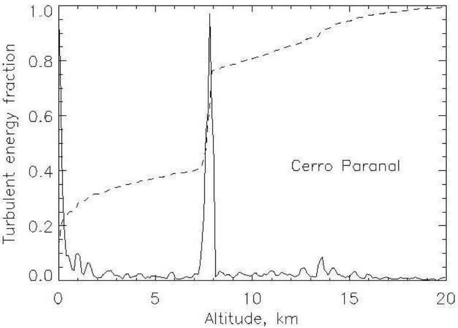
Aberrations

- From the Merriam-Webster online dictionary: **ab·er·ra·tion** (*n*):
  1. The fact or an instance of being aberrant (straying from the right or normal way) especially from a moral standard or normal state.
  2. Failure of a mirror, refracting surface, or lens to produce exact point-to-point correspondence between an object and its image.
  3. Unsoundness or disorder of the mind.

  plus other definitions. . .
- We described the aberration a of a wave in terms of the distance along a ray direction of the actual wave crest relative to the ideal wave crest.
- You saw from Prof. Walter how the Hubble Space Telescope was fabricated with spherical aberration, which was corrected for with the COSTAR upgrade.

Atmospheric turbulence

- Why do the stars twinkle? Because of dynamic rearrangements of the optical path length through the atmosphere. Dynamic aberrations. . .
- Refractive index depends on density. Air has $n = 1 + 8 \times 10^{-5}$, with $n - 1$ varying linearly with pressure and temperature.
- Timescale for rearrangement within aperture of typical telescopes is ~ 5 msec, or 200 Hz.

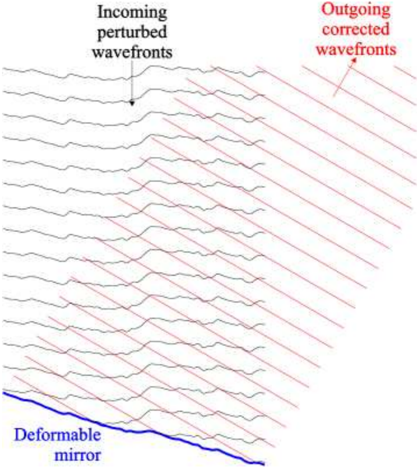


Atmospheric disturbance versus altitude at Cerro Paranal, Chile. Dashed line: altitude-integrated disturbance.

http://www.ctio.noao.edu/~atokovin/tutorial/part2/dm.html

Wavefront correction

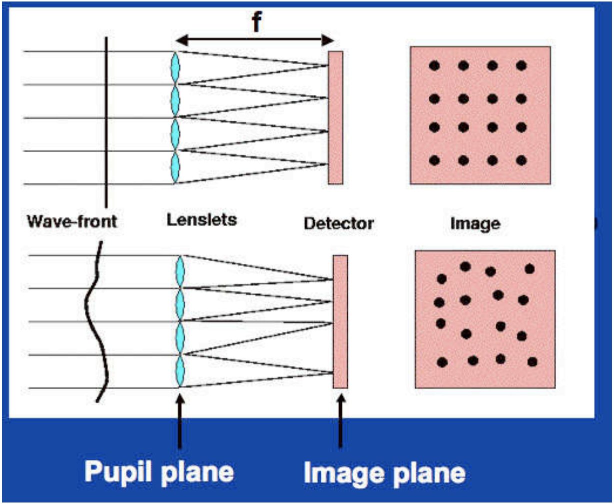
This is what we'd like to do, using a "guide star" (small, isolated star near what we want to image):



From Wikipedia

See this movie which is from http://cfao.ucolick.org/ao/.

