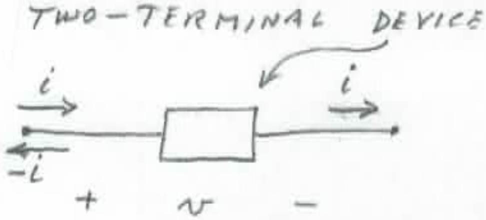
PART 1 - DC CIRCUITS

CHARGE q AND CURRENT $i = \frac{dq}{dt}$

IN A TWO-TERMINAL DEVICE

(ALSO CALLED A "ONE-PORT")

FIG. 1



(- (-v) +)

($q = q(t)$ AND $i = i(t)$ CAN VARY WITH TIME t.)

CHARGE q IS THE AMOUNT OF ELECTRIC QUANTITY MEASURED IN UNITS OF <u>COULOMBS</u>

(CHARGE OF 1 ELECTRON $= 1.602 \times 10^{-19}$ COULOMBS)

CURRENT i IS THE FLOW OF ELECTRIC CHARGE ALONG A WIRE - OR THROUGH THE DEVICE.

$$i = \frac{dq}{dt} \quad \text{AMPERES} \quad (= \text{COULOMBS/SECOND})$$

TOTAL CHARGE FLOWING THROUGH THE DEVICE IN THE TIME INTERVAL t_0 TO t_1 IS

$$q(t) - q(t_0) = \int_{t_0}^{t_1} i(x)\,dx \qquad (t_1 > t_0)$$

VOLTAGE v IS THE FORCE PRODUCING THE CURRENT i.
$v = v(t)$ IS MEASURED IN VOLTS.

SO THE POWER p PRODUCED AT ANY INSTANT OF TIME t IS

$$p = vi \qquad \text{OR} \qquad p(t) = v(t)i(t)$$

p IS MEASURED IN WATTS W AND IS POWER DISSIPATED WHEN v AND i HAVE THE POLARITIES OF FIG.

SO, THE TOTAL ENERGY $\mathcal{E}$ PRODUCED BY p DURING t_0 TO t_1 ($t_1 > t_0$)

IS

$$\mathcal{E}(t_0, t_1) = \int_{t_0}^{t_1} v(x)\, i(x)\, dx = \int_{t_0}^{t_1} p(x)\, dx$$

$\mathcal{E}$ IS MEASURED IN UNITS OF JOULES

THUS, $\dfrac{1 \text{ JOULE}}{1 \text{ SECOND}} = 1 \text{ WATT}.$

IF $v(-\infty) = i(-\infty) = 0$, THEN

$$\mathcal{E}(-\infty, t) = \int_{-\infty}^{t} v(x)\, i(x)\, dx = \int_{-\infty}^{t} p(x)\, dx$$

A PASSIVE ONE-PORT IS A ONE-PORT SUCH THAT

$$\mathcal{E}(-\infty, t) = \int_{-\infty}^{t} p(x)\, dx \geq 0 \qquad \text{FOR ALL } t.$$

AN ACTIVE ONE-PORT IS A ONE-PORT FOR WHICH

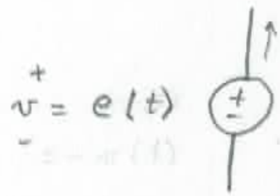
$$\mathcal{E}(-\infty, t) < 0 \quad \text{AT SOME } t \text{ AND FOR SOME } v(t).$$

(AN ACTIVE ONE-PORT HAS A SOURCE OF ELECTRICAL ENERGY WITHIN IT --- SUCH AS A BATTERY.)

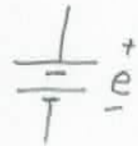
Two important active one-port

1. The <u>independent</u> voltage source:

Here, $e(t) = -v(t)$ is specified and is unaffected by $i(t)$.

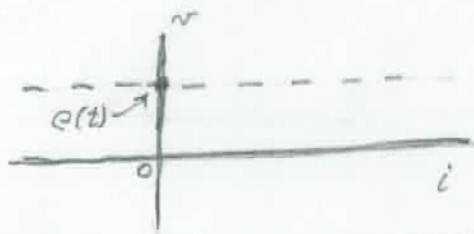


Special case: the ideal battery



"Symbols"

So, in a plot of v versus i we have:



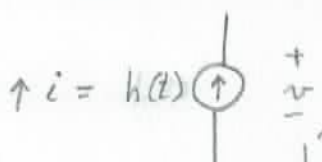
$e(t)$ can vary up and down with respect to t.

