Musical Meaning: Larson's Forces and Schlenker's Semantics

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Abstract Larson's theory of musical forces links music to metaphors of physical motion, while Schlenker's music semantics is similarly based on associations with the physical world. This thesis investigates the connection between the two approaches. The aim is to incorporate Larson's forces as constraints within a formal treatment of music semantics, while adhering to Schlenker's main objective of being able to say when a denotation is true of a musical expression. The effort is also computational, since logic programs are provided that implement part of the resulting framework.

1 Introduction

This thesis[1] is about the semantics of music. Semantics is the study of what words and expressions mean, and is a branch of linguistics as well as philosophy. In the latter case it is part of the philosophy of language. Semantics in (the philosophy of) language is usually about the exchange of information, where one would like the speaker's intended meaning and the hearer's interpretation to match. It is less clear, however, whether music involves an exchange of information, and whether such an alignment is required, desired, or even possible. It is nevertheless worthwhile to study musical meaning in terms of truth (or satisfaction) conditions.

In this study, analogies between language and music are considered for the formu-

[1] A significant part of the thesis was published in the *15th Baltic International Yearbook of Cognition, Logic and Communication* [dN23].

lation of ideas on music and meaning. A core assumption is that music is a kind of language. More specifically, this thesis looks at formal semantics as a methodology: in terms of semantics where linguistic phenomena considered relevant for meaning are specified in formal definitions, which are then used to construe models in which the expressions of the language are true. This is the primary objective: to study music semantics from the perspective of truth – or the truth conditions of musical expressions, although other takes on musical meaning, in particular relating to emotions, will also be looked at.

The main aim of this thesis is to connect Philippe Schlenker's music semantics [Sch19] with Steve Larson's work on musical forces [Lar97a]. Both authors establish metaphorical links between music and the physical world. In Schlenker's case musical features are associated with events or situations in the world, while Larson relates musical pitch motion to physical motion. This suggests that, at least for pitch and harmony, the approaches can be linked, in the sense that Larson's musical forces can be used to constrain Schlenker's semantics, or that Larson's framework can be interpreted within that of Schlenker. Both approaches are broadly explained below, with concepts used by the authors formalised in subsequent sections.

The reasons for explicit formalisation are as follows: first of all Schlenker sketches a broad framework but only deals with voice and musical truth. Secondly, while Larson's approach is narrower in the sense of being more about individual note movements, his forces tend to be described rather than defined. Finally, with the status of music semantics as yet unclear – e.g. should it depend on how a listener interprets music or not? – Hamm, Kamp, and Van Lambalgen insist that in either case, a semantics should be explicitly formalised, *"to ensure the computability which is fundamental to cognition"* ([HKvL06], page 3). As in Hamm *et al.*, the formalisation will result in a (partial) implementation using logic programming.

The objective of the thesis is to test the consistency of Schlenker's and Larson's ideas about music and meaning, and to see to what extent they can be conceived to complement one another. More specifically, the goals can be formulated as follows:

Goals

1. To put Schlenker's claim that a formal semantics of music can be developed to the test, in order to see what such a semantics might look like.

2. To give provisory formalisations of Larson's musical forces, so that they can be used within the formal semantics mentioned above.

3. To demonstrate the formalisations are indeed computable by providing logic programs and their outputs for (part of) the definitions.

4. To explore Schlenker and Larson's shared idea that music has a metaphorical connection to the physical world, and in particular the way in which truth conditions in a Schlenkerian music semantics may be constrained by Larson's forces.

1.1 Schlenker's music semantics

Schlenker's *Prolegomena to Music Semantics* [Sch19] explores the idea that music has a semantics, which to him consists in music having a meaning that relates it to something which is external to the music itself. This semantics, according to the author, is a rule-governed system which licenses inferences about a music-external reality (*ibid*, page 36). The basic idea is that inferences are drawn about actions or features of so-called virtual sources (after Bregman [Bre90]), which are imagined to be responsible for or represent the music's sounds. For example, lower-pitched sounds might be associated with larger entities, and if the sound gets louder (crescendo) then it may be inferred that the entity or entities are getting closer (viz. [Sch19]2, pages 50-52). As Schlenker puts it, music semantics starts as sound semantics. See figure 1 for an example of the latter feature.[2]

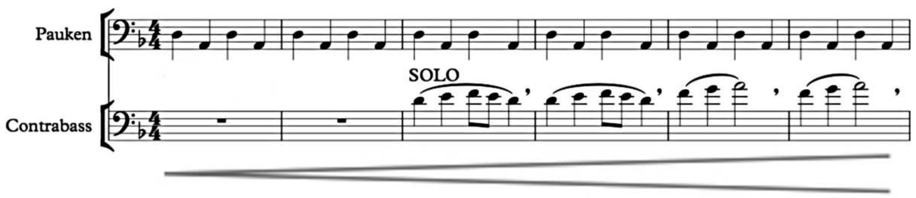


Figure 1: Mahler's *Frère Jacques* – First Symphony, 3rd movement, example 12 b (with added crescendo) in [Sch19], page 51, sound: bit.ly/2m9WnIS

Schlenker remarks that because of the crescendo, the piece could be interpreted as an approaching procession, which, as the author adds, is intended to be playing funeral music according to the composer (Gustav Mahler). The score in the figure shows two instruments, or voices, which may be associated with two virtual sources, in this case percussion (timpani) and bass.[3] The idea here is that the first source is responsible for the procession inference, and the second is self-referential in that its inference in this case is simply the music itself. This particular example is

[2] Note that in the figure – and elsewhere – a hyperlink is included. If this thesis is read in a web browser, it is advised to open the links in a new tab or window so the document stays open.

[3] Schlenker points out that it is also possible for several instruments to play a single voice, or for a single one to play several ([Sch19], page 37). In this study, each example is a single voice.

intended to illustrate the effect of the crescendo, loudness being one of several properties of music about which one could in principle draw inferences. It is not difficult to imagine that if the music were instead to go softer (descrescendo), then it would rather represent a procession moving away (as in example 12 c, *ibid*, page 51). Schlenker does not go into the question why it would be funeral music, but an attempt at explaining this will be given in section 6.6.

Like loudness, pitch and harmony are also features of music from which inferences might be drawn, and it is these that this study is focused on. Similarly, rhythm and velocity or speed are musical features. In the main examples of this thesis, the latter will be kept largely regular and constant, while pitches will be mostly kept within an octave, i.e. the chief aim is to consider the interaction of notes within relatively small tonal intervals, rather than to look at pitch in the sense of ‘very high’ vs. ‘very low’ – which as noted is another way of considering pitch, but from the above ‘sound semantics’ rather than a ‘tonal’ perspective. That said, sound semantics will not be completely ignored but the primary focus is tonal.

Figure 2 is an example from [Sch19] (page 44) where tonal inferences are used, and shows the score of the beginning of Richard Strauss’ *Also Sprach Zarathustra*. Schlenker links this with Stanley Kubrick’s 1968 film *2001: A Space Odyssey*, where this music is used. His aim is to explain why the music is appropriate for the motion picture’s imagery. This effectively boils down to the claim that a description of the events depicted in the imagery qualifies as one of the snippet’s possible denotations,[4] as will be explained now.

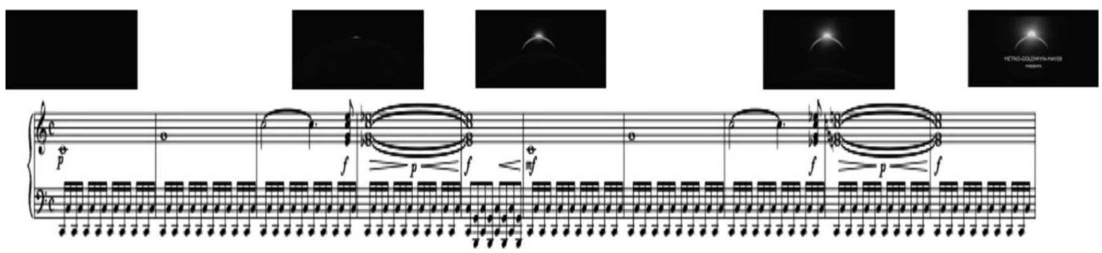


Figure 2: Opening of Strauss’ *Also Sprach Zarathustra* annotated with imagery from Kubrick’s *2001: A Space Odyssey* (example 6 in [Sch19]), video: bit.ly/2DfiE3m

The concept of possible denotation is defined by Schlenker on page 66:

[4] According to Schlenker, the motion picture’s imagery synchronises with a two-stage appearance of a star from behind a planet, with the first five measures corresponding to the first stage, and the latter ones to the second, but only part of this intuition is worked out formally.

Definition 1.1.1. *Let M be a voice, with $M = \langle M_1, \ldots, M_n \rangle$. A possible denotation for M is a pair $\langle O, \langle e_1, \ldots, e_n \rangle\rangle$ of a possible object and a series of n possible events, with the requirement that O be a participant in each of $e_1, \ldots, e_n$.*

This definition assumes that a musical voice (which is associated with a virtual source as explained on page 3 and in footnote 3 here) is split into n musical events and posits an object O. It says that for each event M_i there is an (imagined) event e_i in the world, in which O participates. The resulting event sequence may be considered as the music's denotation (or meaning), in case it is 'true of' the music.

Each M_i can contain a number of features from which inferences may be drawn. For *Zarathustra*, Schlenker considers two: harmony and loudness. The analysis being limited to the first three measures, he renders the musical events as $M = \langle\langle I, 70db\rangle, \langle V, 75db\rangle, \langle I, 80db\rangle\rangle$, with harmony moving from stable to less stable and back (viz. [Sch19] pp. 65-66 and section 3.1 here), while loudness increases. The idea is that there is a corresponding sequence of virtual (imagined) events which are 'true of' the music, and Schlenker gives the following definition to make this precise (*ibid*, page 67 – things will be further clarified in section 3).

Definition 1.1.2. *Let $M = \langle M_1, \ldots, M_n \rangle$ be a voice, and let $\langle O, \langle e_1, \ldots, e_n \rangle\rangle$ be a possible denotation for M. M is true of $\langle O, \langle e_1, \ldots, e_n \rangle\rangle$ if it obeys the following requirements.*

(a) *Time: The temporal ordering of $\langle M_1, \ldots, M_n \rangle$ should be preserved, i.e. it should be the case that $e_1 < \ldots < e_n$, where $<$ is ordering in time.*

(b) *Loudness: If M_i is less loud than M_k, then either*
   - (1) *O has less energy in e_i than in e_k; or*
   - (2) *O is further from the perceiver in e_i than in e_k.*

(c) *Harmonic stability: If M_i is less harmonically stable than M_k, then O is in a less stable position in e_i than it is in e_k.*

The above would allow for the denotation *Sun-rise* = ⟨*sun,* ⟨*minimal-luminosity, rising-luminosity, maximal-luminosity*⟩⟩ to be true of M, but as Schlenker points out, there are more options, e.g. *Boat-approaching* = ⟨*boat,* ⟨*maximal-distance, approach, minimal-distance*⟩⟩, while their opposites *Sun-set* and *Boat-departing* are among the denotations that do not qualify (*ibid*, page 68).

Musical meanings in Schlenker's view are the possible denotations that are true of the musical events, or the 'world' events that qualify given definition 1.1.2. Schlenker emphasises that any meaning thus obtained is merely one among many possibilities ([Sch19], page 67). This is presumably because the information music conveys is more abstract than language (*ibid*, page 36).

The above points to some underlying presuppositions in Schlenker's music semantics: meaning is truth-conditional and extra-musical, i.e. music is about some reality that is external to itself. So music is not about harmony or associated properties of tension (instability) and relaxation (stability), as this would be a so-called 'internal' semantics. Neither is it about expectations or emotions that might be aroused within the listener. Instead, it is a structure-preserving mapping between musical events and events in the world.

1.2 Larson's musical forces

In Larson's *Musical Forces and Melodic Patterns* [Lar97a] it is claimed that the way (experienced) listeners hear music is aided by three metaphorical musical forces: gravity, magnetism, and inertia. The idea is that because of these forces, music is heard as purposeful, because they link music to phenomena that are familiar from the physical world.

Larson specifies the forces as follows. Gravity is a note's tendency to descend given some stable threshold or ceiling, magnetism is the tendency to be attracted to a (more) stable pitch, and inertia the tendency of notes to continue in the same pattern. Larson's basic idea is that a piece of music may be viewed (or rather heard) as having been constructed level by level, from a simple level where movement is towards stable notes to more complex ones where movement can also be towards less stable notes. These more complex levels are called embellishments, and the transitions from lower to higher levels are 'controlled' by the forces ([Lar04], page 457), with the *"increasingly more-detailed levels leading ultimately to the piece itself"* ([LV05], page 132).

Figure 3 below (with sound file links as elsewhere)[5] is a simple example to illustrate this idea as well as Larson's three forces. The basic first level, containing the notes *[e, c]*, has motion from the mediant (or 'third') to the tonic. At the more complex second level, these notes have been embellished with the less stable subdominant (the 'fourth') and supertonic (the 'second'), ultimately obtaining *[f, e, d, c]*. So while motion is only towards the tonic, i.e. to the most stable goal at the first level, it is also towards the less stable mediant at the second level (see section 3.1 for more on stability).

The whole sequence is an example of gravity (marked g in the figure) for its down-

[5] If this thesis is read in print, go to `https://mickdeneeve.github.io/ac/ma/ex/` for a listing of all mp3 sound file links.

Level 1:
4321.1.mp3

Level 2:
4321.2.mp3

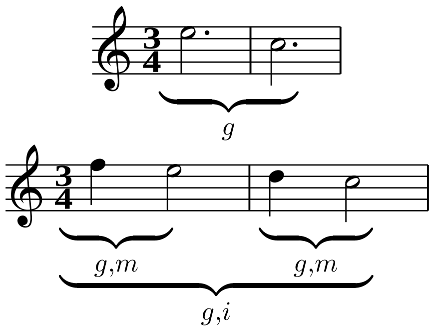


Figure 3: Embellishing to *4-3-2-1*

ward motion (both levels), and of inertia (marked i, level 2) because of continued motion and pattern repetition: *[f, e]* is reproduced in *[d, c]*. These subsequences (again at level 2) are also gravitational. Moreover they are magnetic (m) since the less stable f is attracted to the more stable e, and the less stable d is similarly attracted to the (most) stable c. But note the attraction *[f, e]* is stronger, since these notes differ by a semitone, while the final two notes are a whole tone apart.

Larson staged prediction experiments to test the psychological reality of his forces, e.g. in *Measuring Musical Forces* [LV05], where subjects were presented with cues and asked to predict the next note. Similar to the figure 3 example, they might be given the middle two notes *[e, d]*, and asked to complete the pattern. In case the answer is c (viz. *ibid*, page 122), the pattern is said to 'give in' to gravity as well as inertia. It should strictly also give in to magnetism, but Larson has tended to restrict this to semitone attraction in [Lar02] and beyond (page 381, footnote 6).

In [LV05], patterns or note sequences are stepwise (viz. definition 3.1.4 here), meaning the next note always differs by at most a whole tone (page 121). Stepwise motion is considered central to musical perception in Larson's view; in case larger intervals (i.e. leaps) occur, the listener is left expecting a stepwise completion ([Lar97b], pages 105-6). See figure 4 for an example with leaps leading to a stepwise connection (completion) between the first and last notes (but see also section 5.1).

leap.trace.mp3

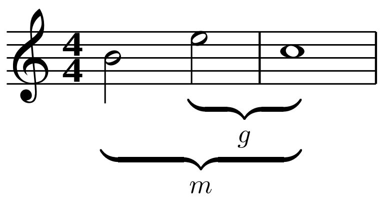


Figure 4: Displacement of a trace

In the figure, the interval between the first two notes *[b, e]* is a fourth, which gives rise to a 'trace' on the *b* in the listener's mind that gets 'displaced', in the author's terminology, upon hearing the final note *c*, since the interval *[b, c]* is stepwise (a semitone). The idea is that the listener hears this interval, or is aware of the connection between these notes.

The figure's first and last notes could be viewed as the first level, i.e. the pattern starts out by 'giving in' to magnetism with *[b,c]*. At the second level, the note *e* is inserted – as an 'operation' (cf. [Lar04], page 466) controlled by gravity – and subsequently *[b,e,c]* is obtained (note that both levels use the tonic *c* as the goal of the motion). In other words, the magnetism that is 'heard' by virtue of the displacement is determined not at the final but at an earlier (i.e. the first) level.

The example illustrates how forces may be assigned at multiple levels, and indicate (or 'control') musical motion towards more (harmonic) stability. This implies that forces may be assigned at such points in a partioning of musical events, and that such events may be analysed hierarchically. This in turn implies the possibility of a music semantics with at least a compositional flavour.

1.3 Plan for the rest of the thesis

The remainder of this study is organised as follows. In section 2, a short history of musical philosophy is given, and Schlenker's and Larson's conceptions of semantics are described. Then in section 3, Larson's forces are used as an abstract stage in the analysis of a few melodic patterns, from two of his papers. There, force assignments will be mapped to events which can be considered as denotations for Schlenker's semantics. This will involve the specification of several formal definitions, the chief aim being the extension of definition 1.1.2.

Section 4 will then extend some of these ideas, including the specification of a different notion of magnetism and a broader conception of inertia. Additionally, modulation is considered, as well as the meaning of so-called levels of embellishment. Moreover, dynamic interpretation is suggested as an alternative or augmentative system, since this may be more in line with Meyer's [Mey56] ideas on affect. After that, section 5 gives a partial logic programming implementation of Schlenker/Larson framework, and additionally, some propositions on the relation between harmony and note partitionings are proved.

Subsequently in section 6, the framework and its merits are discussed, including issues concerning forces, events, compositionality and affect. Finally in section 7, the conclusions of the thesis are summarised.

2 Background

2.1 History

Western philosophical thinking about music starts in ancient Greece, first of all in the 6th century BCE with Pythagoras, who is said to have explored the idea of musical intervals as mathematical ratios.[6] For Pythagoras, these musical ratios were a reflection of a cosmological or natural order. A similar idea is found in Plato (4th century BCE), for whom art, including music, were forms of so-called mimesis, which is Greek for imitation and refers to the idea that art 're-presents' nature. In *The Republic* [Pla00], Plato argues that musical education instills an understanding of order or 'grace' in the mind which in turn aids the understanding of nature (Book III, §401), with the implication that music should express virtues such as temperance, though it can also express emotions like sorrow.[7] A similar conception appears in the work of his pupil Aristotle, who wrote about music's ability to imitate nature and emotions, and affect the latter (*Poetics* [Ari96], Part I; *Politics* [Ari05], Book VIII, Part V). These early ideas about imitation and resemblance are relevant here as they are essentially also what Larson and Schlenker allude to when characterising musical meaning using physical metaphor.[8]

As is not uncommon in science, the classical era is followed by a long period of relative silence. During medieval times, books on music including Ptolemy's *Harmonics* continued to be read, but only after the invention of printing did further ideas develop and gain traction. In the 18th century, Jean-Jacques Rousseau argued that speech did not arise to express needs, but feelings (*On the Origin of Language* [Rou86], chapter 2). According to Rousseau, there was poetry before prose – early writing was in verse – but in general, *"all voices speak under the influence of passion"* (*ibid*, chapter 12). Music, firstly in song form, developed out of recital, with emphasised or heightened speech to stress the emotional content.

For Friedrich Nietzsche too, the development of language and music are connected. Nietzsche held that before there was language, people's main sense of self was in terms of communal connections, and like Rousseau a century earlier, he believed language was first of all meant for communicating emotions – or more specifically those relevant to survival (*On Music and Words* [Nie16]). It, however, allowed for

[6] No accurate historical records survive for this claim, but the attribution is down to, among others, Ptolemy [Pto00].

[7] Curiously but perhaps predictably Plato thought the harmonies associated with this should be banished because he considered them useless (§398).

[8] It might also relate to Schlenker's ideas on emotional affect as 'experienced events' ([Sch19], section 10).

the development of consciousness as a side-effect, thus changing people's sense of self to move towards the individual rather than the communal. This distinction ultimately boils down to what Nietzsche called Apollonian versus Dionysian, and it is the latter mode he thinks music is closer to, as it most strongly allows for conveying emotions (or the 'heart of the world', viz. *The Birth of Tragedy* [Nie08], §21). 'Apollonian music', on the other hand, is an "imitative counterfeit of phenomena" without poetic power (*ibid*, §17), so one might say that the function of music is to, as it were, lose one's sense of self. Note how Nietzsche's 'Apollonian' characterisation of music is close to Schlenker's – in fact Nietzsche would likely reject the path taken in this thesis.

Further down the road – halfway through the 20^{th} century – in *Emotion and Meaning in Music* [Mey56], Leonard Meyer sought to explain how music can communicate emotional and aesthetic meanings while lacking explicit symbols that refer to a non-musical world. Meyer did not deny that music may in fact have such symbols in an implicit way, but he chose to avoid the debate on whether music is a language and whether its stimuli may be viewed as signs or symbols. Instead, he conjectured that meaning, which he considered as consisting mainly if not exclusively of emotional affect, arose out of listener's expectations. Hence it is the task of the composer to manage these by either satisfying them or by thwarting their fulfillment – i.e. to play with the expectations. Meyer thought that particularly delay was a significant tool, as it would catch the listener's attention while expectations were left unfulfilled, which allow objectified meanings to be assigned, i.e. meanings that ensue from a listener wondering about the intention behind the music. According to Meyer this can result in either an intellectual or an affective response, depending on the listener's disposition or training, e.g. those technically skilled in music would be more likely to respond intellectually by conscious reflection. That being said, Meyer did not go into questions concerning the precise nature of musical meaning in great depth, i.e. what it might designate, as his stated goal was to study how meanings arise from relationships between musical elements and listener expectations. But according to Meyer, both absolutist and referentialist notions of meaning (broadly speaking 'internal' and 'external' semantics in Schlenker's terminology) are consistent with his views.

While in part inspired by psychological as well as probabilistic methods, Meyer's work is mainly discursive in nature, and formal studies into music cognition did not appear until almost three decades later. Notable examples are Eugene Narmour's *Implication-Realization Model*, and *A Generative Theory of Tonal Music* by Fred Lerdahl and Ray Jackendoff. Narmour was a student of Meyer's and tried to continue and formalise part of his teacher's work on musical expectation, by reducing possible melodies into a limited number of archetypes – for note-to-note relationships [Nar90] as well as those between non-contiguous notes [Nar92]. Narmour's model is a perception-oriented account of cognition, aimed at describing how melodies are heard in terms of ongoing realisations and denials of expectations. According to Larson and VanHandel [LV05], Narmour's concept of musical 'process' is similar to Larson's musical force of inertia.

Lerdahl and Jackendoff's theory [LJ83] is an attempt to get at what might constitute a grammar of western art music in terms of hierarchical grouping structures which aim to make explicit how musical elements from larger to smaller chunks depend on other musical elements. Crucially though, the authors implicitly assume the logical soundness perspective, which means that every derivable formula of a logic is also true – or put differently, if the syntax is properly taken care of, the semantics comes for free.[9] But because, as Lerdahl and Jackendoff have it, music has no denotations or syntactic categories such as nouns or verbs, there is no sensible sense of meaning to be associated with their grouping structures, a point they emphasise in the book's first chapter. Despite the fact that they do not deny that music might have meaning, it is not something they feel could be usefully wedded to their theory. Roger Scruton [Scr74] advanced the stronger version of this view by claiming that music cannot be given a semantics as it cannot be endowed with the same sort of truth-preserving rules as natural language has.

