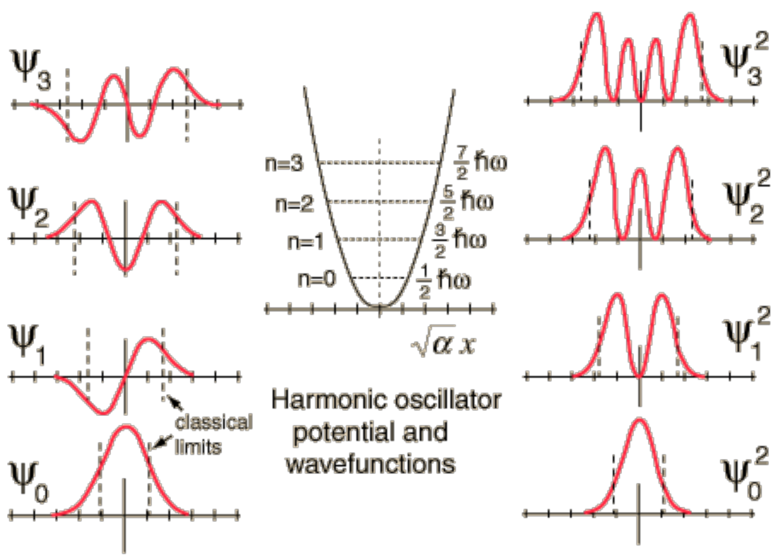
Quantum Harmonic Oscillator: Wavefunctions

The Schrodinger equation for a harmonic oscillator may be solved to give the wavefunctions illustrated below.



For the simple harmonic oscillator (the spring) the potential is

$$V = \frac{1}{2}kx^2 \tag{1}$$

and the classical oscillation frequency is

$$\omega_o = \sqrt{\frac{k}{m}} \qquad \omega_o = 2\pi f \tag{2}$$

We used the uncertainty prinicple to estimate that the particle at the bottom of the well oscillates over a length scale

$$L = \left(\frac{\hbar^2}{mk}\right)^{1/4} \tag{3}$$

The lowest energies are

$$E_n = \left(\frac{1}{2} + n\right)\hbar\omega_o \qquad n = 0, 1, 2, 3\ldots \tag{4}$$

The lowest wave functions are

$$\Psi_0 = \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} e^{-y^2/2} \tag{5}$$

$$\Psi_1 = \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} \sqrt{2}ye^{-y^2/2} \tag{6}$$

$$\Psi_2 = \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} \frac{1}{\sqrt{2}}(2y^2 - 1)e^{-y^2/2} \tag{7}$$

where

$$y \equiv \frac{x}{L} \tag{8}$$