ESE 271 Third Exam Name:

Fall, 2003 ID Number:

Do not place your answers on this front page.

Prob. 1: (10 points)

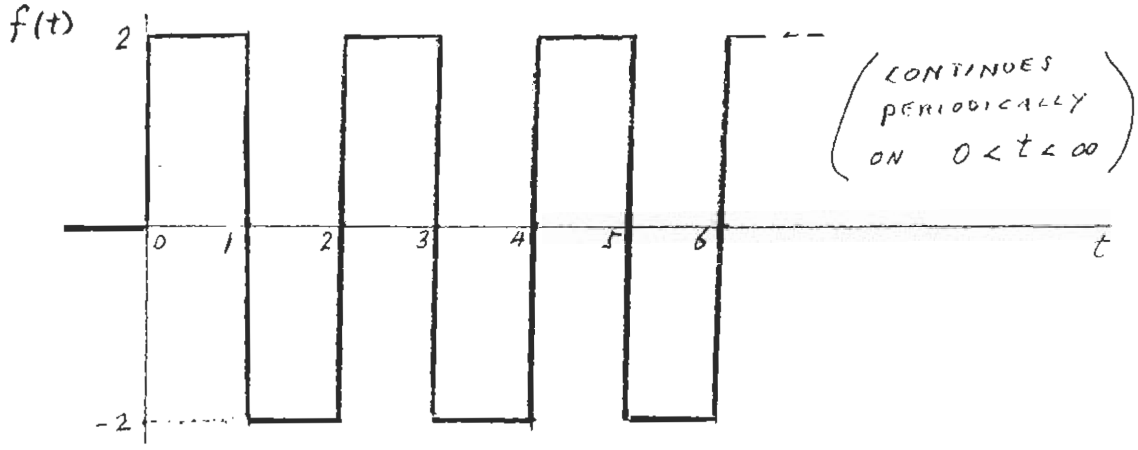
Prob. 2: (30 points)

Prob. 3: (30 points)

Prob. 4: (30 points)

Prob. 1. (10 points):

Find the Laplace transform of the following periodic wave.



FOR $0 < t < 2$, $\quad f(t) = 2u(t) - 4u(t-1) + 2u(t-2)$

THE LAPLACE TRANSFORM OF THIS IS

$$\frac{2}{s} - \frac{4}{s}e^{-s} + \frac{2}{s}e^{-2s}$$

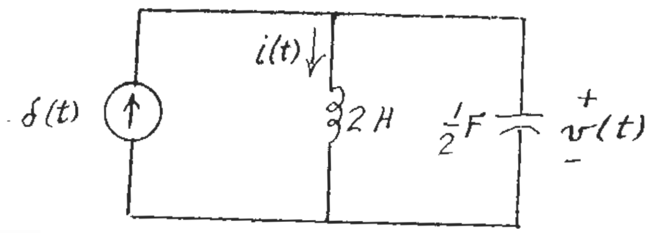
So,

$$F(s) = \mathcal{L}\, f(t)$$

$$= \left(\frac{2}{s} - \frac{4}{s}e^{-s} + \frac{2}{s}e^{-2s}\right)\frac{1}{1-e^{-2s}}$$

Prob. 2. (30 points)

Find $v(t)$ for $t > 0$.



$i(0+) = 6\ A$

$v(0+) = 4\ V$

FIRST METHOD: USE INTEGRO DIFFERENTIAL EQUATIONS,

KCL: $\delta(t) = \frac{1}{2}\int_0^t v(x)\,dx + i(0+) + \frac{1}{2}\frac{dv}{dt}$

So,

$V = V(s)$

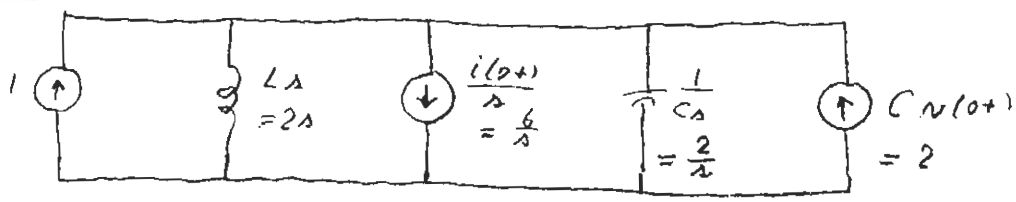
$$1 = \frac{V}{2s} + \frac{6}{s} + \frac{1}{2}(sV - 4)$$

$$V\left(\frac{1}{2s} + \frac{s}{2}\right) = 1 - \frac{6}{s} + 2$$

$$V = \frac{6s - 12}{s^2+1} = 6\frac{s}{s^2+1} - 12\frac{1}{s^2+1}$$

$$v(t) = 6\cos t - 12\sin t, \quad t > 0$$

SECOND METHOD: USE TRANSFORMED CIRCUIT.



