ESE 271 Third Exam Name:

Spring, 2003 ID Number:

Do not place your answers on this front page.

Prob. 1 (20 points):

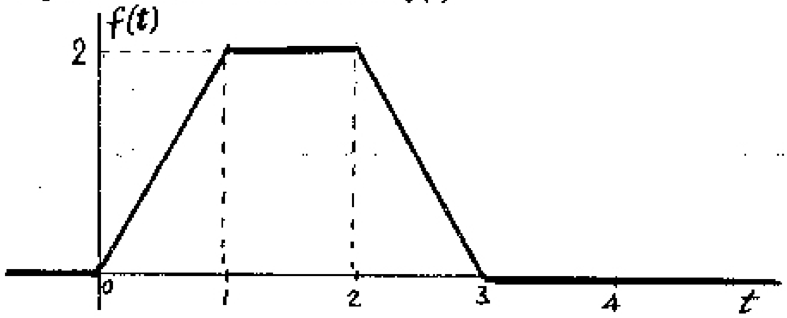
Prob. 2 (20 points):

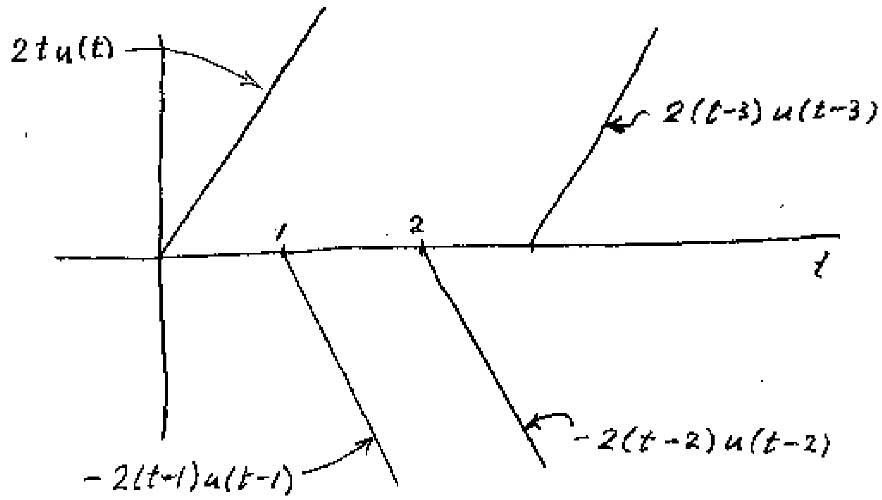
Prob. 3 (30 points):

Prob. 4 (30 points):

Prob. 1 (20 points):

Find the Laplace transform of the function $f(t)$.





So,

$$F(s) = \mathcal{L}\left(2t\,u(t) - 2(t-1)\,u(t-1) - 2(t-2)u(t-2) + 2(t-3)u(t-3)\right)$$

$$= \frac{2}{s^2}\left(1 - e^{-s} - e^{-2s} + e^{-3s}\right)$$

Prob. 2 (20 points):

Given an input-output system, when the input $f(t) = u(t)$, we have the output $g(t) = e^{-2t}$ for $t > 0$. Find $g(t)$ for $t > 0$ when $f(t) = tu(t)$. ($u(t)$ is the unit step function.)

FIRST CASE: $\quad F(s) = \frac{1}{s}$, $\quad G(s) = \frac{1}{s+2}$

THEREFORE, $\quad H(s) = \frac{G(s)}{F(s)} = \frac{s}{s+2}$

SECOND CASE:

$$G(s) = F(s)\ H(s) = \frac{1}{s^2} \times \frac{s}{s+2} = \frac{1}{s(s+2)}$$

$$= \frac{A}{s} + \frac{B}{s+2}$$

$A = \frac{1}{2}$, $\quad B = -\frac{1}{2}$

So, $\quad g(t) = \frac{1}{2} u(t) - \frac{1}{2} e^{-2t} u(t)$

OR

$g(t) = \frac{1}{2} - \frac{1}{2} e^{-2t}$ $\quad$ FOR $t > 0$.

