PHY 300, Spring 2006, Exam 2, April 19, 2006

Please show all work in your exam book. Calculators are allowed. You have been given an equation sheet. Do the easiest problem first, and the hardest last.

1. There's an old Far Side (Gary Larson) cartoon that shows a giant monster's eyeball viewed in the right-side mirror of a car, along with the discaimer printed on those mirrors of "Objects in mirror are closer than they appear." Steven Spielberg stole this joke in the movie "Jurassic Park" when *T. rex* was chasing people in a car. Those side mirrors present you with a non-inverted virtual image with a demagnified view of the scene behind you so you can see a greater angular range to the right side of your car.

   OK, on to the problem: you see a monster's hairy eyeball that appears to be 10 m back, and which appears to be 10 cm in diameter when you know it's really 20 cm in diameter. How far back is the monster? What's the radius of curvature of your side mirror? Show what the mirror looks like in a sketch.

2. A linear polarizer with TA horizontal (0°) is followed by by one with TA vertical (90°). Unpolarized light is incident upon it. A) Describe the transmitted light. B) Now insert a linear polarizer with TA at 45° between the two original polarizers, and describe the transmitted light. C) Now add a quarter wave plate after the 45° polarizer but before the final vertical polarizer, and describe the transmitted light. Use Jones matrices throughout.

3. A person whose eyes have a resting focusing strength of 50 diopters can see objects clearly only over a distance range of 0.40 to 1.10 m. Find A) their eye length; B) their range of accomodation; C) the focusing strength of eyeglasses needed to correct for relaxed viewing at large distances; D) their near point with these distance-correcting eyeglasses on; and E) the focusing strength of reading glasses (or bifocal inserts) needed to let them read things at 25 cm distance and no accommodation (eye relaxed).

4. You are given two glasses with the following properties:

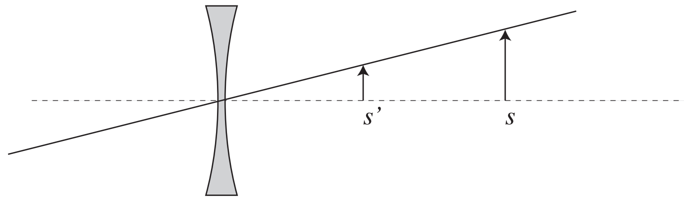
	n_F	n_D	n_C
Glass 1	1.505	1.500	1.495
Glass 2	1.612	1.600	1.588

   Calculate V for each class, and give its catalog code ($n_D - 1/10V$). Design an achromat with a focal length of 20 cm at λ_D. Sketch what the lens looks like, indicating the radius of curvature of all surfaces and which lens is made out of which glass. Calculate the focal length at λ_D of each lens separately.

5. A Keplerian telescope has an aberration-free objective lens with $f = 30$ cm, and a convex-plano eyepiece lens with $f = 5$ cm (in both cases, glass with $n = 1.50$ is used). Calculate the aberration function for the eyepiece for rays parallel to the optical axis. Now use this aberration function to calculate the lateral ray aberration at the intermediate image plane for rays exiting the eyepiece at 5°. What absolute angular error, and what fractional error, does this correspond to for viewing stars in the sky with this telescope?

Solutions:

1. The imaging situation is as follows:



The image height is $h' = 10$ cm when the object height is $h = 20$ cm. The image is non-inverted, and it's one you view with your eye rather than project onto a screen so it's a virtual image, so the image distance is negative (*i.e.*, it's on the same side of the lens as the object). The magnification is $m = -(-10\text{ cm})/(20\text{ cm}) = 1/2$ and it's a positive number because the image is not inverted. From

$$m = -\frac{s'}{s} = -\frac{h'}{h} \qquad \text{we get} \qquad s = s'\frac{h}{h'} = (-10\text{ m})\frac{20\text{ cm}}{-10\text{ cm}} = 20\text{ m}$$

so that's how far back the monster is. The focal length of the "lens" is found from

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{20} + \frac{1}{-10} = \frac{1}{20} - \frac{2}{20} = \frac{1}{-20}$$

so $f = -20$ cm. This is not a double-concave lens but a convex mirror with a radius of curvature of

$$f = R2 \qquad \Rightarrow R = 2f = -40\text{ m}$$

2. A) In the first case we have

$$\begin{aligned}\begin{bmatrix} A' \\ B' \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\end{aligned}$$

so no light emerges. For B), we have

$$\begin{aligned}\begin{bmatrix} A' \\ B' \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 \\ B \end{bmatrix}\end{aligned}$$

so we have vertical lienarly polarized light emerging. For C) we have

$$\begin{aligned}\begin{bmatrix} A' \\ B' \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -i & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 0 & 0 \\ -i & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \frac{-i}{2}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \frac{-i}{2}\begin{bmatrix} 0 \\ A \end{bmatrix}\end{aligned}$$

so the input horizontally polarized light becomes the output vertically polarized light with a $90°$ phase shift and a factor of 2 reduction in electric field.

