ESE 271 Final Exam Name:

Spring, 2003 ID Number:

Do not place your answers on this front page.

Prob. 1 (30 points):

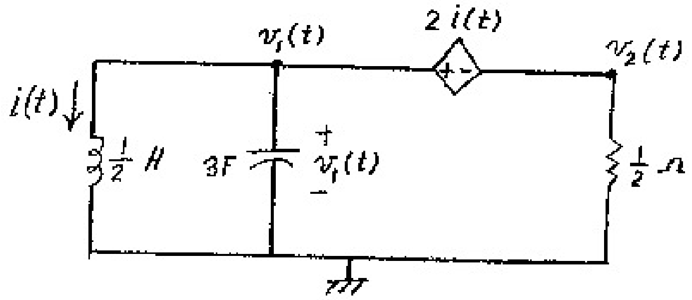
Prob. 2 (25 points):

Prob. 3 (30 points):

Prob. 4 (15 points):

Prob. 1 (30 points):

Using a nodal analysis, find the Laplace transform $V_1(s)$ of the node voltage $v_1(t)$. Use Cramer's rule and write your answer as a determinant over a determinant. (You may do this either by first writing the integrodifferential equations or first making the transformed network. Write your answer neatly.)



$i(0+) = 3\,A$

$v_1(0+) = 4\,V$

BY INTEGRO DIFFERENTIAL EQUATIONS:

INSIDE BALLOON AROUND THE DEPENDENT SOURCE:

$$v_1 - v_2 = 2i = 2\left(\frac{1}{\frac{1}{2}}\int_0^t v_1(x)\,dx + 3\right)$$

So, $V_1 - V_2 = \frac{4}{s}V_1 + \frac{6}{s}$ (or) $V_1\left(1 - \frac{4}{s}\right) - V_2 = \frac{6}{s}$

KCL ON THAT BALLOON:

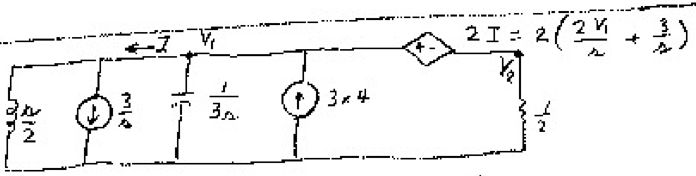
$$\frac{1}{\frac{1}{2}}\int_0^t v_1(x)\,dx + 3 + 3\frac{dv_1}{dt} + \frac{v_2}{\frac{1}{2}} = 0$$

So, $\frac{2}{s}V_1 + \frac{3}{s} + 3(sV_1 - 4) + 2V_2$ (or) $V_1\left(3s + \frac{2}{s}\right) + 2V_2 = 12 - \frac{3}{s}$

THUS,

$$V_1 = \frac{\begin{vmatrix} \frac{6}{s} & -1 \\ 12 - \frac{3}{s} & 2 \end{vmatrix}}{\begin{vmatrix} 1 - \frac{4}{s} & -1 \\ 3s + \frac{2}{s} & 2 \end{vmatrix}}$$

USING THE TRANSFORMED CIRCUIT:



INSIDE BALLOON: $V_1 - V_2 = \frac{4}{s}V_1 + \frac{6}{s}$ (or) $V_1\left(1 - \frac{4}{s}\right) - V_2 = \frac{6}{s}$

ON BALLOON $\frac{2V_1}{s} + \frac{3}{s} + 3sV_1 - 12 + 2V_2 = 0$ (or) $V_1\left(\frac{2}{s} + 3s\right) + 2V_2 = 12 - \frac{3}{s}$

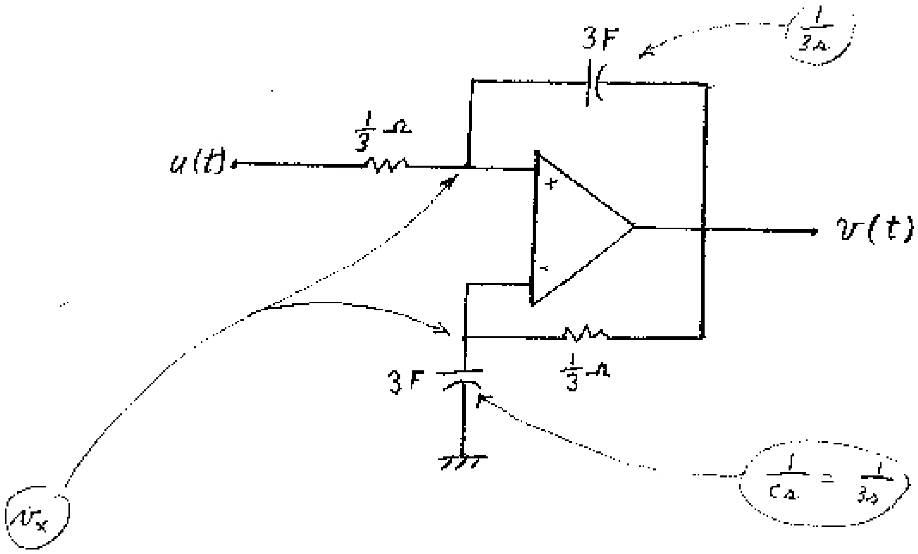
ANOTHER CORRECT ANSWER IS GOTTEN BY MULTIPLYING THE EQUATIONS BY s:

