# Water pump modelling

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## Project description

The aim of this project is to model a system for the purpose of pumping water from a deep well to a open-air reservoir for later use. The water will be used mostly for irrigation purposes.

The main points tackled in this work are the required hydraulic calculations for extracting the water from the well and storing it into the reservoir. The required electrical power will be calculated too.

#### Hydraulic calculations for extracting water

This preliminary step includes all the calculations and considerations needed to assess the feasibility of the project and to accurately select the proper pump according to the head needed.

#### Electrical calculations

This second step analyses how much electrical power is needed to power the pump.

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## Hydraulical modelling

### Pumping water from the well to the reservoir

As far as the pump modelling is concerned, we are going to make the following assumptions:

1. The problem can be tackled as one-dimensional.
2. We are going to assume stationary conditions.
3. The fluid (water) is assumed to be incompressible and at a temperature ranging from 25 to 35 degrees Celsius.
4. We assume that the pressure at both ends a and b (see the picture below) can be approximated to be roughly equal to the atmosferic pressure (101325 Pa).
5. We assume that the speed of the water at the end points a and b is close to zero and therefore can be neglected.

After having gathered some information on the system, the rough sketch below can be used for modelling purposes:

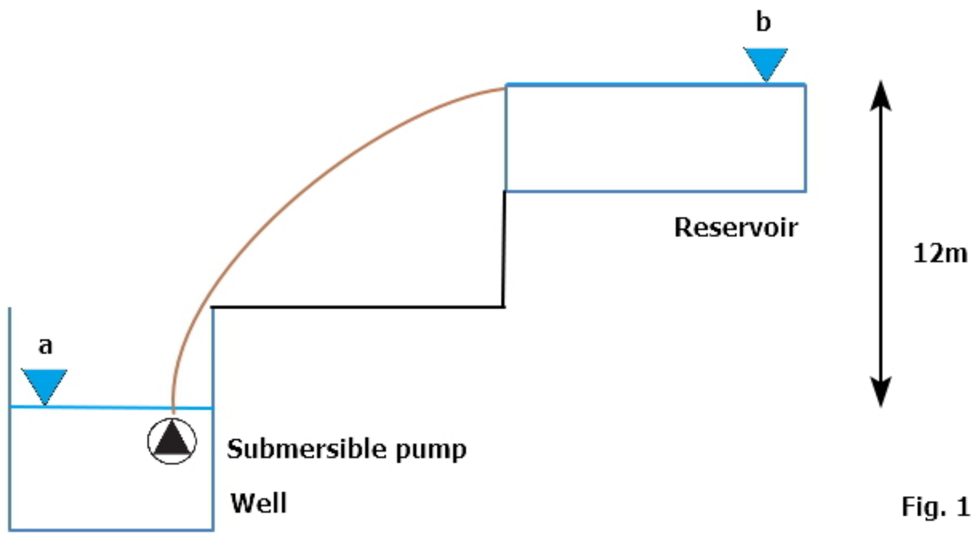



Fig. 1

**Problem data**:

1. The water is available in a well at a depth of about 10 m.
2. The reservoir is located at about a 2 m height from the ground level.
3. The round section pipes to be used are made of PVC.
4. The number of turns in the pipeline is the following: 2 x 45° turns.
5. The minimum required flow rate is

$$20\frac{l}{s} = 0.02\frac{m^3}{s} = 72\frac{m^3}{h}$$

. This requirement is not strict, anything above is fine.

Since the water is available in a well at a depth of 10 m and should be pumped up to a reservoir on a nearby storage tank about 2 m above the ground level, the head required due to the height difference is approximately 12 m.

The complete equation used for modelling the total head, given the assumptions 1,2 and 3, is the conservation of energy equation for incompressible fluids:

$$E_{id} = L - y_p = \frac{P_b - P_a}{\rho} + g(z_b - z_a) + \frac{v_b^2 - v_a^2}{2} + y_t$$

where $E_{id}$ is the ideal energy needed, $y_p$ is the extra-energy needed due to the losses in the pump, $P$ is the preassure at a specified point, $z$ is the height relative to ground, $v$ is the velocity of the fluid and $y_t$ is the pressure drop in the connecting pipes. Note that each element has dimensions of $\frac{J}{kg}$.

Assuming that the pump is working at water level and that the output pipe is spilling water directly into the reservoir, if both the well and the reservoir are big enough (assumption 5), we can assume that the water is almost still, furthermore given assumption 4, we obtain:

$$H_p = H_g + Y_t$$

where $H_g$ is the head (m) required for lifting the water from the well up to the height of the reservoir and $Y_t$ (m) represents the pressure drop in the pipes due to friction and other losses.

## Calculating pressure drop

As far as $H_g$ is concerned, we already have that value which is about 12 m, while $Y_t$ needs to be calculated.

We can break down the pressure drop in the pipes in two main components:

$$Y_t = \frac{\lambda}{2g} v^2 \frac{L}{D} + \xi \frac{v^2}{2g}$$

The first component represents the distributed pressure drop due to friction between water molecules and the pipes, while the second component represents the pressure drop due to changes of direction of the flow such as, for instance, 90 degrees turns in the pipeline.

