ESE 271 Final Exam Name:

Spring, 2002 ID Number:

Do not place your answers on this front page.

Prob. 1 (15 points):

Prob. 2 (25 points):

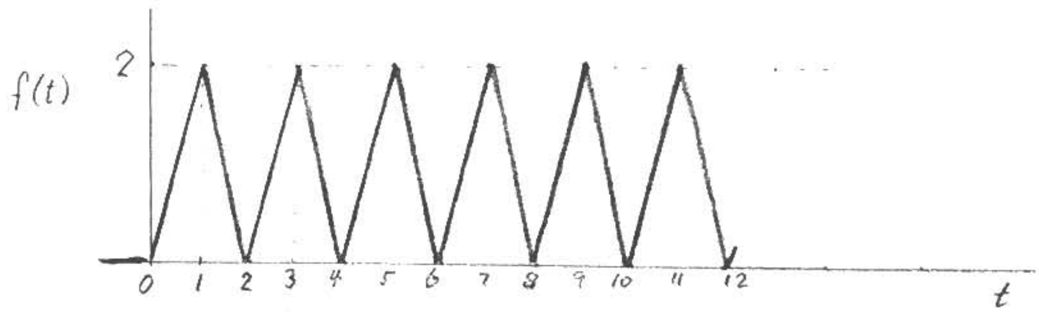
Prob. 3 (30 points):

Prob. 4 (30 points):

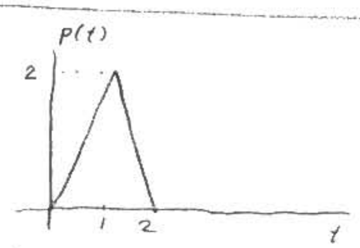
Prob. 1. (15 points):

(a) Find the Laplace transform of the single triangular pulse:

(b) Find the Laplace transform of the pulse train, which continues infinitely to the right.



(a) TO FIRST GET THE TRANSFORM OF



$$p(t) = 2t\,u(t) - 4(t-1)\,u(t-1) + 2(t-2)\,u(t-2)$$

$$P(s) = \frac{2}{s^2} - \frac{4}{s^2}e^{-s} + \frac{2}{s^2}e^{-2s}$$

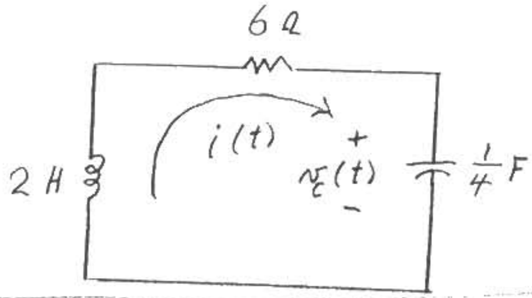
(b) THEN,

$$F(s) = P(s)\left(1 + e^{-2s} + e^{-4s} + e^{-6s} + \cdots\right)$$

$$= \frac{P(s)}{1 - e^{-2s}}$$

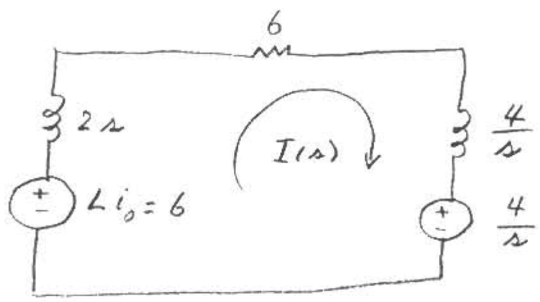
Prob. 2: (25 points):

Find the time function $i(t)$ by using the Laplace-transformed circuit with the initial-condition sources in Thevenin form.



$i(0+) = 3\ A$

$v_c(0+) = 4\ V$



$$I(s) = \frac{6 - \frac{4}{s}}{2s + 6 + \frac{4}{s}} = \frac{6s - 4}{2s^2 + 6s + 4} = \frac{6s-4}{2(s+1)(s+2)}$$

$$\left.\begin{matrix} p_1 \\ p_2 \end{matrix}\right\} = \frac{-6 \pm \sqrt{36 - 32}}{4} = \frac{-6 \pm 2}{4} = -1,\ -2$$

$$I(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \frac{-6-4}{2 \times 1} = -5$$

$$B = \frac{-12-4}{2(-1)} = 8$$

$$i(t) = -5e^{-t} + 8e^{-2t} \quad \text{FOR } t > 0.$$

Prob. 3: (30 points):

Let $F(s)$ be the Laplace transform of $f(t)$, where

$$F(s) = \frac{18s^3 + 1}{(3s^2+3)^2}$$

(a) Find $f(0+)$.

(b) Find $f^{(1)}(0+)$. ($f^{(1)}$ denotes the first derivative of $f(t)$.)

