Ruin forest

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1 Introduction

Trees are known for their remarkable longevity and enduring lifespan, as they can withstand harsh climatic conditions within a given year. Recently, climate change has caused a surge in natural disasters. More specifically, frequent heatwaves and droughts are a real threat as they cause the death of forests in the world. It becomes crucial to pay more attention to these phenomena and estimate precisely how they influence risk of forest collapse.

Throughout their life, trees accumulate a reserve of nonstructural carbohydrates (NSCs), crucial for resilience and rapid leaf expansion. Previous studies have highlighted the impact of natural disasters on tree growth: dry days deplete NSC reserves. They concluded that trees cannot recover below a threshold of NSCs, effectively meaning that without enough NSC, trees die (Adams et al., 2009; Bigler et al., 2007; Bréda and Badeau, 2008; Villalba and Veblen, 1998; Matusick et al., 2018).

Nonetheless, these studies were mostly focused on real-life observations. They were inherently limited by their observations that were fixed over precise time periods. In this report, we aim at adapting a Cramer–Lundberg ruin for modeling forest collapse. This mathematical model is widely used in insurance and risk management to assess the probability of ruin for financial institutions. It provides a framework to estimate the likelihood of an insurance company running out of funds to pay claims and going bankrupt. In our case we define the ruin as the moment when the amount of NSCs drops below a critical level, and takes both physiological parameters of trees and natural disaster parameters to estimate the NSC reserves. This adaptation of Crame-Lundberg thus represents a simplified ruin model for forests that evaluates the impacts of climate hazards (heat and drought stress) through multiple simulations.

Section 2 and section 3 explore the interpretation of model parameters and presents a simple tree growth model based on the Cramer–Lundberg ruin model for insurance. In contrast to the analytically solvable Cramer–Lundberg ruin model, the tree model requires extensive numerical simulations. Section 4 shows the results of numerical simulation. This improves the understanding of forest ruin modeling dynamics, providing insights into the potential impacts of extreme events on tree growth within a specified time period.

2 Cramer-Lundberg model for ruin forest

2.1 Cramer-Lundberg in ruin theory

The Cramer-Lundberg model is a mathematical framework used in insurance and risk management to model the risk of financial ruin (i.e. bankruptcy) for an insurance company. The model was developed in the early 20th century.

The insurance capital at time t, denoted as $R(t)$, is modeled by the equation :

$$R(t) = R_0 + pt - S(t)$$

where $R_0 > 0$ represents the initial capital, $p > 0$ is the annual premium rate collected at each time (in year) t, and $S(t) \geq 0$ is a damage function accounting for random losses due to hazards up to time t. Only the damage function $S(t)$ is random.

Ruin occurs when $R(\tau) \leq 0$, with ψ being the ruin time (i.e. the first positive time when ruin occurs).

$$\psi(R_0, p, T) = \inf\{t > 0 | R(t) \leq 0\}$$

The damage $S(t)$ is represented as a random sum of random variables: $S(t) = \sum_{k=1}^{N(t)} X_k$ where $N(t)$ is a Poisson random variable accounting for the number of hazards occurring up to time t, and

X_k are random variables representing the cost of each hazard. The probability distribution of X_k can be modeled by an extreme value law, such as the generalized Pareto distribution (GPD).

If the damage function $S(t)$ is equal to 0 every year, the capital grows indefinitely.

Actuaries in insurance companies aim to estimate the minimum premium rate p that prevents ruin. But in several scenarios, finding an acceptable value for p to prevent ruin is challenging. This predicament leads insurance companies to turn to reinsurance as a safeguard against bankruptcy following unexpectedly substantial losses $S(t)$. Reinsurance companies function as insurers for insurance companies.

2.2 Ruin forest

In this subsection, we adapt the Cramer–Lundberg model to create a simplified tree growth model that explicitly incorporates the impact of damage $S(t)$ resulting from climate hazards such as droughts or heatwaves.

