Equation sheet for PHY 300 final exam (this version as of December 14, 2008). You will be given this sheet in class.

$$A\cos\omega_1 t + A\cos\omega_2 t = 2A\cos\Big(\frac{\omega_1+\omega_2}{2}t\Big)\cos\Big(\frac{\omega_1-\omega_2}{2}t\Big) \;\Rightarrow\; 2Ae^{i\bar{\omega}t}\cos(\Delta\omega\, t)$$

Random phases: $|R| = \sqrt{\sum_{i=1} NA_i^2} \Rightarrow \sqrt{N}|A|$ Identical phases: $|R| = N|A|$

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t \qquad \text{(4-7)}$$

$$\omega_0^2 = \frac{k}{m} = \frac{g}{\ell} \qquad \gamma = \frac{b}{m}\ \text{(3-30)} \qquad Q = \frac{\omega_0}{\gamma}\ \text{(3-37)}$$

$$A(\omega) = \frac{F_0/m}{\Big[(\omega_0^2-\omega^2)^2 + (\gamma\omega)^2\Big]^{1/2}} = \frac{F_0}{k}\frac{\omega_0/\omega}{\Big[(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0})^2 + \frac{1}{Q^2}\Big]^{1/2}}\ \text{(4-11; 4-14)}$$

$$\tan\delta(\omega) = \frac{\gamma\omega}{\omega_0^2-\omega^2} = \frac{1/Q}{\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}}\ \text{for } A\cos(\omega t - \delta)\ \text{(4-11; 4-14)}$$

$$P_{\text{max}} = \frac{QF_0^2}{2m\omega_0}\ \text{(4-24)} \qquad \bar{P}(\omega) = \frac{F_0^2\omega_0}{2kQ}\frac{1}{(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0})^2 + \frac{1}{Q^2}}\ \text{(4-23}$$

$$\omega^2 = \omega_0^2 - \frac{\gamma^2}{4}\ \text{(3-34)} \qquad \Delta\omega = \frac{\omega_0}{2Q}\ \text{(4-27)} \qquad E(t) = E_0 e^{-\gamma t}\ \text{(3-36)}$$

$$\omega'^2 = \omega_0^2 + 2\omega_c^2 \qquad \omega_n = 2\omega_0\sin\Big[\frac{n\pi}{2(N+1)}\Big]\ \text{(5-25)} \qquad A_{pn} = C_n\sin\Big[\frac{pn\pi}{N+1}\Big]\ \text{(5-26)}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v_p^2}\frac{\partial^2\psi}{\partial t^2}\ \text{(7-9)} \qquad v_p = \frac{\omega}{k}\ \text{(7-27)} \qquad v_g = \frac{d\omega}{dk}\ \text{(7-28)} \qquad v_p = \sqrt{\frac{T}{\mu}}$$

$$\sin\theta \simeq \theta - \frac{\theta^3}{3!} \qquad \cos\theta \simeq 1 - \frac{\theta^2}{2!} \qquad e^x \simeq 1 + x \qquad \sin^2\frac{\beta}{2} = \frac{1}{2}(1-\cos\beta)$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

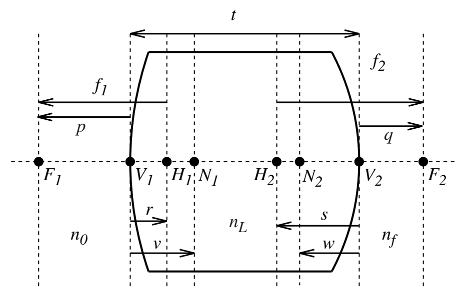
$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$+R$ is with center of curvature "downstream"

