Image Editing

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Introduction

Image editing tasks concern either global changes (color/intensity corrections, filters, deformations) or local changes confined to a selection. The goal here is to perform seamless blending of an object or a texture from a source image (captured by a mask image) to a target image. We want to create a photomontage by pasting an image region onto a new background.

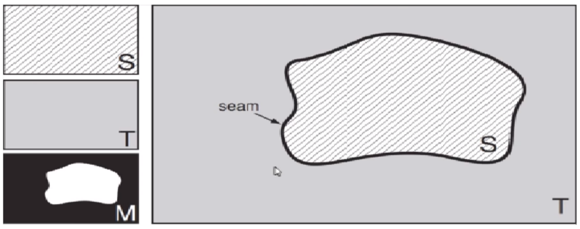

Figure 1: T for Target, S for Source and M for Mask

1. Theory

1.1. Optimization Problem

Unknown function u interpolates in domain Ω of the destination T, under guidance of vector field v, which might be or not the gradient field of a source S. A guidance field is a vector field v used in an extended version of the minimization.

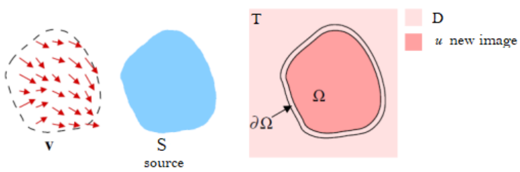

Figure 2: Guided interpolation notations

1.2. First formulation of the Problem

$$\min_u \int_\Omega ||\nabla u - v||^2,$$

under the constraint $f_{D \setminus \Omega} = T$.

1.3. Second formulation of the Problem

This problem can also be stated as follow :

$$\min_u \int_\Omega ||\nabla u - v||^2 + \iota_K(u)$$

where K is the set of images which coincide with the target out of the mask. We can observe that the set K is a closed convex set.

1.4. Discrete formulation of the Problem

The finite difference discretization yields the following discrete, quadratic optimization problem:

$$\min_{f|_\Omega} \sum_{\langle p,q\rangle \cap \Omega \neq \emptyset} (f_p - f_q - v_{pq})^2, \text{ with } f_p = f_p^*, \text{for all } p \in \partial\Omega$$

p for pixel.

2. Algorithms and Methods

In order to solve the optimization problem we are going to use the Forward-Backward algorithm and later on FISTA (Fast Iterative Thresholding Algorithm) algorithm.

Reminder:

if $F = f + g$ is differentiable and ∇f is L-Lipschitz, for $\gamma > 0$ we define :

$$P = prox_{\gamma g}(Id - \gamma \nabla f)$$

For this study :

$$f(u) = \|\nabla u - v\|^2 \qquad g = \iota_K$$

Knowing that $prox_g = Proj_K$, P can be written to :

$$P = Proj_K(Id - \gamma \nabla f)$$

Forward-Backward algorithm:

The FB algorithm is defined, if $\gamma < \frac{1}{L}$ by:

$$x_{n+1} = P x_n$$

Remark : The sequence $F(x_n)_{n \in \mathbf{N}}$ is non increasing.

FISTA algorithm :

The algorithm is based on the same operator T.
It is defined for $\gamma < \frac{1}{L}$ and $\alpha \geqslant 3$ by :

$$x_{n+1} = Py_n$$

$y_n = x_n + \frac{n}{n+\alpha}(x_n - x_{n-1})$

Remarks :
The sequence $F(x_n)_{n\in\mathbf{N}}$ is non increasing.
A choice of α not loo large, close to 3 is often relevant.
FISTA is often faster than FB.

3. The choice of the guidance field v

With the previous algorithms combined with different definitions of the guidance field v we can edit images in many ways.
Here are some examples :
- Transparency,
- Texture flattening,
- Color intensity;
- Local color changes,...