The soundness perspective on natural language semantics is effectively due to Richard Montague (*English as a Formal Language* [Mon70]), who associated partial semantic functions with parts of speech and syntactic categories in such a way that meanings could be propagated and conjoined up to the top node of a grammar tree (a sentence) so that a complete logical expression could be obtained and instantiated in such a way that the construction of meaning follows the construction of syntax (such that 'the semantics comes for free'). One noteworthy attempt to apply this idea to music is due to Kari Kurkula [Kur87], whose main objective was the analysis of a segment of music notation: a single note e on the treble clef, modified by a diminuendo (getting less loud) operation, that was rendered in terms of partial semantic expressions which effectively boiled down to nothing more fancy than to express how the notation denoted a particular sound event. In other words, no other referential properties, let alone affect, or what a combination of notes might potentially signify beyond a string of such events. Nonetheless the effort is relevant in that it illustrates a limit to what may be expressed from the soundness perspective in terms of musical semantics: this is apparently not very

[9] It should be clear that despite the musical context of this thesis, 'soundness' is a logical concept, and as such is unrelated to the 'sound' of music.

far from how people tend to describe what music means to them, which is indeed frequently in terms of emotions (see for instance Koelsch [Koe10]).

Another application of Montague's ideas on the semantics of language to those of music that may similarly be viewed as an illustration of how the soundness perspective limits or perhaps prohibits a fruitful perspective on (formal) music semantics may be found in Mark Granroth-Wilding's *Harmonic Analysis of Music Using Combinatory Categorial Grammar* [GW13] (which is incidentally also referenced by Schlenker). Here, musical meaning is effectively reduced to a journey in pitch space, which is again what Schlenker terms an internal semantics and hence not a 'bona fide' notion of meaning due to the requirement that reference should be to something external to the music itself (due to David Lewis [Lew70] among others, although Scruton makes essentially the same case). These examples indicate why Schlenker says that his proposal for a music semantics constitute *"a source-based semantics rather than a compositional semantics"* ([Sch19], page 39). This is because a compositional semantics, at least if viewed strictly from the soundness perspective, is incapable of expressing the richness that is apparently needed to capture musical meaning. Hence Schlenker seeks meaning mediated by virtual sources to step beyond internal meaning or meaning restricted to something as trivial as reference to sound.

2.2 Schlenker's and Larson's notions of meaning

While Schlenker has an explicit programme geared towards the semantics of music, this is not the case for Larson, who does not even mention meaning very often. Larson views musical meaning as a suggested quality of music, such that it allows a listener to experience feelings, action, or motion ([Lar97b], page 101). According to the author, such suggestions arise in perception because of the interplay of his musical forces, which are viewed as musical motion that is in turn heard as a mapping of physical gesture onto musical space. Meaning arises not only from the musical objects at the musical surface, so to speak, but also from the creative perception of an experienced listener, which allows the listener to hear a fragment of music x as y, with y an assigned meaning, for example an ascending gesture (*ibid*, page 102). There may be a complex interplay between musical elements for meanings to emerge, for instance a C below the magnetic pull from F to E is not part of the musical force as such, but it does give it context by providing information that the melody has landed on the stable third of *C Major* (page 131).

Despite setting out such basic views about the nature of musical meaning, Larson does not delve into the matter deeply. Instead his focus is on the forces, rather than on meanings that may be assigned by a listener because of their interplay.

Put differently, he focuses on how the magnetism in the motion from F to E prolongs *C Major*, rather than the traversal through physical space it suggests, or an emotion it might instill in a listener. In other words, even though Larson puts his work on musical forces in the context of a physical motion based conception of musical meaning, he stops short of exploring the nature of this idea of meaning.

For Schlenker on the other hand, musical meaning and semantics is the core concept in [Sch19]. He considers this to be part of a wider research programme of 'Super Semantics', that he says might also be called formal semiotics. According to the author, for any representational form it might be said that to know the meaning is to know the truth conditions (viz. [Sch18], page 366), and consequently, he suggests formal semantics could be developed for representational systems including pictures, gestures, music, or dance.

Beside his portrayal of music semantics as truth-conditional as well as part of a wider theory, Schlenker also attempts to characterise it within the classic (semiotic) Peirceian tripartition [Pei98], where signs can be iconic, indexical, or symbolic. He claims musical signs may be considered as indices because his semantics results from positing causal relations between sound and (virtual) sources ([Sch19], page 37). But he does admit things may not be so straightforward: a musical sign could also be viewed as iconic if it resembles its denotation, which hangs on how one views the concept of resemblance.[10] According to the author, a sufficiently abstract notion of iconicity could make the sunrise denotation of figure 2 iconic since it fulfills structure-preserving conditions (his truth definition 1.1.2), which could then be viewed as satisfying the idea of resemblance – even if a sunrise is not a sound-producing event (*ibid*, page 74). Possibly, adding Larson's forces to the mix will push the semantics more in the direction of iconicity if this then means that musical motion more closely resembles the resulting denotations.

A notable difference between Schlenker and Larson is how they view the direction of the mapping between musical and physical space. The idea behind Schlenker's truth definition is to specify a structure-preserving mapping between musical and physical events. Larson on the other hand says that physical gesture is heard as a mapping onto musical space. Essentially, both are talking about a homomorphism between two spaces, i.e. a structure-preserving mapping. Nevertheless, there may be an interesting if subtle difference in emphasis. As noted, Larson's claim with respect to his forces is that these control how experienced listeners hear music, while there is no such restriction for Schlenker, so plausibly for Larson, there is an accent on 'active' listening (viz. his focus on prediction), and given the embel-

[10] For completeness, a sign is symbolic if its denotation is arbitrary, or conventional.

lishment levels (figure 3) even on the compositional process itself. The difference in direction then, might be compared to the difference in linguistics between production (Larson) and comprehension (Schlenker) – or, in other linguistic terms, between expression and interpretation.

3 Larson's forces in Schlenker's semantics

In the present section, Larson's forces will be made (more) precise, and assigned to a few simple musical examples, including a stepwise example as in figure 3 but also a leapwise (i.e. non-stepwise) one, like in figure 4. The idea is that by ascribing forces at points of harmonic stability, a partitioning of the example into musical events ensues. These events are treated as features, i.e. they have the same form as Schlenker's events such as where he reduces a piece to its loudness in decibels and harmonic motion (see section 1.1, and [Sch19], page 66).

Here, harmonic motion will also be a feature due to Larson's reliance on scales and degrees (viz. [Lar97a], page 59), which is because of said stability considerations. The events thus obtained may, like Schlenker's musical events, be viewed as abstract states of affairs, with which a potentially large number of situations in the world could possibly be consistent. Examples of such states of affairs in the world will be given, involving a virtual source and events the source participates in, given an adapted version of Schlenker's truth definition (i.e. definition 1.1.2).

To begin with, definitions for scales, note sequences, harmonic role substitutions and stability are given. This is to be able to give musical examples in terms of numerical sequences, which can then be partitioned at their points of stability because, according to Larson, this is where his forces 'control' the music towards. After this, the forces themselves can be defined, to be assigned to these partitions in a first concrete musical example. Finally, mappings will be specified from the resulting musical events to 'world' events. Note that these mappings will be stipulative, i.e. based on intuitions about the relation between music and the world.

First, for the sake of illustration, two well-known scales are the major and minor ones. They are shown in figure 5, and are *C Major* and *A Minor* respectively. The latter was picked to highlight the relation between the two: *A Minor* may be constructed by starting on the sixth note of *C Major*, and it is consequently also known as mode `VI` of the key of *C Major*. Sound files (mp3) are linked in the figure, and the scales are ended with their triads (chords), which are the first, third and fifth notes sounded together – which are also Larson's points of stability. The numbers at the top of the figure are to indicate how so-called note positions

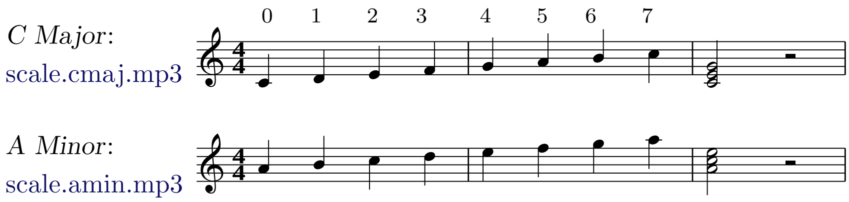


Figure 5: The *C Major* and *A Minor* scales with their triads

are represented here: as 0-based numerical lists, i.e. the root of the harmony gets the number 0, and so what was earlier specified as being stable first, third, and fifth notes, will be rendered as numbers 0, 2, and 4, respectively. This is to enable modular or clock arithmetic (division remainder), since the final note 7 is the first one repeated an octave higher, and 7 *mod* 7 = 0. The effect is that one can continue to count into higher – or lower – octaves, e.g. in the coming example the note below the root is used, rendered as -1, and -1 *mod* 7 = 6.

A first example, more comprehensively analysed later, is adapted from [Lar97a] (where it is example 5, pattern 2), and is shown in figure 6 below, with forces assignments as well as harmonic motion. In the following section, elements of musical structure are defined formally, and illustrated with aspects of this example.

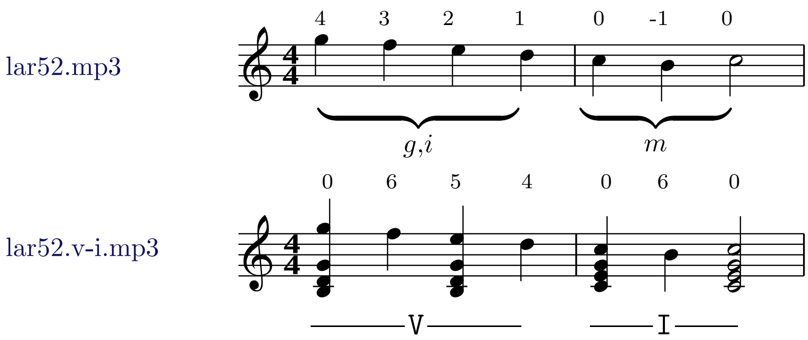


Figure 6: Larson's example 5.2 from [Lar97a]

The first row of numbers are note positions as above, and the second row are their so-called relative scale degrees. The Roman numerals are scale modes to which these degrees are relative, as will be explained. The forces gravity (g), magnetism (m) and inertia (i) are assigned based on these relative scale degrees on the second row, i.e. to note groups with stable endings (0, 2 or 4, as indicated above).

3.1 Musical structure

Scales, sequences and triads Scales are essentially given by structures indicating the number of half steps (or semitones) from the starting note (root). The spaces or intervals between the notes of a scale are usually either a whole or a half step, e.g. *C Major* above has half steps between notes 2 and 3, and between 6 and 7, while *A Minor* has them between 1 and 2, and 4 and 5 – with the remaining intervals whole steps (two semitones).[11]

But in order to produce a concrete scale description from such ratios, a collection of pitches is needed.

Definition 3.1.1. *A pitch collection is a 12-element list of pitch names. The one used here is* $P = [c, c\sharp, d, d\sharp, e, f, f\sharp, g, g\sharp, a, a\sharp, b]$*, with the following enharmonic ('same-sounding') equivalences:* $c\sharp = d\flat, d\sharp = e\flat, f\sharp = g\flat, g\sharp = a\flat$*, and* $a\sharp = b\flat$*. These equivalences may be substituted for each other. Pitches may be referred to by their (0-based) indices, e.g.* $P[4] = e$*.*

While it is possible to generate the minor scale from the major scale since it is one of its modes (illustrated in figure 5 – which more generally holds for other scales as well), this is not done here. Instead, the so-called scale measurements used in this study are given 'hard-coded' as distance lists instead.

Definition 3.1.2. *A scale measurement is a 7-element list of semitone distances counted from the first element, or* $root(0)$*. Specific measurements are firstly* $Ionic\ Major = [0, 2, 4, 5, 7, 9, 11]$*,* $Lydian\ Major = [0, 2, 4, 6, 7, 9, 11]$*,* $Mixolydian\ Major = [0, 2, 4, 5, 7, 9, 10]$*, and*$Aeolian\ Minor = [0, 2, 3, 5, 7, 8, 10]$*. The latter three are respectively modes 3, 4, and 5 of* $Ionic\ Major$*. Secondly there is* $Harmonic\ Minor = [0, 2, 3, 5, 7, 8, 11]$*, and* $Melodic\ Minor = [0, 2, 3, 5, 7, 9, 11]$*. These are derived from Aeolian Minor by respectively raising the last note, 6, and additionally raising note 5 as well. Except for the first, called prime, intervals in scales are named after their ordinals. If the third has distance 3, the scale's aspect is* $minor$*, if this is 4 it is* $major$*.*

Note that even though the definition gives harmonic modes as 0-based numbers as this is computationally convenient, in the musical examples harmonic motion is given in traditional (1-based) Roman numeral notation (as in figure 6). So in the sequel, if `V-I` motion is indicated (cadential motion as explained later), then this may be considered as shorthand for $mode(4) \rightarrow mode(0)$.

With pitch collections and scale measurements, concrete scales can be produced:

[11] For further illustration consider a piano keyboard: a whole tone would be where going from one white key to another involves skipping over a black key. The white key areas have two spots without a black key in between. These are precisely the half steps or semitones mentioned.

Definition 3.1.3. *A scale instantiation or simply scale is generated from a pitch collection P and scale measurement M as follows. Pick a root pitch $P[r]$, and fill a list S as follows. For each $i = 0 \ldots 6 : S[i] = (M[i] + r)$ mod 12.*

This will generate the *A (Aeloic) Minor* scale in figure 5 from $P = [c, c\sharp, d, d\sharp, e, f, f\sharp, g, g\sharp, a, a\sharp, b]$ and $M = [0, 2, 3, 5, 7, 8, 10]$ as follows: the root pitch is $P[9] = a$, so the indices for picking the pitches from P are $[9, 11, 12, 14, 16, 17, 19]$ *mod* $12 = [9, 11, 0, 2, 4, 5, 7]$, so $S = [a, b, c, d, e, f, g]$.

While this may all look rather involved, the purpose is to later be able to compute Larson's forces, particularly magnetism, from a given scale plus a note sequence. In figure 5 the note sequence for the above scale was given as $[0, 1, 2, 3, 4, 5, 6, 7]$, which as indicated is the scale plus its root repeated an octave higher. It is also the indices *mod* 7 for the pitches in S above. Since 7 *mod* $7 = 0$ (i.e. 7 has degree 0; see below),the pitches in the figure are $[a, b, c, d, e, f, g, a]$.[12] A note sequence in general is specified as follows.

Definition 3.1.4. *A note sequence is a numerical list of note positions. A note position is the location of a note in a 7-note scale (relative to a root) defined as 0, and may be either positive, i.e. higher than the root, or negative. In case for each adjacent position pair, $abs(p_1 - p_2) = 0$ or 1, the sequence is stepwise. For any position p, its degree is given by p mod 7. Degrees are traditionally called tonic, supertonic, mediant, subdominant, dominant, submediant, and subtonic.*

As indicated, the idea is that note sequences are to be partitioned with forces assigned at points of stability, but before that concept can be made precise, triads (or chords), chord progressions and cadences (a special kind of progression) should be defined, since stability also applies to these. This will then be applied to figure 6's harmony to illustrate things further, after the following definition.

Definition 3.1.5. *A triad is a list of three note degrees $[d_0, d_1, d_2]$ with $(d_1 - d_0)$ mod $7 = (d_2 - d_1)$ mod $7 = 2$ (in traditional terms they differ by the interval of a third). The above order being termed root position, the order $[d_1, d_2, d_0]$ is called the first inversion, and the order $[d_2, d_0, d_1]$ the second inversion. Given a scale S, the triad's aspect can be determined: this is given by $(S[d_1] - S[d_0])$ mod 7, which is the number of semitones between d_0 and d_1. This may be 3 or 4. In the first case the aspect is minor, in the second it is major(these intervals are called minor and major thirds). The root d_0 gives the triad's mode, and it is customary to use this to represent a triad as a 1-based Roman numeral (i.e. $d_0 + 1$), usually written in capitals though frequently in lower case if the underlying scale is minor.*

[12] Pitches as used here are names and do not distinguish between octaves. That the start and end of the sequence are an octave apart is instead read off from the numerical representation.

The harmonised part of figure 6 is repeated below as figure 7, to help illustrate triadic harmonic motion as just specified.

lar52.v-i.mp3

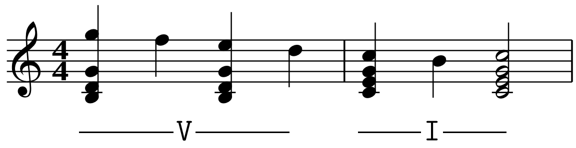


Figure 7: Larson's example 5.2 from [Lar97a] harmonised

According to Schlenker cadences in the form `V-I` are typically used to end a piece ([Sch19], page 59), and to assume that here, there are two ways to arrive at the respective triads (chords) in the above figure.

In both cases, the last triad is mode 0 (i.e. `I`) of the *C Ionic Major* scale, and given a triad is constructed from the stable tones $[0, 2, 4]$ and that *C Ionic Major* $= [c, d, e, f, g, a, b]$, the final triad is the tonic triad in root position: $[c, e, g]$. Then `V` is based on the *Mixolydian Major* scale, which has $[0, 2, 4, 5, 7, 9, 10]$ as its measurement, so given pitch collection $P = [c, c\sharp, d, d\sharp, e, f, f\sharp, g, g\sharp, a, a\sharp, b]$, find `V` $= mode(4) = g$ in *C Ionic Major*, which is $P[7]$, and which generates *G Mixolydian Major* $= [g, a, b, c, d, e, f]$ according to definition 3.1.3. Then using the stable tones $[0, 2, 4]$ in first inversion $[2, 4, 0]$ yields the triad $[b, d, g]$ as it appears in the figure.

An easier way to get from `I` to `V` is to use the stable tones $[0, 2, 4]$, and note that since `V` $= mode(4)$, the new triad tones are the old ones $+4$ *mod* $7 = [4, 6, 1]$, or $[6, 1, 4]$ in first inversion position, yielding $[b, d, g]$ directly from *C Ionic Major*.

Harmonic motion and substitution The previous definition 3.1.5 clarified how triads may be built on any note of a particular scale, and that scale as well as starting root may determines a chord's character. But the subsequent example underlined that what makes triads or more generally harmony interesting, is its motion. To make this more precise, the concepts of progression and cadence – which were illustrated above – as well as the concept of substitution, are specified below in order to describe the effect of progressions on note degrees. This will end up modifying Larson's conception of triadic stability.

Definition 3.1.6. *Given a scale S, a triad series T is the list of triads using all the scale's degrees $0, 1, \ldots, 6$ as modes, so $T = [[0, 2, 4], [1, 3, 5], \ldots, [6, 1, 3]]$, or in Roman numerals, $[$`I`$,$ `II`$, \ldots,$ `VII`$]$. A progression is any sequence of triads drawn from some scale's triad series. A cadence is a progression which ends on the modes $[4, 0]$, or in convential Roman numeral notation, on* `V-I`.

The notions of progression and cadence are more general in music theory than as presented above, since a progression is not limited to having chords relative to a single scale: it is possible to use so-called modulation and switch to a chord drawn from a different scale (see section 4.3 for an example where this happens). Furthermore, there are cadences that do not end on `I`, notably the so-called deceptive cadence – this 'pretends' to end on `I`, i.e. to *resolve*, but then serves up `VI` instead (see [Sch19], page 59). Nonetheless, the concepts as specified are as they will be used in this section, for the sake of keeping the musical domain simple.

While the examples in this thesis are essentially single (monophonic) musical lines, it is assumed that they are heard harmonically, i.e. that in perception the listener imposes a harmonic structure over them, with a preference for cadences, as already suggested in figure 7's harmonisation. This implies that the listener internally fills in sounds not physically present, a process Larson calls auralisation ([Lar04], page 467) – plausibly because exposure to music has led to particular expectations. Figure 8 shows the previously introduced melody again, with its note sequence and cadential harmony, in order to illustrate how the latter modifies the former.

lar52.mp3
lar52.v-i.mp3

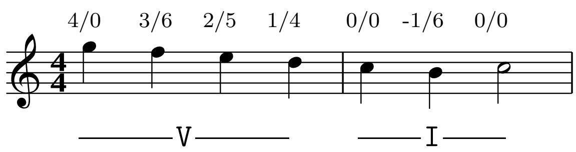


Figure 8: Larson's example 5.2 from [Lar97a] with harmonic substitutions

The note sequence in figure 8 is in the form 'position/relative' (cf. figure 6), as the usual note degrees undergo a substitution, i.e. take on a different role, given a harmonic mode different from mode 0. Finding such a relative note role involves modularly subtracting harmonic modes from the absolute note degrees. For instance, for the fourth note in the example, 4 tonal steps are needed to get from C to G (i.e. from `I` to `V` or mode 0 to 4), and subtracting this (modulo 7) from the old degree 1 gives the new role 4. This is specified below, where for the above figure, the note sequence N is $[4, 3, 2, 1, 0, -1, 0]$, the harmonic sequence H is $[4, 4, 4, 4, 0, 0]$ (i.e. $[\mathtt{V}, \mathtt{V}, \mathtt{V}, \mathtt{V}, \mathtt{I}, \mathtt{I}]$), and the resulting substitution R_s is $[0, 6, 5, 4, 0, 6, 0]$.

Definition 3.1.7. *For a note sequence N and accompanying harmonic sequence H, the modal role substitution sequence R is a list of pairs n/s such that for each $n \in N$ and its associated $h \in H$, $s = (n - h) \bmod 7$. R may also be given as R_s with just the substitutes.*

That given the above, the fourth note in the figure might now be viewed as having degree (role) 4 rather than 1 is significant, because as indicated this is a degree that Larson considers stable (see the paragraph following figure 5). This concept is looked at next in more general terms.

Tonal and harmonic stability As indicated in section 1, a central notion for Schlenker and Larson is stability. For the latter this is firstly due to motion of gravity and magnetism towards more stable notes, but also because embellishments at higher levels tend to have less stable notes, i.e. as Larson would have it, musical compositions are constructed towards having more instable notes present. Neither author gives unambiguous (i.e. total) orderings from least to most stable, but both give some clues for the degrees – the notes within a scale – as well as for the chords (triads), i.e. for the modes. This is explained below, and the definition following it will be treated as what Schlenker calls 'pitch space'.

It should firstly be noted that total orderings of tonal and harmonic stability may not ultimately be realistic, but their existence is assumed here to make the mappings from musical to physical events more straightforward. Furthermore, Larson makes a distinction between inherent and contextual stability (viz. [Lar97b], page 106), which is not made here – stability in this study is Larson's inherent stability.

First of all the tonal degrees.[13] According to Larson, the tonic, mediant, and dominant are most stable, i.e. the notes of the major tonic triad ([Lar97a], page 59), while the subtonic is (inherently) unstable ([Lar97b], page 128) and dissonant. Moreover, the subdominant is considered less stable than both mediant and submediant (*ibid*, page 111). Additionally, Larson remarks that the dominant is particularly stable. That leaves the supertonic, plus some educated guesses. Assuming the subtonic is the most unstable because its triad contains an inherently unstable tritone interval (also known as 'flat fifth'; *ibid*, page 110),[14] the supertonic should be somewhere between subtonic and submediant, which leaves the question whether the subdominant is more or less stable than the supertonic. It is arguably unstable since it is known as an 'avoid note' that signals change to the harmony, i.e. modulation (viz. [Hon97], page 14), so it will be considered second-least stable.

Secondly, the harmonic modes, for which it is Schlenker that gives several clues in [Sch19]. On page 59 he gives the Roman numeral ordering `IV` < `V` < `I` due to the common occurence of `IV-V-I` cadences, because *"this provides a gradual path towards tonal repose"*, but also notes that substituting `VI` for `I` in the aforementioned deceptive cadence results in lower stability, which implies `VI` < `I`, and so

[13] Traditional names are used here, see definition 3.1.4.
[14] In major as well as harmonic and melodic minor scales.

