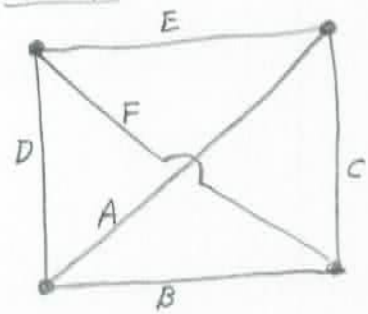
Mesh Analysis for Planar Networks

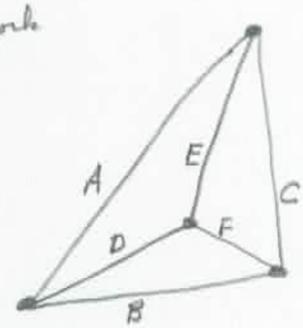
A planar network is a network that can be drawn in a plane in such a fashion that the branches touch only at nodes. (That is, the branches do not jump over each other.)
This is called a "planar embedding"

Example

Same network



This is not a planar embedding



This is a planar embedding

(There may be many planar embeddings for the same network.)

In mesh analysis we will choose a particular planar embedding, and then will analyze based on that.

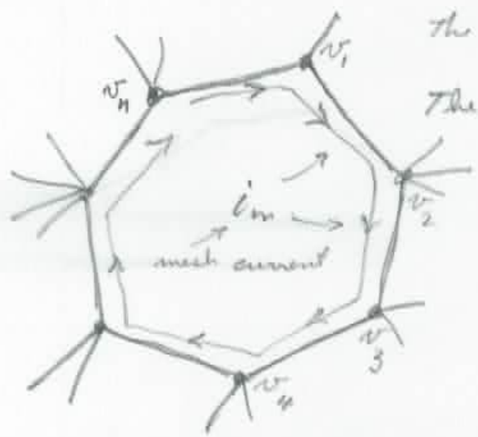
There are networks that do not have any planar embeddings.
We will not do a "mesh analysis" for those.
(A nodal analysis will always work for those as well.)
(There are analyses that generalize mesh analyses and work for those.)

In nodal analysis, KVL was automatically satisfied, and we applied KCL at the nodes.

In mesh analysis, KCL is automatically satisfied, and we apply KVL around the "meshes".

– So, what is a "mesh"?

A <u>mesh</u> is a tracing of branches bounding an area of a planar embedding with the area having no branches within it. For example:



The v_k are <u>node</u> voltages.
The mesh current is i_m.

No branches inside

Let's add <u>branch</u> voltage drops around the mesh. We get

$$\cancel{v_1} - \cancel{v_2} + \cancel{v_2} - \cancel{v_3} + \cancel{v_3} - v_4 + \cdots + v_n - \cancel{v_1} = 0$$

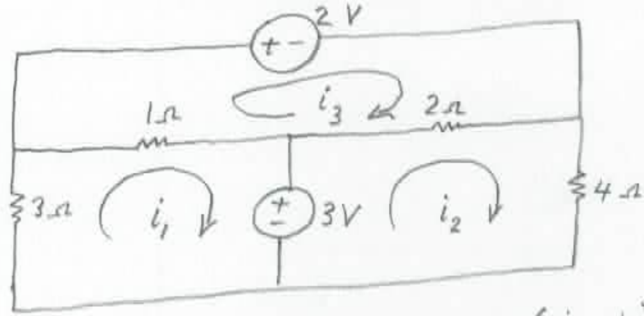
This equals 0 because all the node voltages cancel out.

Thus, in a mesh analysis, we apply KVL around every mesh, using mesh currents as the unknowns.

<u>Important note</u>: At each node, KCL is <u>automatically</u> satisfied by the mesh currents, because each mesh current enters and leaves each node through which it goes:

So we only need to apply KVL.

Example. In this example, we only have resistors and independent voltage sources. (The simplest case.)



KVL around i_1 mesh: $3i_1 + 1(i_1 - i_3) + 3 = 0$

$$\boxed{4i_1 - i_3 = -3}$$

KVL around i_2 mesh: $-3 + 2(i_2 - i_3) + 4i_2 = 0$

$$\boxed{6i_2 - 2i_3 = 3}$$

KVL around i_3 mesh: $2 + 2(i_3 - i_2) + 1(i_3 - i_1) = 0$

$$\boxed{-i_1 - 2i_2 + 3i_3 = -2}$$

In matrix form:

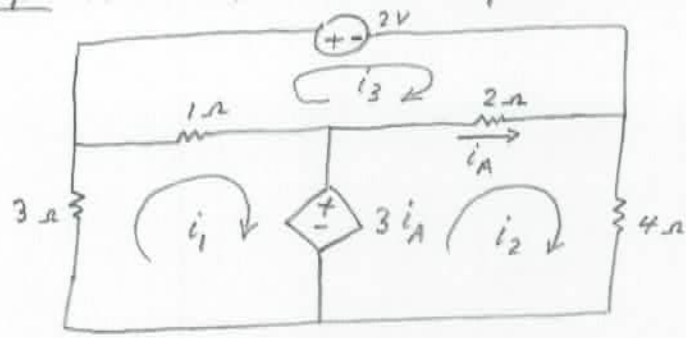
$$\begin{bmatrix} 4 & 0 & -1 \\ 0 & 6 & -2 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ -2 \end{bmatrix}$$

This can be solved to get i_1, i_2, and i_3.