3. Recall that 1 diopter is equal to an inverse focal length of 1 m^{-1}. With the eye at rest, the net focusing strength of the eye is 50.0 D. A) If the person sees an object at a distance of 1.1 m in that condition, then their eye length is given by

$$\frac{1}{s'} = \frac{1}{f_0} - \frac{1}{s} \qquad \text{or} \qquad \frac{1}{s'} = 50 - \frac{1}{1.1} = 49.09 \qquad \text{or} \qquad s' = 2.037 \text{ cm.}$$

B) At maximum accommodation, they can see clearly at a distance of 0.40 m in which case they have

$$\begin{aligned}\frac{1}{s} + \frac{1}{s'} &= \frac{1}{f_0} + \frac{1}{f_a} \\ \frac{1}{f_a} &= \frac{1}{s} + \frac{1}{s'} - \frac{1}{f_0} = \frac{1}{0.40} + \frac{1}{0.02037} - 50 = 2.5 + 49.09 - 50 = 1.59 \text{ D}\end{aligned}$$

so this is an old person with a poor range of accommodation. C) To make for relaxed distance viewing, we want to add a corrective lens f_c:

$$\begin{aligned}\frac{1}{\infty} + \frac{1}{s'} &= \frac{1}{f_0} + \frac{1}{f_c} \\ \frac{1}{f_c} &= \frac{1}{s'} - \frac{1}{f_0} = 49.09 - 50 = -0.91 \text{ D}\end{aligned}$$

which is a mildly diverging corrective lens. D) With this corrective lens, their near point becomes

$$\begin{aligned}\frac{1}{s} + \frac{1}{s'} &= \frac{1}{f_0} + \frac{1}{f_c} + \frac{1}{f_a} \\ \frac{1}{s} &= \frac{1}{f_0} + \frac{1}{f_c} + \frac{1}{f_a} - \frac{1}{s'} = 50 + (-0.91) + 1.59 - 49.09 \qquad \text{or} \qquad s = 0.21 \text{ m}\end{aligned}$$

or $s = 63$ cm which is a bit awkward (newspaper at arm's length). E) To let them see at 25 cm with no accommodation (eyes relaxed), they should have reading glasses (or bifocal inserts) with f_r given by

$$\begin{aligned}\frac{1}{s} + \frac{1}{s'} &= \frac{1}{f_0} + \frac{1}{f_r} \\ \frac{1}{f_r} &= \frac{1}{s} + \frac{1}{s'} - \frac{1}{f_0} = \frac{1}{0.25} + 49.09 - 50 = +3.09 \text{ D}\end{aligned}$$

or a positive correcting lens.

4. Let's start by calculating V for each glass:

$$\begin{aligned}V_1 &= \frac{n_D - 1}{n_F - n_C} = \frac{1.500 - 1}{1.505 - 1.495} = \frac{0.500}{0.010} = 50.0 \\ V_2 &= \frac{1.600 - 1}{1.612 - 1.588} = \frac{0.600}{0.024} = 25.0\end{aligned}$$

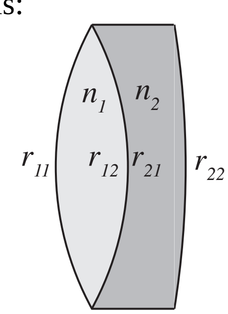
For glass 1, we have $n_{1D} - 1$=0.500 and $V_1 = 50.0$, so the catalog code of $(n_D - 1)/V$ is 500/500. For glass 2, we have $n_{2D} - 1$=0.600 and $V_2 = 25.0$, so the catalog code is 600/250. We then want

$$\begin{aligned} \frac{1}{f_D} &= (n_{1D} - 1)\frac{2}{|r_1|}\frac{V_1 - V_2}{V_1} \\ |r_1| &= (n_{1D} - 1)\, 2 f_D \frac{V_1 - V_2}{V_1} = (1.500 - 1) \cdot 2 \cdot (20 \text{ cm}) \cdot \frac{50 - 25}{50} = 10 \text{ cm} \end{aligned}$$

and

$$\frac{1}{r_{22}} = \frac{1}{|r_1|}\left[2\frac{n_{1D} - 1}{n_{2D} - 1)}\frac{V_2}{V_1} - 1\right] = \frac{1}{10 \text{ cm}}\left[2\frac{(1.500 - 1)}{(1.600 - 1)}\frac{25}{50} - 1\right] = \frac{1}{-60 \text{ cm}}$$

so r_{22} is convex. The lens looks like this:

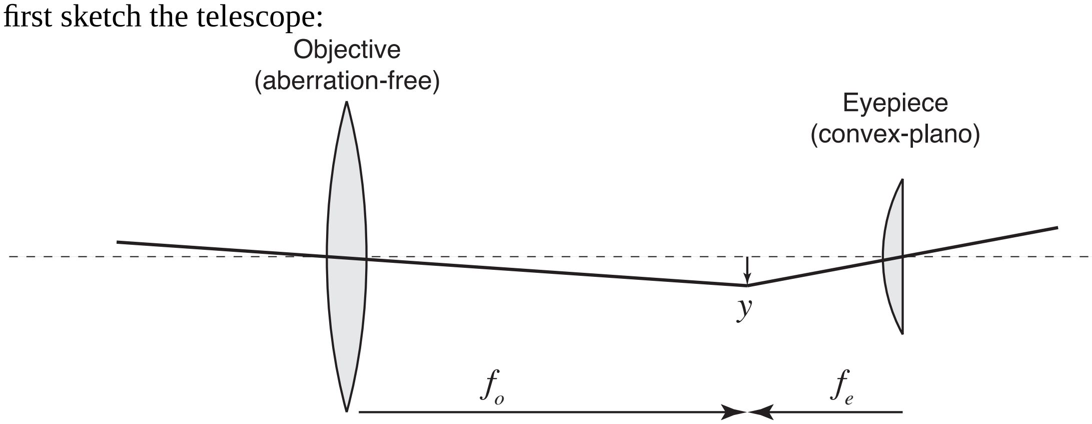


We have $r_{11} = +10$ cm, $r_{12} = -10$ cm, $r_{21} = -10$ cm, and $r_{22} = -60$ cm. The focal lengths for the two individual lens elements are

$$\begin{aligned} \frac{1}{f_{1D}} &= \frac{1.500 - 1}{1}\left(\frac{1}{10} - \frac{1}{-10}\right) = \frac{1}{10 \text{ cm}} \\ \frac{1}{f_{2D}} &= \frac{1.600 - 1}{1}\left(\frac{1}{-10} - \frac{1}{-60}\right) = \frac{1}{-20 \text{ cm}} \end{aligned}$$

and $1/10 + 1/(-20) = 1/20$ so we have $f_D = 20$ cm as desired.

5. Let's first sketch the telescope:



We are told that the objective is aberration-free, so we only need to worry about aberrations of the eyepiece. The eyepiece has a focal length of 5 cm, and $R_2 \to \infty$, so we can find R_1

for the eyepiece lens from

$$\begin{aligned} \frac{1}{f} &= \frac{1.5-1}{1}\left(\frac{1}{R_1}-\frac{1}{\infty}\right) \\ R_1 &= \frac{1.5-1}{1} f = 0.5\cdot(5\text{ cm}) = 2.5\text{ cm} \end{aligned}$$

For the eyepiece, we have the object one focal length away or $s = 5$ cm, and the image at infinity. Now let's look at the aberration function:

$$a(Q) = -\frac{y^4}{8}\left[\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2 + \frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right]$$

For rays parallel to the optical axis, at the second, planar surface of the eyepiece we have $s \to \infty$, $s' \to \infty$, and $R \to \infty$ so the aberration function from the second optical surface is zero. We only have to worry about the first optical surface, where $n_1 = 1$, $s = f = 5$ cm, $s' \to \infty$, and $R = R_1 = 2.5$ cm. In this case the aberration function becomes

$$\begin{aligned} a(Q) &= -\frac{y^4}{8}\left[\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2 + \frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right] \\ &= -\frac{y^4}{8}\left[\frac{1}{5\text{ cm}}\left(\frac{1}{5\text{ cm}}+\frac{1}{2.5\text{ cm}}\right)^2 + \frac{1.5}{\infty}\left(\frac{1}{\infty}-\frac{1}{2.5\text{ cm}}\right)^2\right] \\ &= -\frac{y^4}{8}\left[\frac{1}{5}\left(\frac{3}{5}\right)^2\right] = \frac{y^4}{8}\frac{9}{125} = y^4\frac{9}{1000} \end{aligned}$$

The lateral ray aberration is normally given by

$$b_y = \frac{s'}{n_2}\frac{da}{dy}$$

but in this case we're better off turning it around and considering the aberration in terms of the object (the intermediate image), so we have

$$b_y = \frac{s}{n_1}\frac{da}{dy} = \frac{5\text{ cm}}{1}\frac{d}{dy}\left(y^4\frac{9}{1000}\right) = 5\cdot 4\cdot y^3\frac{9}{1000} = y^3\frac{9}{50}$$

Now for rays exiting the eyepiece at an angle of $\theta' = 5°$, the height at the intermediate image plane should be $y = (5\text{ cm})\cdot\tan(5°) = 0.44$ cm. However, the lateral ray aberration is $b_y = (9/50)y^3 = (9/50)(0.44)^3 = 0.015$ cm. These heights divided by the focal length of the objective give angles in the sky, so we have

$$\begin{aligned} \theta &= \arctan\left(\frac{0.44}{30}\right) = 0.84° \\ \theta_{b_y} &= \arctan\left(\frac{0.015}{30}\right) = 0.029° \end{aligned}$$

and the fractional angular error is $(0.029°/0.84°) = 0.034$ or 3.4%.