We model the risk of ruin of a forest within a finite time (horizon) T. The objective is to generate numerous finite sequences of $R(t)$ corresponding to a sample of all possible T-long trajectories.

Here $R(t)$ denotes the quantity of nonstructural carbohydrates in a tree, referred to as "reserves" (in kg.C.m^{-2}), which quantify tree growth. The reserve is dependent on its previous state.

$$R(t) = \min\left[(1-b)R(t-1) + p(t)\Delta t - \sum_{i=N(t-1)+1}^{N(t)} S(t), R_{\max}\right]$$

$$\begin{cases} R_{\max} = 100 : \text{ maximum NSC reserve} \\ b = 0.05 : \text{ percentage of NSC consumed during a year} \\ N(t) : \text{ the number of years with hazards, modelled by a homogeneous} \\ \qquad \text{Poisson process with parameter } \Lambda \end{cases}$$

where the net primary production (NPP) $p(t)$ and the resulting damage $S(t)$ during the hazards years are expressed as:

$$p(t) = p_0 - \frac{BS(t-1)}{\Delta t}$$

$$\begin{cases} p_0 : \text{ optimum average yearly NPP allocated to NSC reserves} \\ B \geq 0 : \text{ memory factor of the previous damage} \\ \Delta t = 1 \text{ year} \end{cases}$$

$$S(t) = A_h \sum_{k=1}^{M(t)} Y_k$$

$$\begin{cases} Y_k : \text{ hazards occurring during year } t \text{ which follow a GPD with} \\ \qquad \text{scale parameter} \sigma \text{ and shape parameter } \xi \\ A_h : \text{ normalizing constant} \\ M(t) : \text{ number of hazards during year } t\text{, following a Poisson distribution} \\ \qquad \text{with parameter } \lambda \end{cases}$$

The challenge lies in estimating scales for the values of A_h and B. Different tree species exhibit varied strategies to cope with heat and drought stress, leading to distinct interannual memory effects B (Adams et al., 2009; Teuling et al., 2010). One strategy involves maintaining growth during a hazard at the expense of impacts in subsequent years, representing anisohydric behavior with a positive value of B. Another strategy involves reducing growth during a hazard, with less impact in subsequent years, representing isohydric behavior with a B value close to 0. The values of A_h and B can be chosen to scale with the expected behavior of trees and can be estimated from observations or expert knowledge.

In the next section, we calculate the net profit condition (NPC) and the ruin probability of this model (with some simplicity).

3 Theoretical insights

The goal of this section is to compute the net profit condition (NPC) and the ruin probability in some special cases, because their computation in the original ruin model is analytically complex. The net profit condition (NPC) is met when expected value of $R(t)$ consistently increases over time. The probability of ruin given that the initial reserve $r = R_0$ is defined as:

$$\psi(r) = \mathbb{P}\left(\exists t > 0,\ R(t) < 0 \mid R_0 = r\right)$$

We can also define the ruin probability before T years as:

$$\psi(r,T) = \mathbb{P}\left(\exists t \in [0,T],\ R(t) < 0 \mid R_0 = r\right)$$

To simplify, we do not take into account the memory of previous climate hazards B, so the reserve is written by:

$$R(t) = (1-b)R(t-1) + p_0 - \sum_{i=N(t-1)+1}^{N(t)} S_i$$

3.1 Computation of net profit condition

Remark 3.1. *If climate hazards never occurred, the NPC is $p_0 > bR_{max}$, in this case, $R(t)$ converges to R_{max} when t tends to infinity.*

