ESE 271 Third Exam Name:

Fall, 2009 ID Number:

Do not put your answers on this front page.

Each problem is worth 25 points.

Prob. 1:

Prob. 2:

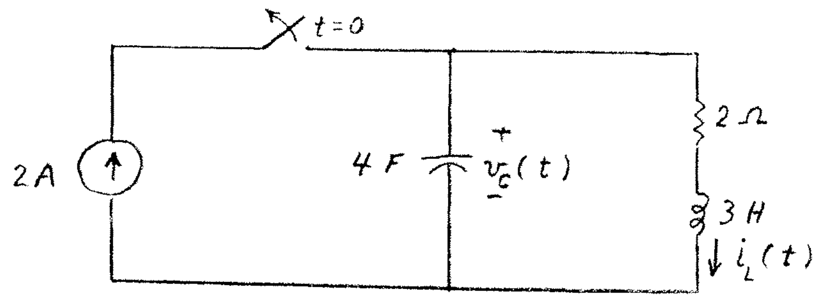
Prob. 3:

Prob. 4:

3rd exam

Prob. 1(a) (12 points):

This circuit is in the DC steady state for $t < 0$ with the switch closed. The switch is opened at $t = 0$. Find $i_L(0+)$ and $v_C(0+)$.



AT DC, CAPACITOR IS AN OPEN AND INDUCTOR IS A SHORT.

So,

$$v_C(0+) = V_C(0-) = 2 \times 2 = 4 \text{ VOLTS}$$

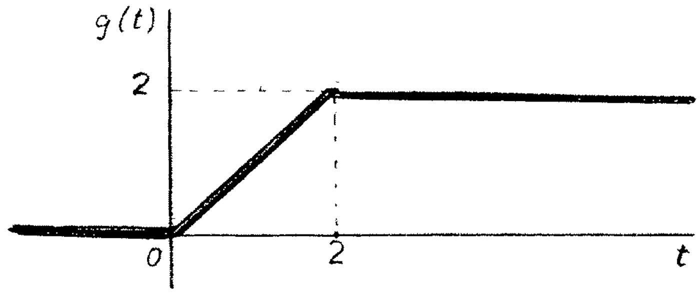
$$i_L(0+) = i_L(0-) = 2 \text{ AMPERES}$$

BECAUSE THERE ARE NO "JUMPS" AT $t = 0$.

3rd exam

Prob. 1(b) 13 points:

Find the Laplace transform of the second generalized derivative $g^{(2)}(t)$ of $g(t)$.



$$g(t) = t\,u(t) - (t-2)\,u(t-2)$$

$$g^{(1)}(t) = u(t) - u(t-2)$$

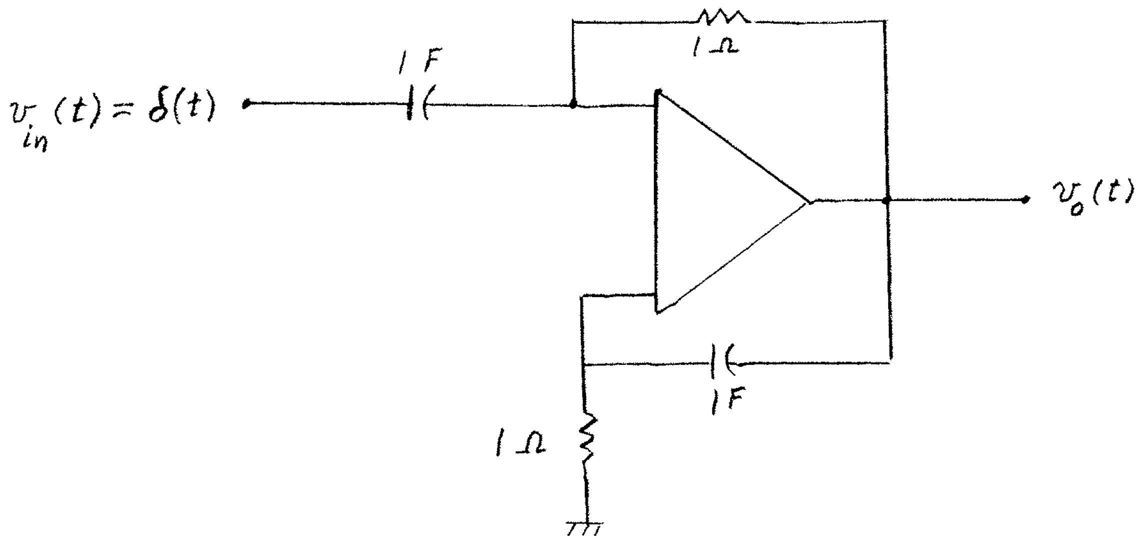
$$g^{(2)}(t) = \delta(t) - \delta(t-2)$$

$$\mathcal{L}\, g^{(2)}(t) = 1 - e^{-2s}$$

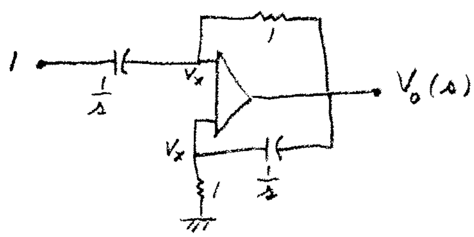
3rd exam

Prob. 2 (25 points):