> **Note.** v is now taken with a polarity opposite to that of Fig. 1

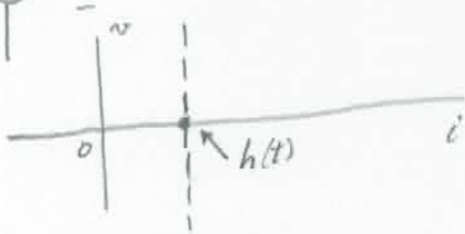
Now, $v = e(t)$ is measured as a voltage <u>rise</u> with respect to direction of current i.

So, if $i(t) > 0$ and $e(t) > 0$ at some t, we get power <u>generated</u> (rather than dissipated).

2. The <u>independent</u> current source:



So now:



$h(t)$ can vary left and right with respect to t.

> **Note.** i is now taken with a polarity opposite to that of Fig. 1

So, if $v(t) > 0$ and $h(t) > 0$ at some t, we again get power <u>generated</u> (rather than dissipated).

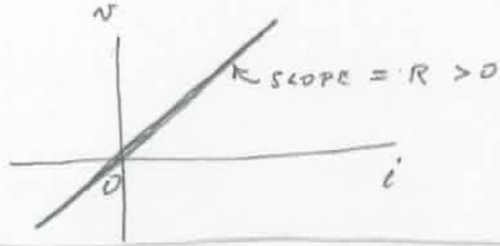
(Note. Current sources are unusual, in contrast to voltage sources.)

RESISTORS R (SYNONYMOUSLY, RESISTANCES)

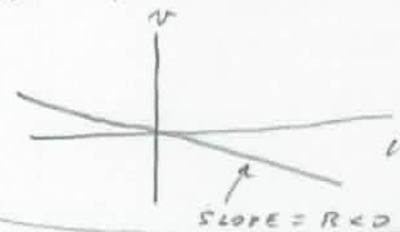
OHM'S LAW: $v(t) = R\, i(t)$, WHERE R IS A CONSTANT

USUALLY POSITIVE.

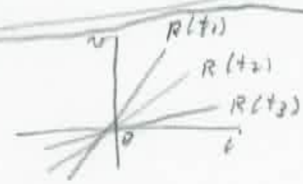
FIG. 2



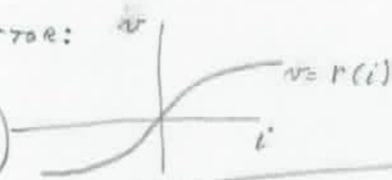
WE COULD ALSO HAVE A NEGATIVE RESISTOR $R < 0$



WE COULD ALSO HAVE A TIME-VARYING RESISTOR:



OR A NONLINEAR RESISTOR:



(WE WILL IGNORE THESE POSSIBILITIES IN THIS COURSE)

UNITS

$$v \text{ (IN VOLTS V)} = R \text{ (IN OHMS } \Omega) \; i \text{ (IN AMPERES A)}$$

$$(V = \Omega A)$$

FOR A FIXED (i.e., CONSTANT) RESISTOR R, POWER DISSIPATED (SEE POLARITIES OF FIG. 2)

IS:

$$p(t) = v(t)\, i(t) = R(i(t))^2 = \frac{(v(t))^2}{R}$$

FOR $R > 0$, R IS PASSIVE: $\mathcal{E}(t) = \int_{-\infty}^{t} p(x)\,dx = \int_{-\infty}^{t} R(i(x))^2 dx \geq 0$

CONDUCTANCE G IS THE RECIPROCAL OF RESISTANCE R:

$$G = \frac{1}{R} \quad \text{(UNITS SIEMENS} = \frac{1}{\text{OHMS}}.$$

SIEMENS = MHOS (OLDER NAME)

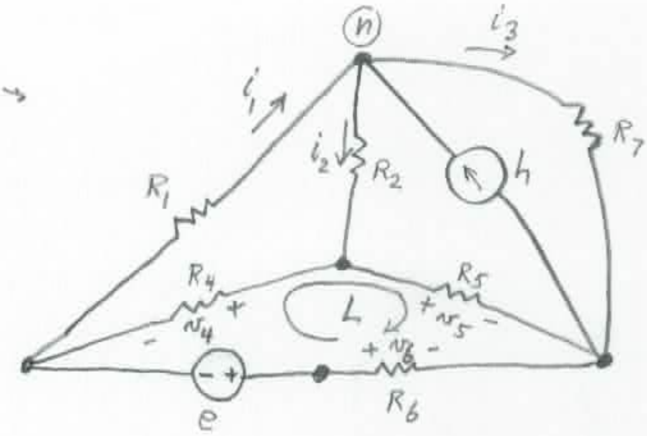
THUS,

$$i(t) = G\, v(t)$$

$$p(t) = \frac{i(t)^2}{G} = G\, v(t)^2$$

KIRCHOFF'S LAWS

A "NETWORK" →



A NODE IS WHERE LINES MEET. (EXAMPLE: n)

A BRANCH IS AN ELEMENT BETWEEN ADJACENT NODES.