A rough sketch of the pipeline layout is given below in fig.2:

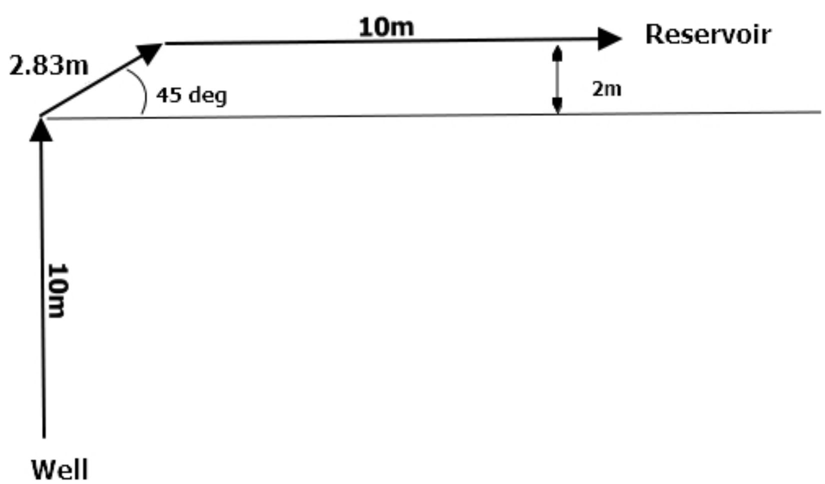



Fig. 2

As you can see, the number of 45° turns is 2, while the length of the pipeline is approximately 22.83 m.

As far as the distributed pressure drop goes, we can use the required flow rate and once we have the fluid velocity, given by

$$v = \frac{4\dot{Q}}{\pi D^2}$$

and the $\lambda$ coefficient that can be estimated using Reynolds' number[1]

$$Re = \frac{\rho v D}{\mu}$$

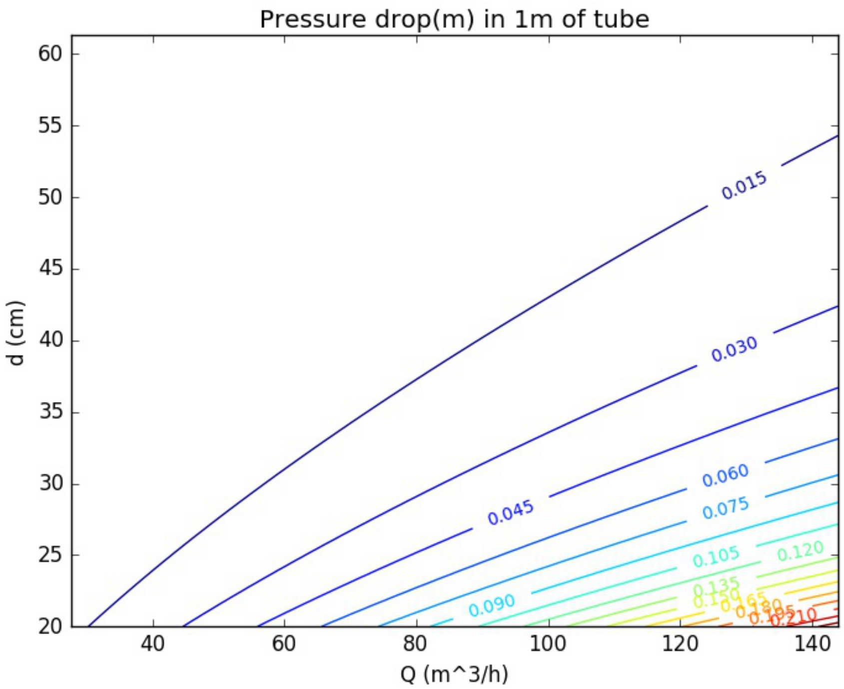
we can choose the appropriate pipe's diameter in order to keep the pressure drop due to friction as low as convienent.