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2-n_1}{R_1} \qquad \frac{1}{f} = \frac{n_2-n_1}{n_1}\Big(\frac{1}{R_1} - \frac{1}{R_2}\Big) \qquad \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \frac{1}{f} = \frac{2}{R} \qquad m = -\frac{n_1 s'}{n_2 s}$$

$$\mathcal{T} = \begin{bmatrix} 1 & 0 \\ L & 1 \end{bmatrix},\ \mathcal{R} = \begin{bmatrix} \frac{n}{n'} & \frac{1}{R}\left(\frac{n}{n'} - 1\right) \\ 0 & 1 \end{bmatrix},\ \mathcal{L} = \begin{bmatrix} 1 & \frac{2}{R} \\ 0 & 1 \end{bmatrix},\ \mathcal{F} = \begin{bmatrix} 1 & -\frac{1}{f} \\ 0 & 1 \end{bmatrix}\ \text{for}\ \begin{bmatrix} \alpha \\ y \end{bmatrix}$$

$$
\begin{aligned}
f_1 &= \frac{n_0/n_f}{B} \\
f_2 &= \frac{-1}{B} \\
r &= \frac{A - n_0/n_f}{B} \\
s &= \frac{1-D}{B} \\
v &= \frac{A-1}{B} \\
w &= \frac{n_0/n_f - D}{B} \\
1 &= -\frac{f_1}{s} + \frac{f_2}{s'} \\
m &= -\frac{n_0 s}{n_f s'}
\end{aligned}
$$



$$
\begin{bmatrix} A & B \\ C & D \end{bmatrix}
$$

$$
a(Q) = -\frac{y^4}{8}\Big[\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2 + \frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\Big] \qquad b_y = \frac{s'}{n_2}\frac{da}{dy} \qquad b_z = \frac{s'}{y} b_y \qquad \sigma = \frac{R_2+R_1}{R_2-R_1}
$$

$$
\frac{1}{f_D} = (n_{1D}-1)\frac{2}{|r_1|}\frac{V_1-V_2}{V_1} \qquad \frac{1}{r_{22}} = \frac{1}{|r_1|}\left[2\frac{(n_{1D}-1)}{(n_{2D}-1)}\frac{V_2}{V_1} - 1\right] \qquad V \equiv \frac{n_D - 1}{n_F - n_C}
$$

$$
\lambda_F = 486.1 \text{ nm} \qquad \lambda_D = 587.6 \text{ nm} \qquad \lambda_C = 656.3 \text{ nm}
$$

$$
\begin{aligned}
\text{TE: } r_\perp &= \frac{\cos\theta - \sqrt{n^2-\sin^2\theta}}{\cos\theta + \sqrt{n^2-\sin^2\theta}} = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)} \\
\text{TM: } r_\parallel &= \frac{n^2\cos\theta - \sqrt{n^2-\sin^2\theta}}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)} \\
\text{TE: } t_\perp &= \frac{2\cos\theta}{\cos\theta + \sqrt{n^2-\sin^2\theta}} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)} \\
\text{TM: } t_\parallel &= \frac{2n\cos\theta}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}
\end{aligned}
$$

$$
R = r^2 \qquad T = n\frac{\cos\theta_t}{\cos\theta_i}t^2 \qquad \theta_c = \arcsin(n) \qquad \theta_p = \arctan(n) \qquad r = -r' \qquad t't = 1 - r^2
$$

Beyond critical angle:

$$
\begin{aligned}
\text{TE: } r_\perp &= \frac{\cos\theta - i\sqrt{\sin^2\theta - n^2}}{\cos\theta + i\sqrt{\sin^2\theta - n^2}} \\
\text{TM: } r_\parallel &= \frac{n^2\cos\theta - i\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta + i\sqrt{\sin^2\theta - n^2}}
\end{aligned}
$$

$$\varphi_{\text{TE}} = 2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}\right) \text{ for } \theta > \theta_c$$

$$\varphi_{\text{TM}} = 2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}\right) \text{ for } \theta > \theta_c$$

$$I_r = I_0\frac{4r^2}{(1+r^2)^2} \qquad I_t = I_0\frac{1}{1+F\sin^2(\delta/2)} \qquad F \equiv \frac{4r^2}{(1-r^2)^2} \qquad \delta = 4\pi\frac{\ell}{\lambda}\frac{n_t}{\sqrt{1-(n/n_t)^2\sin^2\theta_i}}$$