Proof. In fact, if climate hazards never happened, there would be no memory of the previous hazard, thus, the damage $p(t) = p_0$ and the reserve is written as:

$$R(t) = \min\left[(1-b)R(t-1) + p_0, R_{\max}\right]$$

so,

$$\begin{aligned}
\mathbb{E}\left[R(t)\right] &= \mathbb{E}\left[(1-b)R(t-1) + p_0\right] \\
&= (1-b)\mathbb{E}\left[R(t-1)\right] + p_0 \\
&= \mathbb{E}\left[R(t-1)\right] + p_0 - b\mathbb{E}\left[R(t-1)\right]
\end{aligned}$$

the NPC in this case is:

$$\begin{aligned}
\forall t,\ p_0 - b\mathbb{E}\left[R(t-1)\right] > 0 &\Leftrightarrow \forall t,\ p_0 > b\mathbb{E}\left[R(t-1)\right] \\
&\Leftrightarrow p_0 > b\max_t \mathbb{E}\left[R(t)\right] \\
&\Leftrightarrow p_0 > bR_{max}
\end{aligned}$$

when the NPC is respected, $R(t)$ become an increasing and bounded function at R_{max}. So that $\lim_{t\to\infty} R(t) = R_{max}$. □

We now take into account the damage of climate hazard, thus, the reserve is written as:

$$R(t) = (1-b)R(t-1) + p_0 - \sum_{i=N(t-1)+1}^{N(t)} S_i$$

$$\begin{aligned}
\mathbb{E}\left[R(t)\right] &= \mathbb{E}\left[(1-b)R(t-1) + p_0 - \sum_{i=N(t-1)+1}^{N(t)} S_i\right] \\
&= (1-b)\mathbb{E}\left[R(t-1)\right] + p_0 - \mathbb{E}\left[\sum_{i=N(t-1)+1}^{N(t)} S_i\right] \\
&= (1-b)\mathbb{E}\left[R(t-1)\right] + p_0 - \mathbb{E}\left[\sum_{i=1}^{N(t)-N(t-1)} S_i\right] \\
&= (1-b)\mathbb{E}\left[R(t-1)\right] + p_0 - \mathbb{E}\left[S_1\right]\mathbb{E}\left[N(t)-N(t-1)\right] \\
&= (1-b)\mathbb{E}\left[R(t-1)\right] + p_0 - \mathbb{E}\left[\sum_{k=1}^{M} Y_k\right]\mathbb{E}\left[N(t)-N(t-1)\right] \\
&= (1-b)\mathbb{E}\left[R(t-1)\right] + p_0 - \mathbb{E}\left[M\right]\mathbb{E}\left[Y_1\right]\mathbb{E}\left[N(t)-N(t-1)\right] \\
&= (1-b)\mathbb{E}\left[R(t-1)\right] + p_0 - \Lambda\lambda\mathbb{E}\left[Y_1\right]
\end{aligned}$$

thus, for all t, we have:

$$\begin{aligned}
\mathbb{E}\left[R(t)\right] > \mathbb{E}\left[R(t-1)\right] &\Leftrightarrow p_0 - b\mathbb{E}\left[R(t-1)\right] - \Lambda\lambda\mathbb{E}[Y_1] > 0 \\
&\Leftrightarrow p_0 - \Lambda\lambda\mathbb{E}[Y_1] > b\mathbb{E}\left[R(t-1)\right]
\end{aligned}$$

The expected value of $R(t)$ consistently increases over time when $p_0 - \Lambda\lambda\mathbb{E}[Y_1] > b\mathbb{E}\left[R(t-1)\right]$ for all time t, therefore, $p_0 - \Lambda\lambda\mathbb{E}[Y_1] > b\max_t \mathbb{E}\left[R(t-1)\right]$. On the other hand, we have $\max_t \mathbb{E}\left[R(t-1)\right] = R_{max}$, the NPC is written as:

$$\begin{aligned}
&p_0 > \Lambda\lambda\mathbb{E}[Y_1] + bR_{max} \text{ , with } Y \sim GDP(u,\sigma,\xi) \\
&p_0 > bR_{max} + \Lambda\lambda\left(u + \frac{\sigma}{1-\xi}\right)
\end{aligned}$$