KCL: $-1 + \frac{V}{2s} + \frac{6}{s} + \frac{V}{2/s} - 2 = 0$, THIS GIVES $V = \frac{6s-12}{s^2+1}$ AGAIN.

CONTINUE AS IN FIRST METHOD

ANOTHER SOLUTION

$$V = \frac{6s-12}{(s-j)(s+j)} = \frac{A}{s-j} + \frac{A^*}{s+j}, \quad A = \frac{j6-12}{2j} = 3 + 6j = \sqrt{3^2+6^2}\,\angle \tan^{-1}\frac{6}{3}$$

$$= 6.708\angle 63.43^\circ$$

So, $v(t) = 13.416\cos(t + 63.43^\circ)$

EITHER SOLUTION IS CORRECT.

AN APPLICATION OF THE FORMULA FOR THE COSINE OF A SUM OF TWO ANGLES SHOWS THEY ARE THE SAME ANSWER.

Prob. 3. (30 points

Solve the following convolution equation for $g(t)$.

$$\int_0^t f(x)\, g(t-x)\, dx = w(t)$$

where $f(t) = e^{-6t}$, $w(t) = \sin 2t$, and $t > 0$.

APPLY $\mathcal{L}$:

$$\frac{1}{s+6} G(s) = \frac{2}{s^2+4}$$

$$G(s) = \frac{2s+12}{s^2+4} = 2 \frac{s}{s^2+4} + 6 \frac{2}{s^2+4}$$

APPLY $\mathcal{L}^{-1}$:

$$g(t) = 2 \cos 2t + 6 \sin 2t$$

ANOTHER SOLUTION:

$$G(s) = \frac{2s+12}{s^2+4} = \frac{A}{s-j2} + \frac{A^*}{s+j2}$$

$$A = \frac{j4+12}{j4} = 1-j3 = \sqrt{1^2+3^2} \angle \tan^{-1} \frac{-3}{1}$$

$$A = 3.162 \angle -71.57^\circ$$

So,

$$g(t) = 2 \times 3.162 \cos(2t - 71.57^\circ)$$

$$= 6.324 \cos(2t - 71.57^\circ)$$

(THE TWO SOLUTIONS ARE THE SAME.)

Prob. 4. (30 points)

Find the inverse Laplace transform of

$$F(s) = \frac{3s^3 + 6s^2 - s + 4}{s^2 + 2s + 1}$$

$$\begin{array}{r|l} & \quad 3s \\ s^2 + 2s + 1 & \overline{3s^3 + 6s^2 - s + 4} \\ & \underline{3s^3 + 6s^2 + 3s} \\ & \qquad\qquad -4s + 4 \end{array}$$

$$F(s) = 3s + \frac{-4s + 4}{s^2 + 2s + 1} = 3s + \frac{-4s + 4}{(s+1)^2}$$

$$= 3s + \frac{A}{(s+1)^2} + \frac{B}{s+1}$$

$$A = (-4s+4)\Big|_{s=-1} = 8$$

$$B = \frac{d}{ds}(-4s+4)\Big|_{s=-1} = -4$$

So,

$$f(t) = 3\,\delta^{(1)}(t) + 8te^{-t} - 4e^{-t} \qquad \text{for} \quad t > 0.$$

Also

$$f(t) = 3\,\delta^{(1)}(t) + (8te^{-t} - 4e^{-t})\,u(t)$$

Either answer is correct.

It is incorrect to write

$$f(t) = \left(3\,\delta^{(1)}(t) + 8te^{-t} - 4e^{-t}\right)u(t)$$

because $\delta^{(1)}(t)\,u(t)$ is not meaningful.

Another way to get B: after getting A, choose $s = 0$ and solve for B

$$F(s) = \frac{8}{(s+1)^2} + \frac{B}{s+1}$$

at $s = 0$: $\quad 4 = 8 + B$. So $B = -4$