A LOOP IS A TRACING THROUGH THE NETWORK THAT ENDS AT THE NODE WHERE IT STARTED. (EXAMPLE: L)

A MESH IS A SPECIAL CASE OF A LOOP (TO BE EXPLAINED)

KIRCHHOFF'S CURRENT LAW (KCL):

THE "ALGEBRAIC SUM" OF ALL CURRENTS FLOWING IN TOWARD A NODE EQUALS 0.

EXAMPLE: AT NODE n: $i_1 - i_2 + h - i_3 = 0$

(A GENERALIZATION: THE ALGEBRAIC SUM OF ALL CURRENT ENTERING A CLOSED SURFACE EQUALS 0.)

KIRCHOFF'S VOLTAGE LAW (KVL):

THE "ALGEBRAIC SUM" OF ALL VOLTAGE DROPS AROUND ANY LOOP EQUALS 0.

EXAMPLE: AROUND LOOP L (TRACING CLOCKWISE):

$$-v_4 + v_5 - v_6 + e = 0$$

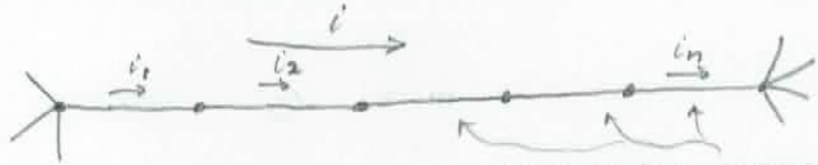
A BASIC FACT:

OHM'S LAW + KVL + KCL DETERMINE ALL THE CURRENTS AND VOLTAGES IN THE NETWORK
($v=Ri$)

(BUT THERE ARE UNUSUAL EXCEPTIONS).

SERIES RESISTANCE AND VOLTAGE DIVISION

A SERIES CIRCUIT OCCURS WHEN EVERY NODE – EXCEPT FOR POSSIBLY THE TWO END NODES – HAVE EXACTLY 2 BRANCHES CONNECTED TO IT.

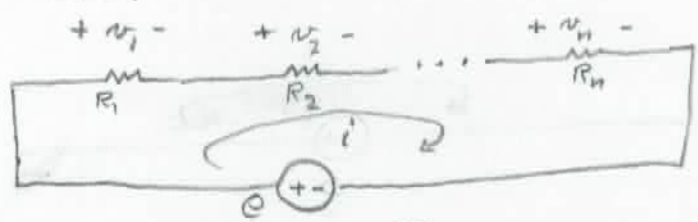


THERE ARE ELEMENTS IN EACH BRANCH.

BECAUSE OF KCL: THE CURRENTS IN EACH BRANCH ARE ALL THE SAME

$$i_1 = i_2 = \cdots = i_n = i$$

A PARTICULAR CASE:



KVL: $e = v_1 + v_2 + \cdots + v_n$

OHM'S LAW: $v_1 = R_1 i, \ \ldots, \ v_n = R_n i$

THUS, $e = R_1 i + \cdots + R_n i = (R_1 + \cdots + R_n)\, i$

SET $R_S = R_1 + \cdots + R_n$

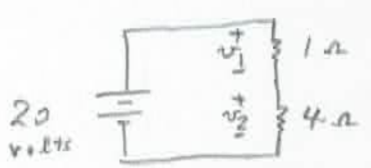
R_S = EQUIVALENT RESISTANCE OF THE SERIES CIRCUIT OF RESISTORS

So, $i = \dfrac{e}{R_S}$ AND $v_1 = \dfrac{R_1}{R_S} e, \ \ldots, \ v_n = \dfrac{R_n}{R_S} e.$

THESE ARE THE "VOLTAGE DIVISION" EQUATIONS.

THAT IS, THE TOTAL VOLTAGE e SPLITS UP ALONG THE RESISTORS R_k $(k = 1, \ldots, n)$ IN PROPORTION TO THOSE RESISTORS R_k.

EXAMPLE:



$v_1 = \frac{1}{5} \times 20 = 4$ volts

$v_2 = \frac{4}{5} \times 20 = 16$ volts

SERIES RESISTORS CONTINUED

LET'S ADD UP ALL THE POWERS DISSIPATED IN THE RESISTORS:

$$\frac{v_1^2}{R_1} + \cdots + \frac{v_n^2}{R_n} = \frac{1}{R_1}\left(\frac{R_1}{R_S}e\right)^2 + \cdots + \frac{1}{R_n}\left(\frac{R_n}{R_S}e\right)^2$$

$$= \frac{R_1}{R_S^2}e^2 + \cdots + \frac{R_n}{R_S^2}e^2 = \frac{R_1 + \cdots + R_n}{R_S^2}e^2$$

$$= \frac{R_S}{R_S^2}e^2 = \frac{e^2}{R_S} = ei$$ = POWER GENERATED BY THE SOURCE e.

