7.13

a $\psi_{432} \Rightarrow$ a) $E = \frac{-13.6\, eV}{4^2} =$

b) $R_{43} =$ Hmm I don't know how you could have known this

c) $\overline{L^2} = \ell(\ell+1)\hbar^2$

$\sqrt{\overline{L^2}} = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{3\cdot 4}\,\hbar$

d) $\overline{L_z} = 3\hbar$

7.14

It Suffice to work piece by piece: look at Table 7-2

$|\psi_{300}|^2$ = sperically symmetric since ψ_{300} indep of angle

$|\psi_{310}|^2 + |\psi_{311}|^2 + |\psi_{31-1}|^2 \propto 2\cos^2\theta + \sin^2\theta\, |e^{+i\phi}|^2$

$+ \sin^2\theta\, |e^{-i\phi}|^2$

$\propto 2(\cos^2\theta + \sin^2\theta)$ ~~[illegible]~~ spherical

Thus:

$$|\psi_{300}|^2 + |\psi_{311}|^2 + |\psi_{31-1}|^2 \propto 2$$ ← sperical symmetric

$$|\psi_{322}|^2 + |\psi_{32-2}|^2 + |\psi_{321}|^2 + |\psi_{32-1}|^2 + |\psi_{320}|^2 =$$

$$\frac{1}{4}\sin^4\theta + \frac{1}{4}\sin^4\theta + \sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta + \frac{1}{6}(3\cos^2\theta - 1)^2$$

$$= \text{———————}\,\|\,\text{———————} + \frac{1}{6}(9\cos^4\theta - 6\cos^2\theta +$$

Now one needs some methodoloy

write $\sin^2\theta = 1 - \cos^2\theta$

$\sin^4\theta = (1 - \cos^2\theta)^2$ and chug away

Straight forward algebra gives $\quad c \equiv \cos\theta$

$$= \frac{1}{12}(1-c^2)^2 + 2(1-c^2)c^2 + \frac{1}{6}(3c^2-1)^2$$

$$= \frac{1}{2}(1-c^2)^2 + 2(1-c^2)c^2 + \frac{1}{6}(3c^2 - 1)^2 = 2/3$$

$$= \frac{1}{2}(1-2c^2+c^4) + 2c^2 - 2c^4 + \frac{9}{6}c^4 - \frac{6}{6}c^2 + \frac{1}{6} = 2/3$$

So

$$|\psi_{322}|^2 + |\psi_{32-2}|^2 + |\psi_{321}|^2 + |\psi_{32-1}|^2 + |\psi_{320}|^2 \propto 2/3$$

which is indep of angle

This last part got a little involved

Problems

① Schrödinger Equation $\quad \frac{u}{r} = R$

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} R = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}\left(\frac{u}{r}\right)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \left[\frac{-1}{r^2}u + \frac{1}{r}\frac{\partial u}{\partial r}\right]$$

$$= \frac{-1}{r^2}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$$

$$= \frac{-1}{r^2}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial^2 u}{\partial r^2} = \frac{1}{r}\frac{\partial^2 u}{\partial r^2}$$

as claimed!

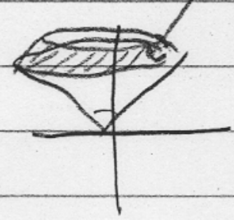
7.11

For $l=0 \quad m=0$

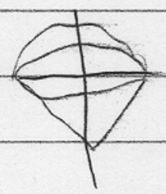
$$\frac{d\mathcal{P}}{d\Omega} = \frac{|\Theta_{lm}|^2}{4\pi} = \frac{1}{4\pi}$$

Integrating

$d\Omega = 2\pi \sin\theta \, d\theta$



$$\mathcal{P} = \int_{con} \frac{d\mathcal{P}}{d\Omega} \, d\Omega$$



$$\mathcal{P} = \int_0^{\Theta_0} \frac{1}{4\pi} \, 2\pi \sin\theta \, d\theta$$

$$= \frac{1}{2} \left(-\cos\Theta\right) \Big|_0^{\Theta_0}$$

$$\mathcal{P} = \frac{1}{2}\left(1 - \cos\Theta_0\right)$$

(a) $\simeq 0.041 \simeq 4.1\%$

(b) Then for $l=1 \quad m=0$

$$\mathcal{P} = \int_{\substack{\text{polar}\\ \text{region}}} dV \, |\psi|^2 = \int_{\substack{\text{polor}\\ \text{region}}} |R|^2 \, |\Theta_{lm}|^2 \, |\Phi|^2 \, r^2 \, dr \, d\Omega$$

