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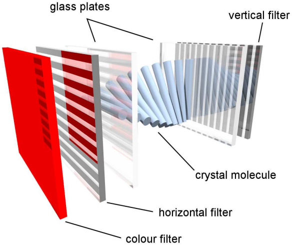
LCDs: liquid crystal displays

- When discussing polarization, we should have mentioned liquid crystal displays or LCDs!



- They involve crossed polarizers with a birefringent material in between.
  - No applied voltage: quarter wave plate, so light is transmitted.
  - Applied voltage: less circularly polarizing, so light is blocked.
  - Voltage is applied using optical transparent, lithographically-patternable conductor: indium tin oxide.
- For color displays, place red, green, and blue filters on top of successive pixels, and make the pixels small enough to not be viewable.
- Explains why LCD displays can look funny when you have polarizing sunglasses on. . .

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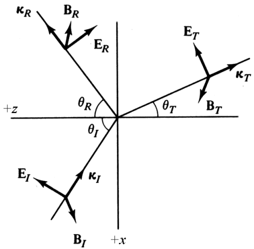
Light at a refractive interface

- At a refractive interface, we have an incoming wave, a reflected wave r, and a transmitted wave t. We might be able to divide up the incident electric field E into E_r and E_t, but right at the interface a maximum in the incident electric field will certainly produce an extremum in E_r and in E_t.



- In other words, the phase of the electric waves must be equal at the interface, or

$$(\vec{k}_i \cdot \vec{r} - \omega_i t)|_{z=0} = (\vec{k}_r \cdot \vec{r} - \omega_r t)|_{z=0} = (\vec{k}_t \cdot \vec{r} - \omega_t t)|_{z=0} \qquad (1)$$

- Use a coordinate system centered on the boundary and consider the point $\vec{r} = 0$:

$$\omega_i t = \omega_r t = \omega_t t \qquad \text{or} \qquad \omega_i = \omega_r = \omega_t \qquad (2)$$

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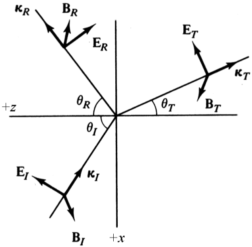
Light at a refractive interface II

- Having $\omega_i = \omega_r = \omega_t$ means that the $\vec{k} \cdot \vec{r}$ products must satisfy

$$(\vec{k}_i \cdot \vec{r})|_{z=0} = (\vec{k}_r \cdot \vec{r})|_{z=0} = (\vec{k}_t \cdot \vec{r})|_{z=0} \quad (3)$$



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Relating k across interfaces

- We will want to compare values for k across different interfaces. Recall that the phase velocity is

$$v_p = \frac{\omega}{k} \qquad \text{so} \qquad k = \frac{\omega}{v_p} \tag{4}$$

- Also remember that $v_p = c/n$ in a refractive medium.
- We can then say

$$k = n\frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi n}{\lambda} \tag{5}$$

and therefore relate k across interfaces according to the refractive index on either side of the interface.

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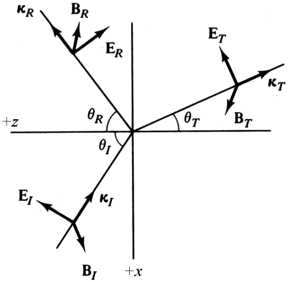
Incident and reflected rays

Continuing with Eq. 3 of $(\vec{k}_i \cdot \vec{r})|_{z=0} = (\vec{k}_r \cdot \vec{r})|_{z=0} = (\vec{k}_t \cdot \vec{r})|_{z=0}$, let's consider it in terms of vector components, keeping in mind the coordinate system shown at right:



$$\begin{aligned} -|k_{ix}|\cdot x - |k_{iz}|\cdot z &= -|k_{rx}|\cdot x + |k_{rz}|\cdot z \\ &= -|k_{tx}|\cdot x - |k_{tz}|\cdot z \end{aligned} \tag{6}$$

Consider first the incident and reflected rays:

$$\begin{aligned} -|k_{iz}|\cdot z &= +|k_{rz}|\cdot z \\ -\frac{2\pi n_i}{\lambda_i}\sin\theta_i &= +\frac{2\pi n_r}{\lambda_r}\sin\theta_r \end{aligned} \tag{7}$$

Is this of any use?

