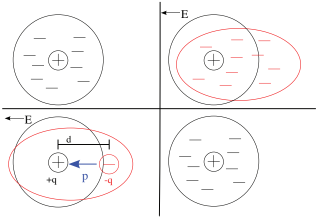
Linear media

- Back when we derived the form of the refractive index, we said $n \equiv \sqrt{\mu\epsilon}/\sqrt{\mu_0\epsilon_0}$.
- We found that $\mu \simeq \mu_0$ for most situations in optics.
- We said that media experience a bulk polarization in response to an electric field of
$$\vec{P} = \epsilon_0 \chi_e \vec{E} \tag{1}$$
where $\epsilon = \epsilon_0(1+\chi_e)$ is an electric permittivity and χ_e is an electric susceptibility. Also the dielectric constant is $K = \epsilon/\epsilon_0$.
- The dielectric response picture that we drew was as follows:



P and n

We then tackled the problem of an incident EM wave on the electron orbitals:

$$\begin{aligned} F_{\text{total}} &= F_{\text{binding}} + F_{\text{damping}} + F_{\text{driving}} \\ m_e\ddot{x} &= -m_e\omega_j^2 x - m_e\gamma_j\dot{x} + q\,E_0 e^{i\omega t} \end{aligned} \tag{2}$$

This gave us

- a mean displacement x of the orbitals;
- thus a dipole moment $p = qx$;
- thus a volume polarization of $P = n_a p$ where $n_a = \rho N_A/A$ is the atom number density.

We then used Eq. 1 to write $\chi_e = \vec{P}/(\epsilon_0\vec{E})$ and found

$$n \simeq \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{1+\chi_e} = \sqrt{1+\frac{\vec{P}}{\epsilon_0\vec{E}}} \simeq 1 + \frac{1}{2}\frac{\vec{P}}{\epsilon_0\vec{E}} \tag{3}$$

all under the assumption of linear dielectric response.

Linear, and beyond

- Again, we made an assumption of linear response of the bulk polarization P in terms of the driving electric field E of $\vec{P} = \epsilon_0 \chi_e \vec{E}$ (Eq. 1).
- What if the response is nonlinear? We might then have

$$\chi_e = \chi_1 + \chi_2 E + \chi_3 E^2 + \dots \tag{4}$$

- Consider the χ_2 term which multiplies E^2. This implies

$$P_2(+E) = \epsilon_0 \chi_2 (+E)^2 \text{ and } P_2(-E) = \epsilon_0 \chi_2 (-E)^2.$$

  - Now if we have a symmetric optical medium, reversing the direction of the electrical field is the same as reversing atom positions $\vec{r} \to -\vec{r}$.
  - Yet $I \propto E^2$; that is, E^2 corresponds to a light irradiance (like intensity).
  - If a medium is symmetric, we expect to get the polarization response for the same irradiance irrespective of a 180° flip of the medium.
  - Therefore we demand $\chi_2 = 0$ in a symmetric medium! (Also $\chi_4 = 0$, $\chi_6 = 0, \dots$)

Calcite

But not all media are symmetric! Calcite (figures from Hecht):

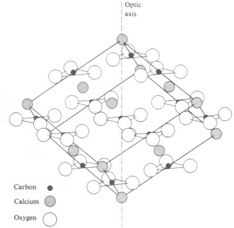


Figure 8.16 Arrangement of atoms in calcite.

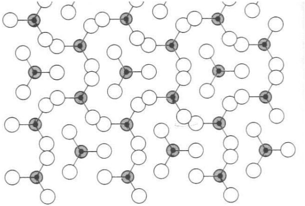


Figure 8.17 Atomic arrangement for calcite looking down the optical axis.

