PHY300 Practice Midterm Exam

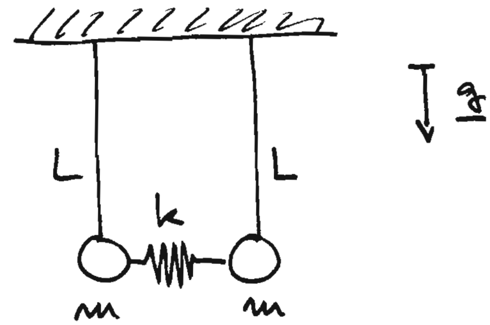
1. Military specifications often require electronic devices to be able to withstand accelerations of $10\ g = 98.1\ \mathrm{m/s^2}$. Manufacturers test their devices using a shaking table that can oscillate at various frequencies and amplitudes. If a device is given an oscillation with amplitude 2 cm, what should its frequency be?

2. A simple pendulum of length L is released from rest from an angle θ_0. a) Assuming the motion of the pendulum is simple harmonic motion, find its speed as it passes through $\theta = 0$. b) Using the conservation of energy, find this speed exactly. c) Show that for small θ_0 the results in a) and b) are the same.

3. The wave function for a traveling harmonic wave on a string is $y(x,t) = 0.001\sin(62x + 300t)$, where y and x are in meters and t is in seconds. In which direction does this wave travel, and what is its speed? What is its wavelength and frequency? What is the maximum displacement of any string segment?

4. A uniform string of length 2m and mass 0.01 kg is clamped at both ends and placed under a tension of 10 N. Sketch the 4th normal mode. What is its frequency?

5. Find the normal modes for the following system:



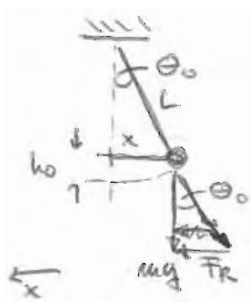
PROBLEM 1

$$x(t) = x_0 \sin \omega t$$
$$\dot{x}(t) = x_0 \omega \cos \omega t$$
$$\ddot{x}(t) = -x_0 \omega^2 \sin \omega t$$

$$\Rightarrow a_{max} = x_0 \omega^2 \qquad \Rightarrow \omega = \sqrt{\frac{a_{max}}{x_0}} = 70/s$$

$$\Rightarrow \nu = \frac{\omega}{2\pi} = 11.1 \text{ Hz}$$

PROBLEM 2



restoring force in x-direction:

$$F_R = -mg \tan\theta_0 \approx -mg\theta_0$$

$$\theta_0 = \frac{x}{L}$$

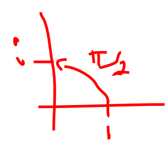
$$m\ddot{x} = -mg\theta_0 = -mg\frac{x}{L} \Rightarrow x(t) = x_0 e^{i\omega t}, \ \omega = \sqrt{\frac{g}{L}}$$

a.) Pendulum released from θ_0

$$x(t) = x_0 e^{i\sqrt{\frac{g}{L}}t}, \quad x_0 = \theta_0 \cdot L$$

$$v(t) = \underbrace{i x_0 \sqrt{\frac{g}{L}}} e^{i\sqrt{\frac{g}{L}}t}$$

$$i = e^{i\pi/2}$$



Speed when pendulum goes through $\theta = 0$ is $v_{max} = \theta_0 L \sqrt{\frac{g}{L}} = \theta_0\sqrt{gL}$

b.) Potential energy converts fully into kinetic energy as pendulum passes through $\theta_0 = 0$.

$$P.E.(\theta_0) = mgh_0 = mg(L - L\cos\theta_0) = mgL(1-\cos\theta_0)$$

$$P.E.(\theta_0) \overset{!}{=} K.E.(\theta = 0)$$

$$\Rightarrow \frac{1}{2}\cancel{m} v_{max}^2 = \cancel{m}gL(1-\cos\theta_0) \Rightarrow v_{max} = \sqrt{2gL(1-\cos\theta_0)}$$

c.) Small θ_0: $\cos\theta_0 \approx 1 - \frac{1}{2}\theta_0^2$

$$\Rightarrow v_{max} \approx \sqrt{2gL(\cancel{1}-\cancel{1}+\tfrac{1}{2}\theta_0^2)} = \theta_0\sqrt{gL}$$