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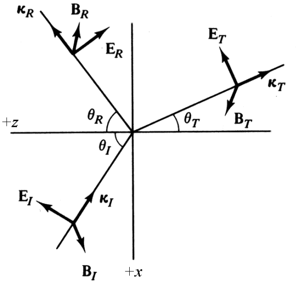
Incident and reflected II

Again, we had from Eq. 7 the expression

$$-\frac{2\pi n_i}{\lambda_i}\sin\theta_i = +\frac{2\pi n_r}{\lambda_r}\sin\theta_r$$



If we realize that $n_i = n_r$ because it's the same medium, and $\lambda_i = \lambda_r$ for the same reason, we can boil the above down to

$$-\theta_i = \theta_r \tag{8}$$

which is really just the law of reflection!

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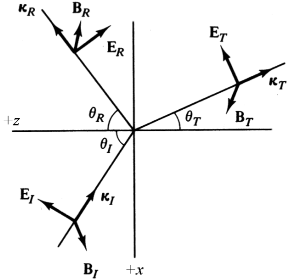

Let's now look at the terms

$$\begin{aligned} -|k_{iz}| \cdot z &= -|k_{it}| \cdot z \\ n_i \frac{\omega_i}{c} \sin\theta_i &= n_t \frac{\omega_t}{c} \sin\theta_t \end{aligned}$$

and remind ourselves that the electric fields must be in phase-lock at the interface, or $\omega_i = \omega_t$. We have therefore shown that

$$n_i \sin\theta_i = n_t \sin\theta_t \tag{9}$$

which is Snell's law!

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E&M at interfaces

We now want to remind ourselves of some relationships (see *e.g.*, Eq. 7.46 in Sec. 7.3.5 of Griffiths, *Introduction to Electrodynamics*, first edition, 1981) for boundaries between media in electrodynamics:

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp} \tag{10}$$

$$B_{1\perp} = B_{2\perp} \tag{11}$$

$$E_{1\|} = E_{2\|} \tag{12}$$

$$\frac{1}{\mu_1} B_{1\|} = \frac{1}{\mu_2} B_{2\|}. \tag{13}$$

That is, the dielectric constant of a material affects the electric field perpendicular to a surface, but not along the surface plane; and the magnetic permeability μ affects the field B along the surface plane but not perpendicular to it. We remind ourselves that $\mu = \mu_0$ for most cases that we care about with light, and also (see *e.g.*, Griffiths 1981 Eq. 8.77, Sec. 8.2.4) that for waves traveling along $\hat{x}$ we have

$$E_y = \frac{c}{n} B_z \tag{14}$$

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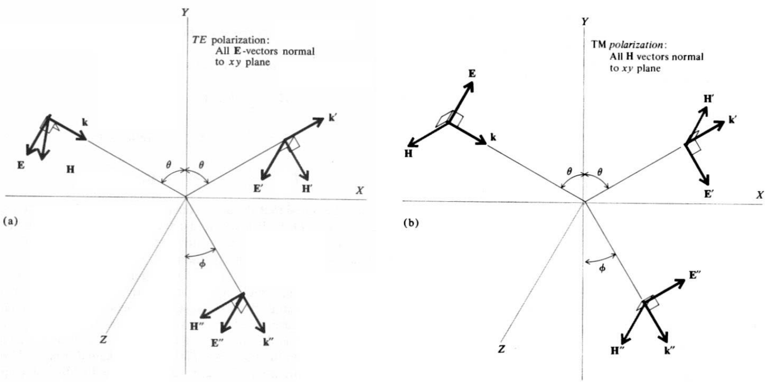
TE and TM

