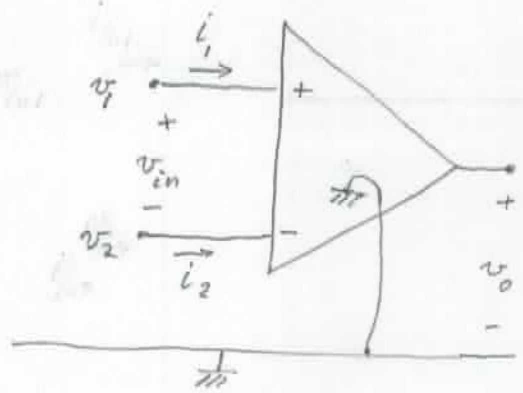
Operational Amplifiers – The DC Case

Operational Amplifiers (called OP-AMPS) are specialized amplifiers having extremely high voltage gains (typically, 10^5) and very large input resistance (perhaps $10^6\,\Omega$ when the other resistors connected to the op-amp have values in the 10^3 range).

The circuit symbol and its relation to the ground node is:

FIG. 1:

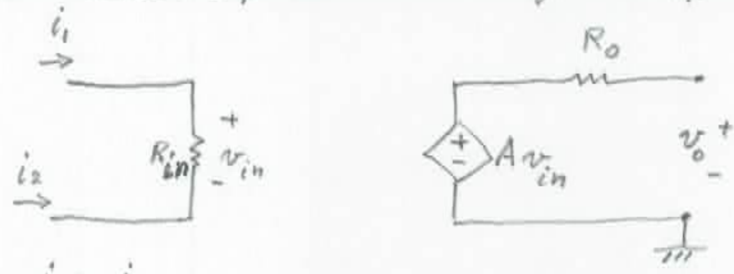


There is a ground connection within the op amp, as indicated

v_1 and v_2 are node voltages measured with respect to ground.

Thus, $v_{in} = v_1 - v_2$

Case 1: A quite accurate equivalent circuit for the op amp is:



Hence, $i_1 = -i_2$

Possible parameter values are:

$R_{in} = 1\ M\Omega = 10^6\ \Omega$

$R_o = 30\ \Omega$

$A = 10^5$

(OR)

$R_{in} = 10^9\ \Omega$

$R_o = 300\ \Omega$

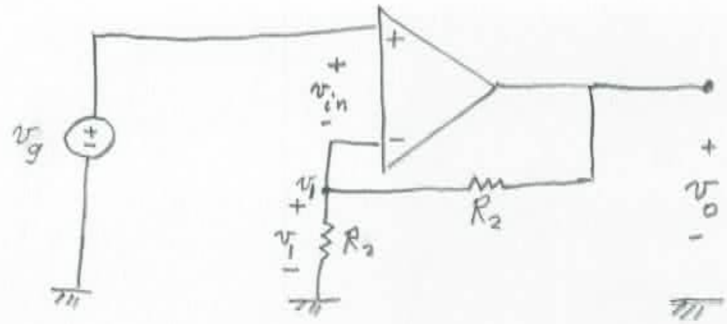
$A = 10^5$

Case 2: An ideal case arises when we set $R_{in} = \infty$, $R_o = 0$, $A = 10^5$