3.2 Computation of ruin probability

In the considered ruin model, we assume that the damage S_i during the climate hazard year is $S_i = \sum_{k=1}^{M} Y_k$ for every $i \in [1, N(t)]$ where M is a Poisson random variable with parameter λ, $N(t)$ is number of hazard years until year t and $Y_k \sim GPD(u,\sigma,\xi)$. Therefore, the damage $S(t)$ is a sum of $M(t)$ independent generalized Pareto variables whose distribution is very complex. We decide to change the distribution of Y_k to exponential distribution (i.e. $Y_k \sim \mathcal{E}(\lambda_Y)$ with $\lambda_Y > 0$). Yet, $\forall k \in [1, M(t)]$ where $M(t) \sim \mathcal{E}(\lambda)$, the Y_k are pairwise independent, thus, the damage during the climate hazard year is

$$S_i = \sum_{k=1}^{M(t)} Y_k \sim \Gamma(M(t), \lambda_Y), \text{ for every } i \in [1, N(t)]$$

We introduce the following property:

Property 3.1. *For every $a \in \mathbb{N}^*$, we consider the ruin model defined below, where the costs (damages) follow $\Gamma(a, \lambda_Y)$. We denote $\psi^{(a)}(r)$ is the ruin probability for this model. Thus, for any u positive and for every $a \in \mathbb{N}^*$, we have*

Proof. We consider 2 ruin models: one where the costs (damages) follow $\Gamma(a,\lambda_Y)$ and the other follows $\Gamma(a+1,\lambda_Y)$. We denote $R^{(a)}(t)$, $R^{(a+1)}(t)$ their reserves and $\psi^{(a)}(u)$, $\psi^{(a+1)}(u)$ their ruin probabilities. Their total damages until year t are

$$S^{(a+1)}_{total}(t)=\sum_{i=1}^{N(t)} S_i^{a+1} \text{ and } S^{(a)}_{total}(t)=\sum_{i=1}^{N(t)} S_i^{a}$$

We have

$$S^{(a+1)}_{total}(t)=\sum_{i=1}^{N(t)} S_i^{a+1}=\sum_{i=1}^{N(t)}\sum_{k=1}^{a+1} Y_k=\sum_{i=1}^{N(t)}\left(\sum_{k=1}^{a} Y_k+Y_{a+1}\right)=S^{(a)}_{total}(t)+\sum_{i=1}^{N(t)} Y_{a+1}\geq S^{(a)}_{total}(t)$$

thus, $R^{(a+1)}(t)<R^{(a)}(t)$ i.e. $\psi^{(a+1)}(r)\geq\psi^{(a)}(r)$ □

In our case, a is denoted by M which is a Poisson random variable with parameter λ. This property implies that increasing the value of λ (i.e. $\mathbb{E}[M]=\lambda$) leads to an increase in the ruin probability. This implication is verified by numerical simulation in Section 4.2.

We return to the computation of ruin probability of our model. We recall that

$$R(t)=(1-b)R(t-1)+p_0-\sum_{i=N(t-1)+1}^{N(t)} S_i$$

Due to the model's complexity, we cannot compute directly the ruin probability $\psi(r)$. Instead, we calculate the probability $\mathbb{P}(R(t)<0)$ based on the reserve value at the previous instant $(t-1)$.