PROBLEM 3

$$y(x,t) = \underbrace{y_0}_{y_0 = 0.001\,m} \sin(\underbrace{k}_{k = 62/m}x + \underbrace{\omega}_{\omega = 300/s} t)$$

wave travels in negative x direction:

$$\varphi = kx + \omega t \stackrel{!}{=} \text{const. for point with constant phase (i.e. point that travels along with wave)}$$

$$\Rightarrow \quad x = \frac{\text{const}}{k}\,m - \underbrace{\frac{\omega}{k}}_{\ominus\, v = -\frac{\omega}{k}} t \qquad ! \qquad t \nearrow \; x \searrow$$

NEGATIVE.

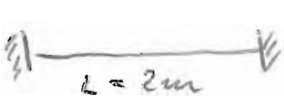
SPEED $\quad v = + \frac{300/s}{62/m} = +4.8\ m/s$

WAVELENGTH $\quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{62} m = 0.1\ m$

FREQUENCY $\quad \nu = \frac{\omega}{2\pi} = \frac{300}{2\pi} Hz = 47.7\ Hz$

MAX DISPL. $\quad y_0$

PROBLEM 4

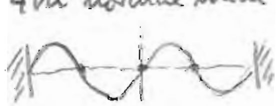


$$\mu = \frac{m}{L} = \frac{0.01}{2}\frac{kg}{m} = 0.005\ kg/m$$

$$T = 10\ N$$

$$\Rightarrow \quad v = \sqrt{\frac{T}{\mu}} = 44.7\ m/s$$

4th normal mode

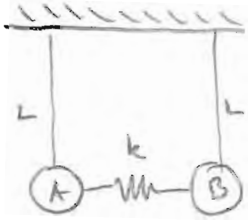


$$k_4 = \frac{2\pi}{\lambda_4} = \frac{2\pi}{L/2}$$

$$\omega_4 = v \cdot k_4 = 2\pi \nu_4 \Rightarrow \nu_4 = \frac{v k_4}{2\pi} = \frac{v \cdot 2\pi}{L/2\ 2\pi}$$

$$= \frac{2v}{L} = 44.7\ Hz.$$

PROBLEM 5



Equations of motion:

$$m\ddot{x}_A = -mg\frac{x_A}{L} + k(x_B - x_A)$$

$$m\ddot{x}_B = -mg\frac{x_B}{L} + k(x_A - x_B)$$

Try normal mode ansatz:

$$x_A = Ae^{i\omega t}, \quad x_B = Be^{i\omega t}$$

Put ansatz into equations of motion:

$$-m\omega^2 A = -mg\frac{A}{L} + k(B-A)$$

$$-m\omega^2 B = -mg\frac{B}{L} + k(A-B)$$

First, eliminate ω^2 to get relationship between A and B:

$$\omega^2 = +\frac{g}{L} + \frac{k}{m}\left(1 - \frac{B}{A}\right) \qquad (*)$$

$$\omega^2 = \frac{g}{L} + \frac{k}{m}\left(1 - \frac{A}{B}\right)$$

$$\Rightarrow \left(1 - \frac{B}{A}\right) = \left(1 - \frac{A}{B}\right)$$

$$\Rightarrow \frac{B}{A} = \frac{A}{B}$$

$$\Rightarrow A = \pm B$$

Insert this result into (*):

$$\omega^2 = \frac{g}{L} + \begin{cases} 0 & \text{for } A = B \\ \frac{k}{m}(1+1) & \text{for } A = -B \end{cases}$$

$$\Rightarrow \omega = \begin{cases} \sqrt{\frac{g}{L}} & \text{for } A = B \\ \sqrt{\frac{g}{L} + \frac{2k}{m}} & \text{for } A = -B \end{cases}$$

This gives the normal mode solutions:

Normal mode 1:

$$x_A = A e^{i\sqrt{\frac{g}{L}}\,t}$$

$$x_B = A e^{i\sqrt{\frac{g}{L}}\,t}$$

$\rightarrow \rightarrow$

Normal mode 2:

$$x_A = A e^{i\sqrt{\frac{g}{L} + \frac{2k}{m}}\,t}$$

$$x_B = -A e^{i\sqrt{\frac{g}{L} + \frac{2k}{m}}\,t}$$

$\rightarrow \leftarrow$