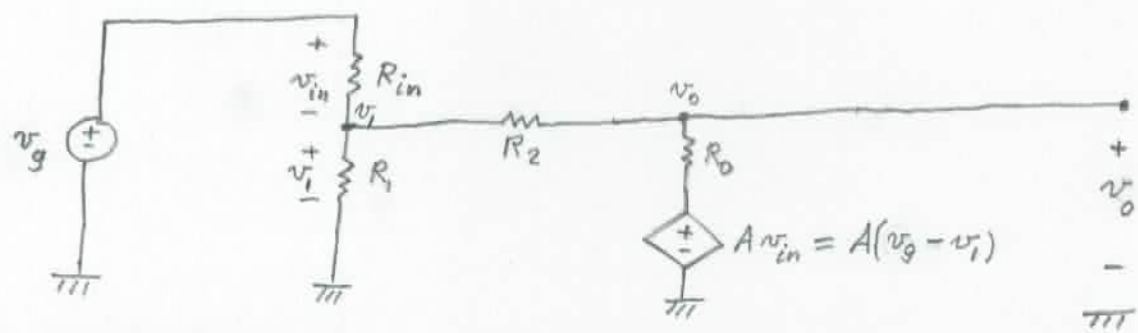
Case 3: A still more ideal case (called the virtual-short virtual-open model) arises when $v_{in} = 0$ and $i_1 = i_2 = 0$ and v_o is determined by the circuit connected around the op amp.

Example for Case 1:

Let us first consider a circuit where the op amp has the equivalent circuit of Case 1:



Upon replacing the op-amp by its equivalent circuit (Case 1), we get



Case 1:

KCL at the v_1 node:

$$\text{①} \quad \frac{v_1 - v_g}{R_{in}} + \frac{v_1}{R_1} + \frac{v_1 - v_o}{R_2} = 0$$

KCL at the v_o node:

$$\text{②} \quad \frac{v_o - v_1}{R_2} + \frac{v_o - Av_g + Av_1}{R_o} = 0$$

v_1 and v_o are the only two unknowns in these two equations. So, we can solve for v_o to get v_o in terms of v_g and the resistors and A.

Case 2: We will solve for v_o assuming $R_{in} = \infty$ and $R_o = 0$

(actually, R_{in} is very big and R_o is very small compared to other resistors.)

From ①: $0 + v_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v_o}{R_2}$. Thus, $v_o = v_1\left(1 + \frac{R_2}{R_1}\right)$ ← ③

From ②: Since $\frac{1}{R_o}$ is much bigger than $\frac{1}{R_2}$, we can neglect $\frac{v_o - v_1}{R_2}$ in ②.

Thus, $v_o = A\underbrace{(v_g - v_1)}_{v_{in}}$. So, $v_1 = v_g - \frac{v_o}{A}$ ← ④

Now, put ④ into ③: We get $v_o = \left(v_g - \frac{v_o}{A}\right)\left(1 + \frac{R_2}{R_1}\right)$

Solve for v_o. See next page.

$$\text{So, } v_o = v_g \frac{\left(1+\frac{R_2}{R_1}\right)}{1+\frac{1+\frac{R_2}{R_1}}{A}}$$

This is v_o in Case 2. (The ideal op amp.)

Case 3: The virtual-short virtual open case: $A = \infty$.

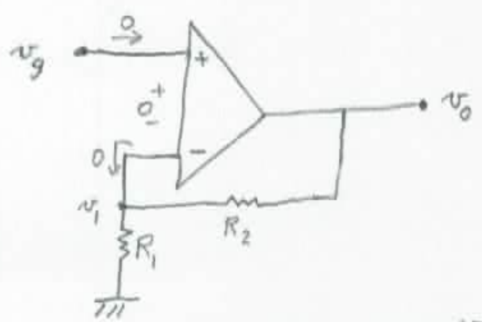
(5) $$\text{So, } v_o = v_g\left(1+\frac{R_2}{R_1}\right)$$

This same result occurs when we take $i_1 = i_2 = v_{in} = 0$ in Fig. 1 (on page OP AMP 1).

$v_{in} = 0$ is the "virtual short" condition.

$i_1 = i_2$ is the "virtual" open condition

We can get (5) directly as follows:



So by virtual-short condition, $v_1 = v_g$.

By virtual-open condition, R_1 and R_2 are effectively in series.