So

$$\mathcal{P} = \underbrace{\int_0^\infty |R|^2 \, 4\pi r^2 \, dr}_{1} \int_{\substack{\text{polar} \\ \text{region}}} \frac{d\Omega}{4\pi} \, |\Theta|^2 \, \overbrace{|\Phi|^2}^{\Phi_m = e^{im\phi}}$$

$$\mathcal{P} = \int_{\substack{\text{polar} \\ \text{region}}} \frac{d\Omega}{4\pi} \, |\Theta_{10}|^2$$

$$\mathcal{P} = \int_0^{\theta_0} \frac{2\pi \sin\theta \, d\theta}{4\pi} \; 3\cos^2\theta$$

$$= \frac{1}{2} \left[-\cos^3\theta \right]_0^{\theta_0}$$

Note let $x = \cos\theta$

$$I = -\int^{\cos\theta_0} dx \; 3x^2$$

$$I = \int_{\cos\theta_0}^{1} dx \; 3x^2$$

$$= x^3 \Big|_{\cos\theta_0}^{1}$$

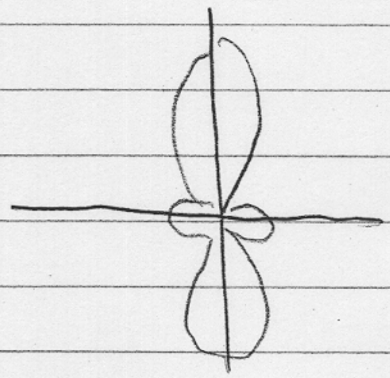
$$= 1 - \cos^3\theta_0$$

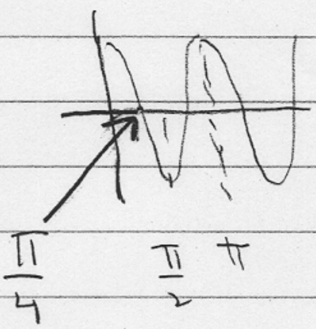
$$\mathcal{P} = \frac{1}{2} \left(1 - \cos^3\theta_0 \right)$$

$$\mathcal{P} \simeq 11\%$$

7.12

$$|\Theta_{20}|^2 = \frac{5}{16}(3\cos 2\theta + 1)^2$$





b) So from figure

maximum at $\theta = 0$ & $\theta = \pi$

c) $$\frac{dP}{d\Omega} = \frac{|\Theta_{20}|^2}{4\pi} = \frac{5}{16}(3\cos 2\theta + 1)^2 \frac{1}{4\pi}$$

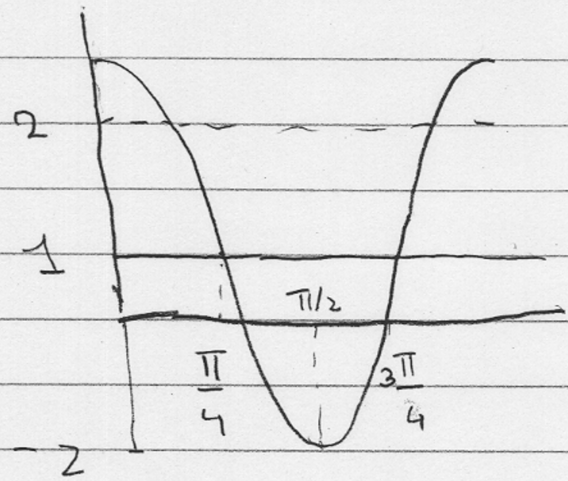
At max $(1 + 3\cos 2\theta)^2 = 4^2 = 16$

So at $\frac{1}{4}$ of max we must have

$$(1 + 3\cos 2\theta)^2 = \frac{1}{4} \cdot 16 = 4$$

$$1 + 3\cos 2\theta = \pm 2$$

Graph



So for +2 case find two solutions from graph

$$1 + 3\cos 2\theta = +2 \Rightarrow \cos 2\theta = +\frac{1}{3}$$

$$\theta \simeq \frac{1}{2}\cos^{-1}\left(\frac{1}{3}\right) \simeq 35^\circ = 0.61\text{rad}$$

<u>and</u>

$$\theta \simeq 180^\circ - \frac{1}{2}\cos^{-1}\left(\frac{1}{3}\right) \approx 145^\circ$$

$$= 2.52 \text{ rad}$$

For -2 case we have

$$\theta = 90^\circ = \pi/2 \text{ rad}$$

20) $KE = \frac{1}{2} I\omega^2 = \frac{L_z^2}{2I}$

So the Schrödinger eq:

$$(KE + \cancel{PE}^{\,0})\Psi = +i\hbar \frac{\partial \Psi}{\partial t}$$

$$\frac{L_z^2}{2mR^2} = +i\hbar \frac{\partial \Psi}{\partial t} \qquad L_z = -i\hbar \frac{\partial}{\partial \varphi}$$

$$-\frac{\hbar^2}{2I} \frac{\partial^2 \Psi}{\partial \varphi^2} = +i\hbar \frac{\partial \Psi}{\partial t}$$

7.21 - replacement)

Let $\Psi(\varphi, t) = \Phi(\varphi) e^{-iEt/\hbar}$

Then:

$$+i\hbar \frac{\partial}{\partial t} \Psi(\varphi, t) = \Phi(\varphi)\, +i\hbar \frac{\partial}{\partial t} e^{-iEt/\hbar}$$

$$= E\, \Phi(\varphi) e^{-iEt/\hbar}$$

So the Schrodinger Eq:

$$-\frac{\hbar^2}{2I} \frac{\partial^2 \Phi}{\partial \varphi^2} e^{-iEt/\hbar} = E\, \Phi e^{-iEt/\hbar}$$

or $\boxed{-\frac{\hbar^2}{2I} \frac{\partial^2 \Phi}{\partial \varphi^2} = E\, \Phi(\varphi)}$