$$\begin{aligned}
\mathbb{P}(R(t)<0)&=\mathbb{P}\left((1-b)R(t-1)+p_0-\sum_{i=N(t-1)+1}^{N(t)} S_i<0\right)\\
&=\mathbb{P}\left((1-b)R(t-1)+p_0<\sum_{i=N(t-1)+1}^{N(t)} S_i\right)\\
&=\mathbb{P}\left(u_{t-1}<\sum_{i=N(t-1)+1}^{N(t)} S_i\right)\text{, with } u_{t-1}=(1-b)R(t-1)+p_0\\
&=\mathbb{P}\left(u_{t-1}<S_{N(t)}\mid N(t)=N(t-1)+1\right)+\mathbb{P}\left(u_{t-1}<0\mid N(t)=N(t-1)\right)\\
&=\mathbb{P}\left(u_{t-1}<S_{N(t)}\mid N(t)=N(t-1)+1\right)+0\\
&=\mathbb{P}(u_{t-1}<S_{N(t)})\text{ (because } S_{N(t)} \text{ and } N(t) \text{ are independent)}\\
&=\mathbb{P}\left(u_{t-1}<\sum_{k=1}^{M} Y_k\right)\\
&=\mathbb{P}\left(u_{t-1}<\sum_{k=1}^{m} Y_k\mid M=m\right)\\
&=\mathbb{P}\left(u_{t-1}<\sum_{k=1}^{m} Y_k\right)\text{ (because } Y_k \text{ and } M \text{ are independent)}\\
&=\mathbb{P}\left(u_{t-1}<X\right)\text{ where } X\sim\Gamma(m,\lambda_Y)\\
&=1-\int_0^{u_{t-1}}\frac{\lambda_Y^m x^{m-1}}{(m-1)!}e^{-\lambda_Y x}dx
\end{aligned}$$

4 Numerical simulation

After establishing the theoretical framework of the model, it becomes clear that with its multiple parameters, the analytical analysis of the model is rather complicated, if not impossible without model simplification. In response to these complexities, this section aims to analyse the model numerically, with many simulations as a practical approach to explore the model's behaviour and estimate the impact of each parameter to the NSC reserve and the ruin probability.

As the true value of R_{max} is unknown, we standardise $R(t)$ to be a percentage of R_{max}, so a value of 100 means an ideal and maximum value of $R(t)$ and the ruin occurs when $R(t) = 0$. We presume that each year 5% of the total NSC produced is allocated and consumed, thus, we set $p_0 = 5\% R_{max} = 5$ and $b = 0.05$.

In order to simulate the damage resulting from climate hazards, we need several other factors: the number of years $N(t)$ in which the climate hazard occurs during T years, the duration of each hazard in days $M(t)$, and the extent of damage Y_k incurred for each day of the hazard. $N(t)$ is modeled by a homogeneous Poisson processes with parameters $\Lambda = 5$ years. The duration $M(t)$ is a Poisson random variable with parameter $\lambda = 10$ days and finally, Y_k is modeled by the generalized Pareto distribution (GPD) with scale parameter $\sigma = 0.1$, shape parameter $\xi = -0.2$ and the threshold $u = 1$.

4.1 Simulation of ruin model

We generate 10^4 simulations using these parameters in the fixed time interval $[0, 100]$ (i.e. $T = 100$ years). For each simulation, we calculate the average reserve $R(t)$ until the forest ruins ($R(t) = 0$). Then, we select four simulations: those corresponding to the 95^{th} quantile, the median, and the 5^{th} quantile of the average reserve function, and one simulation where ruin occurs (i.e. the ruin time τ is less than 100 years). We repeat these simulations for 2 cases: $B = 0$ (isohydric) when the forest has no memory of hazard and $B = 0.4$ (anisohydric) when it has.

The damage values $S(t)$ is bounded because the shape parameter ξ is negative. The statistical characteristics of all $S(t)$ simulations are expected to be alike, derived from the same underlying distributions. Therefore, in Figure 1, we present only two simulations of damage: one leading to ruin (in red) and the other representing the median value of the average reserve $R(t)$. We observe that when adding the memory of the previous climate hazard B into yearly NPP allocated to reserves, the ruins may occur earlier. The growth is similar for both models, depicting that trees are resilient to damage over multiple years.

4.2 Study of the impact of hazard parameters on ruin year

The hazard parameters of this ruin model are the parameters of a Poisson distribution (Λ and λ) and the scale σ, shape ξ and threshold u parameters of GPD. The goal of this section is to evaluate the influence of each hazard parameter to the ruin year and the ruin probability.