Because we have different relationships for E and B across interfaces, we'll consider two cases: transverse electric or TE, and transverse magnetic or TM:

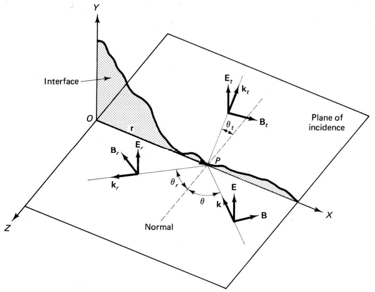


Fowles Fig. 2.11

TE waves at the interface

For TE waves, we use Eq. 12 ($E_{1\|} = E_{2\|}$) to obtain

$$E_i + E_r = E_t. \qquad (15)$$



TE waves: Pedrotti and Pedrotti Fig. 20-1. See also Fowles Fig. 2.11.

Meanwhile, Eq. 11 of $B_{1\perp} = B_{2\perp}$ and Eq. 14 of $E_y = (c/n)B_z$ together give

$$\begin{aligned} B_i \cos\theta_i - B_r \cos\theta_r &= B_t \cos\theta_t \qquad (16) \\ n_i E_i \cos\theta_i - n_i E_r \cos\theta_i &= n_t E_t \cos\theta_t \end{aligned}$$

where we have made use of the fact that $n_i = n_t$ and $\theta_i = \theta_r$.

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TM waves at the interface

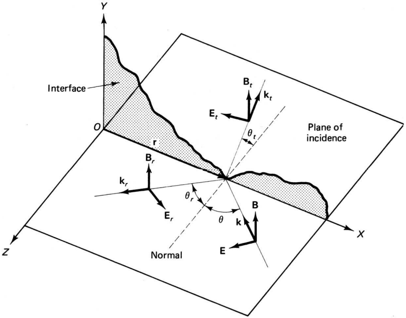
For TM waves, we use Eq. 10 of $\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$ to obtain

$$-E_i \cos\theta_i + E_r \cos\theta_r = -E_t \cos\theta_t \tag{17}$$

Meanwhile, Eq. 13 of $B_{1\|}/\mu_1 = B_{2\|}/\mu_2$, along with Eq. 14 of $E_y = (c/n)B_z$, together give

$$\begin{aligned} B_i + B_r &= B_t \\ n_i E_i + n_i E_r &= n_t E_t \end{aligned} \tag{18}$$

where again we have $n_i = n_r$ and $\theta_i = \theta_r$.



TM waves: Pedrotti and Pedrotti Fig. 20-2. See also Fowles Fig. 2.11.

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TE: consolidating results

If we substitute Eq. 15 of $E_i + E_r = E_t$ into Eq. 16 of $n_i E_i \cos\theta_i - n_i E_r \cos\theta_i = n_t E_t \cos\theta_t$, we find

$$\begin{aligned} n_i E_i \cos\theta_i - n_i E_r \cos\theta_i &= n_t (E_i + E_r) \cos\theta_t \\ E_i (n_i \cos\theta_i - n_t \cos\theta_t) &= E_r (n_i \cos\theta_i + n_t \cos\theta_t) \end{aligned}$$

or

$$\text{TE: } r_\perp \equiv \frac{E_r}{E_i} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \tag{19}$$

We can also solve for E_t/E_i to find

$$\text{TE: } t_\perp \equiv \frac{E_t}{E_i} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}. \tag{20}$$

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TE: further consolidation

It is often convenient to write these expresions in terms of a relative index of refraction n of

$$n \equiv \frac{n_t}{n_i}$$

and refer only to the incident angle $\theta_i \Rightarrow \theta$ since we can always find the transmitted angle from it. Using Snell's law $n_i \sin\theta_i = n_t \sin\theta_t$, we find (see *e.g.*, Eq. 2.22 of Pedrotti and Pedrotti)

$$\frac{n_t}{n_i}\cos\theta_t = n\sqrt{1-\sin^2\theta_t} = \sqrt{n^2 - (\frac{n_t}{n_i})^2(\frac{n_i}{n_t})^2\sin^2\theta_i} = \sqrt{n^2-\sin^2\theta_i} \tag{21}$$