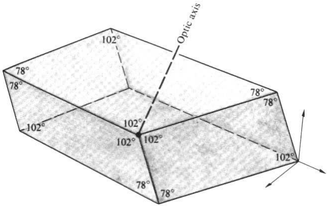
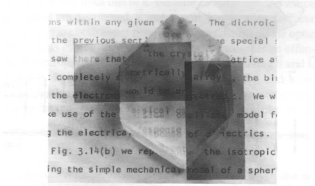


Figure 8.18 Calcite cleavage form.



A calcite crystal (blunt corner on the bottom). The transmission axes of the two polarizers are parallel to their short edges. Where the image is doubled the lower, undeflected one is the ordinary image. Take a long look: there's a lot in this one. (Photo by E.H.)

Second harmonic generation

- If we can allow χ_2 to be nonzero, then we can have a volume polarization response to an applied electric field $E = E_0 \cos(\omega t)$ as follows:

$$
\begin{aligned}
P_2 &= \epsilon_0 \chi_2 E_0^2 \cos^2(\omega t) = \epsilon_0 \chi_2 E_0^2 \left[\frac{1}{2} \Big(1 + \cos(2\omega t) \Big) \right] \\
&= \frac{1}{2} \epsilon_0 \chi_2 E_0^2 + \frac{1}{2} \epsilon_0 \chi_2 E_0^2 \cos(2\omega t)
\end{aligned} \tag{5}
$$

- That is, if we focus a light beam down to a small spot and thus crank up E_0^2 in a nonlinear medium, we can generate polarization changes at twice the driving frequency 2ω!
- Second harmonic generation (SHG)! Frequency doubling! Green light from infrared light!

SHG: dispersion

- With SHG, we now have two different waves in the medium: one at the driving frequency ω, and one at the doubled frequence 2ω.
- But recall that $n \simeq 1 + A + B\omega^2$. The frequencies ω and 2ω will travel at different velocities within the crystal, and thus will eventually get out of phase!
- Since waves travel like e^{-iknz}, the Rayleigh quarter wave criterion for dephasing ($\pi/2$ phase shift) gives a *coherence length* $\ell_{c,\mathrm{SHG}}$ for maximum SHG:

$$kn_\omega(\ell_{c,\mathrm{SHG}}) - kn_{2\omega}(\ell_{c,\mathrm{SHG}}) = \frac{2\pi}{\lambda}|n_\omega - n_{2\omega}|(\ell_{c,\mathrm{SHG}}) = \frac{\pi}{2}$$

$$\text{giving} \qquad \ell_{c,\mathrm{SHG}} = \frac{\lambda}{4|n_\omega - n_{2\omega}|} \tag{6}$$

  This suggests an optimum second harmonic generation medium length.

- In fact, *coherence length* is a somewhat poor choice of words, in that we'll use the same term to describe partial coherence phenomena. . .

SHG $\ell_{c,\mathrm{SHG}}$

Figure from Hecht:

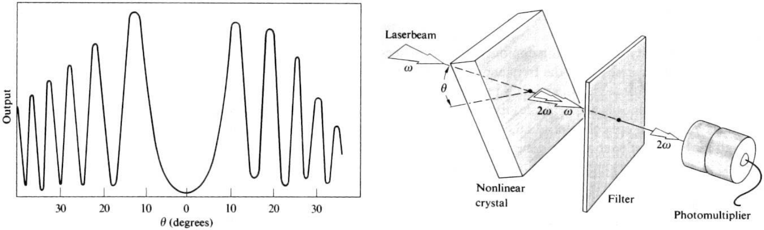


Figure 13.56 Second-harmonic generation as a function of θ for a 0.78-mm thick quartz plate. Peaks occur when the effective thickness is an even multiple of ℓ_c. [From P. D. Maker, R. W. Terhune, M. Nisenoff, and C. M. Savage, *Phys. Rev. Letters* **8**, 21 (1962).]