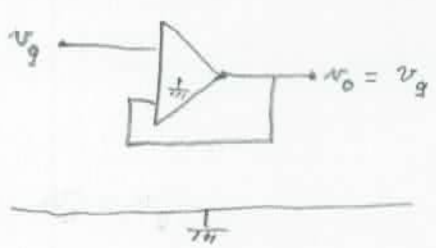
So, by the voltage-divider rule,

$$v_g = v_1 = v_o \frac{R_1}{R_1 + R_2}. \quad \text{Therefore, } v_o = v_g\left(1+\frac{R_2}{R_1}\right).$$

DONE.

The voltage follower: Also called the unity gain amplifier:

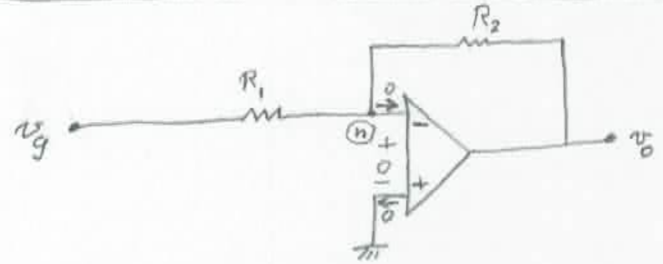
This occurs in Case 3 when we set $R_1 = \infty$ and $R_2 = 0$.



We will now analyze some more $\underbrace{\text{input - output equations}}$

(i.e., "transfer functions")

for other circuits containing the virtual-short virtual-open model of the OP AMP.



Note: The "feedback" is to the negative terminal of the OP AMP

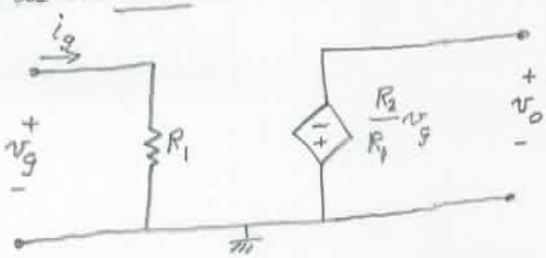
at node (n), the node voltage is 0.

So, by KCL at (n): $\dfrac{v_g - 0}{R_1} + \dfrac{v_o - 0}{R_2} + 0 = 0$ (current into OP AMP = 0)

So $\underline{\underline{v_o = -\dfrac{R_2}{R_1} v_g}}$ (The "transfer function" is $\dfrac{v_o}{v_g} = -\dfrac{R_2}{R_1}$)

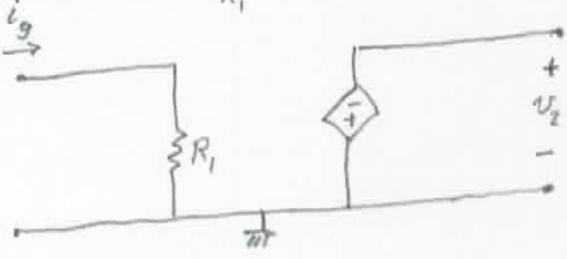
This is called an "INVERTER"

It is a $\underline{\text{VCVS}}$ and has the equivalent circuit:



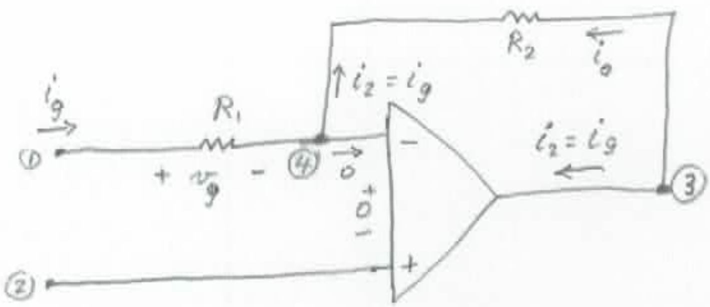
$+ \; v_o \; - = -\dfrac{R_2}{R_1} v_g$ (μ = voltage gain $= -\dfrac{R_2}{R_1}$)