To accomplish this, we take into account the ranges of values for the parameters. In this report, these ranges are estimated from the meteorological data, details in (Yiou and Viovy, 2021). We then use the Monte Carlo method, renowned for its robustness in stochastic simulations, to estimate the influence of parameters within our model. This method works by employing a series of random samplings to simulate the behaviour of uncertain variables, enabling a comprehensive exploration of the parameter space. We thus repeatedly execute 100000 Monte Carlo simulations under varying parameter configurations and then aggregate the results to discern the impact of the different parameters.

The influence of each parameter is depicted in Fig. 2. Each boxplot represents the probability distribution of the ruin time for a given value of one parameter and a random combination of the other parameters. We can clearly see that the GPD scale and the GPD shape have no effects on

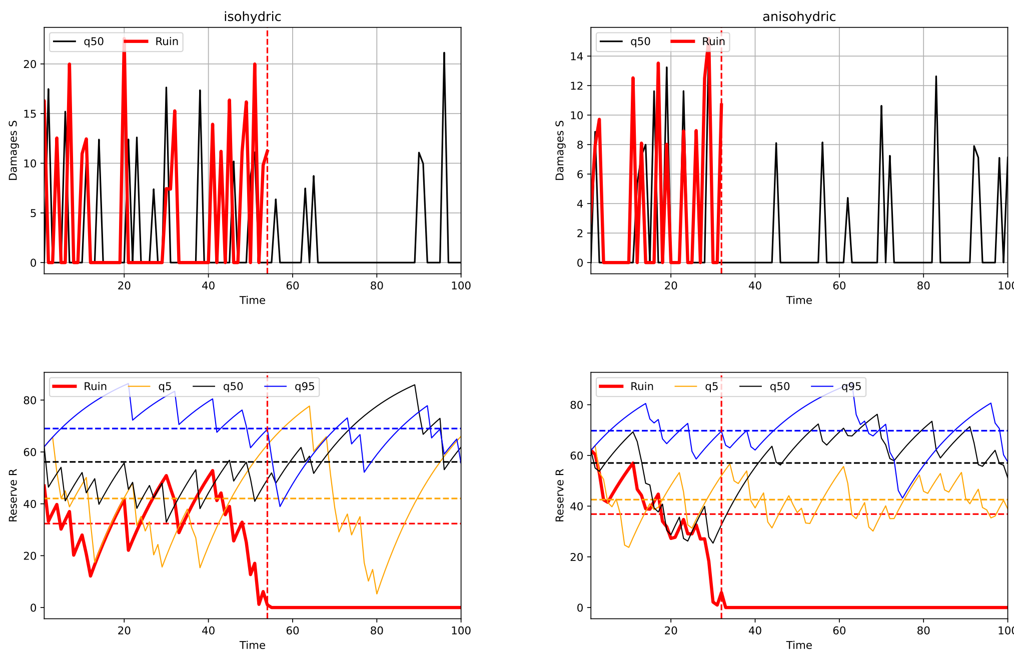

Figure 1: Simulations of the damage $S(t)$ (upper) and the reserve $R(t)$ (lower) in 2 cases: isohydric (on the left) and anisohydric (on the right). The orange, black and blue lines represent $R(t)$ trajectories reaching the 5^{th} quantile, the median and the 95^{th} quantile of 10^4 simulation means of reserve. The red lines showcase simulation where the ruin occurs (i.e. $R(t)$ reaches 0 before 100 years), the dashed-dotted line indicating the ruin time. The horizontal dashed lines denote the means of the reserve before ruin.

