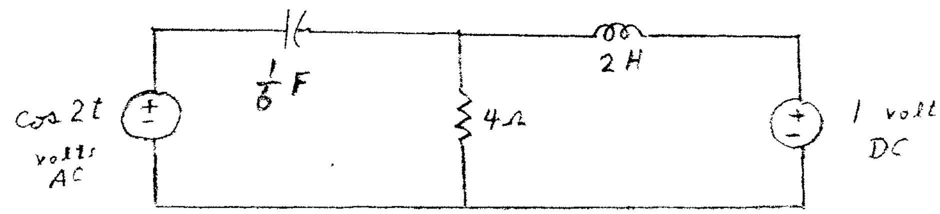
Prob. 2:

Prob. 3:

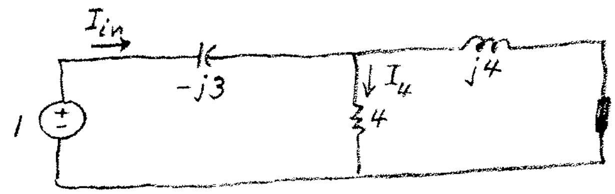
Prob. 4:

Prob. 1:

This circuit is an AC-DC steady-state condition. Find the average power dissipated in the 4 Ω resistor.



For $\omega = 2$:



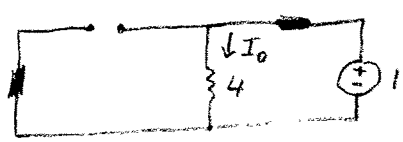
$$I_{in} = \frac{1}{-j3 + \frac{4(j4)}{4+j4}} = \frac{1}{2-j}$$

$$I_4 = I_{in} \frac{j4}{4+j4} = \frac{j}{3+j}$$

$$|I_4| = \frac{1}{\sqrt{3^2+1}} = \frac{1}{\sqrt{10}}$$

$$P_{av_2} = \frac{|I_4|^2 R}{2} = \frac{\frac{1}{10} \times 4}{2} = .2 \text{ WATTS}$$

For DC:



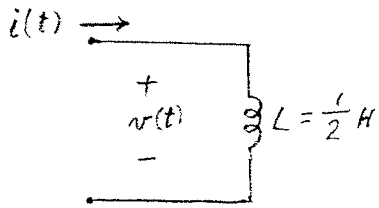
$$I_0 = \frac{1}{4}$$

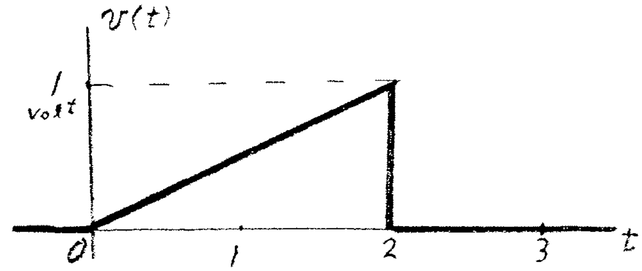
$$P_{av_0} = \left(\frac{1}{4}\right)^2 4 = .25 \text{ WATTS}$$

TOTAL $P_{av} = .2 + .25 = .45$ WATTS.

Prob. 2:

Find the Laplace transform $I(s)$ of the current $i(t)$ in the inductor $L = 1/2$ H, when $i(0+) = 2$ A, and $v(t)$ is the single triangular pulse shown.

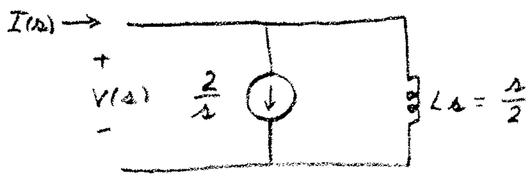




$$v(t) = \frac{t}{2}u(t) - \frac{(t-2)}{2}u(t-2) - u(t-2)$$

$$V(s) = \frac{1}{2s^2} - \frac{1}{2s^2}e^{-2s} - \frac{1}{s}e^{-2s}$$

NORTON EQUIVALENT CIRCUIT:



$$I(s) = \frac{2}{s} + \frac{V(s)}{s/2} = \frac{2}{s} + \frac{2}{s}\left(\frac{1}{2s^2} - \frac{1}{2s^2}e^{-2s} - \frac{1}{s}e^{-2s}\right)$$