If we use $i_g = \dfrac{v_g}{R_1}$ as the input variable, we get a $\underline{\text{VCCS}}$:



$+ \; v_2 \; - = - R_2 i_1$ (ρ = transresistance $= - R_2$)

Here is a special way of getting a CCCS:

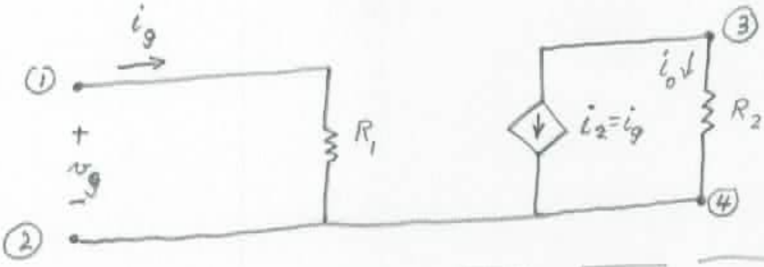


$$i_2 = i_g = \frac{v_g}{R_1}$$

The equivalent circuit is:

$$i_o = -i_2 = -i_g$$

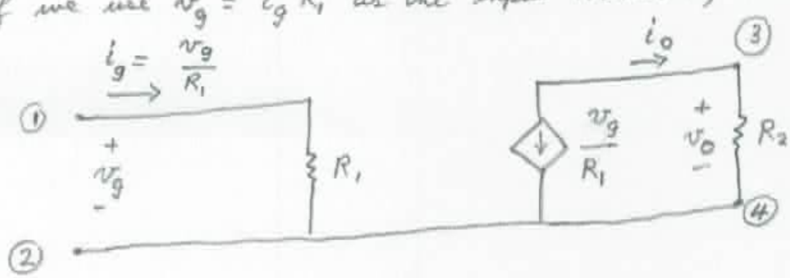
$$\left(\begin{aligned} \beta &= \text{current gain} \\ &= -1 \end{aligned} \right)$$



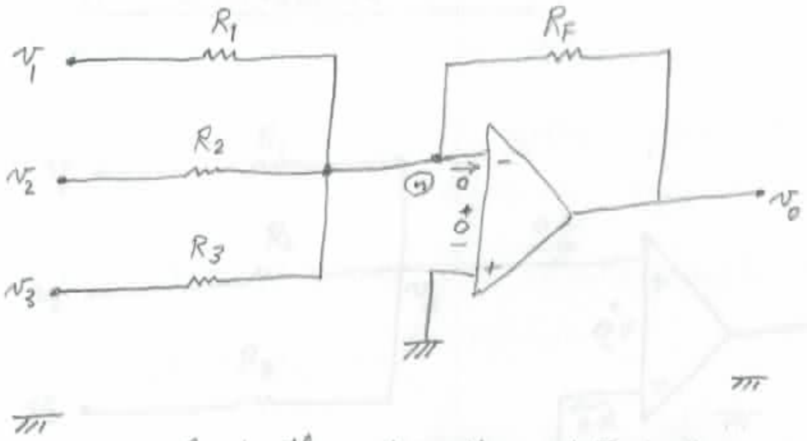
If we use $v_g = i_g R_1$ as the input variable, we get a VCCS:

$$i_o = -\frac{v_g}{R_1}$$

$$\left(\begin{aligned} g &= \text{transconductance} \\ &= -\frac{1}{R_1} \end{aligned} \right)$$



AN INVERTING SUMMER



Again, the node voltage at n is 0.

By KCL at node n:

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \frac{v_o}{R_F} = 0$$

So,
$$v_o = -\frac{R_F}{R_1} v_1 - \frac{R_F}{R_2} v_2 - \frac{R_F}{R_3} v_3$$

If we choose $R_F = R_1 = R_2 = R_3$, we get

$$v_o = -v_1 - v_2 - v_3$$