(c) Determine $f(\infty)$ as a single number, if possible. If not possible, state why it is not possible.

(a)

$$F(s) = \frac{18s^3+1}{9(s^2+1)^2} = \frac{18s^3+1}{9(s^4+2s^2+1)}$$

$$f(0+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{18s^4+s}{9(s^4+2s^2+1)} = 2$$

(b)

$$f^{(1)}(0+) = \lim_{s\to\infty} s\left(sF(s) - f(0+)\right)$$

$$= \lim_{s\to\infty} s\left(\frac{18s^4+s}{9s^4+18s^2+9} - 2\right)$$

$$= \lim_{s\to\infty} s\left(\frac{\cancel{18s^4}+s-\cancel{18s^4}-36s^2-18}{9s^4+18s^2+9}\right)$$

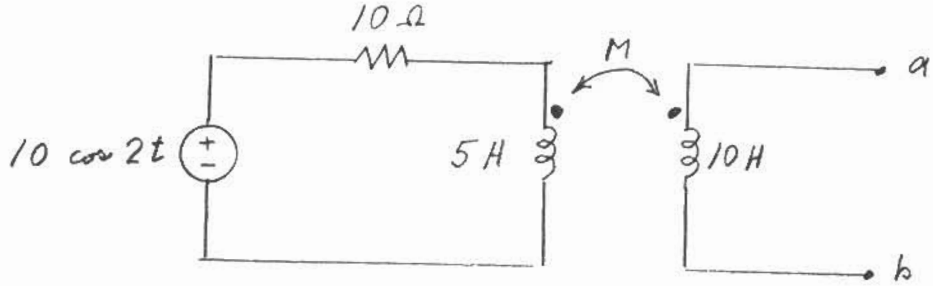
$$= 0$$

(c) ROOTS OF DENOMINATOR ARE AT $s = \pm j$ (THEY ARE DOUBLE ROOTS)

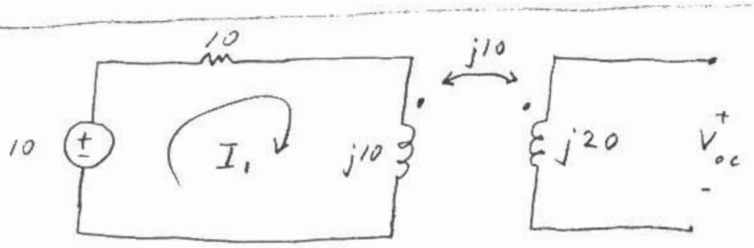
THUS, THE FINAL-VALUE THEOREM DOES <u>NOT</u> HOLD.

Prob. 4: (30 points):

Using AC steady state analysis, draw Thevenin's equivalent circuit as seen from the terminals a and b. Determine Thevenin's impedance Z_{TH} and open-circuit voltage V_{oc} between terminals a and b as complex numbers in polar form.



$M = 5\ H$

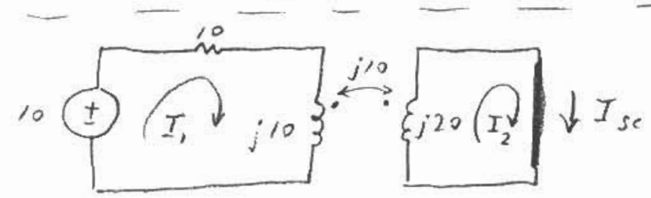


To GET Voc:

$$I_1 = \frac{10}{10+j10} = \frac{1}{1+j} = \frac{1}{\sqrt{2}}\angle -45^\circ$$

$$V_{oc} = j10\, I_1 = \frac{10}{\sqrt{2}}\angle 45^\circ = 7.071\angle 45^\circ$$

To GET I_{sc}:



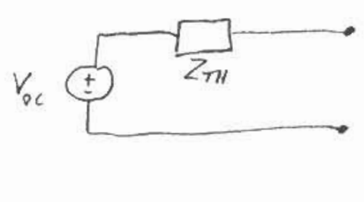
1st LOOP $\quad I_1(10+j10) - j10\, I_2 = 10$

2nd LOOP $\quad -j10\, I_1 + j20\, I_2 = 0 \implies I_1 = 2\, I_2$

So, $\quad 2 I_2 (10+j10) - j10\, I_2 = 10$

$$I_2 = \frac{10}{20+j10} = \frac{1}{2+j} = \frac{1}{\sqrt{5}\angle 26.57^\circ} = .4472\angle -26.57 = I_{sc}$$

THUS, $\quad Z_{TH} = \dfrac{V_{oc}}{I_{sc}} = \dfrac{7.071\angle 45^\circ}{.4472\angle -26.57^\circ} = 15.81\angle 71.57^\circ$



← THEVENIN'S CIRCUIT.