$$E = E_0 e^{-x/\alpha} \text{ with } \alpha = \frac{\lambda}{2\pi\sqrt{\sin^2\theta/n^2 - 1}} \qquad \sigma \ll \omega\epsilon\text{: } \alpha = \frac{1}{\sigma}\sqrt{\frac{\epsilon}{\mu}} \qquad \sigma \gg \omega\epsilon\text{: } \alpha = \frac{1}{2\sqrt{2\sigma\mu\omega}}$$

$$\text{LHC: } \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} \qquad \text{RHC: } \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix} \qquad \text{QWP, SA horizontal: } \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$

$$\text{Linear polarizer, TA } \theta\text{: } \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$z^3 \gg \frac{r^4}{2\lambda} \qquad z \gg 4\frac{r^2}{\lambda} \qquad f = \frac{\theta}{\lambda} \qquad f_{\max} = \frac{1}{2\Delta}$$

$$\begin{aligned}\psi(x,y,z) &= \psi_0\frac{\lambda}{z}\frac{1}{A}\exp\left[-i\frac{2\pi z}{\lambda}\right]\int_{x_0}\int_{y_0}\tilde{g}(x_0,y_0)\exp\left[-i\pi\frac{(x-x_0)^2+(y-y_0)^2}{\lambda z}\right] \\ &= \psi_0\frac{\lambda}{z}\frac{1}{A}\exp\left[-i\frac{2\pi z}{\lambda}\right]\exp\left[-i\pi\frac{x^2+y^2}{\lambda z}\right]\int_{x_0}\int_{y_0}\tilde{g}(x_0,y_0)\exp\left[-i\pi\frac{x_0^2+y_0^2}{\lambda z}\right]\exp\left[i2\pi\frac{xx_0+yy_0}{\lambda z}\right]\end{aligned}$$

$$\psi(x,y,z) \simeq \psi_0\frac{\lambda}{z}\frac{1}{A}\int_{x_0}\int_{y_0}\tilde{g}(x_0,y_0)\exp\left[i2\pi(\frac{xx_0}{\lambda z}+\frac{yy_0}{\lambda z})\right]dx_0\,dy_0$$

$$\psi = \psi_0\mathcal{F}^{-1}\left\{\mathcal{F}\left\{\tilde{g}(x_0,y_0)\right\}\cdot\exp[i\pi\lambda z(f_x^2+f_y^2)]\right\}$$

$$\frac{\sin^2\beta}{\beta^2}\cos^2\alpha \text{ with } \beta = \pi b f \text{ and } \alpha = \pi a f \qquad \sin\theta = \frac{\lambda}{b} \qquad \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin(N\alpha)}{\sin\alpha}\right)^2$$

$$\left(\frac{2J_1(\nu)}{\nu}\right)^2 \text{ with } \nu = 2\pi a f \text{ and } \nu_{\text{first min}} = 1.22\pi \qquad r_{\text{first min}} = 0.61\frac{\lambda}{\text{N.A.}} \text{ with N.A.} = n\sin\theta$$

$$m\lambda = 2a\sin\theta_b \text{ (Littrow)} \qquad m\lambda = a\sin(2\theta_b) \text{ (normal)}$$

$$\text{N.A.} = \sqrt{n_1^2 - n_2^2} \qquad L_s = d\sqrt{\left(\frac{n_1}{n_0\sin\theta}\right)^2 - 1}$$

$$m\left[\varphi_r + 2\pi\frac{n_1}{\lambda}(\sqrt{L_{s,\text{ max}}^2 + d^2} - L_s)\right] \le \frac{\pi}{2}$$

$$\ell_{c,\text{SHG}} = \frac{\lambda}{4|n_\omega - n_{2\omega}|} \qquad \Phi = \frac{2\pi}{\lambda}rn_0^2V \qquad 2\lambda_s\sin(\theta) = 2\lambda_s\sin(\Phi_D/2) = \lambda$$

$$\epsilon_0 = 8.85\times10^{-12}\ \text{C}^2\cdot\text{N}^{-1}\cdot\text{m}^{-2} \qquad \mu_0 = 4\pi\times10^{-7}\ \text{N}\cdot\text{A}^{-2} \qquad g = 9.8\ \text{m/sec}^2$$