ESE 271 Second Exam Name:

Spring, 2004 ID Number:

Do not place your answers on this front page.

Every problem is worth 25 points.

Prob. 1:

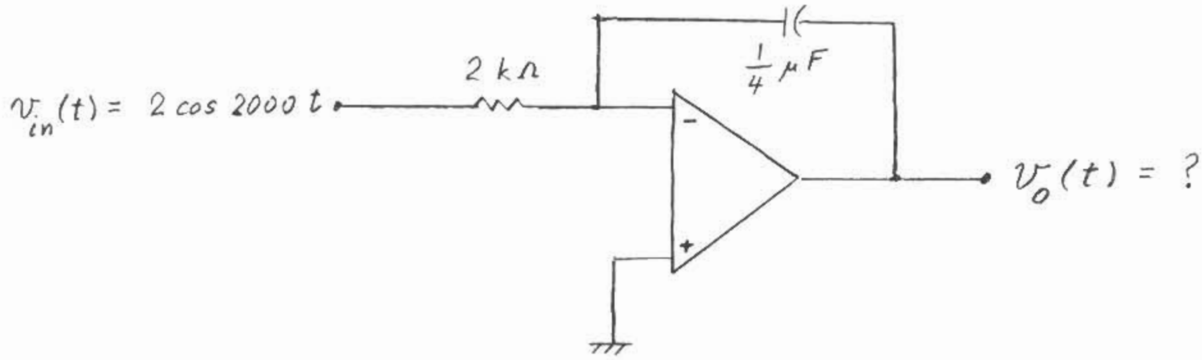
Prob. 2:

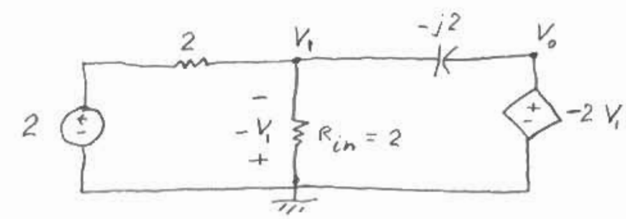
Prob. 3:

Prob. 4:

Prob. 1:

Find the output voltage $v_o(t)$ as a single cosinusoid. The gain of the opamp is $A = 2$, its input resistance is $R_{in} = 2\ k\Omega$, and its output resistance is $R_o = 0$. (These are not typical values.) Do NOT use the virtual-open/virtual-short model of the opamp.





$V_o = -2V_1$ or $V_1 = -\frac{V_o}{2}$

$$\frac{V_1 - 2}{2} + \frac{V_1 - V_o}{-j2} + \frac{V_1}{2} = 0$$

$$V_1(2+j) - jV_o = 2$$

$$-\frac{V_o}{2}(2+j) - jV_o = 2$$

$$V_o(-2-3j) = 4$$

$$V_o = \frac{4}{\sqrt{2^2+3^2}\ \angle \tan^{-1}\frac{-3}{-2}} = \frac{4}{3.606\ \angle 180° + 56.3°} = 1.109\ \angle -236.3°$$

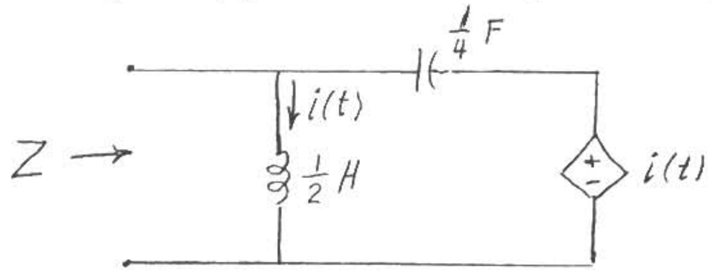
$$= 1.109\ \angle 123.7°$$

So,

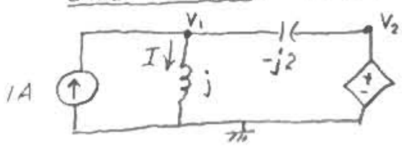
$$v_o(t) = 1.109\cos(2000t - 236.3°) = 1.109\cos(2000t + 123.7°)$$

Prob. 2:

Find the input impedance (i.e., the Thevenin impedance) Z of the network when the angular frequency is $\omega = 2$. (You may give Z in either rectangular form or polar form.)



