Quiz + a bit extra. Excited sodium atoms at rest emmit yellow light.

- (numerical) Estimate the wavelength of this light, λ_o. Estimate a typical room-temperature speed v of an atom.
- (analytic) When the atom is moving, the wavelength measured by a stationary observer is not the same – it is "doppler" shifted. Suppose that an atom is moving away from you with speed v . Determine the shifted wavelength, λ , in terms of v and λ_o
- (analytic) Determine a power series expansion in v/c for the for λ to first order v/c.
- Substitute the the numbers estimated in part (a) and estimate the percent shift $\Delta\lambda/\lambda$.

Solution

- The wavelength is inbetween blue and red, but is closer to red (red, yellow, green, blue is the order). $\lambda \sim 550\,\mathrm{nm}$. But I would take anything in the visible. The speed is of order the sound speed, $v \sim v_{\mathrm{sound}} \sim 300\,\mathrm{m/s}$
- We have since the atom is moving away:

$$f = f_o\sqrt{\frac{1 - v/c}{1 + v/c}} \tag{1}$$

So

$$\begin{aligned}\lambda = \frac{c}{f} =& \frac{c}{f_o}\sqrt{\frac{1 + v/c}{1 - v/c}} && (2)\\ =& \lambda_o\sqrt{\frac{1 + v/c}{1 - v/c}} && (3)\\ & && (4)\end{aligned}$$

- Then expanding

$$\begin{aligned}\lambda =& \lambda_o\sqrt{1+\beta}\frac{1}{\sqrt{1-\beta}} && (5)\\ \simeq& \lambda_o(1 + \frac{1}{2}\beta + O(\beta^2))(1 - \frac{1}{2}(-\beta) + O(\beta^2)) && (6)\\ \lambda \simeq& \lambda_o(1 + \beta + O(\beta^2)) && (7)\end{aligned}$$

- So

$$\frac{(\lambda - \lambda_o)}{\lambda_o} \simeq \beta \simeq \frac{300\,\mathrm{m/s}}{3\,\times 10^8\mathrm{m/s}} \tag{8}$$

Two other ways to arrive at Eq. (7). Based on errors in class.

1. Expand like this

$$\lambda = \lambda_o \sqrt{1+\beta}\frac{1}{\sqrt{1-\beta}} \tag{9}$$

$$\simeq \lambda_o \frac{1+\frac{1}{2}\beta + O(\beta^2)}{1-\frac{1}{2}\beta + O(\beta^2)} \tag{10}$$

   The $O(\beta^2$ indicates an error of order β^2. This is not a power series in v/c. So Keep going

$$\frac{1}{1-\frac{1}{2}\beta + O(\beta^2)} \simeq 1 + \frac{1}{2}\beta + O(\beta^2) \tag{11}$$

   where we use $1/(1-x) \simeq 1 + x + x^2 + \ldots$. So

$$\lambda = \lambda_o \frac{1+\frac{1}{2}\beta + O(\beta^2)}{1-\frac{1}{2}\beta + O(\beta^2)} \tag{12}$$

$$= \lambda_o (1 + \frac{1}{2}\beta + O(\beta^2))(1 + \frac{1}{2}\beta + O(\beta^2)) \tag{13}$$

   Ther rest is the same.

2. Or use the general expression, though this is arguably more difficult. At linear order we have

$$f(\beta) = f(0) + f'(0)\beta + O(\beta^2) \tag{14}$$

   where

$$f(x) = \sqrt{\frac{1+x}{1-x}} \tag{15}$$

   Then

$$f(0) = 1 \tag{16}$$

   and differentiating using the chane rule

$$f'(x) = \frac{1}{(1+x)^{1/2}(1-x)^{3/2}} \tag{17}$$

   so

$$f'(0) = 1 \tag{18}$$

   Finally

$$f(\beta) = 1 + \beta + O(\beta^2) \tag{19}$$

3. Or use maple or mathematica, but not on test: `series(f(`β`),` β`)`

$$f(\beta) = 1 + \beta + \frac{1}{2}\beta^2 + \frac{1}{2}\beta^3 + \frac{3}{8}\beta^4 + \frac{3}{8}\beta^5 + O\left(\beta^6\right) \tag{20}$$