THUS, WE SEE THAT THE TOTAL POWER DISSIPATED IN ALL THE RESISTORS EQUALS THE POWER GENERATED IN THE SOURCE e.

This is a general principle – called Tellegen's Theorem:

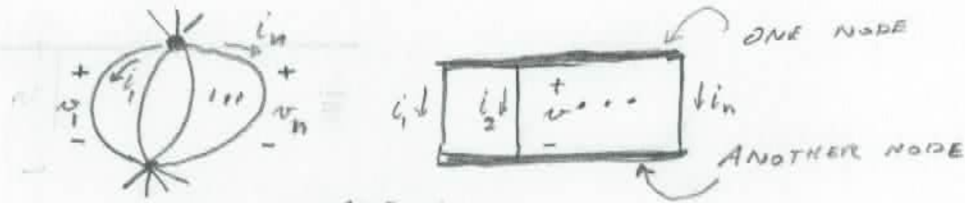
In any network of positive resistors and independent sources, the total power dissipated in the the resistors equals the total power generated in all the sources.

(but some of the sources may be dissipating instead of generating.)

PARALLEL RESISTORS AND CURRENT DIVISION

(This is the "dual" of series resistance!)
$R \to G, \; v \to i, \; i \to v$

A PARALLEL CIRCUIT OCCURS WHEN TWO OR MORE BRANCHES ARE CONNECTED TO THE SAME TWO NODES.



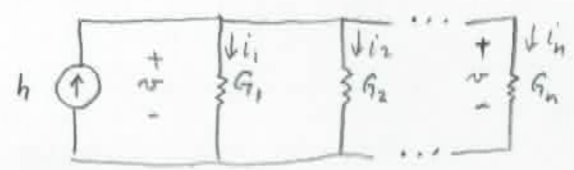
← SAME →
CIRCUIT

BECAUSE OF KVL: THE VOLTAGES ON EACH BRANCH ARE ALL THE SAME.

$$v_1 = v_2 = \cdots = v_n = v$$

A PARTICULAR CASE:

FIG. A



$$G_k = \frac{1}{R_k}, \quad k = 1, \dots, n.$$

KCL: $h = i_1 + i_2 + \cdots + i_n$

OHM'S LAW: $i_1 = G_1 v, \; \dots, \; i_n = G_n v$

THUS, $h = (G_1 + \cdots + G_n) v = G_p v$

SET $G_p = G_1 + \cdots + G_n$ (2) →

G_p = EQUIVALENT CONDUCTANCE OF THE PARALLEL CIRCUIT OF RESISTORS.

So, $v = \dfrac{h}{G_p}$ AND $i_1 = \dfrac{G_1}{G_p} h, \; \dots, \; i_n = \dfrac{G_n}{G_p} h$

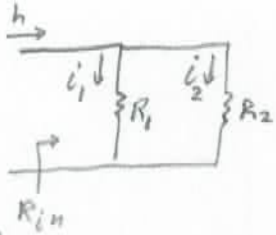
THESE ARE THE "CURRENT DIVISION" EQUATIONS.

THAT IS, THE TOTAL CURRENT h SPLITS UP AMONG THE CONDUCTANCES G_k IN PROPORTION WITH THOSE CONDUCTANCES.

USUALLY, WE WORK WITH RESISTANCES, (NOT CONDUCTANCES).
So, EQUATION (2) IS ALSO GIVEN BY

$$R_p = \frac{1}{G_p} = \frac{1}{G_1 + \cdots + G_n} = \frac{1}{\frac{1}{R_1} + \cdots + \frac{1}{R_n}}$$

EXAMPLE: TWO RESISTORS IN PARALLEL.



$$i_1 = \frac{G_1}{G_1 + G_2} h = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} h = \frac{R_2}{R_1 + R_2} h$$

$$R_{in} = \frac{1}{G_1 + G_2} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

SIMILARLY, $i_2 = \frac{R_1}{R_1 + R_2} h$

THESE EQUATIONS WORK ONLY FOR 2 RESISTORS ONLY.