(among others) VI < IV < V < I. Moreover, on the next page the author mentions the so-called perfect cadence II-V-I, which similarly implies II < V < I. In minor scales however, the perfect cadence would be given by the modes VII-III-VI, i.e. VII < III < VI is also obtained. This implies VII < III < VI < II as well as VII < III < II < VI, but the latter is prefered according to Krumhansl's harmonic hierarchy ([Kru83], page 46).[15]

The above may then be combined into the following orderings for tonal and harmonic (inherent) stability:

Definition 3.1.8. *For note degrees (or roles) within a scale, the following tonal stability ordering is adopted:* $O_t = 6 < 3 < 1 < 5 < 2 < 4 < 0$*. In traditional terms, this means that* subtonic < subdominant < supertonic < submediant < mediant < dominant < tonic*. For triad modes built on a scale, the following harmonic stability ordering is used:* $O_h = 6 < 2 < 1 < 5 < 3 < 4 < 0$*, which written traditionally, means VII < III < II < VI < IV < V < I.*

Alphabets and levels Finally before giving the example of figure 6 (page 15) in full, the basics of levels of embellishment are specified. A brief example was already given in figure 3 of section 1.2. The core idea is that a musical line is constructed at a given level by choosing notes from a so-called reference alphabet such that they move to notes that have in turn been picked from a goal alphabet, which is a subset of the reference alphabet of more stable notes.

Figure 2 in Larson's *Musical Forces and Melodic Expectations* [Lar04] (page 471) gives some examples of possible reference and goal alphabets, but here a constant reference alphabet is assumed: the musical sequence's basic scale according to definition 3.1.3, with the goal alphabets incrementally augmented following definition 3.1.8, by adding increasingly less stable notes to them at each level.[16] As mentioned in section 1.2, Larson's view of musical composition is that it is built up as several levels of embellishment *"leading ultimately to the piece itself"*. Taken together, things may be specified as follows.

Definition 3.1.9. *An* alphabet *is a set of note degrees for a given scale. If G and R are alphabets while $G \subseteq R$, and G has no fewer stable degrees than R, then R is called a* reference alphabet*, and G a* goal alphabet*. For any note sequence S, its last note $last(S) \in G$. If L_1 and L_2 are sequences such that $set(L_1) \subseteq set(L_2)$ and $last(L_2)$ is not more stable than $last(L_1)$, then L_2 is an* embellishment *of L_1 while L_1 and L_2 are called* levels*.*

[15] IV < V < I and VI < I imply not only VI < IV < V < I but also IV < VI < $V < I$ and $IV < V < VI < I$; the first is similarly prefered according to [Kru83] (viz. [dN23], endnote 10).

[16] But it is possible to have goal notes as stable as the previous level; viz. the definition (3.1.9).

3.2 Musical forces

Figure 9 shows Larson's example 5.2 from [Lar97a] built up in three levels, with forces assignments as well as harmonisation. It elaborates the aforementioned figure 6.

- Level 1: lar52.1.mp3
- Level 2: lar52.2.mp3
- Level 3 (V-I): lar52.3.mp3

lar52.3.v-i.mp3

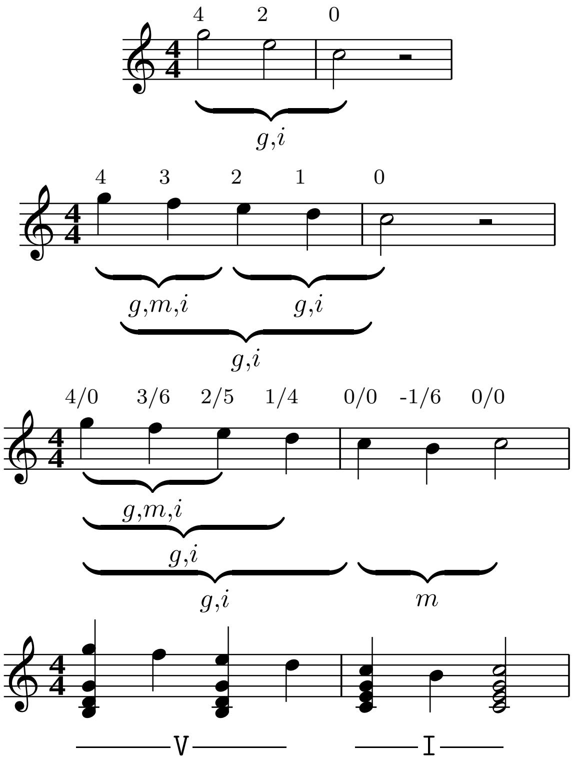


Figure 9: Larson's example 5.2 [Lar97a] with embellishments, forces and harmony

This section's purpose is to explain how the forces as shown in the figure are to be specified: how they 'control' the transitions between levels, and how the perceived harmony affects this. The example in figure 9 is initially built from the *C Ionic Major* triad notes. In other words, *C Ionic Major* is the reference alphabet while the goal alphabet consists of merely *c*. At the second level, the melody is filled in with intermediary notes to make it descend stepwise, and at the final level, it does what Larson calls a 'crouching recovery' ([Lar97b], page 103), i.e. it dips below the tonic and then returns.

Digging a bit deeper and assuming a constant *C Ionic Major* reference alphabet, the second level introduces the less stable *e* (the mediant) as additional goal alphabet member for the commencing musical motion, with the concluding one ending on the tonic *c* as before. For the third level consider the second bracket encompassing the first four notes: this indicates the addition of d (the supertonic) as further less stable member of the goal alphabet. But if the musical line is heard as a `V-I` cadence, that d becomes the dominant instead of the supertonic, and is then hence (contextually) stable.

As can be seen, the additions of goals to the melody are increasingly less stable (viz. definition 3.1.8). The bottom shows the example harmonised according to a `V-I` cadence, while the staff above it indicates the effects on the note degrees – i.e. what roles they take on – in case the melody is perceived like this. It also assigns forces at the ensuing relative (i.e. contextual) points of stability. These will now be defined, as well as how they direct the transitions from level to level. After that, it is specified how the musical events as given by the forces and the harmony are mapped to physical ('world') events, which comprises the semantics.

Partitions and events As figure 9 shows, forces are assigned to groups of notes where stable points occur, which implies that these are partition boundaries within note sequences. As can be seen, such groupings may be separate or notes may share a boundary (cf. [Sch19], page 77, figure 36 e). While conceivably, some possible underlying harmonic motions might be derived from a melodic line (e.g. the example starting on the dominant suggests mode `V`), it is assumed here that such motions are given, or rather that there is a role substitution sequence (viz. definition 3.1.7) on the basis of one. Hence to partition a note sequence, all that is needed is the substitution sequence.

Definition 3.2.1. *Given a role substitution sequence R for a note sequence N, a partitioning P is a list of sublists called partitions, such that preferably for the first, but certainly for the last item n/s of each, $s \in \{0, 2, 4\}$, i.e. s is one of the stable triad tones. Moreover, intermediary notes should be less stable than these boundaries. P may be split into P_v and P_s so as to separate the partitions into one for note positions (values) and one for role substitutions.*

The above prefers partitions according to Larson's aforementioned 'simple motion' without requiring so. Some subsequent partitions have two notes (the implied minimum) and may start on an unstable one (an example was given in figure 3). Recall Schlenker's definition (1.1.1 here) of a possible denotation, which assumes a musical line, or voice, is divided into musical events. The above definition serves to specify how this might be done. In figure 9 there are two such events, while there are also overlaps which underscore that there may be more than one way to partition a line (i.e. the above definition is non-deterministic). How forces are assigned and work across levels is clarified next.

Forces and partitions Larson's forces of gravity, magnetism and inertia are assigned to partitions of the musical line, and depend on information about stability and direction. The relevant definitions below apply to some partition p in a partitioning P, where p may be given as p_v or p_s, as in definition 3.2.1. The idea for gravity is that it applies to a partition where musical motion is towards a stable threshold (or 'platform', viz. [LV05], page 122). It may then be specified as follows.

Definition 3.2.2. *Let p_v be a partition of length ≤ 2, with x, y and z their first, penultimate and last elements (so x and y may coincide). Then p_v is gravitational (marked g) in case $y > z$, and if $x \neq y$ then also $x > z$.*

This definition is meant to be consistent with table 5 in [LV05], page 122, tentatively generalising to note patterns larger as well as smaller, than the listed three-note ones – which include patterns that start and end on the same note, hence the second condition. Considering figure 9 the situations are more straightforward in that in all cases, it suffices to note that the partitions marked g are all descending, i.e. $x > y > z$.

As for magnetism, it was mentioned earlier (in section 1.2) that while Larson has in the past regarded it as a measure of attraction to a more stable pitch analogous with physics, he has later come to define the concept in a binary fashion: as the question whether or not a note resolves via semitone resolution – because particular statistical regression experiments gave 'better results' (as indicated in [Lar02], page 381, footnote 6). However, the core idea, like physical magnetism, is as follows: the closer a note gets to its goal, the stronger magnetic attraction becomes. It is one way to characterise goal directedness in music, and hence the concept is specified and applied in general here, using Larson's physics measure in terms of semitone distances.

The idea behind Larson's (general) magnetism definition ([Lar04], page 463) then, is to gauge the attraction ('pull') from some unstable note to its resolution. This is measured in terms of where it actually goes, called the attractor, and where it might have gone if it had moved the other way, which is termed the opponent ('opposing attractor' in [Lar04]). These notions are made precise below.

Definition 3.2.3. *Given a partition p_v, a scale S, its instantiation I and a mode m, the relative stabilities q are $([0, 2, 4]+m)$ mod 7. Let t and u be the penultimate and last elements of p_v, and set $t_s = t$ mod 7 and $u_s = u$ mod 7 (i.e. their respective note values are $I[t_s]$ and $I[u_s]$). Then for each element i in q determine candidate semitone distances x_i and y_i as follows: $x_i = (t_s - S[q_i])$ mod 12, and $y_i = 12 - x_i$. Now put each $min(x_i, y_i)$ in D, and then the attractors A are note value / distance* *pairs v/d with each n given by $I[q_i]$, and d by its associated d in D. With the partition's last note and distance $I[u_s]/d_u \in A$, this will be called the partition's attractor a_p. If this element is discarded from A, the remaining element with the smallest d is its opponent o_p.*

The above definition entails that each partition has attractors and opponents, and consequently each has a magnetism value. Once the attractor and opponent are known, this value is computed as follows.

Definition 3.2.4. *Given some partition p_i, its attractor $a_i = v_i/a_i$ and opponent $o_i = w_i/o_i$, the magnetic value of p_i is given as $m_{p_i} = \frac{1}{a_i^2} - \frac{1}{o_i^2}$. Given its penultimate note value v_p, the partition is binary magnetic (marked $\overline{m}$) if $abs(v_i - v_p) = 1$. If $v_i - v_p < 0$ then p_i is downward magnetic ($m_\downarrow$), and else upward magnetic ($m_\uparrow$). In the latter case, if p_i resolves to the tonic and is binary magnetic, it has crouching recovery ($m^\star$).*

The last binary magnetism part refers to semitone resolution (see section 1.2). To illustrate the more general numeric case, magnetic values will be computed for the two upper partitions in level 3 of figure 9 (although only the latter has been marked as such since it is binary).

The first partition $p_1 = [4, 3, 2, 1]$, while the scale $S = [0, 2, 4, 5, 7, 9, 11]$, its instantiation $I = [c, d, e, f, g, a, b]$, and its mode $m = 4$, i.e. even though the partition is in mixolydian mode (`V`), the scale perspective is nonetheless *C Ionic Major* (this may incidentally also be called the *key*). The relative stabilities are $q = ([0, 2, 4] + 4)\ mod\ 7 = [4, 6, 1]$, and $t = t_s = 2$ and $u = u_s = 1$. These are respectively $I[2]$ and $I[1]$ i.e. e and d. Now $x_0 = (I[2] - I[q_0])\ mod\ 12 = (I[2] - I[4])\ mod\ 12 = (4 - 7)\ mod\ 12 = 9$ and $y_0 = 12 - 9 = 3$. Then similarly $x_1 = (4 - I[6])\ mod\ 12 = (4 - 11)\ mod\ 12 = 5$ and $y_1 = 12 - 5 = 7$, and $x_2 = (4 - I[1])\ mod\ 12 = (4 - 2)\ mod\ 12 = 2$ and $y_2 = 12 - 2 = 10$. Then the smallest distances to the attractors $D = [3, 5, 2]$ and so the attractors $A = [g/3, b/5, d/2]$. The attractor is the last note d, and removing this leaves $g/3$ with the smallest distance, hence g is the opponent.

As for the second partition $p_2 = [0, -1, 0]$, the scale and instantiation are as above, but now the mode $m = 0$, so $q = [0, 2, 4]$. The penultimate $t = -1$ and $t_s = -1\ mod\ 7 = 6$, while for the last note $u = u_s = 0$. Their values are $I[6]$ and $I[0]$, i.e. b and c. Now $x_0 = (I[6] - I[q_0])\ mod\ 12 = (11 - 0)\ mod\ 12 = 11$ and $y_0 = 12 - 11 = 1$, $x_1 = (11 - 4)\ mod\ 12 = 7$ and $y_1 = 5$, and $x_2 = (11 - 7)\ mod\ 12 = 4$ and $y_2 = 8$. So $D = [1, 5, 8]$ and $A = [c/1, e/5, g/4]$. The attractor being $c/1$, discarding this in A leaves opponent $g/4$.

So for p_1 the attractor $a_{p_1} = d/2$ and the opponent $o_{p_1} = g/3$, while for p_2, $a_{p_2} = c/1$ and $o_{p_2} = g/4$. Then $m_{p_1} = \frac{1}{a_1^2} - \frac{1}{o_1^2} = \frac{1}{4} - \frac{1}{9} = \frac{5}{36} \approx 0.14$. As for p_2, the attractor $a_{p_2} = c/1$ and the opponent $o_{p_2} = g/4$, and so $m_{p_2} = \frac{1}{a_2^2} - \frac{1}{o_2^2} = 1 - \frac{1}{16} = \frac{15}{16} \approx 0.94$. This demonstrates that while indeed magnetism values may be computed for any partition, they tend to be (considerably) higher for partitions that resolve by a semitone (as mentioned in the paragraphs preceding definition 3.2.3).

Finally, inertia. This broadly speaking means to continue in the same manner, which may be a direction or more generally a pattern. The latter being too broad for convenience, only upward, downward, still, and alternating are specified.

Definition 3.2.5. *A partition P_v is inertic if it is still, moves continuously upward or downward, or is alternating. In the first case (marked i), its length > 1 and for all $x_1, \dots, x_n \in P_v$, $x_1 = \dots = x_n$. In the next two cases ($i_\uparrow$ and $i_\downarrow$), P_v has length > 2 and for all $x_1, \dots, x_n \in P_v$, either $x_1 < \dots < x_n$ or $x_1 > \dots > x_n$. In the final case ($i_\updownarrow$), the length of $P_v \geq 3$ and for each consecutive pair $(x, y), (y, z) \in P_v$, either $x < y > z$ or $x > y < z$.*

In figure 9, just the downward cases appear but the others can presumably be imagined. In the next section, the focus is on the relationship between the forces and transitions between musical levels.

Forces and transitions As already highlighted, according to Larson, musical pieces are to be viewed as constructed level by level, with higher levels figuring as embellishments of the lower ones, and transitions from lower to higher controlled by the forces ([Lar04], page 457). Transitions are non-deterministic, i.e. there may be multiple solutions.

Definition 3.2.6. *Given two partitions P and P' such that P' is an embellishment of P, the pair (P, P') is a transition. A sequence $T = [P_1, \dots, P_n]$ is transitional if each pair $(P_i, P_{i+1}) \in T$ is a transition, in which case each $P \in T$ is a level.*

Even if the above does not say precisely how one note embellishes another note, how embellishments are 'controlled by the forces' can now be specified if left-branching is assumed, following Lerdahl and Jackendoff [LJ83], page 181, meaning the most significant note is at a partition's end and musical motion is towards this.

Definition 3.2.7. *Given a force F, a transitional sequence T and a transition $t = (P_i, P_{i+1}) \in T$, t is controlled by F if P_{i+1} is F, and strongly controlled by F if P_{i+1} is F but not P_i.*

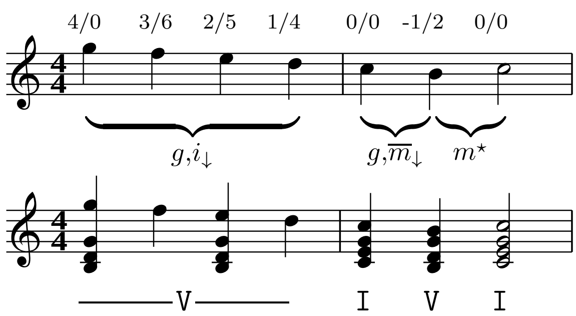
Note particularly that magnetism is in this case to be considered as semitone resolution, i.e. binary like the other forces, instead of following definition 3.2.4 which assigns it numerically to any partition (viz. section 1.2). Other than this, note how in figure 9, gravity controls how the notes are filled in between levels 1 and 2. The next section serves to connect Larson's forces to Schlenker's semantics.

3.3 Musical meaning

The goal of this section is to associate musical forces with situations and events in the world, following what was set out about Schlenker's work in sections 1.1 and 2.2, particularly concerning the idea of a 'bona fide' semantics, which is a relation between music and a reality external to the music itself.

Musical example To begin with, the example depicted in figure 9 will be extended and assigned semantics. The extension (level 4) is shown in figure 10, with postfix markings as specified in the previous section. So figures 9 and 10 together constitute four levels.

- Level 4 (V-I-V-I):
  lar52.4.mp3



lar52.4.v-i-v-i.mp3

Figure 10: Additional level for Larson's example 5.2 [Lar97a] with forces and harmony

Note that while figure 9's second measure comprises a single partition, in figure 10, that partition has been split in two. This reflects the addition of the unstable *b* (i.e. the subtonic) to the goal alphabet, and explains the addition of the extra harmonic function `V` (or mode 4) – see also section 5.2.

The partitions may now be viewed as musical events, written as (sequences of) quadruples $\langle harmony, gravity, magnetism, inertia\rangle$ as in equation 3.3.1, which gives the events for all four levels $L_1, \ldots, L_4$ – see the next paragraph, definition 3.3.2, for the format.

Equation 3.3.1.
$M_{L_1} = \langle\langle \mathtt{I}, 1, -0.05_\downarrow, 1_\downarrow\rangle\rangle$
$M_{L_2} = \langle\langle \mathtt{I}, 1, \overline{0.75}_\downarrow, 1_\downarrow\rangle, \langle \mathtt{I}, \varnothing, 0_\downarrow, \varnothing\rangle\rangle$

$M_{L_3} = \langle\langle \mathtt{V}, 1, 0.14_{\downarrow}, 1_{\downarrow}\rangle, \langle \mathtt{I}, \emptyset, \overline{0.94^{\star}}, \emptyset\rangle\rangle$
$M_{L_4} = \langle\langle \mathtt{V}, 1, 0.14_{\downarrow}, 1_{\downarrow}\rangle, \langle \mathtt{V}, 1, \overline{0.89}_{\downarrow}, \emptyset\rangle, \langle \mathtt{I}, \emptyset, 0.94^{\star}, \emptyset\rangle\rangle$

The magnetism values for M_{L_3} and the outer ones for M_{L_4} are as in the example computation after definition 3.2.4. M_{L_4}'s middle partition (following definition 3.2.3) resolves to b from c by 1 semitone with its opponent at d and 3 semitones. For M_{L_1} note that the attractor is c with a distance of 4 semitones (from the predecessor b), and the opponent is g with a distance of 3. For M_{L_2}'s first partition, the attractor is e which has distance 1, with opponent g that has distance 3.

Voice and truth Next, musical events M are to be mapped to world events W. The idea is to arrive at a denotation *bird-landing* by mapping *harmony* to altitude, *gravity* to wind pressure, *inertia* to wing power, and *magnetism* to gravitional force. The latter may seem odd or confusing, but the mapping only needs to preserve structure and need not correspond to musical features (as Schlenker puts it, a virtual source does not have to be sound-producing – [Sch19], page 38). First however, the truth definition will be given by adapting definition 1.1.2 from section 1.1, preceded by the specification of a voice (i.e. equation 3.3.1 contains voices).

Definition 3.3.2. *A voice is a sequence of musical events rendered as quadruples with values for harmony, gravity, magnetism and inertia. Harmony values are written in Roman numeral notation while gravity is 1 if true. Magnetism is given in terms of attraction value (viz. definition 3.2.4), marked with suffixes ($v_{\downarrow}$ if downward, $v_{\uparrow}$ if upward, and $v^{\star}$ if crouching recovery), or overline ($\overline{v}$ if binary magnetic). Inertia is similary marked for downward or upward, or for alternating ($v_{\updownarrow}$). Absence of one of Larson's forces is indicated by $\emptyset$ (as magnetism may be 0).*

So according to the above, M_{L_4} in equation 3.3.1 is a voice, and the next definition serves to enable mappings $M \longrightarrow W$, i.e. turn musical events into 'world' events.

Definition 3.3.3. *Let $M = \langle M_1, \ldots, M_n\rangle$ be a voice, and $\langle O, \langle e_1, \ldots, e_n\rangle\rangle$ a possible denotation for M (see definition 1.1.1). M is true of $\langle O, \langle e_1, \ldots, e_n\rangle\rangle$ if it obeys the following requirements.*

(a) Time: The temporal ordering of $\langle M_1, \ldots, M_n\rangle$ should be preserved, i.e. it should be the case that $e_1 < \ldots < e_n$, where $<$ is ordering in time.

(b) Forces: If M contains the following forces then these act to move O to a position of – relative – stability.

(1) If M is gravitational, then O moves, under the influence of gravity or a similar constant force, to a stable position of less energy, i.e. e_n has less energy than e_1 (this and subsequent clauses assume Schlenker's ideas on pitch inferences, [Sch19], pages 52-53, where higher pitch or frequency is associated with more events, i.e. more energy or energy potential).

(2) *Since all M are magnetic, O is assumed to be attracted to a stable position to some degree, with values interpreted as 'closeness to a goal'. If M is downward magnetic, O will be in a position of less energy (e_n has less energy than e_1), and if M is upward magnetic, it will be in a position with more energy – unless e_n resolves to tonic position, in which case energy is assumed to be absorbed ('crouching recovery').*

(3) *If M is inertic then O follows a pattern to a stable position – which may be voluntary through action or involuntary via the effect of a force (with the latter case comparable to gravity). If M is downward inertic then O moves steadily to a less energetic position, and if M is upward inertic, O follows a steady pattern to a position with higher energy. If M is still inertic then O remains in the same position. Finally, if M is alternating inertic then O follows a regular but alternating pattern, which is to a less energetic position if M is also gravitational, or to a position with more energy if M is not gravitational and has even length; else O ends up in the same position (as specified above).*

(c) *Harmonic stability: If M_i is less harmonically stable than M_k, then O is in a less stable position in e_i than it is in e_k.*

Mapping to denotation On the above definition, the following denotation is true of M_{L_4}, as a mapping $M_{L_4} \longrightarrow W_{L_4}$. This mapping is stipulated as being $harmony \longrightarrow altitude$, $gravity \longrightarrow wind\,pressure$, $magnetism \longrightarrow gravitational\,force$ and $inertia \longrightarrow wing\,power$, with these terms abbreviated somewhat in equation 3.3.4 below (where the idea is that W_{L_4} = *bird-landing*).

Equation 3.3.4.

$$W_{L_4} = \left\langle bird, \left\langle \begin{matrix} altitude{=}high, \\ wind{=}high, \\ force{=}low, \\ power{=}high \end{matrix} \right\rangle, \left\langle \begin{matrix} altitude{=}high, \\ wind{=}low, \\ force{=}medium, \\ power{=}low \end{matrix} \right\rangle, \left\langle \begin{matrix} altitude{=}low, \\ wind{=}low, \\ force{=}high, \\ power{=}low \end{matrix} \right\rangle \right\rangle$$

This equation is a mapping like the one Schlenker makes from his equation (23_a) in [Sch19] on page 66 to (26_a), page 68 – i.e. from $M = \langle\langle \mathtt{I}, 70db\rangle, \langle \mathtt{V}, 75db\rangle, \langle \mathtt{I}, 80db\rangle\rangle$ to *Sun-rise* = ⟨*sun*, ⟨*minimal-luminosity*, *rising-luminosity*, *maximal-luminosity*⟩⟩, except here, the denotation consists of attribute/value pairs. Moreover, rather than tacitly mapping combinations of musical events to single events (e.g. Schlenker maps `I` and 70*db* to *minimal-luminosity*), mappings of individual musical events have been made explicit, hence there are multiple sub events in the above equation.