ONE METHOD: APPLY A CURRENT SOURCE. DO A NODAL ANALYSIS.



$I = \frac{V_1}{j} = -jV_1$

AT NODE 1: $-1 + \frac{V_1}{j} + \frac{V_1 - V_2}{-j2} = 0 \Rightarrow V_1 + V_2 = j2$

ALSO: $V_2 = -jV_1$

SO: $V_1 = \frac{j2}{1-j} = \sqrt{2}\,\angle 135^\circ$

THUS, $Z = \frac{V_1}{1} = \sqrt{2}\,\angle 135^\circ$

ANOTHER METHOD: APPLY A CURRENT SOURCE. DO A MESH ANALYSIS.

$I_1 = 1\ A$. ALSO, AROUND I_2-LOOP: $j(I_2 - I_1) - j2\,I_2 + I = 0$

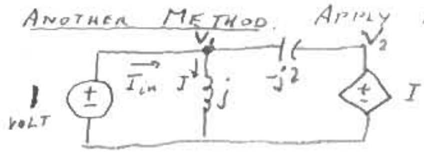
$I_2(j - j2 - 1) = j - 1$

$I_2 = \frac{-1+j}{-1-j} = -j$

SO, $V_1 = jI = j(I_1 - I_2)$

$= j - jI_2 = j - 1 = \sqrt{2}\,\angle 135^\circ$, $Z = \frac{V_1}{1} = \sqrt{2}\,\angle 135^\circ$

ANOTHER METHOD. APPLY A VOLTAGE SOURCE. DO A NODAL ANALYSIS.

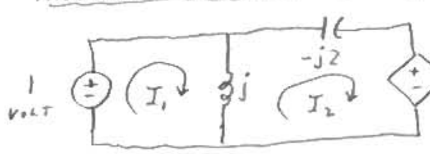


$V_1 = 1$, $V_2 = I = \frac{1}{j} = -j$

$I_{in} = I + \frac{V_1 - V_2}{-j2} = -j + \frac{1-(-j)}{-j2} = \frac{-1-j}{2}$

$Z = \frac{1}{I_{in}} = \frac{2}{-1-j} = -1 + j = \sqrt{2}\,\angle 135^\circ$

ANOTHER METHOD: APPLY A VOLTAGE SOURCE. DO A MESH ANALYSIS:



$I = I_1 - I_2$

$-1 + j(I_1 - I_2) = 0 \Rightarrow I_2 = j + I_1$

$j(I_2 - I_1) - j2\,I_2 + I_1 - I_2 = 0 \Rightarrow I_1(1-j) + I_2(-1-j) = 0$

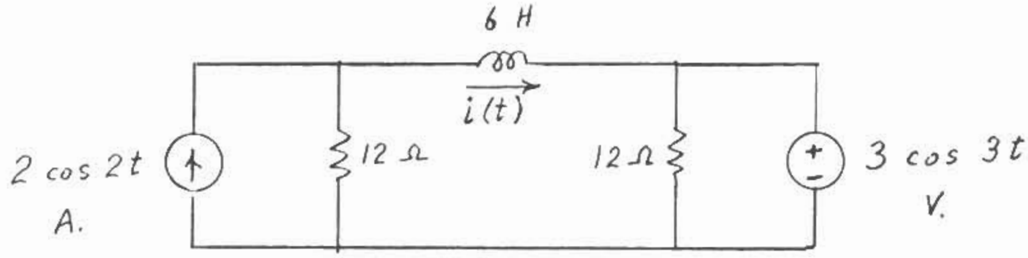
SO, $I_1(1-j) + (j + I_1)(-1-j) = 0$

$I_1 = \frac{-1+j}{-2j}$

$Z = \frac{1}{I_1} = -1 + j = \sqrt{2}\,\angle 135^\circ$

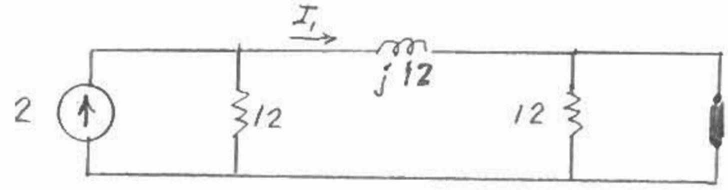
Prob. 3:

Find the current $i(t)$ in the inductor as a function of time t.



WE MUST USE SUPERPOSITION:

CASE 1: THE CURRENT SOURCE ALONE.

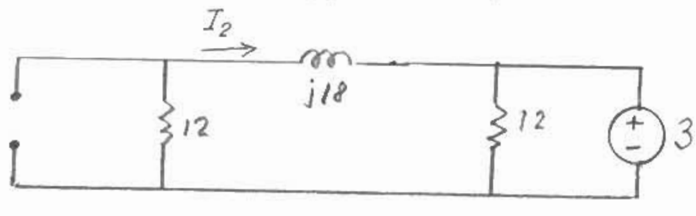


CURRENT DIVISION →

$$I_1 = 2\,\frac{12}{12+j12} = 2\cdot\frac{1}{1+j} = \sqrt{2}\,\angle -45^\circ$$

$$i_1(t) = \sqrt{2}\cos(2t - 45^\circ) = 1.414\cos(2t-45^\circ)$$

CASE 2: THE VOLTAGE SOURCE ALONE.



$$I_2 = -\frac{3}{12+j18} = -\frac{1}{4+j6} = \frac{-1}{\sqrt{4^2+6^2}\,\angle \tan^{-1}\frac{3}{2}} = .139\,\angle -236.3 = .139\,\angle 123.7$$

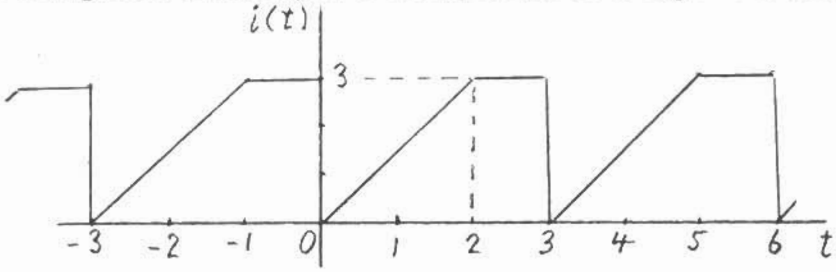
$$i_2(t) = .139\cos(3t + 123.7)$$

$$i(t) = i_1(t) + i_2(t)$$

$$= 1.414\cos(2t-45^\circ) + .139\cos(3t+123.7^\circ)$$

Prob. 4:

Find the effective value (i.e., the rms value) of the following periodic wave. Give your answer as a single real number (not as an expression involving two or more numbers.)



$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i(t)^2\,dt}$$

$$= \sqrt{\frac{1}{3}\left(\int_0^2 \left(\frac{3}{2}t\right)^2 dt + \int_2^3 3^2\,dt\right)}$$

$$= \sqrt{\frac{1}{3}\left(\frac{9}{4}\cdot\frac{t^3}{3}\Big|_0^2\right) + \frac{1}{3}\cdot 9}$$

$$= \sqrt{\frac{1}{3}\left(\frac{9}{4}\cdot\frac{8}{3}\right) + 3}$$

$$= \sqrt{2+3} = \sqrt{5} = 2.24$$