$$= \frac{2}{s} + \frac{1}{s^3} - \frac{1}{s^3}e^{-2s} - \frac{2}{s^2}e^{-2s}$$

ANOTHER WAY: USE THE INTEGRAL EXPRESSION

$$i(t) = \frac{1}{L}\int_0^t v(x)\,dx + i(0+)$$

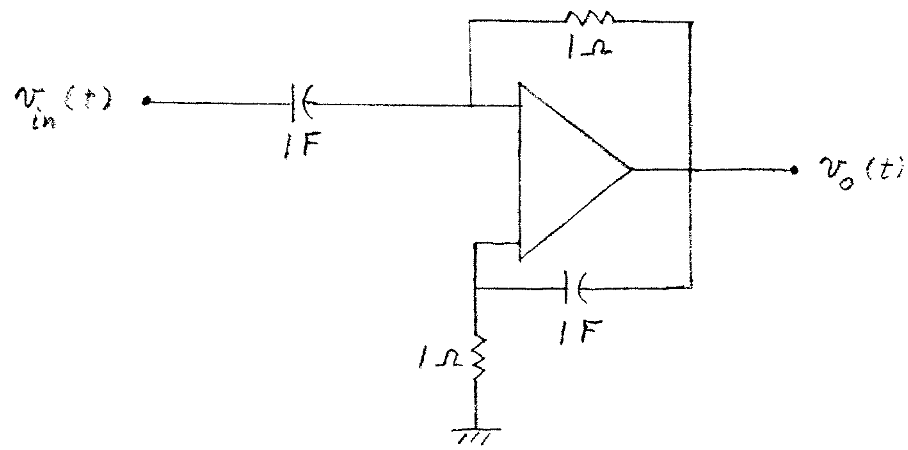
NOW, APPLY $\mathcal{L}$:

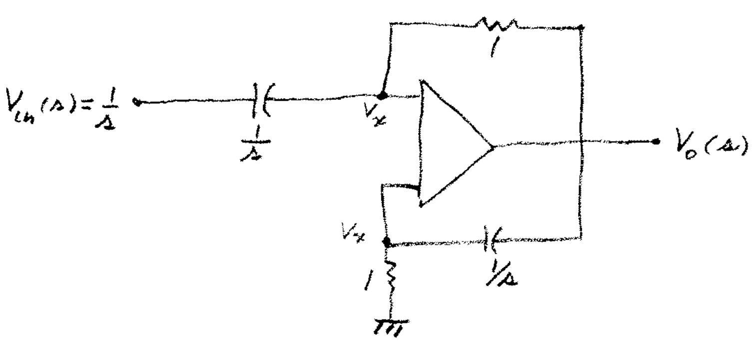
$$I(s) = \frac{1}{Ls}V(s) + \frac{i(0+)}{s} = \frac{2}{s}V(s) + \frac{2}{s}$$

SAME RESULT OCCURS.

Prob. 3:

Find the output voltage $v_o(t)$ for $t > 0$ when $v_{in}(t) = 1$ for $t > 0$. The initial charges on the capacitors are both 0 at $t = 0+$. Use the virtual-short virtual-open model for the op-amp.





KCL at upper V_x node:

$$\frac{V_x - \frac{1}{s}}{\frac{1}{s}} + \frac{V_x - V_o}{1} = 0$$

$$V_x = \frac{V_o + 1}{s + 1}$$

KCL at Lower V_x node:

$$V_x = \frac{1}{\frac{1}{s} + 1} V_o = \frac{s}{s+1} V_o$$

$$V_o = \frac{s+1}{s}\left(\frac{V_o + 1}{s+1}\right)$$

$$V_o = \frac{1}{s-1}$$

So,

$$v_o(t) = e^t \quad \text{for } t > 0$$

Prob. 4:

For $t > 0$ determine the time function $f(t)$ given by the convolution $f(t) = g(t) * h(t)$, where $g(t) = u(t) - u(t-1)$ and $h(t) = te^{-t}u(t)$.

$$G(s) = \frac{1-e^{-s}}{s}$$

$$H(s) = \frac{1}{(s+1)^2}$$

$$F(s) = G(s)\,H(s) = \frac{1-e^{-s}}{s(s+1)^2}$$

$$F(s) = \left(\frac{A}{s} + \frac{B_1}{(s+1)^2} + \frac{B_2}{s+1}\right)(1-e^{-s})$$

$$A = \left.\frac{1}{(s+1)^2}\right|_{s=0} = 1$$

$$B_1 = \left.\frac{1}{s}\right|_{s=-1} = -1$$

$$B_2 = \left.\frac{d}{ds}\frac{1}{s}\right|_{s=-1} = \left.-\frac{1}{s^2}\right|_{s=-1} = -1$$

$$F(s) = \left(\frac{1}{s} - \frac{1}{(s+1)^2} - \frac{1}{s+1}\right)(1-e^{-s})$$

$$f(t) = u(t) - te^{-t}u(t) - e^{-t}u(t) - u(t-1) + (t-1)e^{-(t-1)}u(t-1)$$
$$- e^{-(t-1)}u(t-1)$$