The example shows two particular instances of changing energy levels where one decreases while the other increases. The former is altitude as mediated by *harmony*, which decreases because height implies energy potential of an object. The latter is gravitational force, mediated by *magnetism*, which increases as the object approaches the earth, with attraction maximal on the ground.

Additional features that have been brought into the equation are wind pressure (*gravity*) and wing power (*inertia*). The former is absent in the last partition and the latter in the final two, meaning their last values have been copied into the subsequent partition(s). In both cases this follows energy considerations from definition 3.3.3 (b_1 and b_3, respectively). Of further interest are the altitude and gravitational force features, particularly the latter, since the increasingly stronger magnetism (in equation 3.3.1's M_{L_4}) implies increasing closeness to a goal – where the final magnetic force is a 'crouching recovery' with the landing absorbing (kinetic) energy. As for the prior levels, note that levels 1 and 2 might be interpreted as *bird-moving*, which can be considered as a partial interpretation, where it also conveniently holds that *bird-landing* $\models$ *bird-moving*.

Even though not all forces were assigned in all partitions of figure 10, their mapped images are all in equation 3.3.4. To underpin this, three final concepts should be specified: event map, energy update and eventful denotation (i.e. equation 3.3.1).

Definition 3.3.5. *An* event map *is a pair* $\langle \mathcal{M}, \mathcal{W} \rangle$ *where* $\mathcal{M}$ *is a list of mappings* $\mathcal{F}$ *from musical features to features in the world* $\mathcal{G}$*, and* $\mathcal{W}$ *is a list of partial orderings for the values of* $\mathcal{G}$*, ordered from higher to lower levels of energy, with potential incomparable values written as* v°*.*

So with $\mathcal{M}$ for the mapping in equation 3.3.1 being given above it, $\mathcal{W} = [\{high, low\}, \{high, low\}, \{high, medium, low\}, \{high, low\}]$.

Definition 3.3.6. *Given an event map* $\langle \mathcal{M}, \mathcal{W} \rangle$*, a force* f *and an attribute/value pair* $a = v$ *(with* $a \in \mathcal{M}$ *and* $v \in \mathcal{W}$*), an* energy update *gives a new value* v' *for* a *(i.e.* $a = v'$*) picked from* $\mathcal{W}$ *under the following rules:*

(1) If f *is downward or gravitational, then if available,* v *is the next comparable value smaller than* v*.*

(2) If f *is upward, if available,* v *is the next comparable value greater than* v*.*

(3) If neither of the above is the case or if no such comparable value, then $v' = v$*.*

It can now be specified how to exhaustively map musical events into a denotation, thereby completing the semantics.

Definition 3.3.7. *Given a voice* M*, a denotation* $\langle O, E \rangle$ *is* eventful *in case* $M \longrightarrow E$ *is an injective homomorphism (i.e. every* $m \in M$ *is covered by some distinct* $e \in E$*), which holds in case the following does:*

(1) *If a force is absent in a partition but present in its predecessor partition, the predecessor value is used after an energy update. For successors the value is subsequently constant. If it is present in the successor the energy update is in reverse, with further predecessors having constant values.*
(2) *If a force is absent in a partitioning but present in the encompassing partition, its value is applied to the last subpartition, and to their predecessors with reverse energy update.*
(3) *If a force is absent in an encompassing partition but present in a subpartition, the encompassing partition receives the value of the last subpartition.*

By way of illustration how this applies, consider the second measure in figure 10, which consists of two partitions where inertia has not been assigned (though it strictly could have been in case the last three notes had been viewed as a single partition). Given $inertia \longrightarrow wing\ power \in \mathcal{M}$ and $\{high, low\} \in \mathcal{W}$, the last two values in equation 3.3.4 follow from definition 3.3.7's first clause. Encompassing partitions will crop up in this section's final two examples.

Minimal pairs As mentioned in section 1.1, Schlenker's methodology includes constructing so-called minimal pairs, where a change is applied to an instance to make a conjectured meaning disappear ([Sch19], page 62). For the example of figure 10, such a change is depicted in figure 11 below, and rendered in equation 3.3.8, so that the minimal pair is (M_{L_4}, M'_{L_4}).

- Level 4 (V-I-V-I): lar52.4.mp3

Level 4′ (V-I-V): lar52.4p.mp3

lar52.4p.v-i-v.mp3

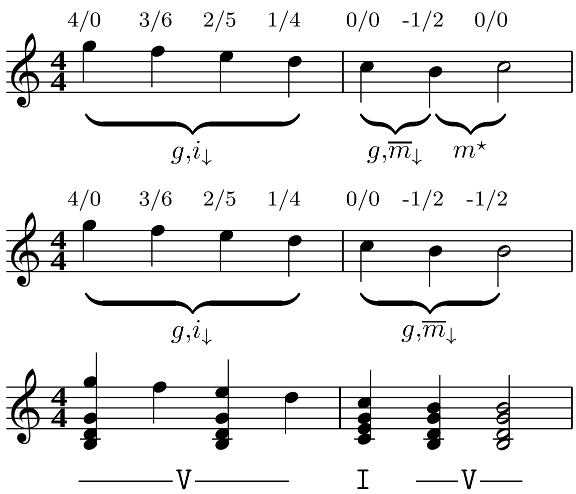


Figure 11: 'Minimal pair' change to Larson's example 5.2 [Lar97a]

Equation 3.3.8.
$M_{L_4} = \langle\langle \mathtt{V}, 1, 0.14_{\downarrow}, 1_{\downarrow}\rangle, \langle \mathtt{V}, 1, \overline{0.89}_{\downarrow}, \varnothing\rangle, \langle \mathtt{I}, \varnothing, 0.94^{\star}, \varnothing\rangle\rangle$
$M'_{L_4} = \langle\langle \mathtt{V}, 1, 0.14_{\downarrow}, 1_{\downarrow}\rangle, \langle \mathtt{V}, 1, \overline{0.89}_{\downarrow}, \varnothing\rangle\rangle$

The minimal change involves repeating the penultimate note. The idea is that since the music no longer resolves, the altitude does not reach the level 'low', and so the bird does not land – hence W_{L_4} is no longer valid. Note that the presence of the subtonic b in the goal alphabet is the same for level $4'$ as it is for level 4.

This section will end with two further examples (ignoring minimal pairs): a melody in major and minor to shows the effect of different scales, and a leapwise example.

Major and minor example Consider figure 12 below. Like the previous one, it is from example 5 in [Lar97a] – this example is the eleventh pattern.

(1) Ionic Major:
lar511.maj.mp3

lar511.maj.har.mp3

(2) Aeolian Minor:
lar511.min.mp3

lar511.min.har.mp3

(3) Harmonic Minor:
lar511.hmin.mp3

lar511.hmin.har.mp3

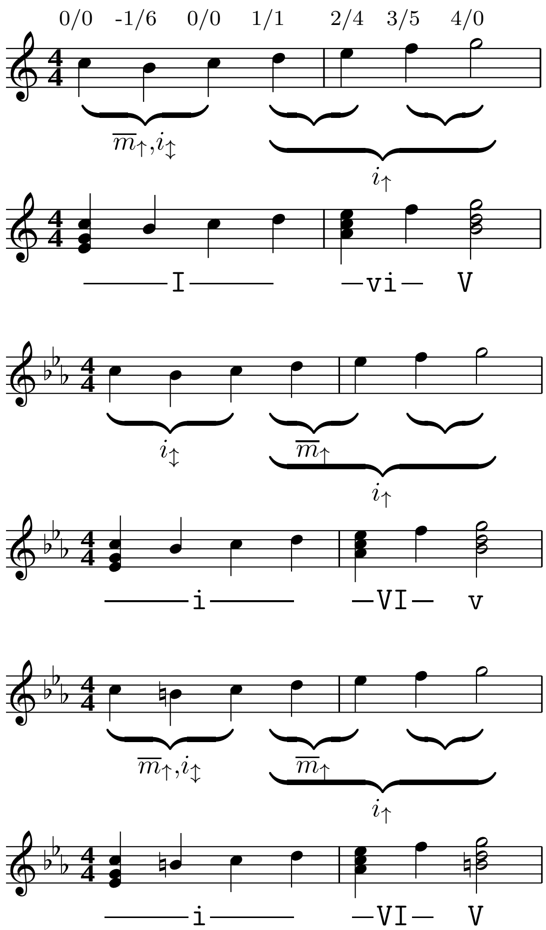


Figure 12: Larson's example 5.11 [Lar97a] in three different scales

The example is the non-resolving pattern $[0, -1, 0, 1, 2, 3, 4]$ (given 0-based unlike in Larson's paper, and in fact the preceding example reversed). It is rendered in

C Ionic Major, *C Aeolian Minor* and *C Harmonic Minor*, and harmonised using first, sixth and fifth modes. Brackets where no force could be assigned are blank.

In the harmonisations, the convention is used to write major chords in uppercase Roman numerals, and minor ones in lowercase (where first mode `i` should not be confused with inertia i), to highlight how the choice of scale affects this. The point is to demonstrate how this affects distances in a note sequence, and ultimately the assignment of forces and semantics. The scale and harmony effects are made explicit below for the harmonic minor case, with musical and world events.

The scale measurement for harmonic minor is $[0, 2, 3, 5, 7, 8, 11]$ (definition 3.1.2). With pitch collection $[c, d\flat, d, e\flat, e, f, g\flat, g, a\flat, a, b\flat, b]$ (enharmonic, viz. definition 3.1.1), this instantiates *C Harmonic Minor* as $S = [c, d, e\flat, f, g, a\flat, b]$ (definition 3.1.3). Given `i-VI-V`, this implies the following triad sequence: $[[0, 2, 4], [5, 0, 2], [4, 6, 1]]$ (definition 3.1.6). These index values are then picked from S to yield $[[c, e\flat, g], [a\flat, c, e\flat], [g, b, d]]$, where the first has minor, and the other two have major aspect (definition 3.1.5). The triads at the bottom of figure 12 are $[[e\flat, g, c], [a\flat, c, e\flat], [b, d, g]]$, with the outer ones as first inversions and the middle in root position.

Equation 3.3.9 shows the musical events for the *Harmonic Minor* case in figure 12. It is a (hierarchically) structured event because the upward inertic force runs across the last four notes that are in turn subdivided into groups of two. The elementary events are triples $\langle harmony, magnetism, inertia \rangle$.

Equation 3.3.9.

$$M_{L_3} = \left\langle \left\langle \mathtt{i}, \overline{0.94}_{\uparrow}, 1_{\updownarrow} \right\rangle, \left\langle \begin{matrix} \left\langle \mathtt{V}, 0, 1_{\uparrow} \right\rangle, \\ \left\langle \left\langle \mathtt{VI}, \overline{0.75}_{\uparrow}, \varnothing \right\rangle, \left\langle \mathtt{V}, 0, \varnothing \right\rangle \right\rangle \end{matrix} \right\rangle \right\rangle$$

The mapping is similar to that of the previous example with $\mathcal{M} = [harmony \longrightarrow altitude,\ magnetism \longrightarrow gravitational\ force, inertia \longrightarrow wind]$, with values $\mathcal{W} = [\{high, medium, low\}, \{strong, medium, weak\}, \{swirles^{\circ}, strong, weak\}]$.

Equation 3.3.10.

$$W_{L_3} = \left\langle leaf, \left\langle \begin{matrix} altitude{=}low, force{=}strong, \\ wind{=}swirles \end{matrix} \right\rangle, \left\langle \left\langle \begin{matrix} \left\langle \begin{matrix} altitude{=}high, force{=}weak, \\ wind{=}strong \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} altitude{=}medium, force{=}medium \\ wind{=}weak \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} altitude{=}high, force{=}weak \\ wind{=}strong \end{matrix} \right\rangle \end{matrix} \right\rangle \right\rangle \right\rangle$$

The proposed denotation is $W_{L_3} = \textit{leaf-floating}$, or, a leaf floating upward in the wind, with decreasing magnetism meaning gravitaty getting increasingly less grip. Regarding definitions concerning the previous example (notably 3.3.6 and 3.3.7), note the value *swirles* in $\mathcal{G}$ is not comparable, and also that the second and third note groups in figure 12 (3) are encompassed by upward inertia, allowing them to be assigned respectively weak and strong wind ($i_\uparrow$ implies rising energy).

In *Aeolian Minor*, magnetism for the first three-note partition is 0.14, while in *Harmonic Minor* it is 0.94. In both cases, the other values are 0.75 and 0, so in *Aeolian Minor* the *wind* pattern $[strong, medium, weak]$ does not obtain; it would instead be $[weak, medium, weak]$, i.e. a different meaning. This might explain why motion appears somewhat more graceful in *Harmonic Minor* – which incidentally borrows its half-step leading tone (and hence binary magnetism) from *Ionic Major*.

Leapwise example Consider the present section's final example in figure 13.

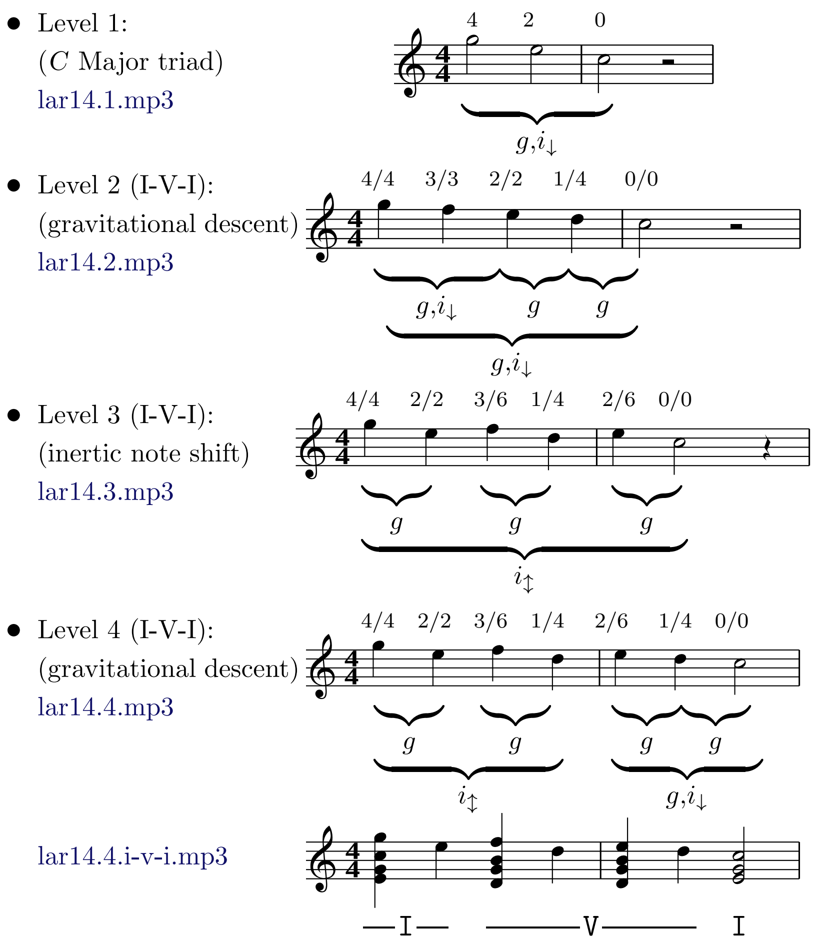


Figure 13: Larson's example 14 [Lar04] with embellishments, forces and harmony

The example was taken from [Lar04], table 14, pattern 14, with note sequence

$[4, 2, 3, 1, 2, 1, 0]$. As can be seen from the first four notes $[4, 2, 3, 1]$, the example is not stepwise (it intervals larger than a whole tone), but there are nonetheless stepwise connections between the notes. To make these explicit, consider particularly level 2 which is stepwise, and the complete sequence in level 4.

The idea is that the stepwise second level arises from the basic first one (the major triad) by a transforming function that adds less stable notes, in this case completing the descent to the tonic – i.e. the transformation is 'controlled by' gravitational descent. At the third level, the mediant (e) is shifted back into second position to ultimately create an inertic repeating pattern of notes separated by an interval of a third (a 'leap', whereby the sequence becomes leapwise). While the initial shift creates the downward interval of a third from g to e, the consequence of this is that the third note f is now followed by d – also a downward third interval – and so to complete the pattern as being controlled by inertia as Larson woud have it, another e is inserted at position five. Finally, gravitational descent is applied once more to the last measure, yielding Larson's example in full.[17]

As for the goal alphabet, this is extended with an unstabler note in levels 2 and 3: first with mediant e and then with supertonic d. In level 4 however, rather than adding a new further note, the unstable d is added (controlled by gravity) to the last measure, prompting the extension of the harmonic function `V` (mode 4) in order to 'restabilise' it and add a new partition (as previously indicated, more on this in section 5.2).

Musical and world events will now be given for the second and fourth levels, in equations 3.3.11 and 3.3.12 respectively. These should be viewed as two levels of perception or perspectives on the same situation. The first one consists of (structured) quadruples $\langle harmony, gravity, magnetism, inertia\rangle$.

Equation 3.3.11.

$$M_{L_2} = \left\langle \left\langle \substack{\langle \mathtt{I}, 1, 0_{\downarrow}, 1_{\downarrow}\rangle, \\ \langle\langle \mathtt{I}, 1, 0.75_{\downarrow}, 1_{\downarrow}\rangle, \langle \mathtt{V}, 1, 0.89_{\downarrow}, 1_{\downarrow}\rangle\ \langle \mathtt{I}, 1, 0_{\downarrow}, 1_{\downarrow}\rangle\rangle} \right\rangle \right\rangle$$

$$M_{L_4} = \left\langle \left\langle \substack{\langle \mathtt{V}, \varnothing, -0.14_{\downarrow}, 1_{\updownarrow}\rangle, \\ \langle\langle \mathtt{I}, 1, 0.07_{\downarrow}, \varnothing\rangle, \langle \mathtt{V}, 1, -0.14_{\downarrow}, \varnothing\rangle\rangle} \right\rangle, \left\langle \substack{\langle \mathtt{I}, 1, 0_{\downarrow}, 1_{\downarrow}\rangle, \\ \langle\langle \mathtt{V}, 1, 0.14_{\downarrow}, \varnothing\rangle, \langle \mathtt{I}, 1, 0_{\downarrow}, \varnothing\rangle\rangle} \right\rangle \right\rangle$$

As before, the events are structured to reflect larger and smaller note partitions. The event map $\langle \mathcal{M}, \mathcal{W}\rangle = \langle [harmony \longrightarrow lean, gravity \longrightarrow speed, magnetism \longrightarrow tide, inertia \longrightarrow wind], [\{false, true\}, \{high, medium, low, none\}, \{strong, medium, still\}, \{strong, weak, swirles^{\circ}\}]\rangle$. The intended denotation is *sailboat-arriving*.[18]

[17] NB. It is not given level by level in [Lar04].

[18] After Schlenker's *Boat-approaching* mentioned in section 1.1; see also [Sch19], page 68.

Equation 3.3.12.

$$W_{L_2} = \left\langle \textit{sailboat}, \left\langle \left\langle \begin{array}{c} \langle \textit{lean=false}, \textit{speed=none}, \textit{tide=still}, \textit{wind=still} \rangle, \\ \langle \textit{lean=false}, \textit{speed=high}, \textit{tide=medium}, \textit{wind=strong} \rangle, \\ \langle \textit{lean=true}, \textit{speed=low}, \textit{tide=strong}, \textit{wind=weak} \rangle, \\ \langle \textit{lean=false}, \textit{speed=none}, \textit{tide=still}, \textit{wind=still} \rangle \end{array} \right\rangle \right\rangle \right\rangle$$

$$W_{L_4} = \left\langle \textit{sailboat}, \begin{array}{c} \left\langle \begin{array}{c} \langle \textit{lean=true}, \textit{speed=medium}, \textit{tide=still}, \textit{wind=swirles} \rangle, \\ \left\langle \begin{array}{c} \langle \textit{lean=false}, \textit{speed=high}, \textit{tide=still}, \textit{wind=swirles} \rangle, \\ \langle \textit{lean=true}, \textit{speed=medium}, \textit{tide=still}, \textit{wind=swirles} \rangle \end{array} \right\rangle \end{array} \right\rangle, \\ \left\langle \begin{array}{c} \langle \textit{lean=false}, \textit{speed=none}, \textit{tide=still}, \textit{wind=weak} \rangle, \\ \left\langle \begin{array}{c} \langle \textit{lean=true}, \textit{speed=low}, \textit{tide=still}, \textit{wind=strong} \rangle, \\ \langle \textit{lean=false}, \textit{speed=none}, \textit{tide=still}, \textit{wind=weak} \rangle \end{array} \right\rangle \end{array} \right\rangle \end{array} \right\rangle$$

W_{L_2} represents the broad perspective of the boat's movement towards the shore (or harbour, land, etc.), while W_{L_4} zooms in on some more details. For instance, the tidal motion in W_{L_2} is a factor bringing the boat in, because the associated magnetism values are large and increase (and end up zero). In W_{L_4}, magnetism stays around zero due to leap motion and partitioning, which may be interpreted as an observer focusing on the boat and the water around it, which would appear still relative to the vessel. This shows how both perspectives may be used for the complete picture, just as the levels in figure 13 are, with the second one needed to bring out the 'hidden' stepwise structure of the example's melody.

Things being 'still' in the top part of W_{L_2} in equation 3.3.12 (the bottom bracket in figure 13) may seem odd, but this is due to the choice of viewing Larson's forces as applying to the end of an event, so what is being referred to is the boat having arrived. What happens during the event is taken care of by the three subpartitions, where things are (partly) in motion. While for the first example (figure 9), there is an entailment relation between levels 2 and 4 (*bird-landing* $\models$ *bird-moving*), this is not the case here, since as indicated, the levels are complementary. However, if as stated W_{L_4} is more detailed, then one would actually want that $W_{L_4} \models W_{L_2}$, but this is not strictly the case here, although it would be a desirable property for a semantics involving levels. See section 4.4 for more on this issue.

4 Extensions

4.1 Krumhansl magnetism

Larson's notion of magnetism is not entirely satisfying, for instance in figure 10 and equation 3.3.1 the motion from c to b is on the same order as from b to c, while intuitively, moving from the subtonic to the tonic should reflect a substantially stronger attraction than going the other way. Krumhansl and Kessler conducted experiments to determine how listeners judged the 'fit' of all twelve pitches within major and minor tonal contexts ([KK82], page 343; the precise values are in [Kru90], page 80). The ratings recorded can be used to specify relative 'from/to' attraction values. There are probably a number of ways to do this, but here, simple division is used in order to obtain an asymmetric measure, while Larson's idea of measuring magnetism using an opposing attractor is dropped: only the originating and target notes matter.

Definition 4.1.1. *Following Krumhansl [Kru90], the fit of the twelve notes from tonic through to subtonic for major harmonies is given by* $\mathcal{F}_{maj} = [6.35, 2.23, 3.48, 2.33, 4.38, 4.09, 2.52, 5.19, 2.39, 3.66, 2.29, 2.88]$, *and for minor ones by* $\mathcal{F}_{min} = [6.33, 2.68, 3.52, 5.38, 2.60, 3.53, 2.54, 4.75, 3.98, 2.69, 3.34, 3.17]$. *Given some partition p with final note pair $[n, m]$, the relative fit from n to m is decided by the harmonic function for the landing note m (in keeping with the convention used here for forces assignments). Let $\mathcal{F} = \mathcal{F}_{maj}$ or $\mathcal{F}_{min}$ according to the aspect of p's last triad, and let x and y be n and m's substitutes (in the sense of definition 3.1.7), but with x substituted according to y's harmonic function. Then p's magnetic value $m_p = \frac{\mathcal{F}[m]}{\mathcal{F}[n]}$.*

For the partitions p_1, p_2 and p_3 in figure 10, the above definition yields the following magnetism values. In *G Major*, the note pair $[e, d]$ gives rise to $m_{p_1} = 5.19/3.66 = 1.42$, and $[c, b]$ to $m_{p_2} = 4.38/4.09 = 1.07$. In *C Major*, the pair $[b, c]$ has it that $m_{p_3} = 6.35/2.88 = 2.20$. This means that now, $m_{p_1} > m_{p_2}$, while the reverse holds for equation 3.3.1[19] So this formulation appears to have an advantage over Larson's definition in that one would expect the motion from c to b to lead to less stability – and hence the attraction to be lower – than from e to d. This is because in the overall key *C Major*, the subtonic b is less stable than the supertonic d, but in the context of the local harmonic function `V` (*G Major*), b is also less stable in its role as mediant than d which is then dominant (see definition 3.1.8). Additionally, note that with the alternative magnetism definition, m_{p_3} is substantially greater than m_{p_2}, while in equation 3.3.1 using Larson's specification, the difference is far smaller. This should also be intuitively expected, since the melody reaches the tonic on the fifth note and then moves away from it, after which it returns: this final resolution should be particularly powerful rather than being merely somewhat more powerful.