Find the Laplace transform $V_o(s)$ of the output voltage $v_o(t)$. The initial charges on both capacitors at $t = 0+$ are 0.



The transformed circuit:



At the upper V_x node:

$$\frac{1 - V_x}{\frac{1}{s}} + \frac{V_o - V_x}{1} = 0$$

So, $$V_x = \frac{s + V_o}{s+1}$$

At the lower V_x node:

$$V_x = \frac{1}{1 + \frac{1}{s}} V_o = \frac{s}{s+1} V_o$$

So, $$V_x = \frac{s + V_o}{s+1} = \frac{s V_o}{s+1}$$

$$V_o = \frac{s}{s-1}$$

3rd exam

Prob. 3 (25 points):

Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}F(s)$ **for** $t > 0$, **where**

$$F(s) = \frac{2s+1}{(s+1)(s+2)^2}$$

$$F(s) = \frac{A}{s+1} + \frac{B_1}{(s+2)^2} + \frac{B_2}{s+2}$$

$$A = \left.\frac{2s+1}{(s+2)^2}\right|_{s=-1} = -1$$

$$B_1 = \left.\frac{2s+1}{s+1}\right|_{s=-2} = 3$$

$$B_2 = \left.\frac{d}{ds}\frac{2s+1}{s+1}\right|_{s=-2} = \left.\frac{(s+1)2-(2s+1)}{(s+1)^2}\right|_{s=-2} = 1$$

So,

$$F(s) = \frac{-1}{s+1} + \frac{3}{(s+2)^2} + \frac{1}{s+2}$$

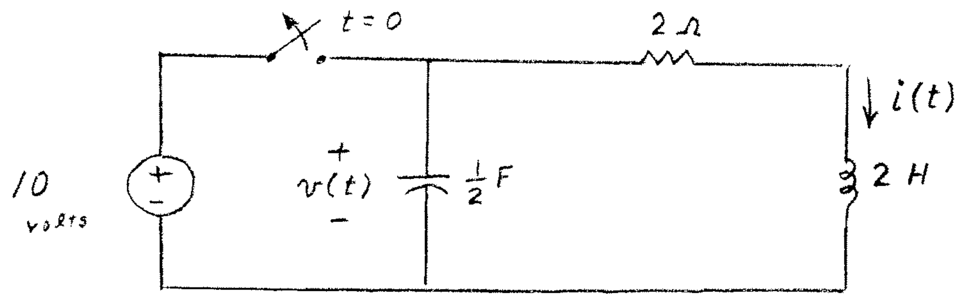
Therefore, for $t > 0$:

$$f(t) = -e^{-t} + 3te^{-2t} + e^{-2t}$$

3rd exam

Prob. 4 (25 points).

At $t = (0+)$, $v(0+) = 10$ volts and $i(0+) = 5$ amperes. Find the Laplace transform of the current $i(t)$ where $t > 0$.



THE INTEGRODIFFERENTIAL EQUATION FOR $t > 0$ IS:

$$-v(t) + R\,i(t) + L\frac{d\,i(t)}{dt} = 0$$

(THIS IS KIRCHHOFF'S VOLTAGE LAW.)

SUBSTITUTING FOR $v(t)$, WE GET

$$\frac{1}{C}\int_0^t i(x)\,dx - v(0+) + R\,I(s) + L\frac{d\,i(t)}{dt} = 0$$

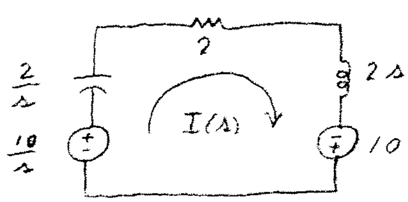
APPLYING THE LAPLACE TRANSFORM, WE GET

$$2\,\frac{I(s)}{s} - \frac{10}{s} + 2\,I(s) + 2\big(s\,I(s) - 5\big) = 0$$

THIS YIELDS

$$I(s) = \frac{5s + 5}{s^2 + s + 1}$$

ANOTHER WAY TO GET THIS IS TO USE THE TRANSFORMED CIRCUIT:



THIS YIELDS THE SAME TRANSFORMED EQUATION.