Quantity	**Symbol**	**Value**
Coulombs Constant	$\frac{1}{4\pi\epsilon_o}$	$8.98 \times 10^9\ \mathrm{Nm^2/C^2}$
Electron Mass	m_e	9.1×10^{-31} kg
Proton Mass	m_p	1.67×10^{-27}kg
Electron Charge	e	-1.6×10^{-19} C
Electron Volt	eV	1.6×10^{-19} J
Permitivity	ϵ_o	$8.85 \times 10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{Nm^2}}$
Magnetic Permeability	μ_o	$4\pi \times 10^{-7}\ \mathrm{N \cdot A^2}$
Speed of Light	c	3.0×10^8 m/s
Planck's Constant	h	$6.6 \times 10^{-34}\mathrm{m^2kg/s}$
Planck's Constant/2π	$\hbar$	$1.05 \times 10^{-34}\mathrm{m^2kg/s}$

Integrals	**Value**	
$\int_{-\infty}^{\infty} du\, e^{-\alpha u^2}$	$\sqrt{\frac{\pi}{\alpha}}$	
$\int_{-\infty}^{\infty} du\, u^2 e^{-\alpha u^2}$	$\frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}}$	
$\int_{0}^{\infty} du\, u^n e^{-\alpha u}$	$\frac{n!}{\alpha^{n+1}}$	
$\int du\ \sin^2(\alpha u)$	$\frac{u}{2} - \frac{\sin(2\alpha u)}{4\alpha}$	
$\int du\ \cos^2(\alpha u)$	$\frac{u}{2} + \frac{\sin(2\alpha u)}{4\alpha}$	
$\int_{-\frac{1}{2}}^{+\frac{1}{2}}\ du u^2 \sin^2(n\pi u)$	$\frac{-6+n^2\pi^2}{24n^2\pi^2}$	$n = 2, 4, 6, 8$
$\int_{-\frac{1}{2}}^{+\frac{1}{2}}\ du u^2 \cos^2(n\pi u)$	$\frac{-6+n^2\pi^2}{24n^2\pi^2}$	$n = 1, 3, 5, 7$
$\int (\cos(\theta))^\alpha \sin(\theta) d\theta$	$\frac{-1}{\alpha+1}\ (\cos(\theta))^{\alpha+1}$	
$\int (\sin(\theta))^\alpha \cos(\theta) d\theta$	$\frac{+1}{\alpha+1}\ (\sin(\theta))^{\alpha+1}$	

n	ℓ	m	$\Phi_m(\varphi)$	$\Theta_{lm}(\theta)$	$R_{nl}(r)$	Ψ_{nlm}
1	0	0	1	1	$\frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o}$	$\frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o}$
2	0	0	1	1	$\frac{1}{\sqrt{32\pi a_o^3}} \left(2 - \frac{r}{a_o}\right) e^{-r/2a_o}$	$\frac{1}{\sqrt{32\pi a_o^3}} \left(2 - \frac{r}{a_o}\right) e^{-r/2a_o}$
2	1	0	1	$\sqrt{3}\cos(\theta)$	$\frac{1}{\sqrt{96\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o}$	$\frac{1}{\sqrt{32\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o} \cos(\theta)$
2	1	± 1	$e^{\pm i\varphi}$	$\sqrt{\frac{3}{2}}\sin(\theta)$	$\frac{1}{\sqrt{96\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o}$	$\frac{1}{\sqrt{64\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o} \sin(\theta) e^{\pm i\varphi}$

For potential $V = \frac{1}{2}\, k\, x^2$ the lowest wave functions and energies

$$\begin{aligned}
\Psi_0 &= \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} e^{-y^2/2} \\
\Psi_1 &= \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} \sqrt{2} y e^{-y^2/2} \\
\Psi_2 &= \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} \frac{1}{\sqrt{2}} (2y^2 - 1) e^{-y^2/2}
\end{aligned}$$

where

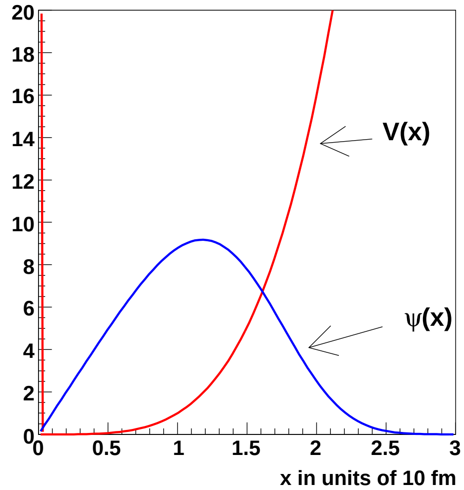
$$y \equiv \frac{x}{L} \qquad L = \sqrt{\frac{\hbar}{M\omega_o}} \tag{1}$$