FOR 3 OR MORE RESISTORS, WE HAVE TO USE

$$i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1} + \cdots + \frac{1}{R_n}} h.$$

WITH REGARD TO POWER RELATIONS, WE AGAIN HAVE THAT THE TOTAL POWER DISSIPATED IN ALL THE RESISTORS EQUALS THE POWER GENERATED IN THE CURRENT SOURCE h:

SEE FIG. A:

$$i_1^2 R_1 + \cdots + i_n^2 R_n = \left(\frac{G_1}{G_p} h\right)^2 R_1 + \cdots + \left(\frac{G_n}{G_p} h\right)^2 R_n$$

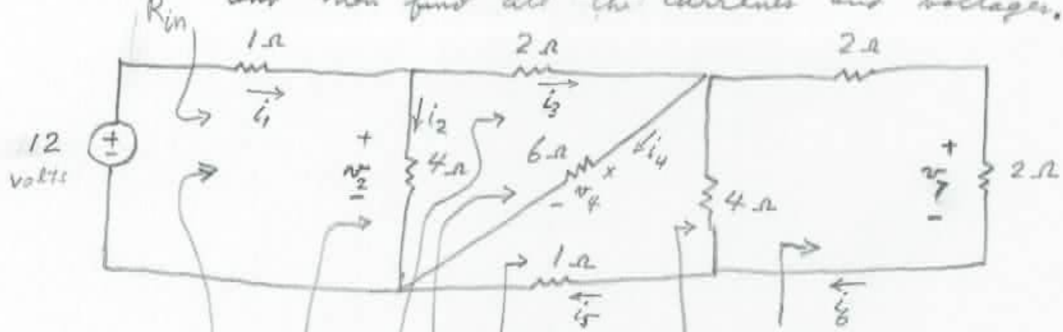
$$= \frac{G_1}{G_p^2} h^2 + \cdots + \frac{G_n}{G_p^2} h^2$$

$G_1 R_1 = 1$ $\qquad$ $G_n R_n = 1$

$$= \frac{G_1 + \cdots + G_n}{G_p^2} h^2 = \frac{h^2}{G_p} = v h = \text{POWER GENERATED IN THE CURRENT SOURCE } h.$$

$$\frac{h}{G_p} = R_p h = v$$

Example Find the input resistance R_{in} of the following series-parallel circuit, and then find all the currents and voltages.



$2+2 = 4\ \Omega$

$\frac{4\times4}{4+4} = 2$ (SEE "BOX" ON p. 9)

$1+2=3$

(ALL NUMBERS ARE RESISTANCES IN OHMS Ω)

$\frac{6\times3}{6+3} = 2$

$2+2=4$

$\frac{4\times4}{4+4} = 2$

$1+2=3 = R_{in}$

TO GET SOME OF THE CURRENTS AND VOLTAGES: (ALL THE OTHERS CAN BE OBTAINED SIMILARLY.)

$i_1 = \frac{12}{R_{in}} = \frac{12}{3} = 4\ A$

KVL: $v_2 + 1\times i_1 = 12, \quad v_2 = 12-4 = 8$

OHM'S LAW: $i_2 = \frac{v_2}{4} = 2\ A$

KCL: $i_3 = i_1 - i_2 = 4-2 = 2A$

KVL: $v_2 = 2i_3 + v_4, \quad 8 = 2\times2 + v_4, \quad v_4 = 4$

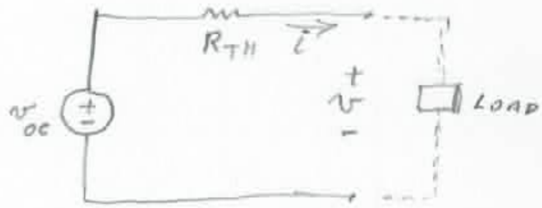
KCL: $i_5 + i_4 + i_2 = i_1, \quad i_5 + \frac{v_4}{6} + i_2 = i_1, \quad i_5 + \frac{4}{6} + 2 = 4, \quad i_5 = 4-2-\frac{2}{3} = \frac{4}{3}$