Frequency mixing

- Let's put two waves of different frequencies into a nonlinear medium: $E = E_{01}\cos\omega_1 t + E_{02}\cos\omega_2 t$.
- Look at the second harmonic term:

$$\epsilon_0\chi_2\left[E_{01}^2\cos^2\omega_1 t + E_{02}^2\cos^2\omega_2 t + 2E_{01}E_{02}\cos\omega_1 t\cos\omega_2 t\right]$$

- Because
  $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ and
  $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$,
  you can see that we'll generate signals at $2\omega_1$ and $2\omega_2$ as before (from the $\cos^2$ terms) and also at $\omega_1 + \omega_2$ and $\omega_1 - \omega_2$.
- This is known as parametric amplification, and it's how we can use a lower frequency but intense laser as a pump for a higher frequency laser (such as in erbium-doped fiber amplifiers).

Pockels effect

- One way to drive nonlinear optical effects is to use a strong DC electric field to bias a medium into being nonsymmetric!
- This happens in crystals that show the piezoelectric effect, where an applied voltage can distort (*i.e.*, expand or contract) the crystal.
- One can change the refractive index along the direction of the applied voltage, as opposed to the orthogonal direction, and thus produce a quarter wave or half wave plate as desired!

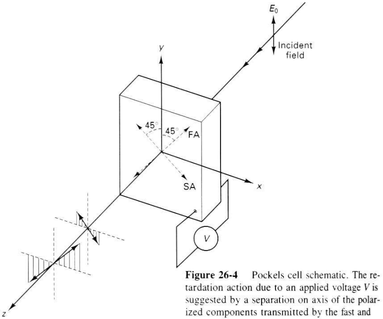


Figure 26-4 Pockels cell schematic. The retardation action due to an applied voltage V is suggested by a separation on axis of the polarized components transmitted by the fast and slow axes of the crystal.

Pockels cell

The net phase shift Φ between the two polarization directions is given by

$$\Phi = \frac{2\pi}{\lambda} r n_0^2 V \tag{7}$$

where r is a linear electrooptic coefficient:

TABLE 26-2 LINEAR ELECTRO-OPTIC COEFFICIENTS FOR REPRESENTATIVE MATERIALS

Material (wavelength if not 633 nm)	Linear electro-optic coefficient[a] r (pm/V)	Refractive index n_0
KH_2PO_4 (KDP)	11	1.51
KD_2PO_4 (KD*P)	24.1	1.51
$(NH_4)H_2PO_4$ (ADP) $\lambda = 0.546$ nm	8.56	1.48
$LiNbO_3$ (lithium niobate)	30.9	2.29
$LiTaO_3$ (lithium tantalate)	30.5	2.18
GaAs (gallium arsenide) $\lambda = 10.6\ \mu m$	1.51	3.3
ZnS (zinc sulfide) $\lambda = 0.6\ \mu m$	2.1	2.36
Quartz	1.4	1.54

[a] Depending on crystalline symmetry, materials have more than one electro-optic coefficient. Only one has been listed here for use in a Pockels cell. These and others may be found in [3].

A Pockels cell uses this with a pair of linear polarizers to make a voltage-controlled optical transmission device:

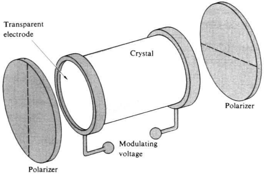


Figure 8.57 A Pockels cell.

Pockels cell: Q switch

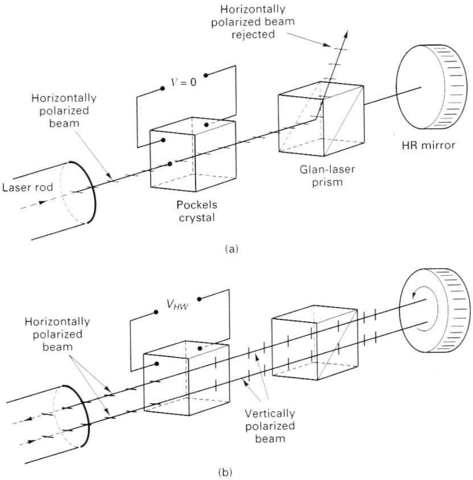


Figure 26-7 Light-controlling action of a Pockels cell, used as a Q-switch. The configuration in (a) produces low transmission at zero-cell voltage and in (b) high transmission at half-wave voltage. In (b), the incident and reflected beams are separated for clarity. Repeated re-entries of the beam into the laser cavity initiates stimulated emission that produces the laser pulse.