Prob. 3 (~~25~~ 30 points):

Find the inverse Laplace transform of

$$F(s) = \frac{1}{s^2(s^2+4s+8)}$$

POLES AT $P_0 = 0$ AND:

$$\left.\begin{matrix} P_1 \\ P_2 \end{matrix}\right\} = \frac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$$

So,

$$F(s) = \frac{1}{s^2(s+2-j2)(s+2+j2)} = \frac{A_1}{s^2} + \frac{A_2}{s} + \frac{B}{s+2-j2} + \frac{B^*}{s+2+j2}$$

$$A_1 = \left.\frac{1}{s^2+4s+8}\right|_{s=0} = \frac{1}{8}$$

$$A_2 = \left.\frac{d}{ds}\left(\frac{1}{s^2+4s+8}\right)\right|_{s=0} = (-1)\left.\frac{2s+4}{(s^2+4s+8)^2}\right|_{s=0} = -\frac{4}{64} = -\frac{1}{16}$$

$$B = \frac{1}{(-2+j2)^2(-2+j2+2+j2)} = \frac{1}{(-j8)(j4)} = \frac{1}{32}$$

$$\left(\begin{matrix} (-2+j2) = 2\sqrt{2}\angle{-4} \\ (-2+j2)^2 = 8\angle{-90^\circ} \end{matrix} \right.$$

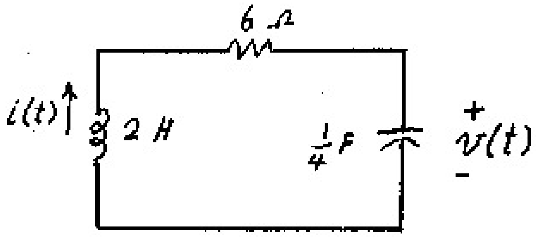
So,

$$f(t) = \frac{1}{8}\, t\, u(t) - \frac{1}{16}\, u(t) + 2\left|\frac{1}{32}\right| e^{-2t} \cos 2t \; u(t)$$

$$= \left(\frac{1}{8} t - \frac{1}{16} + \frac{1}{16} e^{-2t} \cos 2t\right) u(t)$$

Prob. 4 (30 points):

Find $i(t)$ for $t > 0$ when $i(0+) = 2$ A and $v(0+) = 3$ V.



THE INTEGRO DIFFERENTIAL EQUATION IS:

$$2\frac{di}{dt} + 6i + \frac{1}{\frac{1}{4}}\int_0^t i(x)\,dx + 3 = 0$$

APPLY $\mathcal{L}$

$I = I(s)$

$$2sI - 2\times 2 + 6I + \frac{4}{s}I + \frac{3}{s} = 0$$

$$I\left(2s + 6 + \frac{4}{s}\right) = 4 - \frac{3}{s}$$

$$I = \frac{4s-3}{2s^2+6s+4} = \frac{4s-3}{2(s+1)(s+2)}$$

$$\left.\begin{matrix}P_1\\P_2\end{matrix}\right\} = \frac{-6 \pm \sqrt{36-32}}{4} = -1, -2$$

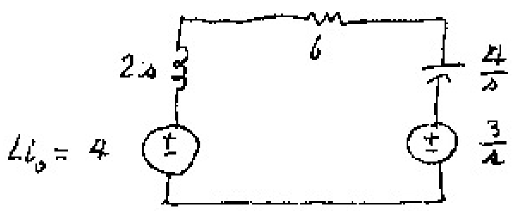
$$= \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \frac{-7}{2}, \quad B = \frac{-11}{-2} = \frac{11}{2}$$

So,

$$i(t) = -\frac{7}{2}e^{-t} + \frac{11}{2}e^{-2t} \quad \text{for } t > 0$$

ANOTHER WAY: USE THE TRANSFORMED CIRCUIT:



By KVL:

$$-4 + 2sI + 6I + \frac{4}{s}I + \frac{3}{s} = 0$$

$$I = \frac{4s-3}{2s^2+6s+4}$$

ETC.