For instance, by Cramer's rule:

$$i_3 = \frac{\begin{vmatrix} 4 & 0 & -3 \\ 0 & 6 & 3 \\ -1 & -2 & -2 \end{vmatrix}}{\begin{vmatrix} 4 & 0 & -1 \\ 0 & 6 & -2 \\ -1 & -2 & 3 \end{vmatrix}} = -\frac{21}{25}$$

Example where there is a dependent voltage source:



i_A is a branch current

$i_A = i_2 - i_3$

KVL on i_1 mesh: $3i_1 + 1(i_1 - i_3) + \underbrace{3i_A}_{3i_2 - 3i_3} = 0$

$$\boxed{4i_1 + 3i_2 - 4i_3 = 0}$$

KVL on i_2 mesh:

$$\underbrace{-3i_A}_{-3i_2 + 3i_3} + 2(i_2 - i_3) + 4i_2 = 0$$

$$\boxed{3i_2 + i_3 = 0}$$

KVL on i_3 mesh: $2 + 2(i_3 - i_2) + 1(3i_3 - i_1) = 0$

$$\boxed{-i_1 - 2i_3 + 5i_3 = -2}$$

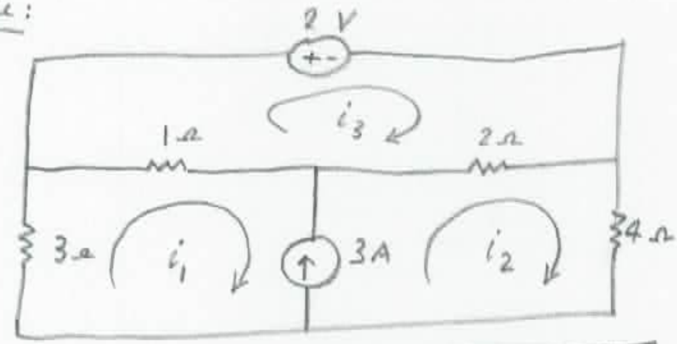
These can be solved to get the mesh currents i_1, i_2, and i_3.

These in turn will yield any branch current and any branch voltage.

When there is a current source, we need to do something more: We first equate the current source to the mesh currents going through it. Then, we write KVL around "loops" that do not go through that current source.

(A loop is a tracing around branches that closes on itself. It is a generalization of a mesh.)

Example:



First, for the 3 A source, write $\boxed{3 = i_2 - i_1}$

Then, think of removing that source, and write KVL around the "new meshes" that arise.

For instance, with 3A source removed, write KVL around the 3 Ω, 1 Ω, 2 Ω, 4 Ω "mesh". But, keep the original mesh currents as is.

$$3 i_1 + 1 (i_1 - i_3) + 2 (i_2 - i_3) + 4 i_2 = 0$$

$$\boxed{4 i_1 + 6 i_2 - 3 i_3 = 0}$$

Then KVL around the i_3 mesh is as before, because it does not go through the current source.

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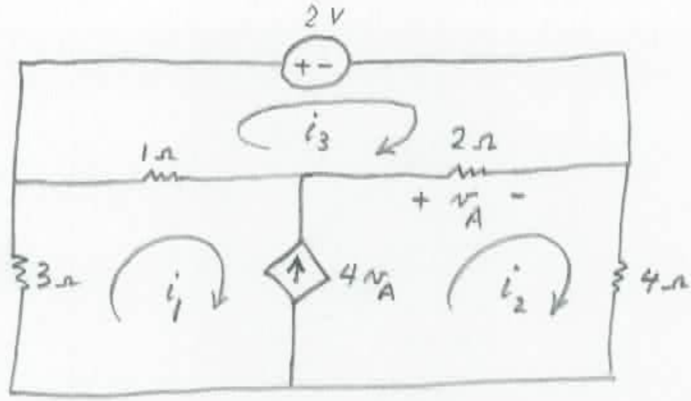
$$2 + 2 (i_3 - i_2) + 1 (i_3 - i_1) = 0$$

$$\boxed{- i_1 - 2 i_2 + 3 i_3 = -2}$$

These can be solved for i_1, i_2, i_3.

Here's an example where the current source is dependent.

Example:



For the dependent current source:

$$i_2 - i_1 = 4v_A \quad , \quad v_A = 2(i_2 - i_3)$$

So, $i_2 - i_1 = 8i_2 - 8i_3$

$$\boxed{-i_1 - 7i_2 + 8i_3 = 0}$$

For the 3Ω, 1Ω, 2Ω, 4Ω "new mesh", we get again

$$3i_1 + 1(i_1 - i_3) + 2(i_2 - i_3) + 4i_2 = 0$$

$$\boxed{4i_1 + 6i_2 - 3i_3 = 0}$$

For the i_3 mesh, we get again

$$\boxed{-i_1 - 2i_2 + 3i_3 = -2}$$