Sensing wavefront distortions



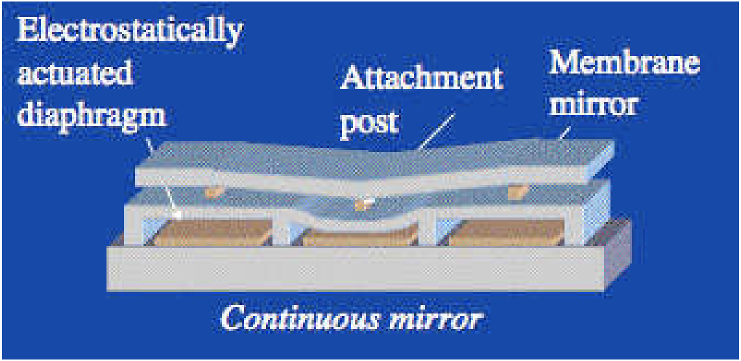
Shack-Hartmann wavefront sensing scheme.

http://www.ucolick.org/~max/289C/

Rubber mirrors

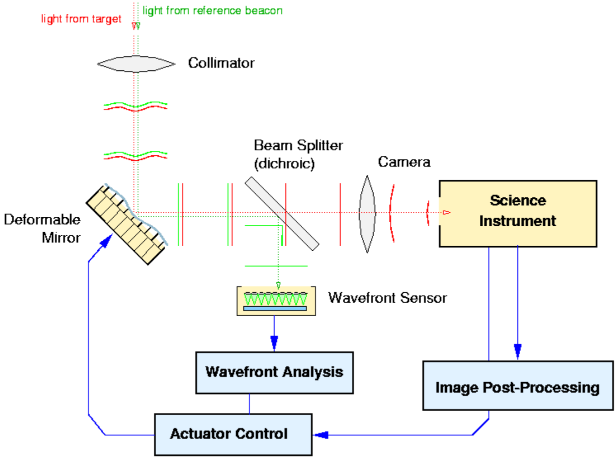
Keck telescope adaptive mirror.

Mini rubber mirrors



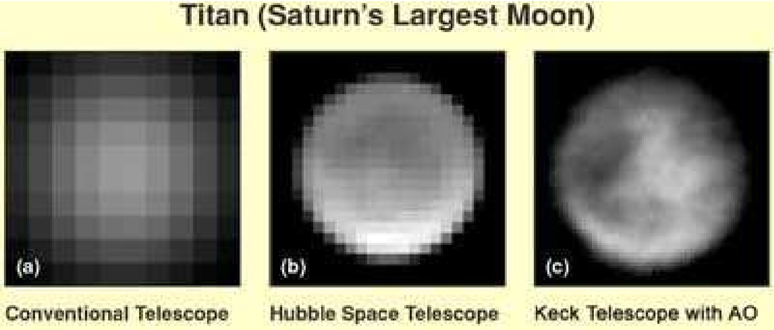
http://www.ucolick.org/~max/289C/

Adaptive optical systems



From http://cfao.ucolick.org/ao/

Adaptive optics successes



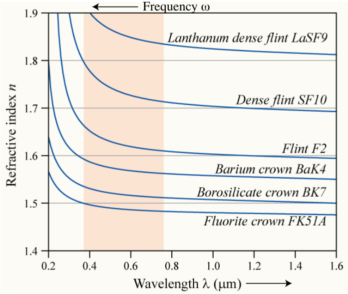
From http://cfao.ucolick.org/ao/

Chromatic dispersion and lenses

Recall that we found for the refractive index in the visible range the expression

$$n \simeq 1 + A\left(1 + \frac{B}{\lambda^2}\right)$$

This was based on a damped, driven harmonic oscillator model with a resonance at a particular frequency. Different glasses have different resonant frequencies in the UV and thus different A and B in the visible.



http://en.wikipedia.org/wiki/Image:Dispersion-cu

Dispersion in glasses

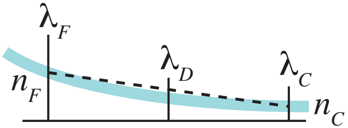
Glasses are often characterized in terms of response at three particular Fraunhofer spectral lines:

$$\lambda_F = 486.1 \text{ nm} \qquad \lambda_D = 589.2 \text{ nm} \qquad \lambda_C = 656.3 \text{ nm}$$

It's common to specify things in terms of the center wavelength λ_D and then do a linear approximation of the dispersion curve:

$$\frac{\partial n}{\partial \lambda} = \frac{n_F - n_C}{\lambda_F - \lambda_C} \qquad \text{or alternatively} \qquad V \equiv \frac{n_D - 1}{n_F - n_C} \tag{1}$$

where V is called the Abbe number or constringence.