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The Fresnel equations

By using the expression of Eq. 21, and a bit more elbow grease, one can come up with the following expressions which are known as the Fresnel equations (see Pedrotti and Pedrotti Eqs. 20-23 – 20-26, or Fowles Eqs. 2-56 – 2-59):

$$\text{TE: } r_\perp = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \tag{22}$$

$$\text{TM: } r_\parallel = \frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \tag{23}$$

$$\text{TE: } t_\perp = \frac{2\cos\theta}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)} \tag{24}$$

$$\text{TM: } t_\parallel = \frac{2n\cos\theta}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} \tag{25}$$

The case of $n_i > n_t$ or $n < 1$ is that of *internal reflection*, while the case of $n_t > n_i$ or $n > 1$ is that of *external reflection*. These relationships tell us a lot!

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Brewster's angle

What happens when $\theta_i + \theta_t = \pi/2$? Look at $r_\parallel$ in the second form given in Eq. 23:

$$\text{TM: } r_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

If $r_\parallel$ goes to zero and we have no reflection of TM waves, then all reflected light at this angle is TE polarized! This angle is known as Brewster's angle or θ_p. We can find it from Snell's law as

$$\begin{aligned} n_i \sin\theta_{i=p} &= n_t \sin\theta_t \\ \sin\theta_{i=p} &= n \sin\theta_t = n\sin(\pi/2 - \theta_{i=p}) = n\cos\theta_{i=p} \\ \sin\theta_p &= n\cos\theta_p \\ \tan\theta_p &= n \end{aligned} \tag{26}$$

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Surface glare

- Consider the refractive boundary between air and water, where $n = 1.3$: we find that $\theta_p = 52^\circ$, while for glass with $n = 1.5$ we find $\theta_p = 56^\circ$.
- Imagine that you're on a boat when the sun is 53° above the horizon: direct sunlight will be reflected off the water's surface at Brewster's angle, so only the TE mode relative to the water surface (horizontal linear polarization) will survive.
- Sunglasses with a linear polarizer with a vertical fast axis (FA) will block the glare!

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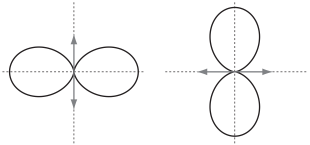
Scattering and dipole radiation

- Light that is singly-scattered by a refractive medium can be thought of as being re-radiated by a collection of damped, driven harmonic oscillators.
- With vertical linear polarization, the oscillator is driven up and down; with horizontal linear polarization, the oscillator is driven sideways.
- In your E&M class you will learn that dipole radiation for a vertical oscillator follows a $\sin^2\theta$ angular distribution (drawn at right).



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Scattering and dipole radiation II

- Apply the rules of dipole radiation to light scattering.
- Polarized sunglasses with a vertical fast axis will transmit light scattered from the left and right of the sun, but not from above and below.

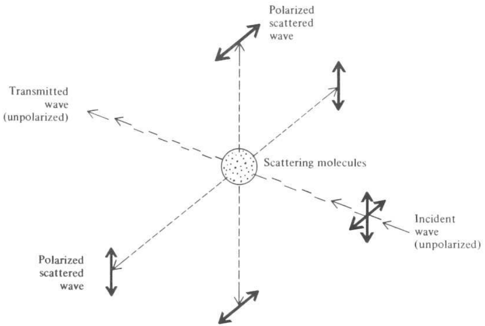

Figure 2.4. Illustrating polarization in the molecular scattering of light. The **E** vectors for the incident and scattered waves are indicated.

Fowles Fig. 2.4

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A diversion: why the sky is blue

- Another feature of dipole radiation that we should mention: its strength scales as $\omega^4 \propto \lambda^{-4}$, so blue light is scattered much more strongly than red light is.
- At midday, this means that when we look at the sky (but not straight at the sun!), we see mostly blue light.
- At high altitude, the sky looks darker blue because of less scattering.

