PHY 251 Fall 2009: Exam 1 (Thursday, October 1).

1. While staring out the window, you are somewhat astonished to see a spear go flying past, and you snap a photograph. From your photo you see that it is 2 meter long and tilted up at an angle of 25° from the x direction, and it goes flying past at $v_x = 1.5 \times 10^8$ m/sec and $v_y = 0$. What is length, and tilt angle, of the spear in its own inertial reference frame?
Answer: Because of relative motion in the x direction, we see a contracted version of the spear's length. There's no motion in the y direction, so we see the true length in that direction. That is, we see $x' = x_0/\gamma$ and $y' = y_0$ and from that we can say $r' = \sqrt{x'^2 + y'^2}$ and $\tan(\theta') = y'/x'$. Note that

$$\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1 - (\frac{1.5\times 10^8}{3\times 10^8})^2} = 1/\sqrt{1-(\frac{1}{2})^2} = 1/\sqrt{3/4} = 2/\sqrt{3} = 1.155.$$

We can relate these to dimensions in the spear's frame of reference as follows:

$$\begin{aligned}
r_0 &= \sqrt{x_0^2 + y_0^2} = \sqrt{x'^2\gamma^2 + y'^2} \\
&= \sqrt{[2\cos(25°)]^2(1.155)^2 + [2\sin(25°)]^2} = 2.258 \text{ meters} \\
\tan\theta_0 &= \frac{y_0}{x_0} = \frac{y'}{\gamma x'} = \frac{1}{\gamma}\tan\theta' \\
\text{or } \theta_0 &= \tan^{-1}[\tan(\theta')/\gamma] = \tan^{-1}[\tan(25°)/(1.155)] = 22.0°.
\end{aligned}$$

That is, the meter stick is longer than 2 meters, and is rotated by a smaller angle when viewed in its own frame.

2. An observer on Earth observes two spacecraft moving in the *same* direction toward the earth. Spacecraft A appears to have a speed of 0.50c, and spacecraft B appears to have a speed of 0.80c. What is the speed of spacecraft A measured by an observer in spacecraft B?
Answer: From our frame, we have spacecraft A moving at $v_A = 0.50c = v_{1,x}$, and spacecraft B moving at $v_B = 0.80c$. If we make a velocity shift $v = v_B$ to go into spacecraft B's frame, the velocity $v_{2,x}$ that spacecraft B sees spacecraft A traveling at is given by

$$v_{2,x} = \frac{v_{1,x} - v}{1 - vv_{1,x}/c^2} = \frac{0.50c - 0.80c}{1 - (0.80c)(0.50c)/c^2} = \frac{-0.30c}{1 - 0.4} = -0.5c.$$

3. An electron is found to have a momentum of 250 keV/c. A photon is also found to have the same momentum. What's the kinetic energy and velocity of the electron? The photon?
Answer: For the photon, $p = E/c = (250 \text{ keV}/c)/c = 250 \text{ keV}/c^2$ and it travels at the speed of light. For the electron, $p = \gamma\beta m_e c$ and γ is a function of β so we have

$$\begin{aligned}
\gamma\beta &= \frac{p_e}{m_e c} = \frac{\beta}{\sqrt{1-\beta^2}} \\
(\frac{p_e}{m_e c})^2 &= \frac{\beta^2}{1-\beta^2} = \frac{1}{1/\beta^2 - 1} \\
\frac{1}{\beta^2} - 1 &= (\frac{m_e c}{p_e})^2 \\
\beta &= 1/\sqrt{1 + (\frac{m_e c}{p_e})^2} = 1/\sqrt{1 + (511/250)^2} = 0.4395
\end{aligned}$$

giving $\gamma = 1/\sqrt{1-\beta^2} = 1.113$, and $E_k = (\gamma - 1)m_e c^2 = (1.113-1)(511 \text{ keV}) = 57.7$ keV.

4. When cesium metal is illuminated with light of wavelength 300 nm, the photoelectrons emitted have a maximum kinetic energy of 2.23 eV. Find (a) the work function of cesium and (b) the stopping potential if the incident light has a wavelength of 400 nm.
Answer: The work function of cesium is found from

$$\varphi = \frac{hc}{\lambda} - E_k = \frac{1240 \text{ eV}\cdot\text{nm}}{300 \text{ nm}} - (2.23 \text{ eV}) = 1.90 \text{ eV}.$$

If we instead use a wavelength of 400 nm, the kinetic energy is

$$E_k = \frac{hc}{\lambda} - \varphi = \frac{1240 \text{ eV}\cdot\text{nm}}{400 \text{ nm}} - (1.90 \text{ eV}) = 1.20 \text{ eV}$$

so a stopping potential of 1.20 volts is required to counter the electron's kinetic energy.

5. A 100 keV photon undergoes Compton scattering at an angle of 40°. Find the energy of the scattered photon, and the energy and angle of the recoil electron.
Answer: The energy of the scattered photon can be found from the Compton formula:

$$\begin{aligned}
\lambda_s - \lambda_0 &= \frac{h}{m_e c}(1-\cos\theta) \\
\frac{hc}{E_s} - \frac{hc}{E_0} &= \frac{hc}{m_e c^2}(1-\cos\theta) \\
E_s &= 1/\Big[\frac{1}{E_0} + \frac{1-\cos\theta}{m_e c^2}\Big] = 1/\Big[\frac{1}{100 \text{ keV}} + \frac{1-\cos 40^\circ}{511 \text{ keV}}\Big] = 95.6 \text{ keV}
\end{aligned}$$

The energy of the scattered electron is then 100-95.6=4.4 keV, so that it has $\gamma = 1 + E_k/(m_e c^2) = 1.0086$ and

$$\beta = \sqrt{1 - (1 + \frac{E_k}{m_e c^2})^{-2}} \simeq \sqrt{1 - (1 - \frac{2E_k}{m_e c^2})} \simeq \sqrt{\frac{2E_k}{m_e c^2}} = \sqrt{\frac{2\cdot 4.4 \text{ keV}}{511 \text{ keV}}}$$

or $\beta \simeq 0.131$ so that the electron's net momentum is

$$p_e = \gamma\beta m_e c^2 = (1.0086)(0.131)(511 \text{ keV}/c^2)c = 67.5 \text{ keV}/c.$$

Finally, we can get the angle φ of the scattered electron setting the $\hat{y}$ momenta to be equal and opposite:

$$\begin{aligned}
(E_s/c)\sin\theta &= p_e \sin\varphi \\
\varphi &= \arcsin\Big[\frac{E_s/c}{p_e}\sin\theta\Big] = \arcsin\Big[\frac{95.6 \text{ keV}/c}{67.5 \text{ keV}/c}\sin 40^\circ\Big] = 65.6^\circ
\end{aligned}$$