Dispersion II

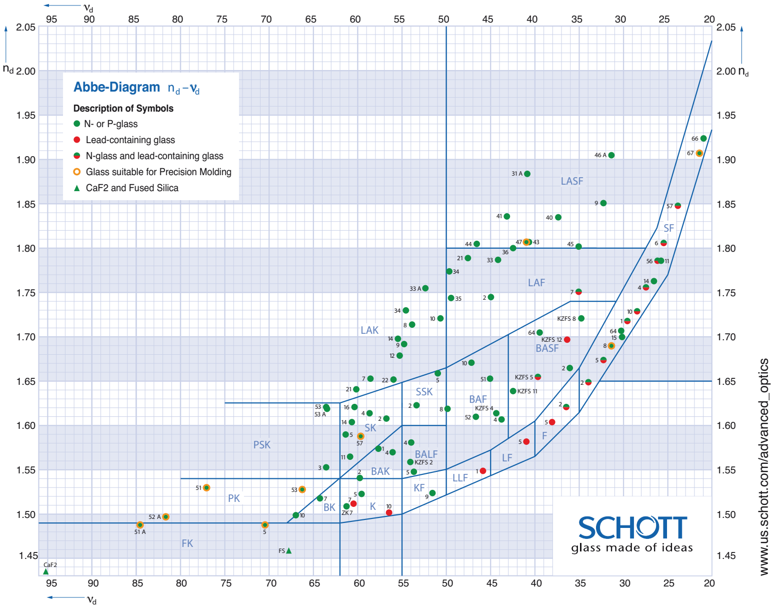
Again, we have:

$$\frac{\partial n}{\partial \lambda} = \frac{n_F - n_C}{\lambda_F - \lambda_C} \qquad \text{or alternatively} \qquad V \equiv \frac{n_D - 1}{n_F - n_C}$$

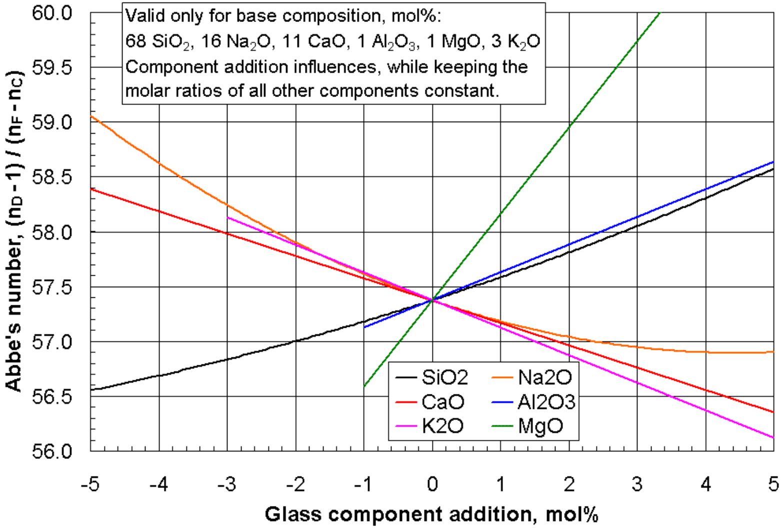
Some example glasses, including their catalog code $n_D - 1/10V$:

	n_C	n_D	n_F	V	$n_D - 1/10V$
Borosilicate crown	1.51461	1.51707	1.52262	64.55	517/645
Flint	1.61564	1.62045	1.63198	37.97	620/380
Fused silica	1.45637	1.45846	1.46313	67.83	458/678

Abbe diagram



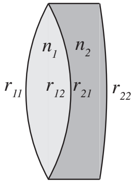
Modifying Abbe's number



http://glassproperties.com/abbe_number/

Achromat I

- Chromatic: of color.
- Achromat: no color-dependence in imaging.
- Let's see if we can put two lenses together to have their dispersions cancel.



