Quantity	**Symbol**	**Value**
Coulombs Constant	$\frac{1}{4\pi\epsilon_o}$	$8.98 \times 10^9\,\mathrm{Nm^2/C^2}$
Electron Mass	m_e	$9.1 \times 10^{-31}\,\mathrm{kg}$
Proton Mass	m_p	$1.67 \times 10^{-27}\mathrm{kg}$
Electron Charge	e	$-1.6 \times 10^{-19}\,\mathrm{C}$
Electron Volt	eV	$1.6 \times 10^{-19}\,\mathrm{J}$
Permitivity	ϵ_o	$8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{Nm^2}}$
Magnetic Permeability	μ_o	$4\pi \times 10^{-7}\,\mathrm{N \cdot A^2}$
Speed of Light	c	$3.0 \times 10^8\,\mathrm{m/s}$
Planck's Constant	h	$6.6 \times 10^{-34}\mathrm{m^2kg/s}$
Planck's Constant/2π	$\hbar$	$1.05 \times 10^{-34}\mathrm{m^2kg/s}$

Integrals	**Value**	
$\int_{-\infty}^{\infty} du\, e^{-\alpha u^2}$	$\sqrt{\frac{\pi}{\alpha}}$	
$\int_{-\infty}^{\infty} du\, u^2 e^{-\alpha u^2}$	$\frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}}$	
$\int_0^{\infty} du\, u^n e^{-\alpha u}$	$\frac{n!}{\alpha^{n+1}}$	
$\int du\ \sin^2(\alpha u)$	$\frac{u}{2} - \frac{\sin(2\alpha u)}{4\alpha}$	
$\int du\ \cos^2(\alpha u)$	$\frac{u}{2} + \frac{\sin(2\alpha u)}{4\alpha}$	
$\int_{-\frac{1}{2}}^{+\frac{1}{2}} du u^2 \sin^2(n\pi u)$	$\frac{-6+n^2\pi^2}{24n^2\pi^2}$	$n = 2, 4, 6, 8$
$\int_{-\frac{1}{2}}^{+\frac{1}{2}} du u^2 \cos^2(n\pi u)$	$\frac{-6+n^2\pi^2}{24n^2\pi^2}$	$n = 1, 3, 5, 7$
$\int (\cos(\theta))^\alpha \sin(\theta) d\theta$	$\frac{-1}{\alpha+1}\,(\cos(\theta))^{\alpha+1}$	
$\int (\sin(\theta))^\alpha \cos(\theta) d\theta$	$\frac{+1}{\alpha+1}\,(\sin(\theta))^{\alpha+1}$	

1. Consider the hydrogen atom.

1. (Numbers) An electrons starts in the $n = 3$ orbit of the hydrogen atom. Draw an energy level diagram exhibiting the subsequent decays to the ground state. Determine the energies of all the photons which are emitted.

2. (Number and Picture) Determine the wavelength of the most energetic photon which is emitted. Draw a schematic picture (or set of pictures if needed) which is *approximately to scale* showing the wavelength of the light, the atom, and the electron Compton length.

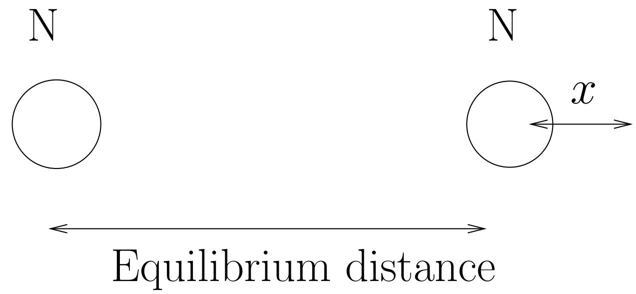
3. (Symbol) Starting from the Coulomb Law and the Bohr Quantization condition $L = n\hbar$ where L is the angular momentum of the electron, determine the allowed quantized radii in terms of fundamental constants. Consider the proton to be infinitely heavy compared to the electron.

4. (Symbol) Derive the velocity (or v/c if you prefer) of the electron in the n-th orbit in terms of fundamental constants

5. (Symbol) Determine the time it takes an electron in the n-th orbit to complete to make one complete revolution in terms of fundamental constants

6. (Numbers) For $n = 1$ evaluate your answers for parts 3,4, and 5 numerically.