7.23)

$$-\frac{\hbar^2}{2I}\frac{\partial^2 \Phi}{\partial \varphi^2} = E\,\Phi$$

$$\frac{\partial^2 \Phi}{\partial \varphi^2} = -\left(\frac{2IE}{\hbar^2}\right)\Phi$$

$$\frac{\partial^2 \Phi}{\partial \varphi^2} = -m^2\,\Phi \qquad \text{where} \quad m^2 = \frac{2IE}{\hbar^2}$$

This has the general solution

$$\Phi = A e^{im\varphi} + B e^{-im\varphi} \qquad m = \sqrt{\frac{2IE}{\hbar^2}}$$

We can take $A = 1$ $B = 0$ to find a particular solution

$$\Phi = e^{im\varphi}$$

7.24) For a singe valued function

$$\Phi(0) = \Phi(2\pi)$$

$$e^{im0} = e^{im2\pi}$$

So,

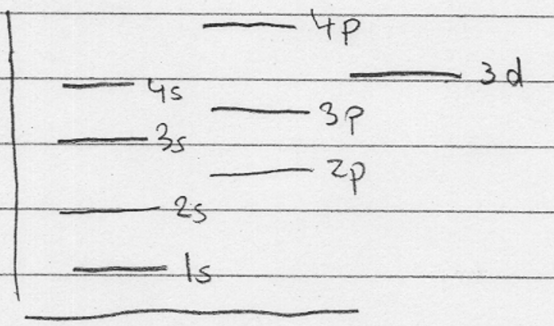
$$1 = e^{im2\pi} \quad \text{or} \implies \quad m = 0, \pm 1, \pm 2, \ldots$$

Then

$$E = \frac{\hbar^2 m^2}{2I} \qquad m = 0, \pm 1, \pm 2 \ldots$$

Problem 3

Ne $\Rightarrow$ Ne has 10 protons & 10 electrons



Ne: $1s^2\ 2s^2\ 2p^6$ ← its an inert gas

O: $1s^2\ 2s^2\ 2p^4$

The spins in the last level are

$$\underbrace{\uparrow\downarrow}_{1s^2}\ \underbrace{\uparrow\downarrow}_{2s^2}\ \underbrace{\uparrow\uparrow\uparrow\downarrow}_{2p^4}$$

Problem 4: In the middle of periodic table we are filling 3d leves:

$m = \overbrace{-2, -1, 0, 1, 2}^{5\text{ states}}$ then there is spin: So $5 \times 2 = 10$

Problem 5

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + \frac{l(l+1)\hbar^2}{2mr^2} + V(r) \right] u_{nl} = E\, u_{nl} \qquad (Eq *)$$

$$u_{nl} = C\,\frac{r}{a_0}\cdot\frac{r}{a_0}\; e^{-r/a_0}$$

Differentiating Carefully

$$\frac{\partial^2}{\partial r^2} C\left(\frac{r}{a_0}\right)^2 e^{-r/2a_0} = \frac{\partial}{\partial r}\left[\frac{2}{a_0}\left(\frac{r}{a_0}\right) e^{-r/2a_0} - \left(\frac{r}{a_0}\right)^2 e^{-r/2a_0}\,\frac{1}{2a_0}\right]$$

$$= \frac{2}{a_0^2} e^{-r/2a_0} + \frac{1}{a_0}\frac{2r}{a_0} e^{-r/2a_0}\left(\frac{-1}{2a_0}\right)$$

$$- \frac{1}{2a_0}\, 2\left(\frac{r}{a_0}\right)\frac{1}{a_0} e^{-r/2a_0}$$

$$+ \left(\frac{r}{a_0}\right)^2 e^{-r/2a_0}\,\frac{1}{(2a_0)^2}$$

$$= \underbrace{\frac{e^{r/2a_0} r^2}{a_0^2}}_{=u_{21}}\left[\frac{2}{r^2} + \frac{-2}{r a_0} + \frac{1}{4a_0^2}\right]$$

Then Eq * becomes (with $l=1$)

$$u_{21}\left[-\frac{\hbar^2}{2m}\left(\frac{2}{r^2}-\frac{2}{ra_0}+\frac{1}{4a_0^2}\right)+\frac{2\hbar^2}{2mr^2}+\frac{-e^2}{4\pi\varepsilon_0 r}\right]$$

$$= E u$$

Note:

$$+\frac{\hbar^2}{ma_0} = \frac{+e^2}{4\pi\varepsilon_0}$$

So find

$$u_{21}\left(-\frac{\hbar^2}{2ma_0^2}\,\frac{1}{4}\right) = E\,u_{21}$$

Which is true provided E

$$E = -\frac{\hbar^2}{2ma_0^2}\,\frac{1}{2^2} = \frac{-13.6\,eV}{2^2}$$