- Let the lenses be thin so that we add their refractive powers:

$$\frac{1}{f_D} = \frac{1}{f_{1D}} + \frac{1}{f_{2D}} \tag{2}$$

$$= (n_{1D} - 1)\left(\frac{1}{r_{11}} - \frac{1}{r_{12}}\right) + (n_{2D} - 1)\left(\frac{1}{r_{21}} - \frac{1}{r_{22}}\right)$$

$$= (n_{1D} - 1)K_1 + (n_{2D} - 1)K_2 \tag{3}$$

We want to have $1/f_D$ be constant over a given wavelength range.

Achromat II

Again, we want to have $1/f_D$ be constant over a given wavelength range, or

$$\frac{\partial}{\partial\lambda}\left(\frac{1}{f}\right) = 0 = K_1\frac{\partial n_1}{\partial\lambda} + K_2\frac{\partial n_2}{\partial\lambda} = K_1\frac{n_{1F}-n_{1C}}{\lambda_F-\lambda_C} + K_2\frac{n_{2F}-n_{2C}}{\lambda_F-\lambda_C} \tag{4}$$

Look at one of these terms in more detail:

$$\begin{aligned} K_1\left(\frac{n_{1F}-n_{1C}}{\lambda_F-\lambda_C}\right)\left(\frac{n_{1D}-1}{n_{1D}-1}\right) &= K_1\left(\frac{n_{1F}-n_{1C}}{n_{1D}-1}\right)\left(\frac{n_{1D}-1}{\lambda_F-\lambda_C}\right) \\ &= (n_{1D}-1)K_1\frac{1}{V_1(\lambda_F-\lambda_C)} \\ &= \frac{1}{f_{1D}}\frac{1}{V_1(\lambda_F-\lambda_C)} \end{aligned} \tag{5}$$

which means we can write our achromat condition as

$$\frac{1}{f_{1D}}\frac{1}{V_1} + \frac{1}{f_{2D}}\frac{1}{V_2} = 0 \tag{6}$$

Achromat III

We now have two equations and two unknowns:

$$\frac{1}{f_{1D}}\frac{1}{V_1}+\frac{1}{f_{2D}}\frac{1}{V_2}=0 \qquad \Rightarrow \qquad \frac{1}{f_{1D}}=-\frac{1}{f_{2D}}\frac{V_1}{V_2}$$

$$\frac{1}{f_D}=\frac{1}{f_{1D}}+\frac{1}{f_{2D}} \qquad \Rightarrow \qquad \frac{1}{f_{1D}}=\frac{1}{f_D}-\frac{1}{f_{2D}}$$

so we can combine them:

$$\begin{aligned}
-\frac{1}{f_{2D}}\frac{V_1}{V_2} &= \frac{1}{f_D}-\frac{1}{f_{2D}} \\
\frac{1}{f_D} &= \frac{1}{f_{2D}}\left(1-\frac{V_1}{V_2}\right) && (7) \\
\text{or} \qquad f_{2D} &= f_D\left(1-\frac{V_1}{V_2}\right) \qquad \text{or} \qquad \frac{1}{f_{2D}}=\frac{1}{f_D}\frac{V_2}{V_2-V_1} && (8)
\end{aligned}$$

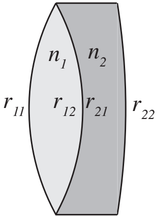
Achromat IV

We can then find $1/f_{1D}$ from

$$\begin{aligned} \frac{1}{f_{1D}} &= \frac{1}{f_D} - \frac{1}{f_{2D}} = \frac{1}{f_D} - \frac{1}{f_D}\frac{V_2}{V_2 - V_1} \\ &= \frac{1}{f_D}\left(\frac{V_2 - V_1 - V_2}{V_2 - V_1}\right) = \frac{1}{f_D}\frac{V_1}{V_2 - V_1} \end{aligned} \tag{9}$$

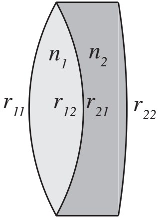
OK, so we have expressions for f_{1D} and f_{2D}. Can we determine the radii of curvature?