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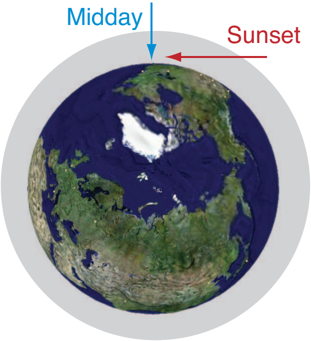
Sunrise, sunset. . .

- At sunset, rays from the sun traverse even more of the atmosphere, so the blue light is all gone and red light is all that remains!
- If we had a thicker atmosphere, we might see red light at noon. . .



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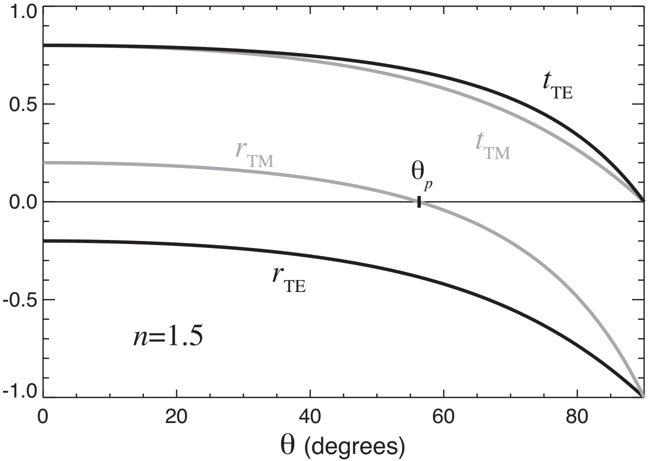
Coefficients for $n = 1.5$

Let's get back to the Fresnel equations. Here's a plot for $r_{\mathrm{TE}} = r_{\perp}$, $r_{\mathrm{TM}} = r_{\parallel}$, $t_{\mathrm{TE}} = t_{\perp}$, and $t_{\mathrm{TM}} = t_{\parallel}$ for $n = 1.5$:



The negative values of r describe conditions where the phase is inverted by 180° upon reflection.

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Reflectivity and transmittance

- We've determined the electric field amplitude coefficients r and t on either side of an interface.
- We know that irradiance I goes like electric field squared, so it's tempting to say that the reflection is $R = r^2$ and the transmittance is $T = t^2$.
- However, to quote Gershwin, "it ain't necessarily so." What we really expect is $R + T = 1$ to conserve energy.

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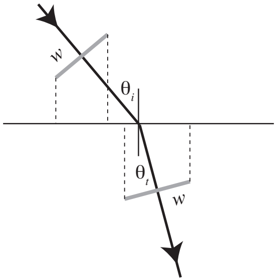
Reflectivity and transmittance II

- Let's then consider a beam which is circular with diameter w along its propagation direction, and consider the elliptical area it projects onto the refractive interface.
- The figure at right shows incident and transmitted beam widths; the reflected beam width is the same as the transmitted beam width.



- The conservation of energy (Watts/m^2 times m^2) becomes

$$\begin{aligned} I_i \frac{\pi}{4} w \cdot w \cos\theta_i &= I_r \frac{\pi}{4} w \cdot w \cos\theta_i + I_t \frac{\pi}{4} w \cdot w \cos\theta_t \\ I_i \cos\theta_i &= I_r \cos\theta_i + I_t \cos\theta_t \end{aligned} \tag{27}$$

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Reflectivity and transmittance III