2. A schematic of diatomic nitrogen are shown below.



For simplicity we will keep one of the Nitrogen nucleus as fixed and consider only the vibrations of the other nucleus. As the two nuclei are compressed from there equilibrium position the electrons rapidly adjust giving rise to a net force due to the Coulomb interaction, which tends to restore the nuclei to their equilibrium position. This force is proportional to the displacement x of the nuclei from there equilibrium position, i.e. $F = -kx$ and the potential energy of the nucleus is $\frac{1}{2}kx^2$. We can estimate the magnitude of the spring constant as follows: When the molecule is compressed a distance of order the Bohr radius the coulomb potential energy of the compressed diatomic molecule is energy is of order

$$\frac{e^2}{4\pi\epsilon_o}\frac{1}{a_o}$$

For the symbolic results of this problem express your symbolic results in terms of fundamental constants, and M the nuclear mass, and the Bohr Radius a_o.

1. (Symbol) Estimate the magnitude of the spring constant.
2. (Number) Determine the ratio $\frac{m}{M}$ were m_e is the electron mass and M is the nuclear mass of ${}^{14}_{7}N$ (which therefore has 14 nucleons).
3. (Symbol) Due to quantum mechanics, the position of the vibrating nucleus is never exactly at one place but rather nuclear the wave function is spread out over a certain length scale.) Determine this length scale.
4. (Symbol and Number) Determine the ratio between the length scale found in part #3 and the Bohr radius. Evaluate part this ratio numerically

If you have time do the following:

6. (Symbol) Estimate the time it takes for one molecular oscillation.
7. (Symbol and then Number) Estimate the ratio between this time scale and the time it takes for an electron to complete one Bohr orbit for the $n = 1$ state. You can use the results of problem **1**. for instance. Evaluate this ratio numerically

If you still have time do the following:

10. (Symbol and Number) Determine the ratio between the vibrational energy of the nuclei and the Rydberg energy $R_\infty = \frac{\hbar^2}{2ma_o^2}$ and evaluate it numerically.

3. Consider an electron in the box. For symbolic results express your results in terms of fundamental constants and L the length of the box found in part 1.

1. (Number) When an electron decays from the $n = 2$ state down to the $n = 1$ state it emits a photon. Determine the size of the box, L for which the energy of this photon equals the energy of the analogous $(n = 2) \to (n = 1)$ transition in Hydrogen.

2. (Graph) For this box draw the second excited state of the electron and associated probability distribution.

3. (Number) Based on your graph estimate the probability that the electron is between $-0.1L$ and $0.1L$. Explain your reasoning .

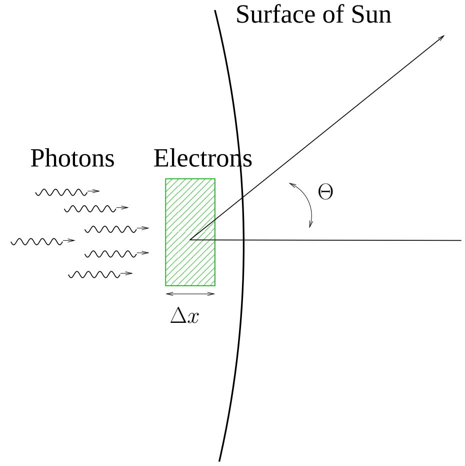
4. (Symbol) For the second excited state there is a time dependent wave function which is a solution to the time dependent Schrödinger equation. Write down this wave function. Ask me if you do not know the answer to this.

5. (Symbol) Verify that it is a solution to the the time dependent Schrödinger equation and determine the appropriate energy.

6. (Symbol) Rewrite the time dependent second excited state as a sum of complex exponentials $e^{i\phi}$

7. (Symbol and then number) Determine the average kinetic energy of the second excited state and evaluate it numerically.