$$\begin{cases} V_1(s-4) - sV_2 = 6 \\ V_1(3s^2+2) + 2sV_2 = 12s - 3 \end{cases}$$

$$V_1 = \frac{\begin{vmatrix} 6 & -s \\ 12s - 3 & 2s \end{vmatrix}}{\begin{vmatrix} s-4 & -s \\ 3s^2+2 & 2s \end{vmatrix}}$$

Prob. 2 (25 points):

The initial charges on the capacitors are 0. The input voltage is the unit step function $u(t)$. Find the output voltage $v(t)$ as a function of time $t > 0$.



At upper node for v_x:

$$\frac{V_x - \frac{1}{s}}{\frac{1}{3}} + \frac{V_x - V}{\frac{1}{3s}} = 0$$

$$V_x(3 + 3s) - 3sV = \frac{3}{s}$$

At lower node for v_x:

$$\frac{V_x}{\frac{1}{3s}} + \frac{V_x - V}{\frac{1}{3}} = 0$$

$$V_x(3 + 3s) - 3V = 0$$

Subtract 2^{nd} equation from the first one:

$$3V - 3sV = \frac{3}{s}$$

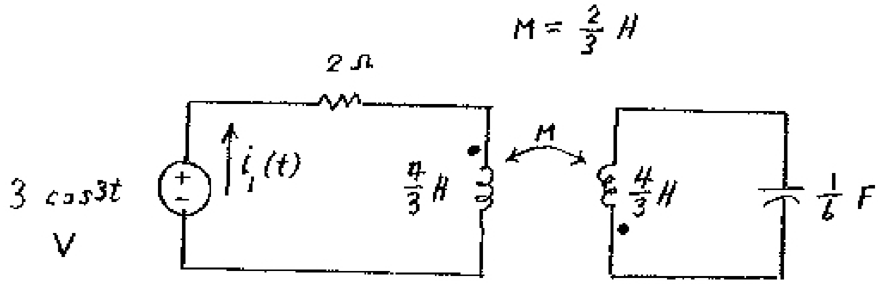
$$V = \frac{\frac{1}{s}}{1-s} = \frac{-1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$A = \frac{-1}{-1} = 1, \quad B = -1$$

So, $v(t) = 1 - e^{t}$ for $t > 0$.

Prob. 3 (30 points):

This circuit having a nonideal transformer is in the AC steady state. Find the current $i_1(t)$ flowing upward through the source. Write your answer as a cosinusoid.



USE A MESH ANALYSIS WITH MESH CURRENTS

1ST LOOP: $-3 + I_1(2+j4) + j2 I_2 = 0$

2nd LOOP: $j4 I_2 + \frac{1}{j3 \times \frac{1}{6}} I_2 + j2 I_1 = 0$

FROM 2nd EQUATION: $I_2(j4 - j2) + I_1(j2) = 0$ THUS, $I_2 = -I_1$

FROM 1st EQUATION:

$$-3 + I_1(2+j4) - j2 I_1 = 0$$

$$I_1 = \frac{+3}{2+j2} = \frac{+3}{2\sqrt{2}\,\angle 45^\circ} = \frac{3}{2\sqrt{2}} \angle -45^\circ$$

$$i_1(t) = \frac{3}{2\sqrt{2}} \cos(3t + 135^\circ) = 1.06 \cos(3t - 45^\circ)$$

Prob. 4 (15 points):

Find the initial value $f(0+)$ and the initial slope $f^{(1)}(0+)$ of the function $f(t)$ whose Laplace transform is

$$F(s) = \frac{4s^3 + 3s^2 - 2s + 6}{2s^4 + 6s^3 + 8s^2 + 4s}$$

This $f(t)$ has a final value $f(\infty)$. (Take my word for it.) Find $f(\infty)$.

$$f(0+) = \lim_{s \to \infty} s F(s) = \lim_{s \to \infty} \frac{4s^4 + \cdots}{2s^4 + \cdots} = 2$$

$$f^{(1)}(0+) = \lim_{s \to \infty} s \left(s F(s) - f(0+) \right)$$

$$= \lim_{s \to \infty} s \left(\frac{\cancel{4s^4} + 3s^3 - 2s^2 + 6s - 2(\cancel{2s^4} + 6s^3 + 8s^2 + 4s)}{2s^4 + 6s^3 + 8s^2 + 4s} \right)$$

$$= \lim_{\to \infty} \frac{-9s^4 + \cdots}{2s^4 + \cdots} = -4.5$$

$$f(\infty) = \lim_{s \to 0+} s F(s) = \frac{6}{4} = \frac{3}{2}$$