ESE 271 Third Exam Name:

Fall, 2008 ID Number:

Do not place your answers on this front page.

Each problem is worth 25 points.

Prob. 1:

Prob. 2:

Prob. 3:

Prob. 4:

Prob. 1:

Given that $f(t) = \cos 3t$ for $t > 0$, Find the Laplace transform of

$$\frac{d}{dt}\, t f(t)$$

You may leave your answer in factored form.

$$F(s) = \mathcal{L} f(t) = \frac{s}{s^2 + 3^2}$$

$$\mathcal{L}\, t f(t) = -\frac{d}{ds}\left(\frac{s}{s^2+9}\right) = \frac{s^2-9}{(s^2+9)^2}$$

$$\mathcal{L}\, \frac{d}{dt}\, t f(t) = s\left(\frac{s^2-9}{(s^2+9)^2}\right) - 0 = \frac{s(s^2-9)}{(s^2+9)^2}$$

ANOTHER WAY:

$$\frac{d}{dt}\, t \cos 3t = \cos 3t - 3t \sin 3t$$

$$\mathcal{L}\, \frac{d}{dt}\, t \cos 3t = \frac{s}{s^2+9} - 3\left(-\frac{d}{ds}\right)\frac{3}{s^2+9}$$

$$= \frac{s}{s^2+9} + 9\,\frac{(-2s)}{(s^2+9)^2}$$

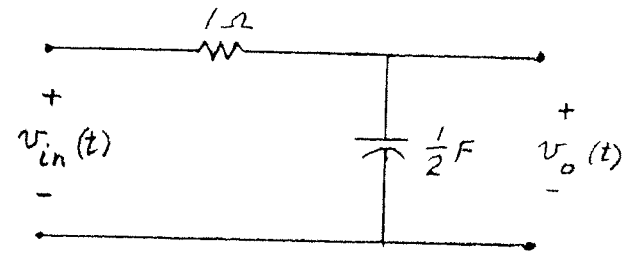
$$= \frac{s(s^2-9)}{(s^2+9)^2}$$

Prob. 2:

In the following circuit, the input voltage is

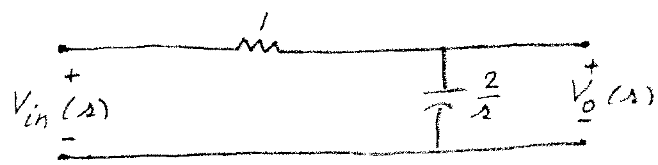
$$v_{in}(t) = u(t) - u(t-2)$$

Find the output voltage $v_o(t)$ for $t > 0$.



$v_o(0+) = 0$

THE TRANSFORMED CIRCUIT IS :



$$V_{in}(s) = \frac{1 - e^{-2s}}{s}$$

$$V_o(s) = V_{in}(s) \frac{\frac{2}{s}}{1 + \frac{2}{s}}$$

$$V_o(s) = \frac{2}{s+2} \cdot \frac{1 - e^{-s}}{s} = \left(\frac{A}{s} + \frac{B}{s+2}\right)\left(1 - e^{-s}\right)$$

$$A = 1, \quad B = -1$$

So,

$$v_o(t) = 1 - e^{-2t} - u(t-2) + e^{-2(t-2)} u(t-2)$$

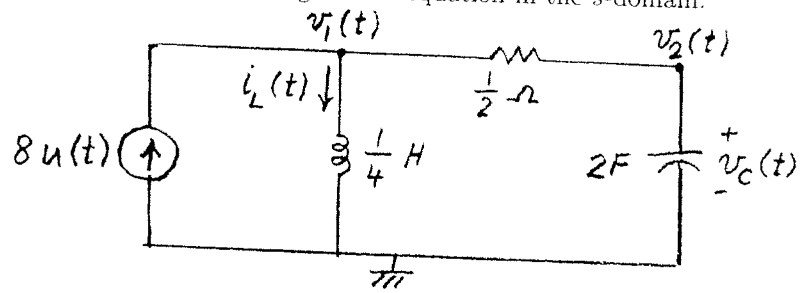
WHERE $t > 0$

OR

$$v_o(t) = u(t) - e^{-2t} u(t) - u(t-2) + e^{-2(t-2)} u(t-2)$$

Prob. 3:

At $t = 0+$, the initial current $i_L(0+)$ and initial voltage $v_C(0+)$ are as shown. Using a nodal analysis for $t > 0$, find two equations involving $V_1(s) = \mathcal{L}\, v_1(t)$ and $V_2(s) = \mathcal{L}\, v_2(t)$, and then fill in the following matrix equation in the s-domain.