Again, we have from Eq. 27 the relationship $I_i \cos\theta_i = I_r \cos\theta_i + I_t \cos\theta_t$. Now we need to remind ourselves of the relationship between irradiance and electric field of

$$I = \sqrt{\frac{\epsilon}{\mu}}\langle E\rangle^2 \tag{28}$$

and remember that the electric fields are in phase across either side of the refractive index boundary so we can just as well talk about E^2 as $\langle E\rangle^2$. We then have

$$\begin{aligned} E_{0i}^2\sqrt{\frac{\epsilon_i}{\mu_i}}\cos\theta_i &= E_{0r}^2\sqrt{\frac{\epsilon_i}{\mu_i}}\cos\theta_i + E_{0t}^2\sqrt{\frac{\epsilon_t}{\mu_t}}\cos\theta_t \\ E_{0i}^2\sqrt{\frac{\epsilon_i}{\mu_i}}\cos\theta_i &= (rE_{0i})^2\sqrt{\frac{\epsilon_i}{\mu_i}}\cos\theta_i + (tE_{0i})^2\sqrt{\frac{\epsilon_t}{\mu_t}}\cos\theta_t \\ 1 &= r^2 + t^2\sqrt{\frac{\epsilon_t}{\epsilon_i}\frac{\mu_i}{\mu_t}}\frac{\cos\theta_t}{\cos\theta_i} \end{aligned} \tag{29}$$

as our conservation of energy expression thus far.

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Reflectivity and transmittance IV

Again, we have from Eq. 29 the relationship

$$1 = r^2 + t^2 \sqrt{\frac{\epsilon_t}{\epsilon_i} \frac{\mu_i}{\mu_t} \frac{\cos\theta_t}{\cos\theta_i}}$$

so our guess that $R = r^2$ for reflectivity has turned out to be true. However, the transmittance is a bit more complicated expression. To evaluate it, let's remind ourselves of the definition of the refractive index:

$$n \equiv \frac{\sqrt{\epsilon\mu}}{\sqrt{\epsilon_0\mu_0}} \qquad \text{which gives} \qquad \frac{n_t}{n_i} = \sqrt{\frac{\epsilon_t\mu_t}{\epsilon_i\mu_i}} \tag{30}$$

As a result, in the case where magnetic effects can be ignored so we can say $\mu_i \simeq \mu_t \simeq \mu_0$, we find that the transmittance T is given by

$$T \simeq \frac{n_t}{n_i} \frac{\cos\theta_t}{\cos\theta_i} t^2 = n \frac{\cos\theta_t}{\cos\theta_i} t^2 \tag{31}$$

where in the second case we have used the relative refractive index $n = n_t/n_i$.

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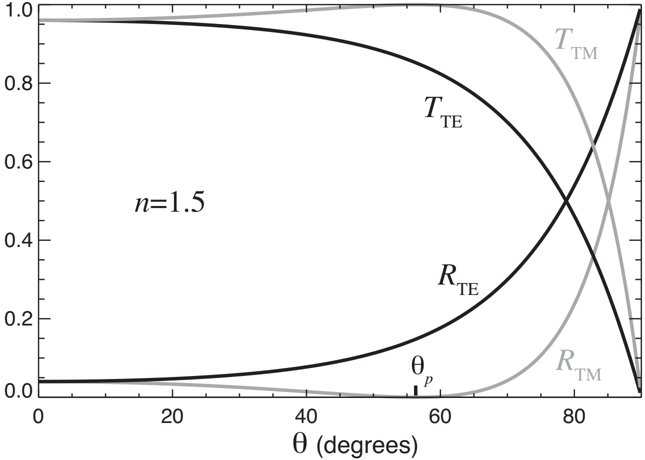
Reflectivity and transmittance V

Now that we know $R = r^2$ and $T \simeq n(\cos\theta_t/\cos\theta_i)t^2$, we can plot the reflectivity and transmittance for $n = 1.5$:



Notice Brewster's angle $\theta_p = \arctan(n)$ (Eq. 26)