1. State an order of magnitude size for the following objects:
   (a) The hydrogen atom.
   (b) The compton wavelength of the electron.
   (c) The nucleus.
   (d) The wavelenth of. light emitted when a hydrogen atom decays from a $2p$ state to a $1s$ state.
2. Draw a schematic figure (or figures if it helps to explain things) showing rougly the relative sizes of these things. Not art, just enough to convince me that you understand what the numbers mean. You may have to use words to explain your figure or figures
3. Estimate or state how energetic a photon would need to be before it does the following:
   (a) Excite a hydrogen atom from its ground to an excited state
   (b) Knock an electron sufficiently hard to take it from rest to 30% of the speed of light.

Consider making a simple model of an atom.

- Estimate the size an atom
- Suppose we place an electron in a (1 Dimensional) box with which has a Length L exactly equal to the size you estimated in part (a). Determine the energy of the photon emitted when the electron in the box decays from its first excited state down to its ground state. Work symbolically and then substitute numbers.
- Make a sketch of the ground state and first and second excited state wave functions of this electron in the box. Also make a sketch of the associated probability density $P(x)$ for the ground, first, and second excited states. (There is a total of six graphs in this problem).
- Show that the second excited state obeys the Schrödinger equation and determine the corresponding energy. (Work analytically numbers not necessary)
- Consider the electron in its first excited state. Lets agree to place the left hand side of the box at $-L/2$ Determine the probability that an electron in the first excited state is between $-L/4$ and $L/4$.
- Estimate v/c for the electron in the box. (Work analytically and then substitute numbers)

A schematic plot of a stationary state wave function of a proton inside a nucleus is shown below together with the corresponding potential $V(x)$. The units on the y axis are abitrary since the wave functions are not normalized. The units on the x axis are: $1\,\text{unit} = 10\,\text{Fm}$.



a. Make an order or magnitude estimate for the Kinetic, Potential , and total energies based on this figure. Is this the ground, first, second, ... state of the potential? Explain.

b. Treat the nucleus as 1D box of size L, determine the L which would give the same energy as you estimated in part a.

c. Suppose the proton were in this 1D box and decayed from the first excited state down to the ground state, what would the energy be of the emitted photon.

Consider the vibrations of a hydrogen nucleus in a harmonic potential $V(x) = \frac{1}{2}kx^2$. We previously estimated in a homework problem that the size of the ground state wave function is $L = \left(\frac{\hbar^2}{Mk}\right)^{1/4}$. For arguement sake Lets take a spring constant of $k = 13.6\,\mathrm{eV}/\mathring{A}^2$

1. What's the mass M ?
2. Show that the following wave function for the electron is a solution to the time independent Schrödinger equation and determine the energy

$$\psi(x) = Ae^{-u^2/2} \qquad \text{with} \qquad u \equiv \frac{x}{L} \tag{2}$$

3. Make a graph of the probability distribution associated with this wave function. Be sure to indicate units on the x-axis. The units are the most important part of this – see the figures in problem #2 and #5 to if you don't know what I'm talking about.
4. Compute the normalziation constant A.
5. Compute the following quantities.
   (a) $\overline{x}, \overline{p}, \overline{x^2}, \overline{p^2}$
6. Determine the standard deviation Δx and Δp for momentum and position. Compute the product $\Delta x \Delta p$ and comment on the result.

Let us now address the electronic structure of the atom of ${}^{27}_{13}\text{Al}$. Consider the electrons orbiting around the nucleus this (If you don't know how many protons and electrons there are ask me).

1. Write down the electronic structure of this atom.
2. The electrons in the last unfilled shell are known as valence electrons. Determine the total angular momentum squared of the valence electrons and list the possible values for the average angular momentum in the z direction of the valence electrons.

Now strip away all of the electrons from Aluminum nucleus except for one, and consider the orbit of this single remaining electron circling the nucleus using the Bohr Model. For the symbolic parts of this problem refer to the number of protons as Z and label the discrete orbits by $n = 1, 2, 3, \ldots$ express your results in terms of fundamental constant and if you wish the Bhor radius.