Let's chose $r_{11} = +|r_1|$, $r_{12} = -|r_1|$, and $r_{21} = -|r_1|$.



Achromat V

Again, we want $r_{11} = +|r_1|$, $r_{12} = -|r_1|$, and $r_{21} = -|r_1|$. This lets us find a condition for the focal length of the first lens:



$$\begin{aligned} \frac{1}{f_{1D}} &= \frac{1}{f_{1D}} \frac{V_1 - V_2}{V_1} = (n_{1D} - 1)\left(\frac{1}{r_{11}} - \frac{1}{r_{12}}\right) \frac{V_1 - V_2}{V_1} \\ &= (n_{1D} - 1) \frac{2}{|r_1|} \frac{V_1 - V_2}{V_1} \end{aligned} \tag{10}$$

so we want $V_1 > V_2$.

Achromat V

What about the second lens?

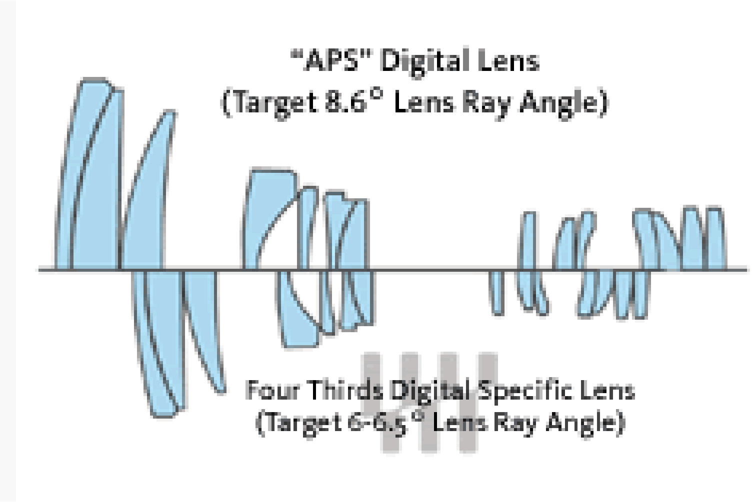
$$\frac{1}{f_{2D}} = (n_{2D}-1)\left(\frac{1}{r_{21}} - \frac{1}{r_{22}}\right) = (n_{2D}-1)\left(-\frac{1}{|r_1|} - \frac{1}{r_{22}}\right) = \frac{1}{f_D}\frac{V_2}{V_2 - V_1} \tag{11}$$

But $1/f_D = (n_{1D} - 1)(2/|r_1|)(V_1 - V_2)/V_1$ so

$$\begin{aligned}
-(n_{2D} - 1)\left(\frac{1}{|r_1|} + \frac{1}{r_{22}}\right) &= (n_{1D} - 1)\frac{2}{|r_1|}\frac{V_1 - V_2}{V_1}\frac{V_2}{V_2 - V_1} \\
&= -(n_{1D} - 1)\frac{2}{|r_1|}\frac{V_2}{V_1} \\
\frac{1}{|r_1|} + \frac{1}{r_{22}} &= \frac{(n_{1D} - 1)}{(n_{2D} - 1)}\frac{2}{|r_1|}\frac{V_2}{V_1} \\
\frac{1}{r_{22}} &= \frac{1}{|r_1|}\left[2\frac{(n_{1D} - 1)}{(n_{2D} - 1)}\frac{V_2}{V_1} - 1\right]
\end{aligned} \tag{12}$$

So we've done it! We've found that one can design a lens which has no chromatic aberrations at least for a narrow wavelength range about a single wavelength λ_D.

Complex lens design



This shows an Olympus zoom lens set for two different focal lengths. High quality zoom lenses have a very complicated design! Notice the achromat doublets in there?