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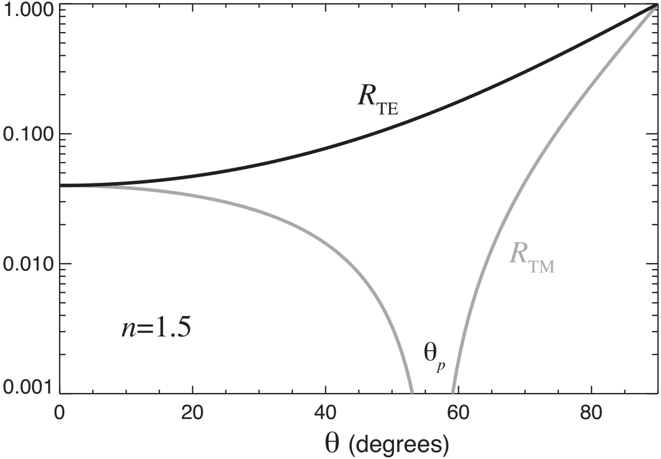
Reflectivity and transmittance V

And here's a plot of the reflectivity on a log scale for $n = 1.50$:



Notice Brewster's angle $\theta_p = \arctan(n)$ (Eq. 26)

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What about $n < 1$?

What happens when the relative refractive index is $n < 1$? Let's start by looking at the denominator term in Eqs. 22–25 of $\sqrt{n^2 - \sin^2\theta}$. In the situation where $\sin\theta < n$ we have no problems with this term. Now remember that $\theta = \theta_i$, and $n = n_t/n_i$. We can therefore consider the limiting case of $n = \sin\theta$ as

$$\begin{aligned} \sin\theta &= n \\ n_i \sin(\theta_i = \theta_c) &= n_t \sin(\theta_t = 90^\circ) \\ \theta_c &= \arcsin\left(\frac{n_t}{n_i}\right) = \arcsin(n) \end{aligned} \tag{32}$$

where we recognize the middle expression as Snell's law. The final expression allows us to calculate the critical angle θ_c at which light undergoes total internal reflection when going from materials of higher to lower refractive indices ($n_i > n_t$, or $n < 1$).

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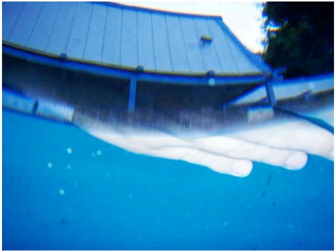
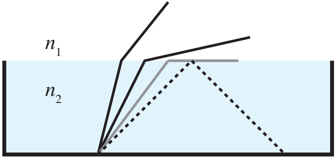
Internal reflection

- Again, the critical angle is given by $\theta_c = \arcsin(n)$
- With $n = 1/1.33$ we have $\theta_c = 48.8^\circ$ for water
- With $n = 1/1.5$ we have $\theta_c = 41.8^\circ$ for a representative glass.
- We also have a Brewster's angle $\theta_p = \arctan(n)$ of 36.9° and 33.7° for water and glass, respectively.
- TIR=total internal reflection

TIR and swimming pools

- A reminder from Lecture 8: you're looking up from the bottom of a swimming pool.
- You're going to see a circle of light with a semi-angle of $\theta_{c,\text{water}} = 48.8^\circ$ that is coming from the sky.
- Beyond that angle you'll see rays reflecting from the sides and bottom of the pool.





Pool photo: http://lifshitz.ucdavis.edu/~dmartin/phy7/7C/Refraction/Refraction.html

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Immersion lenses

- Goal: maximize light gathering power of a lens (also its resolution, as we'll see later) used to image a wet cell.
- Air gap leads to TIR when rays exit solution.
- Solution: use a cover slip, and then an immersion oil to avoid total internal reflection.

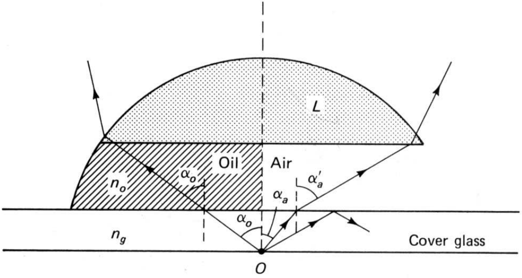

Figure 6-25 Microscope objective, illustrating the increased light-gathering power of an oil-immersion lens.

This is also now being used for printing the finest possible features in integrated circuit manufacturing.