3. (Symbol) Starting with Newton's Law and the Bohr Quantization condition determine the allowed orbital radii for this electron in terms of fundamental constants n and Z
4. (Symbol) Determine the kinetic energy of the electron in the n^{th} orbit.
5. (Symbol) Determine the potential *and* total energies of the n^{th} orbit.
4. (Symbol) Determine the momentum and de Broglie wavelength of the electron in the n^{th} Bohr orbit
5. (Symbol) What is the ratio between the circumfrence of the n^{th} orbit and the de Broglie wavelength of that orbit. Interpret the result.

Consider an electron orbiting a proton forming a hydrogen atom.

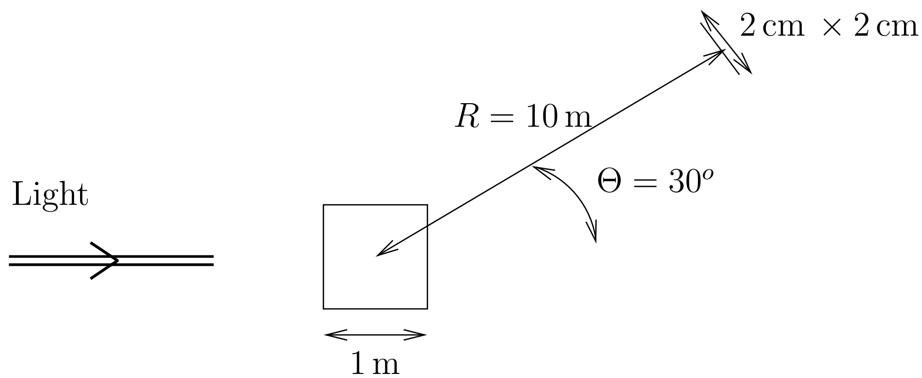
1. Consider an electron in the $3d$ state of the coulomb potential. Determine the energies that would be emitted as the electron decays down to its ground state. Draw an energy level diagram and show the possible transitions.
2. List all of the states and their quantum numbers (i.e. nlm and spin) that are degenerate with the $3d$ state
3. Draw a graph of the $1s$, $2s$ and $2p$ radial wave functions and the associated radial probability distributions. Sketch the effective potential for these wave functions.
4. Determine the most probable radius of the $2p$ orbital. Indicate this radius on the appropriate graph in part 3.
5. Determine the variance in the radius for the $2p$ state. Indicate roughly what this corresponds to in your graph of part 3.
6. Show that the $2p$ orbital satisfies the radial Schrödinger equation and determine the associated energy. (You will need to differentiate carefully to get this to work – check your work and write as neatly as you can to get this right!)

When light travels through the atmosphere it scatters off the molecules in the air in different directions. The process is known as Rayleigh scattering and explains why the sky is blue at mid-day and red at sunset. When N_γ photons pass through a slab of air of thickness t with ρ molecules per volume, the number of photons scattered per solid angle is

$$\frac{dN_{\mathrm{scatt}}}{d\Omega} = [N_\gamma \rho_{\mathrm{air}} t] \; A \left(\frac{450\,\mathrm{nm}}{\lambda}\right)^4 (1+\cos^2\Theta) \qquad A = 0.3\times 10^{-31} m^2 \tag{3}$$

Here A has the dimension of area and λ is the wavelength of the light. The number scattered is wavelength dependent and therefore short wavelengths (such as blue $\lambda = 450\,\mathrm{nm}$) scatter more than long wavelengths (such as red $\lambda = 625\,\mathrm{nm}$). We wish to calculate how far the light will travel in our atmosphere before scattering off the molecules in the air.

To estimate this, imagine you exist in a vacuum, and are scattering a beam of light off a small glass cell consisting of one meter of air as shown below



1. Using $pV = nRT$ we first estimate the number of molecules per unit volume for the air cell. The gas which may be treated as an ideal gas at a temperature of $293^o K$ (room temperature) and pressure of one atmosphere (1 atm=$10^5 N/m^2$). The ideal gas constant is $R = 8.32\, J/(\mathrm{mol}^o K)$. What's ρ_{air} ?

2. Work with blue light. As the light traverses the cell it scatters according to formula given above. For argument sake assume that there are a total of $N_\gamma = 10^{20}$ photons in the light source. You place a photodector a distance of 10 m from the cell at an angle of thirty degrees. The area of of the collecting tube of the photodector is 2cm × 2cm square. Determine the number of photons which are collected by the photodector as the beam of photons passes through the cell.