As for the semantics, the adapted definition need not change it overall; it can

[19] These values were obtained using the local harmony of *G Major* for the first two partitions and *C Major* for the last, but noting that the Krumhansl and Kessler experiments yielded so-called key profiles, one might instead opt to evaluate everything from the global key of *C Major* perspective (where *G Major* is fifth mode), in which case the magnetism values for the first two partitions are 0.79 and 0.45, with the last one unchanged.

still be *bird-landing*, but the mapped 'force' variable in equation 3.3.4 might be reinterpreted as 'absorbed force' instead of 'gravitational force'. The new values for W_{L_4} in equation 3.3.4 would then be $\langle \textit{bird}, \langle \textit{force=low} \rangle, \langle \textit{force=low} \rangle, \langle \textit{force=high} \rangle \rangle$, to reflect that the force of the landing is absorbed upon contact with the ground.

4.2 Motivic inertia

Larson's definition of inertia is very general: *"the tendency of a pattern of musical motion to continue in the same fashion"* ([Lar04], page 461). So far, inertia has been restricted to cases of upward, downward, and alternating motion (viz. definition 3.2.5). However, repeating or developing a musical motif or theme could arguably also count as continuing musical motion in the same fashion. In this section, a simple example will be considered: four measures of Ludwig van Beethoven's *Ode to Joy* from his *Symphony No. 9*. This also happens to feature as an example in Schlenker ([Sch19], page 42). Here, a denotation will be suggested, using melodic similarity, for an effect not unlike Schlenker aims to illustrate with his *Zarathustra* example (*ibid*, page 44), namely the development of a phenomenon in stages. In Schlenker's paper, the phenomenon is a sunrise; here, it will be the (re-)appearance of the moon as a (partial) cloudcover moves across it.

An exhaustive characterisation of using inertia would require one to define some measure of melodic similarity where two instances would need to remain within certain bounds (see for example [VVJ16] for an overview). But this would go well beyond the aims of this study, hence here it is simply assumed that the first and third measures of *Ode to Joy* are sufficiently similar. The music and forces are depicted in figure 14 below.

otj.mp3

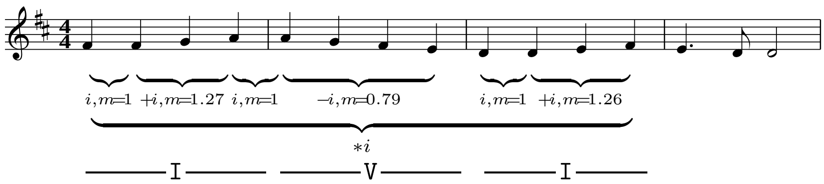


Figure 14: Annotated second half of Schlenker's example 5 (*Ode to Joy*)

Some limitations: the forces in the figure (and the subsequent denotation) are only about the first three measures. Moreover, gravity is ignored so beside harmony, only inertia and magnetism are used, with the latter according to the Krumhansl method from section 4.1 (using global harmony, see footnote 19).

Ode to Joy has a simple repetitive structure which the partitioning in figure 14 highlights and encompasses with the large bracket labeled $*i$: this is supposed to capture the repeating motivic inertic structure. The idea is that the same situation is represented in different contexts with different results, e.g. the moon's luminosity goes from full to none and back, but the stages are preceded by a dimmed stage as the cloudcover tends to be thinner around a gap. Meanwhile, the harmony moves from stable to less stable to stable again, which is associated with the moon's visibility.

Equation 4.2.1 gives the elementary musical events for the (upper) partitions in figure 14, for triples $\langle harmony, magnetism, inertia\rangle$.

Equation 4.2.1.
$M = \langle\langle \mathtt{I}, 1, 1\rangle, \langle \mathtt{I}, 1.27, 1_{\uparrow}\rangle, \langle \mathtt{V}, 1, 1\rangle, \langle \mathtt{V}, 0.79, 1_{\downarrow}\rangle, \langle \mathtt{I}, 1, 1\rangle, \langle \mathtt{I}, 1.26, 1_{\uparrow}\rangle\rangle$

The mappings are as follows, with values in curly brackets: *harmony* $\longrightarrow$ *visibility* {*decreasing*, *increasing*}, *magnetism* $\longrightarrow$ *luminosity* {*none*, *dimmed*, *bright*}, *inertia* $\longrightarrow$ *cloudcover* {*none*, *partial*, *full*}. The resulting denotation ('world events') $W =$ *moon-appearing* is then as in equation 4.2.2. Note that magnetism values 1.27 and 1.26 are both mapped to *bright* (and see also section 5.4).

Equation 4.2.2.

$$W = \left\langle moon, \begin{array}{c} \left\langle \begin{array}{c} visibility{=}increasing, \\ luminosity{=}dimmed, \\ cloudcover{=}partial \end{array} \right\rangle, \left\langle \begin{array}{c} visibility{=}increasing, \\ luminosity{=}bright, \\ cloudcover{=}none \end{array} \right\rangle, \left\langle \begin{array}{c} visibility{=}decreasing, \\ luminosity{=}dimmed, \\ cloudcover{=}partial \end{array} \right\rangle, \\ \left\langle \begin{array}{c} visibility{=}decreasing, \\ luminosity{=}none, \\ cloudcover{=}full \end{array} \right\rangle, \left\langle \begin{array}{c} visibility{=}increasing, \\ luminosity{=}dimmed, \\ cloudcover{=}partial \end{array} \right\rangle, \left\langle \begin{array}{c} visibility{=}increasing, \\ luminosity{=}bright, \\ cloudcover{=}none \end{array} \right\rangle \end{array} \right\rangle$$

The point of motivic inertia is to generalise the idea of 'continuing in the same fashion' to repetitions and near-repetitions, i.e. to motivic / thematic development. The idea is that the semantics can be seen to be mirrored by or develop along with the music, so as to denote the development of a phenomenon in stages – i.e. like in Schlenker's *Zarathustra* example. While the example given here may not be perfect, it serves to illustrate this basic idea.

4.3 Modulation

For the examples in this thesis, it has so far been assumed that music is restricted to a single key – which may be broadly specified as the (instantiated) scale on which the music is based. For instance, definition 3.1.6 expounds the concept of a (harmonic) progression in terms of triads related to a given scale, while in fact, music may be built on more than one scale. The same definition also specifies the notion of cadence, but these can be appropriately presupposed to be confined to some key. However, it is common for successive cadences to be associated to different keys. See for example figure 15, in which the start of the jazz standard *How High the Moon* is depicted.[20]

Note though that the harmonic functions in the figure do not precisely follow those as given in the jazz musician's standard reference *The Real Book* ([stu94], page 202), because this would make some of the ascending three-note groups end on degree 1 (i.e. a second in 1-based usage), while partitions are required to end on 0, 2 or 4 here (viz. definition 3.2.1). This has been resolved by adding three fifth modes (shown underlined), of which two can also act as minor modes as explained later. One of these added modes is in the first (pickup) measure and two follow what would otherwise have been an prolonged mode `I`. Additionally, the two instances of mode `II` have been extended to cover the tied note in the next measure. So to be clear, the original ('Real Book') harmony is mode `I` in *G Major* for measures 2 and 3 followed by a `II-V-I` cadence resolving on *F major* in measure 6, and another `II-V-I` resolving on *E♭ Major* in the last measure.

For Schlenker, one interpretation of modulation is the source moving to a new location or environment; another is the perception of a new source ([Sch19], page 61). The example in figure 15 is to reflect the latter.

hhtm.mp3

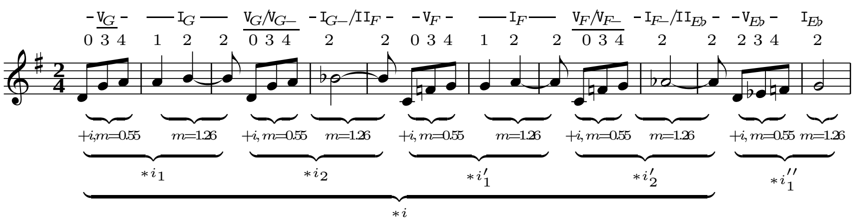


Figure 15: Beginning of *How High the Moon* (Lewis/Hamilton)

In the figure, the magnetism values given are computed using the 'Krumhansl' method from section 4.1.[21] The motivic inertia assignments at the second level are intended to group together similar segments of melody, and have been given the subscript 1 in case the next group repeats the last note at the first level, and 2 otherwise (with $*i'_1$, $*i'_2$ and $*i''_1$ to indicate the variations). Note that in

[20] The first ten measures are shown, but to keep these on a single line to facilitate the semantic assignments, the note lengths were halved – i.e. crotchets (or quarter notes) become quavers (eighth notes); and to suit this the meter was changed from $\frac{4}{4}$ to $\frac{2}{4}$. Moreover, the original last measure's four *g*s were changed into a single long *g*. No explicit harmonisation has been included as this would also cut the line, but for this the reader is referred to recordings such as Chet Baker's (from the 1959 album *Chet*: youtu.be/GRuEjG8iU4A).

[21] In the top-level forces assignments, partitions 3 and 4 as well as 7 and 8 are intended to have a note-sharing boundary which was hard to typeset into the figure – see page 23.

each case except the last, having the same inertic and magnetism assignments at the first motivic inertia level implies the same subscript assignment at the second level.[22] The third level with the big $*i$ group is meant to indicate the alternation of subscripts 1 and 2, but also happens to capture the identical first-level assignments.

The Krumhansl values for the a of group 4 and g of group 8 were drawn from G and F, respectively, and those for group 4's $b\flat$ and group 8's $a\flat$ respectively from F and $E\flat$ (these keys are annotated at the top of figure 15). This was done in order to apply modulation 'actively', and yields the magnetism values for both groups as $4.09/3.48 \approx 1.26$ – i.e. the same as for groups 2, 6 and 10. More choices are in fact possible, but the current choice might indicate that the Krumhansl magnetism reflects something essential about the way the composition works (see also section 6.3). Note the double harmonic annotations above groups 3, 4, 7 and 8; this indicates the presence of so-called pivot chords that can be analysed as belonging to the old as well as the new key – and incidentally also illustrates the use of the `II-V-I` progression in modulation (viz. [Smi08], page 67).

The double harmonic annotations moreover underscore how various choices can be made with respect to harmonic interpretation, and hence also regarding modulation. Most basically, the piece could be viewed as modulating from G to F, and then to $E\flat$. Here however, it was decided to view it as modulating from G to G- (i.e. to 'G *Minor*'), then to F and F-, and finally to $E\flat$. This is justified by noting that even though G- can be viewed as simply the second mode of F (also known as 'Dorian minor'), it is also the sixth mode ('natural' or Aeolian minor) of $B\flat$ – with the annotated $\mathtt{V}_{G-}$ in the figure the D- 'Phrygian' minor third mode.[23] Consequently a real modulation can arguably be posited at this point. Moreover, that attractions within minor are heard relative to a different attraction vector was demonstrated by Krumhansl (viz. [Kru83] page 39, or [Kru90] page 80).[24]

So the music depicted in figure 15 is to denote four related events, i.e. one for each key, which are indicated by the motivic inertia tags $*i_1$, $*i_2$, $*i'_1$ and $*i'_2$ (the last $*i''_1$ is ignored). Equation 4.3.1 gives the relevant musical events in terms of triples

[22] Also note that in case Larson's method for assigning magnetism had been used, these equivalences at the first level would not have obtained, since the sequence of assignments would instead have been $[\{\{+i, m = -0.75\}, \{m = 0.21\}\}, \{\{+i, m = 0.21\}, \{m = 0.75\}\}, \{\{+i, m = -0.75\}, \{m = 0\}\}, \{\{+i, m = 0.75\}, \{m = 0.75\}\}, \{\{+i, m = -0.75\}, \{m = 0\}\}]$.

[23] In other words, the repeating three-note patterns in measures 1, 3, 5 and 7 are all consistent with both major and minor – their middle notes are an interval of a fourth up from the first note, which makes them so-called 'suspended' chords (or more accurately arpeggios) since these may resolve to either a major or minor third, i.e. they are (intentionally?) ambiguous.

[24] However, these values are not used here but they are discussed in section 6.3.

$\langle harmony, magnetism, inertia \rangle$.

Equation 4.3.1.

$$M_{*i_1} = \langle \langle \mathtt{V}, 0.55, 1_{\uparrow} \rangle, \langle \mathtt{I}, 1.26, \varnothing \rangle \rangle$$
$$M_{*i_2} = \langle \langle \mathtt{V}, 0.55, 1_{\uparrow} \rangle, \langle \mathtt{I}, 1.26, \varnothing \rangle \rangle$$
$$M_{*i'_1} = \langle \langle \mathtt{V}, 0.55, 1_{\uparrow} \rangle, \langle \mathtt{I}, 1.26, \varnothing \rangle \rangle$$
$$M_{*i'_2} = \langle \langle \mathtt{V}, 0.55, 1_{\uparrow} \rangle, \langle \mathtt{I}, 1.26, \varnothing \rangle \rangle$$

The mappings along with their possible values can then be specified as follows: $harmony \longrightarrow motion\{lower, higher\}, magnetism \longrightarrow distance\{lower, higher\}$, $inertia \longrightarrow altitude\{lower, higher\}$. The values could have been more fine-grained but were picked for simplicity. For equation 4.3.2 below, definitions 3.3.6 and 3.3.7 (energy update and eventfulness) were used where inertia is unassigned in figure 15. Since the musical events are equivalent, only one instance is given.

Equation 4.3.2.

$$W_{*i_1} = \left\langle bird_{*i_1}, \left\langle \begin{array}{c} motion{=}higher, \\ distance{=}higher, \\ altitude{=}lower \end{array} \right\rangle, \left\langle \begin{array}{c} motion{=}lower, \\ distance{=}lower, \\ altitude{=}higher \end{array} \right\rangle \right\rangle$$

The intended single-instance denotation is W_{*i_1} = *bird-landing-on-tree*, but then W_{*i} = *birds-landing-on-tree* – i.e. the idea is that of a bird flying (upward) to land on a tree branch, and for each modulation a new bird is perceived that performs the same action. As can be gathered from the music in figure 15, the second bird might land somewhat differently compared to the first, but here such details are abstracted away from, since the point is to illustrate how Schlenker's view of modulation may be wedded to Larson's ideas on musical forces.

While it appears to be the case that modulation can in principle be handled, the semantics is somewhat sketchy and leaves a few things to be desired. For instance, only a single bird is specified formally, but in fact it should be a collection of them. Moreover, even in the single bird case, there is only the bird participating in the event while it would be desirable to have the tree as well, in order to characterise the interaction between the two. But given this thesis has been confined to single musical lines – or voices, this is a general limitation here due to Schlenker's association of voices with objects ([Sch19], page 66). This issue is addressed in section 4.5.

4.4 Level-by-level semantics

In section 3.3 it is noted that for the *bird-landing* example, levels 1 and 2 in figure 9 (section 3.2) might be interpreted as the more general situation *bird-moving*. This relates to the idea that as more notes – less stable ones – are added to a given musical level, it is desirable that a more specific situation may arise as a denotation. In this section, this idea will be illustrated by starting with a minimal base level for the example and enriching its semantics to obtain *bird-landing* $\models$ *bird-descending* $\models$ *bird-moving* – i.e. each more complex musical level is to entail its simpler level semantically (which might possibly be viewed as a form of compositionality).

It is possible to have one further level down from level 1 in figure 9; this minimal level 0 would be the first and last notes of level 1, i.e. the notes *g* and *c*, or the motion from the dominant to the tonic (`V-I`), or in 0-based notation, the notes sequence $[4, 0]$ – which are the most stable according to definition 3.1.8 – and so then at level 1, the less stable note *e* (the mediant, 2) would be added. The reason this minimal level has two notes rather than one is firstly because intuitively, a single note on its own without any context is meaningless if, as is the case here, there is no significant notion of 'sound semantics' (viz. section 1.1). Secondly, two notes is the minimum for which it is possible within Larson's framework – which forms the basis of the semantics here – to assign a musical force. So while it might be possible in principle to have a base level with just the tonic (*c* or 0), this would effectively be deemed meaningless in the sense that one cannot assign forces and hence no denotation to it. So within the framework adopted in this thesis, there is a minimal musical and situational event level consisting of two notes.

Then considering only the bottom groupings for levels 1-3 in figure 9 (on page 22), the following denotations including the new level 0 might be proposed. Level 0: *bird-moving*, levels 1-2: *bird-descending*, and level 3: *bird-landing*; so that indeed level 3 $\models$ level 2 $\ldots$ $\models$ level 0. The semantics hinges on using the Krumhansl magnetism from section 4.1, according to which the following values obtain for each level's last-notes pair: 1.22, 1.45, 1.82, and 2.20, i.e. level-by-level, the final magnetism values get increasingly stronger. Although this is also the case if Larson's method is used (the values for levels 0-3 would then be -0.07, -0.05, 0, and 0.94), the increases follow a more intuitively fitting pattern using Krumhansl magnetism. Moreover, magnetic values may then be interpreted as altitude by noting that 2.20 is the maximum attraction value for notes that are in the major scale, and subtracting each value from 2.20. Additionally multiplying the values by 100, the following 'bird altitudes' could then be associated with levels 0-3: $98m$, $75m$, $38m$, and $0m$.[25]

Musical events for the levels are triples $\langle gravity, inertia, magnetism' \rangle$ (the apos-

[25] In case the top grouping of level 2 is also considered, this would yield an extra altitude value of $113m$ for the first group, so this level might then be interpreted as *bird-descending-rapidly*.

trophe signifies the derived magnetism altitude) – and may be specified as follows:

Equation 4.4.1.
$M_{L_0} = \langle\langle 1, 0, 98\rangle\rangle$
$M_{L_1} = \langle\langle 1, -1, 75\rangle\rangle$
$M_{L_2} = \langle\langle 1, -1, 38\rangle\rangle$ [26]
$M_{L_3} = \langle\langle 1, -1, 38\rangle, \langle 1, -1, 0\rangle\rangle$

The mappings being *gravity* $\longrightarrow$ *wind pressure*, *inertia* $\longrightarrow$ *wing power*, and *magnetism′* $\longrightarrow$ *altitude*, the following denotations may then be obtained (the abbreviations should be obvious):

Equation 4.4.2.
$W_{L_0} = \textit{bird-moving} = \langle\langle \textit{wind=high, alt=98m, power=low}\rangle\rangle$
$W_{L_2} = \textit{bird-descending} = \langle\langle \textit{wind=high, alt=38m, power=high}\rangle\rangle$
$W_{L_3} = \textit{bird-landing} = \langle\langle \textit{wind=high, alt=38m, power=high}\rangle,$
$\langle \textit{wind=low, alt=0m, power=low}\rangle\rangle$

So this gives the desired entailment relationship $W_{L_3} \models W_{L_2} \models W_{L_0}$, but note that it is entailment as understood intuitively, except for $W_{L_3} \models W_{L_2}$ because the values of M_{L_2} in equation 4.4.1 are contained in M_{L_3}. The example rendered in this way also serves to illustrate that levels reflect aspects of the music that may be perceived simultaneously, as was already pointed out in the example of the displacement of a trace in figure 4. Indeed, according to Larson (and VanHandel, [LV05], page 131), levels may be associated with the perceptions of skilled listeners, which implies that meanings associated with different levels may reflect different aspects of the same situation, or, in this case, simpler versions of the same event.

4.5 Multiple voices

In this thesis, the focus has been on single musical lines, which affects the denotations that may be associated with ('be true of') such a line, due to Schlenker linking musical voices with virtual sources (see [Sch19] page 66, plus definitions 1.1.1 and 1.1.2 here). For instance, the *bird-landing* example of figure 10 gets its denotation in equation 3.3.4 based only on the properties relating to the bird: its altitude, the wind it experiences, the gravitational force and its wing power. But where it lands, the ground, is ignored, like the tree in the example of section 4.3.

The choice to focus on single voices ensued from the wish to use simple examples and patterns used by Larson like the one in figure 10, but it can in fact quite readily be augmented with an extra voice. Consider figure 16 below.

[26] Or $\langle\langle 1, -1, 113\rangle, \langle 1, -1, 38\rangle\rangle$ when considering the top grouping.

lar52.2v.mp3

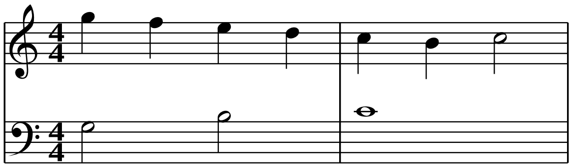


Figure 16: Larson's example 5.2 [Lar97a] with additional bass voice

This version has the basic `I-V-I` cadence as before, except that it has now been cast expicitly into a bass voice. The figure has been left without further forces assignments or a denotation, which are instead described: the idea of the lower voice is indeed to represent the earth, but from the perspective of the bird. In other words, inertia is upward in the bass, but in fact it represents the earth rising – relatively – towards the landing animal.

So while the choice made here throughout has been to view the harmony as a (perceived) property of a singe musical line in order to try to describe a single line as exhaustively as possible within the frameworks provided by Schlenker and Larson, the consequence of this decision is the restriction of denotations to single objects or entities, and hence to potentially describe secondary objects in terms of the primary ones rather than specifying their interaction. One method to represent such complex musical events involving more than one object might incidentally be Cooper's [Coo13].

4.6 Dynamic interpretation

Schlenker's *Zarathustra* example was considered in section 1.1 of this thesis. According to Schlenker, one reason why Strauss' music is appropriate for the imagery (see figure 2) is that its antecedent-consequent structure *"certainly evokes the development of a phenomenon in stages"* ([Sch19], page 44). Schlenker gives a denotation *Sun-rise* on page 66 (or see page 5 here), which does not seem entirely satisfactory when one asks whether it yields a credible cognitive rendition of the listening experience. While an attempt at better capturing the development of a phenomenon in stages was already made – using inertia – in section 4.2 of this study, this section considers a different take on the matter.

The intention of the composer, by having just a tonic and a fifth interval sounding, i.e. no major or a minor third, appears to be to leave the listener in the dark as to the character of the tonality (cf. section 4.3, footnote 23), but what is a core feature of the music is its surprise switch from major to minor harmony: right at the stage where major is actually established. This calls, at least, for a way to render musical events in terms of what seems ‘normal’ or expected, in order to encode deviation and surprise. But these are themselves not properties of observed events, but rather effects on the listener, and so the question becomes whether these can (or should) be considered as part of the music’s meaning.[27] Figure 17 gives a simplified version of the music but also extends it to reflect what eventually happens in Strauss’ composition, and equation 4.6.1 renders the music logically in an attempt to capture the expectation and surprise that it evokes.