$i_L(0+) = 5\ A$

$v_C(0+) = v_2(0+) = 3\ V$

THE INTEGRODIFFERENTIAL EQUATIONS:

AT $v_1(t)$ NODE:

$$-8u(t) + \frac{1}{1/4}\int_0^t v_1(x)\,dx + 5 + \frac{v_1(t) - v_2(t)}{1/2} = 0$$

AT $v_2(t)$ NODE:

$$\frac{v_2(t) - v_1(t)}{1/2} + 2\,\frac{dv_2(t)}{dt} = 0$$

APPLY $\mathcal{L}$ AND REARRANGE EQUATIONS:

$$\frac{4}{s}V_1(s) + 2V_1(s) - 2V_2(s) = \frac{8}{s} - \frac{5}{s} = \frac{3}{s}$$

$$2V_2(s) - 2V_1(s) + 2\left(sV_2(s) - 3\right) = 0$$

$$\begin{bmatrix} \frac{4}{s} + 2 & -2 \\ -2 & 2s+2 \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} \frac{3}{s} \\ 6 \end{bmatrix}$$

(WE CAN MULTIPLY THE FIRST EQUATION BY s (OR ANOTHER FUNCTION OF s) TO GET ANOTHER CORRECT ANSWER.)

Prob. 4:

Determine the inverse Laplace transform $f(t) = \mathcal{L}^{-1}F(s)$ for $t > 0$, where

$$F(s) = \frac{s+2}{s^3 + 4s^2 + 5s}$$

$$F(s) = \frac{s+2}{s(s-p_1)(s-p_2)} = \frac{s+2}{s(s^2+4s+5)}$$

$$\left.\begin{matrix} p_1 \\ p_2 \end{matrix}\right\} = \frac{-4 \pm \sqrt{4^2 - 4\times 5}}{2} = -2 \pm j$$

So,

$$F(s) = \frac{A}{s} + \frac{B}{s+2-j} + \frac{B^*}{s+2+j}$$

$$A = \frac{2}{5}$$

$$B = \frac{-2+j+2}{(-2+j)(-2+j+2+j)} = \frac{1}{2(-2+j)} = \frac{1}{2\sqrt{5}\ \angle \tan^{-1}\frac{1}{-2}}$$

$$B = \frac{1}{2\sqrt{5}} \angle -153.43^\circ = \frac{1}{2\sqrt{5}} \angle 206.57^\circ$$

So, for $t > 0$:

$$f(t) = \frac{2}{5} + 2\,|B|\,e^{-\alpha t}\cos(\beta t + \theta)$$

WHERE $2|B| = \frac{1}{\sqrt{5}}$, $\alpha = 2$, $\beta = 1$

(OR) $f(t) = .4 + .4472\, e^{-2t}\cos(t + 206.57^\circ)$

ANOTHER EQUIVALENT ANSWER IS OBTAINED AS FOLLOWS:

$$f(t) = \frac{2}{5} + B\,e^{-(2-j)t} + B^*\,e^{-(2+j)t}, \quad \text{WHERE } B = \frac{1}{10}(-2-j)$$

$$= \frac{2}{5} + \frac{1}{10}(-2)\,e^{-2t}\left(e^{jt} + e^{-jt}\right) + \frac{1}{10}\,e^{-2t}\left(-j e^{jt} + j e^{-jt}\right)$$

$$= \frac{2}{5} - \frac{4}{10}\,e^{-2t}\cos t + \frac{2}{10}\,e^{-2t}\sin t$$