Optimisation du 23/03/20

↳ Exercice 1)

$f(x,y) = xy - \ln(x+y)$

$$\left.\begin{aligned} f'(x) &= y - \left(\frac{1}{(x+y)}\right) \\ f'(y) &= x - \left(\frac{1}{(x+y)}\right) \end{aligned}\right\} = 0$$

$y - \left(\frac{1}{(x+y)}\right) = 0 \qquad y = \left(\frac{1}{x+y}\right)$

$x - \left(\frac{1}{(x+y)}\right) = 0 \qquad x = \left(\frac{1}{x+y}\right)$

$y^2 + xy = 1 \rightarrow y(x+y) = 1$

$x^2 + xy = 1 \rightarrow x(x+y) = 1$

$x + y = \frac{1}{y} \rightarrow y / \frac{x+y}{y} = 1 \rightarrow \frac{x}{y} = 1$

↳ $x = y$

$x + y = \frac{1}{x} \rightarrow x / \frac{x+y}{x} = 1 \rightarrow \frac{y}{x} = 1 \rightarrow y = x.$

$$y(y+y)=1 \qquad x(x+x)=1$$

$$2y^2=1 \qquad 2x^2=\underline{1}$$

$$y^2=\frac{1}{2} \qquad x^2=\frac{1}{2}$$

$$y=\sqrt{\tfrac{1}{2}}=\frac{1}{\sqrt{2}} \qquad x=\sqrt{\tfrac{1}{2}}=\frac{1}{\sqrt{2}}$$

suite

$$f'(x)=Y-\left(\frac{1}{(x+y)}\right)$$

$$f''_{x^2}(x)=\cancel{1+2\left(\frac{2}{(x+y)^2}\right)}\ \cancel{}\left(\frac{1}{(x+y)^2}\right)$$

$$f'(y)=\cancel{}\left(\frac{1}{(x+y)}\right)$$

$$f''_{y^2}(y)=\cancel{}\left(\frac{1}{(x+\ )^2}\right)$$

$$f''(x,y)=1+\frac{1}{(x+y^2)}$$

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suite du 23/03/20 ex 2

3) $f(x,y) = y(x-2)$ s/c $x+y^2+1=0$

$\frac{d(g)}{d(x)} = (x+y^2+1)$

$= 1$

$x+y^2+1=0$

$x=-y^2-1$

$x=-1$

$(-1,0) \,!= (0,0)$ condi° remplie

$L(x,y,\lambda) = f(x-y) - \lambda \times g(x,y)$

$= y(x-2) - \lambda(x+y^2+1)$

$= xy - 2y - \lambda x - \lambda y^2 - \lambda$

$L'(x) = y-1$

$L'(y) = x-2-2\lambda y$

$L'(\lambda) = x-y^2-1$

$y - \lambda = 0 \rightarrow y = \lambda$

$x = 2 + 2\lambda y$

$y^2 = -x - 1$

$x = 2 + 2y^2$

$x = 2 + 2(-x-1)$

$x = 2 - 2x - 2$

$x = -2x$

$2x + x = 0$

$x(2+1) = 0$

$\cancel{x(3)=0}$ $3x = 0$

$x = 0$..

$\cancel{y^2 = -x - 1}$

$y^2 = \frac{(x-2)}{2}$

$y^2 = -1$

$y^2 = i^2 \rightarrow$ nombre complexe

$y = 1$ ou $y = -1$

$\lambda = i$ ou $\lambda = -i$

$A(0, i, i)$

$B(0, -i, -i)$

Suite du exercice

$$\frac{1}{(x+y)^2} = 0 \rightarrow 1 = (x+y)^2$$

$$\sqrt{1} = \sqrt{(x+y)^2}$$

$$1 = x^2 + 2xy + y^2$$

$$-2xy = x^2 + y^2$$

$$f''(x) = \frac{1}{(x+y)^2} = \frac{1}{2}$$

$$f''(y) = \frac{1}{(x+y^2)} = \frac{1}{2}$$

$$f''(x,y) = 1 + \frac{1}{(x+y)^2} = \frac{3}{2}$$

$$D = \begin{Bmatrix} \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \rightarrow D = \frac{1}{4} - \frac{3}{4}$$

$$\boxed{= -2.}$$