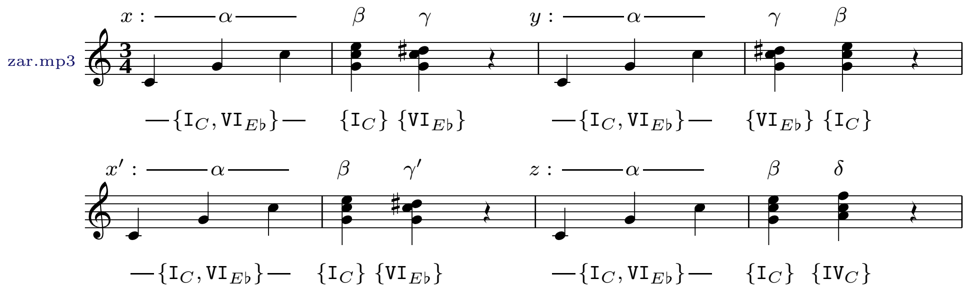


Figure 17: *Zarathustra* rendered as alternating between possibly *C Major* or *E♭ Minor* before modulating to *F Major* (also the fourth degree of *C Major*)

The notation in equation 4.6.1 below follows Veltman [Vel96]. It serves to express that an information state ϕ can be updated with an expression ψ to yield a new state $\phi[\psi]$, where what ψ brings to the interpretation process has been accomodated.[28]

Equation 4.6.1.

$$
\begin{aligned}
x :\ & \emptyset[\alpha] = \{\mathtt{I}_C, \mathtt{VI}_{E\flat}\} \\
& \alpha[\beta] = \{\mathtt{I}_C\} \\
& \beta[\gamma] = \{\mathtt{VI}_{E\flat}\} \qquad [\![\gamma]\!]_x \approx \textit{surprise} \qquad \textit{(expectation blocked/delayed)}
\end{aligned}
$$

$$
\begin{aligned}
y :\ & x[\alpha] = \{\mathtt{I}_C, \mathtt{VI}_{E\flat}\} \\
& \alpha[\gamma] = \{\mathtt{VI}_{E\flat}\} \\
& \gamma[\beta] = \{\mathtt{I}_C\} \qquad [\![\beta]\!]_y \approx \textit{suspense} \qquad \textit{(block/delay persists)}
\end{aligned}
$$

[27] While Schlenker acknowledges the harmonic uncertainty in *Zarathustra* (stating major or minor is initially ‘underspecified’ – [Sch19], page 65), he does not consider this as being of semantic interest in itself.

[28] The notation is sloppy because strictly, in the first x part, $\alpha[\beta]$ should read $\emptyset[\alpha][\beta]$, and $\beta[\gamma]$ is actually $\emptyset[\alpha][\beta][\gamma]$ – and similar elsewhere, for else the alphas, betas and gammas are musical expressions as well as information states – so consider the expressions as ‘shorthand’.

$$x' : \; y[\alpha] = \{\mathtt{I}_C, \mathtt{VI}_{E\flat}\}$$

$$\alpha[\beta] = \{\mathtt{I}_C\}$$

$$\beta[\gamma'] = \{\mathtt{VI}_{E\flat}\} \qquad [\![\gamma']\!]_{x'} \approx \textit{suspense} \quad \textit{(block/delay persists still)}$$

$$z : \; x'[\alpha] = \{\mathtt{I}_C, \mathtt{VI}_{E\flat}\}$$

$$\alpha[\beta] = \{\mathtt{I}_C\}$$

$$\beta[\delta] = \{\mathtt{IV}_C\} \qquad [\![\delta]\!]_z \approx \textit{fulfillment} \quad \textit{(expectation met)}$$

In equation 4.6.1, x, y, x', z and the greek letters refer to sections or stages of the *Zarathustra* version in figure 17. The example expresses how a listener's information about the piece gets updated as it progresses, with an information state rendered as a set of possibilities. So at the beginning, an empty state updated with the first three notes yields the possibilities that the harmony could be *C Major*, or C Minor – the latter being the $\mathtt{VI}^{\text{th}}$ mode of $E\flat$. [29] At β, the $\mathtt{VI}_{E\flat}$ possibility is eliminated, expressed as the update $\alpha[\beta] = \{\mathtt{I}_C\}$, at which point one possibility remains, i.e. 'certainty', but this idea is immediately thwarted by the composer (Strauss), who serves up $\mathtt{VI}_{E\flat}$ (*C Minor*) after all. The 'meaning' (between double square brackets) of stage x is then 'surprise', since the expectation that the piece would be in the key of C has been blocked.

At stage y, Strauss does the same but reversed, i.e. now $\mathtt{VI}_{E\flat}$ gets retracted in favour of $\mathtt{I}_C$. This could be viewed as a form of suspense, since there is still no 'settled' key. Stage x' is a repetition of x, where the suspense continues, until finally at z there is a resolution: Strauss settles the piece in *C Major* – although he does move to its $\mathtt{IV}^{\text{th}}$ mode (F), so the notation is admittedly somewhat sloppy in that respect – but the result is nevertheless a sense of fulfillment that the expectation is finally met, i.e. that something happens which is considered possible given one's information.

According to Leonard Meyer, it is at such points that listeners become inclined to assign meaning to music. To paraphrase the basic idea in *Emotion and Meaning in Music* [Mey56], chapter 4: The mind tends to respond unconsciously to patterns completed according to expectation, but when expectations are inhibited or delayed, response becomes conscious, and affect or the objectification of meaning may follow. In other words, music which carries on predictably does not by and

[29] Note that for Schlenker, the `I-V-I` progression is actually about the first three notes, while here, these are considered as part of a single harmonic function or chord.

large capture a listener's attention, so even if one manages to identify musical features which reflect 'the development of a phenomenon in stages', this may not suffice: the listener's attention will also need to be drawn.

Equation 4.6.1 could possibly provide the beginning of a formalisation to identify these 'points of interest'. An idea that is incidentally implicit in Meyer is that the fulfillment of a delayed expectation will lead to stronger affect, and this is plausibly why the final stage (z) of the example is particularly powerful, or, why the music 'works'.

5 Computational syntax and semantics

This section has two main aims. One relates to a tacit assumption in this thesis so far, namely that the harmony of a musical line is simply given, while it can in fact be said to be imposed onto the music by embellishments with unstable goals. So the first aim is to prove that embellishments can increase the partitionability of a musical sequence (i.e. the syntax) while expanding its harmony. The other aim is complementary; it is to illustrate how some of the definitions given in the thesis may be implemented (in Prolog, see explainer 1 on page 52 and beyond, or [Fla94] for more in-depth Prolog explanation) – to demonstrate the validity of the point made with regard to [HKvL06] on page 2. This involves the automatic production of partitions, as well as the assignment of harmony, forces and semantics.[30]

While a simple statement on musical lines in mode 0 (i.e. `I`) is to be given a universal proof, this section will also embellish two examples using a Prolog program, one of which is to serve as an existential proof.[31] Some of the Prolog code will be given in this section; the full code can be found in the appendix (A, page 75).[32] But first, an intermezzo harking back to Larson's notion of displacement is required.

5.1 Displacement of traces

The example of figure 4 in section 1.2 illustrated Larson's conception of trace and displacement: within a note sequence which contains leaps (intervals of a minor

[30] To recap, Hamm, Kamp, and Van Lambalgen argue for formalisation to ensure theories of cognition are computable, but actual implementation takes things one step further, however, in [HKvL06] the idea is to do just that, see for instance page 9.

[31] According to Igor Rivin's principle *'A computer program is a proof'* ([Riv14], page 598).

[32] The source code is also available at `https://mickdeneeve.github.io/ac/ma/src/` and can be run using SWI Prolog (`https://www.swi-prolog.org`).

third or larger), the listener will nevertheless hear stepwise motion if present. But while the example assumes a starting level *[b, c]*, this might in principle also be *[e, c]* instead, with the *b* added last by virtue of (alternating) inertia. In other words, it may not be the case that there is a level *[b, c]* – to assign magnetism to – because like partitioning, 'leveling' is non-deterministic. The following definition can nonetheless make the displacement explicit.

Definition 5.1.1. *Given a non-stepwise partition Q, a displacement $Q_1, \ldots, Q_n$ is a decomposition of Q into non-contiguous but sequential sublists such that at least one of these is stepwise.*

In figure 18, the sequence is depicted with the forces including (Krumhansl) magnetism in full, with arrows to indicate the extracted notes for which the forces are determined – since the listener should hear the stepwise motion from *b* to *c*.

leap.trace.mp3

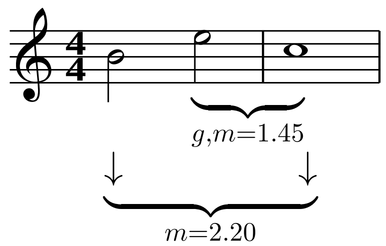


Figure 18: Trace displacement with stepwise magnetism

While one might 'disembellish' a musical line to deconstruct it and find partitions for forces assignment, this could be challenging for more complex lines with several possible origins. With the above definition, displacement may be used as an alternative, which is relevant for the next section.

5.2 Partitions and harmony

Two examples will now be presented, both of which get embellished with an unstabler goal note (controlled respectively by magnetism and inertia), such that augmenting the harmony with an extra harmonic function allows the examples to be partitioned further.

For the first example, a standard `I` harmony – i.e. mode 0 – is assumed throughout first, which after embellishment is augmented with `V` (mode 4), and becomes `I-V-I`. The first example is depicted in figure 19 (with the partitions shown bracketed). Harmonic expansion lets the sequence be partitioned further. This observation, limited to cases as above (with initial harmony `I`) can be expressed as a proposition:

Original: css1.mp3

Embellished: css1.add.mp3

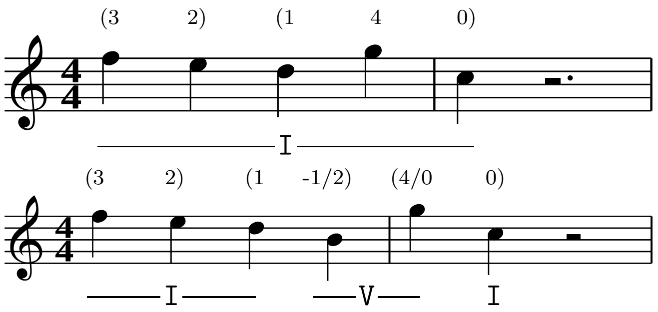


Figure 19: Embellishing a given line by magnetism

Proposition 5.2.1. *For any note sequence N where the accompanying harmonic function sequence H is mode 0 throughout and the notes or degrees* 0*,* 4 *and* 2 *are already present, continued embellishment with less stable notes increases the number of partitions by having an expanded harmonic function sequence H'.*

Proof. *Because N already has* 0*,* 4 *and* 2*, it has length $l_N \geq 3$, while the substitution sequence $R = N$. If l_R is odd, it has partitions P of length $l_P = 3$, and if l_R is even, it has partitions with length $l_P = 2$. In either case, adding an unstable note $u \in \{5, 1, 3, 6\}$ as second note blocks the appearance of a point of partition, because if $l_P = 2$ there cannot be a subsequent partition of length* 1*, and if $l_P = 3$, then even though the new length is* 4*, partitioning fails because the first partition cannot end on* 0*,* 4 *or* 2*. Moreover, any further element $e \in \{5, 1, 3, 6\}$ likewise added will not have further partitioning succeed by the same token.*

Now consider table 1 containing substitutions s, according to definition 3.1.8, with note degrees left to right and harmonic modes top to bottom, from more to less stable, with substitutions in accordance with definition 3.1.7; $s = (n - h)$ mod 7*.*

$h \backslash n$	0	4	2	5	1	3	6
0	0*	4*	2*	5	1	3	6
4	3	0*	5	1	4*	6	2*
3	4*	1	6	2*	5	0*	3
5	2*	6	4*	0*	3	5	1
1	6	3	1	4*	0*	2*	5
2	5	2*	0*	3	6	1	4*
6	1	5	3	6	2*	4*	0*

Table 1: *Note substitution table following definitions 3.1.8 and 3.1.7*

With the stabilities starred for convenience, it can now be read off from the table that for any embellishments, the following substitutions s in R' and expansions h' for H' can be made for any resulting added unstable notes $n \in \{5, 1, 3, 6\}$:

For $n = 5$: set $s = 2$ and $h' = 3$, or $s = 0$ and $h' = 5$, or $s = 4$ and $h' = 1$;
For $n = 1$: set $s = 4$ and $h' = 4$, or $s = 0$ and $h' = 1$, or $s = 2$ and $h' = 6$;
For $n = 3$: set $s = 0$ and $h' = 3$, or $s = 2$ and $h' = 1$, or $s = 4$ and $h' = 6$;
For $n = 6$: set $s = 2$ and $h' = 4$, or $s = 4$ and $h' = 2$, or $s = 0$ and $h' = 6$.

This means that R' differs from R by being augmented with only stable points of partition, so that if $l_{R'} \geq 3$, then R' has at least two partitions (the total being $\lfloor \frac{l_{R'}}{2} \rfloor$). So the number of partitions as well as the number of harmonic functions have increased. $\square$

Note that in figure 19's example, b is indeed added as second note – it is added to the original's second partition *[d,g,c]* to create the embellished *[d,b,g,c]* which can only be partitioned to *[[d,b],[g,c]]* upon restabilisation by expanding the harmony. This is demonstrated below with the aid of Prolog code. Consider the predicate `partitions/2` in coding 1 below.

```
/* partition(+Degrees, -Partitions)
      Given a lists of Degrees, find all partitions; returning
      them in lists of the form DegreePartitions; with smallest
      last and excluding the original ('self partition').
*/
partition(Subs, Partitions) :-
   findall(SubParts, subparts(Subs,SubParts), Parts),
   reverse(Parts, [_|Partitions]).

/* subparts(+DegreesList, -DegreesPartitions)
      List of lists DegreesPartitions requires partitions end
      stable and are at least 2 elements (for forces assigments).
*/
subparts([], []).
subparts([Sub|STail], [[Sub|NewSTail]|SubParts]) :-
   append(NewSTail, NewSub, STail),
   length(NewSTail, SLength),
   last(NewSTail, Final),
   SLength > 0,
   member(Final, [0, 4, 2]),
   subparts(NewSub, SubParts).
```

Coding 1: Predicates `partition/2` and `subparts/2`, part of program 1 (`partition.pl`)

> ***Explainer 1*** *Prolog is a logic programming language. Its statements are of the form predicate(Variables) $\leftarrow$ conditions (with the arrow expressed as ':-') or simply fact(literals) (without conditions). Conditions separated by a comma are conjunctions, and if separated by a semicolon they are disjunctions. Several predicates (or facts) of the same name terminated by a period in a program are also disjunctions as they*

are considered different versions of the same statement (with different conditions). A call to a predicate (program) is to prove that it is true by proving that its conditions are. During this process, the predicate's variables (which start with capitals) are instantiated (or 'returned') to (or as) literals consistent with the program's specified conditions.

Prolog's basic data type is a list. It has the form `[Head|Tail]`*, where* `Head` *is an element subject to the predicate conditions, and* `Tail` *is another list for which the predicate is commonly further specified recursively. This requires a so-called stop condition (a fact), often involving the empty list (*`[]`*). Comments (between '/*' and '*/') describe a predicate, with input arguments prefixed by '+' and outputs with '-'.*

The original sequence of figure 19 can indeed only be partitioned into two parts, and by adding the b which is degree 6, there are still just two partitions (or parts), yet when degree 6 is restabilised to degree 2 with the introduction of harmonic function `V` (mode 4: $(6-4)$ mod $7 = 2$), there are three possible partitionings, one of which has three partitions, as shown in output 1 (see also explainer 2).

```
?- partition([3,2,1,4,0], P).
P = [[[3,2], [1,4,0]]].

?- partition([3,2,1,6,4,0], P).
P = [[[3,2], [1,6,4,0]]].

?- partition([3,2,1,2,4,0], P).
P = [[[3,2,1,2], [4,0]], [[3,2], [1,2,4,0]], [[3,2], [1,2], [4,0]]].

?- partition([3,2,1,2,0,0], P).
P = [[[3,2,1,2], [0,0]], [[3,2], [1,2,0,0]], [[3,2], [1,2], [0,0]]].
```

Output 1: Partitioning figure 19 using coding 1

Explainer 2 *Output 1 is essentially a call to* `subparts/2` *from coding 1. This predicate has two arguments (hence '*`/2`*') and it is true in case the second argument constitutes a splitting of the first argument into sublists (*`append/3` *can split as well as concatenate) – such that each respective last element is stable (i.e. it needs to end on* $\{0, 4, 2\}$*). Since there may be several such solutions,* `partition/2` *uses the built-in* `findall/3` *to gather all of these and output them as lists.*

In output 1, the first Prolog query relates to figure 19's original, and the second one to the figure's embellishment if there were no harmonic expansion – i.e. in case the embellishment is still to be read as if it were `I` (mode 0) throughout. In the third query the -1 has been flipped, i.e. from degree 6 to 2, by applying mode 4 in the manner indicated above.[33] In the final query the same was done to the next note – thereby completing the harmony of figure 19.

The final query's input was adapted with a 'crude' method to push the new harmonic function to be active around the note that is to receive it. The idea is that any note which is also stable under this function is also flipped towards it. It is 'crude' because the start of a section to be interpreted under a new harmonic function need not be stable according to it as long as the last note (the goal) is stable – but in order to not get too many possible solutions for the boundaries of a new harmony, the crude way was adopted here. It is shown in coding 2.

```
/* subs(+DegreesList, +HarmonyList, +NewMode, +Index, -Subs)
     Subs is a list of substitutions, i.e. degrees to which
     NewMode can be applied, such that the substitutions are
     contiguous from Index and stable given NewMode.
*/
subs(Degs, Harms, NewMode, Index, ISubs) :-
   index(Degs, 0, IDegs),
   index(Harms, 0, IHarms),
   append(_Degs1, Degs2, IDegs),
   append(ISubs, _Degs3, Degs2),
   member(Index:_, ISubs),
   select(ISubs, IHarms, SubHarms),
   stable(ISubs, SubHarms, NewMode).

/* update(+Degs, +Harms, +Subs, +NewMode, -NewDegs, -NewHarms)
     Apply NewMode and Subs (according to indeces I therein) to
     Degs and Harms so new substituted degrees NewDegrees are
     consistent with new harmonic functions NewHarms.
*/
update(Degs, Harms, [], _, Degs, Harms) :- !.
update(Degs, Harms, [I:Sub|Subs], NewMode, NewDegs, NewHarms) :-
   index(Harms, 0, IHarms),
   member(I:Harm, IHarms), !,
   NewSub is ((Sub+Harm)-NewMode) mod 7,
   replace(I, NewSub, Degs, NextDegs),
   replace(I, NewMode, Harms, NextHarms),
   update(NextDegs, NextHarms, Subs, NewMode, NewDegs, NewHarms).
```

Coding 2: Predicates `subs/5` and `update/6`, part of program 2 (`subs.pl`)

Explainer 3 *The predicate* `subs/5` *tries to push the new harmony as far left and right as possible. If notes to the left or to the right of*

[33] Unlike in program 2 (`subs.pl`), the use of a predicate like degree/3 is omitted here, so the *b* of figure 19 (bottom) is immediately rendered as 6 instead of -1.

the target note are stable under the new harmonic function, `subs/5` *succeeds (i.e. it is true). Solutions for the predicate may then be passed to* `update/6` *which replaces the old harmonic functions accordingly.*

Output 2 does just this; it shows the application of coding 2 to the augmented note degree sequence at the bottom of figure 19. The call to `subs/5` requests solutions `S`: what regions around the new (0-based) index 3 are stable given the new harmonic function `V` (mode 4). The call to `update/5` then takes the last output solution, and given this `S = [3:6,4:4]` (i.e. index:degree) requests the new substituted degree sequence `D`, plus the new harmonic function sequence `H`. The resulting `D` and `H` are the degrees and harmony of figure 19's bottom line.

```
?- subs([3,2,1,6,4,0], [0,0,0,0,0,0], 4, 3, S).
S = [2:1, 3:6] ;
S = [2:1, 3:6, 4:4] ;
S = [3:6] ;
S = [3:6, 4:4] .

?- update([3,2,1,6,4,0], [0,0,0,0,0,0], [3:6,4:4], 4, D, H).
D = [3, 2, 1, 2, 0, 0],
H = [0, 0, 0, 4, 4, 0].
```

Output 2: Harmonising figure 19 using coding 2

This section's second example, in figure 20, is a `V-I` that expands to `IV-V-I`.

Original:
css2.mp3

Embellished:
css2.add.mp3

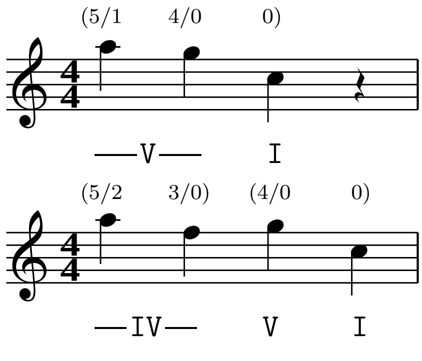


Figure 20: Embellishing a given line by inertia

Output 3's second query shows figure 20's embellishment cannot be partitioned until `6` is reharmonised to `0` in the third query by reinterpreting it as part of `IV` (mode 3) instead of `V`, i.e. mode 4 (NB: the input is in degrees, see footnote 33).

```
?- partition([1,0,0], P).
P = [].

?- partition([1,6,0,0], P).
P = [].

?- partition([1,0,0,0], P).
P = [[[1, 0], [0, 0]]].

?- partition([2,0,0,0], P).
P = [[[2, 0], [0, 0]]].
```

Output 3: Partitioning figure 20 using coding 1

The final query shows the output with extended harmonisation as in the bottom part of figure 20. This is again determined using `subs/5` with `update/6` as before. The degrees from the second query in output 3 are taken as the basis for this.

```
?- subs([1,6,0,0], [4,4,4,0], 3, 1, S).
S = [0:1, 1:6] ;
S = [1:6] .

?- update([1,6,0,0], [4,4,4,0], [0:1,1:6], 3, D, H).
D = [2, 0, 0, 0],
H = [3, 3, 4, 0].
```

Output 4: Harmonising figure 20 using coding 2

As can be seen from the last query, the new harmony (`H`) aligns with the bottom line of figure 20.

While this example is simpler than the previous one by being shorter, it is more complex by already having an initial harmony beyond mode 0 (`I`). But because the stabilities specified in definition 3.1.8 have been assumed to relate to mode 0 as so-called inherent stabilities, there is no appropriate notion of 'contextual' or 'relative' stability available here to say what is to be less stable given that the music is heard as mode 4, i.e. beyond mode 0 (see also page 20). Therefore only the following existential proposition will be formulated – as indicated at the start of this section, codings/programs and their outputs are to be considered as proof.