3. Determine the number of photons which scatter as they traverse the cell.

4. What fraction of the blue photons are scattered per meter of air? What fraction of the red photons are scattered per meter of air?

Since the molecules scatter blue light more effectively than red, the strength of the blue light is greatly reduced on its way to reaching our eyes during sunset. Only the red light from the sun travels and reach our eyes during these scenic moments.

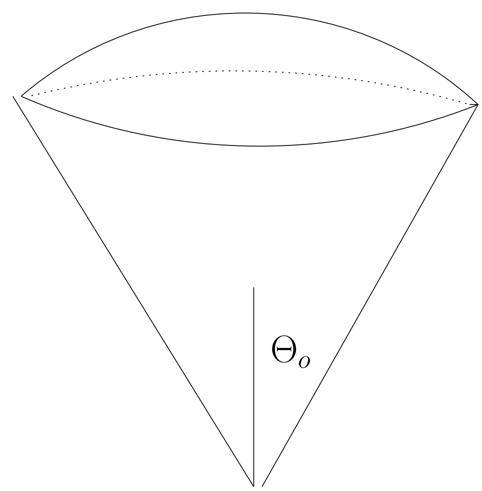
Consider the angular distribution of the $2p$ states of hydrogen

1. Determine the probability that a $m = 0$ electron will be within $\Theta_o = 45^o$ of the azimuth as shown below

2. Determine the probability that a $m = 1$ electron will be within $\Theta_o = 45^o$ of the azimuth as shown below.

3. Explain the differences between your results by sketching the angular probability distribution P_Ω either as a regular graph or as a polar plot.

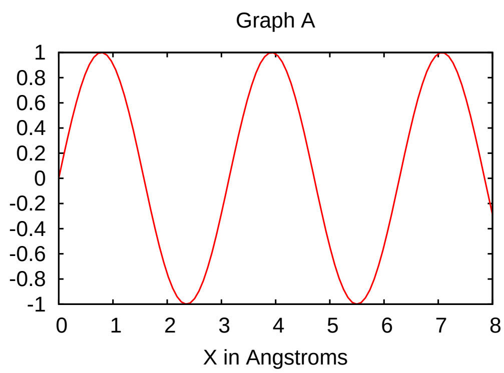
4. Using the wave functions explain why six $2p$ electrons make a “closed shell”.

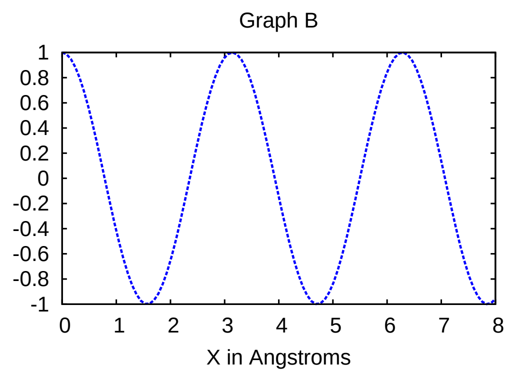


The wave function of an electron moving freely without external forces (so its potential energy is zero) is described by the wave function

$$\Psi(x,t) = Ae^{+ikx}e^{-i\omega t}$$

where A is a real constant. At time $t = 0$ the real and imaginary imaginary parts of this function are shown below.





1. Which figure corresponds to the real part and which figure corresponds to the imaginary part. Explain your reasoning.

2. Determine numerical values of the momentum in eV/c and the wave number k in $\AA^{-1}$ for the wave function given in the function above.

3. Make a sketch of the probability distribution associated with this wave function $\Psi(x,t)$.

4. Show that it is a solution to the time dependent Schrödinger equation and determine the energy.

Now consider a super position of two waves. At time $t = 0$ the wave function is

$$\Psi(x,0) = Ae^{+ik_1 x} + Ae^{+ik_2 x}$$

with $k_1 = 1.0\,\AA^{-1}$ and $k_2 = 1.1\,\AA^{-1}$.

6. What is the imaginary part of this function.

7. Make a sketch of the imaginary part of this function All wavelengths visible in your graph should be clearly labeled and below the graph you should tell what are the numerical values of these wavelengths in angstroms. *Neatness counts and messy graphs will cause negative points.*