CURRENT DIVISION: $i_6 = i_5 \frac{4}{4+4} = \frac{2}{3}$

OHM'S LAW: $v_7 = 2\, i_6 = \frac{4}{3}$ volts

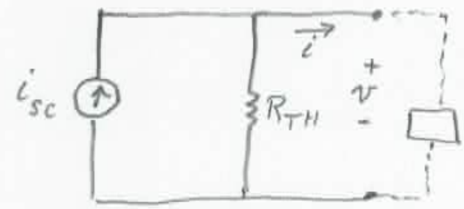
THEVENIN AND NORTON EQUIVALENT CIRCUITS

THEVENIN'S CIRCUIT



$$v = -R_{TH}\, i + v_{oc}$$

NORTON CIRCUIT

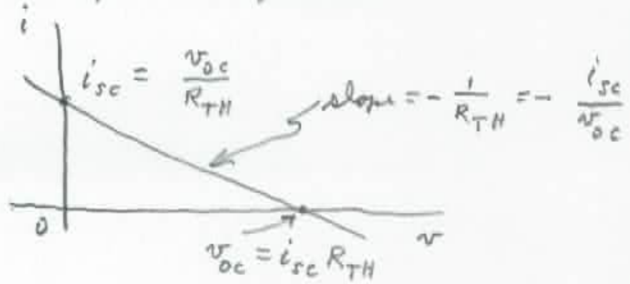


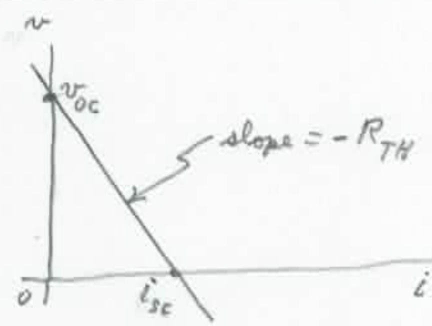
$$i_{sc} = i + \frac{v}{R_{TH}}$$

(OR) $v = -R_{TH}\, i + R_{TH}\, i_{sc}$

These circuits and equations are the same so far as the terminal v and i are concerned, *if* $v_{oc} = R_{TH}\, i_{sc}$.

Graphs of the equations:

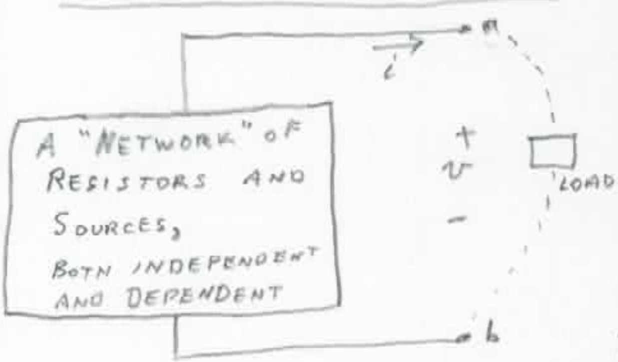




R_{TH} = INPUT RESISTANCE WHEN SOURCES ARE REMOVED.

SHORT v_{oc}
OPEN i_{sc}

A USE OF THESE CIRCUITS:



Measure v when $i = 0$. This gives $v = v_{oc}$. (open circuit at a, b.)

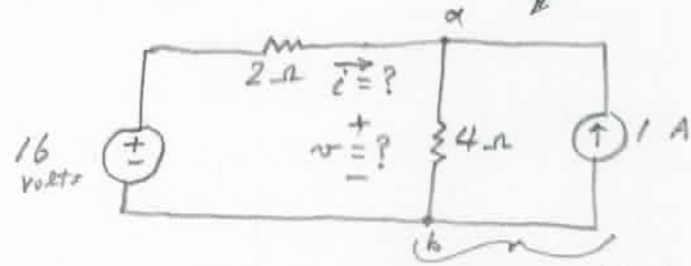
Measure i when $v = 0$. This gives $i = i_{sc}$. (Short circuit at a, b.)

Set $R_{TH} = \frac{v_{oc}}{i_{sc}}$

Then this "network" can be replaced by either Thevenin's circuit or Norton's circuit to get the same v and i at the terminals a, b whatever the load happens to be. This simplifies the calculation of v and i, as we keep changing the load.

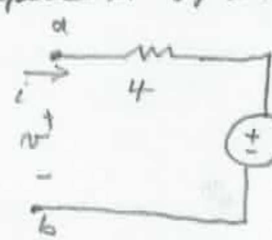
Example: Here's another use of the equivalence of the Thevenin and Norton circuits to analyze a circuit.

Problem: Get v and i in this circuit



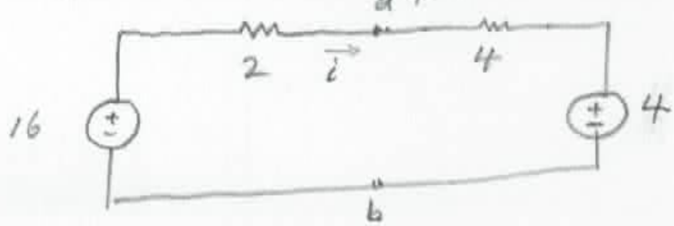
This is a Norton form of circuit: $R_{TH} = 4$, $i_{sc} = 1$ A.