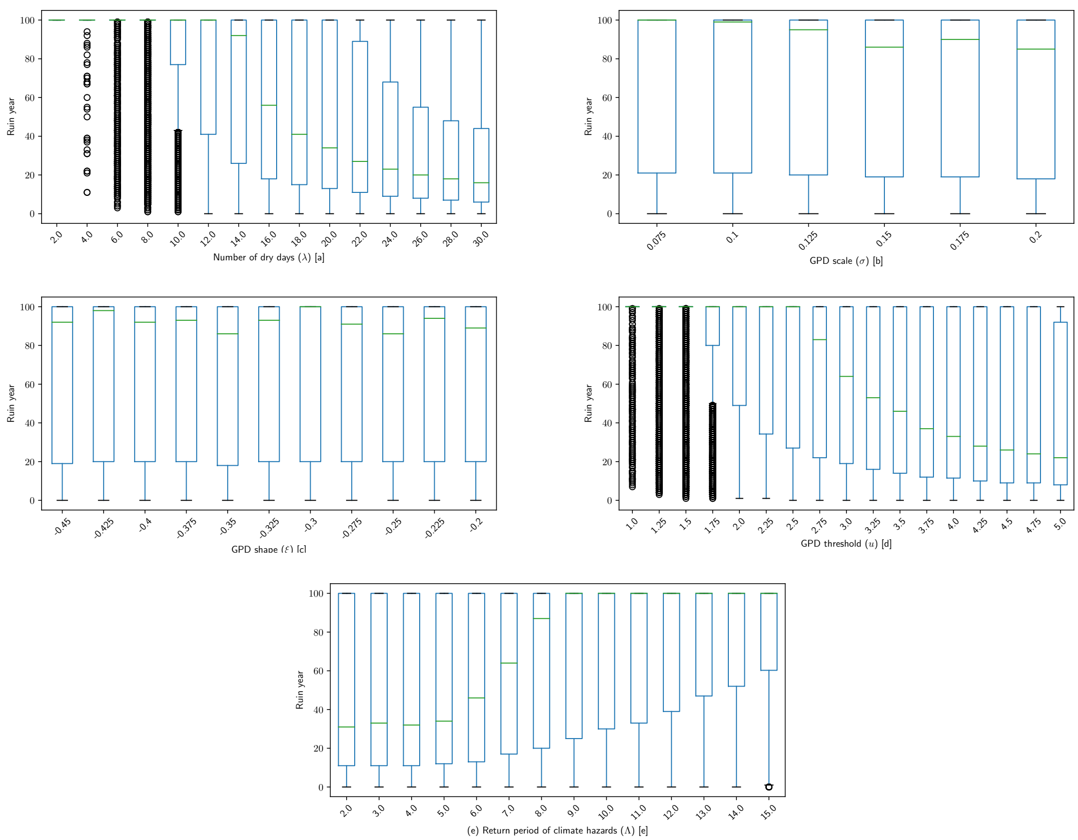

Figure 2: Probability distributions of the ruin year as functions of the number of dry days [a], GPD fit scale [b], GPD fit shape [c], the threshold u above which damage S(t) is triggered [d] and the hazard return period [e]. B = 0.4 in all experiments. For each value of the control variable, a boxplot is given.

Table 1: Nonlinear Model Results

Parameter	Interpretation	Average value	Range
u	GPD threshold	1.7	[1, 5]
σ	GPD scale	0.15	[0.08, 0.2]
ξ	GPD shape	-0.32	[-0.45, -0.2]
λ	number of hazard days	15	[2, 30]
Λ	return period for climate hazard	8	[2, 15]

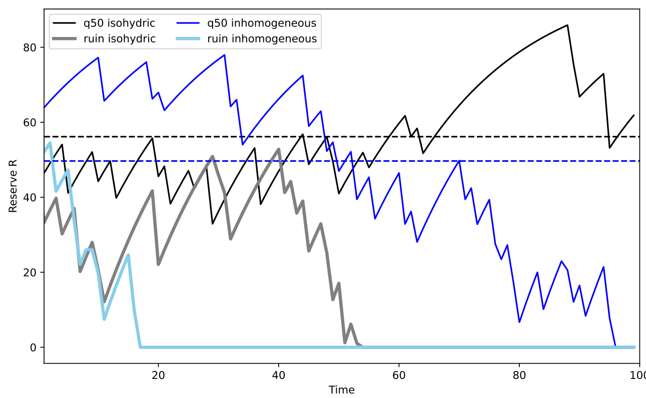

Figure 3: Comparison between the median obtained from the hPP model and the one obtained from the iPP model