4. Thompson scattering is when a low energy photon scatters off an electron at rest. This scattering process is of central importance in addressing whether a photon escapes from the sun. A schematic of a beam of photons escaping the sun is shown below



If $N_{\rm in}$ photons scatter into a slab of electrons of thickness Δx. The number of photons dN scattered into a solid angle $d\Omega$ is given by Thompson scattering

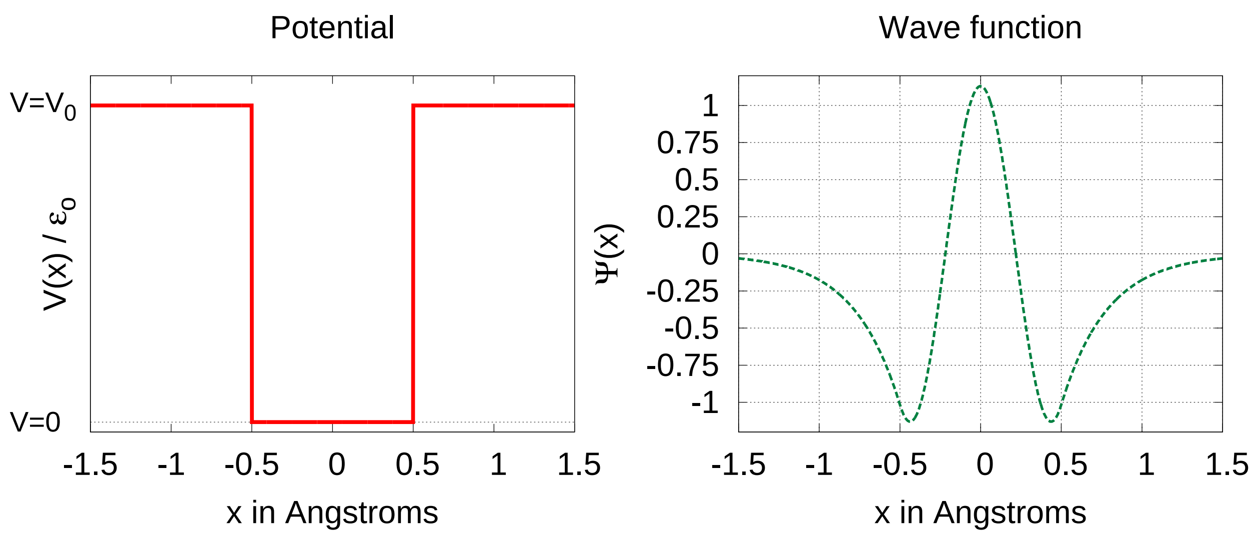
$$dN = [N_{\rm in}\rho\Delta x] \times A(1-\cos^2\Theta) \times d\Omega \qquad \text{with} \qquad A = 8.0\times 10^{-30}\,\mathrm{m}^2 = \left(\frac{\alpha\hbar}{mc}\right)^2$$

Here A is a constant dimensions of area. ρ is the number of electrons per unit volume. The sun is made up predominantly of ionized hydrogen with a mean mass density of $10^3\,\mathrm{kg/m^3}$. There is of course one electron per ionized hydrogen and the photons scatter predominantly off these electrons. For the symbolic answers express your results in terms of fundamental constants and, $A, \Theta, \Delta\Omega, \Delta x$, and ρ.

1. (Number) Determine the number of electrons per volume ρ of electrons in the sun.
2. (Number) If we place the sun at the origin, estimate the solid angle subtended by the earth given that it takes 8 minutes for light from the sun to reach the earth and that the earths radius is about equal to the distance from here to California 6000 km.

Now consider what happens when a beam of $N_{\rm in}$ photons traverse a distance short distance Δx through the edge of the sun as shown below.

3. (Symbol) Determine the total number of photons $N_{\rm back}$ which are scattered backwards (i.e with $\Theta > 90^o$) back into the sun as the beam of N photons traverses the plasma of size Δx.
4. (Symbol then Number) $\Delta N_{\rm back}/N$ is the fraction of photons in the beam which scatter backwards as the beam of photons traverses a distance Δx through the the sun. Estimate the length over which this fraction approaches 100%. Work analytically and then substitute numbers. Only photons which are produced within a distance of a couple of these lengths will actually escape the sun.



The figure above shows a potential and a corresponding stationary wave functionfor an electron in the potetial well. (Stationary wave functions also known as a stationary state or an `eigenfunction`, but these are just words) .

1. Is this the ground, first, second, third, fourth, or . . . excited state
2. Draw the ground state and the first excited state and the corresponding probability densities.
3. Estimate the energy (in eV) of this state from this figure.
4. Estimate the potential V_o (in eV) from this figure.
5. Estimate the probability of finding the electron in the classically forbidden region.
6. Where are the inflection points of the wave function, i.e. where does the concavity change sign. Why does the wave function have an inflection point at these points – explain using the Schrödinger equation