Proposition 5.2.2. *Given a note sequence N where the accompanying harmonic function sequence H contains a mode $h > 0$, there is an embellishment which increases the number of partitions by having an expanded harmonic function sequence H', i.e. with an $h' > h$.*

Proof . *The note sequence is the one of figure 20, and with reference to footnote 31, the second and third queries of output 3 demonstrate the existence of the required embellishment, since the third query produces a partitioning by virtue of an expanded harmonic sequence.* □

5.3 Assigning forces

Both examples in the preceding section were augmented according to Larson's notion of embellishing being *"controlled by the forces"* (viz. section 1.2). This will be made explicit by computing the forces on the partitions that ensued from the embellishments in Prolog. For the augmentation of figure 19, a displacement procedure is needed as indicated in section 5.1.

```
/* displace(+Notes, -Displaced)
     Displaced is a two-way non-contiguous but sequential
     partitioning of Notes that has a stepwise partition.
*/
displace(Notes, Displaced) :-
   divide(Notes, Displaced),
   member(Steps, Displaced),
   stepwise(Steps),
   length(Displaced, 2).

/* divide(+Notes, -Divided)
     Divided is a non-contiguous but sequential list of lists
     partitioning of Notes. Auxiliary divide/3 makes subdivisions
     among two lists first.
*/
divide([], []).
divide([Note|Notes], [[Note|DNotes]|Divided]) :-
   divide(Notes, DNotes, Rest),
   length(DNotes, L),
   L >= 1,
   divide(Rest, Divided).
divide([], [], []).
divide([Note|Notes], [Note|DNotes], Divided) :-
   divide(Notes, DNotes, Divided).
divide([Note|Notes], DNotes, [Note|Divided]) :-
   divide(Notes, DNotes, Divided).
```

Coding 3: Predicates `displace/2` and `divide/2`, part of program 3 (`subs.pl`)

Output 5 shows `displace/2` applied to figure 19 (embellished). Forces are assigned to the first solution's last partition and the third solution's first, in output 6.

```
?- displace([3,2,1,-1,4,0], D).
D = [[3,2,1,4], [-1,0]] ;
```

```
D = [[3,2,1,0], [-1,4]] ;
D = [[3,2,1], [-1,4,0]] ;
D = [[3,2,-1,4], [1,0]] ;
D = [[3,2], [1,-1,4,0]] ;
D = [[3,-1,4,0], [2,1]] ;
D = [[3,-1,4], [2,1,0]] ;
D = [[3,4], [2,1,-1,0]] .
```

Output 5: Displacements for figure 19's embellishment using coding 3

Explainer 4 *Output 5 shows a common way to interact with Prolog: ask for a solution and request more by entering the disjunction operator (';'). The* **`displace/2`** *predicate from coding 3 divides elements into two lists before checking for a stepwise partition. If it is not, Prolog 'backtracks' and creates a new division. Requesting more solutions as in output 5 boils down to explicitly asking Prolog to backtrack.*

Coding 4 shows predicates implementing some of Larson's forces from section 3.2, but with magnetism according to Krumhansl (viz. section 4.1). These will be applied to the examples (figures 19 and 20) of this section.

```
 /* gravitational(+Notes)
       Succeeds if the final note descends and is lower than the
       first. This is a crude version of Steve Larson's notion, e.g.
       no ceiling is specified beyond the partition's first note.
       Auxiliary gravitational/3 applies this first-note ceiling.
 */
gravitational([First|Notes]) :-
   reverse([First|Notes], [Final,Penultimate|_]),
   gravitational(First, Penultimate, Final), !.
gravitational(Note, Note, Final) :-
   Note > Final.
gravitational(First, Penultimate, Final) :-
   First >= Final,
   Penultimate > Final.

/* kmagnetic(+Scale, +Partition, -Kvalue)
      Gives a Krumhansl magnetic value determined between the
      sequence's penultimate and last notes, which is set to be
      last/penultimate so it is asymmetric unlike Larson's notion.
*/
kmagnetic(Scale, Notes, K) :-
   reverse(Notes, [Final,Penultimate|_]),
   kmagnetic(Scale, Penultimate, Final, K).
kmagnetic(Scale, Penultimate, Final, K) :-
   kmagnetic(Scale, Values),
   PRole is Penultimate mod 7,
```

```
    FRole is Final mod 7,
    nth0(PRole, Values, PValue),
    nth0(FRole, Values, FValue),
    KF is FValue / PValue,
    round(KF, 2, K).

/* inertic(+Notes, -Type)
       Determines inertia for a note sequence, following Larson,
       (if present); which is defined as 'following a pattern',
       and specified here as either repeating its notes ('still'),
       ascending ('up'), descending ('down'), or 'alternating'.
*/
inertic(Notes, Type) :-
    length(Notes, L),
    (   L < 1,
        next(==, Notes),
        Type = still
        ;
        L > 2,
        (   next(<, Notes),
            Type = up
            ;
            next(>, Notes),
            Type = down   )
        ;
        L >= 3,
        alt(Notes),
        Type = alt   ), !.
```

Coding 4: `gravitational/1`, `kmagnetic/3` and `inertic/2` from program 4 (`forces.pl`)

> ***Explainer 5*** *In coding 4, `inertic/2` has nested disjunctions which might instead have been coded as different versions of the predicate (see explainer 1). The disjunctions (separated by ';') may be read as 'if-then-else' statements: if length is smaller than 1, some conditions need be true, else if it is greater than 2, others must hold, and so on.*

The first three queries in output 6 refer to the beginning and end of figure 19's embellished line, and the last query to figure 20's embellishment.

```
?- gravitational([3,2,1]).
true.

?- kmagnetic(imaj, [-1,4,0], K).
K = 1.22.

?- kmagnetic(imaj, [-1,0], K).
K = 2.20.
```

```
?- inertic([5,3,4,0], T).
T = alt.
```

Output 6: Force assignments for embellishments of figures 19 and 20 using coding 4

While the `[3,2,1]` in output 6's first query is not a partition in figure 19 since it does not end on a stable note, it could be one given output 2's second solution to the `subs/5` call: if the `V` (mode 4) in the figure were to start one note earlier, the note d would be the dominant (degree 4), since $(1 - 4)$ mod $7 = 4$. Under this interpretation, gravity could be assigned – though the query has been included mostly for illustration. The more important point concerns the middle two calls to `kmagnetic/3`: with magnetism determined by the transition from penultimate to last note such as in the second query, the attraction value is markedly lower than when measured using the displaced stepwise attraction like in the third query.

So the stronger third query's `kmagnetic` value should be what is meant when it is said that figure 19's embellishment is controlled by the force of magnetism (see e.g. section 1.2). Similarly, the fourth and last query represents what is meant when saying that the embellishment of figure 20 is controlled by inertia.

5.4 Assigning semantics

Finally, the creation of musical events M from partitions and their assigned forces, and mapping these to a denotational 'world' events W, is to be illustrated with an implementation. This will be $W =$ *moon-appearing* from section 4.2, using the Beethoven *Ode to Joy* example (figure 14).

Coding 5 shows the Prolog predicates that specify the mapping, produce the musical events by computing forces, and apply the mapping to create the denotations.

```
/* map(+Name, -ValueList, -MapList)
     Specifies mapping between a feature in the world and one
     of Larson's forces, and/or the music's harmonic function.
*/
map(visibility, [4,0], [decreasing,increasing]).
map(luminosity, [0.79,1,1.26,1.27], [none,dimmed,bright,bright]).
map(cloudcover, [down,still,up], [none,partial,full]).

/* map(+Name=ForceValue, -MapValue)
     Assigns a value specified in map/3 to a named feature.
*/
map(FName=FVal, FMap) :-
   map(FName, FVals, FMaps),
```

```
    nth0(NF, FVals, FVal),
    nth0(NF, FMaps, FMap).

/*  events(+NotePartitions, +HarmonyPartitions, -MusicalEvents)
        Creates musical events by assigning Larson's forces and
        the harmonic function to note partitions.
*/
events([], [], []).
events([Notes|NParts], [Harms|HParts], [[H,K,I]|Events]) :-
    last(Harms, H),
    kmagnetic(imaj, Notes, K),
    inertic(Notes, I),
    events(NParts, HParts, Events).

/*  events(+MusicalEvents, -WorldEvents)
        Applies mappings to musical events to create world events.
*/
events([], []).
events([[K,I,H]|E], [[KName=KMap,IName=IMap,HName=HMap]|W]) :-
    [visibility,luminosity,cloudcover] = [KName,IName,HName],
    map(KName=K, KMap),
    map(IName=I, IMap),
    map(HName=H, HMap),
    events(E, W), !.
```

Coding 5: `map/3`, `map/2`, `events/3` and `events/2` from program 5 (`otj.pl`)

Explainer 6 *As coding 5 shows, different predicates may have the same name: Prolog distinguishes between them by the number of arguments. So while `map/3` relates the values from the musical events equation 4.2.1 to the mappings (see page 39), `map/2` does the actual value assignments. Similarly, `events/3` outputs the value of equation 4.2.1 while `events/2` assigns the 'world' denotations of equation 4.2.2.*

The inputs to `map/3` are the note partitions `N` and associated harmonic function partitions `H`, for the partitioning as shown in figure 14: `N = [[2,2], [2,3,4], [4,4], [4,3,2,1], [0,0], [0,1,2]]`, and `H = [[0,0], [0,0,0], [0,4], [4,4,4,4], [0,0], [0,0,0]]`. Assuming these inputs for `events/3`, and its output as input for `events/2`, output 7 shows the result.

```
?- events(N, H, E).
E = [[0,1,still], [0,1.27,up], [4,1,still],
     [4,0.79,down], [0,1,still], [0,1.26,up]].

?- events(E, W), list(W).
[visibility=increasing,luminosity=dimmed,cloudcover=partial]
[visibility=increasing,luminosity=bright,cloudcover=full]
```

```
[visibility=decreasing,luminosity=dimmed,cloudcover=partial]
[visibility=decreasing,luminosity=none,cloudcover=none]
[visibility=increasing,luminosity=dimmed,cloudcover=partial]
[visibility=increasing,luminosity=bright,cloudcover=full]
```

Output 7: Musical and world events for figure 14's partitions using coding 5

The output arguments `E` and `W` indeed align with equations 4.2.1 and 4.2.2, respectively. Note though that `E` uses 0-based modes (e.g. 4 instead of `V`), and that due to its length, `W` is output using `list/1` (viz. program 6, `lists.pl`) which prints list elements line by line.

As noted in section 3 (on page 14), mappings to denotations in this thesis are stipulative, and while coding 5 implements the semantics, it also makes this (more) explicit, since the program would equally work with less intuitive elements for the denotations. At the same time, the partitionings of section 5.2 appear to indicate that the musical syntax is autonomous in that it does not depend on the semantics. These points will be discussed further in the next section (6.1).

6 Discussion

6.1 Meaning

The general idea underpinning Schlenker's music semantics is that meaning is a relation between the symbols of a language and a language-external reality, and so under the assumption of music being a kind of language, it is required that any semantics establish a relation between a musical piece and some music-external reality ([Sch19], page 41). What Schlenker says about his *Sun-rise* example (mentioned here in section 1.1, see also *ibid*, page 68) is that his truth definition *"can deliver a notion of truth"*, before going on to list denotations. The underlying idea for this thesis – having been adopted from Schlenker – is the same, but it begs the question: is 'a notion of truth' also 'a good notion of meaning'?

This is perhaps the hardest question to answer, but the main point is that it draws on a general view of semantics as truth-conditional. This is the idea behind Schlenker's more general ideas on 'Super Semantics' (viz. section 2.2 and [Sch18]): studying other representative systems, including music, to see if they can also be given truth conditions. But one of the attractions of a truth-conditional account of meaning in language is that it is reasonable to assume that both speaker and hearer will tend to agree on the conditions under which a statement is true, which forms an important foundation for language as a means of communication: the hearer understands what the speaker intends. But it is less clear if this is also the case here: are the denotations of the musical sequences given here what the composer or musician intends them to mean, and given they mean this or anything for that matter, could the hearer be expected to agree?

While it seems unlikely that composer and hearer would agree on denotations as they have been instantiated here and in [Sch19], Schlenker's way out of the conundrum is his claim that the information conveyed by music is more abstract than that conveyed by language (page 36), and the more abstract the information the more possible denotations – as witnessed by the various denotations highlighted here in section 1.1 (see also *ibid*, page 68). As the author points out, even a statement like *"It is raining"* can refer to a multitude of possible situations (page 67), which boils down to saying that this, too, is in some way abstract in that saying it does not fill in all the details of the situation, however, it does constrain the nature of the possible situations that may be true.

The musical events in Schlenker (such as in *(26)* on page 68) or the attribute/value pairs in the musical event equations here should then be similarly deemed 'abstract' in that they allow several situations in the world to cohere with them, but not any. Larson's forces are to be viewed as constraining the possible ways of 'filling in the details'. So basically from the point of view of communication from composer to hearer, what is being communicated are constraints of physical metaphor, but ultimately this squares with Schlenker's truth-conditional perspective, given his view that natural language can similarly refer to multiple realities.

Schlenker advocates a notion of meaning involving inferences that may be drawn about virtual sources participating in 'world' events (viz. section 1.1), such that a structure-preserving mapping between musical and 'world' events may be made ([Sch19], page 38). In this thesis, Larson's forces were used in an attempt to provide an underpinning for the musical events on which such a mapping may be specified. This has in some way taken the place of 'inferences' in Schlenker's sense, since his method relies on a listener's intuitions about what may be sensible denotations – to be demonstrated by altering a musical element to make the effect disappear (see page 31 of this thesis). But other than that, Larson's forces might provide a way of simplifying the music such that categories of meaning can result.

Larson's notion of musical levels was brought to bear to build up a musical line 'controlled by the forces' (viz. section 1.2). The idea of inferences however, is considered here to be something that would desirably hold between levels L_1 and L_2 such that $L_2 \models L_1$ (see sections 4.4 and 6.4), even if Larson's idea of operations on alphabets (cf. [Lar04], page 466) is broader and would allow for level transitions for which such entailments would not hold.

Finally, Schlenker's semantics associates a musical voice with a virtual source, but the musical examples in this thesis have been restricted to single voices (viz. footnote 3). However, section 4.5's aim has been to demonstrate that the methods described here could in principle be extended to handling multiple musical voices, and hence denotations with multiple virtual sources.

6.2 Formalisation

At the beginning of [Sch19], Schlenker states his aim: to argue that a formal semantics of music can be developed, yet the paper is mostly informal. It has two formal definitions, namely voice and truth (*(24)* and *(25)* on pages 66-67), plus formal write-ups of some denotations (*(26)* on page 68), but other than that it relies on an informal appeal to intuitions about semantics and music. This is not a problem given the goal of arguing for the possibility of formalisation which is not the same as the act of it. Nevertheless, it is one reason that in the present study, an attempt was made to express things formally where they could.

Another motivation is that only (numerical) magnetism was defined precisely by Larson ([Lar04], page 463); in other cases forces were informally specified in the text. But moreover there is the underlying theme in Larson's work that forces act to move music to points of stability, which implies these stabilities can be exploited to partition a musical line into musical (sub) events. Schlenker tacitly acknowledges such partitions, but it is rather arbitrarily left up to the listener or interpreter to decide what these are. Hence it was decided here that stabilities determine partitions and therefore events.[34]

A third reason is the argument already mentioned at the start of this thesis and set out in Hamm, Kamp and Van Lambalgen [HKvL06]: regardless of whether a semantics is in the realist or cognitive tradition, it requires formalisation to ensure the computability of cognition. It should be noted here that Schlenker's music semantics has features of both traditions. It is realist in its appeal to truth conditions, which constitute the relation between the symbols of the language and aspects of the world, and are considered independent of meanings as grasped by a mind (*ibid*, page 1). But in Schlenker's semantics, these aspects are fictional ([Sch19], page 37), and in this sense it would be hard to deny a cognitive dimension to the semantics. Moreover, it is a semantics of events, with events explicitly

[34] This ties in with Rothstein's [Rot89] idea of musical phrases being directed by tonal motion.

marked as a category within conceptual structures in cognitive semantics, which are generated algorithmically ([HKvL06], page 2). This holds for the musical and the ‘world’ events in [Sch19] as well as here.

An attempt was made in section 5 to algorithmically generate at least part of the musical – or ‘syntactic’ – and the semantic structures used in this thesis,[35] first and foremost to demonstrate that the ideas presented here really can be computed, and that the formal definitions of the thesis may indeed be implemented in a computer program. In section 1.2 it was noted that Larson aimed to use his forces to predict melodic continuations, and he does in fact mention using computer models/programs for this, for instance in [Lar04] – however, it appears no implementations were made available for others to use. Here, the aim of the (Prolog) implementations (which are available) is not prediction, but the partitioning of note sequences (section 5.2), and the assignment of forces (section 5.3) and semantics (5.4). The programs are not intended to be comprehensive or work on any conceivable input, for instance overlapping partitions (viz. section 3.2) are not handled, but neither are multiple voices, modulation (see sections 4.5 and 4.3) or features like rhythm (viz. section 1.1) – the inputs are simple numerical note sequences confined to a single key.

One of the motivations for the programs was to demonstrate that (the perception of) musical harmony is intrinsically tied to Larson’s notion of musical embellishment, aiding the proofs of two (limited) propositions in section 5.2, to the effect that harmonic complexity increases with adding embellished musical levels. This might further underline the idea that embellishments not only enrich the syntax but also the semantics – see sections 4.4 and 6.4. But if embellishments effectively enrich the harmony, and harmony is a semantic category (definition 3.3.3 here as well its basis in Schlenker [Sch19], viz. page 67’s truth definition), then seemingly harmony lies at the syntax/semantics interface and the musical syntax cannot be said to be autonomous.

Finally, the formalisations in this thesis are not as far removed from the sort of qualitative introspective intuitions that are central to Schlenker as one might think,[36] since as pointed out, the mappings are stipulated by being based on such intuitions – see section 3 as well as the computational semantics section 5.4.

[35] Like in [HKvL06], this was done using logic programming.
[36] See [dN23], foot/endnote 13.

6.3 Forces

Larson's idea that composing and listening to music is controlled by three forces is interesting in that it only posits three and dubious for the same reason: how can the plethora of music be explained by just three forces? Perhaps it cannot, but the point is: he might be praised for his attempt at rather severe reductionism, since each additional force is essentially an extra assumption from the perspective of Occam's razor,[37] and if Larson is right about his forces then they are certainly quite productive. So while this study is indebted to Larson for said reductionism since it suggests what may be viewed as a semantic primitives proposal, that is not to say the forces should be accepted 'as is', as was highlighted in section 4.1.

lar14.2.min.mp3

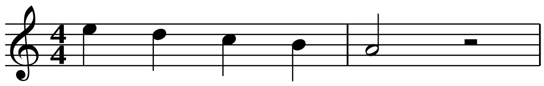


Figure 21: Larson's example 14 (level 2) in *A Minor*

Figure 21 serves to provide a further illustration, in particular what happens to magnetism in case the second level from the example from figure 13 is, instead of in *C Major*, rendered in *A Minor*. The point is that computing magnetism for the last three notes $[c, b, a]$ has attractor a with a distance of 2 (from 'pivot' b), and opponent c has distance 1. In other words, there is a stronger attraction from the pivot to mediant c than to tonic a – to be precise, in the first case the value is $\frac{1}{4}$, and in the second it is $-\frac{1}{4}$.

As noted in section 4.1 the stable tones tonic, mediant and dominant are treated equivalently according to Larson's notion of magnetism, while the Krumhansl-inspired alternative introduced there does not: it explicitly treats the tonic as the strongest attractor, followed by dominant and mediant in major, and mediant and dominant in minor.[38] According to the Krumhansl magnetism conception, the attraction value to the tonic amounts to 1.80, while for the mediant it is 0.56. So one aspect that this alternative overcomes in minor is that having a semitone distance to the mediant but a whole tone to the tonic no longer implies a stronger attraction to the mediant.

But another perhaps more compelling case is depicted in figure 22 below.

[37] Named after William of Ockham (1287 – 1347), this is the principle sometimes referred to as KISS ('keep it simple, stupid'), which states that in case there is more than one theory competing to explain the same phenomenon, the one that makes the fewest assumptions is to be preferred.

[38] Note that the major case follows the stability ordering specified in definition 3.1.8 here, while minor does not.

mag.sym.mp3

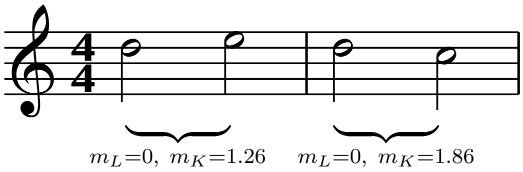


Figure 22: Magnetism example with symmetry

As pointed out in section 4.1, Larson's notion of magnetism appears to suffer from an unfortunate case of symmetry in certain instances. A case in point would be if a melody were to move from d to e, and then from d to the tonic c, just as depicted in figure 22.

In both halfs of the melody, the magnetism following Larson m_L is 0, since in the first half the opponent would be e and in the second half it would be c, with both attractor and opponent two semitones away from the pivot d, and so $m_L = \frac{1}{2^2} - \frac{1}{2^2} = 0$. But while for the first half, $m_K = \frac{4.38}{3.48} \approx 1.26$, for the second one $m_K = \frac{6.35}{3.48} \approx 1.82$. This is in line with the intuition that attraction to the tonic should be stronger than to the mediant.

As for inertia, it was pointed out in section 4.2 that 'continuation of a pattern' is very general and hinges on what is meant by a pattern as well as how this is identified. The section suggests motivic or thematic development might be viewed as being driven by inertia, and contains an example (*Ode to Joy*) where a pattern of forces assignments may be considered as having a large encapsulating so-called motivic inertia assignment. It is suggested that the equivalence or similarity (the magnetism values almost match but not exactly) of the forces mediate melodic similarity, and that this provides a way, or at least provides a direction for generalising the notion of inertia used in this thesis – which has been limited to upward, downward, still and alternating patterns. Though it must be said that no further semantics were developed for this encapsulating label $*i$: it was assigned in the usual fashion (and incidentally implemented computationally in section 5.4).

Section 4.3 on modulation illustrates how motivic inertia encapsulations may be assigned on more than one level. This section is intended to address a limitation in the thesis, namely that the musical examples have to that point been restricted to harmonically moving within modes of a single key, while in fact much music tends to move from key to key, visiting modes within these. In the example of the section, *How High the Moon*, there is a similar repetition of forces assignments to warrant assigning encapsulating $*i$ labels, but additionally, the harmony repeats `V-I` patterns in different keys to further corroborate these assignments. This is then used to illustrate an intuition of Schlenker's: one way to interpret modulation is the perception of a new source. This new source, another bird in this case, then participates in the world in the same way as the sources perceived earlier. But like with the aforementioned *Ode to Joy* example, no methods have been developed here to specify these encapsulations formally – this would have to be a topic of further inquiry.

Concerning gravity finally, it must be pointed out that while introduced in section 1.2 as the 'tendency to descend given some stable threshold or ceiling', gravity was in fact assigned in this thesis to any descending partition without regard for how a threshold or ceiling might be specified, except that a partition's beginning sets it, i.e. descendency (gravity) holds when the partition ends lower than it starts (even if it ascends in between).

6.4 Compositionality

As noted in section 1.2, Schlenker calls his semantics 'source-based rather than compositional' ([Sch19], page 39). The only other reference to compositionality is in the appendix (page 97) in the context of so-called internal semantics, i.e. the sort of semantics not considered 'bona fide' (page 41). While this does not boil down to an immediate claim that music semantics is necessarily non-compositional, it does at least suggest the point is not very important. But this is at odds with the idea that compositionality in semantics is a methodological principle rather than an empirical hypothesis (see for instance [GS91]), which makes the issue hard to ignore or dismiss.

However, Larson's conception of musical levels seems to be able to provide a version of compositionality. If level μ' embellishes level μ such that $\mu' \models \mu$ (as in section 4.4 where *bird-landing* $\models$ *bird-descending* $\models$ *bird-moving*), then this is compositionality in an intuitive sense: an increase in syntactic informativity is reflected in semantic informativity (i.e. each embellishment reflects a semantic enrichment). This would be in an incremental rather than a linear sense, although compositionality could conceivably also be considered linearly by taking events $\langle O, E\rangle = \langle O, \langle e_1, \ldots, e_n\rangle\rangle$ with $\langle O, E\rangle \models \langle O, e_m\rangle$ for any $e_m \in E$. Possibly, this is how thematic development as described in section 4.2 might work, which would imply that incrementality and linearity may sometimes go hand in hand.

6.5 Affect

Music semantics has been frequently described in terms of emotional affect, with Meyer's *Emotion and Meaning in Music* [Mey56] a well-known example. A central idea there is that the mind responds unconsciously to patterns in music completed according to expectation, but is drawn towards them when these are inhibited or delayed, in which case affect or the objectification of meaning may follow (chapter 4) – section 4.6 of this thesis works out a formal version of the 'objectification' part. Schlenker would however view such aspects of music as part of the pragmatic rather than the semantic dimension, for instance on page 42 of [Sch19] he gives an example of a single-note change at the end of the repetition of Beethoven's famous *Ode to Joy* theme that indeed draws attention to it, but this is considered to be an instance of choosing a particular message or expressing it in a certain way (*ibid*, page 82).