the ruin time, as the median ruin year stays the same for all values of these parameters. On the contrary, the number of dry days λ per year and the GPD threshold u have a important impact on the ruin time, as the median shifts from 100 for lowest values to 20 for the highest. The last parameter Λ (return period for climate hazards) also impacts heavily the ruin time in a logical sense. When the period is short (2 years) the ruin year is sooner (30 years) than when it is longer, with no ruin when the period is at least 9 years.

4.3 Refining the model with inhomogeneous Poisson processes

In the preceding stages of simulation, we have modeled $N(t)$, the temporal distribution of hazard events, by a homogeneous Poisson process, characterized by a constant rate λ. This approach, while foundational, simplifies the underlying dynamics of hazard occurrences by assuming a consistent likelihood over time. However, to better mirror the evolving nature of these hazards, and particularly to offer a simplified model of global warming effects, we transition to an inhomogeneous Poisson process. This refined model introduces a variable intensity function $\Lambda(t)$ (see 4) which decreases over time:

$$\Lambda(t) = \frac{1}{\lceil 6 - exp(\frac{x}{60}) \rceil}$$

This modification suggests an increasing frequency of hazard events as time progresses, a pattern consistent with the progressive impact of global warming. By integrating this time-dependent intensity function, the model gains enhanced predictive accuracy. Fig 3 compares the two models (hPP and iPP), and we can clearly see the decreasing trend of the reserver $R(t)$ of the model using the iPP, as a result of the period between two hazard years being shorter over time.

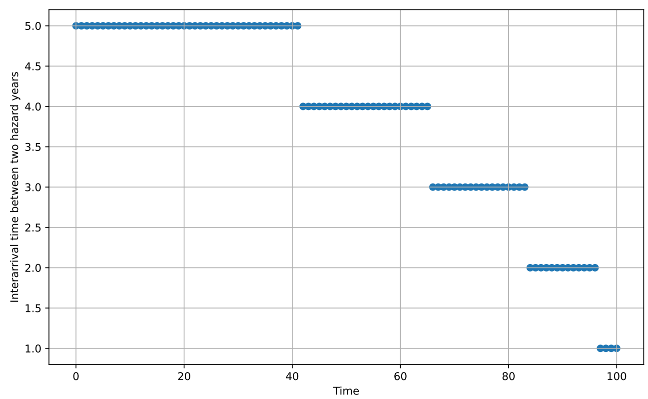

Figure 4: Inverse of intensity of hazard years ($\Lambda(t)$) over time. This function graph represents the inter-arrival times between hazard years. The curve reflects decreasing period, meaning an increasing trend in the frequency of hazards over time.

5 Conclusion & future work

This report explains a novel approach grounded in ruin theory to explore ruin tipping points for trees within a statistical framework. It demonstrates how frequent extreme events can lead to irreversible damage in trees. We modeled the vulnerability of an (idealized) forest by its probability of mortality through the depletion of nonstructural carbohydrate reserves.

With several simulations, we identified climate parameters influencing the ruin time which are the Pareto threshold u, the number of dry days per year λ and the return period of hazard years Λ.

The model has limitations due to the simplicity of our growth-ruin model. Certain parameters, particularly the impact scale factor A_h, were chosen arbitrary, highlighting the need for more detailed expertise to refine them for each tree species. Although simulations were efficient, future research could explore nonstationary simulations (without taking a fixed horizon T) to explicitly consider climate change. Nonetheless our results were still useful to determine the correlation between regular growth and occasional abrupt damage caused by climatic conditions.

A possible work to improve our results is to incorporate real data from climate model simulations. This would lead to a more accurate assessment of the potential impacts of extreme heat on forest ecosystems, thus deepening our insights into ruin tipping points.

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