Acousto-optical modulators

Consider an acousto-optical modulator (AOM):

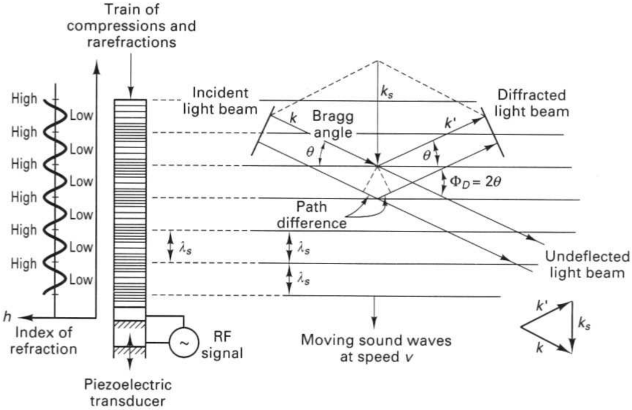


Figure 26-12 Variations in the refractive index of a medium due to the passage of a harmonic acoustic wave (left) and the scattering of an incident optical beam by the induced "planes" (right). The inset shows the relationship of wave vectors required by momentum conservation when the acoustic wave has the direction indicated.

Path difference is $2\lambda_s \sin\theta = 2\lambda_s \sin(\Phi_D/2)$

AOM: Bragg condition

- Constructive interference when $m\lambda =$path difference, or $m\lambda = 2\lambda_s \sin\theta$.
- Conservation of momentum requires $\vec{k_0} = \vec{k'} + \vec{k_s}$ where k_0 is the incident wave, k' is the scattered wave, and k_s is the acoustical wave in the modulator.
- But k_s is much smaller than k_0 or k'! In fact, $k_s \simeq 2k_0 \sin\theta$ or $\lambda_s = \lambda_0/(2\sin\theta)$.
- In addition, conservation of energy says $\hbar\omega_0 = \hbar\omega' + \hbar\omega_s$. That is, not only do we diffract the beam, but we give it a very small adjustment in photon energy or wavelength!

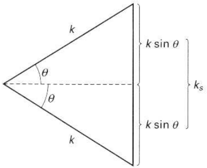


Figure 26-13 Wave vector triangle in the approximation $|k'| = |k| = k$. The angle θ is the Bragg angle. The geometrical relationship of sides is equivalent to the Bragg diffraction condition.

AOM modulators and dumps

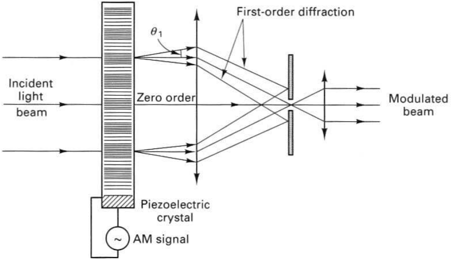


Figure 26-14 Modulation of a light beam by an acousto-optic grating in the Raman-Nath regime. The modulated signal driving the piezoelectric crystal is transferred to the output beam in zero order. Only the zero- and first-order diffracted beams are shown.

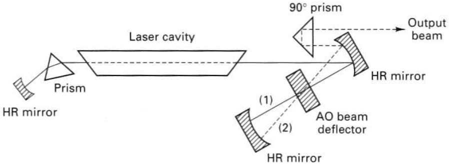


Figure 26-15 Cavity dumping of a laser using an acousto-optic beam deflector. Turning on the acoustic wave deflects the beam outside the laser cavity and initiates cavity dumping.