1. A person with a normal, $+50$ diopter eye strength at rest has a far point of 70 cm and a near point of 20 cm. What's their range of accommodation? What power of eyeglasses is needed to correct their far point? What is their near point with glasses on?
Answer: This person is myopic or nearsighted. Let's assume their eye focusing strength at rest is 50 diopters, and calculate the eye length:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \qquad \Rightarrow \qquad \frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=(50\ \text{D})-\frac{1}{.7\ \text{m}}=(50-1.4)\frac{1}{\text{cm}}$$

so their eye length is $s'=1/48.6=2.057$ cm. We can find their range of accommodation $1/f_a$ from using this eye length with the near point:

$$\begin{aligned}\frac{1}{s_{\text{np}}}+\frac{1}{s'} &= \frac{1}{f_0}+\frac{1}{f_a}\\ \frac{1}{s_{\text{np}}}+\frac{1}{s'}-\frac{1}{f_0} &= \frac{1}{f_a}\\ \frac{1}{0.2\ \text{m}}+48.6-50 &= 5.+48.6-50=3.6\ \text{D}=\frac{1}{f_a}\end{aligned}$$

so this person has a moderate range of accommodation and is therefore likely to be around 40 years old (see slide 13 of `l9.pdf`). We want to give them a corrective lens $1/f_c$ to give them vision at a distance of infinity:

$$\begin{aligned}\frac{1}{s\to\infty}+\frac{1}{s'} &= \frac{1}{f_0}+\frac{1}{f_c}\\ 0+48.6 &= 50+\frac{1}{f_c} \qquad \Rightarrow \qquad \frac{1}{f_c}=-1.4\ \text{D}\end{aligned}$$

We now want to calculate their near point with eyeglasses on:

$$\frac{1}{s_{\text{np}}}=\frac{1}{f_0}+\frac{1}{f_a}+\frac{1}{f_c}-\frac{1}{s'}=50+3.6+(-1.4)-48.6=3.6 \qquad \Rightarrow \qquad s_{\text{np}}=0.278\ \text{m}$$

or 28 cm.

2. Consider a double convex lens with radii of curvature of 30 and 40 cm for front and back surfaces, respectively, and a thickness of 2 cm. Calculate the transfer matrix for this lens. Calculate the positions of the principal planes and nodal points, and show them in a sketch. Use $n=1.5$ for the glass, and $n=1$ for air.
Answer: Our sign convention says that positive radii correspond to having the center of curvature be to the right of the refractive interface. That is, in this case we have $R_1=+30$ cm and $R_2=-40$ cm; we also have $t=2$ cm, $n_0=1$, $n_L=1.5$, and $n_f=1$. The transfer matrix is found from

$$\mathcal{R}_2\mathcal{T}\mathcal{R}_1 = \begin{bmatrix}\frac{n}{n'} & \frac{1}{R_2}\left(\frac{n}{n'}-1\right)\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ L & 1\end{bmatrix}\begin{bmatrix}\frac{n}{n'} & \frac{1}{R_2}\left(\frac{n}{n'}-1\right)\\ 0 & 1\end{bmatrix}$$

$$
\begin{aligned}
&= \begin{bmatrix} 1.5 & \frac{1}{-40}(\frac{1.5}{1}-1) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{1.5} & \frac{1}{+30}(\frac{1}{1.5}-1) \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{3}{2} & -\frac{1}{80} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{3} & -\frac{1}{90} \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{3}{2} & -\frac{1}{80} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{3} & -\frac{1}{90} \\ \frac{4}{3} & \frac{44}{45} \end{bmatrix} = \begin{bmatrix} \frac{59}{60} & -\frac{13}{450} \\ \frac{4}{3} & \frac{44}{45} \end{bmatrix} = \begin{bmatrix} 0.9833 & -0.0289 \\ 1.333 & 0.9778 \end{bmatrix}
\end{aligned}
$$

To get the positions of the principal planes and nodal points, we need to calculate r, v, s, and w:

$$
\begin{aligned}
r &= \frac{A - n_0/n_f}{B} = \frac{59/60 - 1/1}{-13/450} = \frac{-1/60}{-13/450} = \frac{450}{780} = \frac{15}{26} = 0.577 \\
v &= \frac{A-1}{B} = \frac{15}{26} = 0.577 \text{ as with } r \\
s &= \frac{1-D}{B} = \frac{1-44/45}{-13/450} = -\frac{1/45}{13/450} = -\frac{450}{13 \cdot 45} = -\frac{10}{13} = -0.769 \\
w &= \frac{n_0/n_f - D}{B} = -\frac{10}{13} = -0.769 \text{ as with } s
\end{aligned}
$$

That is, we have H_1 and N_1 at the same place which is 0.577 cm to the right of the left surface of the 2 cm thick lens, and H_2 and N_2 at the same place which is 0.769 cm to the left of the right surface of the lens.