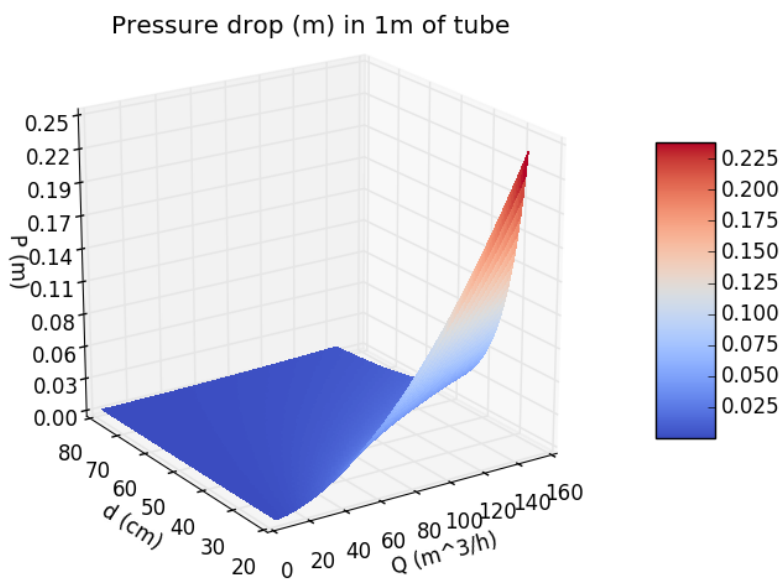
This simulation plots pressure drop for 1m of tube agains flow rate ($\dot{Q}$) and the pipe's diameter ($D$):

```
# Imports
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm
from matplotlib.ticker import LinearLocator, FormatStrFormatter
from numpy import vectorize

# Let's define the variables
Q_MIN = 0.001              # Min flowrate m^3/s
Q_MAX = 0.04               # Max flowrate m^3/s
D_MIN = 0.2
D_MAX = 0.8
N = 100

MU = 0.001                          # Water viscosity @25 °C Pa s
RHO = 1000                          # Water density kg/m^3
L = 1                               # Pipe's length
E = 0.000005                        # Pipe's Rugosity
Q = np.linspace(Q_MIN, Q_MAX, N)    # Flowrate m^3/s
d = np.linspace(D_MIN, D_MAX, N)    # Pipe's diameter m

# This function calculates velocity of the fluid
def vel(Q, d):
    v = 4*Q/(np.pi * d**2)
    return v

# This function calculates friction coefficient Lambda
def lbd(v, d, mu=MU, rho=RHO, e=E):
    reynolds = rho*v*d/mu
    # If RE < 2100 flow is laminar, otherwise it is turbolent
    if reynolds <= 2100:
        lbda = 64/reynolds
    else:
        lbda = 1.325/(np.log(e/(3.7*d)+5.74/(reynolds**0.9)))**2
    return lbda

# This function calculates the pressure drop
def pressureDrop(Q, d, L=L):
    v = vel(Q, d)
    pDrop = 0.5 * lbdv(v,d) * v**2 * d / L
    return pDrop

# Plot functions: contour and 3d plot
def contour(X, Y, Z, N_=20):
    plt.figure()
```

```
    CS = plt.contour(X, Y, Z, N_)
    plt.clabel(CS, inline=1, fontsize=10)
    plt.title('Pressure drop(m) in 1m of tube')
    plt.xlabel('Q (m^3/h)')
    plt.ylabel('d (cm)')
    plt.show()

def dplot(X, Y, Z):
    fig = plt.figure()
    ax = fig.gca(projection='3d')
    surf = ax.plot_surface(Q, d, pdrop, rstride=1, cstride=1, cmap=cm.coolwarm,
                           linewidth=0, antialiased=False)
    ax.zaxis.set_major_locator(LinearLocator(10))
    ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))
    ax.set_xlabel('Q (m^3/h)')
    ax.set_ylabel('d (cm)')
    ax.set_zlabel('P (m)')
    ax.set_title('Pressure drop (m) in 1m of tube')
    fig.colorbar(surf, shrink=0.5, aspect=5)
    plt.show()


# Vectorize function for calculating lambda
lbdv = vectorize(lbd)

# Grid
Q, d = np.meshgrid(Q, d)

# Calculate pressure drop
pdrop = pressureDrop(Q, d)

# Convert pressure drop in meters
pdrop = pdrop * RHO / 9.81
# Convert Q in m^3/h and d in cm
Q = Q * 3600
d = d * 100

# Plot
#dplot(Q,d,pdrop)

# Contour plot
contour(Q, d, pdrop)
```

The output of the simulation can be explored below:




From the simulation we find that the expected pressure drop due to distrubuted pressure losses in a pipe of length 1 m, at the required flowrate, is about 0.045 m using pipes with a diamter of 23.5 cm. Note that we can use smaller pipes but then we would need more head and consequently, a bigger pump. Since there are close to no space constraints, it is convenient to use bigger pipes.

Since we are using a total length of 22.83 m of pipes, the total expected distributed pressure drop is about 1.03 meters.

As far as concentrated pressure drop losses go, each 45° turn accounts for a $\xi$ coefficient of around 0.6. Given that there are 2 turns, and that using the pipes diameter and the flowrate we can calculate the fluid speed (0.46 m/s), the total pressure drop estimated is

$$2\xi\frac{v^2}{2g} = 0.01m$$

note that it is almost negligible. Nevertheless, we will include it in the calculations.

Summing up the two, we obtain the following total pressure drop:

$$Y_t = 1.04m$$

therefore, the total head required by the plant, at the required flowrate, is 13.04 m.

Using the submersible pump **DN 80 KCM080L** with 2 poles (characteristic curve number 1 page 37 of the attached datasheet), we have at the required head, a flowrate slightly above the required $20\frac{l}{s}$.

The expected power consumptio is less than 5 kW while the pump, according to the datasheet, works at its peak efficiency of 52.7 %.

# Conclusions

This is a preliminary study for a water pumping system. We hope that the results can provide an interesting insight to the project. Further measurements and simulations will be provided if required by the customer.

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Notes:

[1] Reynold's number was estimated using this formula