

ESE 271

Third Exam

Name:

Spring, 2002

ID Number:

Do not place your answers on this front page.

Prob. 1:

Prob. 2:

Prob. 3:

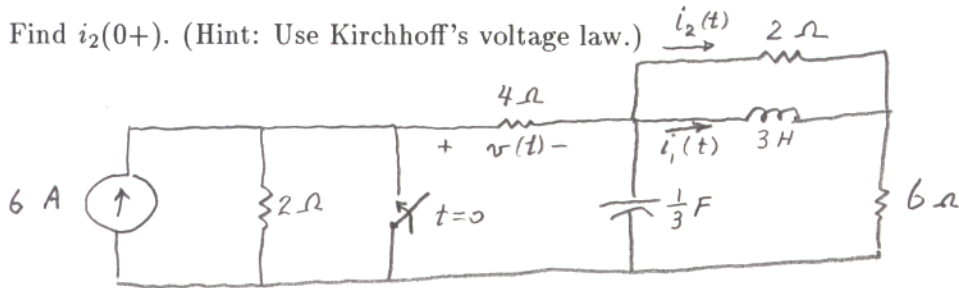
Prob. 4:

Prob. 1. (25 points):

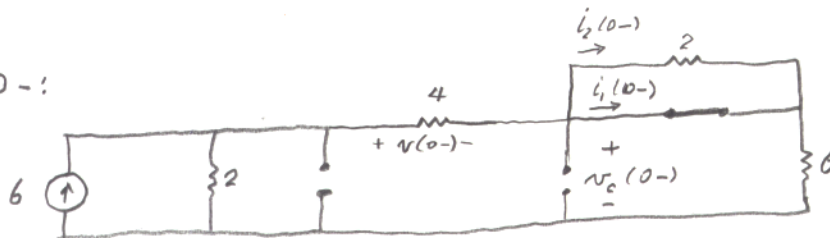
The network is in the DC steady state at $t = 0^-$.

(a) Find $v(0+)$ and $i_1(0+)$

(b) Find $i_2(0+)$. (Hint: Use Kirchhoff's voltage law.)



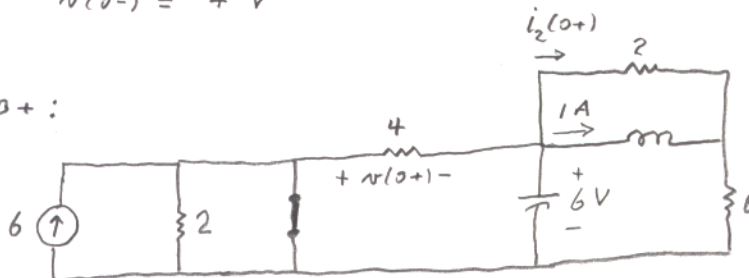
At $t = 0^-$:



$$i_1(0^-) = 6 \frac{2}{2+10} = 1 \text{ A}, \quad v_c(0^-) = 6 \text{ V}$$

$$v(0^-) = 4 \text{ V}$$

At $t = 0^+$:



$$i_1(0^+) = 1 \text{ A} \quad (\text{CURRENT IN INDUCTOR DOES NOT JUMP.})$$

$$v_c(0^+) = 6 \text{ V} \quad (\text{VOLTAGE ON CAPACITOR DOES NOT JUMP.})$$

By KVL:

$$6 = 2i_2(0^+) + (i_2(0^+) + 1)6$$

$$0 = 8i_2(0^+)$$

$$i_2(0^+) = 0 \text{ A}$$

$$v(0^+) = -6 \text{ V}$$

Prob. 2: (20 points):

Solve the following convolution equation to determine the Laplace transform $F(s)$ of $f(t)$ as a polynomial over a polynomial. The initial value of $f(t)$ is $f(0+) = 2$.

$$\frac{df}{dt} = \int_0^t f(t-\tau) e^{-3\tau} d\tau$$

$$sF(s) - 2 = F(s) \frac{1}{s+3}$$

$$F(s) \left(s - \frac{1}{s+3} \right) = 2$$

$$F(s) = \frac{2}{s - \frac{1}{s+3}} = \frac{2(s+3)}{s^2 + 3s - 1}$$

Prob. 4: (25 points):

Determine the function of time t that is the inverse Laplace transform of

$$\frac{s+4}{(s+1)(s+2)^3}$$

$$F(s) = \frac{A}{s+1} + \frac{B_0}{(s+2)^3} + \frac{B_1}{(s+2)^2} + \frac{B_2}{s+2}$$

$$A = \left. \frac{s+4}{(s+2)^3} \right|_{s=-1} = \frac{3}{1^3} = 3$$

$$B_0 = \left. \frac{s+4}{s+1} \right|_{s=-2} = -2$$

$$B_1 = \left. \frac{d}{ds} \frac{s+4}{s+1} \right|_{s=-2} = \left. \frac{s+1 - (s+4)}{(s+1)^2} \right|_{s=-2} = \left. \frac{-3}{(s+1)^2} \right|_{s=-2} = -3$$

$$B_2 = \left. \frac{1}{2} \frac{d^2}{ds^2} \frac{s+4}{s+1} \right|_{s=-2} = \left. \frac{1}{2} \frac{d}{ds} \left(\frac{-3}{(s+1)^2} \right) \right|_{s=-2} = \frac{1}{2} (-3)(-2) \left. \frac{1}{(s+1)^3} \right|_{s=-2} = -3$$

So,

$$f(t) = 3e^{-t} - 2 \frac{t^2}{2!} e^{-2t} - 3te^{-2t} - 3e^{-2t}$$

$$= 3e^{-t} - t^2 e^{-2t} - 3te^{-2t} - 3e^{-2t} \quad \text{For } t > 0$$