3. Using Jones matrices, take unpolarized light, run it through a polarizer at $135°$, and run it through a quarter wave plate with SA vertical (SA=slow axis). What is the polarization type of the resulting light?
Answer: Let's take some unspecified polarization $[A, B]$ and run it through these devices (first a $135°$ linear polarizer, and then a quarter wave plate with a vertical slow axis). First of all, a $135°$ linear polarizer has a matrix of

$$
\begin{bmatrix} \cos^2(135°) & \sin(135°)\cos(135°) \\ \sin(135°)\cos(135°) & \sin^2(135°) \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}.
$$

We then have a net effect of

$$
\begin{aligned}
\begin{bmatrix} A' \\ B' \end{bmatrix} &= \begin{bmatrix} -i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} \\
&= \begin{bmatrix} -i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} (1/2)(A-B) \\ -(1/2)(A-B) \end{bmatrix} = \frac{1}{2}(A-B) \begin{bmatrix} -i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\
&= -\frac{1}{2}(A-B) \begin{bmatrix} i \\ 1 \end{bmatrix} = (-i)(-\frac{1}{2})(A-B) \begin{bmatrix} 1 \\ -i \end{bmatrix} = (\text{stuff})\text{RHC}.
\end{aligned}
$$

That is, we get right-hand circular (RHC) polarized light out of unpolarized light.

4. Use 517/645 borosilicate crown glass and 620/380 flint glass to design an achromat doublet with $f_D = 12$ cm. (Recall that XXX/YYY means $n_D = 1$+XXX/1000, and V =YYY/10).
Answer: We have for the glass properties

517/645 borosilicate crown	$n_D = 1.517$	$V = 64.5$
620/380 flint glass	$n_D = 1.620$	$V = 38.0$

with $V \equiv (n_D - 1)/(n_F - n_C)$ for the wavelengths $\lambda_F = 486.1$ nm, $\lambda_D = 587.6$ nm, and $\lambda_C = 656.3$ nm. The first lens has a radius of curvature found from

$$\frac{1}{f_D} = (n_{1D} - 1)\frac{2}{|r_1|}\frac{V_1 - V_2}{V_1}$$

giving
$$|r_1| = (n_{1D} - 1)\,2f_D\,\frac{V_1 - V_2}{V_1} = (1.517 - 1)\,2\,(12\text{ cm})\,\frac{64.5 - 38.0}{64.5} = 5.098\text{ cm}$$

For the second surface of the second lens, we have

$$\frac{1}{r_{22}} = \frac{1}{|r_1|}\left[2\frac{(n_{1D} - 1)}{(n_{2D} - 1)}\frac{V_2}{V_1} - 1\right] = \frac{1}{5.098\text{ cm}}\left[2\frac{(1.517 - 1)}{(1.620 - 1)}\frac{38.0}{64.5} - 1\right] = \frac{1}{-292.1\text{ cm}}$$

so the second lens has a second surface that is only very slightly convex.

5. Determine the critical angle and polarizing angles for external and internal reflections at an interface between one medium with $n_1 = 1.2$ and another medium with $n_2 = 1.8$. Calculate r and R for both modes of polarization at half the critical angle.
Answer: The normalized refractive index is $n = 1.8/1.2 = 1.5$. The critical angle for internal reflection is $\theta_c = \arcsin(1/1.5) = 41.8°$. The polarizing angle for internal incidence is $\theta_{p,i} = \arctan(1./1.5) = 33.7°$ while for external incidence it is $\theta_{p,e} = \arctan(1.5) = 56.3°$. If we're speaking of half the critical angle, then we are speaking of the $n = 1/1.5 = 2/3$ case only, so we want to calculate r and R for the angle $\theta = 41.8/2 = 20.6°$ where we have the following results:

$$\begin{aligned}
\text{TE: } r_\perp &= \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} \\
&= \frac{\cos(20.6°) - \sqrt{(2/3)^2 - \sin^2(20.6°)}}{\cos(20.6°) + \sqrt{(2/3)^2 - \sin^2(20.6°)}} = 0.246 \\
\text{TM: } r_\parallel &= \frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \\
&= \frac{(2/3)^2\cos(20.6°) - \sqrt{(2/3)^2 - \sin^2(20.6°)}}{(2/3)^2\cos(20.6°) + \sqrt{(2/3)^2 - \sin^2(20.6°)}} = -0.153
\end{aligned}$$

Then $R_{\mathrm{TE}} = 0.246^2 = 0.0606$, and $R_{\mathrm{TM}} = (-0.153)^2 = 0.0234$.