Schlenker is quite explicit that while expecting certain musical content may have emotional effects, this essentially boils down to music conveying information about itself rather than a music-external reality, and hence cannot constitute a 'bona fide' semantics (page 41). But if emotions are themselves external to music, then the question whether a semantics can be based on what emotions a piece of music expresses may not be considered settled on the basis of that argument alone. In other words, Schlenker's dismissal of expectations as music-internal information may not be sufficient to dismiss emotions as semantic, since these are arguably external. This is particularly significant given that music's ability to affect emotion and mood is a chief reason for producing as well as listening to it (viz. the aforementioned [Koe10]). Or to push the case further, Schlenker's appeal to truth-conditional semantics is inspired by the informative use of natural language, i.e. on the idea that giving or receiving information is the chief reason for producing as well as listening to language. But then requiring that music also express information about the world may be barking up the wrong tree.

At issue here is what the primary elements of music semantics really are. While Schlenker does not deny the emotional effects of music, he believes that rather than being primary, they are mediated via the musical events to 'world' events mapping, in other words, the world events constitute the primary semantic ontology so to speak, and emotions are a result of this. The basic tenet is the idea of experienced events ([Sch19], page 86): the listener recognises or experiences emotions associated with the events a virtual source undergoes as brought out by the mapping from musical events to a denotation.

Schlenker does give a modified truth definition – or rather a first draft – to reflect this idea on page 94, to the effect that if a musical event gets less harmonically stable, then the source gets to be in a less stable emotional state, but he admits that what 'less emotionally stable' is then to mean would still have to be unpacked. Moreover, the adapted definition states that the event is what causes the less stable state, and this once again takes the semantics to the Peircian indices perspective by virtue of assuming a causal connection between signal and source (viz. *ibid*, page 74). But the event, i.e. the cause, is an imagined one, and the music is what plausibly causes the imagination on Schlenker's account, and if this is in turn to cause the emotion, then there might possibly be a bit more to be unpacked to avoid circularity.

But another way to elicit emotions in music is expectation: when something unexpected happens a feeling of distress may occur, and an expected musical event could lead to a sense of satisfaction – though Meyer [Mey56] is rather cautious about the latter: continued expectational fulfillment disengages the listener (chapter 4). Schlenker is quite dismissive of the role of expectations for a so-called 'bona fide' theory of musical semantics, as to him, expectations can be captured syntactically (see [Sch19], section 2.1) in a language which only conveys information about its own form and is hence an internal semantics (cf. page 6 of this thesis). But this ignores the effect that the incremental dissipation of information can have on a listener, and essentially treats a piece of music as given in full – for semantic analysis 'after the fact'.

Hence the addition of section 4.6 to this thesis, which gives an alternative analysis of Schlenker's *Zarathustra* example (*ibid*, section 3.2) in stages, in an attempt to model what the listener might expect given what has been heard, following ideas from Veltman's update semantics [Vel96]. As noted in section 4.6, this means of analysis may serve to identify the points at which listeners are disposed to assigning meaning to music in the first place. But it might also point to a (partial) informal characterisation of musical semantics along the following lines. Whereas Schlenker's truth-conditional semantics would have it that one knows the meaning of a musical expression if one knows the conditions under which it is true, the dynamic alternative would be that to know the meaning is to know the change it brings about in the information state of a listener that has accomodated the music (cf. *ibid*, page 221).

6.6 Folk semantics: major and minor

As a final 'bonus' example, the first example, in figure 1, is revisited. In section 1.1, it was remarked that Mahler's *Frère Jacques* theme is intended by the composer to represent a funeral march, with the question why this might be so left unanswered, so an attempt will be made to explain this in terms of pitch. For simplicity, the question is reduced to the well-known issue in music that major keys tend to be viewed as 'happy', and minor keys as 'sad'. Given that Mahler's theme is in minor, it can then be assumed that it is 'sad', i.e. that it can appropriately represent a sad event such as a funeral, but the question remains how this can be a property of the pitch relations.

Curtis and Bharuga [CB10] have sought an explanation in a comparison to speech, broadly put: someone who is sad tends to speak less energetically than someone who is happy, which translates to fewer vibrations of the vocal cords, and consequently a lower pitch, with the difference approaching the tonal difference between a minor third and a major third interval. However, an alternative explanation may be possible in terms of Larson's magnetism force. To this end, the theme has been depicted with magnetism in figure 23, rendered in both its original *Aeolic Minor* and in *Ionic Major*.

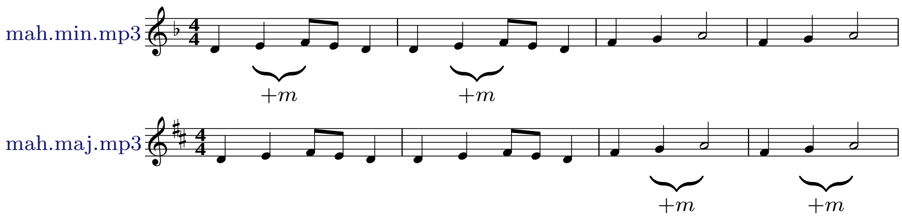


Figure 23: Mahler's *Frère Jacques* theme in original *D Minor*, and in *D Major*, with magnetism assignments

In minor as well as in major, there are upward magnetic pulls, but these occur at different points within the tonal pitch space. In the minor case, the pull is to the mediant (the third), which indicates a prolongation, i.e. that the music stays within the given key. But in major, the magnetic pull is towards the subdominant, i.e. to the fourth. As noted in section 3.1, the subdominant is a so-called 'avoid note', which, when emphasised (i.e. not merely a 'passing note'), signals the modulation to a different key, which in this case would be to *G Major*. The reason is that the subdominant is not a chord note, but it is however a fourth removed from the tonic – or one step back along the so-called circle of fifths (see for instance [BWWW17], figure 1), which is how modulations frequently occur because of closeness: a sharp is lost or a flat is gained, i.e. the modulated scale differs by just a single note.

So then the difference in terms of the magnetic pull between the Mahler theme in minor and in major might possibly be characterised as follows: in minor, the melody is pulled towards a chord note, i.e. the mediant, which prolongs the key of *D minor*, but in major on the other hand, the pull is towards a non-chord note that is also a common modulation note. The conjecture put forward here is then that *D minor* represents a state of 'staying put', while the *D major* alternative, even if it is eventually prolonged, more strongly implies at least the possibility of motion – towards a different key. The difference between 'staying put' and 'motion potential' might then be viewed as one aspect of the difference between sadness and happiness.

7 Concluding remarks

This thesis has established a way to integrate Steve Larson's musical forces into Philippe Schlenker's music semantics by viewing each force (gravity, magnetism and inertia) as a metaphor for attraction to states of stability in the physical world, such that laws governing energy potential are coherently adhered to. Musical states of stability have been considered as end points of musical events, and to be homomorphic with physical events via a truth definition.

So this constitutes an essential extension of Schlenker's semantics, which has no natural way to decide what a musical event is. It exploits Larson's theory such that for a given note sequence and harmonic interpretation, a partitioning labeled with musical forces can be obtained – and subsequently, as demonstrated, a denotation. But equally importantly, Larson's forces provide a way to constrain the possible denotations in a determined way, since while Schlenker claims that music provides more abstract information than language, there must be certain restrictions on what might be allowed.

Since Larson's framework is about pitch and harmony, this thesis was confined to these features of music, even though Schlenker's semantics involves more properties including loudness. The result adapts Larson's magnetism and inertia (in section 4). It is a semantics where an extra-musical reality can be causally associated with a musical expression, as required by Schlenker. This has been formalised in order to ensure computability, and also to allow criticism of the present effort to be made equally precise.

Part of the resulting system was subsequently implemented in section 5 using logic programming. This showed not only that the definitions can be rendered as computer programs – thus actually demonstrating said computability – but also that (basic) harmony increases in complexity along with musical embellishments, in order to retain stability and partitionability.

The framework presented thus follows Schlenker's approach but is not strictly wedded to it. Particularly emotional affect may turn out to be renderable as extra-musical reality despite Schlenker's insistence that this is an 'internal semantics' when viewed from the perspective of musical expectancy. At the end of section 4 a different take on expectancy was explored, from the viewpoint of dynamic semantics. It appears that at least, a framework of music semantics adopting this angle can help identify listeners' points of interest where meanings may be assigned, which could be in the mould of Schlenker/Larson or some other theory. Possibly, more might be gained from a dynamic perspective, e.g. assigning forces accross partition boundaries – which would have to be part of a future work effort.

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A Prolog source files

This appendix lists the source files containing the Prolog code used in this thesis. As indicated, they are also available at `https://mickdeneeve.github.io/ac/ma/src/`, and require SWI Prolog (`https://www.swi-prolog.org`).

To load a source file, e.g. the first one listed here called `partition.pl`, enter '`[partition].`' in the Prolog console (i.e. without quotes but with the full stop).

```
/* FILE:        partition.pl
   PURPOSE:     Create partitions of note sequences
   REQUIRES:    SWI Prolog (https://www.swi-prolog.org)
   AUTHOR:      Mick de Neeve <mick@live.nl>
   INSTITUTE:   University of Amsterdam
   DATE:        December 3, 2024
*/

/* partition(+Notes/Degrees, -Partitions)
      Given lists Notes/Degrees, i.e. notes with their degree
      substitutions, find all partitions; returning them in lists
      of the form NotePartitions/DegreePartitions; with smallest
      last and excluding the original ('self partition').

*/
partition(Notes/Subs, Partitions) :-
   findall(  NoteParts/SubParts,
             (   subparts(Subs,SubParts),
                 noteparts(SubParts,Notes,NoteParts)
             ), Parts),
   reverse(Parts, [_|Partitions]).

/* partition(+Degrees, -Partitions)
      Given a lists of Degrees, find all partitions; returning them
      in lists of the form DegreePartitions; with smallest last and
      excluding the original ('self partition').

*/
partition(Subs, Partitions) :-
   findall(  SubParts,
             subparts(Subs,SubParts),
             Parts),
   reverse(Parts, [_|Partitions]).

/* subparts(+DegreesList, -DegreesPartitions)
      List of lists DegreesPartitions requires partitions end
      stable and are at least 2 elements (for forces assigments).
*/
subparts([], []).
subparts([Sub|STail], [[Sub|NewSTail]|SubParts]) :-
   append(NewSTail, NewSub, STail),
   length(NewSTail, SLength),
   last(NewSTail, Final),
   SLength > 0,
   member(Final, [0, 4, 2]),
```

```
    subparts(NewSub, SubParts).

/* noteparts(+DegreePartitions, +Notes, -NotePartitions)
       NotePartitions is a partitioning of Notes such that it
       aligns with DegreePartitions.
*/
noteparts([], _, []).
noteparts([Part|Parts], Notes, [NotePart|NoteParts]) :-
    append(NotePart, NoteRest, Notes),
    length(Part, L),
    length(NotePart, L),
    noteparts(Parts, NoteRest, NoteParts), !.
```

Program 1: Prolog file `partition.pl`

```
/* FILE:        subs.pl
   PURPOSE:     Apply harmonic mode substitutions to note sequences
   REQUIRES:    SWI Prolog (https://www.swi-prolog.org)
   AUTHOR:      Mick de Neeve <mick@live.nl>
   INSTITUTE:   University of Amsterdam
   DATE:        December 3, 2024
*/

:- ensure_loaded([lists]).

/* subs(+NoteList/HarmonyList, +NewMode, +Index, -DegreesHarmLists)
       DegreesHarmLists is a list of lists with all solutions to
       finding contiguous stable applications of NewMode from Index
       using subs/5, and applying the result to update/6 to produce
       the new harmony lists.
*/
subs(Notes/Harms, NewMode, Index, NewDegsHarms) :-
    degrees(Notes, Harms, Degs),
    findall(   NewDegs/NewHarms,
               (   subs(Degs,Harms,NewMode,Index,ISubs),
                   update(Degs,Harms,ISubs,NewMode,NewDegs,NewHarms)
               ), NewDegsHarms   ).

/* subs(+DegreesList, +HarmonyList, +NewMode, +Index, -Subs)
       Subs is a list of substitutions, i.e. degrees to which
       NewMode can be applied, such that the substitutions are
       contiguous from Index and stable given NewMode.
*/
subs(Degs, Harms, NewMode, Index, ISubs) :-
    index(Degs, 0, IDegs),
    index(Harms, 0, IHarms),
    append(_Degs1, Degs2, IDegs),
    append(ISubs, _Degs3, Degs2),
    member(Index:_, ISubs),
    select(ISubs, IHarms, SubHarms),
```

```
    stable(ISubs, SubHarms, NewMode).

/* degrees(+Notes, +Harms, -Degrees)
       Degrees is a list of harmonic roles of the notes given the
       harmonic function numbers in Harms.
*/
degrees([], [], []).
degrees([Note|Notes], [Harm|Harms], [Degree|Degrees]) :-
    Degree is (Note-Harm) mod 7,
    degrees(Notes, Harms, Degrees).

/* stable(+Subs, +Harms, +NewMode)
       Succeeds if all suggested substitutions Subs, given the
       current harmonic sequence Harms, are stable under the new
       harmonic mode NewMode.
*/
stable([], [], _).
stable([_:Deg|DTail], [_:Harm|HTail], Mode) :-
    Sub is ((Deg+Harm)-Mode) mod 7,
    member(Sub, [0,4,2]), !,
    stable(DTail, HTail, Mode).

/* update(+Degs, +Harms, +Subs, +NewMode, -NewDegs, -NewHarms)
       Apply NewMode and Subs (according to indeces I therein) to
       Degs and Harms so new substituted degrees NewDegrees are
       consistent with new harmonic functions NewHarms.
*/
update(Degs, Harms, [], _, Degs, Harms) :- !.
update(Degs, Harms, [I:Sub|Subs], NewMode, NewDegs, NewHarms) :-
    index(Harms, 0, IHarms),
    member(I:Harm, IHarms), !,
    NewSub is ((Sub+Harm)-NewMode) mod 7,
    replace(I, NewSub, Degs, NextDegs),
    replace(I, NewMode, Harms, NextHarms),
    update(NextDegs, NextHarms, Subs, NewMode, NewDegs, NewHarms).
```

Program 2: Prolog file `subs.pl`

```
/* FILE:       displace.pl
   PURPOSE:    Find stepwise successions of notes in a sequence
   REQUIRES:   SWI Prolog (https://www.swi-prolog.org)
   AUTHOR:     Mick de Neeve <mick@live.nl>
   INSTITUTE:  University of Amsterdam
   DATE:       December 3, 2024
*/

/* displace(+Notes, -Displaced)
       Displaced is a two-way non-contiguous but sequential
       partitioning of Notes that has a stepwise partition.
*/
```

```
displace(Notes, Displaced) :-
   divide(Notes, Displaced),
   member(Steps, Displaced),
   stepwise(Steps),
   length(Displaced, 2).

/* divide(+Notes, -Divided)
      Divided is a non-contiguous but sequential list of lists
      partitioning of Notes. Auxiliary divide/3 makes subdivisions
      among two lists first.
*/
divide([], []).
divide([Note|Notes], [[Note|DNotes]|Divided]) :-
   divide(Notes, DNotes, Rest),
   length(DNotes, L),
   L >= 1,
   divide(Rest, Divided).
divide([], [], []).
divide([Note|Notes], [Note|DNotes], Divided) :-
   divide(Notes, DNotes, Divided).
divide([Note|Notes], DNotes, [Note|Divided]) :-
   divide(Notes, DNotes, Divided).

/* stepwise(+Notes)
      Succeeds if Notes has no successor transitions greater than 1.
*/
stepwise([N1, N2]) :-
   stepwise(N1, N2), !.
stepwise([N1,N2|Notes]) :-
   stepwise(N1, N2),
   stepwise([N2|Notes]).
stepwise(N1, N2) :-
   S is abs(N1-N2),
   S =< 1.
```

Program 3: Prolog file `displace.pl`

```
/* FILE:       forces.pl
   PURPOSE:    Compute Larson's forces for note sequences
   REQUIRES:   SWI Prolog (https://www.swi-prolog.org)
   AUTHOR:     Mick de Neeve <mick@live.nl>
   INSTITUTE:  University of Amsterdam
   DATE:       December 3, 2024
*/

:- ensure_loaded([maths]).

/* gravitational(+Notes)
      Succeeds if the final note descends and is lower than the
      first. This is a crude version of Steve Larson's notion, e.g.
```

```
      no ceiling is specified beyond the partition's first note.
      Auxiliary gravitational/3 applies this first-note ceiling.
*/
gravitational([First|Notes]) :-
   reverse([First|Notes], [Final,Penultimate|_]),
   gravitational(First, Penultimate, Final), !.
gravitational(Note, Note, Final) :-
   Note > Final.
gravitational(First, Penultimate, Final) :-
   First >= Final,
   Penultimate > Final.

/* kmagnetic(+Scale, -Kmagnetism)
      Gives the Ionic major or Aeloic minor scale vectors from
      Krumhansl's Cognitive Foundations of Musical Pitch (1990).
*/
kmagnetic(imaj, [6.35,3.48,4.38,4.09,5.19,3.66,2.88]).
kmagnetic(amin, [6.33,3.52,5.38,3.53,4.75,3.98,3.34]).

/* kmagnetic(+Scale, +Partition, -Kvalue)
      Gives a Krumhansl magnetic value determined between the
      sequence's penultimate and last notes, which is set to be
      last/penultimate so it is asymmetric unlike Larson's notion.
*/
kmagnetic(Scale, Notes, K) :-
   reverse(Notes, [Final,Penultimate|_]),
   kmagnetic(Scale, Penultimate, Final, K).
kmagnetic(Scale, Penultimate, Final, K) :-
   kmagnetic(Scale, Values),
   PRole is Penultimate mod 7,
   FRole is Final mod 7,
   nth0(PRole, Values, PValue),
   nth0(FRole, Values, FValue),
   KF is FValue / PValue,
   round(KF, 2, K).

/* inertic(+Notes, -Type)
      Determines inertia for a note sequence, following Larson,
      (if present); which is defined as 'following a pattern',
      and specified here as either repeating its notes ('still'),
      ascending ('up'), descending ('down'), or 'alternating'.
*/
inertic(Notes, Type) :-
   length(Notes, L),
   (  L < 1,
      next(==, Notes),
      Type = still
      ;
      L > 2,
```

```
    (   next(<, Notes),
        Type = up
        ;
        next(>, Notes),
        Type = down   )
    ;
    L >= 3,
    alt(Notes),
    Type = alt   ), !.

/*  next(+Operator, +Notes)
        Succeeds if each successive note in Notes progresses
        according to Operator. Auxiliary notes/3 compares each note
        to its successor.
*/
next(_, []).
next(Operator, [Note|Notes]) :-
    next(Operator, Note, Notes).
next(_, _, []) :- !.
next(Operator, Note, [Next|Notes]) :-
    Comparison =.. [Operator,Note,Next],
    Comparison,
    next(Operator, Next, Notes).

/*  alt(+Notes)
        Succeeds if all notes in Notes go alternatingly up and down
        or vice versa.
*/
alt([]).
alt([N1,N2|Notes]) :-
    alt(N1, N2, Notes).
alt(_, _, []) :- !.
alt(N1, N2, [N3|List]) :-
    (   N1 < N2,
        N2 > N3
        ;
        N1 > N2,
        N2 < N3   ), !,
    alt(N2, N3, List).
```

Program 4: Prolog file `forces.pl`

```
/*  FILE:       otj.pl
    PURPOSE:    Compute semantics for Beethoven's Ode to Joy
    REQUIRES:   SWI Prolog (https://www.swi-prolog.org)
    AUTHOR:     Mick de Neeve <mick@live.nl>
    INSTITUTE:  University of Amsterdam
    DATE:       December 3, 2024
*/
```

```
:- ensure_loaded([forces, lists]).

/* otj(-NotePartitions, -HarmonyPartitions)
       Note partitions and accompanying harmony for the first
       three measures of the second half of Schlenker's example 5
       (Ode to Joy) in his Prolegomena to Music Semantics (2019)
*/
otj([[2,2], [2,3,4], [4,4], [4,3,2,1], [0,0], [0,1,2]],
    [[0,0], [0,0,0], [0,4], [4,4,4,4], [0,0], [0,0,0]]).

/* map(+Name, -ValueList, -MapList)
       Specifies mapping between a feature in the world and one
       of Larson's forces, and/or the music's harmonic function.
*/
map(visibility, [4,0], [decreasing,increasing]).
map(luminosity, [0.79,1,1.26,1.27], [none,dimmed,bright,bright]).
map(cloudcover, [down,still,up], [none,partial,full]).

/* map(+Name=ForceValue, -MapValue)
       Assigns a value specified in map/3 to a named feature.
*/
map(FName=FVal, FMap) :-
   map(FName, FVals, FMaps),
   nth0(NF, FVals, FVal),
   nth0(NF, FMaps, FMap).

/* events(+NotePartitions, +HarmonyPartitions, -MusicalEvents)
       Creates musical events by assigning Larson's forces and
       the harmonic function to note partitions.
*/
events([], [], []).
events([Notes|NParts], [Harms|HParts], [[H,K,I]|Events]) :-
   last(Harms, H),
   kmagnetic(imaj, Notes, K),
   inertic(Notes, I),
   events(NParts, HParts, Events).

/* events(+MusicalEvents, -WorldEvents)
       Applies mappings to musical events to create world events.
*/
events([], []).
events([[K,I,H]|E], [[KName=KMap,IName=IMap,HName=HMap]|W]) :-
   [visibility,luminosity,cloudcover] = [KName,IName,HName],
   map(KName=K, KMap),
   map(IName=I, IMap),
   map(HName=H, HMap),
   events(E, W), !.
```

Program 5: Prolog file `otj.pl`

```
/* FILE:        lists.pl
   PURPOSE:     Provide utility predicates for lists
   REQUIRES:    SWI Prolog (https://www.swi-prolog.org)
   AUTHOR:      Mick de Neeve <mick@live.nl>
   INSTITUTE:   University of Amsterdam
   DATE:        December 3, 2024
*/

/* index(+List, +StartIndex, -IndexedList)
       IndexedList is List with its elements prefixed in the form
       index:element, starting from StartIndex.
*/
index([], _, []).
index([H|T], I, [I:H|NT]) :-
   J is I + 1,
   index(T, J, NT).

/* select(+IndexedPart, +IndexedList, -IndexedSelected)
       IndexedList is full list; IndexedPart is subset of its indeces
       but may have different elements, and IndexSelected gets the
       elements of IndexedList with matching indeces in IndexParts.
*/
select([], _, []).
select([I:_|PTail], IList, [I:Elt|STail]) :-
   member(I:Elt, IList), !,
   select(PTail, IList, STail).
select([_|PTail], IList, STail) :-
   select(PTail, IList, STail).

/* replace(+Index, +Element, +List, -ReplacedList)
       ReplacedList is List with Element substituted for whatever was
       originally at Index (0-based); using the auxiliary replace/5
       that includes a starting index as its second argument (so
       replace/5 replaces only from this index).
*/
replace(Index, Elt, List, Replaced) :-
   length(List, Len),
   Index < Len,
   replace(Index, 0, Elt, List, Replaced), !.
replace(Index, Index, Elt, [_|List], [Elt|List]).
replace(Index, Lower, Elt, [Head|List], [Head|Replaced]) :-
   Next is Lower + 1,
   replace(Index, Next, Elt, List, Replaced).

/* list(+List)
       Print list elements to standard output.
*/
list([]).
list([Head|Tail]) :-
```

```
    writeln(Head),
    list(Tail).
```

Program 6: Prolog file `lists.pl`

```
/* FILE:        maths.pl
   PURPOSE:     Provide utility predicates for maths
   REQUIRES:    SWI Prolog (https://www.swi-prolog.org)
   AUTHOR:      Mick de Neeve <mick@live.nl>
   INSTITUTE:   University of Amsterdam
   DATE:        December 3, 2024
*/

/* round(+Float, +Decimals, -Rounded)
      Round Float to a given number of decimal places.
*/
round(Float, Dec, Rounded) :-
   Num is Float * 10^Dec,
   round(Num, Int),
   Rounded is Int / 10^Dec.
```

Program 7: Prolog file `maths.pl`