Replace it by its equivalent Thevenin circuit:



$v_{oc} = R_{TH}\, i_{sc} = 4 \times 1 = 4$

So equivalently with respect to v and i, we have



Apply KVL: $-16 + 2i + 4i + 4 = 0$

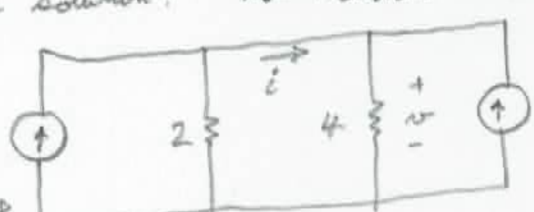
$$i = \frac{16-4}{2+4} = 2 \text{ A.}$$

So,

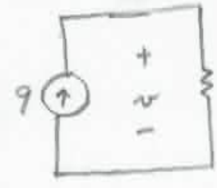
$$v = 16 - 2i = 12 \text{ Volts}$$

Alternative solution: Use Norton's circuits:

$i_{sc} = \frac{v_{oc}}{R_{TH}} = \frac{16}{2} = 8$



$\equiv$



$\frac{2 \times 4}{2+4} = \frac{8}{6} = \frac{4}{3}$

$$v = 9 \times \frac{4}{3} = 12 \text{ Volts}$$

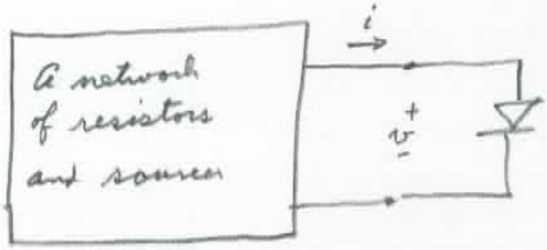
$$i = \frac{v}{4} - 1 = 3 - 1 = 2 \text{ A}$$

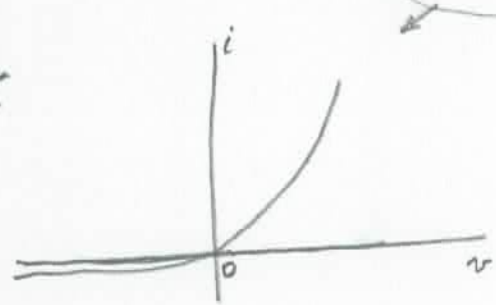
Another example

Load-line analysis for a p-n junction diode:

a "nonlinear" element

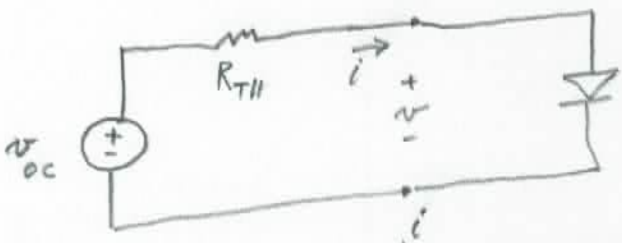
It's current-voltage curve



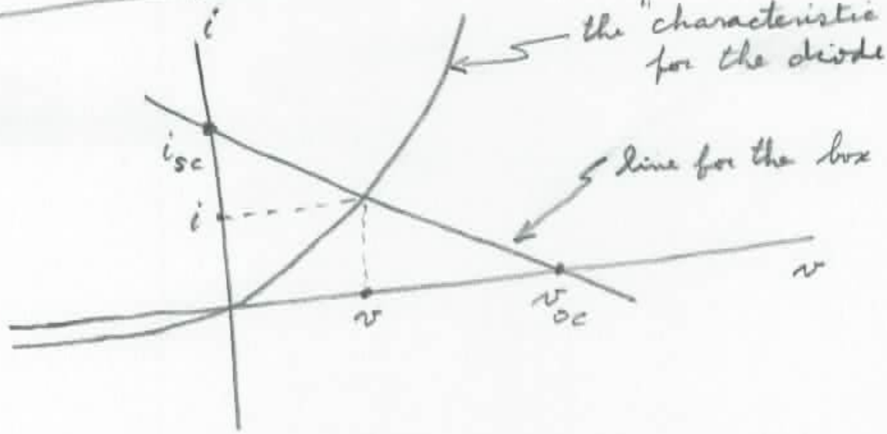


FIND v and i:

Use Thevenin's equivalent for this "box":



$$R_{TH} = \frac{v_{oc}}{i_{sc}}$$



the "characteristic curve" for the diode.

line for the box

So, draw the curve for the diode,

draw the line for the box,

then find the point of intersection.

(i, v)

DEPENDENT SOURCES

These are very different from independent sources.

(shown on page 3)

VCVS = voltage-controlled voltage source:

v_1 + −

Somewhere in circuit: v is voltage between two nodes.

μv_1

Elsewhere in circuit.

μ = constant, called the "voltage gain."

(μ is any real number, fixed)

CCVS = current-controlled voltage source:

Somewhere:

i_1

i_1 is a current in some branch.

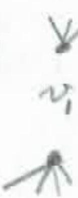
Elsewhere:

$v = r i_1$

r = a constant, called the "transresistance"

VCCS = voltage-controlled current source:

Somewhere:



v_1 + −

Elsewhere:

$i = g v_1$

g = a constant, called the "transconductance"

CCCS = current-controlled current source:

Somewhere:

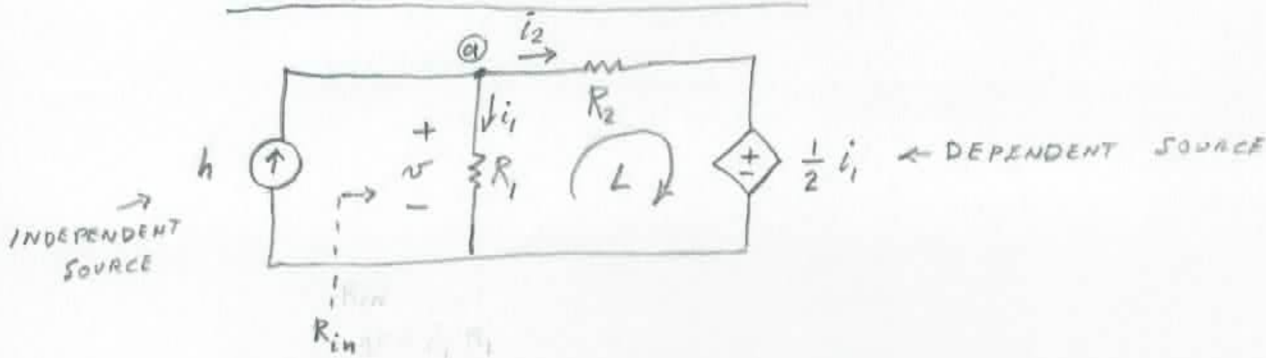
i_1

Elsewhere:

$i = \beta i_1$

β = a constant, called the "current gain"

AN ANALYSIS FOR A CIRCUIT:



PROBLEM: FIND i_1, THEN FIND v, THEN FIND $R_{in} = \dfrac{v}{h} = \dfrac{v}{i_1 + i_2}$

SOLUTION: WRITE TWO EQUATIONS

(1) KCL AT NODE (a): $-h + i_1 + i_2 = 0$

(2) KVL AROUND LOOP L: $-i_1 R_1 + i_2 R_2 + \frac{1}{2} i_1 = 0$

SOLVING: $i_2 = \dfrac{R_1 - .\frac{1}{2}}{R_2} i_1$

$$i_1 \left(1 + \frac{R_1 - \frac{1}{2}}{R_2}\right) = h, \text{ so } i_1 = \frac{R_2}{R_1 + R_2 - \frac{1}{2}} h$$

DON'T NEED i_2 } → SO, $i_2 = \dfrac{R_1 - \frac{1}{2}}{R_2}\left(\dfrac{R_2}{R_1 + R_2 - \frac{1}{2}} h\right) = \dfrac{R_1 - \frac{1}{2}}{R_1 + R_2 - \frac{1}{2}} h$

THUS, $R_{in} = \dfrac{i_1 R_1}{h} = \dfrac{R_1 R_2}{R_1 + R_2 - \frac{1}{2}}$

Another way to get i_1 is to use matrices – See Appendix A of book.

Equations (1) and (2) can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 \\ \frac{1}{2} - R_1 & R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h \\ 0 \end{bmatrix}$$

Using Cramer's rule to get i_1:

$$i_1 = \frac{\begin{vmatrix} h & 1 \\ 0 & R_2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ \frac{1}{2} - R_1 & R_2 \end{vmatrix}} = \frac{h R_2 - 0 \times 1}{R_2 - \left(\frac{1}{2} - R_1\right)} = \frac{R_2}{R_1 + R_2 - \frac{1}{2}} h \quad \left(\text{same answer}\right)$$

"determinants" →

Some numerical results for this example:

Let $R_1 = R_2 = 1$. We get $R_{in} = \dfrac{R_1 R_2}{R_1 + R_2 - \frac{1}{2}} = \dfrac{1}{3/2} = \dfrac{2}{3}\ \Omega$

a positive resistor

Let $R_1 = R_2 = \frac{1}{10}$. We get $R_{in} = \dfrac{\frac{1}{10} \cdot \frac{1}{10}}{\frac{2}{10} - \frac{5}{10}} = -\dfrac{1}{30}\ \Omega$

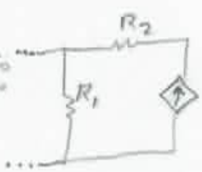
a negative resistor!

Input power from the source h is

$$p = h^2 R_{in} = h^2\left(-\tfrac{1}{30}\right) = -\frac{h^2}{30}$$

a negative number

This means that (positive) power is flowing from this



toward the left into the source h:

Question: Where does that power come from?

Answer: From the dependent source.

I will discuss this further.